the standard neoclassical growth model, with labor-augmenting technical progress and a constant rate of labor force participation, this is also the gross investment necessary to maintain a constant ratio of capital to the effective or augmented labor force and a constant ratio of capital to output. The conventional net investment needed for this purpose we call the growth requirement (Table A.5, line 10). Net MEW investment (line 11) is change in capital stock less the growth requirement.

If NNP is a desirable measure of social income in a stationary economy, sustainable MEW is a natural analogue for a growing economy.[11] Indeed, in the special case of zero population growth and no technological change, sustainable MEW and NNP are identical. NNP, it will be recalled, is the amount of consumption that leaves the capital stock "intact." The reason for keeping *capital* intact in a stationary economy is that the same amount of consumption, in aggregate and per capita, will be available in future years. The reason for keeping the *capital-output ratio* intact in a growing economy is that per capita consumption will grow at the rate of technological progress.

An alternative concept of social income would be sustainable per capita consumption, which will be larger than sustainable MEW when there is technological progress. Per capita consumption can be sustained by technological advance even while the capital-output and capital-labor ratios steadily decline. With a production function that allows factor substitution, today's consumption standard could eventually be produced with a capital-labor ratio asymptotically approaching zero. During this process the marginal productivity of capital would steadily rise. Our proposed measure of social income is more austere and, we believe, more consonant with revealed social preference. We do not observe current generations consuming capital on the grounds that their successors will reap the benefits of technological progress.

A guiding principle for a definition of social income is the following: The social income is that amount of consumption that is consistent with the social valuation of investment at its current opportunity cost in terms of consumption. The social value of giving up an extra dollar of current consumption in favor of capital accumulation is the sum of the resulting increments to future consumption, each discounted by the appropriate social discount rate. When this value exceeds a dollar, investment is less than optimal and consumption should be reduced until

[11] See P. A. Samuelson, "The Evaluation of 'Social Income,'" in F. A. Lutz and D. C. Hague, eds., *The Theory of Capital*, London, Macmillan, 1961.

lowered capital yield and increased social discount rates combine to lower the value of investment to par. Similarly, when the stream of returns from a marginal dollar of investment sums to less than a dollar, current investment is too large and consumption too small. The amount of current consumption at which the marginal social value of investing a dollar at the expense of consumption is precisely a dollar may be regarded as the social income. It follows that the optimal amount of MEW net investment—defined as social income less actual consumption—is zero.

How do sustainable MEW and NNP relate to this principle? Under what conditions will these be the definitions of social income that follow from the valuation principle given above? Sufficient conditions can be presented formally. Let $c(t)$ be consumption per worker at time t and $L(t)$ the size of the work force. We assume that the labor force is a fixed proportion of the population; therefore, $c(t)$ can also be regarded as an index of per capita consumption. The labor force L is growing exponentially at rate n. Labor-augmenting technical progress is occurring at rate γ; so $L(t)e^{\gamma t}$ is the effective labor force, which is growing at rate $g = n + \gamma$. Gross output per worker is $e^{\gamma t}f(k)$, where k is the ratio of capital stock to effective labor force $K/Le^{\gamma t}$ and k' is the rate of change of k. Capital depreciates at the exponential rate δ.

The equation relating consumption, output, capital, and investment at every moment of time is:

$$c(t) = e^{\gamma(t)}\{f[k(t)] - (g + \delta)k(t) - k'(t)\}. \qquad \text{(A.1)}$$

Consider a feasible and efficient consumption plan: a sequence $c(t)$ for $t \geqq 0$, feasible in the sense that it is consistent with (A.1), given the initial capital stock, $k(0)$, efficient in the sense that it would not be possible to increase any $c(t)$ without diminishing some other $c(t)$. We can then ask: What is the increase in per capita consumption at time θ that can be obtained by a unit reduction of per capita consumption at time 0—the present—keeping the rest of the plan unchanged?

Let $r(t) = f'[k(t)] - \delta$, the net marginal productivity of capital at time t. Since the population is growing exponentially at rate n, the rates that transform per capita saving and investment today into per capita consumption in the future are $r(t) - n$; that is, a unit reduction of the rate of per capita consumption at time 0 will yield an increase of per capita consumption at time θ of

$$\exp\left\{\int_0^\theta [r(t) - n]dt\right\}$$

if consumption rates at all other times before and after θ are unchanged.[12]

If the consumption plan corresponds to a neoclassical growth equilibrium, k and r are constants and per capita consumption is growing at rate γ. The marginal trade-off of later for earlier consumption is $e^{(r-n)\theta}$ and depends only on the intervening time θ.

We turn now to the other half of the story, the social valuation of increments of future consumption yielded by current saving. Suppose that society's intertemporal preferences, at any current date designated by 0, can be described by a social welfare function,

$$U = \int_0^\infty u[c(t)]e^{-\rho t}dt,$$

where u is the one-period utility of consumption, and ρ is the constant pure rate of time preference at which utility is discounted. Let the one-period utility function be of the form $A + Bc^{1-\alpha}$ so that marginal utility $u'(c) = (1-\alpha)Bc^{-\alpha}$, where α and $(1-\alpha)B$ are positive. Furthermore, the elasticity of marginal utility with respect to consumption is $u''c/u' = -\alpha$. Holding U constant, the marginal rate of substitution between per capita consumption rates at θ and 0 is

$$\frac{u'[c(0)]}{u'[c(\theta)]} = \left[\frac{c(0)}{c(\theta)}\right]^{-\alpha} e^{\rho\theta}.$$

Thus the slope of any indifference curve between $c(\theta)$ and $c(0)$ is $-e^{-\rho\theta}$ along the 45° ray and $-e^{(\rho+\alpha\gamma)\theta}$ along the ray $c(\theta) = c(0)e^{\gamma\theta}$ (see Figure A.1).

[12] The rate at which incremental saving at time t can increase k, the ratio of capital to effective labor, is $r(t) - g$. Over the interval $(0, \theta)$ continuous reinvestment of the proceeds of incremental saving at time 0 will compound the increase in k to

$$\exp\left\{\int_0^\theta [r(t) - g]dt\right\}.$$

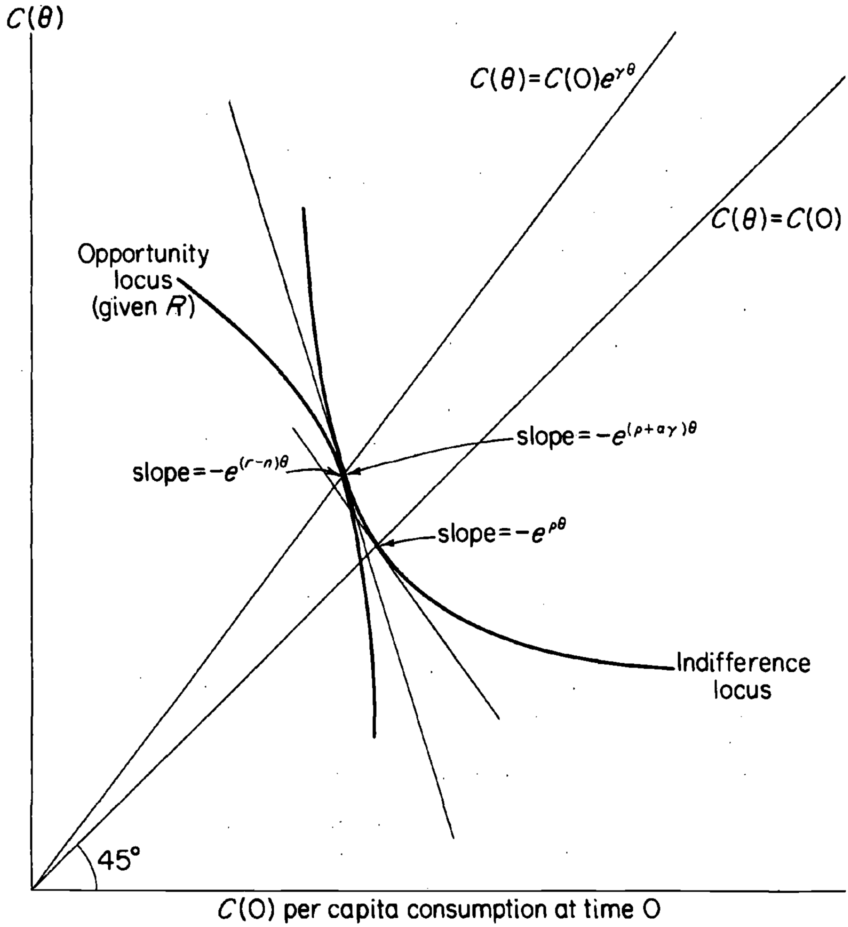
The increase of the aggregate capital stock will then be

$$L(0)e^{g\theta} \cdot \exp\left\{\int_0^\theta [r(t) - g]dt\right\} = L(0)\exp\left[\int_0^\theta r(t)dt\right].$$

This increment can be consumed during a small interval following time θ while leaving subsequent values of $k(t)$ at their original values, so that the initial consumption plan can be executed thereafter. Divided among the population $L(0)e^{n\theta}$ this gives an increment of per capita consumption of

$$\exp\left\{\int_0^\theta [r(t) - n]dt\right\}.$$

FIGURE A.1
Illustration of Balanced Growth as Optimal Consumption Plan
(ρ = *pure rate of social time preference;* α = *—elasticity of marginal utility;* γ = *rate of technological progress*)



Under these assumptions the basic condition that must be met in order that the social valuation of investment equal its cost in current consumption is equality of the two intertemporal substitution rates, the one reflecting production possibilities and the other social preferences. They must be equal for every time interval θ:

$$\left[\frac{c(0)}{c(\theta)}\right]^{-\alpha} e^{\rho\theta} = \exp\left\{\int_0^\theta [r(t) - n]dt\right\}.$$

For a consumption path in equilibrium growth at rate γ, the condition reduces to $e^{(\rho+\alpha\gamma)\theta} = e^{(r-n)\theta}$. This will be true for all θ provided that $\rho + \alpha\gamma = r - n$.

If this condition is met, as illustrated in Figure A.1, the path of sustainable MEW—per capita consumption growing at rate γ—fulfills the basic principle for definition of social income.[13]

In the absence of technological progress and population growth, the condition is simply $\rho = r$. The path of NNP—constant per capita and aggregate consumption—meets the condition that the net marginal productivity of capital equal the pure social rate of time preference.

To summarize, social income is the amount society can consume without shortchanging the future. Thus social income refers to a consumption path along which saving and investment are, according to social valuations of their future yields, just worth their cost in current consumption. Under special conditions this path may be one with per capita consumption growing steadily at the rate of technological progress, and sustainable MEW is then the appropriate measure of social income. In our economy revealed social preference seems to support our inference that the consumption plan is one of ever-growing consumption per capita and our use of social valuations that are consistent with steady growth.

A.3 Imputation for Nonmarket Activities: Time Components of Consumption

Only a fraction of a lifetime is spent in gainful employment, but it is that fraction alone that shows up in output and consumption as ordinarily measured. Leisure and nonmarket work grow steadily in importance, and their omission can bias downward estimates of trends

[13] The result can also be derived by explicitly maximizing U with respect to $k'(t)$, given $k(0)$, using (A.1). The first-order conditions are:

$$\int_t^\infty u'[c(v)]e^{\gamma v}\{f'[k(v)] - (g+\delta)\}e^{-\rho(v-t)}dv = e^{\gamma t}u'[c(t)] \text{ for all } t \geqslant 0.$$

Differentiating this with respect to t gives

$$-u'[c(t)]e^{\gamma t}[r(t) - g]e^{-\rho t} = (\gamma - \rho)e^{(\gamma-\rho)t}u'[c(t)] + e^{(\gamma-\rho)t}u''[c(t)]c'(t).$$

Using $-\alpha = u''c/u'$ we have the general requirement that

$$r(t) - g = \rho - \gamma + \alpha\frac{c'(t)}{c(t)}.$$

An equilibrium growth path will meet this condition if and only if the constant value of k that characterizes it produces a value of r such that $r - n = \rho + \alpha\gamma$.

of per capita consumption. Imputation of the consumption value of leisure and nonmarket work presents severe conceptual and statistical problems. Since the magnitudes are large, differences in resolution of these problems make big differences in overall MEW estimates.

A.3.1 Conceptual Issues. Consider an individual dividing a fixed endowment of time R among gainful employment W, leisure L, and nonmarket productive activity H. From the earnings of his employment he purchases consumption C. Let v_t be the real wage; $v_t p_t^C$, the money wage; and p_t^C, the price of market consumption goods, all for year t. These prices can be observed. Let p_t^L be the price of an hour of the consumption good leisure, and p_t^H the price of an hour's worth of the consumption good produced by home activity. These prices cannot be observed, and this is the source of the problem. Take all base-period prices, v_0, p_0, p_0^C, p_0^L, p_0^H, to be 1.

