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# 1 Performance of Operational Policy Rules in an Estimated Semiclassical Structural Model

Bennett T. McCallum and Edward Nelson

## 1.1 Introduction

In a series of studies on monetary policy rules, McCallum (1988, 1990, 1993, 1995) has utilized and promoted a research strategy that emphasizes *operationality* and *robustness*. The first of these properties intentionally limits consideration to policy rules (i) that are expressed in terms of instrument variables that could in fact be controlled on a high-frequency basis by actual central banks and (ii) that require only information that could plausibly be possessed by these central banks. Thus, for example, hypothetical rules that treat, say, M2 as an instrument or that feature instrument responses to current-quarter values of real GDP are ruled out as nonoperational. The second property focuses on a candidate rule's tendency to produce at least moderately good performance in a variety of macroeconomic models rather than "optimal" performance in a single model. The idea behind this criterion is that there exists a great deal of professional disagreement over the appropriate specification of crucial features of macroeconomic models, and indeed even over the appropriate objective function to be used by an actual central bank.

Most of the models used in McCallum's own studies have, however, been nonstructural vector autoregression or single-equation atheoretic constructs that are quite unlikely to be policy invariant. Even the so-called structural models in McCallum (1988, 1993) are essentially small illustrative systems that are not based on well-motivated theoretical foundations. Thus these studies have

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not contributed any proposed models of their own to be used in a profession-wide exploration of the robustness of candidate rules' properties.

In the present study, accordingly, we formulate, estimate, and simulate two variants of a model of the U.S. economy that is intended to have structural properties. The model is quite small—following in the line of work previously contributed to by Fuhrer and Moore (1995), Yun (1996), Ireland (1997), and Rotemberg and Woodford (1997) among others—but is based on aggregate demand and supply specifications that are designed to reflect rational optimizing behavior on the part of the economy's private actors. Our formulations pertaining to demand are rather orthodox, but in terms of aggregate supply—that is, price adjustment behavior—we consider two alternatives, one of which is not standard. In particular, we begin with the formulation of Roberts (1995), which is based on the well-known models of Calvo (1983), Rotemberg (1982), and Taylor (1980). In addition, however, we develop a modification of the Mussa-McCallum-Barro-Grossman "P-bar model," whose theoretical properties are arguably more attractive. Although we consider only two simple variants of our macroeconomic model, we suggest that its design makes it an attractive starting point for a more extensive robustness study. Our estimation is conducted by instrumental variables and utilizes quarterly U.S. data for 1955–96.

With our estimated model we carry out stochastic and counterfactual historical simulations not only with the class of policy rules promoted in McCallum's previous work but also with rules that are operational versions of the Taylor (1993) type and others with an interest rate instrument. Some of the issues that we explore in these simulations are the following:

- Is it true that response coefficients in a rule of the Taylor type should be much larger than recommended by Taylor (1993)?
- Is there any tendency for adoption of a nominal GDP target rule to generate instability of real GDP and inflation?
- In studying questions such as these, how important is it quantitatively to recognize that actual central banks do not have complete information when setting instrument values for a given period?
- How sensitive to measures of "capacity" output are rules that feature responses to output gaps?
- Do interest rates exhibit extreme short-run volatility when base money rules are utilized?

Organizationally, we begin in section 1.2 with a discussion of several important background issues. Then sections 1.3 and 1.4 are devoted to specification of the macroeconomic model to be utilized, with the former pertaining to the model's aggregate demand sector and the latter to aggregate supply. Section 1.5 describes data and estimation and reports estimates of the model's basic structural parameters. Simulation exercises with various policy rules are then conducted in sections 1.6 and 1.7 for the two variants of the model, and conclusions are summarized in section 1.8.

## 1.2 Monetary Policy Rules: Alternatives and Issues

We begin by discussing various forms of possible monetary policy rules and some issues raised by the differences among them. In the previous research by McCallum, quarterly data have been utilized and the principal rule specification has been

$$(1) \qquad \Delta b_t = \Delta x^* - (1/16)(x_{t-1} - b_{t-1} - x_{t-17} + b_{t-17}) + \lambda(x^*_{t-1} - x_{t-1}),$$

