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APPENDIX I

The Measurement of Probable Seasonal Fluctuations by Means of Operations on the Deviations of the Data from Graduated Curves. The Elimination of Seasonal Fluctuations before Graduation.

One object of the methods of graduation described in this book is the complete *elimination* of seasonal fluctuations. The problem of *measuring*—rather than *eliminating*—seasonal fluctuations has not been discussed. However, the problem of measurement must not be assumed necessarily divorced from that of elimination. Paradoxical as it may sound, the elimination of seasonal fluctuations is generally the best first step in the process of measuring them. For such measurement, operations on the deviations of the data from a smooth "seasonal eliminating" curve are theoretically more desirable than operations on the raw data. It is, of course, true that, as the number of years increases, extreme delicacy of smoothness or fit in the graduated curve becomes less and less practically important.

A few years ago the writer was approached by the statistical department of a government bureau and asked to propose a good but simple method of discovering any seasonal fluctuations which might exist in economic time series of moderate length. He replied that, as he did not know of any simple and yet really ideal method, he would suggest graduating the data roughly by means of a 2-months moving average of a 12-months moving average, taking the deviations of the data from this moving average (centered), and arriving at seasonal fluctuations from these deviations. Rough as is the method, it has been widely used and favorably noticed year after year. Moreover, though the method is extremely simple, in most cases the results are quite good. The averaging process used in obtaining a seasonal index for any one month generally makes unnecessary any great excellence in the graduation from which deviations are being measured.

A common method of obtaining seasonal fluctuations is to take the arithmetic average of each nominal month and then adjust for trend. This amounts to using a straight line as a base from which to measure deviations. If the number of years covered be large and if the assumption be made that the seasonal fluctuation is constant throughout the period, even such a crude and simple method often gives good results. Indeed, only if the number of years covered by the seasonal fluctuation is quite small, would it seem worth while to employ any particularly refined graduation—and only then if the erratic fluctuations were so small and the seasonal fluctuations so pronounced and regular as to make any deductions from such a short period legitimate.

Unless the number of years is very small, the results obtained from crude and from delicate graduations tend to be very similar. For example, the seasonal fluctuation of the 97 months of Call Money Rates from January 1886 to January 1894 (see Appendix VIII) is practically identical when computed from the *average* deviations of the data from a 43-term approximately fifth-degree parabolic graduation and when computed from the *average* deviations of the data from a 2-months moving average of a 12-months moving average.

The reader must remember that the arithmetic average of the *deviations* of the original data for successive Januaries from the successive January values given by the 43-term graduation equals the arithmetic average of the original data for successive Januaries minus the arithmetic average of the January values given by the 43-term graduation—similarly for other months than January and for the 2-months moving average of a 12-months moving average as well as for the 43-term graduation. Hence, only if the quasi-seasonal [1] obtained by

[1] The variations in the average values of the nominal months of either the 43-term graduation or the 2-months average of a 12-

averaging nominal months in the 43-term graduation differs appreciably from the quasi-seasonal obtained by averaging nominal months in the 2-months moving average of a 12-months moving average, will the seasonal obtained from *deviations* of the data from the 43-term graduation differ appreciably from the seasonal obtained from *deviations* of the data from the 2 of a 12-months moving average. Now an appreciable difference between the quasi-seasonal fluctuations of the 43-term graduation *itself* and quasi-seasonal fluctuations of the 2 of a 12-months moving average *itself* will occur only when the number of years covered is quite small—for data such as monthly Call Money Rates, say less than eight or nine years.

On the other hand, if the investigator wishes to make a careful study of *changing* seasonal fluctuations, he may well use some more delicate graduation than a 2 of a 12-months moving average, though it would seldom be worth while to use any formula involving much computation. The averaging process used in determining seasonal fluctuations will take care of any slight inadequacies in the smoothing formula. The 27-term formula re-

months moving average do not necessarily constitute any part of a true seasonal. Only by accident does any true seasonal remain in either of these graduations. There would be some variation in the average value of the nominal months of any curve whatsoever—except a straight line.

ferred to on page 28 would seem a highly desirable one to use. It is the last word in simplicity of calculation.[1]

Sometimes it is desirable to obtain an extremely close fit to all the factors in the data except seasonal fluctuations. A method of accomplishing this result is to eliminate seasonal fluctuations from the original data before final smoothing. This permits the use of non-seasonal-eliminating graduation formulas, which will follow the adjusted data extremely closely. For example, a Spencer 15-term formula may be applied to the data after the elimination of seasonal fluctuations and the resulting graduation will only accidentally contain any of the original seasonal fluctuations in spite of the closeness of its fit to all the minor movements of the adjusted data.

Charts VIII and IX show the results of applying a Spencer 15-term formula to the 97 months of Call Money data after adjustment for a constant seasonal fluctuation. On each chart is given a 43-term graduation for purposes of comparison. Chart VIII shows the two graduations and the data *after* adjustment for seasonal fluctuations. Chart IX

[1] Take a 16-months moving average of the data with the following simple weights: —1, 0, 0, 0, +1, +1, +1, +1, +1, +1, +1, +1, 0, 0, 0, —1. Take a 12-months moving total of this weighted 16-months moving total. Divide each of the final results by 72.

CHART VIII

CALL MONEY RATES ADJUSTED FOR SEASONAL FLUCTUATIONS, 43-TERM CYCLICAL GRADUATION AND SPENCER'S 15-TERM GRADUATION OF THE ADJUSTED DATA

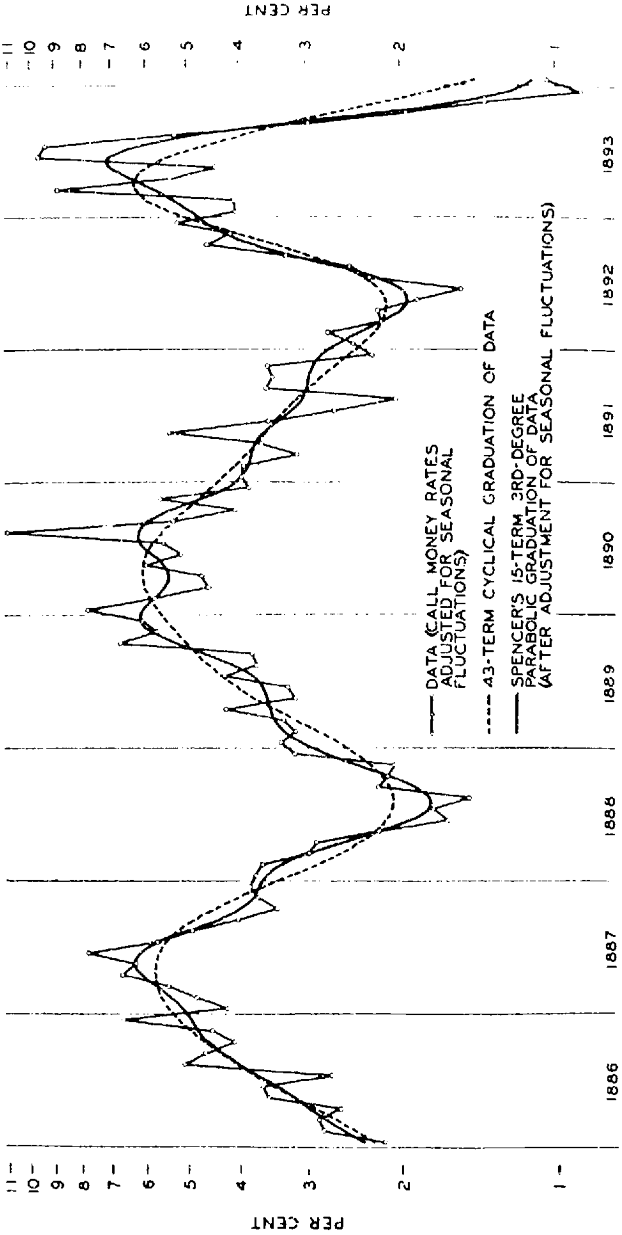
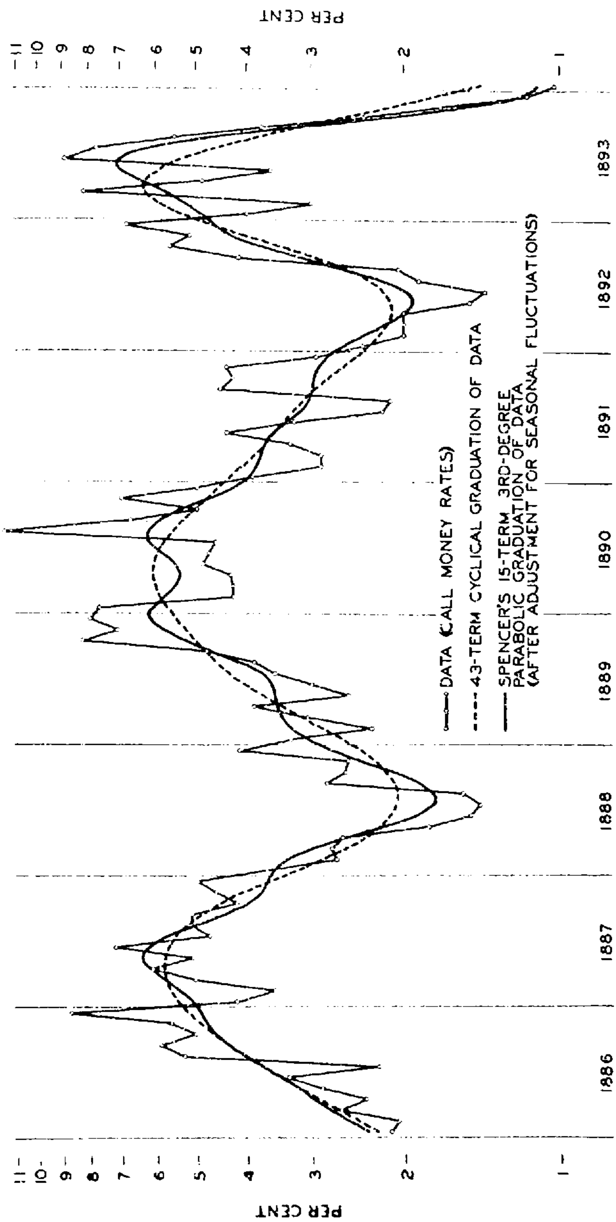


CHART IX

CALL MONEY RATES (NOT ADJUSTED FOR SEASONAL FLUCTUATIONS),
43-TERM CYCLICAL GRADUATION
AND SPENCER'S 15-TERM GRADUATION OF ADJUSTED DATA



shows the same two graduations and the data *before* adjustment for seasonal fluctuations.[1]

The *heavy* solid line in Chart X shows a Spencer 15-term graduation applied to the *adjusted* data. This is the same graduation as that shown in Charts VIII and IX. The broken line in Chart X shows a Spencer 15-term graduation applied to the *unadjusted* data. A moment's inspection will show that, with such pronouncedly seasonal data as Call Money Rates, the results of fitting such a graduation as Spencer's 15-term to the adjusted data are quite different from the results of fitting to the unadjusted data. The "close fit" of the Spencer 15-term graduation shown in Chart IX is a close fit to the "adjusted" data. Only if the adjustment is reasonable is the fit to the unadjusted data reasonable. The degree of reasonableness of the graduation of the unadjusted data depends upon the degree of reasonableness in the "seasonal" which has been eliminated.

