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CHAPTER VII

Desirable Characteristics of Formulas for Graduating Monthly Time Series.

A number of methods of mathematical smoothing have now been described and discussed. Most of these methods were found upon examination to be unsuited to describe such cyclical data as monthly Call Money Rates. However, two methods of graduating were found to give relatively excellent results when applied to such data. One of these methods is to use an approximately fifth-degree parabolic summation formula having simple computation weights. Three examples of such fifth-degree formulas were given—a 39-term formula whose computation weights are 10 and 15, a 43-term formula whose computation weights are 7 and 10, and a 33-term formula whose computation weights are 58 and 100.[1] The other method is to use the Whittaker-Henderson graduation outlined immediately above. For our purposes n would be taken in the Whittaker-Henderson grad-

[1] Third-degree or approximately third-degree parabolic formulas were suggested as desirable if the data series was very short or if the investigator wished to reduce the computation to a minimum. The 27-term formula described on page 28 was presented as the last word in ease of computation.

uation as equal to 5. Considerable preference was expressed for the first of these methods. We did not seriously consider using the Whittaker-Henderson method in spite of the mathematical elegancies inherent in that method. Two drawbacks to the use of the method were emphasized. It does not sufficiently eliminate seasonal fluctuations unless n be taken so large as to smooth the series more than we desired. The computation is not only laborious but its nature is such that mistakes are easily made.

In choosing a method of graduating the monthly series in the interest rate and security price study, two considerations were primary. First, the graduation must be good, that is, the graduated curve must not only be smooth but give a good fit to the data. Second, the computation must be easy, that is, it must not only be simple to understand but take little time to perform. It may be interesting to outline some characteristics which seemed most desirable in any formula to be used for graduating our particular time series.

1. *The graduation must be uninfluenced, or only negligibly influenced by distant observations:*

This requirement excluded any such procedure as harmonic analysis, unless such analysis be applied to successive portions of the curve in some sort of a moving manner. However, no such scheme was considered, as the resulting weight diagrams are not smooth and the computation is extremely laborious.

None of the graduations discussed in this book (with the possible exception of the Rhodes curve) are appreciably influenced by distant observations. Aside from Dr. Rhodes' graduation and the Whittaker-Henderson graduation, no point on any graduation in this book is in the slightest degree influenced by any observation further distant than 22 months.[1]

2. *The graduation must be easy to compute:*

We chose the summation type of computation with only the simplest multiplications. This type of computation is not only easy to perform but easy to understand. It is also extremely easy to check as the work proceeds. The entire elimination of multiplications is not of prime importance when the multiplications are so extremely simple as in the 39-term approximately fifth-degree parabolic formula described on page 71, the 43-term formula described on page 73, or the 33-term formula described on page 68.

[1] This is true even of graduations of data from which seasonal fluctuations have first been eliminated, if we do not consider the elimination of the seasonal fluctuation as a part of the graduation. See Appendix I.

3. *The weight diagram must be as smooth as possible:*

As the graduation was to be computed by means of a summation formula, it could be represented by a "weight diagram." This weight diagram should be as smooth as possible.[1]

4. *The graduation should eliminate 12-months seasonal fluctuations:*

As a summation formula was decided upon, the elimination of monthly seasonal fluctuations was obtained by having, as a possible first computing operation, the taking of a 12-months moving total of the data. All further operations are then to be performed on this 12-months moving total. All the formulas in this book which exactly eliminate 12-months seasonal fluctuations may be so calculated.

Of course a 12-months seasonal fluctuation may sometimes be of such a type that a graduation containing a 6-months or even a 3-months moving average will eliminate most of it. For example, the total dividend payments of American corpora-

[1] In spite of the above statement, the reader must not overemphasize the element of smoothness in the weight diagram. The weight diagram does not need to be superlatively smooth. He must remember that the increase in the smoothness of the graduation which results from using a superlatively smooth weight diagram instead of a merely ordinarily smooth one is negligible. Compare Note 1, page 56.

tions show a pronounced quarterly seasonal. A 3-months moving average will therefore remove a large percentage of the 12-months seasonal.

