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ISSUES IN CONTROLLABILITY AND THE THEORY OF ECONOMIC POLICY

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The paper demonstrates that the concepts of dynamic controllability are useful for the theory of economic policy by establishing four propositions. First, dynamic controllability is a central concept in stabilization policy. Second, the ability to achieve a target state, even if it cannot be maintained, may be of economic interest. Third, dynamic controllability is of special interest for 'historical' models. Fourth, the conditions for any notion of dynamic controllability are distinct from and weaker than those for Tinbergen static controllability.

1. Introduction

A considerable volume of research on extending the theory of economic policy to dynamic models has been summarized in a recent paper by Nyberg and Viotti (1978) (henceforth N–V). It is the purpose of our paper to emphasize certain issues not generally discussed in this literature, thereby supplementing and extending the discussion summarized in N–V. In particular, we focus on an assessment of the N–V conclusion that 'the concept of [dynamic] controllability ... is of limited interest for the theory of economic policy ...'. We find four important reasons for qualifying this statement.

First, dynamic controllability provides a convenient sufficient criterion for determining whether the policy authority has the ability to steer the economy toward an equilibrium state. The consequent importance of controllability for stabilization policy is discussed in section 2. Second, controllability can be relevant for policy even if the state to which the economy is moved cannot be maintained. We provide examples in section 3. Third, controllability is especially interesting for models exhibiting *hysteresis*, i.e. models for which the equilibrium depends upon the initial conditions. An example of such a historical model is given in section 4. Finally, N–V have inferred an overly strict requirement for a system to be perfectly controllable which has led them to an incorrect generalization of Tinbergen's static

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controllability condition to dynamic systems. This issue is discussed in section 5. Before proceeding to sections 2–5, we establish some terminological conventions and recall some important theorems on dynamic systems.

1.1. Dynamic point controllability

Consider a linear system with constant coefficients

$$\dot{z}=Az+Bx. \tag{1}$$

A is an $n\times n$ matrix and B is an $n\times r$ matrix of constants. z is an n-vector of state variables and x is an r-vector of instruments or controls. What N-V call controllability or dynamic controllability we call *dynamic point controllability*.

The system of (1) is dynamically point controllable iff there exists a path for the controls capable of moving the state vector from any initial state in the state space and from any initial time to any other terminal or target state in pre-assigned finite time. The necessary and sufficient condition for the system (1) to be dynamically point controllable is that the $n\times nr$ matrix ϕ have rank n where

$$\phi=[B, AB, A^2B,\dots, A^{n-1}B].^1 \tag{2}$$

Dynamic point controllability of the state vector, z, can be extended to dynamic point controllability of the output (or target) vector y. Let the complete dynamic system be given by

$$\dot{z}=Az+Bx \tag{1'}$$

and

$$y=Cz+Dx, \tag{1''}$$

where y is an m-vector of output (or target) variables and z and x are as in (1). The necessary and sufficient condition for dynamic output point controllability of the system (1′), (1″) is that the rank of the $m\times r(n+1)$ matrix Ω be m, where

$$\Omega=[D, CB, CAB, CA^2B,\dots, CA^{n-1}B].^2 \tag{2'}$$

[1] See, for example, Preston (1974), Buiter (1979), Gersovitz (1975) and Aoki (1976).

[2] See Aoki (1976, p. 89). In fact, (1) is perfectly general. Use the transformation $\dot{q}=y$. This permits us to rewrite (1′) and (1″) as

$$\dot{w}=\bar{A}w+\bar{B}x, \quad \text{with } w=\begin{bmatrix} z \\ q \end{bmatrix},\ \bar{A}=\begin{bmatrix} A & 0 \\ C & 0 \end{bmatrix},\ \bar{B}=\begin{bmatrix} B \\ D \end{bmatrix},$$

i.e. w is an $(n+m)$ vector, $\bar{A}$ is an $(n+m)\times(n+m)$ matrix and $\bar{B}$ is an $(n+m)\times r$ matrix.