If the seasonal fluctuations be uniform from year to year, the whole procedure is quite defensible. If the seasonal fluctuations actually eliminated be little more than some sort of an average of radically unlike movements, both the adjust-

[1] In both Chart VIII and Chart IX, the Spencer 15-term graduation is that obtained by graduating the *adjusted* data. Of course, no such statement need be made in connection with the 43-term graduation, as it gives the same results whether applied to adjusted or unadjusted data.

ment for seasonal fluctuations and the graduation of the adjusted data become difficult to defend. If the seasonal fluctuations seem to be, as in the case of the 97 months of Call Money Rates shown in Charts VIII, IX, and X, moderately though not pronouncedly regular, the graduation of the data. after adjustment for any "average" seasonal, is open to some slight criticism. It might be contended that it is a graduation of data which in sections has been adjusted for a seasonal fluctuation not existing in those sections.

Such a criticism cannot be made of the 43-term graduation. That graduation eliminates all seasonal fluctuation. It is unaffected by the reality or unreality of such seasonal fluctuation. It gives identically the same results when fitted to the unadjusted data as it does when fitted to the data adjusted for any seasonal fluctuation whatever—real or unreal. The adjustment of the data for an absolutely non-existent seasonal does not affect the results of applying the 43-term formula.

If seasonal fluctuations are to be eliminated before graduation, it is, of course, highly desirable that the investigator, before eliminating them, consider carefully whether they exist. For example, Call Money Rates since 1915 show a greatly reduced monthly seasonal fluctuation. To eliminate from such rates a seasonal derived from earlier rates would be quite illegitimate. A Spencer 15-

DESCRIPTION OF CHART X

Chart X is in two parts. The upper part shows Call Money Rates (unadjusted for seasonal fluctuations) and three Spencer 15-term graduations. The lower part of the chart shows a moving seasonal—on the same scale as the upper part of the chart.

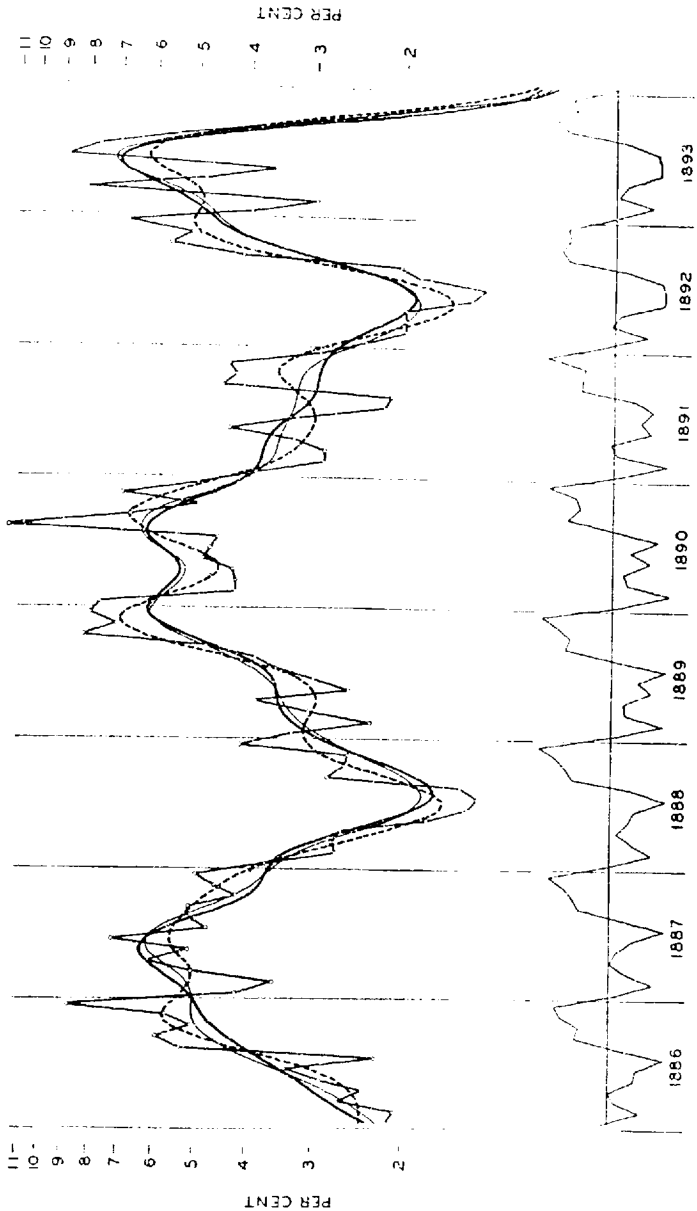
The three Spencer 15-term graduations shown in the upper part of the chart are graduations of three different sets of data. The broken line is a graduation of the data as shown on the chart (unadjusted for seasonal fluctuations). The *heavy* solid line is a graduation of the data after adjustment for a constant seasonal. The *light* solid line is a graduation of the data after adjustment for the moving seasonal shown at the bottom of the chart.

The constant seasonal was calculated by taking the arithmetic average of the deviations of each nominal month in the 97 from the 43-term cyclical graduation and adjusting for trend and to zero.

The construction of the moving seasonal may be illustrated as follows. The deviations of nine consecutive Januaries from the 43-term graduation were listed. The largest and the smallest deviations were then thrown out. The arithmetic average of the remaining seven deviations was taken. For example, the January 1886 seasonal was the arithmetic average of seven January deviations out of the nine Januaries from January 1882 to January 1890 inclusive. A similar procedure for each month gave a moving seasonal. This moving seasonal was corrected for sum by taking a 2-months moving average of a 12-months moving average of the seasonal and calling the deviations of the first moving seasonal from this graduation the final moving seasonal. As the trend is necessarily linear such a straight line formula was admissible.

In the calculations for both the constant seasonal and the moving seasonal, deviations were taken from the 43-term graduation because that graduation had already been computed. A much simpler formula, such as the 27-term formula described on page 28, would give perfectly satisfactory results.

CHART X FOR DESCRIPTION OF THIS CHART SEE OPPOSITE PAGE



term formula applied to the monthly rates since 1915, after they had been adjusted for such a defunct seasonal, would give totally meaningless results. From 1857 to date, the changes in the characteristics of the seasonal fluctuations of Call Money Rates have been great. Generally the changes have been gradual though sometimes they have been sudden. When they have been sudden, the date of the change has usually corresponded with the date of some outstanding economic occurrence—such as the panic of 1873 or the early years of the Federal Reserve System. In such cases a particular seasonal fluctuation may be used up to a particular date, when another seasonal fluctuation will be substituted.

When the nature of the seasonal fluctuation appears to be changing gradually, the natural procedure would seem to be to calculate a *changing* seasonal. At the bottom of Chart X is a picture of such a changing seasonal. The solid *light* line in the upper part of the same chart gives the results of applying a Spencer 15-term graduation to the data after adjustment for this *changing* seasonal. The solid *heavy* line gives the results of applying a Spencer 15-term formula to the data after the elimination of a *constant* seasonal.[1] An examination of these two lines will show how much differ-

[1] For a description of the methods used in obtaining the constant seasonal and the changing seasonal, see page 130.

ence may appear in the graduation because of the elimination of different seasonal fluctuations—even in a period such as January 1886 to January 1894, when changes in the nature of the seasonal were not particularly violent.

If a less sensitive formula than Spencer's 15-term formula were used, the differences in the graduation resulting from differences in the seasonal fluctuations would, of course, be less. For example, Spencer's 21-term formula would be less affected by differences in the seasonal fluctuations than would Spencer's 15-term formula.[1]

[1] Spencer's 15-term formula is quite sensitive. Though it gives a graduation which is, of course, much smoother than that given by a simple 5-months moving average, it fits the data as closely as does the 5-months moving average.

If a formula with only a little greater smoothing power than the Spencer 15-term formula be desired, the following 17-term formula may be used: Take a 12-months moving total of the data with the following weights: -1, 0, 0, $+2$, $+2$, $+2$, $+2$, $+2$, $+2$, 0, 0, -1. (It will be noted that the plus weights constitute two times a simple 6-months moving total.) Take a 2-months moving total of a 5-months moving total of the results. As the total equals 100, division may be performed by merely moving the decimal point. The weight diagram is excellent. This is a very desirable formula, and the reader should not be disturbed by the fact that the graduation falls $\frac{3}{10}$ *outside* the parabola.

If, however, he desires a closer fit to a second degree parabola, he may use the following: Take a 14-months moving total with the following weights: -1, 0, 0, $+1$, $+2$, $+3$, $+4$, $+4$, $+3$, $+2$, $+1$, 0, 0, -1. (The plus weights constitute a 4-months moving total of a 5-months moving total.) Take a 2-months moving total of a 3-months moving total of the results. Divide by 108. The weight diagram is excellent. Falls $\frac{1}{6}$ *outside* the parabola $y = x^2$.

The sum of the squares of the third differences of the weights is, in each of the above formulas, much smaller than in Higham's

A crude, though extremely simple, procedure for eliminating seasonal fluctuations is sometimes used by financial writers and the editors of financial magazines and newspapers. The quotation for the present month is compared with the quotation for the same month in the preceding year. This comparison is made either by subtracting the quotation for the same month in the preceding year from the quotation for the present month or by dividing the quotation for the present month by the quotation for the same month in the preceding year.

The mathematical significance of any such operation may be described in a multitude of ways. Some ways are enlightening, others are not. There is a simple and enlightening way to describe the operation of *subtracting* the quotation for the same month last year from the quotation for the present month and using the resulting figure instead of the raw data. It amounts to taking a 12-months moving total of the data and using the first differences of this moving total instead of the raw data. The

17-term strictly third-degree parabolic formula—a 5 of a 5 of a 5 of −1, +1, +1, +1, −1 divided by 125.