On the principle that the individual can on the margin exchange leisure or nonmarket activity for market consumption at the money wage $v_t p_t^C$, we can estimate the total money value of his consumption as $v_t p_t^C W_t + v_t p_t^C H_t + v_t p_t^C L_t$. But what did he get for his "money"? The three components of consumption must be "deflated" by the relevant prices p_t^C, p_t^H, p_t^L. This gives an expression for real consumption

$$v_t W_t + \frac{v_t p_t^C}{p_t^H} H_t + \frac{v_t p_t^C}{p_t^L} L_t .$$

Since real consumption at time zero is by definition R, the consumption index is:

$$v_t \frac{W_t}{R} + \frac{v_t p_t^C}{p_t^H} \frac{H_t}{R} + \frac{v_t p_t^C}{p_t^L} \frac{L_t}{R} . \qquad \text{(A.2)}$$

The basic issue is whether the consumption prices of nonmarket uses of time have (a) risen with wage rates, or (b) risen with the prices of market consumption goods. On the first assumption, an hour not sold on the market is still an hour, the same in 1965 as in 1929. The only gains in consumption that can be credited on this account are the reductions in hours of work. On the second assumption, an hour not sold in the market has increased in consumption value the same as an hour worked, namely, by the increase in the real wage.

In our numerical estimates below we have calculated three variants:

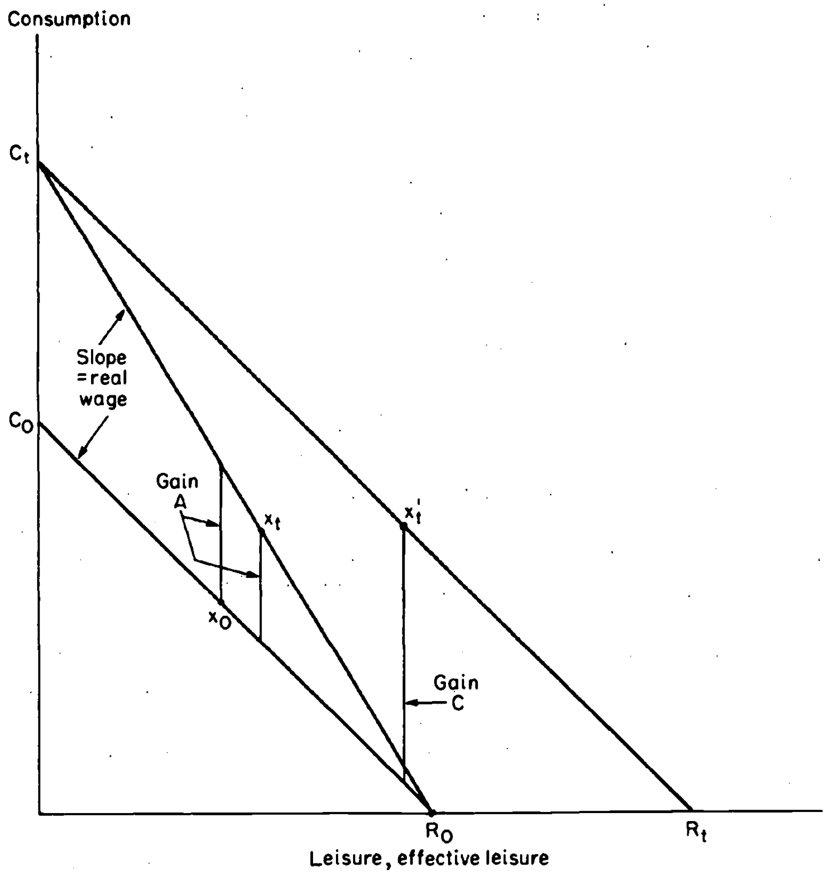
Variant A: $p_t^H = p_t^L = v_t p_t^C$. The index (A.2) is then $1 + (v_t - 1)(W_t/R)$.
Variant B: $p_t^H = p_t^C$; $p_t^L = v_t p_t^C$. The index is $1 + (v_t - 1)[(W_t + H_t)/R]$.
Variant C: $p_t^H = p_t^L = p_t^C$. The index is $1 + (v_t - 1) = v_t$.

Variant A is the most conservative alternative. (C) is the most optimistic alternative.

The essential question is whether nonmarket activities have shared in the technical progress that has raised real wages. If this progress has been time-augmenting, not simply work-augmenting, then the optimistic alternative is correct. But if technology has increased solely the effectiveness of on-the-job work, the pessimistic alternative is correct.

The alternatives can be shown diagrammatically if we confine ourselves to two instead of three uses of time. In Figure A.2, the horizon-

FIGURE A.2
Alternative Interpretations of Welfare Gains Accompanying Wage Increases



tal axis measures leisure; and the vertical axis, market consumption. The line R_0C_0 represents the opportunity locus in the base period; its slope is -1, on the convention that the base-period real wage is scaled at unity. The point x_0 represents the individual's choice. In period t the real wage has increased to v_t, the slope of the new opportunity locus is R_0C_t, and the selected point is x_t. According to the pessimistic interpretation, the gain in welfare, measured in market consumption, is approximated by the vertical difference between the two lines R_0C_0 and R_0C_t, measured either up from x_0 or down from x_t. The individual's time has not increased, and he gains from the higher real wage only in the degree that he works.

The optimistic interpretation is that technological progress has augmented his time, so that in terms of *effective* leisure and consumption the opportunity locus has shifted outward to R_tC_t. The real wage per effective hour is unchanged, although it has increased in terms of natural hours. The point x_t is, in terms of effective leisure, really x_t'. The increment in welfare is approximated by the vertical difference between the parallel lines R_0C_0 and R_tC_t, and is independent of the amount of time the individual works in either period.

Formally, let the individual maximize $U(v_tW_t, h_tH_t, l_tL_t)$,[14] subject to $W_t + H_t + L_t = R$, where h_t and l_t are augmentation indexes for household production and leisure, with $h_0 = l_0 = 1$. The first-order conditions are $v_tU_1 = h_tU_2 = l_tU_3 = \lambda_t$.

If (W_0, H_0, L_0) is the maximizing decision at time zero and (W_t, H_t, L_t) at time t, what is the measure of the change in welfare? The change in utility can be linearly approximated as

$$\begin{aligned} U_1(v_tW_t - W_0) + U_2(h_tH_t - H_0) + U_3(l_tL_t - L_0) \\ = [U_1(W_t - W_0) + U_2(H_t - H_0) + U_3(L_t - L_0)] \\ + [U_1(v_t - 1)W_t + U_2(h_t - 1)H_t + U_3(l_t - 1)L_t]. \end{aligned}$$

The first of the two terms is the substitution effect, which is approximately zero because with $v_0 = h_0 = l_0 = 1$, $U_1 = U_2 = U_3 = \lambda_0$ and $W_t + H_t + L_t = W_0 + H_0 + L_0 = R$. The second term is the income effect, the gain in utility we seek. Dividing by U_1, we convert the utility gain into its equivalent in market consumption:

$$(v_t - 1)W_t + (h_t - 1)H_t + (l_t - 1)L_t.$$

[14] We have assumed that work does not enter directly into the utility function. We do not consider complications that may arise if work is a direct source of satisfaction or pain, nor do we see any way to measure the marginal utility of work.

TABLE A.6
Rise in Three Price Indexes, 1929–65

	Ratio: 1965 to 1929	Average Annual Growth Rate
Consumption deflator	1.97	1.9%
Service deflator	2.06	2.0
Wage index	4.65	4.3

Source: NIP Table 8.1 (see note 6, above) and Table A.11 below.

Expressed as a ratio of base-period consumption R, this gives the results cited above: (A) if $h_t = l_t = 1$, (B) if $h_t = v_t$, $l_t = 1$, (C) if $h_i = l_t = v_t$.

Nonmarket activity. Housework is not directly productive of satisfaction, but rather yields a range of end products (meals, healthy children, gardens, etc.). Given the increase in household equipment and consumer durables, it would be surprising if nonmarket activities did not share in at least part of the advances in technologies that have raised productivity in the market economy.

The proper deflation of housework would be a base-weighted price of the bundle of home-produced services. In the absence of such an index, the closest measure is the deflator for the service component of consumption expenditures in the national accounts. This is compared with the total consumption deflator and the wage index in Table A.6.

It is clear that the price deflators for services and for consumption as a whole moved together over this period, while the wage index rose more than twice as fast. Table A.7 gives the growth of price indexes for important categories of consumption related to housework.

Leisure poses a deeper problem. To the extent that time itself is the final good—daydreaming, lounging, resting—then the conservative interpretation is indicated. But if leisure time is one among several inputs into a consumption process, then it may well have been augmented by technological progress embodied in the complementary inputs—television, boats, cars, sports equipment, etc.

A.3.2 Measurement. We are not aware of any reasonably comprehensive estimates of the use of time over the period 1929–65. Data on the average workweek are available and are used below. Ta-

TABLE A.7
Rise in Prices of Various Household Services, 1929–65

	Ratio: 1965 to 1929	Average Annual Growth Rate
Transportation	2.12	2.1%
Cleaning	2.06	2.0
Domestic service	3.39	3.4
Barbershops	3.03	3.1
Medical care	2.71	2.8
Purchased meals and beverages	2.35	2.4

Source: NIP Table 8.6 (see note 6, above). Note that the index of domestic service is an index of costs rather than a proper price index of output.

ble A.8 gives the results of a large sample survey conducted in 1954. We are doubtful about its reliability, but at present we have no choice but to base our estimates on this survey.

According to this survey leisure time of those surveyed amounted to about 47.6 hours per week for the men and 49.7 hours per week for the women. We will regard personal care and cost of work as instrumental maintenance items and exclude them from consumption. The

TABLE A.8
Use of Time, 1954
(average hours per day, between 6 A.M. *to 11* P.M.)

	Men	Women
Gainful work	6.0	1.5
Cost of work	1.4	0.7
Personal care	0.6	0.9
Housework	2.2	6.7
Leisure	6.8	7.1

Note: Leisure includes time at restaurant, tavern; at friend's or relative's home; in games, sports, church; recreation at home; reading; and sleep during this seventeen-hour period.

Source: A Nationwide Study in Living Habits, cited in Sebastian de Grazia, *Of Time, Work, and Leisure,* New York, Twentieth Century Fund, 1962.

TABLE A.9
Principal Occupation of Population, 14 and Over, Various Years, 1929–65
(*millions of persons*)

	Total Population	Em-ployed	Unem-ployed	Keeping House	School	Other
1929	88.0	47.9	1.5	28.1	6.0	4.5
1935	95.5	42.5	10.6	30.3	6.6	5.5
1945	106.7	64.3	1.0	27.8	4.8	8.8
1947	108.8	59.6	2.4	32.4	6.4	8.0
1954	117.7	64.3	3.6	33.9	6.3	9.6
1958	123.1	66.5	4.7	34.2	7.5	10.2
1965	137.6	74.6	3.5	35.6	11.1	12.8

Source: Economic Report of the President, 1967, Table B-20, for employed and unemployed. Other series from U.S. Department of Commerce, *Statistical Abstract of the United States,* various years; U.S. Department of Commerce, *Historical Statistics of the United States,* various editions. Since series are not always compatible, some adjustments have been made to link them. For 1929 and 1935, the last three columns are estimated from data on female population and employment, school enrollment, and population over 65 years, with the total constrained to equal total population.

important item other than leisure is housework, which takes 46.9 hours a week for women and 15.4 hours per week for men.

Table A.9 makes a breakdown of the population age 14 and over [15] by five time occupations for different years.

Table A.10 estimates the average hours of leisure and nonmarket activity for the five groups of the population described in Table A.9. Table A.11 shows the wage rates applicable to each group.

The general problem of valuation of housework and leisure time was discussed above. In addition, there are some special problems:

Unemployment: In general, time is to be valued at its opportunity cost, the wage rate. Should the unemployed be treated as having zero wage? Clearly this is not the proper treatment for the frictionally or voluntarily unemployed, whose opportunity cost should be close to the market wage rate. On the other hand, during the Great Depression, most unemployed persons could not have obtained work at anywhere near the prevailing wage. Our compromise is to treat unemployment

[15] Why do we exclude children under 14? Because the market value of their time is very low, not because we undervalue the joys of childhood.