1.2. Dynamic path controllability

What N–V refer to as perfect controllability,[3] we call *dynamic path controllability*. The system (1) is dynamically path controllable iff there exists a path for the controls capable of moving the state vector from any initial state and from any initial time along any pre-assigned (target) trajectory for any pre-assigned finite time interval. The necessary and sufficient condition for (1) to be dynamically path controllable is that the $n^2 \times (2n-1)r$ matrix Ψ have rank n^2 where

$$\Psi = \begin{bmatrix} B & AB & \dots & A^{2n-2}B \\ 0 & B & & \vdots \\ \vdots & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & B & A^{n-1}B \end{bmatrix}.^{4} \tag{3}$$

1.3. Static controllability

An equilibrium of the system (1) is any z^* such that

$$0 = Az^* + Bx \tag{4}$$

for constant x. The equilibrium of system (1) is statically controllable iff there exists a $\bar{x}$ such that $0 = Az^* + B\bar{x}$ for any z^*. If A is of full rank (n), the equilibrium of (4) is statically controllable iff the rank of $B = n$, i.e. there should be as many linearly independent instruments as there are linearly independent targets.[5]

2. Dynamic point controllability of target states that are equilibria

N–V emphasize that if the target state is not an equilibrium of the system, dynamic point controllability only ensures that there is an adjustment path that will make the system *pass through* the target state at a pre-assigned point in time (T). Thus, this concept does not indicate whether it is possible to keep the system at the target state beyond T. However, if the state is an equilibrium of the system, it is clearly possible to keep the system there.

From the viewpoint of stabilization policy, dynamic point controllability is therefore a more important property of the system than stability. If the

[3]The literature also refers to it as *functional reproducibility*, e.g. Brockett and Mesarovic (1965) and Basile and Marro (1971).

[4]See Aoki (1975, 1976). For the criterion for dynamic path controllability of the output vector y when $y = Cz + Dx$; $\dot{z} = Az + Bx$, see footnote 2.

[5]See, for example, Tinbergen (1955).

system is stable, it returns to an equilibrium after a perturbation with the policy authority rigidly adhering to whatever *fixed* values of its controls are consistent with the original equilibrium. Stability analysis therefore assumes x is *fixed* and considers only the eigenvalues of the matrix A. Dynamic point controllability, by contrast, implies that there exists a *trajectory* for the x capable of returning the system to an equilibrium after a perturbation.[6] Consequently, we consider dynamic point controllability to be a better characterization of the policy options potentially available to the policy authority. This interpretation is strengthened by the connection between dynamic point controllability and the stabilizability of a system.

Stabilizability

A pair of matrices (A, B) is stabilizable if the range space of $[B, AB, \ldots, A^{n-1}B]$, i.e. the space spanned by its columns, contains the subspace spanned by the eigenvectors of A with non-negative real parts [Aoki (1973, p. 134)]. Intuitively, (A, B) is a stabilizable pair if all sources of instability in A can be eliminated by a control matrix B, as the following propositions, advanced without proof,[7] indicate.

Proposition 1. If the dynamic system (1) *is dynamically point controllable,* (A, B) *is a stabilizable pair.*

In other words, dynamic point controllability implies stabilizability. Proposition 2 implies that a system which is stabilizable can always be stabilized in a simple manner.

Proposition 2. If (A, B) *is a stabilizable pair, there exists an* $(r \times n)$ *matrix* Γ *such that* $A + B\Gamma$ *is a stable matrix, i.e. all eigenvalues of* $A + B\Gamma$ *have negative real parts.*

Consequently, dynamic controllability implies that there always exists a set of proportional feedback controls which stabilize the system. Proportional feedback is equivalent to policy behavior characterized by partial adjustment, the simplest and most intuitive form of response to disequilibrium. To know that any system that is dynamically point controllable can be stabilized in so simple a manner is clearly of great interest.

[6]Because all trajectories are considered feasible, dynamic controllability may overstate the options available since there may be outside constraints on the path of the x, e.g. inequality constraints.

[7]For proofs see Wonham (1967) and Heymann (1968). See also Aoki (1973, 1976).

3. Dynamic point controllability of disequilibrium states

The previous section focussed on the usefulness of the dynamic point controllability concept with respect to equilibrium states. In this section we briefly consider the usefulness of this concept when the state which is reached at T is not an equilibrium and it is therefore not possible to say, at least on the basis of point controllability (but see section 5), whether the system can be made to stay in this state.

N–V dismiss the importance of reaching a state which cannot be maintained: 'In economics, it is usually not sufficient to reach the desired position; we must also be able to stay there.' While we are in general agreement with this statement we wish to emphasize exceptions to this position. For instance, models of the political business cycle emphasize that governments may try to bring the economy to a point on, or just before, the election which ensures re-election. Problems of sustainability after re-election may be of secondary importance, especially if the favorable situation can be reconstructed by the next election. Clearly, controllability is the natural analytical device for this purpose. Other examples of the usefulness of point controllability may well be developed by the consideration of other problems in political economy. For instance, tariff retaliation could be formulated as a dynamic game where the ability to reach a state in which one's opponent capitulates may be important.