If, on the other hand, a formula is desired which will follow the data even more closely than Spencer's 15-term formula, the following simple 13-term formula may be used: Take an 11-months moving total with the following weights: −1, 0, +1, +2, +3, +4, +3, +2, +1, 0, −1. (It will be noted that the plus weights constitute a 4-months moving total of a 4-months moving total.) Take a 3-months moving total of the results. Divide by 42. When applied to $y = x^2$, falls $\frac{1}{21}$ *outside* the parabola. The weight diagram is comparatively well-shaped.

procedure which consists of *dividing* the quotation for the present month by the quotation for the same month last year amounts to using the antilogarithms of the first differences of a 12-months moving total of the logarithms of the data—instead of the raw data.

In either case the results are based upon month to month changes (first differences) of a crude graduation, namely, a 12-months moving average. Hence, even if an extremely good method of graduation were used instead of a 12-months moving average, the results would still be of the nature of a first derivative of the graduation. Moreover, as the 12-months moving average does not extend to the end of the data, its first differences do not tell whether, at the present time, the underlying curve of the data is high or low or whether it is rising or falling, but simply *whether it was rising or falling six months ago.*

Now, if the cycle were of unchanging period and shape such information would tell us something definite about the present. For example, if the data were a sine curve of 24 months period, we would know that, when the ratio of the present month to the same month last year began to increase, the sine curve had just passed through a low. The low of a 24-months sine curve occurs six months later than the point of inflection on the downward movement. However, if the data were a 48-months sine curve, we would know that when the ratio of the present month to the same month last year began to increase, the bottom of the sine curve was still six months ahead.

If the underlying curves are not sine curves, but less simple curves, the conclusions to be derived from the actions of such comparison of the present month with the same month last year must be considered most carefully. Though this type of operation on the data sometimes yields interesting results, such results must be carefully interpreted. The fact that the final figures are of the nature of a derivative curve must never be forgotten. The procedure seems likely to lead to misunderstandings when its results are intended for general public information.

APPENDIX II

A Computation Sheet Illustrating Graduation by Spencer's 15-Term Third-Degree Parabolic Formula.

Some readers of this book may be interested in examining a computation sheet for calculating a summation graduation. A computation sheet for graduating Rhodes' data (see Appendix V) by means of Spencer's 15-term formula (see page 55) appears below. The moving totals are calculated before applying the weights. This procedure makes the discovery of errors much easier. The moving totals are self-checking. Any mistake in the weight multiplications or their additions tends to stand out on the graduation like a sore thumb.

There are eleven columns in the paradigm given below. Columns 6, 8 and 9 may, of course, be eliminated, reducing the total number of columns to eight. This is, however, not desirable. The extra concentration needed in computation more than offsets the labor involved in copying out these columns as in the paradigm given below.

The columns in the computation sheet are:

1. Dr. Rhodes' data.
2. 5-year moving totals of column 1.
3. 4-year moving totals of column 2.

A Spencer 15-Term Graduation

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
137										
137										
131	669									
131	670	2654								
133	661	2640								
138	654	2619	10512							
128	655	2599	10434							
124	649	2576	10350	31302	30798	41400	− 31536	− 30564	41400	129.38
132	641	2556	10266	31050	30564	41064	− 31302	− 30378	40998	128.12
127	631	2535	10188	30798	30378	40752	− 31050	− 30252	40626	126.96
130	635	2521	10126	30564	30252	40504	− 30798	− 30207	40315	125.98
118	628	2514	10084	30378	30207	40336	− 30564	− 30225	40132	125.41
128	627	2514	10069	30252	30225	40276	− 30378	− 30300	40075	125.23
125	624	2520	10075	30207	30300	40300	− 30252	− 30402	40153	125.48
126	635	2527	10100	30225	30402	40400	− 30207	− 30537	40283	125.88
127	634	2539	10134	30300	30537	40536	− 30225	− 30720	40428	126.34
129	634	2548	10179	30402	30720	40716	− 30300	− 30948	40590	126.84
127	636	2565	10240	30537	30948	40960	− 30402	− 31197	40846	127.64
125	644	2588	10316	30720	31197	41264	− 30537	− 31416	41228	128.84
128	651	2615	10399	30948	31416	41596	− 30720	− 31542	41698	130.31
		2615								

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
135	657	2631	10472	31197	31542	41888	− 30948	− 31521	42158	131.74
136	663	2638	10514	31416	31521	42056	− 31197	− 31371	42425	132.58
133	660	2630	10507	31542	31371	42028	− 31416	− 31119	42406	132.52
131	658	2608	10457	31521	31119	41828	− 31542	− 30816	42110	131.59
125	649	2581	10373	31371	30816	41492	− 31521	− 30504	41654	130.17
133	641	2554	10272	31119	30504	41088	− 31371	− 30201	41139	128.56
127	633	2529	10168	30816	30201	40672	− 31119	− 29886	40684	127.14
125	631	2504	10067	30504	29886	40268	− 30816	− 29550	40292	125.91
123	624	2480	9962	30201	29550	39848	− 30504	− 29160	39935	124.80
123	616	2449	9850	29886	29160	39400	− 30201	− 28680	39565	123.64
120	609	2417	9720	29550	28680	38880	− 29886	− 28119	39105	122.20
119	600	2374	9560	29160	28119	38240	− 29550	− 27462	38507	120.33
118	592	2320	9373	28680	27462	37492	− 29160	− 26742	37732	117.91
114	573	2262	9154	28119	26742	36616	− 28680	− 26001	36796	114.99
115	555	2198	8914	27462	26001	35656	− 28119	− 25269	35731	111.66
107	542	2134	8667							
101	528	2073	8423							
105	509	2018								
100	494									
96	487									
92										
94										

4. 4-year moving totals of column 3.
5. +3 times the values in column 4. The first figure 31302 equals the second figure of column 4 multiplied by 3—i.e., 10434 × 3. Etc.
6. +3 times the values in column 4. The first figure 30798 equals the fourth figure of column 4 multiplied by 3—i.e., 10266 × 3. Etc.
7. +4 times the values in column 4. The first figure 41400 equals the third figure of column 4 multiplied by 4—i.e., 10350 × 4. Etc.
8. −3 times the values in column 4. The first figure −31536 equals the first figure of column 4 multiplied by −3—i.e., 10512 × −3. Etc.
9. −3 times the values in column 4. The first figure −30564 equals the fifth figure of column 4 multiplied by −3—i.e., 10188 × −3. Etc.
10. Algebraic totals of columns 5, 6, 7, 8, 9.
11. Column 10 divided by 320.

APPENDIX III

Three Sets of Twenty-five Term Third-Degree Parabolic Graduation Weights.

For the method of obtaining the weights of column I, see page 54. The weights do not eliminate 12-months seasonal fluctuations. For the appearance of the weight diagram, see Figure 7, Chart I.

The weights of column II, if fitted to a second (or third) degree parabola, exactly fit the parabola. They eliminate 12-months seasonal fluctuations. The sum of the squares of the third differences of the weights is the minimum possible with the preceding restrictions. See page 58 and Figure 15, Chart I.

The weights of column III are rough approximations to those of column II. The resulting graduation falls 1/6 (a negligible distance) *outside* the parabola $y = x^2$. It eliminates 12-months seasonal fluctuations. It is computed by taking 1/144 of a 4-months moving total of a 12-months moving total of an 11-months moving total which has the following eleven simple weights: −1, 0, 0, +1, +1, +1, +1, +1, 0, 0, −1.

I Henderson Ideal Third-Degree Parabolic	II Ideal Seasonal Eliminating Parabolic	III Summation Seasonal Eliminating Parabolic
−.00334	−.00740	−.00695
−.00890	−.01676	−.01389
−.01369	−.02028	−.02083
−.01456	−.01700	−.02083
−.00922	−.00462	−.00695
+.00321	+.01607	+.01389
+.02217	+.04167	+.04167
+.04580	+.06726	+.06944
+.07138	+.08795	+.09028
+.09572	+.10033	+.10417
+.11576	+.10362	+.10417
+.12892	+.10009	+.09722
+.13350	+.09814	+.09722
+.12892	+.10009	+.09722
+.11576	+.10362	+.10417
+.09572	+.10033	+.10417
+.07138	+.08795	+.09028
+.04580	+.06726	+.06944
+.02217	+.04167	+.04167
+.00321	+.01607	+.01389
−.00922	−.00462	−.00695
−.01456	−.01700	−.02083
−.01369	−.02028	−.02083
−.00890	−.01676	−.01389
−.00334	−.00740	−.00695
Total +1.00000	+1.00000	+1.00000

APPENDIX IV

THE WEIGHTS IMPLIED IN VARIOUS GRADUATION FORMULAS.

In the text of this book a number of graduation formulas have been described and discussed as weighted moving averages. The present Appendix contains a table giving weights implied in fifteen of these graduation formulas. The "weight diagrams" for these fifteen graduations (with nine other graduations) are presented in Chart I (pages 77, 78, 79). The column numbers of Appendices IV, VII and VIII and the Figure numbers of Chart I are comparable. For example, Kenchington's 27-term formula is No. 12 in all three Appendices and in Chart I.

The weights given in this Appendix are those *implied* in the graduation formulas. For example, the weights implied in a 12-months simple moving average are 12 in number and all equal. As the total must equal unity, each weight equals 1/12. Chart I, Figure 1, represents such a system of weights. Each ordinate of Figure 1 is 1/12th of a unit high.

On pages 43 to 46, the possibility of describing various smoothing formulas as weighted moving averages was illustrated by discussing the "weights" implied in a 2-months moving average of a 12-months moving average (see Chart I, Figure 2), an 8-months moving average of a 12-months moving average (see Chart I, Figure 3), a 4-months moving average of a 5-months moving average of a 6-months moving average (see Chart I, Figure 4), a set of 13 weights such that, if applied to 13 consecutive and equally spaced observations, the result is the mid ordinate of a third-degree parabola fitted by the method of least squares (see Chart I, Figure 5).

The weight systems given in this Appendix are:

Col. 7. A Henderson Ideal 23-term third-degree parabolic graduation—see page 58, and Appendix VII.

Col. 8. A Henderson Ideal 29-term third-degree parabolic graduation—see Appendix VII.

Col. 9. A Henderson Ideal 33-term third-degree parabolic graduation—see Appendix VII.