with $\lambda \geq 0$. Here $b_t$ and $x_t$ denote logarithms of the (adjusted) monetary base and nominal GNP (or GDP), respectively, for period $t$. The variable $x^*_t$ is the target value of $x_t$ for quarter $t$, with these targets being specified so as to grow smoothly at the rate $\Delta x^*$. This rate is in turn designed to yield an average inflation rate that equals some desired value—for example, a value such as 0.005, which with quarterly data would represent roughly 2 percent per year.[1] Whereas a growing-level target path $x^{*1}_t = x^{*1}_{t-1} + \Delta x^*$ was used in McCallum's early work (1988), his more recent studies have emphasized growth rate targets of the form $x^{*2}_t = x_{t-1} + \Delta x^*$ or weighted averages such as $x^{*3}_t = 0.8x^{*2}_t + 0.2x^{*1}_t$. In equation (1), the rule's second term provides a velocity growth adjustment intended to reflect long-lasting institutional changes, while the third term features feedback adjustment in $\Delta b_t$ in response to cyclical departures of $x_t$ from the target path $x^*_t$, with $\lambda$ chosen to balance the speed of eliminating $x^*_t - x_t$ gaps against the danger of instrument instability.

More prominent in recent years has been the rule form proposed by Taylor (1993), which we write as

$$(2) \qquad R_t = r^* + \pi^{av}_{t-1} + \mu_1(\pi^{av}_{t-1} - \pi^*) + \mu_2\tilde{y}_t .$$

Here $R_t$ is the quarter $t$ value of an interest rate instrument, $\pi^{av}_{t-1}$ is the average inflation rate over the four quarters prior to $t$, $\pi^*$ is the target inflation rate, and $\tilde{y}_t = y_t - \bar{y}_t$ is the difference between the (logs of) real GDP $y_t$ and its capacity or natural rate value $\bar{y}_t$. The policy feedback parameters $\mu_1$ and $\mu_2$ are positive—each of them equals 0.5 in Taylor's (1993) example[2]—so that the interest rate instrument is raised in response to values of inflation and output that are high relative to their targets.

There are two major reasons for the greater prominence of Taylor's rule (2) as compared with rule (1). First, it is specified in terms of an interest rate instrument variable, which is much more realistic.[3] Second, from several studies including Taylor (1993), Stuart (1996), Clarida, Galí, and Gertler (1998), among others, it appears to be the case that actual policy in recent years, say after

1. Whatever the desired quarterly inflation rate, $\Delta x^*$ is set equal to that value plus an estimated long-run average rate of growth of real output, a number assumed to be independent of the policy rule adopted.

2. When annualized values of inflation and the interest rate are used.

3. Virtually all central banks of industrialized countries use some short-term (nominal) interest rate as their instrument or "operating target" variable. For an extensive recent discussion, see Bank for International Settlements (1997).

1986, has been rather well described by a formula such as Taylor's with coefficients quite close to his for some major countries.

As specified by Taylor (1993), however, rule (2) is not fully operational since it assumes unrealistically that the central bank knows the value of real GDP for quarter $t$ when setting the instrument value $R_t$ for that quarter. In fact, there is considerable uncertainty regarding the realized value of real GDP even at the end of the quarter in actual economies.[4] In addition, it is far from obvious how $\bar{y}_t$ should be measured—even in principle—as is emphasized in McCallum (1997), and different measures can imply significantly different instrument settings.[5] The first of these objections can be easily overcome by using the value of $y_t$ expected to prevail at the start of period $t$. Also, in the same spirit, some more rational representation of expected future inflation could be used in place of $\pi^{av}_{t-1}$. Overcoming the second objection, regarding the measurement of $\bar{y}_t$, could be more difficult.

Alterations in rule (1) could also be considered, such as using the expectation of $x^*_t$ (or of $x^*_{t+1}$) rather than actual $x^*_{t-1}$ as the basis for feedback adjustments. More generally, the target values in rules (1) and (2) could be exchanged, to provide rules with (i) a base instrument and $\pi^*$ and $\tilde{y}$ targets and (ii) an interest instrument plus a $\Delta x_t$ target. In the work that follows, we shall explore several such variants of policy rules.

In this regard, some analysts might suggest that the monetary base instrument be discarded since actual central banks are not inclined even to consider the use of a $b_t$ instrument.[6] Several academics have hypothesized that policy could be made more effective if a base instrument were utilized, however,[7] and there are clearly some disadvantages of the interest rate scheme. In particular, there is an observable tendency for an interest instrument to become something of a target variable that is thus adjusted too infrequently and too timidly (see Goodhart 1997). In any event, the question of the comparative merits of $b_t$ and $R_t$ instruments is one that seems to warrant scientific study—indeed, more than is provided below.