4. Dynamic point controllability of historical systems

Hysteresis is the dependence of an equilibrium on the initial state and the path the economy experiences towards the equilibrium.[8] Consider the system (1) when A is not of full rank. In this situation, A^{-1} does not exist and the equilibrium of the system is not uniquely determined, for any given fixed x, by (4). Instead, if (1) converges at all, it will move from any particular initial state to an equilibrium determined by that initial state, the disequilibrium path of x, and the final value of x. Dynamic point controllability of the *equilibrium values* of the target variables of the model implies that the initial values of the instruments, their values during the adjustment towards equilibrium, and their equilibrium values can jointly be used to select the equilibrium values of the target variables. The equilibrium values of those target variables that are not statically controllable can be chosen by the policy authority by leaving the steady state temporarily. The possibility of this type of policy is demonstrated if we can establish dynamic point controllability of an output vector consisting of the target variables plus a set

[8]An interesting example of a (highly nonlinear) model with 'local' hysteresis due to adjustment costs can be found in Kemp and Wan (1974).

of variables capable of assuming values that sustain any values of the target variables as equilibrium values. Similarly, hysteresis of the output variables means that the equilibrium value of the output vector, y, is not uniquely determined by (4) and (1″).

A potentially important example of a historical model in which 'the time path to equilibrium partially shapes that equilibrium' is mentioned in Phelps' *Inflation Policy and Unemployment Theory* (1972, pp. 77–80, 256). If a temporary boom has permanent effects on the attitudes and/or aptitudes of workers, i.e. if a departure from equilibrium produces effects which persist after the return to equilibrium, the long-run or equilibrium natural rate of unemployment is not invariant to the adjustment path towards equilibrium.

A convenient representation of the notion that the natural rate of unemployment, u_N, depends on past values of the actual rate, u, is given in eq. (5). We also assume that some part of government expenditure, G, facilitates search and lowers u_N:

$$u_N(t)=f\left(\int_{-\infty}^{t}[u(s)-u_N(s)]\mathrm{d}s,\,G\right),\qquad f_1>0,\ f_2<0. \tag{5}$$

Differentiating (5) and taking a linear approximation yields

$$\dot{u}_N=\delta(u-u_N)+\theta\dot{G},\qquad \delta>0,\ \theta<0. \tag{5'}$$

We now complement (5′) by a simple macrodynamic model to obtain an example of the role of point controllability in the analysis of a historical system. Let p denote the log of the price level, Π the expected rate of inflation, M the log of the nominal stock of money balances, and G real public spending on goods and services. The structural equations are (5′) and

$$\dot{p}=\alpha(u-u_N)+\Pi,\qquad \alpha>0, \tag{6}$$

$$u=\beta(M-p)+\gamma G,\qquad \beta<0,\ \gamma<0,^{9} \tag{7}$$

$$\dot{\Pi}=\eta(\dot{p}-\Pi),\qquad \eta>0. \tag{8}$$

Eq. (6) is an expectations-augmented price Phillips curve or a Phelps–Friedman–Lucas supply function. Eq. (7) expresses the rate of unemployment as a decreasing function of the stock of real money balances $m=M-p$, and the level of public spending. Eq. (8) gives the adaptive expectations mechanism governing inflation expectations.

[9] The positive constant term in this equation has been omitted for algebraic simplicity.

The steady-state equilibrium conditions are:

$$u_N = u, \tag{9a}$$

$$u = \beta m + \gamma G, \tag{9b}$$

$$\dot{p} = \dot{M}, \tag{9c}$$

$$\Pi = \dot{p}, \tag{9d}$$

$$\dot{G} = 0. \tag{9e}$$

Eqs. (9b) and (9c) demonstrate that the model has the hysteresis property. The steady-state equations do not suffice to uniquely determine the equilibrium unemployment rate. In equilibrium the actual rate of unemployment equals the natural rate, but for each value of G there exists a continuum of equilibrium solutions for m and u, given by eq. (9b). Let $\dot{p}$ and u be the target variables. The target vector $(\dot{p}, u)$ is not statically controllable using the instruments $\dot{M}$ and G. Note that (9e) implies that $\dot{G}$ is not a potential instrument for static control. Only a one-dimensional subspace of the target space (i.e. only $\dot{p}$) is statically controllable. Given an initial condition, however, any solution of the dynamic system which converges to a steady state will generate a well-defined equilibrium value of u and thus also of m.