Col. 11. Spencer's 21-term summation third-degree parabolic graduation—see Appendices VII and VIII.

Col. 12. Kenchington's 27-term summation third-degree parabolic graduation—see Appendices VII and VIII.

Col. 13. A 29-term summation approximately third-degree parabolic graduation (if fitted to parabola $y = x^2$, falls 7/6 outside). See Appendices VII and VIII.

Col. 14. A 29-term summation non-parabolic graduation (if fitted to parabola $y = x^2$ falls 3 1/2 outside). See Appendices VII and VIII.

Col. 15. A 25-term "Ideal" 12-months seasonal eliminating third-degree parabolic graduation. See page 58 and Appendix VII.

Col. 18. A 35-term summation 5th-degree parabolic graduation. See Appendices VII and VIII.

Col. 19. A 41-term summation fifth-degree parabolic graduation. See Appendices VII and VIII.

Col. 20. A 43-term summation fifth-degree parabolic graduation. See Appendices VII and VIII.

Col. 21. A 45-term summation fifth-degree parabolic graduation. See page 66 and Appendices VII and VIII.

Col. 22. Another 45-term summation fifth-degree parabolic graduation. See page 65 and Appendices VII and VIII.

Col. 23. A 39-term summation approximately fifth-degree parabolic graduation. See Appendices VII and VIII.

Col. 24. A 43-term summation approximately fifth-degree parabolic graduation—this is the graduation used in the study of interest rates and security prices to eliminate 12-months seasonal and minor erratic fluctuations. See Appendices VII and VIII.

The first five weight systems of this Appendix (columns 7, 8, 9, 11, 12) do not eliminate 12-months seasonal fluctuations; the remaining weight systems do eliminate such 12-months seasonal fluctuations.

THE WEIGHTS IMPLIED IN VARIOUS GRADUATION FORMULAS

(The various graduation formulas have been given the same numbers in Appendices IV, VII and VIII and in Chart I.)

(7) Henderson 25-term Ideal 3rd-degree Parabolic	(8) Henderson 29 term Ideal 3rd-degree Parabolic	(9) Henderson 33 term Ideal 3rd-degree Parabolic	(11) Spencer's 21-term 3rd-degree Parabolic	(12) Kenchington's 27-term 3rd-degree Parabolic
		−.00140		
		−.00418		
	−.00211	−.00754		
	−.00602	−.01031		−.00260
−.00334	−.01018	−.01137		−.00779
−.00890	−.01271	−.00981		−.01299
−.01369	−.01193	−.00509	−.00286	−.01558
−.01456	−.00678	+.00294	−.00857	−.01299
−.00922	+.00311	+.01400	−.01429	−.00260
+.00321	+.01733	+.02745	−.01429	+.01299
+.02217	+.03484	+.04240	−.00571	+.03377
+.04580	+.05413	+.05774	+.01714	+.05714
+.07138	+.07338	+.07234	+.05143	+.07792
+.09572	+.09075	+.08506	+.09429	+.09331
+.11576	+.10455	+.09493	+.13429	+.10649
+.12892	+.11341	+.10118	+.16286	+.11429
+.13350	**+.11646**	**+.10332**	**+.17142**	**+.11688**
+.12892	+.11341	+.10118	+.16286	+.11429
+.11576	+.10455	+.09493	+.13429	+.10649
+.09572	+.09075	+.08506	+.09429	+.09351
+.07138	+.07338	+.07234	+.05143	+.07792
+.04580	+.05413	+.05774	+.01714	+.05714
+.02217	+.03484	+.04240	−.00571	+.03377
+.00321	+.01733	+.02745	−.01429	+.01299
−.00922	+.00311	+.01400	−.01429	−.00260
−.01456	−.00678	+.00294	−.00857	−.01299
−.01369	−.01193	−.00509	−.00286	−.01558
−.00890	−.01271	−.00981		−.01299
−.00334	−.01018	−.01137		−.00779
	−.00602	−.01031		−.00260
	−.00211	−.00754		
		−.00418		
		−.00140		

The total of each column is unity.

(13) 29-term Approximately 3rd-degree Parabolic	(14) 29-term Non-parabolic	(15) 25-term Ideal Seasonal Eliminating 3rd-degree Parabolic	(18) 35-term 5th-degree Parabolic	(19) 41-term 5th-degree Parabolic
				+.00086
				+.00216
				+.00352
			+.00264	+.00367
			+.00600	+.00261
			+.00818	−.00050
−.0027$\dot{7}$	−.00232		+.00460	−.00528
−.0083$\dot{3}$	−.00694		−.00471	−.01131
−.0138$\dot{8}$	−.01389	−.00740	−.01789	−.01670
−.0166$\dot{6}$	−.01852	−.01676	−.02849	−.01955
−.0138$\dot{8}$	−.01852	−.02028	−.03007	−.01795
−.0055$\dot{5}$	−.01157	−.01700	−.02071	−.01064
+.0083$\dot{3}$	+.00231	−.00462	−.00306	+.00151
+.02500	+.02083	+.01607	+.01836	+.01827
+.0416$\dot{6}$	+.04167	+.04167	+.04167	+.03815
+.0583$\dot{3}$	+.06250	+.06726	+.06233	+.05922
+.07500	+.08102	+.08795	+.08039	+.07835
+.0888$\dot{8}$	+.09491	+.10033	+.09587	+.09448
+.10000	+.10417	+.10362	+.10880	+.10656
+.1083$\dot{3}$	+.10880	+.10009	+.11653	+.11420
+.1111$\dot{1}$	**+.11110**	**+.09814**	**+.11912**	**+.11674**
+.1083$\dot{3}$	+.10880	+.10009	+.11653	+.11420
+.10000	+.10417	+.10362	+.10880	+.10656
+.0888$\dot{8}$	+.09491	+.10033	+.09587	+.09448
+.07500	+.08102	+.08795	+.08039	+.07835
+.0583$\dot{3}$	+.06250	+.06726	+.06233	+.05922
+.0416$\dot{6}$	+.04167	+.04167	+.04167	+.03815
+.02500	+.02083	+.01607	+.01836	+.01827
+.0083$\dot{3}$	+.00231	−.00462	−.00306	+.00151
−.0055$\dot{5}$	−.01157	−.01700	−.02071	−.01064
−.0138$\dot{8}$	−.01852	−.02028	−.03007	−.01795
−.0166$\dot{6}$	−.01852	−.01676	−.02849	−.01955
−.0138$\dot{8}$	−.01389	−.00740	−.01789	−.01670
−.0083$\dot{3}$	−.00694		−.00471	−.01131
−.0027$\dot{7}$	−.00232		+.00460	−.00528
			+.00818	−.00050
			+.00600	+.00261
			+.00264	+.00367
				+.00352
				+.00216
				+.00086

The total of each column is unity.

(20)	(21)	(22)	(23)	(24)
43-TERM 5TH-DEGREE PARABOLIC	45-TERM 5TH-DEGREE PARABOLIC	45-TERM 5TH-DEGREE PARABOLIC	39-TERM APPROXIMATELY 5TH-DEGREE PARABOLIC	43-TERM APPROXIMATELY 5TH-DEGREE PARABOLIC
	+.00020	+.00033		
+.00056	+.00069	+.00127		+.00073
+.00143	+.00151	+.00272		+.00187
+.00233	+.00237	+.00384	+.00139	+.00312
+.00301	+.00300	+.00350	+.00347	+.00417
+.00321	+.00273	+.00138	+.00556	+.00469
+.00152	+.00108	−.00236	+.00556	+.00292
−.00177	−.00226	−.00668	+.00347	−.00083
−.00642	−.00688	−.01082	−.00208	−.00625
−.01179	−.01197	−.01393	−.01042	−.01271
−.01639	−.01612	−.01523	−.01875	−.01854
−.01821	−.01771	−.01398	−.02431	−.02135
−.01603	−.01542	−.00961	−.02431	−.01979
−.00923	−.00835	−.00237	−.01806	−.01323
+.00292	+.00364	+.00796	−.00347	−.00063
+.01947	+.01986	+.02143	+.01597	+.01698
+.03865	+.03867	+.03816	+.03819	+.03750
+.05832	+.05837	+.05668	+.06042	+.05854
+.07746	+.07711	+.07501	+.08125	+.07917
+.09378	+.09324	+.09112	+.09792	+.09667
+.10579	+.10543	+.10343	+.10972	+.10937
+.11333	+.11302	+.11125	+.11806	+.11739
+.11612	**+.11558**	**+.11380**	**+.12084**	**+.12042**
+.11333	+.11302	+.11125	+.11806	+.11739
+.10579	+.10543	+.10343	+.10972	+.10937
+.09378	+.09324	+.09112	+.09792	+.09667
+.07746	+.07711	+.07501	+.08125	+.07917
+.05832	+.05837	+.05668	+.06042	+.05854
+.03865	+.03867	+.03816	+.03819	+.03750
+.01947	+.01986	+.02143	+.01597	+.01698
+.00292	+.00364	+.00796	−.00347	−.00063
−.00923	−.00835	−.00237	−.01806	−.01323
−.01603	−.01542	−.00961	−.02431	−.01979
−.01821	−.01771	−.01398	−.02431	−.02135
−.01639	−.01612	−.01523	−.01875	−.01854
−.01179	−.01197	−.01393	−.01042	−.01271
−.00642	−.00688	−.01082	−.00208	−.00625
−.00177	−.00226	−.00668	+.00347	−.00083
+.00152	+.00108	−.00236	+.00556	+.00292
+.00321	+.00273	+.00138	+.00556	+.00469
+.00301	+.00300	+.00350	+.00347	+.00417
+.00233	+.00237	+.00384	+.00139	+.00312
+.00143	+.00151	+.00272		+.00187
+.00056	+.00069	+.00127		+.00073
	+.00020	+.00033		

The total of each column is unity.

APPENDIX V

Seven Graduations of Dr. E. C. Rhodes' Test Mortality Data.

The data are rates of infantile mortality from causes other than diarrheal diseases, for the 42 years from 1870 to 1911, inclusive. See page 82, note 1.

