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PROGRAMMING SOFTWARE NOTES

A PROGRAM FOR THE ESTIMATION OF DYNAMIC ECONOMIC RELATIONS FROM A TIME SERIES OF CROSS SECTIONS*

BY ALAN FREIDEN

An IBM FORTRAN IV level G computer program has been developed by the author which can be used to estimate parameters of the simple variance component model suggested by Marc Nerlove (1). This model is designed to treat data available on a large number of individuals but on each individual for only a short period of time. Examples of areas in which the program may be useful are the analysis of successive decennial censuses, short period longitudinal studies, and behavioral relationships involving lags or other forms of autoregressive processes.

The model can be described as follows. Let,

$$y_{it} = \alpha y_{it-1} + \beta x_{it} + u_{it}, \qquad i = 1, \ldots, N \qquad t = 1, \ldots, T \tag{1}$$

where

$$y = (y_{11}, \ldots, y_{1T}, \ldots, y_{N1}, \ldots, y_{NT})'$$

$$y_{-1} = (y_{10}, \ldots, y_{1T-1}, \ldots, y_{N0}, \ldots, y_{NT-1})'$$

$$x = (x_{11}, \ldots, x_{1T}, \ldots, x_{N1}, \ldots, x_{NT})'$$

$$u = (u_{11}, \ldots, u_{1T}, \ldots, u_{N}, \ldots, u_{NT})'.$$

The u_{it} are unobserved random variables such that $u_{it} = \mu_i + v_{it}$

$$E\mu_i = Ev_{it} = 0, \qquad \text{all } i \text{ and } t,$$

$$E\mu_i\mu_{i'} = \begin{cases} \sigma_\mu^2, & i = i' \\ 0, & i \neq i' \end{cases}$$

$$Ev_{it}v_{i't'} = \begin{cases} \sigma_v^2, & i = i', \quad t = t' \\ 0, & \text{otherwise}. \end{cases}$$

The error term can be interpreted as the sum of an individual effect and an effect assumed to vary over both individuals and time.[1] The variance-covariance matrix of the error terms is

$$Euu' = \sigma^2 \begin{bmatrix} A & 0 & \ldots & 0 \\ 0 & A & \ldots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \ldots & A \end{bmatrix}$$

* The author wishes to thank Marc Nerlove for his helpful comments on an earlier version of this communication and for his advice on developing the computational procedure.

[1] A third component representing period specific and individually invariant effects may be added so that $u_{it} = \mu_i + \lambda_t + v_{it}$. The question of the effect on parameter estimates when the period specific effect is erroneously assumed absent (as it is in this version of the model), still is being investigated. See (3) for a preliminary analysis.

where $\sigma^2 = \sigma_\mu^2 + \sigma_\nu^2$, and

$$A = \begin{bmatrix} 1 & \rho & \dots & \rho \\ \rho & 1 & \dots & \rho \\ \vdots & \vdots & & \vdots \\ \rho & \rho & \dots & 1 \end{bmatrix},$$

and $\rho = \sigma_\mu^2/\sigma^2$.[2]

Nerlove (1, 2) has performed Monte Carlo experiments on the model described above in order to investigate the small sample properties of five different methods of estimation. He concludes that "the two-round procedure, using a value of $\hat{\rho}$ estimated from first-round regressions including individual constant terms, compares favorably with all the other techniques investigated."[3] Computationally, the procedure involves two regressions. First, the estimation of the slopes in a least-squares regression of deviations of the dependent variable from individual means $(y_{it} - \bar{y}_{i.})$[4] on similar deviations of the independent variables. Letting b_k be the estimates of these slopes and s^2 be the sum of squared residuals from this regression, the estimate of ρ is derived as:

$$\hat{\rho} = \frac{\hat{\sigma}_\mu^2}{\hat{\sigma}_\mu^2 + s^2/NT}$$

where

$$\hat{\sigma}_\mu^2 = \frac{1}{N}\sum_{i=1}^{N}\left\{(\bar{y}_{1.} - \bar{y}_{..}) - \sum_k b_k[\bar{x}_{i.}(k) - \bar{x}_{..}(k)]\right\}^2.$$

This estimate, $\hat{\rho}$, then is used to compute two weights,

$$\xi = (1 - \hat{\rho}) + T\hat{\rho},$$
$$\eta = (1 - \hat{\rho}),$$

[2] ρ is the so-called "intra class correlation coefficient" of the classical random effects model in the analysis of variance.

[3] Nerlove (2), p. 381.

[4] $\bar{y}_{i.}$ represents the individual mean, i.e.,

$$\bar{y}_{i.} = \frac{1}{T}\sum_{t=1}^{T} y_{it}.$$

Also, the grand mean,

$$\bar{y}_{..} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T} y_{it} = \frac{1}{N}\sum_{i=1}^{N} \bar{y}_{i.}.$$

which are used to transform the variables as follows:[5]

$$y_{it}^{*} = \frac{y_{it} - \bar{y}_{i.}}{\sqrt{\eta}} + \frac{\bar{y}_{i.}}{\sqrt{\xi}}.$$

The second round provides least-squares estimates of parameters of the original model from,

$$y_{it}^{*} = \alpha y_{it-1}^{*} + \beta x_{it}^{*} + u_{it}^{*}. \tag{2}$$

The program to implement the model has been written to provide for dynamic storage allocation of variably dimensioned arrays. The user must supply a main program which assigns a block of storage for these arrays, control cards, and input data organized by region and by time within regions. The present version of the program is set up to handle twenty (20) or fewer variables. This restriction, however, can be relaxed easily. Copies of the program and more complete documentation regarding its requirements, limitations and user instructions may be obtained by writing

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References

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[2] ———. "Further Evidence on the Estimation of Dynamic Economic Relations from a Time Series of Cross Sections," *Econometrica*, XXXIX, No. 2 (March, 1971), 359–382.
[3] ———. "A Note on Error Components Models," *Econometrica*, XXXIX, No. 2 (March, 1971), 383–396.

[5] ξ and η are the characteristic roots of the matrix

$$\frac{1}{\sigma^2}Euu' = \begin{bmatrix} A & 0 & \dots & 0 \\ 0 & A & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & A \end{bmatrix}$$

Nerlove (1) has shown that the least-squares estimates from the transformed variables in (2) are equivalent to generalized least-squares estimates of (1).