The foregoing paragraphs have been concerned with policy rules from a normative perspective. In estimating and evaluating a macroeconomic model, however, it is useful to consider what policy rule or rules have in fact been utilized during the sample studied. In that regard, it might be argued that no

4. In the United States, e.g., the recent study of Ingenito and Trehan (1997) indicates that the "forecast" error for real GDP at the end of the quarter is about 1.4 percent, implying that annualized growth rates for the quarter would have a 95 percent confidence interval of about ±2.8 percent, thereby possibly ranging from boom to deep recession values. This result is based on revised data, so it abstracts from the problem of data revision.

5. These two objections to rule (2) should not be understood as criticisms of Taylor's (1993) paper, which was written mainly to encourage interest in monetary rules on the part of practical policymakers—and was in that regard extremely successful.

6. Goodhart (1994) has claimed that tight monetary base control is essentially infeasible.

7. Among these academics are Brunner and Meltzer (1983), Friedman (1982), McCallum (1988), and Poole (1982).

rule has been in place, that the Federal Reserve has instead behaved in a discretionary manner. But we believe that there has clearly been a major component of Fed behavior that is *systematic*, as opposed to random, and this component can be expressed in terms of a feedback formula.[8] Of course, there can be little doubt but that there have been changes during our 1955–96 sample in the systematic component's specification, with prominent dates for possible changes including October 1979, late summer 1982, August 1987, and a few others.[9] Thus we have experimented with both slope and constant-term dummy variables. After considerable empirical investigation we have ended with an estimated rule of the form

$$(3)\qquad R_t = \mu_0 + \mu_1 R_{t-1} + \mu_2 E_{t-1}\Delta x_t + \mu_3 E_{t-1}\tilde{y}_t + \mu_4 d_{1t} + \mu_5 d_{2t} E_{t-1}\Delta x_t + e_{Rt},$$

where $\tilde{y}_t$ is the output gap (the log deviation of output from its flexible-price level), $d_{1t}$ and $d_{2t}$ are dummy variables that take on the value 1.0 in 1979:4–82:2 and 1979:4–96:4 respectively, and $e_{Rt}$ is a serially independent disturbance. Thus our estimated rule for 1955:1–96:4 is one that combines the interest rate instrument from rule (2) with a nominal GDP target as in rule (1), as well as an extra countercyclical term. The rule is operational because the monetary authority responds to period $t - 1$ forecasts of $\Delta x_t$ and $\tilde{y}_t$, not their realized values. The inclusion of dummies in equation (3) allows for shifts in the policy rule occurring in late 1979, presumably due to the change in operating procedures and anti-inflationary emphasis that was announced on 6 October. Of these, the dummy $d_{1t}$ captures a possible intercept shift occurring during the period of nonborrowed reserves targeting, and the interactive dummy $d_{2t}E_{t-1}\Delta x_t$ reflects a permanent shift in the Federal Reserve's objectives after 1979. The empirical results of our investigation are reported below in section 1.5.[10]

Returning to the normative topic of effective rule design, several prominent issues concerning target variables will be studied in sections 1.6 and 1.7. One of these involves the claim, expressed by Ball (1997) and Svensson (1997), that targeting of nominal GDP growth rates (or growing levels) will tend to induce undesirable behavior of inflation and output gap variables. It is not difficult to show that Ball's drastic result of dynamic instability of $\pi_t$ and $\tilde{y}_t$ holds only under some highly special model specifications, but it is possible that much greater volatility would obtain than with alternative target variables, so a quantitative examination of the issue is needed.

8. On this topic, see Taylor (1993), McCallum (1997), and Clarida et al. (1998).

9. The study by Clarida et al. (1998) considers one possible break, in October 1979, and finds significant differences in estimated policy rule coefficients before and after that date.

10. As the experiments in this paper are concerned with counterfactual policy rules, we do not use rule (3) in our simulations in sections 1.6 and 1.7. Our reason for nevertheless estimating and reporting eq. (3) is to demonstrate that rulelike behavior is a reasonable characterization of postwar data and to indicate the importance of the regime dummies $d_{1t}$ and $d_{2t}$, which we include in our instrument set when estimating our structural model in section 1.5.