GRADUATIONS OF DR. RHODES' TEST DATA

I	II	III	IV	V	VI	VII	VIII
Rhodes' Original Data	Rhodes' 15-point Curve	Rhodes' 13-point Curve	Rhodes' 11-point Curve	Rhodes' 9-point Curve	Sheppard's Graduation	Spencer's 15-term Formula	Whittaker-Henderson n = 2
137	137.2	137.7	137.7	138.5	136.5	137.11	136.72
137	135.0	134.6	134.3	132.7	135.4	135.10	135.27
131	133.6	133.1	133.1	132.7	134.3	133.77	134.07
131	132.8	132.6	132.9	134.1	133.2	132.84	133.11
133	132.1	132.4	132.8	134.2	132.2	132.07	132.27
138	131.5	132.0	132.1	132.2	131.2	131.29	131.37
128	130.7	131.2	130.7	128.7	130.2	130.41	130.31
124	129.7	130.0	129.2	125.6	129.2	129.38	129.19
132	128.5	128.4	127.8	125.3	128.4	128.12	128.11
127	127.2	126.7	126.7	127.1	127.1	126.96	127.08
130	126.1	125.5	126.0	128.1	125.7	125.98	126.17
118	125.4	124.7	125.2	127.7	125.2	125.41	125.50
128	125.2	124.3	124.6	127.0	126.1	125.23	125.21
125	125.3	124.5	125.0	126.3	126.0	125.48	125.27
126	125.6	125.4	126.2	125.5	125.0	125.88	125.62
127	126.1	126.5	127.4	124.6	125.6	126.34	126.20
129	126.8	127.3	127.7	124.5	126.7	126.84	126.98
127	127.8	128.1	127.6	125.3	128.8	127.64	127.96
125	129.0	129.2	128.0	127.0	129.7	128.84	129.15
128	130.4	130.8	129.3	130.6	136.3	130.31	130.45
135	131.8	132.3	131.4	134.8	131.2	131.74	131.57
136	132.7	133.1	133.0	137.2	131.7	132.58	132.17
133	133.1	133.0	133.2	135.4	131.9	132.52	132.08
131	132.6	131.9	132.1	130.9	131.7	131.59	131.37
125	131.3	130.1	130.4	126.9	130.5	130.17	130.25
133	129.5	128.0	128.9	125.0	129.1	128.56	128.94
127	127.6	126.3	127.4	124.7	127.2	127.14	127.55
125	125.9	125.1	125.8	124.9	126.0	125.91	126.19
123	124.4	124.1	124.1	125.4	124.7	124.80	124.89
123	122.9	123.1	122.7	126.3	123.8	123.64	123.57
126	121.3	121.9	121.5	126.0	122.6	122.20	122.04
119	119.5	120.7	120.2	122.9	119.5	120.35	120.12
118	117.4	118.9	118.4	117.8	117.5	117.91	117.75
114	114.9	116.1	115.8	112.7	115.0	114.99	114.95
115	112.1	112.7	112.6	108.4	112.1	111.66	111.79
107	109.1	109.1	109.1	105.1	108.3	108.09	108.40
101	105.9	105.4	105.5	103.4	105.5	104.55	105.00
105	102.7	101.8	101.9	103.2	102.6	101.21	101.78
100	99.6	98.6	98.6	102.7	99.8	98.24	98.83
96	96.7	96.1	95.7	100.7	97.0	95.82	96.30
92	94.2	94.7	93.8	96.7	94.2	94.11	94.36
94	92.2	94.9	93.0	90.7	91.4	93.30	93.15

APPENDIX VI

Paradigm for Graduating by Robert Henderson's Method.

In the immediately following paradigm n is taken as equal to 3. For general discussion of the method see Chapter VI.

Let there be 13 observations equally spaced on the x axis. Let the ordinates of the observations be 36009, 22009, 27018, 4027, 18045, 7054, 14054, 9045, 29036, 8027, 55036, 34036, and 62054. It is desired to graduate these observations in such a manner that $\frac{9}{1000}$[1] times the sum of the squares of the deviations of the data from the graduated curve plus the sum of the squares of the third differences of the graduated curve shall be a minimum. The actual computation is in three steps. Mr. Henderson calls the three steps, *the preliminary*, *the first half*, and *the second half*. The paradigm[2] on pages 155 and 156 should be followed when reading the instructions below.

[1] The reader will note from page 91 that $k = \frac{9}{1000}$ when $n = 3$.

[2] For the paradigm the data above were so chosen that all ordinates of the graduated curve would be integers. In actual computation, the calculations would be carried to as many decimals as were desired in the graduated curve.

The Preliminary:

1. *Guess* three points on the graduated curve some distance from the beginning. The three points in the paradigm were chosen in such a manner as to give immediate results in the graduation.[1] Their ordinates are 26000, 18907, and 14213. These values are supposed to be guesses at the 10th, 9th and 8th ordinates of the graduated curve.

2. Find the first and second differences of the above three ordinates. Use —4694 as the first difference (*i.e.*, 14213 — 18907). The second difference is +2399.
3. Calculate the figures in the paradigm by filling each column before beginning on the next. Beginning with the first figure of the first column, we notice:

   (a) +2399 is the second difference mentioned above.
   (b) —4694 is the first difference mentioned above.
   (c) +14213 is the guess at the 8th ordinate of the graduated curve. (See paragraph 1.)
   (d) —18776 = —4694 × 4. (See note 1, page 153.)
   (e) +23990 = +2399 × 10. (See note 1, page 153.)
   (f) +19427 = +14213 — 18776 + 23990
   (g) The first figure (+4027) of the second column is the 4th *datum* ordinate.
   (h) —15400 = +4027 — 19427

[1] The preliminary portion of the Henderson method of computation is used simply to obtain suitable estimated figures with which to begin *the first half*.

If the three preliminary points are badly chosen, the preliminary operation will be lengthened. In our illustration, it might have to be extended through *the first half* or even through another return operation. For final results accurate to any particular number of decimals, it would be necessary to work backwards and forwards until at two separate stages three values which were identical to the required number of decimals were obtained at one of the ends.

It is highly desirable to choose 3 preliminary values a considerable distance along the curve (say 20 units when n = 3 and more when n is larger) and to choose them as well as possible. Mr. Henderson advocates fitting a second-degree parabola to eleven consecutive points some distance along the curve, and using the 4th, 5th and 6th terms of the parabola as the three preliminary points.

(i) $-924 = -15400 \times .06$[1]
(j) $+1475 = -924 + 2399$
(k) $-3219 = +1475 - 4694$
(l) $+10994 = -3219 + 14213$
(m) $-12876 = -3219 \times 4$
et cetera

4. The twelve figures at the end of *the preliminary* are calculated as follows: The first three figures are the 6th figures of the last three columns. The other 9 figures are obtained from these 3 figures by assuming the 3 figures to lie on a second-degree parabola and extrapolating the parabola by means of first and second differences.

The First Half:

1. The first three figures of column one are obtained from the last three of the twelve figures just discussed in *the preliminary*.
   (a) +2009 is their second difference.
   (b) −19045 is a first difference. As we are now going backwards, it equals +91099 − 110144.
   (c) +91099 is the 10th figure of the last 12 of *the preliminary*.
2. The computation now runs as it did in *the preliminary* except that the first figure of the second column (+36009) is the first datum ordinate instead of the fourth as in *the preliminary*.
3. The last three figures (+35036, +46036, +59054) are extrapolated from +14144, +19090, +26054, by means of first and second differences.

The Second Half:

1. The reader will be able to understand *the second half* quite easily if he will notice that the figures at the tops of the

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[1] If $n = 3$

$$n + 1 = 4$$

$$\frac{(n+1)\ (n+2)}{2} = 10$$

$$\frac{4\ (2n+3)}{(n+1)\ (n+2)^2\ (n+3)} = .06$$

For the theory back of all this, see the Henderson articles referred to in note 1, page 91.

columns are not data ordinates (as in *the first half*) but are the 6th figures of columns in *the first half* taken in reverse order.

2. The first three figures of the first column of *the second half* are obtained from the last three (extrapolated) figures of *the first half* in the same manner that the first three figures of *the first half* were obtained from *the preliminary*.
3. The 13 ordinates of the graduated curve are the sixth figures of the columns of *the second half* (taken in reverse order) to which are added the last three figures of *the first half*. They are +35009, +26009, +19018, +14027, +11045, +10054, +11054, +14045, +19036, +26027, +35036, +46036, +59054. If the reader will calculate the sixth differences of these figures, he will discover that each of them equals minus $\frac{9}{1000}$ times the corresponding deviation of a datum ordinate from the graduated curve, and hence the sum of the squares of the third differences of the graduated curve plus $\frac{9}{1000}$ times the sum of the squares of the deviations of the data from the graduated curve is a minimum.

The Paradigm

Paradigm for graduating data in such a manner that $\frac{9}{1000}$ times the sum of the squares of the deviations of the data from the graduation plus the sum of the squares of the third differences of the graduation shall be a minimum.[1]

[1] In the following paradigm data ordinates are italicized and ordinates of the final graduation are set in heavy full face type.

Preliminary

	+ 4027	*+27018*	*+22009*	*+36009*	
	−15400	+14150	− 7750	+ 2500	+ 10099
	− 924	+ 849	− 465	+ 150	+ 11063
+ 2399	+ 1475	+ 2324	+ 1859	+ 2009	+ 14036
− 4694	− 3219	− 895	+ 964	+ 2973	+ 19018
+14213	+10994	+10099	+11063	+14036	+ 26009
−18776	−12876	− 3580	+ 3856		+ 35009
+23990	+14750	+23240	+18590		+ 46018
+19427	+12868	+29759	+33509		+ 59036
					+ 74063
					+ 91099
					+110144
					+131198

First Half

	+36009	*+22009*	*+27018*	*+ 4027*	*+18045*
	+ 1000	− 4900	+11150	−15550	+13000
	+ 60	− 294	+ 669	− 933	+ 780
+ 2009	+ 2069	+ 1775	+ 2444	+ 1511	+ 2291
−19045	−16976	−15201	−12757	−11246	− 8955
+91099	+74123	+58922	+46165	+34919	+25964
−76180	−67904	−60804	−51028	−44984	−35820
+20090	+20690	+17750	+24440	+15110	+22910
+35009	+26909	+15868	+19577	+ 5045	+13054

+ 7054	*+14054*	*+ 9045*	*+29036*	*+ 8027*	*+55036*
− 6000	+ 3900	− 7250	+13300	−25200	+32450
− 360	+ 234	− 435	+ 798	− 1512	+ 1947
+ 1931	+ 2165	+ 1730	+ 2528	+ 1016	+ 2963
− 7024	− 4859	− 3129	− 601	+ 415	+ 3378
+18940	+14081	+10952	+10351	+10766	+14144
−28096	−19436	−12516	− 2404	+ 1660	+13512
+19310	+21650	+17300	+25280	+10160	+29630
+10154	+16295	+15736	+33227	+22586	+57286

