Volume Title: The Relation of Cost to Output for a Leather Belt Shop

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because of changes in annual rates which were again unrelated to monthly changes in output or other operating conditions.[21]

Unrectified errors

Several elements of cost were left wholly or partly unrectified, even though their magnitudes were influenced by some irrelevant variation.

The small relative importance of the cost of dies and rings and the difficulty involved in rectification justified the omission of any correction for this cost. Fluctuations in the cost of supplies arising from price changes were ignored both because of the minor importance of the cost and because of the labor involved in correcting for the great diversity of products recorded in the supplies account.

The book figures for water, heat, light, and power were also used. The water, heat, and light data did not appear to need correction, and only a small part of the variation in power cost could be considered irrelevant. It might have been desirable to remove the fluctuations in the cost of power caused by changes in temperature and number of hours of daylight in different periods, but the complexity of any suitable corrective device indicated that attempts at rectification would not be worth the trouble.[22]

5 Methods of Analysis

Selection of Technique

Multiple regression analysis seemed most suitable for investigating the relation of the rectified cost to output and the other operating variables.[23] This approach yields measures of: (1) The relation of cost to each independent variable that influences its behavior after the effects of the other variables have been allowed for,[24] a relation displayed in the form

[21] The minimum coverage was so high and the production cycle so short and uniform that changes in inventory arising from changes in the rate of output did not affect the amount of insurance carried.

[22] Since electricity is produced by the company as a joint product with needed heat and steam, the amount of electricity used may not be closely related to changes in output. Allocations to the various plants are based upon engineering estimates which take account not only of the number of lighting units and the rated power consumption of each machine, but also of the utilization of power plant by-products.

[23] The sample was too small for cross tabulation on a multiple basis in order to reflect the influence of various operating conditions; moreover, well defined measures of any existing relations could not be determined. Confluence analysis did not seem necessary, for reasons discussed later; nor were the factors sufficiently numerous or intercorrelated to justify factor analysis.

[24] The precise meaning of the partial correlation coefficient, such as $Y_{12.3}$, should be pointed out. It measures the closeness of the relation between combined cost (X_1) and output (X_2) after allowing for the effects of average weight (X_3). It shows the correlation between cost and output (as measured by the type of function used) excluding the portion due to the co-variation of cost with weight and of output with weight (as measured by the form of the relation used). Thus it measures the correlation between cost and output which is incremental to any correlation between

of a curve or schedule showing cost for each of a series of values of each independent variable. (2) The importance of the combined effect of the several variables upon cost, a measure needed to indicate the degree to which cost behavior has been accounted for. (3) The reliance that can be placed on the derived curve or function as representative of cost behavior, subject to the limitations implicit in time series data, discussed below. Information such as this is of special value if the cost analysis is to be used as a basis for flexible budgets or to determine marginal cost.

Two methods of multiple regression analysis were employed. The graphic method was used for exploratory purposes, because of its economy and flexibility, and also because the net regression curves so determined serve admirably to present the statistical findings.[25] For the final analysis, fitting by the method of least squares was preferable because of its greater objectivity, the wider acceptability of its error formulas, and the fact that the order in which the variables are considered does not affect the results. The preliminary graphic analysis was intended to determine the various causal factors exerting an influence on cost sufficient to justify their inclusion in the least squares regression analysis and to aid in choosing the general character of the function that best represented the net relation. Although the independent variables were subject to error, the least squares curves and error formulas were computed on the usual assumption that the dependent variable alone is subject to error. This treatment can be justified on two grounds. First, the primary objective was to determine functions that enable prediction of cost from the values of the independent variables, rather than to discover the true functional relation or mutual regression function.[26] For this purpose the procedure is valid, despite errors in these independent variables. Second, it seemed probable that the independent variables finally chosen were subject to less error than cost. The cost data were defective because of recording errors and the possibility of improper rectification in several important respects, notably: (1) omission of certain allocated overheads, (2) stabilization of certain elements whose variation was considered irrelevant, (3) removal of dynamic influences by deflation, (4)

each and average weight (so far as the functions used define their actual relations). This procedure for allowing for the influence of another variable is frequently referred to by a verbal short-cut, by saying that the influence of the other variable is 'held constant'. In this paper, at the suggestion of W. L. Crum, this possibly misleading phrase has been replaced by 'allowing for the effects of'.