5. *If applied to successive points on a sine curve whose period is appreciably greater than the period of the seasonal fluctuation (in our case, 12 "points" or months), the graduation should fall as close as possible to the points on the sine curve:*

The exact fitting to sine curves of many periods cannot, of course, be rigidly fulfilled in practice. No formula, which would rigidly fulfill such a requirement, would, when applied to actual non-mathematical data, give a smooth curve. The element of compromise is introduced by the requirement of smoothness.

If any symmetrical set of weights be applied to an indefinitely extended sine curve, the resulting curve will itself be a sine curve, though not necessarily the same sine curve as the sine curve to which the weights have been applied. In certain limiting cases the resulting graduation may be a straight line. For example, the 43-term approximately fifth-degree parabolic summation formula,[1] if fitted to a sine curve whose period is 2, 3, 4, 5, 6, 8 or 12 months, or such a sine curve on which any straight line has been superposed, will necessarily give a straight line.

[1] See page 73.

For the graduation of most time series, it is much more important that the formula be capable of adequately describing various sine curves than capable of adequately describing any particular degree of parabola, even if such parabola be of as low a degree as the second. The insistence upon an absolute fit to any particular order of parabola is a statistical obsession. The 43-term approximately fifth-degree parabolic formula was designed primarily for fitting cyclical and not parabolic data, though it also gives an extremely close approximation to any parabola of a lower degree than the sixth.

Appendix VII, which is entitled "The Results of Applying Nineteen Different Graduation Formulas to Equidistant Points on Indefinitely Extended Sine Series," contains a table showing the percentage of the amplitudes of sine curves of various periods which are preserved by various graduation formulas. This table merits careful study, though the reader must remember that ability to fit sine curves of various periods is not the only characteristic which might be desired in a formula.

The first row in each column of the table in Appendix VII gives the goodness of fit to a 12-months sine curve. If the elimination of seasonal fluctuations is desired the entry in this column should be zero or close to zero. For this particular length of cycle, goodness of fit is *not* desired. For example, Spencer's 21-term formula which eliminates less than 45 per cent of a 12-months sine curve is distinctly not a formula to be used if seasonal elimination is desired.[1]

Smoothness of the resulting graduation is not considered in Appendix VII. Attention has already been drawn to smoothness of the weight diagram as an important factor leading to smoothness in the graduation. Another factor is the number of terms in the formula. Though the 43-term approximately fifth-degree parabolic formula [2] gives a distinctly closer fit to sine curves of different periods than does the 29-term non-parabolic formula.[3] it also gives a distinctly smoother graduation.

After the reader has studied Appendix VII showing the effects of applying various graduation formulas to sine curves, he should examine Appendix VIII. That Appendix gives the results of applying 14 different graduation formulas to the logarithms of 97 consecutive months of Call Money Rates on the New York Stock Exchange. The figures in Appendix VIII would well merit an adequate chart if it could be easily prepared. However, the various curves interweave so much that

[1] Unless the seasonal fluctuation be eliminated before graduation. See Appendix I.

[2] Column 24 of table in Appendix VII.

[3] Column 14 of table in Appendix VII.

it would be almost impossible to draw them all on one chart in such a manner as to show their respective characteristics unless the chart were made much larger than could be reproduced in this book. If the reader is sufficiently interested in the matter he may have a chart drawn showing the data and the various curves in colored inks. Such a chart, however, must be on an extremely large scale. One lying before me at the time I am writing is 20 inches wide and 39 inches long, and yet it is not on a scale large enough to carry all the 14 graduations without muddling the picture. A chart of about this size might be constructed on which the reader could examine any particular half dozen curves in which he is interested.