+34036	*+62054*			
−23250	+ 7500			
− 1395	+ 450			
+ 1568	+ 2018			
+ 4946	+ 6964	+ 8982	+11000	+13018
+19090	+26054	**+35036**	**+46036**	**+59054**
+19784				
+15680				
+54554				

Second Half

	+10766	+10351	+10952	+14081	+18940
	− 450	+ 450	− 300	0	− 150
	− 27	+ 27	− 18	0	− 9
+ 2018	+ 1991	+ 2018	+ 2000	+ 2000	+ 1991
−11000	− 9009	− 6991	− 4991	− 2991	− 1000
+35036	**+26027**	**+19036**	**+14045**	**+11054**	**+10054**
−44000	−36036	−27964	−19964	−11964	− 4000
+20180	+19910	+20180	+20000	+20000	+19910
+11216	+ 9901	+11252	+14081	+19090	+25964

+25964	+34919	+46165	+58922	+74123
0	0	+ 300	− 150	+ 150
0	0	+ 18	− 9	+ 9
+ 1991	+ 1991	+ 2009	+ 2000	+ 2009
+ 991	+ 2982	+ 4991	+ 6991	+ 9000
+11045	**+14027**	**+19018**	**+26009**	**+35009**
+ 3964	+11928	+19964	+27964	
+19910	+19910	+20090	+20000	
+34919	+45865	+59072	+73973	

Summary

Data	*Graduation*
36009	35009
22009	26009
27018	19018
4027	14027
18045	11045
7054	10054
14054	11054
9045	14045
29036	19036
8027	26027
55036	35036
34036	46036
62054	59054

APPENDIX VII

The Results of Applying Nineteen Different Graduation Formulas to Equidistant Points on Indefinitely Extended Sine Series.

The entries in the table show the percentages of the amplitudes of the various sine curves which are preserved by each formula.

(The various graduation formulas have the same numbers in Appendices IV, VII and VIII and in Chart I.)

(0)	(2)	(7)	(8)	(9)	(11)
Sine Curve Period (Points)	2 of a 12 Simple Moving Average	Henderson 25-term Ideal 3rd-degree Parabolic	Henderson 29-term Ideal 3rd-degree Parabolic	Henderson 33-term Ideal 3rd-degree Parabolic	Spencer's 21-term 3rd-degree Parabolic
12	0	24.54	8.07	− 1.40	55.22
15	23.04	53.53	36.12	20.21	76.09
18	40.93	71.98	58.70	44.31	86.65
20	50.04	79.75	69.22	57.05	90.66
24	63.30	88.93	82.47	74.42	95.10
30	75.41	94.97	91.77	87.54	97.29
36	82.50	97.44	95.73	93.41	98.94
40	85.67	98.28	97.11	95.47	99.29
48	89.89	99.14	98.54	97.70	99.63
60	93.48	99.64	99.38	99.03	99.87
120	98.33	99.98	99.96	99.96	99.95

Notes:

Col. (0) *Sine curve periods*. The first entry in column 2 (zero) means that if the formula of column 2 be applied to equidistant monthly points on an indefinitely extended sine curve whose period is 12 months, such sine curve is entirely eliminated. The formula will give a horizontal straight line. The first entry in column (7) means that if the formula of column (7) be applied to such a 12-months sine curve, whose amplitude is 100 (vertical distance between minimum values and maximum values), the curve resulting from the application of the formula will be a 12-months sine curve whose amplitude will be 24.54.

Col. (2) See page 43.
Col. (7) See page 59.
Col. (8) See Appendix IV.
Col. (9) See page 57 and Appendix IV.
The first entry in this column (*minus* 1.40) may disturb the reader. It signifies that if this particular formula be applied to a 12-months sine curve the resulting smooth curve will be a sine curve whose amplitude will be 1.40 per cent of the original sine curve but which will have maxima where the original curve had minima and vice versa. It will slightly *overcorrect* for a 12-months sine seasonal.
Col. (11) See pages 51, 52, 53.

	(12)	(13)	(14)	(15)	(18)
Sine Curve Period (Points)	Kenchington's 27-term 3rd-degree Parabolic	29-term Approximately 3rd-degree Parabolic	29-term Non-parabolic	25-term Ideal Seasonal Eliminating 3rd-degree Parabolic	35-term 5th-degree Parabolic
12	9.88	0	0	0	0
15	40.86	29.27	33.48	35.29	45.18
18	63.07	54.56	60.05	60.02	72.81
20	72.90	66.52	72.06	70.81	82.90
24	84.88	81.60	86.56	83.83	92.89
30	93.03	92.03	95.86	92.59	97.78
36	96.42	96.32	99.24	96.21	99.20
40	97.58	97.13	100.19	97.44	99.55
48	98.78	99.12	100.92	98.71	99.82
60	99.50	99.86	101.05	99.47	99.97
120	99.94	100.09	100.40	99.95	99.98

Col. (12) See pages 29, 58.
Col. (13) $\frac{1}{360}$ of a 2-months moving total of a 12-months moving total of the results of subtracting a 17-months moving total from a 4-months moving total of an 8-months moving total. This formula is not *rigidly* parabolic. It falls $\frac{7}{6}$ outside the parabola $y = x^2$. See pages 59 and 60.
Col. (14) $\frac{1}{132}$ of a 3-months moving total of a 12-months moving total of the results of subtracting a 16-months moving total from a 4-months moving total of a 7-months moving total. Falls $3\frac{1}{2}$ outside parabola $y = x^2$. See pages 27, 60.
Col. (15) See page 58.
Col. (18) See pages 67, 68.

Sine Curve Period (Points)	(19) 41-term 5th-degree Parabolic	(20) 43-term 5th-degree Parabolic	(21) 45-term 5th-degree Parabolic	(22) 45-term 5th-degree Parabolic	(23) 39-term Approximately 5th-degree Parabolic
12	0	0	0	0	0
15	36.41	34.16	33.13	26.89	43.03
18	65.34	63.22	62.21	56.55	73.25
20	77.27	75.60	74.80	70.41	84.28
24	90.01	89.10	88.67	86.33	94.83
30	96.74	96.39	96.21	95.35	99.35
36	98.78	98.64	98.55	98.21	100.25
40	99.32	99.23	99.19	98.99	100.37
48	99.74	99.71	99.71	99.62	100.34
60	99.94	99.93	99.92	99.90	100.21
120	99.98	100.00	100.00	99.98	100.03

Col. (19) See page 67.
Col. (20) See pages 66, 67.
Col. (21) A 3-months moving total of a 5 of a 5 of an 8 of a 12 of 17 weights. See page 66.
Col. (22) A 2-months moving total of a 3 of a 3 of a 4 of a 6 of an 8 of a 10 of a 12 of 5 weights. See page 65.
Col. (23) $\frac{1}{1440}$ of a 3-months moving total of a 5 of an 8 of a 12 of 15 simple weights: $+2$, -3, 0, 0, 0, 0, 0, $+3$, 0, 0, 0, 0, 0, -3, $+2$. See pages 71, 72, 73, 74, 75.

Sine Curve Period (Points)	(24) 43-term Approximately 5th-degree Parabolic	(25) Whittaker-Henderson n = 3	(26) Whittaker-Henderson n = 4	(27) Whittaker-Henderson n = 5
12	0	31.87	11.75	4.53
15	38.78	63.52	33.13	15.00
18	69.76	83.68	59.34	34.21
20	82.05	90.56	73.19	49.31
24	94.17	96.60	89.00	74.25
30	99.39	99.08	96.84	91.62
36	100.36	99.69	98.92	97.02
40	100.44	99.83	99.42	98.39
48	100.33	99.94	99.81	99.45
60	100.18	99.99	99.95	99.86
120	100.02	100.00	100.00	100.00

Col. (24) $\frac{1}{3600}$ of a 5-months moving total of a 5 of an 8 of a 12 of 17 simple weights: $+7$, -10, 0, 0, 0, 0, 0, 0, $+10$, 0, 0, 0, 0, 0, 0, -10, $+7$. See pages 73, 74, 75.

Col. (25) See Appendix VI.

Col. (26) See Appendix VI.

Col. (27) See Appendix VI.

APPENDIX VIII

FOURTEEN DIFFERENT GRADUATIONS OF CALL MONEY RATES ON THE NEW YORK STOCK EXCHANGE FOR 97 MONTHS—JANUARY 1886 TO JANUARY 1894 INCLUSIVE.

(The various graduations have the same column numbers in Appendices IV, VII and VIII and in Chart I.)

The reader will notice, when examining this Appendix, that the various graduations have been applied to the *logarithms* of the monthly Call Money Rates. In any graduation, the first problem which presents itself is to decide what function of the variable shall be used as raw data for purposes of graduation. This problem cannot be solved by refusing to think about it. There is nothing magical in the form in which the data happen to be originally presented. For example, if the investigator were interested in the history of bond prices and bond yields, it would make an appreciable difference whether he selected prices or yields as the variable to which he would apply a graduation formula. This would be true even if the bonds were all perpetuities—when it would seem legitimate to have averaged their prices. Many economic series are of this type—where it would seem about equally reasonable to select as the raw data for graduation a series or its reciprocals. Of course, if the logarithms of a series be taken, it becomes mathematically indifferent whether the logarithms of the series or the logarithms of its reciprocals be graduated. There are always disadvantages associated with the choice of any particular function of the data—natural numbers, logarithms, reciprocals, etc.

Some of the reasons which led us to graduate the logarithms of monthly Call Money Rates, rather than the natural numbers, are concerned with the nature of the data, while others are concerned with the nature of graduation. The nature of the data is such that the significance of changes would seem to be measured better by ratios than by differences. A change from a 3% Call Money rate to a 4% rate would seem more nearly comparable with a change from a 6% rate to an 8% rate than with a change from a 6% rate to a 7% rate. As an index of change in general money market conditions, a movement from a 3% rate to a 4% rate would seem more important than a movement from a 6% rate to a 7% rate. The nature of graduation is such as to suggest graduating the logarithms of Call Money Rates rather than the natural numbers. The distribution of deviations of the rates from the graduation is more symmetrical when the graduation has been applied to the logarithms than it is when it has been applied to the natural numbers. The Call Money data when charted in the form of natural numbers tend to show flat minimum areas and sharply cusped maximum areas. If the logarithms are charted, there is a tendency for the data to show more of a sine-like appearance with the shapes of maximum areas more nearly the same as those of minimum areas. A graduation applied to the natural numbers will not give as close a fit to the cusped maximum areas as to the flat minimum areas. If the graduation be applied to the logarithms of the data, the closeness of fit will tend to be more nearly the same for maximum and minimum areas.