25 Although the graphic method is useful for preliminary analyses, its reliability is so difficult to assess that it cannot safely be applied in precise analyses. A function fitted by the graphic method may reduce the degrees of freedom but not allow adequately for the reduction in the estimate of the degree of interrelation of the variables. See Wilfred Malenbaum and J. D. Black, 'The Use of the Short-Cut Graphic Method of Multiple Correlation', *Quarterly Journal of Economics*, LII (Nov. 1937); and 'The Short-Cut Graphic Method of Multiple Correlation', a discussion by L. H. Bean, Mordecai Ezekiel, J. D. Black, and Wilfred Malenbaum, *ibid.*, LIV (Feb. 1940), 318-64.

26 To determine the true relation, all variables may be assumed subject to error, and some such technique as confluence analysis used.

reallocation with respect to time periods. The output data, on the other hand, were not subject to fluctuations in price levels, lags in recording, or arbitrary allocation.

Selection of form of cost observations

Preliminary analyses were made first solely for combined cost, i.e., for the aggregate of the various cost elements included in the study. Additional, more detailed analyses covered not only the combined cost function, but also the cost functions for overhead cost, direct cost, and their constituent elements.

Combined cost was analyzed in the form of totals for the accounting period rather than in the form of cost per unit of product. Experimentation with these alternative approaches in previous cost studies has shown that analysis in the form of total rather than average cost yields more convenient and reliable findings.[27] The conversion of cost in total form to average and marginal form, which may be desired for interpretative purposes, is a simple matter. Marginal cost, for example, is the rate of increase in the total function or the slope of the net regression line of total combined cost on output.[28] When the total cost function is linear, marginal cost is simply the coefficient of net regression of total combined cost on output.

Variables causing cost variation

In analyzing the relations of cost to the measurable causal influences, the variables selected for testing were those the management thought might affect cost in some degree. The tests required that the variables have marked independent influences on cost not reflected in other causal forces, i.e., it was necessary for the net regression of cost on the independent variable to be significant. Since certain variables might exhibit a quantitatively significant influence on some cost elements but not on others, this test was applied to various cost elements. Furthermore, the independent variables must account for a significantly large part of the variation in cost.[29]

[27] Several problems are encountered in the statistical analysis of average cost. First, selection of the most suitable specification for the average cost function is more difficult. Second, slight errors in the choice of the function produce magnified errors in the derived marginal cost function. Third, since average cost is a quotient of two variables, each of which is subject to error, the statistical distribution of the quotient may be less likely to conform to the assumptions upon which multiple regression analysis is based.

[28] This involves a special use of the term marginal cost. It is used exclusively hereafter to mean the addition to total cost caused by a unit increment in output equal to one square foot of single-ply equivalent finished belting. An analogous marginal cost could be found for an increment in average weight, average width, or one of the other independent variables, provided that these variables can vary independently of one another.

[29] The coefficients of multiple correlation and multiple determination test this criterion objectively under some circumstances. If the observations are derived from time series, however, there

Each influence selected as relevant was accordingly separately examined in order to ascertain: (1) the reasons for its influence on cost; (2) the best statistical series available for its measurement; (3) its net correlation with cost. The following operating variables were examined to determine their probable effects upon the behavior of cost in the leather belt shop:

1) Output (measured in square feet of single-ply belting)
2) Average weight per square foot of single-ply belting
3) Average width per square foot of single-ply belting
4) Magnitude and direction of change of output from preceding month
5) Percentage of single-ply belting in total output
6) Variability in rate of output within accounting periods
7) Size of manufacturing lot
8) Proportion of special orders
9) Rate of labor turnover