Chart IV shows the Call Money data, a 12-months moving average, and the 43-term approximately fifth-degree parabolic graduation.[1] Chart VI shows the data and two Whittaker-Henderson graduations, one with n = 3, the other with n = 5. Chart VII shows the data with a Spencer 21-term graduation and a Kenchington 27-term graduation. The reader should remember that the Whittaker-Henderson graduation with n = 3 eliminates less than 69 per cent of a 12-months sine curve seasonal and the Spencer 21-term graduation eliminates less than 45 per cent of such a seasonal. The

[1] The relation of Chart V to Chart IV is explained and discussed on page 25.

CHART IV

CALL MONEY RATES, TWELVE MONTHS MOVING AVERAGE AND 43-TERM CYCLICAL GRADUATION

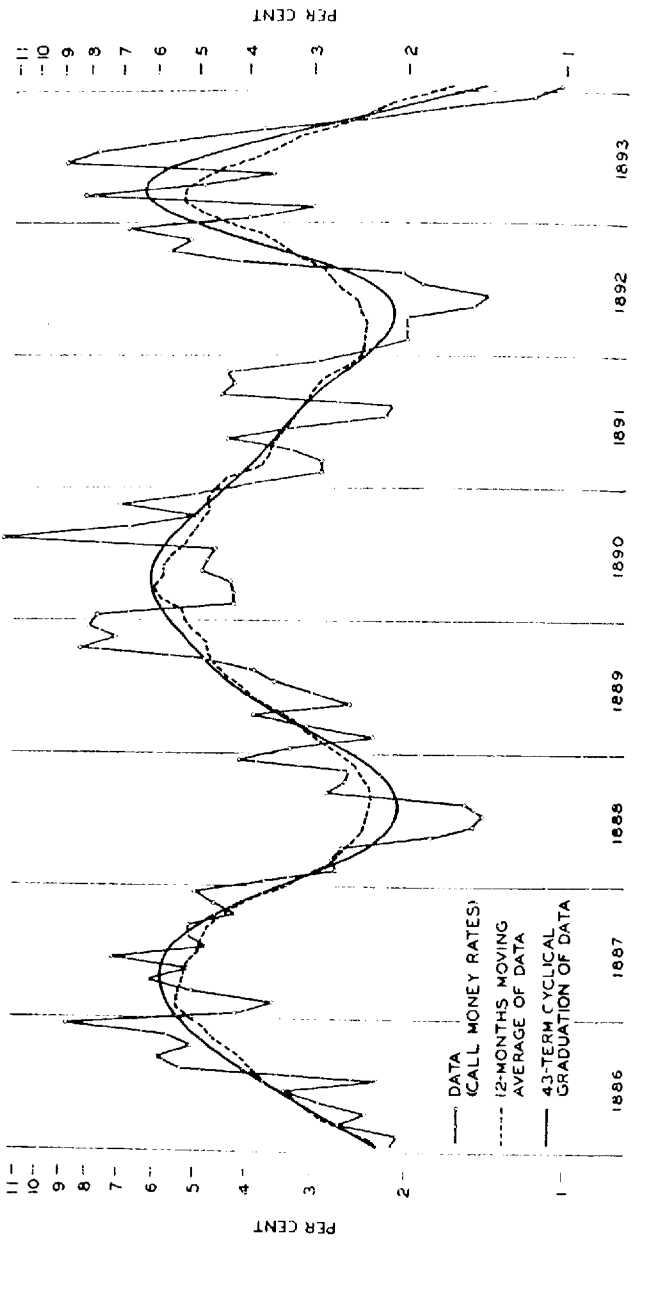

CHART V

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TWELVE MONTHS MOVING AVERAGES OF THE THREE GRADUATIONS IN CHART IV