No function of the data can be chosen such that its graduation will not have peculiarities which, for particular purposes, might be undesirable. For example, a graduation of the logarithms of Call Money Rates will be such that if a borrower had a loan of constant size throughout a period of some years, his interest charges would be somewhat less if he paid the graduated rates than they would be if he paid the actual rates. In the case of most economic data, it is extremely difficult to be sure that some particular function of the data is overwhelmingly more significant than any other function. We decided that the logarithm was the most significant function for our purposes.

In all but two of the graduations below, data outside the range January 1886 to January 1894 have been used. The data (logarithms) for two years before 1886 and two years after 1894 are:

	1884	1885	1894	1895
J	.279	.076	.009	.130
F	.274	.158	.000	.176
M	.243	.117	.037	.352
A	.322	.130	.053	.352
M	1.176	.158	.041	.121
J	.537	.076	.000	.064
J	.279	.130	.000	.146
A	.243	.176	.000	.013
S	.243	.190	.000	.193
O	.290	.328	.000	.336
N	.158	.430	.013	.294
D	.176	.439	.158	.659

		(2)	(11)	(12)	(13)	(14)
	Data (Logarithms)	2 of a 12 Simple Moving Average	Spencer's 21-term 3rd-degree Parabolic	Kenchington's 27-term 3rd-degree Parabolic	29-term Approximately 3rd-degree Parabolic	29-term Non-Parabolic
1886						
J	.328	.3525	.3682	.3501	.3503	.3467
F	.314	.3846	.3827	.3845	.3878	.3856
M	.423	.4317	.3985	.4220	.4264	.4266
A	.377	.4716	.4220	.4613	.4653	.4672
M	.459	.5006	.4587	.4986	.5029	.5059
J	.525	.5349	.5076	.5350	.5383	.5416
J	.352	.5680	.5650	.5709	.5725	.5748
A	.725	.5901	.6216	.6052	.6059	.6070
S	.771	.6117	.6714	.6381	.6380	.6383
O	.704	.6404	.7078	.6704	.6679	.6691
N	.751	.6680	.7288	.6987	.6950	.6981
D	.940	.6923	.7361	.7217	.7180	.7237
1887						
J	.622	.7196	.7366	.7386	.7357	.7435
F	.551	.7326	.7325	.7480	.7479	.7565
M	.703	.7295	.7288	.7509	.7552	.7626
A	.787	.7236	.7275	.7502	.7574	.7627
M	.710	.7165	.7296	.7448	.7542	.7575
J	.857	.7028	.7315	.7364	.7456	.7485
J	.677	.6908	.7308	.7263	.7308	.7348
A	.712	.6839	.7230	.7119	.7089	.7153
S	.710	.6679	.7075	.6896	.6801	.6885
O	.622	.6416	.6829	.6593	.6451	.6535
N	.663	.6074	.6476	.6187	.6044	.6106
D	.699	.5600	.6006	.5691	.5593	.5614

	Data (Logarithms)	(2) 2 of a 12 Simple Moving Average	(11) Spencer's 21-term 3rd-degree Parabolic	(12) Kenchington's 27-term 3rd-degree Parabolic	(13) 29-term Approximately 3rd Degree Parabolic	(14) 29-term Non-parabolic
1888						
J	.576	.5100	.5441	.5156	.5123	.5097
F	.431	.4667	.4789	.4627	.4659	.4594
M	.439	.4340	.4097	.4148	.4230	.4146
A	.420	.4145	.3459	.3769	.3868	.3784
M	.255	.3953	.2974	.3493	.3594	.3523
J	.176	.3812	.2710	.3316	.3420	.3356
J	.158	.3754	.2700	.3251	.3332	.3274
A	.190	.3702	.2926	.3282	.3324	.3260
S	.449	.3694	.3320	.3380	.3377	.3309
O	.415	.3784	.3791	.3545	.3487	.3416
N	.408	.3918	.4232	.3761	.3659	.3591
D	.616	.4106	.4560	.4005	.3896	.3836
1889						
J	.519	.4395	.4752	.4283	.4195	.4147
F	.364	.4725	.4824	.4593	.4550	.4518
M	.486	.4988	.4831	.4920	.4942	.4927
A	.589	.5295	.4877	.5270	.5337	.5343
M	.407	.5690	.5061	.5628	.5710	.5736
J	.477	.5995	.5418	.5967	.6038	.6080
J	.550	.6268	.5941	.6285	.6313	.6364
A	.589	.6531	.6557	.6579	.6550	.6594
S	.682	.6700	.7164	.6832	.6768	.6793
O	.919	.6778	.7643	.7054	.6982	.6986
N	.853	.6913	.7910	.7250	.7189	.7179
D	.903	.7113	.7935	.7419	.7385	.7382
1890						
J	.886	.7244	.7771	.7555	.7562	.7581
F	.628	.7490	.7505	.7668	.7702	.7754
M	.628	.7750	.7255	.7745	.7789	.7873
A	.633	.7719	.7123	.7767	.7822	.7915
M	.688	.7624	.7172	.7729	.7795	.7873
J	.677	.7536	.7371	.7651	.7717	.7763
J	.663	.7328	.7626	.7549	.7602	.7610
A	1.066	.7135	.7818	.7436	.7446	.7439
S	.829	.6994	.7863	.7308	.7255	.7251
O	.699	.6876	.7707	.7145	.7039	.7054
N	.845	.6809	.7348	.6938	.6814	.6850
D	.699	.6720	.6818	.6687	.6590	.6631

	Data (Logarithms)	(2) 2 of a 12 Simple Moving Average	(11) Spencer's 21-term 3rd-degree Parabolic	(12) Kenchington's 27-term 3rd-degree Parabolic	(13) 29-term Approximately 3rd-degree Parabolic	(14) 29-term Non-parabolic
1891						
J	.591	.6518	.6288	.6390	.6365	.6397
F	.459	.6077	.5758	.6065	.6140	.6146
M	.459	.5696	.5321	.5764	.5913	.5889
A	.519	.5593	.5027	.5516	.5684	.5645
M	.641	.5478	.4894	.5324	.5466	.5430
J	.512	.5297	.4914	.5184	.5266	.5249
J	.342	.5113	.5022	.5083	.5073	.5085
A	.328	.4959	.5148	.4980	.4879	.4909
S	.653	.4828	.5220	.4843	.4669	.4701
O	.628	.4671	.5186	.4638	.4433	.4445
N	.641	.4386	.4987	.4361	.4180	.4166
D	.468	.4040	.4611	.4046	.3938	.3891
1892						
J	.380	.3859	.4077	.3755	.3728	.3656
F	.301	.3824	.3477	.3517	.3566	.3484
M	.301	.3802	.2926	.3363	.3478	.3391
A	.301	.3838	.2578	.3324	.3470	.3392
M	.176	.3919	.2531	.3401	.3546	.3481
J	.146	.4100	.2826	.3579	.3706	.3648
J	.274	.4345	.3426	.3869	.3961	.3899
A	.312	.4511	.4222	.4259	.4327	.4247
S	.616	.4840	.5058	.4752	.4797	.4715
O	.751	.5256	.5820	.5343	.5365	.5303
N	.712	.5576	.6427	.5979	.5992	.5971
D	.833	.6068	.6858	.6591	.6590	.6644
1893						
J	.602	.6659	.7151	.7139	.7092	.7231
F	.477	.7093	.7355	.7547	.7451	.7657
M	.914	.7254	.7516	.7745	.7624	.7864
A	.688	.7081	.7655	.7714	.7593	.7817
M	.556	.6724	.7703	.7454	.7354	.7528
J	.948	.6203	.7553	.6991	.6929	.7037
J	.889	.5635	.7127	.6391	.6349	.6398
A	.740	.5190	.6385	.5691	.5647	.5673
S	.574	.4625	.5355	.4919	.4884	.4907
O	.377	.3995	.4149	.4119	.4099	.4126
N	.230	.3516	.2920	.3302	.3303	.3332
D	.064	.2907	.1825	.2473	.2526	.2529
1894						
J	.009	.2141	.0976	.1681	.1787	.1742

	Data (Logarithms)	(18) 35-term 5th-degree Parabolic	(19) 41-term 5th-degree Parabolic	(20) 43-term 5th-degree Parabolic	(21) 45-term 5th-degree Parabolic
1886					
J	.328	.3459	.3519	.3528	.3530
F	.314	.3854	.3877	.3886	.3887
M	.423	.4277	.4257	.4256	.4258
A	.377	.4687	.4640	.4635	.4635
M	.459	.5060	.5014	.5012	.5011
J	.525	.5399	.5379	.5379	.5378
J	.352	.5712	.5731	.5730	.5732
A	.725	.6021	.6066	.6068	.6069
S	.771	.6338	.6383	.6387	.6388
O	.704	.6659	.6679	.6681	.6683
N	.751	.6957	.6947	.6948	.6949
D	.940	.7210	.7179	.7179	.7178
1887					
J	.622	.7399	.7368	.7365	.7365
F	.551	.7521	.7505	.7504	.7504
M	.703	.7575	.7589	.7591	.7592
A	.787	.7586	.7620	.7624	.7625
M	.710	.7561	.7598	.7599	.7601
J	.857	.7497	.7517	.7518	.7517
J	.677	.7380	.7376	.7372	.7369
A	.712	.7204	.7166	.7155	.7150
S	.710	.6946	.6881	.6863	.6857
O	.622	.6598	.6518	.6501	.6493
N	.663	.6167	.6088	.6073	.6066
D	.699	.5665	.5609	.5598	.5595
1888					
J	.576	.5119	.5106	.5107	.5107
F	.431	.4573	.4617	.4627	.4630
M	.439	.4087	.4172	.4188	.4195
A	.420	.3696	.3801	.3818	.3827
M	.255	.3432	.3518	.3535	.3541
J	.176	.3292	.3331	.3341	.3346
J	.158	.3253	.3235	.3238	.3240
A	.190	.3274	.3222	.3221	.3221
S	.449	.3337	.3283	.3282	.3282
O	.415	.3437	.3414	.3414	.3416
N	.408	.3584	.3612	.3618	.3621
D	.616	.3804	.3876	.3887	.3889