TABLE A.10
Hours of Leisure and Nonmarket Work, Persons Over 14, Various Years, 1929–65
(hours per week)

	Employed and Unemployed			Keeping House			School			Other		
	L	NM	Tot	L	NM	Tot	L	NM	Tot	L	NM	Tot
1929	39.4	15.4	54.8	49.7	46.9	96.6	50	13	63	50	10	60
1935	45.5	15.4	60.9	49.7	46.9	96.6	50	13	63	50	10	60
1945	43.1	15.4	58.5	49.7	46.9	96.6	50	13	63	50	10	60
1947	45.7	15.4	61.1	49.7	46.9	96.6	50	13	63	50	10	60
1954	47.6	15.4	63.0	49.7	46.9	96.6	50	13	63	50	10	60
1958	48.6	15.4	64.0	49.7	46.9	96.6	50	13	63	50	10	60
1965	48.1	15.4	63.5	49.7	46.9	96.6	50	13	63	50	10	60

L = leisure hours.
NM = nonmarket hours.
Tot = total hours.
Source: Hours of leisure are obtained by using the benchmark estimates for 1954 and then making estimates using data on average hours worked for other years. Thus the number of leisure hours for any year is obtained by subtracting from 47.6 (the number of hours of leisure for 1954) the difference in hours between the reference year and 1954. Hours data from John W. Kendrick, *Productivity Trends in the United States,* Princeton for NBER, 1961, Table A-X and A-VI. It is assumed that unemployed workers had the same number of hours of leisure and nonmarket work as employed workers. Further, it is assumed that nonmarket activity has stayed the same since 1929. Those keeping house were assumed to have no change from the total number of hours available in 1954 (96.6 per week). Arbitrary numbers were chosen for students and other persons.

as involuntary and thus assign a zero price to the normal working hours of the unemployed. On the other hand, we continue to value their leisure time at the going wage.[16]

Keeping house: The majority of those keeping house are women, and we thus choose the average hourly earnings for women as the proper valuation.

School: Since those in school are primarily under age 20, we use the wage for that age group as the proper valuation of school time.

[16] An alternate imputation is to value *all* time of unemployed workers at zero. For the depression year 1935, this lowers our final estimate of MEW (B variant) by 10 per cent. It makes very little difference for movements over the entire period.

TABLE A.11
Manufacturing Wage Rate and Wage Rate for Different Groups in Population, Various Years, 1929–65
(current dollars per hour)

Year	Employed	Unemployed	Females	Under 20 Years Old	Over 65 Years Old	Wage Index (1958 = 1.00)
1929	0.56	0.56	0.34	0.19	0.49	0.2654
1935	0.54	0.54	0.32	0.18	0.47	0.2559
1945	1.016	1.016	0.61	0.35	0.89	0.4815
1947	1.217	1.217	0.73	0.42	1.06	0.5768
1954	1.78	1.78	1.07	0.61	1.55	0.8436
1958	2.11	2.11	1.27	0.72	1.84	1.000
1965	2.61	2.61	1.57	0.89	2.28	1.237

Source: Basic wage data from *Economic Report of the President* and *Historical Statistics of the United States.* The basic figure is average hourly earnings in manufacturing, which is the only series available back to 1929. (This differs slightly but not appreciably from the ratio of total labor income to Kendrick's man-hour estimate.) Wage rates for females, and for those in the labor force who are under 20 or over 65 years old are calculated as a fraction of the manufacturing wage rate (these numbers being 0.58, 0.34, and 0.81). The data used to calculate the fractions are median incomes of persons who are year-round, full-time workers. Thus the ratio of median incomes of females to males is 4,560/7,814 = 0.58. (Data given in U.S. Department of Commerce, *Current Population Reports, Consumer Income,* Series P-60, No. 66, December 23, 1969, p. 90.)

The wage index is constructed from the data for employed workers with 1958 as the base.

"Other": The final category is "other persons," primarily retired. For this group, we choose the wage rate for persons over 65.

Finally in Table A.12 we calculate the total value of leisure, nonmarket activity, and the sum which we call the "time component" of MEW. Column 1 of Table A.12 gives the current dollar value of the three series. For the reasons given above, two alternative constant-dollar values are calculated for both leisure and nonmarket activity, one using the wage rate as deflator, the other using the consumption price index. Column 2 of Table A.12 shows the result if price deflation is used, while column 3 shows the result of using the wage deflator.

We feel that price deflation is probably superior for nonmarket activity, but that for leisure there is no general presumption. We have, therefore, proceeded with the three variants shown in Table A.12.

TABLE A.12
Value of Leisure and Nonmarket Activity, Various Years, 1929–65
(billions of dollars)

	Current Prices (1)	Deflated by Consumption Deflator (2)	Deflated by Wage Rates (3)
		A. Leisure	
1929	90.1	162.9	339.5
1935	102.7	231.3	401.3
1945	217.0	331.8	450.7
1947	269.3	345.6	466.9
1954	441.4	477.2	523.2
1958	554.9	554.9	554.9
1965	775.5	712.8	626.9
		B. Nonmarket Activity	
1929	47.4	85.7	178.6
1935	48.5	109.2	189.5
1945	99.7	152.4	207.1
1947	124.3	159.6	215.5
1954	195.6	211.5	231.9
1958	239.7	239.7	239.7
1965	321.4	295.4	259.8
		C. Total, Time Component	
1929	137.5	248.6	518.1
1935	151.2	340.5	590.8
1945	316.7	484.2	657.8
1947	393.6	505.2	682.4
1954	637.0	688.7	755.1
1958	794.6	794.6	794.6
1965	1,096.9	1,008.2	886.7

Note: Column 1: For each group, total hours per week times total persons times hourly wage rate times 52, and sumed across all groups. Data are from Tables A.9, A.10, and A.11.

Column 2: Column 1 deflated by consumption deflator.

Column 3: Column 1 deflated by index of wage rate (last column of Table A.11).

TABLE

Preferred County Regression of the Logarithm

(*figures in parentheses*

Area	Constant (α_0)	Log of Population (α_1)	Log of Density (α_2)	Migration Rate (α_3)	Log of % Urban Population (α_4)	Population Negro (α_5)
Mass., R.I.,	7.9 ‡	0.039 †	−0.020 *	0.00045	0.0595 *	−0.0089
Conn.	(17.1)	(1.89)	(0.92)	(0.24)	(0.93)	(−1.0)
New Mexico	2.85 †	0.093 *	−0.087 *	−0.00079	−0.073 †	−0.031 *
	(1.8)	(0.94)	(1.2)	(−0.58)	(1.5)	(1.0)
New York	7.7 ‡	0.010	0.035 ‡	0.0012 ‡	0.035 †	−0.011 ‡
	(15.3)	(0.65)	(2.98)	(0.25)	(1.3)	(2.9)
Wisconsin	7.74 ‡	−0.036 †	0.091 ‡	0.0029 ‡	0.035 ‡	−0.010
	(15.7)	(1.3)	(3.1)	(2.6)	(3.1)	(0.6)
Indiana	7.15 ‡	−0.0014	0.065 ‡	0.0017 ‡	0.0173 †	−0.0072 †
	(22.7)	(0.06)	(2.7)	(2.4)	(1.7)	(1.5)

NA = not available.
* Significant at 75 per cent confidence level.
† Significant at 90 per cent confidence level.
‡ Significant at 99 per cent confidence level.

Variant A: It is assumed that there has been no technological change in the time component, and deflation is therefore by wage rates.

Variant B: This is a hybrid, in which it is assumed that technological change has been occurring at the average rate for nonmarket activity, but that no technological change has taken place in leisure. For this variant, leisure is deflated by the wage index, while nonmarket activity is deflated by the consumption deflator.

Variant C: It is assumed that technological change has been occurring at the average rate for leisure and nonmarket activity, and both are therefore deflated by the consumption deflator.

For most of our discussion below and in the text, our preferred variant is B.

A.13
of Median Income on Selected Variables
are t ratios)

Population over 65 (α_6)	Log of Median Years of Schooling (α_7)	Log of Property Tax per Capita (α_8)	Log of Local Expenditures per Capita (α_9)	Observations	R^2	*F* Test	Mean of Dependent Variable	Standard Error of Estimate	Mean of Median Income per Household
−0.017 (0.021)	0.182 * (0.73)	0.627 ‡ (4.13)	−0.603 ‡ (3.09)	25	.76	5.45	8.72	.061	$6,180
−0.031 * (0.93)	1.86 ‡ (4.21)	0.264 ‡ (1.70)	0.014 (0.035)	22	.91	14.4	8.38	.127	4,614
−0.011 † (1.9)	0.44 ‡ (3.0)	0.17 ‡ (3.6)	−0.22 ‡ (2.9)	62	.85	35.0	8.64	.540	5,761
−0.020 ‡ (2.7)	0.383 ‡ (2.5)	−0.004 (0.061)	0.012 (0.13)	70	.88	49.4	8.46	.074	NA
−0.020 ‡ (4.6)	0.413 ‡ (4.4)	0.114 ‡ (2.2)	−0.038 (0.61)	89	.87	60.6	8.52	.036	NA

A.4 Disamenities and Externalities

In principle those social costs of economic activity that are not internalized as private costs should be subtracted in calculating our measures of economic welfare. The problems of measurement are formidable, and we have been able to do very little toward their solution.

One type of social cost not recorded in the national income accounts is the depletion of per capita stocks of environmental capital. Nonappropriated resources such as water and air are used and valued as if they were free, although reduction in the per capita stocks of these resources diminishes future sustainable consumption. If we had estimates of the value of environmental capital, we could add them to the national wealth estimates of Table A.3 and modify our calculations of MEW net investment accordingly. We have not been able to make this adjustment, but given the size of the other components of wealth, we do not believe it would be significant.

Some unrecorded social costs diminish economic welfare directly rather than through the depletion of environmental capital. The disamenities of urban life come to mind: pollution, litter, congestion, noise, insecurity, buildings and advertisements offensive to taste, etc. Failure to allow for these negative consumption items overstates not only the level but very possibly the growth of consumption. The fraction of the population exposed to these disamenities has increased, and the disamenities themselves may have become worse.

We have attempted to measure indirectly the costs of urbanization. Our measure relies on the assumption that people can still choose residential locations, urban or nonurban, high density or low density. Individuals and families on the margin of locational decisions will, we would expect, require higher incomes to live in densely populated cities than in small towns and rural areas. Urban areas do have higher wage rates and incomes. We interpret this differential as the "disamenity premium" compensating for living in less pleasant surroundings. From the estimated per capita income premium and the locational distribu-

TABLE A.14
Disamenity Estimates

Area	Total Population Effect ($\alpha_1 + \alpha_2$)	Urbanization Effect (α_4)
Massachusetts, Rhode Island, Connecticut	.019	.059
New Mexico	.006	−.073
New York	.045	.035
Wisconsin	.055	.035
Indiana	.064	.017
	Disamenity per Unit Change of Income, 1958 Prices	
	1.75 [a]	3.75 [b]

[a] The coefficient is $1.75 of average household income (1958 prices) per 1 million of population: 1.75 = 0.06 (5,421/180.7) (1.0/1.029), where 5,421 = median family income in the sample states, 180.7 is the population of the United States in millions, 1.0 and 1.029 are consumer deflators for 1958 and 1960, respectively, and 0.06 is the elasticity between income and population change.

[b] The coefficient for urbanization is $3.75 of average household income per percentage point rise in average urbanization: 3.75 = 0.04 (5,421/56.2) (1.0/1.029), where 56.2 is average urbanization, 0.04 is the elasticity between income and the urbanization effect, and all the other figures are as described in note a, above.

TABLE A.15
Corrections for Disamenities of Population and Urbanization, Various Years, 1929–65

	1929	1935	1945	1947	1954	1958	1965
1. Households (no. of mill.)	29.5	32.5	38.9	40.3	46.9	51.0	59.0
2. Disposable personal income per household (1958 prices)	5,105	4,055	5,904	5,409	5,934	6,251	7,389
3. Per cent urbanization	56.2	56.3	58.0	58.6	61.4	62.2	65.1
4. Total population (no. of mill.)	121.8	127.3	140.0	144.1	163.0	174.9	194.6
5. Population density (persons per square mile)	40.3	42.1	46.5	47.9	53.9	57.8	64.4
6. Total correction per household (1958 prices)	425.1	435.1	464.7	474.4	517.2	541.1	586.6
7. Total correction (billions of dollars, 1958 prices)	12.5	14.1	18.1	19.1	24.3	27.6	34.6

Source (for NIP, see note 6, above):

Line
1 *Historical Statistics* and *Statistical Abstract*, various years. Linear interpolation is used to estimate households in noncensus years.
2 Personal disposable income in 1958 prices (NIP Table 2.1) divided by line 1.
3 Same as line 1.
4 *Economic Report of the President, 1968*, Table B-21.
5 Line 4 divided by 3,022,387 square miles.
6 Equals \$1.75 times line 4 plus \$3.75 times line 3.
7 Equals line 6 times line 1.

tion of the population we can compute an aggregate correction and observe its changes over time.

Urban income differentials also reflect, of course, technological productivity advantages. The uncorrected national accounts claim all the gains in productivity associated with urbanization; our correction removes some of them on the ground that they merely offset disamenities. We would not be justified in cancelling out income differentials which are still inducing migration. We have therefore allowed for observed migration and estimated an equilibrium zero-migration differential. We have also attempted to standardize for other factors affecting locational decision besides density and for other sources of income differences.