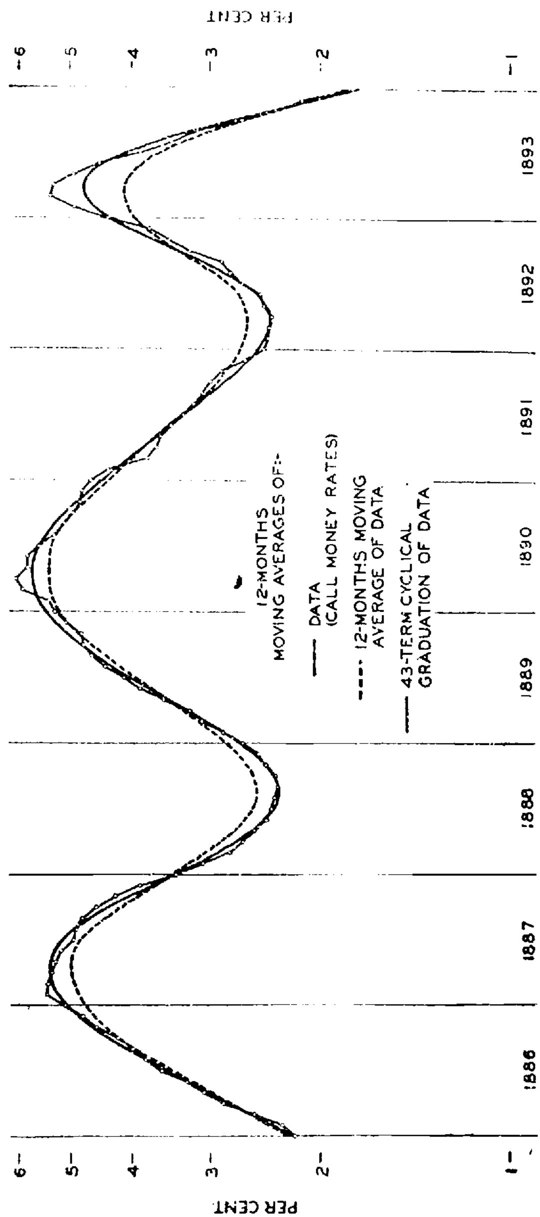


CHART VI

CALL MONEY RATES AND TWO WHITTAKER-HENDERSON GRADUATIONS

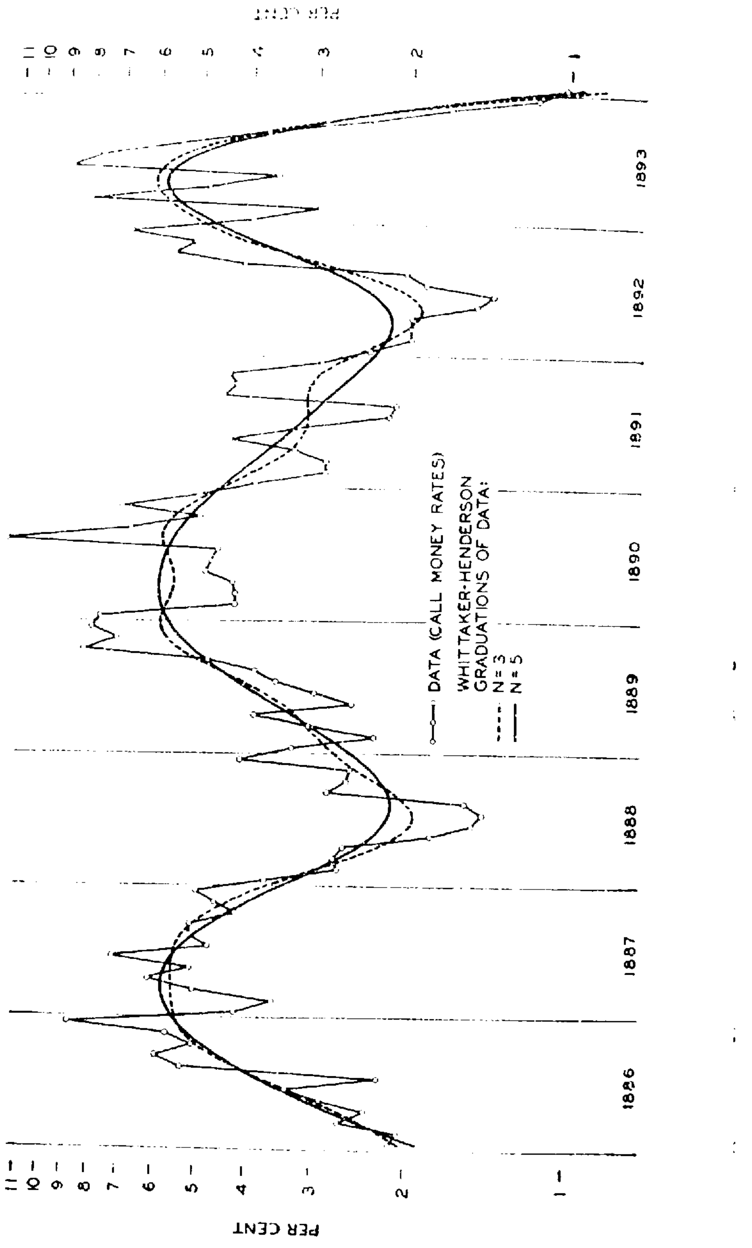
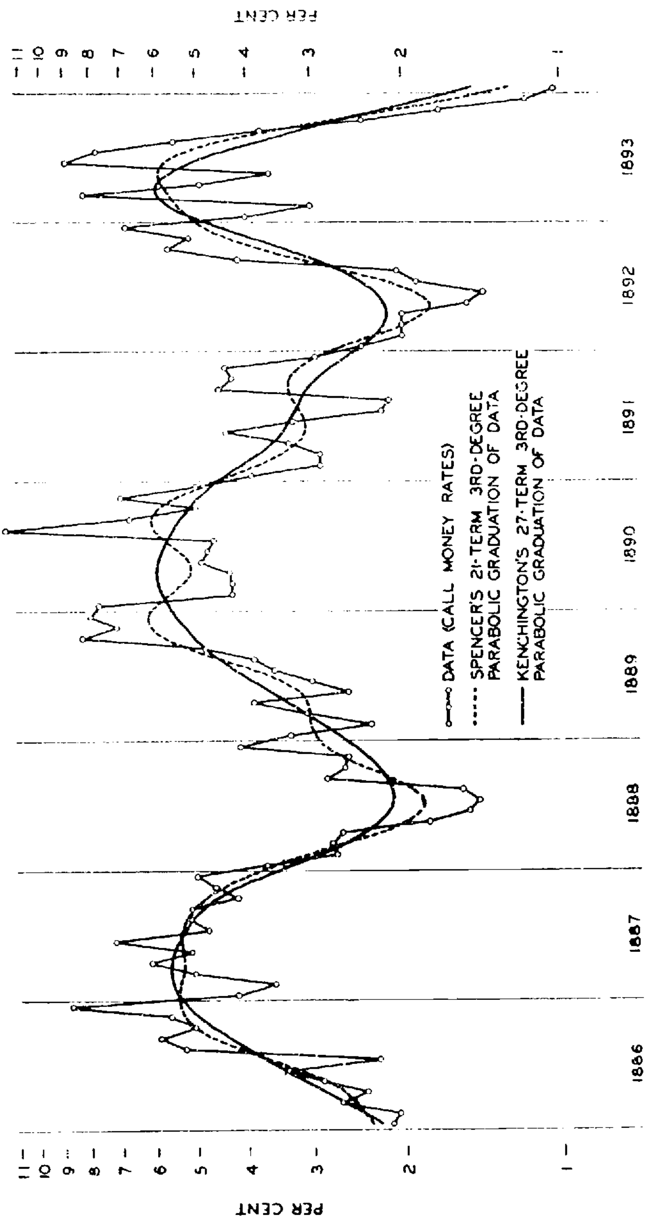


CHART VII

CALL MONEY RATES, KENCHINGTON'S 27-TERM GRADUATION AND SPENCER'S 21-TERM GRADUATION



Kenchington 27-term graduation eliminates over 90 per cent of a 12-months sine curve seasonal and the Whittaker-Henderson graduation, with n = 5, over 95 per cent of such a seasonal.