	Data (Logarithms)	(18) 35-term 5th degree Parabolic	(19) 41-term 5th-degree Parabolic	(20) 43-term 5th-degree Parabolic	(21) 45 term 5th-degree Parabolic
1889					
J	.519	.4119	.4202	.4209	.4212
F	.364	.4517	.4573	.4574	.4575
M	.486	.4973	.4967	.4964	.4961
A	.589	.5438	.5364	.5352	.5346
M	.407	.5854	.5736	.5716	.5710
J	.477	.6180	.6067	.6048	.6041
J	.550	.6413	.6348	.6338	.6332
A	.589	.6586	.6589	.6587	.6587
S	.682	.6737	.6803	.6810	.6813
O	.919	.6906	.7002	.7018	.7022
N	.853	.7108	.7199	.7212	.7218
D	.903	.7337	.7388	.7398	.7403
1890					
J	.886	.7561	.7563	.7569	.7570
F	.628	.7757	.7710	.7709	.7707
M	.628	.7882	.7816	.7802	.7800
A	.633	.7918	.7858	.7845	.7840
M	.688	.7872	.7836	.7827	.7821
J	.677	.7767	.7754	.7747	.7745
J	.663	.7616	.7621	.7619	.7620
A	1.066	.7433	.7450	.7452	.7422
S	.829	.7229	.7251	.7253	.7254
O	.699	.7010	.7033	.7036	.7038
N	.845	.6788	.6805	.6811	.6810
D	.699	.6571	.6573	.6576	.6576
1891					
J	.591	.6359	.6340	.6340	.6343
F	.459	.6119	.6107	.6114	.6115
M	.459	.5871	.5885	.5896	.5901
A	.519	.5632	.5681	.5689	.5695
M	.641	.5424	.5491	.5494	.5499
J	.512	.5259	.5306	.5307	.5308
J	.342	.5132	.5121	.5116	.5114
A	.328	.5000	.4925	.4911	.4908
S	.653	.4816	.4708	.4690	.4685
O	.628	.4567	.4467	.4449	.4412
N	.641	.4268	.4208	.4196	.4189
D	.468	.3952	.3951	.3943	.3942

	Data (Logarithms)	(18) 35-term 5th-degree Parabolic	(19) 41-term 5th-degree Parabolic	(20) 43-term 5th-degree Parabolic	(21) 45-term 5th-degree Parabolic
1892					
J	.380	.3674	.3714	.3712	.3710
F	.301	.3471	.3519	.3516	.3518
M	.301	.3364	.3380	.3382	.3386
A	.301	.3338	.3319	.3328	.3334
M	.176	.3387	.3355	.3370	.3379
J	.146	.3502	.3505	.3524	.3538
J	.274	.3692	.3775	.3804	.3819
A	.312	.4001	.4173	.4213	.4229
S	.616	.4484	.4697	.4737	.4752
O	.751	.5141	.5321	.5350	.5362
N	.712	.5912	.5996	.6008	.6013
D	.833	.6701	.6657	.6651	.6645
1893					
J	.602	.7386	.7235	.7207	.7193
F	.477	.7866	.7660	.7614	.7597
M	.914	.8088	.7876	.7825	.7802
A	.688	.8040	.7856	.7810	.7789
M	.556	.7726	.7602	.7568	.7553
J	.948	.7179	.7141	.7125	.7116
J	.889	.6427	.6517	.6516	.6512
A	.740	.5685	.5777	.5782	.5783
S	.574	.4874	.4962	.4969	.4972
O	.377	.4069	.4109	.4114	.4115
N	.230	.3282	.3248	.3246	.3248
D	064	.2491	.2404	.2401	.2404
1894					
J	.009	.1684	.1608	.1615	.1620

	Data (Loga-rithms)	(22) 45-term 5th-degree Parabolic	(23) 57-term Approxi-mately 5th-degree Parabolic	(24) 43-term Approxi-mately 5th-degree Parabolic	(25) Whittaker-Henderson n = 3	(27) Whittaker-Henderson n = 5
1886						
J	.328	.3477	.3491	.3528	.3076	.2743
F	.314	.3890	.3867	.3885	.3428	.3304
M	.423	.4282	.4259	.4254	.3817	.3845
A	.377	.4652	.4652	.4633	.4245	.4365
M	.459	.4994	.5031	.5013	.4711	.4862
J	.525	.5369	.5388	.5380	.5206	.5333
J	.352	.5737	.5728	.5731	.5710	.5774
A	.725	.6084	.6057	.6071	.6194	.6179
S	.771	.6405	.6374	.6391	.6618	.6541
O	.704	.6694	.6676	.6687	.6951	.6854
N	.751	.6947	.6957	.6956	.7181	.7114
D	.940	.7168	.7201	.7192	.7314	.7319
1887						
J	.622	.7354	.7391	.7381	.7374	.7469
F	.551	.7499	.7525	.7524	.7398	.7566
M	.703	.7596	.7605	.7616	.7415	.7611
A	.787	.7637	.7633	.7654	.7432	.7603
M	.710	.7615	.7609	.7634	.7440	.7540
J	.857	.7523	.7539	.7559	.7422	.7420
J	.677	.7357	.7414	.7419	.7358	.7241
A	.712	.7122	.7219	.7205	.7232	.7003
S	.710	.6811	.6944	.6913	.7029	.6708
O	.622	.6438	.6587	.6545	.6736	.6361
N	.663	.6016	.6145	.6104	.6346	.5971
D	.699	.5564	.5638	.5612	.5860	.5551
1888						
J	.576	.5105	.5104	.5101	.5295	.5117
F	.431	.4657	.4579	.4601	.4689	.4689
M	.439	.4245	.4105	.4145	.4094	.4288
A	.420	.3884	.3720	.3763	.3567	.3935
M	.255	.3590	.3443	.3474	.3163	.3649
J	.176	.3370	.3270	.3279	.2928	.3415
J	.158	.3242	.3194	.3177	.2883	.3331
A	.190	.3206	.3199	.3166	.3015	.3308
S	.449	.3263	.3265	.3233	.3279	.3370
O	.415	.3403	.3387	.3371	.3611	.3505
N	.408	.3637	.3573	.3582	.3951	.3700
D	.616	.3925	.3850	.3861	.4255	.3942

	DATA (LOGARITHMS)	(22) 45-TERM 5TH-DEGREE PARABOLIC	(23) 39-TERM APPROXIMATELY 5TH-DEGREE PARABOLIC	(24) 43-TERM APPROXIMATELY 5TH-DEGREE PARABOLIC	(25) WHITTAKER-HENDERSON n = 3	(27) WHITTAKER-HENDERSON n = 5
1889						
J	.519	.4238	.4159	.4194	.4504	.4219
F	.364	.4593	.4551	.4572	.4709	.4522
M	.486	.4947	.4982	.4976	.4902	.4845
A	.589	.5302	.5408	.5374	.5117	.5183
M	.407	.5649	.5798	.5745	.5385	.5530
J	.477	.5982	.6129	.6079	.5726	.5880
J	.550	.6293	.6395	.6365	.6135	.6225
A	.589	.6581	.6604	.6606	.6580	.6555
S	.682	.6842	.6787	.6821	.7010	.6858
O	.919	.7071	.6970	.7024	.7367	.7123
N	.853	.7267	.7163	.7215	.7603	.7341
D	.903	.7435	.7371	.7402	.7702	.7508
1890						
J	.886	.7574	.7571	.7580	.7683	.7625
F	.628	.7687	.7742	.7727	.7596	.7697
M	.628	.7767	.7856	.7826	.7504	.7732
A	.633	.7804	.7906	.7873	.7457	.7737
M	.688	.7790	.7879	.7855	.7476	.7716
J	.677	.7725	.7783	.7771	.7548	.7669
J	.663	.7616	.7635	.7637	.7634	.7593
A	1.066	.7461	.7451	.7463	.7679	.7484
S	.829	.7270	.7243	.7257	.7630	.7339
O	.699	.7051	.7023	.7033	.7460	.7158
N	.845	.6815	.6800	.6804	.7171	.6945
D	.699	.6578	.6569	.6565	.6789	.6707
1891						
J	.591	.6346	.6331	.6327	.6360	.6453
F	.459	.6125	.6094	.6101	.5937	.6193
M	.459	.5917	.5868	.5887	.5566	.5935
A	.519	.5718	.5658	.5684	.5274	.5684
M	.641	.5519	.5472	.5498	.5070	.5442
J	.512	.5314	.5306	.5320	.4956	.5207
J	.342	.5097	.5142	.5137	.4936	.4977
A	.328	.4865	.4967	.4938	.4955	.4749
S	.653	.4625	.4764	.4718	.4962	.4519
O	.628	.4386	.4520	.4473	.4894	.4285
N	.641	.4149	.4240	.4210	.4698	.4051
D	.468	.3923	.3954	.3943	.4339	.3827

	Data (Logarithms)	(22) 45-term 5th-degree Parabolic	(23) 39-term Approximately 5th-degree Parabolic	(24) 43-term Approximately 5th-degree Parabolic	(25) Whittaker-Henderson n = 3	(27) Whittaker-Henderson n = 5
1892						
J	.380	.3714	.3693	.3693	.3903	.3630
F	.301	.3527	.3477	.3476	.3459	.3481
M	.301	.3391	.3332	.3319	.3087	.3401
A	.301	.3334	.3267	.3245	.2861	.3409
M	.176	.3390	.3290	.3271	.2838	.3519
J	.146	.3579	.3409	.3413	.3049	.3737
J	.274	.3898	.3652	.3693	.3485	.4059
A	.312	.4337	.4037	.4112	.4096	.4471
S	.616	.4869	.4565	.4661	.4803	.4950
O	.751	.5460	.5224	.5309	.5517	.5466
N	.712	.6062	.5967	.6013	.6166	.5988
D	.833	.6625	.6711	.6707	.6712	.6486
1893						
J	.602	.7095	.7364	.7309	.7147	.6931
F	.477	.7459	.7848	.7753	.7483	.7296
M	.914	.7652	.8092	.7988	.7727	.7555
A	.688	.7639	.8057	.7977	.7866	.7682
M	.556	.7465	.7761	.7724	.7872	.7652
J	.948	.7080	.7250	.7259	.7703	.7441
J	.889	.6500	.6576	.6618	.7306	.7028
A	.740	.5816	.5791	.5849	.6642	.6397
S	.574	.5005	.4957	.5004	.5698	.5538
O	.377	.4127	.4103	.4117	.4482	.4446
N	.230	.3238	.3240	.3217	.3011	.3120
D	.064	.2383	.2386	.2341	.1299	.1560
1894						
J	.009	.1611	.1567	.1528	−.0647	−.0234