Our estimates are based on a single cross section, the 1960 census. Consequently we do not know whether the disamenity premium has

TABLE A.16
Measures of Economic Welfare, Actual and Sustainable, Various Years, 1929–65
(billions of dollars, 1958 prices, except lines 14–19, as noted)

	1929	1935	1945	1947	1954	1958	1965
1 Personal consumption, national income and product accounts	139.6	125.5	183.0	206.3	255.7	290.1	397.7
2 Private instrumental expenditures	−10.3	−9.2	−9.2	−10.9	−16.4	−19.9	−30.9
3 Durable goods purchases	−16.7	−11.5	−12.3	−26.2	−35.5	−37.9	−60.9
4 Other household investment	−6.5	−6.3	−9.1	−10.4	−15.3	−19.6	−30.1
5 Services of consumer capital imputation	24.9	17.8	22.1	26.7	37.2	40.8	62.3
6 Imputation for leisure							
B	339.5	401.3	450.7	466.9	523.2	554.9	626.9
A	339.5	401.3	450.7	466.9	523.2	554.9	626.9
C	162.9	231.3	331.8	345.6	477.2	554.9	712.8
7 Imputation for nonmarket activities							
B	85.7	109.2	152.4	159.6	211.5	239.7	295.4
A	178.6	189.5	207.1	215.5	231.9	239.7	259.8
C	85.7	109.2	152.4	159.6	211.5	239.7	295.4
8 Disamenity correction	−12.5	−14.1	−18.1	−19.1	−24.3	−27.6	−34.6
9 Government consumption	0.3	0.3	0.4	0.5	0.5	0.8	1.2
10 Services of government capital imputation	4.8	6.4	8.9	10.0	11.7	14.0	16.6
11 Total consumption = actual MEW							
B	548.8	619.4	768.8	803.4	948.3	1,035.3	1,243.6
A	641.7	699.7	823.5	859.3	968.7	1,035.3	1,208.0
C	372.2	449.4	649.9	682.1	902.3	1,035.3	1,329.5
12 MEW net investment	−5.3	−46.0	−52.5	55.3	13.0	12.5	−2.5
13 Sustainable MEW							
B	543.5	573.4	716.3	858.7	961.3	1,047.8	1,241.1
A	636.4	653.7	771.0	914.6	981.7	1,047.8	1,205.5
C	366.9	403.4	597.4	737.4	915.3	1,047.8	1,327.0
14 Population (no. of mill.)	121.8	127.3	140.5	144.7	163.0	174.9	194.6

(continued)

Table A.16 (concluded)

		1929	1935	1945	1947	1954	1958	1965
	Actual MEW per capita							
15	Dollars							
	B	4,506	4,866	5,472	5,552	5,818	5,919	6,391
	A	5,268	5,496	5,861	5,938	5,943	5,919	6,208
	C	3,056	3,530	4,626	4,714	5,536	5,919	6,832
16	Index (1929 = 100)							
	B	100.0	108.0	121.4	123.2	129.1	131.4	141.8
	A	100.0	104.3	111.3	112.7	112.8	112.4	117.8
	C	100.0	115.5	151.4	154.3	181.2	193.7	223.6
	Sustainable MEW per capita							
17	Dollars							
	B	4,462	4,504	5,098	5,934	5,898	5,991	6,378
	A	5,225	5,135	5,488	6,321	6,023	5,991	6,195
	C	3,012	3,169	4,252	5,096	5,615	5,991	6,819
18	Index (1929 = 100)							
	B	100.0	100.9	114.3	133.0	132.2	134.3	142.9
	A	100.0	98.3	105.0	121.0	115.3	114.7	118.6
	C	100.0	105.2	141.2	169.2	186.4	198.9	226.4
19	Per capita NNP							
	Dollars	1,945	1,205	2,401	2,038	2,305	2,335	2,897
	1929 = 100	100.0	78.0	155.4	131.9	149.2	151.1	187.5

Source (for NIP, see note 6, above):

Line

1 NIP Table 1.2, line 2.
2 NIP Table 2.5, line 52 (personal business), plus one-fifth of line 60 (transportation), deflated by consumption deflator, NIP Table 8.1, line 2.
3 NIP Table 1.1, line 3, deflated by consumption deflator, NIP Table 8.1, line 2.
4 NIP Table 2.5, lines 42 plus 93 less 44, all deflated by consumption deflator, NIP Table 8.1, line 2.
5 Table A.4.
6 Table A.12, part A. Variants B and C from column 3; C, from column 2.
7 Table A.12, part B. Variants B and C from column 2; A, from column 2.
8 Table A.15.
9 Table A.1, line 1.
10 Table A.4.
11 Sum of lines 1–10.
12 Table A.5, line 11.
13 Line 10 plus line 11.
14 *Economic Report of the President, 1971,* Table C-21, p. 221.
15 Line 11 divided by line 14.
17 Line 13 divided by line 14.
19 NNP (NIP Table 1.9) divided by GNP deflator (NIP Table 8.1) times population (Table A.15, line 4).

increased over time. We have simply applied the 1960 premium to population distributions 1929–65.

The unit of observation is the county. It was desired to include sparsely populated areas, and this would not be possible with cities or standard metropolitan statistical areas. The basic data are from the U.S. Department of Commerce *City and County Data Book,* 1960. Regressions were run separately across the counties in each of four states, and in three New England states as a unit. This procedure was followed because we thought that pooling across states and regions would introduce additional sources of variation in locational decision and income choice and obscure the density effects we were seeking to estimate.

The regressions are reported in Table A.13. The dependent variable is the log of median family income for the county. The relevant coefficients are α_1, α_2, and α_4, referring to county population, density, and per cent of county population in urban areas. The other regression variables are included to allow for other sources of income differences. Table A.14 summarizes the regression results for the population variables and shows the values used in the MEW calculations carried out in Table A.15.

The disamenity adjustment is not insubstantial: In 1965 it was about 8 per cent of average family disposable income. If the population were completely urbanized, the adjustment would be about one-third of income. But the correction as a fraction of income has not risen since 1929. Although the population has become more urban and more dense, incomes have grown relative to the disamenity differential.

A.5 Estimates of MEW

We now assemble the components of MEW in Table A.16, which is the same as text Table 1. We also show, in Table A.17, a reconciliation of MEW and GNP. In Table A.18, we show growth rates of NNP and of the three variants of MEW-S. These four series are plotted in Figure A.3.

MEW looks quite different from NNP. It is roughly twice as large. Our preferred variant of MEW-S – variant B, which deflates nonmarket activity by the consumption price index and leisure by the wage rate – has grown somewhat more slowly than NNP: 2.3 per cent per annum compared with 3.0 per cent. The more optimistic variant C has risen faster than NNP. Even the most conservative estimate of MEW-S,

TABLE A.17
Gross National Product and MEW, Various Years, 1929–65
(billions of dollars, 1958 prices)

	1929	1935	1945	1947	1954	1958	1965
1. Gross national product	203.6	169.5	355.2	309.9	407.0	447.3	617.8
2. Capital consumption, NIPA	−20.0	−20.0	−21.9	−18.3	−32.5	−38.9	−54.7
3. Net national product, NIPA	183.6	149.5	333.3	291.6	374.5	408.4	563.1
4. NIPA final output reclassified as regrettables and intermediates							
a. Government	−6.7	−7.4	−146.3	−20.8	−57.8	−56.4	−63.2
b. Private	−10.3	−9.2	−9.2	−10.9	−16.4	−19.9	−30.9
5. Imputations for items not included in NIPA							
a. Leisure	339.5	401.3	450.7	466.9	523.2	554.9	626.9
b. Nonmarket activity	85.7	109.2	152.4	159.6	211.5	239.7	295.4
c. Disamenities	−12.5	−14.1	−18.1	−19.1	−24.3	−27.6	−34.6
d. Services of public and private capital	29.7	24.2	31.0	36.7	48.9	54.8	78.9
6. Additional capital consumption	−19.3	−33.4	−11.7	−50.8	−35.2	−27.3	−92.7
7. Growth requirement	−46.1	−46.7	−65.8	+5.4	−63.1	−78.9	−101.8
8. Sustainable MEW	543.6	573.4	716.3	858.6	961.3	1,047.7	1,241.1

Source (for NIP, see note 6, above):

Line	
1	NIP Table 1.2, line 1.
2	Table A.5, line 6.
3	Line 1 minus line 2.
4a	Table A.1, line 3 plus line 4.
4b	Table A.16, line 2.
5a	Table A.16, line 6.
5b	Table A.16, line 7.
5c	Table A.16, line 8.
5d	Table A.4.
6	Table A.5, line 9 minus line 6.
7	Table A.5, line 10.
8	Sum of lines 3–7; equals Table A.16, line 13.

TABLE A.18
Rates of Growth of NNP and of Sustainable MEW, Various Periods, 1929–65
(average compound growth rate, per cent per year)

	1929–47	1947–65	1929–65
Total			
NNP	2.6	3.6	3.1
MEW variant			
A	2.1	1.5	1.8
B	2.6	2.0	2.3
C	4.0	3.3	3.6
Per capita			
NNP	1.4	2.0	1.7
MEW variant			
A	1.1	−0.1	0.5
B	1.6	0.4	1.0
C	2.3	1.6	2.3
Population	0.96	1.65	1.3

Source: Tables A.16 and A.17.

variant A, shows progress, though only at a rate of 0.5 per cent per year.

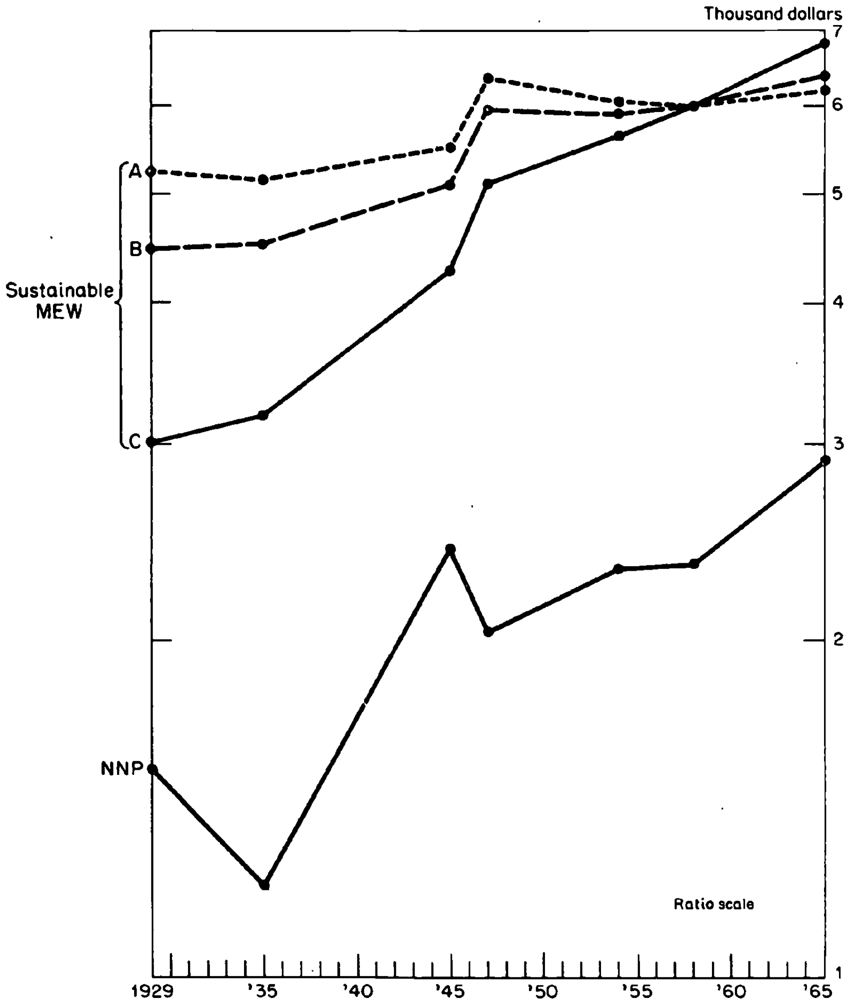
The modifications of the national accounts which make the most difference are the omissions of regrettables and the imputations for leisure and nonmarket work.

The net MEW investment rate was negative before the Second World War and mainly positive since. Since 1945 sustainable MEW has, in the main, exceeded actual MEW (Figure A.4). We have been investing enough to move the economy to a higher consumption path.