Using eqs. (5′) and (6)–(8), the state-space representation of the model can be reduced to

$$\begin{bmatrix} \dot{m} \\ \dot{\Pi} \\ \dot{u}_N \\ \dot{G} \end{bmatrix} = \begin{bmatrix} -\alpha\beta & -1 & \alpha & -\alpha\gamma \\ \eta\alpha\beta & 0 & -\eta\alpha & \eta\alpha\gamma \\ \delta\beta & 0 & -\delta & \delta\gamma \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} m \\ \Pi \\ u_N \\ G \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & \theta \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{M} \\ \dot{G} \end{bmatrix} \tag{10a}$$

or

$$\dot{z} = Az + Bx. \tag{10a′}$$

A policy-maker may wish to move the economy to an equilibrium where $\Pi = \Pi^* = \dot{p}$ and $u_N = u_N^*$. For an equilibrium to prevail, $\dot{m} = \dot{\Pi} = \dot{u}_N = \dot{G} = 0$ is required. As is clear from (10a), $\dot{m} = \dot{\Pi} = \dot{u}_N = 0$ implies $\dot{G} = 0$. Thus, if the outputs $\dot{m}$, $\dot{\Pi}$, $\dot{u}_N$, Π and u_N are controllable, the policy-maker can reach the desired equilibrium. The apparent contradiction with the Tinbergen static controllability criterion which indicates that u is not statically controllable is easily resolved. The variable u can be controlled only by leaving the steady state temporarily and taking advantage of the hysteresis property of the model to select one from the continuum of possible equilibrium values of u.

For the economic system to be in long run equilibrium at t, it is necessary that $u(t)=u_N(t)$, but not that this state of affairs has prevailed at all time in the past. In other words, $\int_{-\infty}^{t}[u(s)-u_N(s)]\mathrm{d}s$ need not equal zero when the system is in equilibrium at t. It is indeed the policy-maker's ability to influence this integral of past deviations of the actual from the natural rate of unemployment that enables him to select alternative equilibrium rates of unemployment. For policy purposes, the comparison of two equilibria, each one of which has been in effect since the beginning of time, is irrelevant. In such a permanent steady state, $u(t)=u_N(t)$ at all times and the natural rate is uniquely determined by $f(0, G)=u_N$. With the transition between steady states specified by our dynamic equations (5)–(8), the dependence of the steady-state rate of employment on the actual non steady-state path of the economy emerges, since u_N need not equal u during the transition.

We have the output system

$$\begin{bmatrix} \dot{m} \\ \dot{\Pi} \\ \dot{u}_N \\ u_N \\ \Pi \end{bmatrix} = \begin{bmatrix} -\alpha\beta & -1 & \alpha & -\alpha\gamma \\ \eta\alpha\beta & 0 & -\eta\alpha & \eta\alpha\gamma \\ \delta\beta & 0 & -\delta & \delta\gamma \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} m \\ \Pi \\ u_N \\ G \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & \theta \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{M} \\ \dot{G} \end{bmatrix} \tag{10b}$$

or

$$y=Cz+Dx. \tag{10b'}$$

Dynamic point controllability of the output vector y requires that the rank of the matrix Ω be 5, where $\Omega=[D, CB, CAB, CA^2B, CA^3B, CA^4B]$. For (10a) and (10b) the first three elements of Ω are given by

$$\left[\begin{array}{cc|cc|cc} 1 & 0 & -\alpha\beta & \alpha(\theta-\gamma) & \alpha\beta(\alpha\beta+\delta-\eta) & -\alpha(\theta-\gamma)(\alpha\beta+\delta-\eta) \\ 0 & 0 & \eta\alpha\beta & -\eta\alpha(\theta-\gamma) & -\eta\alpha\beta(\alpha\beta+\delta) & \eta\alpha(\theta-\gamma)(\alpha\beta+\delta) \\ 0 & \theta & \delta\beta & -\delta(\theta-\gamma) & -\delta\beta(\alpha\beta+\delta) & \delta(\theta-\gamma)(\alpha\beta+\delta) \\ 0 & 0 & 0 & \theta & \delta\beta & -\delta(\theta-\gamma) \\ 0 & 0 & 0 & 0 & \eta\alpha\beta & -\eta\alpha(\theta-\gamma) \end{array}\right]$$

Under our assumptions about the parameters of the model, the first five columns of this matrix are independent and therefore the rank of Ω is five. As a consequence, the policy-maker can bring the system to an equilibrium with any desired level of $\dot{p}=\Pi$ and $u=u_N$.

5. Dynamic path controllability

The necessary and sufficient condition for the system $\dot{z}=Az+Bx$ to be dynamically path controllable is that the matrix Ψ of (3) have rank n^2. N–V state that for a system to be dynamically path controllable, 'it turns out that the number of instruments must be at least as many as the number of targets, i.e. the Tinbergen rule is exactly carried over to the general dynamic case' [Nyberg and Viotti (1978, p. 78)]. In this section we show that dynamic path controllability is possible when the number of linearly independent instruments is less than the number of linearly independent targets.