1) OUTPUT

The rate of output could normally be expected to exert a predominant influence on the magnitude of monthly cost because expenses incurred for materials and direct labor, which vary directly with it, are large. Square feet of single-ply equivalent belting was chosen as the measure of output primarily because the cost of operating the leather belt shop was more closely related to area than to weight, dollar value, or the standard cost of output, which were considered as alternative measures.[30]

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may be an element of serial correlation in the successive observations that accounts for part of the high degree of correlation attributed to the influence of the independent variables. A major defect of many economic time series is positive serial correlation due to lack of independence of successive observations and to the effect of common cyclical influences. In this study, the coefficient of serial correlation of the residuals of the multiple regression equation was found to be -0.311. Since in this instance the coefficient is negative and small, serial correlation did not impair the usefulness of the multiple regression coefficients as estimates of the degree of dependence of cost on the independent variable. John H. Smith has suggested that the explanation for the negative serial correlation may lie in compensatory errors of allocation between adjacent accounting periods.

[30] Weight was defective as a measure of output since it did not reflect certain manufacturing operations performed only upon the surface of the leather. It was, therefore, rejected, even though a measure computed in pounds would have yielded results comparable with price data and with cost computations for other departments and companies.

Dollar value of output was subject to irrelevant fluctuations arising from changes in the prices of finished belting and from variations in the proportions of different qualities of output. Variability in quality was not relevant to cost because the manufacturing processes were approximately identical for all qualities of belting, and the effects of differences in the quality of raw material on cost had already been eliminated by the use of a uniform leather price.

In order to use standard cost as a measure of output it would have been necessary to remove the effect of changes in annual manufacturing cost and material prices. Standard cost is fixed at the beginning of each year on the basis of the average material, labor, and overhead cost of the preceding year. Its raw material cost component reflects differences in quality, but does not vary during the year with changes in hide prices. The annual variation in the differentials between

Production of double- and triple-ply belting is the principal source of fortuitous variation in the area measure of output. To take account of it we converted the area of finished belting into equivalent single-ply belting.[31]

The independent effect of square feet of output on cost was tested by graphic correlation analysis for combined cost and for several of its major elements. In each instance a significant net relation was found. Output was therefore used as an independent variable in the final least squares correlation analysis of each aspect of cost.

2) AVERAGE WEIGHT

The average weight of belting output was believed to influence cost because of its clear relation to raw material cost and its effect upon the cost of certain processing operations. Average weight per square foot of single-ply equivalent was therefore tested as an independent variable in order to have a measure reflecting the effect of weight upon cost independently of the influence of output measured in square feet. The strength of the independent relation was examined by graphic multiple correlation and a marked net correlation was found in the case of both total combined and direct cost. Average weight was therefore selected as another independent variable for the mathematical correlation analysis.

3) AVERAGE WIDTH

For certain manufacturing operations, cost per square foot appeared to be affected by the average width of the belt. The influence of width alone was most accurately reflected by the average width of single-ply equivalent belting. Since graphic correlation indicated that this independent influence was of minor consequence, this variable was omitted from the least squares analysis.

4) MAGNITUDE AND DIRECTION OF CHANGE IN OUTPUT FROM PRECEDING MONTH

In order to detect the influence of two types of factor not already removed in the data rectification, the magnitude and direction of change in output from that of the preceding month were tested as independent variables, both separately and in combination. This procedure served as a rough

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the actual cost of various types of product presented such difficulties in the construction of an accurate deflation device, however, that the use of standard cost as an output measure was abandoned.