A.6 Reliability of the Estimates

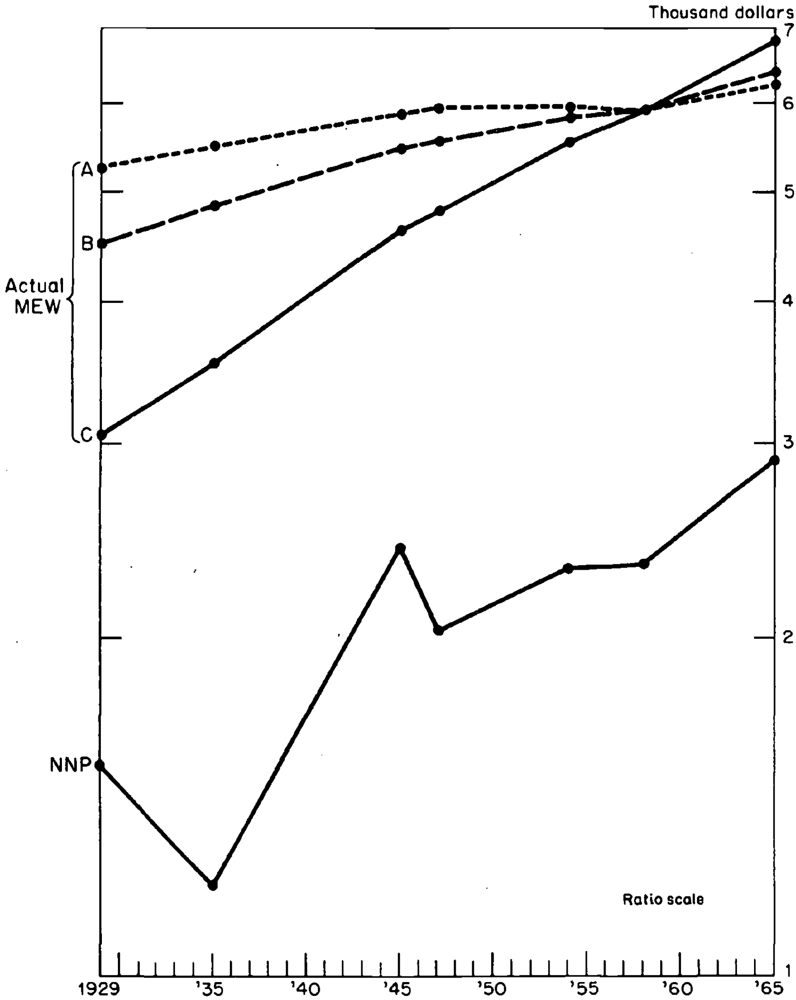
In national accounting, reliability cannot be calculated like statistical sampling error but only judged, for the most part subjectively, by those familiar with the data and the adjustments made in them. We have attempted to estimate very roughly the reliability of our measure of MEW and of its components. These judgments are presented in Table A.19.

FIGURE A.3
Per Capita Net National Product (NNP) and Per Capita Sustainable MEW, 1929–65
(1958 prices)



Source: See Table A.16 and lines 17 and 19.

FIGURE A.4
Per Capita Net National Product (NNP) and Per Capita Actual MEW, 1929–65
(1958 prices)



Source: See Table A.16, lines 15 and 19.

TABLE A.19
Reliability of the Estimates of MEW

Item	Reliability
Consumption expenditures in national accounts	low error
Corrections for	
Instrumental expenditures	medium error
Capital consumption	high error
Growth requirement	high error
MEW net investment	very high error
Imputations for	
Capital services	high error
Leisure	very high error
Nonmarket activity	very high error
Disamenity	very high error
Totals	
GNP	low error
MEW	
Excluding time component	medium error
With time component	high error

Source: Authors' judgment.

We have used as a benchmark the reliability of the gross national product estimates, which we call (for reference) "low error."[17] An item with "medium error" is one with a percentage error we feel to be about twice the percentage error of the GNP. "High error" is about five times the percentage error of GNP. "Very high error" is about ten times the percentage error of GNP.

The sources of unreliability lie both in the data (especially in the case of the time components of MEW) and in the concepts used (such as the proper deflator for leisure or the proper regression for calculating the disamenity premium). There are no independent estimates of comparable totals, as is sometimes the case in the income and product accounts. Totals therefore have all the unreliabilities that their components in combination contribute.

[17] Although no official estimate of the unreliability of GNP exists in the United States, the official estimate in the United Kingdom is that three percentage points either way includes a 90 per cent confidence interval. (See Rita Maurice, ed., *National Accounts Statistics, Sources and Methods,* London, Central Statistical Office, 1969, pp. 42 and 52.)

We must in all candor recognize that in moving away from the conventional accounting framework, we must accept sizable losses in the precision of the estimates.

APPENDIX B: NATURAL RESOURCES

B.1 The Role of Natural Resources in Economic Growth

In this appendix we consider the importance of natural resources in measured economic growth. In comparison with the usual neoclassical growth model, the laws of production are more complex. There are not simply constant returns to scale in capital and labor. The easiest way to view the problem is to assume a constant-returns-to-scale aggregate production function of the form

$$Y = F(A_K K, A_L L, A_R R) \tag{B.1}$$

where Y is output, and K, L, and R are the services from capital, labor, and natural resources, respectively. All technological change is assumed to be factor-augmenting, and A_i is the augmentation level of factor i.

In general, resources might be renewable and augmentable, like capital, or exhaustible, like stocks of minerals. But we shall confine ourselves to the case typified by "land," where the stock is constant —neither augmentable nor destructible—and the services are proportional to the stock.

We can take the logarithmic derivative of (B.1) to obtain:

$$\hat{Y} = \xi_K(\hat{A}_K + \hat{K}) + \xi_L(\hat{A}_L + \hat{L}) + \xi_R(\hat{A}_R + \hat{R}) \tag{B.2}$$

where hats over variables represent proportional rates of growth, and ξ_i is the elasticity of output with respect to factor i.

Since our main interest is the movement of per capita quantities, we define y as per capita income Y/L, k as capital per head K/L, and r as land per head R/L. From (B.2) we derive

$$\hat{y} = \hat{Y} - \hat{L} = \hat{A} + \xi_K(\hat{K} - \hat{L}) + \xi_R(\hat{R} - \hat{L}) + (\xi_K + \xi_R + \xi_L - 1)\hat{L}$$

where $\hat{A} = \hat{A}_K\xi_K + \hat{A}_L\xi_L + \hat{A}_R\xi_R$.

On the assumption of constant returns to scale, the sum of the elasticities of the three factors is unity. If $\hat{L}$ is the constant n, then $\hat{r} = \hat{R} - \hat{L} = -n$, and we have

$$\hat{y} = \hat{A} + \xi_K\hat{k} - \xi_R n, \tag{B.3}$$

and

$$\hat{k} = (sy/k) - n. \tag{B.4}$$

The production function (B.1) can be converted to the intensive form

$$y = A_L f(a_k k,\ a_r r) = A_L F(A_K K/A_L L,\ 1,\ A_R R/A_L L).$$

Balanced growth could occur with constant elasticities ξ_i, constant rates of technical progress, and a constant capital-output ratio k/y. The balanced growth rate is obtained by letting $\hat{k} = \hat{y}$ in (B.3). It is $(\hat{A} - \xi_R n)/(1 - \xi_K)$. The drag due to resource limitation is indicated by the second term in the numerator, as well as by the possibility that ξ_K is smaller than it would be in a two-factor economy.

The share of natural resource owners in national income appears to have fallen over time. This trend is not compatible with balanced growth, and there are several possible interpretations of it. One is the following combination of circumstances: The elasticity of substitution resources for the other two factors taken jointly is greater than 1, and the *effective* quantity of resources per effective worker, $a_r r$, is declining. This implies that the elasticity of output with respect to resources, ξ_R, is falling, and therefore that the drag on growth is progressively diminishing.

A second interpretation is quite the opposite: The elasticity of substitution is less than 1, but effective resources per effective worker are growing, thanks to the speed of resource-augmenting progress.

A third possible mechanism is a shift in demand away from resource-intensive goods, as a result either of income or of price effects. This mechanism cannot be easily described in a one-sector aggregative model. But price-induced shifts of demand are similar in effect to price-induced shifts of factor proportions. A high elasticity of substitution will lower the income shares of resource owners. Inelasticity of demand for resource-intensive products with respect to income growth has the same qualitative effects as rapid land-augmenting progress.

To the central question—How important are natural resources in measured growth?—we seem to get an unambiguous answer: less important than they were. Table B.1, from Denison, indicates that the share of land declined from about 9 per cent to 3 per cent from 1900 to 1950.[18] Denison concludes that while land slowed down the growth rate 0.11 per cent per annum for the period 1909–29, this drag was only

[18] *Sources*, p. 30.

TABLE B.1
Shares of Factors in National Income, Various Periods, 1909–58

				Reproducible Capital Goods					
Period	National Income	Labor	Land	Total	Nonfarm Residential Structures	Other Structures and Equipment	Inventories	U.S. Holdings of Private Assets Abroad	Less: Foreign Holdings of U.S. Private Assets
1909–13	100.0	69.5	8.9	21.6	3.3	13.9	4.6	0.4	.6
1914–18	100.0	67.0	8.8	24.2	3.5	15.3	5.3	0.4	.3
1919–23	100.0	69.5	7.0	23.5	3.4	14.8	4.7	0.8	.2
1924–28	100.0	69.7	6.4	23.9	4.3	14.6	4.3	0.9	.2
1929–33[a]	100.0	69.2	6.2	24.6	4.5	15.3	4.2	1.0	.4
1934–38[a]	100.0	70.4	5.6	24.0	3.6	15.6	4.3	0.8	.3
1939–43[a]	100.0	72.1	4.9	23.0	2.8	15.5	4.3	0.6	.2
1944–48[a]	100.0	74.9	4.0	21.1	2.2	14.6	3.9	0.5	.1
1949–53	100.0	74.5	3.4	22.1	2.5	15.4	3.8	0.5	.1
1954–58	100.0	77.3	3.0	19.7	3.0	13.1	3.0	0.7	.1
1909–58[a]	100.0	71.4	5.8	22.8	3.3	14.9	4.2	0.6	.2
1909–29	100.0	68.9	7.7	23.4	3.7	14.6	4.8	0.6	.3
1929–58[a]	100.0	73.0	4.5	22.5	3.1	15.0	3.9	0.7	.2

Source: Reproduced from Denison, *Sources,* p. 30.

[a] For 1930 through 1940 and 1942 through 1946 these represent interpolated distributions, not the actual distribution for those dates. See text.

0.05 per cent for 1929–57 and would fall slightly more for the next twenty years.[19] In subsequent work, Denison has also examined the extent to which differences in supplies of land and natural resources can account for differences in productivity and growth between the United States and Western European countries. He finds the differences negligible.[20]

A closer look at specific products which are resource-intensive confirms the general suspicion that resources have not been a drag. In a careful study of the relative costs and prices of major categories of

[19] *Ibid.*, p. 270.

[20] See Edward F. Denison, *Why Growth Rates Differ,* Washington, D.C., Brookings, 1967, Chap. 14. The difference ranges between 0.5 and 0.6 per cent of per capita national income.

resource-intensive goods, Barnett and Morse conclude that, with the exception of forestry products, none appears to have become relatively more scarce than goods in general.[21] They examine reasons for this paradox and show that the most important reason is pervasive technological change. Moreover, in those resource-using industries where technology has not come to the rescue of scarcity, substitution of other goods has been significant (substitution away from lead and zinc, from forestry products, from animal power in agriculture).[22]

B.2 Simulations of Three-Factor Production Functions

Our brief review of historical tendencies in resource industries has led us to conclude tentatively that natural resources have not become an increasing drag on economic growth. One possible explanation for this result is that technology allows ample means for substituting away from increasingly scarce natural resources.

In an attempt to make this speculation more concrete, we have studied several three-factor aggregate production functions. Although two-factor (labor-capital) production functions have been widely studied, there does not appear to be comparable work on three-factor (labor-capital-land) functions. Moreover, the only analytical results available are for production functions with constant partial elasticities of substitution between different factors. Consequently, our first step was to examine different functional forms and parameter combinations to see which seemed to exhibit plausible behavior. The final choice between the simulations was on the basis of a comparison of the simulated results with the "revised stylized facts" of growth reviewed above.

B.2.1 Parameters. Four functional forms were tested:

$$Y = [\alpha_1(A_K K)^{-\rho} + \alpha_2(A_L L)^{-\rho} + \alpha_3(A_R R)^{-\rho}]^{-1/\rho} \qquad \text{(PF1)}$$

$$Y = \{\alpha_1[(A_K K)^{1/4}(A_L L)^{3/4}]^{-\rho} + \alpha_2(A_R R)^{-\rho}\}^{-1/\rho} \qquad \text{(PF2)}$$

$$Y = (\alpha_1\{[\beta_1(A_K K)^{1/2} + \beta_2(A_L L)^{1/2}]^{1/2}\}^{-\rho} + \alpha_2(A_R R)^{-\rho})^{-1/\rho} \qquad \text{(PF3)}$$

$$Y = (\alpha_1\{[\beta_1(A_K K)^{-1} + \beta_2(A_L L)^{-1}]^{-1}\}^{-\rho} + \alpha_2(A_R R)^{-\rho})^{-1/\rho} \qquad \text{(PF4)}$$

[21] See Harold J. Barnett and Chandler Morse, *Scarcity and Growth*, Baltimore, Johns Hopkins University Press, 1963, Part III. The other broad sectors were agriculture, extractive industries, and minerals.