The source of N–V's error is a misinterpretation by Aoki of corollary 2 of his proposition 1, stating the necessary and sufficient conditions for dynamic path controllability [Aoki (1975, p. 295)]. The corollary correctly states that the system (1) is dynamically path controllable only if

$$n \leqq (2 - 1/n)r. \tag{11}$$

Aoki then interprets this condition erroneously as 'only if the number of target variables is less than or equal to the number of instrument variables'. [Aoki (1975, p. 295)].

If $n=1$, the system is indeed only controllable if $r \geqq 1$. For $n=2$, (11) implies $r \geqq 4/3$ which, with only integer numbers of controls admissible, requires $r \geqq 2$. When $n=3$, the condition is $r \geqq 9/5$ which is satisfied by $r=2$. In the limit as $n \to \infty$, the right-hand side of (11) tends to $2r$, i.e. the number of instruments must not be less than *half* the number of targets. This condition is clearly much weaker than the Tinbergen condition for static controllability and does not justify the N–V statement quoted above.

First, we note that the Tinbergen condition is *sufficient* for dynamic path controllability. Inspection of (3) shows that Ψ has rank n^2 if B has rank n. Furthermore, if A is the identity matrix, it is clear that B must have rank n for Ψ to have rank n^2. The question is, are there weaker conditions that are still sufficient? We have not been able to state a general condition, but it is possible to find numerical examples for which $n > r$ and yet rank $\Psi = n^2$. As the discussion above implies, the lowest value of n for which this search could be successful is three.

Given the result about A equal to the identity matrix, it is clear that an A matrix which fulfils the Ψ condition is not likely to be sparse. This conjecture is economically intuitive: without n instruments, it seems clear that the instruments ought to affect the states not only directly through the B matrix but also indirectly through the A matrix. A pair of A and B matrices, both of full rank, for which rank $\Psi = 9$ is

$$A = \begin{bmatrix} -3 & -4 & 9 \\ -17 & 2 & 7 \\ -20 & -6 & -4 \end{bmatrix}; \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}. \tag{12}$$

The counterexample demonstrates that the mathematical proposition, i.e. 'dynamic path controllability requires the number of linearly independent instruments to equal the number of linearly independent target variables', is incorrect. However, the behavior of the controls required in order to achieve dynamic path controllability with fewer independent instruments than targets may violate unstated physical or economic constraints, even if it does not violate the simple linear structure to which our mathematical path controllability proposition refers.[10]

6. Conclusion

The aim of this paper has been to demonstrate the usefulness of dynamic controllability concepts for the theory of economic policy. To make our case we establish the following: (1) dynamic controllability is a central concept in stabilization policy; (2) the ability to achieve a target state, even if it cannot be maintained, may be of economic interest; (3) dynamic controllability is of special interest for historical models; and (4) the conditions for any notion of dynamic controllability are distinct from and weaker than those for Tinbergen static controllability. On the basis of these results, we believe that the two notions of dynamic controllability will play a growing role in the dynamic extensions of the theory of economic policy.

[10]In the discrete time formulation of the controllability problem, path controllability is defined both with respect to the number of periods, p, until the system must begin to follow a given target trajectory and the number of periods, P, for which the system is to remain on the target trajectory. Let $z_t = Az_{t-1} + Bx_t$ be the state-space representation of the model. A necessary and sufficient condition for controllability of z_t for P periods, starting p periods from now is that the rank of the $nP \times (p+P-1)r$ matrix Ψ' be nP:

$$\Psi' = \Psi'(A,B,P,p) = \begin{bmatrix} A^{p-1}B & A^{p-2}B & \dots & AB & B & 0 & \dots & 0 \\ A^{p}B & A^{p-1}B & \dots & A^2B & AB & B & \dots & 0 \\ \vdots & & & & & \vdots & \ddots & \vdots \\ A^{p-2+P}B & A^{p-3+P}B & \dots & A^{P}B & A^{P-1}B & A^{P-2}B & \dots & B \end{bmatrix}$$

[See Uebe (1977)]. This condition is harder to satisfy the larger p and P. In the continuous time case, the influence of an initial condition can be undone in an arbitrarily short interval because the controls can be varied continuously. In the discrete time case it may take many periods to 'purge' the influence of the initial state. Even then, however, provided the system has sufficient time to get 'on track' (p is not too small) and provided the control period is not too long (P is not too large), path controllability may be achieved with fewer independent instruments than targets.

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