31 The conversion was performed simply by multiplying double-ply output by 2, and triple-ply output by 3. Although labor and cement cost seemed more than proportionately greater for double-and triple-ply belting, other costs were thought to be less than proportionate, so that the errors tended to compensate. The accuracy of this conversion was tested by introducing the percentage of single-ply belting in total output as an independent variable. No significant net correlation was found to exist between this percentage and cost.

test of the reversibility or continuity of the empirical cost function. Ordinarily static cost functions are assumed to be such that cost is a unique function of output regardless whether the output is attained by a large or small increase or decrease from the preceding period. The observations, however, may not fulfill the conditions assumed, since the cost associated with operating at 60 per cent of capacity after a period of operating at 40 per cent and after a period of operating at 80 per cent may not be the same. A concrete situation, consequently, may fail to conform to the conditions postulated in theory which assumes that adjustments are instantaneous and frictionless.

When the magnitude of the change, regardless of its direction, was used as an independent variable in a graphic correlation analysis, no significant net relation to cost was disclosed. Direction of change was then studied by separate analyses of the cost-output relation for periods of increased and of decreased output, but no noticeable difference was found. A combination of the two influences was then tried by using an independent variable ranging all the way from large increases to large decreases, again without definite indication of a relation. The magnitude of the departure from the preceding month's output was, therefore, excluded from the least squares analysis. In the case of one component, however, a noticeable relation between direction and magnitude of change in output was indicated by the graphic analysis. This variable was accordingly included in the formal regression analysis of overhead cost.

5) PERCENTAGE OF SINGLE-PLY BELTING IN TOTAL OUTPUT

The cost of both finishing and cement is greatly affected by the number of plies. In view of the marked variation from month to month in the proportion of single-ply belting in total output the influence of this factor was considered as an independent variable but was rejected because graphic analysis revealed no significant net correlation, the output measure chosen having adequately reflected the cost changes associated with this variation.

6) VARIABILITY IN RATE OF OUTPUT WITHIN ACCOUNTING PERIODS

Fluctuations in the rate of production were not fully reflected in the output data derived from the records which were kept for four-week accounting periods. These data, therefore, neglect intra-month variation. The same monthly output can be achieved by operating at full capacity for two weeks and then shutting down for two weeks as by operating at half capacity throughout the four weeks. By planning output in advance, however, and scheduling production at an even rate, the management had so reduced intra-month variation that they did not believe it affected cost greatly. A measure of this variation might have been

obtained from the average deviation, standard deviation, or coefficient of variation of daily production, but daily output records for the entire period of analysis were not available. Since a satisfactory measure was unavailable and the effect of this variation was believed to be negligible, no attempt was made to include it in either the graphic or the least squares analysis.

7) SIZE OF MANUFACTURING LOT

The size of the manufacturing lot may markedly affect cost in processes for which the setting-up of machines is expensive. This is especially true when output is composed of diverse products and a different machine set-up is required for each product. Since neither condition existed in the leather belt shop, size of manufacturing lot was rejected as an independent variable.

8) PROPORTION OF SPECIAL ORDERS

Ordinarily special and rush orders cause a certain amount of confusion and inefficiency. It was the opinion of the executives, however, that production in the belt shop is so well scheduled and routed that operations are not significantly disturbed by special orders. Moreover, a suitable measure was difficult to construct because of the lack of data on special orders for the earlier periods. This factor was, therefore, rejected.

9) RATE OF LABOR TURNOVER

Because of the expense involved in the selection and assimilation of new personnel, labor turnover may exert a pronounced influence on cost. Inquiry indicated, however, that in this particular plant the influence of labor turnover had only a negligible influence upon month to month variation in cost. The reasons are first, that the labor turnover rate is approximately constant, and second, that increases in the labor force are achieved mainly by rehiring regular employees previously laid off, rather than by hiring inexperienced workers. This variable, in view of its minor effects, was excluded from consideration.

The conclusion emerges from the preceding discussion of the operating factors affecting cost that only three satisfied the criteria of suitable independent variables for the least squares multiple regression analysis: *output,* measured by square feet of single-ply equivalent belting; *weight,* expressed as average weight per square foot of single-ply equivalent belting; *magnitude and direction of change of output from preceding month,* this last, however, being included solely in the analysis of overhead cost behavior.