[22] "A rough calculation based on Btu's of mineral fuel indicates that if the United States today has to rely upon work animals for its 'horsepower,' the feed would require 15 to 30 times as many acres of cropland as are in use in the country" (*ibid.*, p. 185).

The first one is a general three-factor production function with constant elasticity of substitution (CES). The others are two-stage CES functions, in which production depends on two factors, resources and a capital-labor composite. In PF2 the capital-labor composite is a Cobb-Douglas function of capital and labor, with assumed elasticities of 1/4 for labor and 3/4 for capital. In PF3 and PF4 the composite is itself a CES function of the two "neoclassical" factors, with different elasticities of substitution between them. Unlike PF1, the two-stage functions imply a different partial elasticity of substitution between capital and labor from that between resources and the other two inputs.

In summary, the assumed elasticity between (K, L) and R is the same for all four production functions, namely, $1/(1 + \rho)$. The assumed elasticity between K and L is as follows:

PF1	$1/(1+\rho)$	PF3	2
PF2	1	PF4	1/2

The parameter values tested in simulations were as follows: For ρ, $-9/10$, $-1/2$, $-1/3$, 1. For the rate of labor-augmenting progress, $(g_A)_L$; the rate of capital-augmenting progress, $(g_A)_K$; and the rate of resource-augmenting progress, $(g_A)_R$, the values are 0.0, 0.015, and 0.03.

The numerical specifications were completed with the following parameters: $\alpha_1 = 0.9$; $\alpha_2 = 0.1$; $\beta_1 = 0.25$; $\beta_2 = 0.75$; $s =$ net savings rate $(\Delta K/Y) = 0.1$; $g_L =$ natural growth rate of labor $= 0.01$; $g_R =$ growth rate of resource input $= 0.0$. All values were indexed at 100 at time $t = 0$.

Altogether there were 405 specifications, differing in the form of the function (PF1–PF4) and in the numerical values of their parameters. Each case was simulated for 300 "years." The results were compared with the following stylized facts:

Factor shares are labor 0.73; capital, 0.22; resources, 0.05 (Denison, *Sources*).
Capital growth exceeds output growth by 1 per cent per year.
Output growth is 3.5 per cent per year.
The marginal product of capital (MPK) is constant at 0.15.

Simulations were scored by their conformity to these "facts." Two scoring procedures were used.

The first was based on an arbitrarily weighted sum of squared deviations of simulated results from the facts:

$$(L \text{ share} - 0.73)^2 + (K \text{ share} - 0.22)^2 + 2(R \text{ share} - 0.05)^2 + 3[(g_K - g_Y) - (-0.01)]^2 + 10(g_Y - 0.035)^2 + 0.2(MPK - 0.15)^2. \quad \text{(B.5)}$$

For each simulation, this sum was computed for each period, and its minimum value found. The minimum value was Score I for the simulation. The lower the score, the more acceptable the simulation.

Score II was simply the number of individual criteria met in the year 100 of the simulation, to a maximum of 10 criteria. The criteria were:

(i) $(g_K - g_Y)$ in $[-0.02, 0.005]$ (B.6)
(ii) $(g_{MPL} - g_Y)$ in $[-0.01, 0.01]$
(iii) g_{MPK} in $[0.02, 0.02]$
(iv) g (share of labor) $\geqq 0$
(v) share of labor in $[0.6, 0.8]$
(vi) g (share of K) in $[-0.005, 0.005]$
(vii) share of K in $[0.15, 0.30]$
(viii) (share of R) $\leqq 0$
(ix) share of R in $[0.02, 0.10]$
(x) g_Y in $[0.03, 0.04]$

Conditions (v), (vii), (ix), (i), and (x) in (B.6) are analogous to the first five terms in (B.5) in that order.

B.2.2 Results. The two scoring functions are quite consistent. Score I ranged from 0.001183 to more than 3.0. The 51 lowest scores, ranging from 0.001183 to 0.003998, are analyzed below. None of the 405 cases scored 10 on the second test; ten scored 9. All ten of these cases are among the 51 cited above and listed in Table B.2, below. Other summary compilations appear in Table B.3, below.

Two fairly definite conclusions emerge from these simulations. The elasticity of substitution between resources and the capital-labor composite is greater than 1 in all 51 cases. Secondly, the partial elasticity of substitution between K and L is greater than 1 in the top seven cases, and equal to 1 (Cobb-Douglas) in 35 of the next following cases. Only one out of the 102 substitution elasticities in these 51 cases is less than unity.

The findings relating to the rates of labor- and capital-augmenting technical change are somewhat clouded since in the Cobb-Douglas case factor-augmenting change is indistinguishable from Hicks-neutral

TABLE B.2
Fifty-One Best-scoring Simulations

PF	$\sigma_{(K,L),R} = \frac{1}{1+\rho}$	$\sigma_{K,L}$	$(g_A)_R$	$(g_A)_K$	$(g_A)_L$	Score 1
1	1.5	1.5	0	0	.03	.001183
(1,3)	2	2	0	0	.03	.001250
1	1.5	1.5	.03	0	.03	.001283
3	1.5	2	0	0	.03	.001303
(1,3)	2	2	.015	0	.03	.001325
3	10	2	.015	0	.03	.001344
(1,3)	2	2	.03	0	.03	.001456
2	2	1	.03	0	.03	.001516
1	10	10	0	0	.03	.001531
3	10	2	0	0	.03	.001535
2	1.5	1	.03	0	.03	.001559
2	2	1	0	0	.03	.001634
2	10	1	.03	.03	.015	.001642
2	1.5	1	.015	.03	.015	.001646
2	10	1	.015	.03	.015	.001688
2	10	1	.015	0	.03	.001704
2	1.5	1	0	0	.03	.001719
2	2	1	.015	.03	.015	.001723
2	2	1	.03	.03	.015	.001732
1	10	10	.015	0	.03	.001753
2	2	1	0	.03	.015	.001799
3	1.5	2	.03	0	.03	.001828
2	2	1	.015	0	.03	.001872
2	10	1	0	.03	.015	.001887
2	1.5	1	.03	.015	.03	.001975
2	10	1	0	0	.03	.001994
3	10	2	.03	0	.03	.002125
3	1.5	2	.015	0	.03	.002147
2	10	1	.03	0	.03	.002171
1	1.5	1.5	.015	0	.03	.002208
2	2	1	.03	.015	.03	.002272
2	1.5	1	.015	0	.03	.002285
2	1.5	1	0	.015	.03	.002302
2	1.5	1	0	.03	.015	.002346
2	1.5	1	.015	.015	.015	.002382
1	10	10	.03	0	.03	.002407
2	2	1	.015	.015	.03	.002441
2	1.5	1	.03	.03	.015	.002480

(*continued*)

Table B.2 (concluded)

PF	$\sigma_{(K,L),R} = \frac{1}{1+\rho}$	$\sigma_{K,L}$	$(g_A)_R$	$(g_A)_K$	$(g_A)_L$	Score 1
2	2	1	.015	.015	.015	.002759
2	2	1	0	.015	.03	.002779
2	10	1	.03	.015	.03	.002795
2	1.5	1	.015	.015	.03	.003123
2	1.5	1	.03	.03	.03	.003155
2	10	1	.015	.015	.03	.003288
2	10	1	.015	.015	.015	.003360
2	1.5	1	0	.015	.015	.003462
2	2	1	0	.015	.015	.003588
4	1.5	0.5	.015	0	.015	.003630
2	1.5	1	.015	0	.015	.003634
2	10	1	0	.015	.015	.003883
2	1.5	1	0	.03	.03	.003907

(separable) technical change. There is, however, some reason to favor an estimate of $(g_A)_L$ of 0.03 and of $(g_A)_K$ of 0.0. Of the sixteen cases in Table B.1 which are not Cobb-Douglas, fifteen have $[(g_A)_K, (g_A)_L] = (0, 0.03)$. In 26 of the 35 Cobb-Douglas cases, $(1/4)(g_A)_K + (3/4)(g_A)_L$ was in the range $[2 - (1/8), 2 - (5/8)]$.

No conclusions are possible regarding the growth rate of resource-augmenting change. In all cases effective resources grow less rapidly than effective capital plus effective labor; therefore, with $\sigma_{(K,L),N}$ greater than unity the share of resources declines. If higher rates of g_R had been chosen, this conclusion might have been reversed.

One final note of interest is that the simulations *did* produce a declining capital-output ratio. Since the "apparent" decline of the capital-output ratio has been a puzzle to analysts, it is of some interest to see how this arises in the present model. As is well known, the capital-output ratio in balanced growth is the ratio of the saving rate to the rate of growth of the exogenous factor (usually labor). In a three-factor model, the composite exogenous factor is the combination of labor and resources, weighted by their relative shares. But inputs of resources are growing more slowly than labor inputs, and the share of resources is declining relative to labor's. Therefore, the growth rate of the composite exogenous factor is speeding up over time and the equilibrium capital-output ratio is falling.

B.2.3 The Next Fifty Years? Under the assumption that the models which best correspond to the stylized facts will apply to the future, we can draw inferences about the next few decades. All of the best simulations indicate the same trends; the exact numbers given below are from the best Cobb-Douglas case (PF2), which had $\sigma_{(K,L),R} = 2$, and $[(g_A)_R, (g_A)_K, (g_A)_L] = [0.03, 0, 0.03]$, beginning at year 150.

Briefly, very little changes. The K/Y ratio declines slightly (2.53 to 2.52), while shares of capital and labor increase slightly at the expense of resources (0.237 to 0.240, 0.711 to 0.719, 0.052 to 0.041, respectively). The marginal product of capital rises (0.0936 to 0.0952). The growth rate of output rises slightly (0.0397 to 0.0398), while the rate of change of wages (marginal product per natural worker) approaches 0.03 (up from 0.0296 to 0.0297).

B.3 Production Models Including Natural Resources: Econometric Estimates

The simulations described in the last section are quite optimistic about the effects of natural resources on future growth. They imply that growth will accelerate rather than slow down even as natural resources become more scarce in the future. Since the models used there are only suggestive, it is perhaps useful to check the results with a more formal approach.

One of the best simulations was of the following form, PF2:[23]

$$Y = \{\alpha_1[(A_K K)^{\epsilon}(A_L L)^{1-\epsilon}]^{-\rho} + \alpha_2(A_R R)^{-\rho}\}^{-1/\rho} \tag{B.7}$$

where ϵ was assumed to be 1/4. In this specification, capital and labor are combined with an elasticity of substitution of 1, while the composite capital-labor factor and natural resources are combined with an elasticity of substitution of $1/(1 + \rho)$. Let us designate the composite factor as:

$$N = K^{\epsilon} L^{1-\epsilon} e^{ht} \tag{B.8}$$

where $h = (g_A)_K \epsilon + (g_A)_L(1 - \epsilon)$.

One way to calculate ρ is as follows. The ratio between the shares of the composite factor and natural resources is:

$$z = \frac{\text{share of } N}{\text{share of } R} = \frac{\alpha_1}{\alpha_2}\left(\frac{R}{N}\right)^{\rho} e^{\lambda \rho t} \tag{B.9}$$

where $\lambda = (g_A)_R - h$.

[23] This form won 15 of the top 24 places on Score I.

TABLE B.3
Distribution of Fifty-one Lowest Scores

By Elasticity of Substitution Between Capital and Labor		By Rates of Factor-Augmenting Technical Change			
$\sigma_{(K,L),R}$	No.	Rate	$(g_A)_R$	$(g_A)_K$	$(g_A)_L$
0.5	0	0	17	26	0
1.5	21	.015	19	14	17
2.0	14	.03	15	11	34
10.0	17				

By Production Function			By Combinations of Rates of Technical Change	
Function	$\sigma_{K,L}$	No.	$(g_A)_R$, $(g_A)_K$, $(g_A)_L$	No.
PF1	[a]	9 [b]	(0, 0, .03)	8
PF2	1.0	35	(.015, 0, .03)	8
PF3	2.0	9 [b]	(.03, 0, .03)	8
PF4	0.5	1	All others [c]	27
				51

[a] Same as $\sigma_{(K,L),R}$.
[b] In three cases, PF1 and PF3 are identical.
[c] Fewer than 4 each.

We use data from Denison for both shares and inputs.[24] These are given in Table B.4. The basic estimation is obtained by taking the logarithms of (B.9).

$$\ln z = A + \rho \left[\ln \left(\frac{N}{R} \right) + \lambda t \right] \qquad \text{(B.10)}$$

where A is a constant. N is calculated from (B.8), taking ϵ equal to 0.242 from Table B.1 above.

$$\ln z = \underset{(0.026)}{1.797} - \underset{(.3486)}{.5046} \ln (N/R) - \underset{(.0169)}{.0319} t \qquad R^2 = .9816, \quad SE = .026 \qquad \text{(B.11)}$$

This regression implies an elasticity of substitution between neoclassical factors and resources of about 2 and a value of λ of 0.06. It

[24] One should give the usual caveats about the data. The labor and capital figures are probably good, but Denison assumes that inputs of natural resources are constant due to the domination of land in natural resource inputs. Since the nonland component in resources has certainly been rising, we understate the growth of R, and consequently we probably overstate ρ.

TABLE B.4
Factor Inputs
(1929 = 100;
in each period, resources equal
100.0 on the 1929 base)

	Capital	Labor
1909–13	57.28	67.58
1914–18	65.48	76.10
1919–23	77.00	79.32
1924–28	90.94	92.12
1929–33	101.60	88.74
1934–38	99.44	95.76
1939–43	106.36	132.06
1944–48	114.28	154.14
1949–53	136.92	160.68
1954–58	162.30	174.40

Source: Denison, *Sources*, pp. 85 and 100.

is consistent with the general impression given by the simulation tests —either the elasticity of substitution is high or technological change is relatively resource-saving or both.

APPENDIX C: POPULATION GROWTH AND SUSTAINABLE CONSUMPTION

Equilibrium or Intrinsic Population Growth

A population is in equilibrium when the number of persons of any given age and sex increases at the same percentage rate year after year. This constant rate is the same for all age-sex classes, and therefore for the aggregate size of the population and for the numbers of births and deaths. In equilibrium the relative age-sex composition of the population remains constant.

Such an equilibrium will generally be reached asymptotically if the fertility and mortality structure of the population remains constant. Mortality structure means the vector of death rates by age and sex. Fertility structure means the vector of male and female births as a proportion of the female population of various ages. The equilibrium rate of growth of a population and its equilibrium age distribution will be different for different fertility and mortality structures.

The net reproduction rate, for a given fertility and mortality structure, is the average number of females who will be born to a female baby during her lifetime. For zero population growth (ZPG) this rate must be 1.000. When it is higher, the equilibrium rate of population growth per year will depend also on how early or late in life the average female gives birth.

In the text three equilibrium populations are compared, one corresponding to the 1960 fertility and mortality structure, one to the 1967 structure, and one to an assumed ZPG structure. The 1960 and 1967 structures were obtained from the U.S. Census. The ZPG estimates use the 1967 mortality structure, and a fertility vector obtained by proportionately scaling down the 1967 vector enough to obtain a net reproduction rate of 1.000. Figure C.1 shows the three vectors of birth rates by age of woman: 1960, 1967, ZPG.

The differences in equilibrium age distribution associated with differences in fertility structure are illustrated in Figures C.2, C.3, and C.4. These figures also show actual age distributions for 1960 and 1967. The differences between actual and equilibrium age distributions are, of course, responsible for the considerable discrepancies between actual and equilibrium rates of population growth.

Finally, Figures C.5, C.6, and C.7, show for each of the three structures (a) the hypothetical "projection" which the population would follow if the fertility-mortality structure remained constant, given the initial disequilibrium, and (b) the "constant rate" equilibrium path to which the projected path would converge.

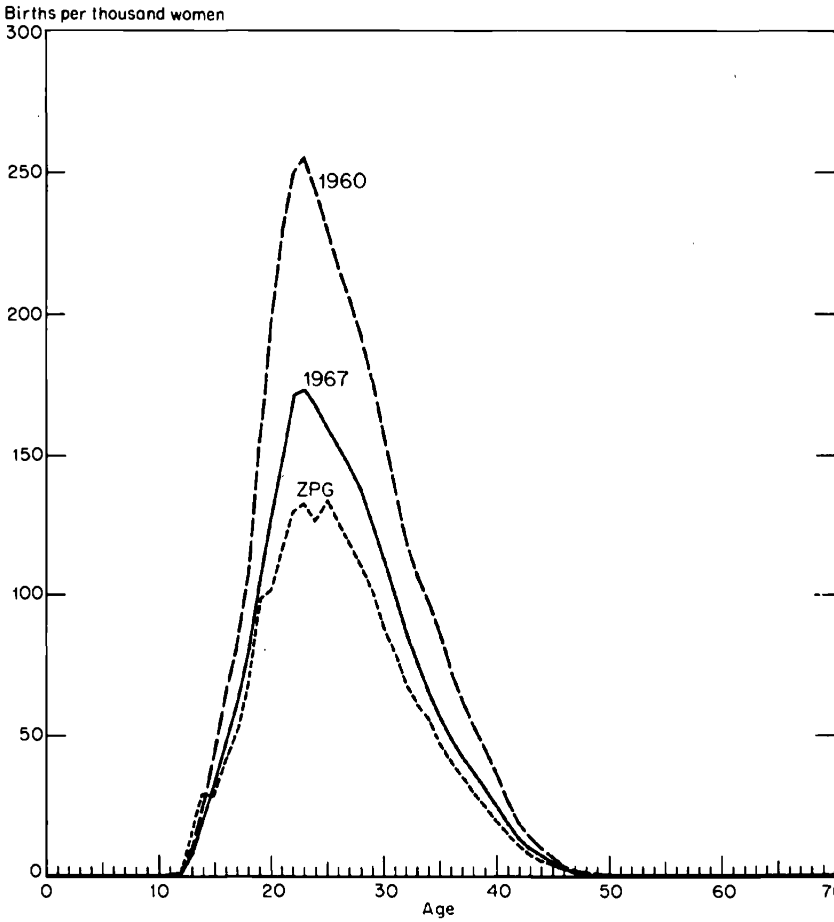
These calculations make no allowance for net immigration, which amounts to 300,000 to 400,000 persons per year under current legislation.

Life Cycle Saving and Aggregate Wealth

As explained in the text, the effect of a change in the equilibrium rate of population growth on sustainable consumption depends in part on the change in the stock of wealth the society desires to hold relative to its income. We have taken the "life cycle" approach to this problem, as described in Tobin's paper "Life Cycle Saving and Balanced Growth."[25]

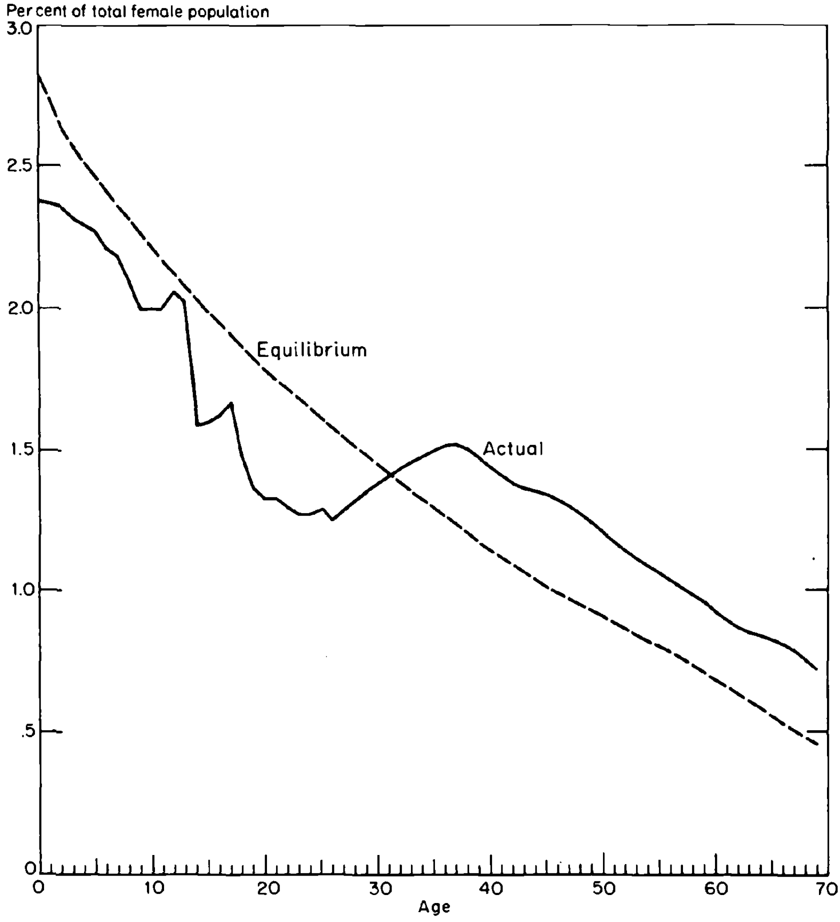
[25] In *Ten Economic Studies in the Tradition of Irving Fisher*, ed. William Fellner, New York, Wiley, 1967, pp. 231–56.

FIGURE C.1
Actual U.S. Birth Rates, 1960 and 1967, and Rates Assuming Zero Population Growth (ZPG)



The population is assumed to be in equilibrium, and the calculations have been made for the three fertility-mortality structures already described: 1960, 1967, ZPG. It is necessary further to group the populations in households. This is done arbitrarily by associating with each female 18 or older: (a) her pro rata share of the living males two years older, and (b) all the surviving children ever born to an average female of her age. Males are children until 20, females until 18; at those ages

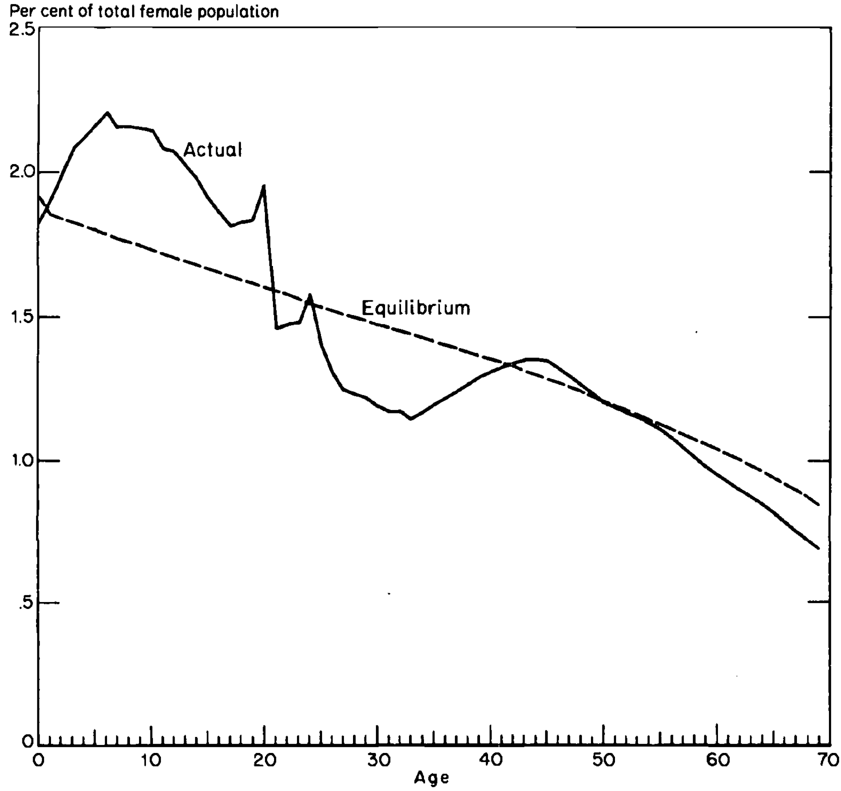
FIGURE C.2
Actual 1960 Age Distribution and Equilibrium Distribution of the U.S. Female Population



they create new households. Over the life of a household its average size varies as births and deaths occur.

The household's income each year is the sum of the incomes of its various members. These vary with age and sex, according to profiles published by the Census Bureau and based on the Current Population Survey. The 1960 profile was used with the 1960 demographic structure, the 1967 profile with the 1967 and ZPG structures. The whole

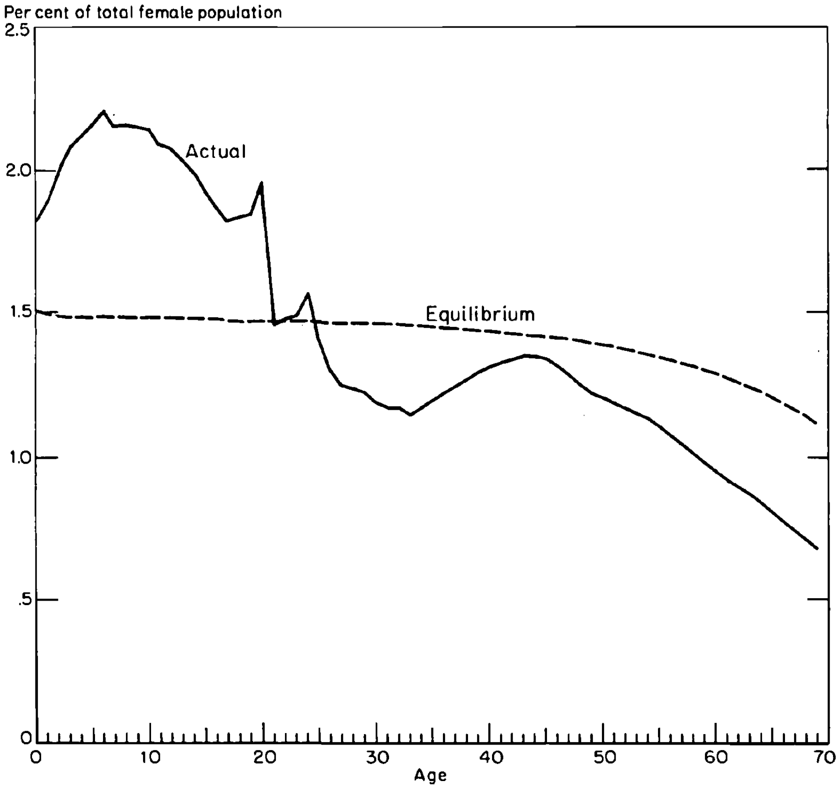
FIGURE C.3
Actual 1967 Age Distribution and Equilibrium Distribution of the U.S. Female Population



profile is assumed to shift upward at 3 per cent per year, the assumed rate of increase of productivity due to labor-augmenting technological progress. Labor inputs of different ages and sexes are assumed to be perfect substitutes, at rates indicated by the profiles.

Each household is assumed to know its future size, n, its labor income, y and the interest rate, r. Over its lifetime the average household consumes all of its income, including interest on any savings accumulated along the way. The household spreads its consumption more evenly than its income, saving in high-income years in order to dissave in low-income years. The utility, u, of consumption at any time is taken to be a function of the consumption, c, per surviving equivalent adult

FIGURE C.4
Actual 1967 Age Distribution and ZPG Equilibrium Distribution of U.S. Female Population

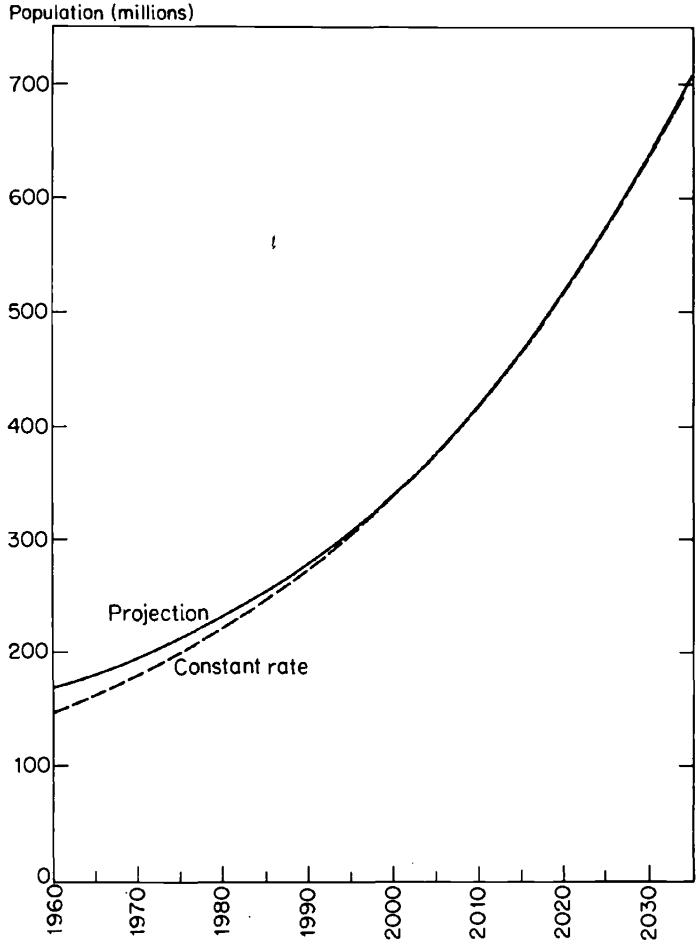


member of the household at that time. The household maximizes over its lifetime the sum of the utilities of this consumption at each age, a, weighted by the expected number of equivalent adult members in the household at that age, $n(a)$, discounted by a subjective rate of time preference, ρ: $\int e^{-\rho a}u[c(a)]n(a)da$, where the limits of integration are from $a = 0$ to $a = A$. This is maximized subject to the budget constraint that expected lifetime income equals expected lifetime consumption:

$$Y = \int e^{-ra}y(a)da = \int e^{-ra}c(a)n(a)da$$

where the integration limits are the same as before and where $y(a)$ is the expected labor income of a household at age a. The calculations

FIGURE C.5
Projected and Equilibrium U.S. Population, 1960 Fertility-Mortality Structure

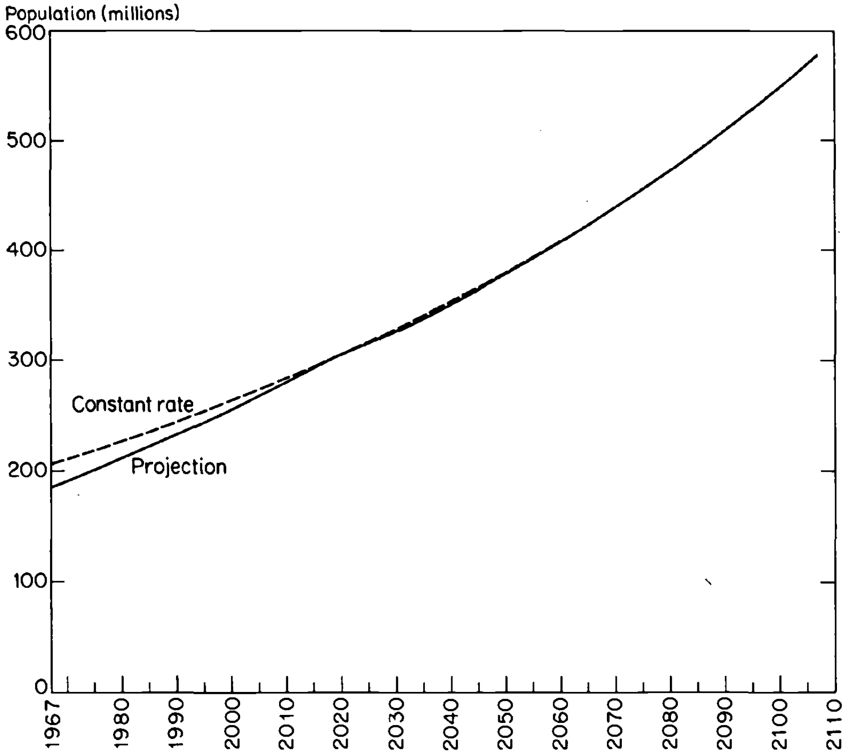


have been made for the specific utility function $u(c) = \ln c$. This leads to the following rule:

$$c(a) = \frac{e^{(r-\rho)a}Y}{\int e^{-\rho a} n(a) da}$$

where the limits of integration are the same as before; Y is the present value, at household age 0, of its expected lifetime labor income; and

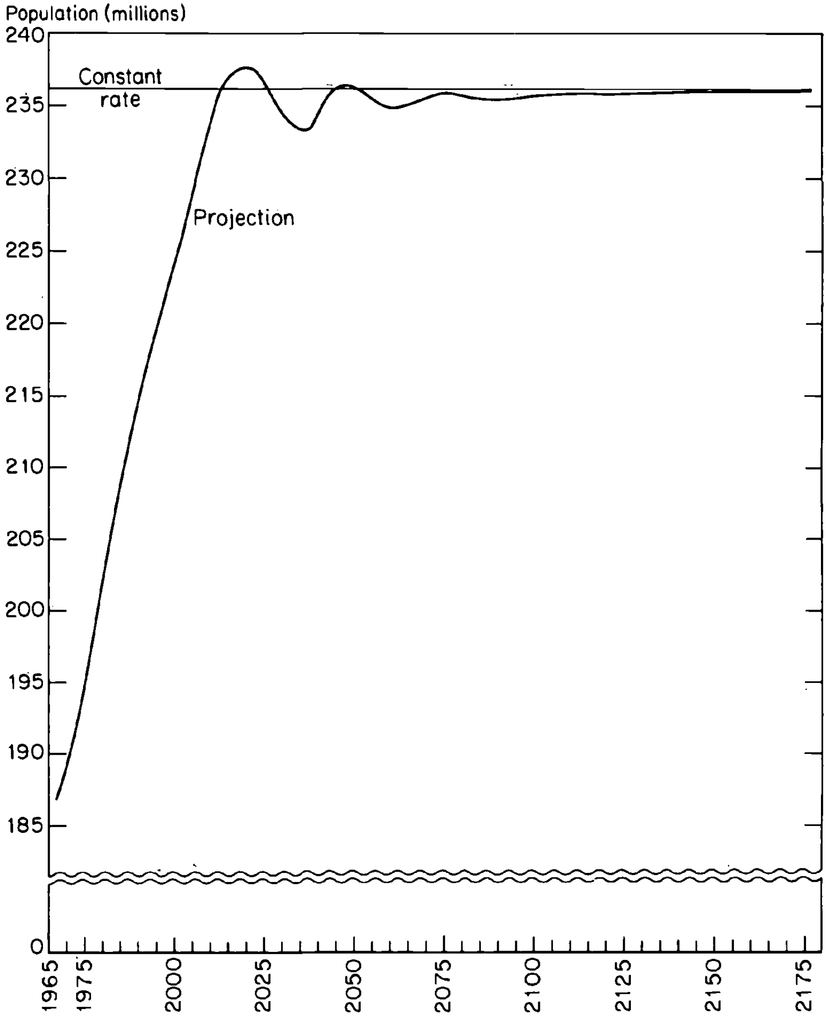
FIGURE C.6
Projected and Equilibrium U.S. Population, 1967 Fertility-Mortality Structure



the denominator is the discounted sum of expected equivalent adult years of household life and consumption. If the market and subjective discount rates were equal, the rule says that lifetime income should be spread evenly in consumption, so that consumption per equivalent adult would be constant. To the extent that r exceeds ρ the household is induced to postpone consumption until later in life.

As this exposition makes clear, the household's consumption pattern depends on (a) the way in which its members are counted—the equivalent adult scale, and (b) the subjective discount rate. Calculations have been made for various equivalent adult scales, ranging from counting teenagers and other children as full members to counting them not at all. In one case the parents are diminishing their old-age consumption in order to increase household consumption during the

FIGURE C.7
Projected and Equilibrium U.S. Population for ZPG Fertility-Mortality Structure



years children are at home; in the other case they are not. Likewise a number of values of subjective discount rate have been assumed. Some of the combinations are shown in text Tables 2–4. For the present purpose, which is to exhibit the effects of changes in the fertility-mortality structure, the assumed equivalent adult scales and subjective discount rate matter very little. They would matter if they were thought to vary systematically with the rate of population growth, but there is no reason to expect that.

On the other hand, the response of consumption patterns to market interest rates does matter. It is this response that makes the aggregate wealth-income ratio respond to market interest rates, as illustrated in the upward sloping curves of text Figure 1.

Household consumption planning is assumed to be actuarial. A given cohort of households breaks even over its lifetime. Some households last longer than average, and some dwindle away sooner. Life insurance and annuities enable the excess consumption of some members of a cohort to be met by the excess saving of other members.

Similarly, households are assumed to be able to borrow, as well as lend, at will at the prevailing interest rate, so long as they have expected future labor income to borrow against. This assumption of a perfect capital market has less effect than might have been supposed, because in most cases households have few or no years of negative net worth.

Given the consumption plan of an average household, it is possible to compute at any time the number, the net worth positions, and the income of households of every age. From this the aggregate wealth-income ratio can be computed. Along a path of equilibrium population and economic growth this ratio will be a constant, dependent on the characteristics of the path but unchanging over time. The reasons that it is a constant of this kind are essentially that (a) the lifetime propensity to consume equals unity regardless of the absolute size of income, and (b) all the demographic and economic variables that determine the pattern of consumption of a household over its lifetime, and the age distribution of households and their members, are constant along an equilibrium path.

As indicated in text Tables 4–6, the key economic variable, the interest rate, is identified with the net marginal productivity of capital and depends on the capital-output ratio. Here we have also made the capital-output ratio and the wealth-income ratio identical. This would not be the case if we allowed for accumulation of wealth in forms other than capital.[26] Then the two ratios would differ, but our conclusions about the effects of population growth would not be affected so long as

[26] See James Tobin, "Money and Economic Growth," *Econometrica,* October 1965, pp. 671–84; and Tobin, "Notes on Optimal Monetary Growth," *Journal of Political Economy,* August 1968, pp. 833–59.

the monetary-fiscal policies that determine the difference remained the same.

How does the fertility-mortality structure affect the aggregate wealth-income ratio? The most obvious way is that it determines the equilibrium age distribution. For example, ZPG puts relatively more households in the retirement years, when wealth declines to zero. On the other hand, it also puts more households in the high-wealth years just before retirement, and fewer in the early, low-wealth years. A less obvious effect is the life cycle of household size. With ZPG, there are fewer children to claim consumption as against the retirement consumption of the adults. When children are counted in the consumption plan, therefore, ZPG raises the peak wealth accumulations of middle-aged households. The upshot is, as reported in text Tables 4–6 and Figure 1, that reduction in fertility raises aggregate wealth-income ratios at all interest rates.