## 1.3 Aggregate Demand Specification

This section describes the aggregate demand side of our model; what follows is essentially a condensed presentation of the derivations in McCallum and Nelson (forthcoming). We assume that there is a large number of infinitely lived households, each of which maximizes

$$E_t \sum_{j=0}^{\infty} \beta^j U(C_{t+j}, M_{t+j}/P_{t+j}^{A}), \tag{4}$$

where $C_t$ denotes the household's consumption in period $t$ and $M_t/P_t^A$ denotes its end-of-period real money holdings, $M_t$ being the nominal level of these money balances and $P_t^A$ the general price level. Real money balances generate utility by facilitating household transactions in period $t$. The instantaneous utility function $U(C_t, M_t/P_t^A)$ is of the additively separable form:

$$\begin{aligned} U(C_t, M_t/P_t^A) &= \sigma(\sigma - 1)^{-1} C_t^{(\sigma-1)/\sigma} \exp \omega_t \\ &\quad + (1 - \gamma)^{-1} (M_t/P_t^A)^{1-\gamma} \exp \chi_t, \end{aligned} \tag{5}$$

with $\sigma > 0$ and $\gamma > 0$. Here $\omega_t$ and $\chi_t$ are both preference shocks, whose properties we specify below.

Each household also acts as a producer of a good, over which it has market power. To this end, it hires $N_t^d$ in labor from the labor market, paying real wage $W_t/P_t^A$ for each unit of labor. With this labor and its own capital stock $K_t$ (which depreciates at rate $\delta$) it produces its output $Y_t$ via the technology $Y_t = A_t K_t^\alpha (N_t^d)^{1-\alpha}$, where $A_t$ is an exogenous shock that affects all households' production. The household sells its output at price $P_t$. Each household consumes many goods, consisting of some of the output produced by other households; the $C_t$ that appears in the household's utility function is an index of this consumption, and $P_t^A$ indexes the average price of households' output.

As is standard in the literature, we assume that the demand function for good $i$ is of the Dixit-Stiglitz form, and that also the producer is obliged to set production equal to this demand:

$$A_t K_t^\alpha (N_t^\alpha)^{1-\alpha} = (P_t/P_t^A)^{-\theta} Y_t^A, \tag{6}$$

with $\theta > 1$ and $Y_t^A$ denoting aggregate output.

The household is also endowed with one unit of labor each period, and supplies $N_t^S$ of this to the labor market. The household's budget constraint each period is then

$$\begin{aligned} &(P_t/P_t^A)^{1-\theta} Y_t^A - C_t - K_{t+1} + (1 - \delta) K_t + (W_t/P_t^A) N_t^S \\ &\quad - (W_t/P_t^A) N_t^d + TR_t - M_t/P_t^A + M_{t-1}/P_t^A \\ &\quad - B_{t+1}(1 + r_t)^{-1} + B_t = 0. \end{aligned} \tag{7}$$

In equation (7), $B_{t+1}$ is the quantity of government bonds bought by the household in period $t$; each of these is purchased for $(1 + r_t)^{-1}$ units of output and redeemed for one unit of output in period $t + 1$. $TR_t$ denotes lump-sum government transfers paid to the household in period $t$. Letting $\xi_t$ denote the Lagrange multiplier on constraint (6) and $\lambda_t$ the multiplier on (7), the household's first-order conditions with respect to $C_t$, $M_t/P_t^A$, $K_{t+1}$, and $B_{t+1}$ are

$$C_t^{-1/\sigma} \exp \omega_t = \lambda_t, \tag{8}$$

$$(M_t/P_t^A)^{-\gamma} \exp \chi_t = \lambda_t - \beta E_t \lambda_{t+1}(P_t^A/P_{t+1}^A), \tag{9}$$

$$\lambda_t = \beta(1 - \delta)E_t\lambda_{t+1} + \alpha\beta E_t \xi_{t+1} A_{t+1} K_{t+1}^{\alpha-1}(N_{t+1}^d)^{1-\alpha}, \tag{10}$$

$$\lambda_t = \beta E_t \lambda_{t+1}(1 + r_t). \tag{11}$$

Because leisure does not enter its utility function, the household's optimal labor supply is $N_t^S = 1$ each period, although, since we assume below that the labor market does not clear, this desired labor supply will not be the realized value of labor utilized.

As an employer of labor, the household's first-order condition with respect to $N_t^d$ is

$$\lambda_t(W_t/P_t^A) = (1 - \alpha)\xi_t A_t K_t^{\alpha}(N_t^d)^{-\alpha}. \tag{12}$$

Equation (12) indicates that, as in Ireland (1997), the markup of price over marginal cost is equal to $\lambda_t/\xi_t$. The household has one more first-order condition, pertaining to its optimal choice for $P_t$. We defer the analysis of this decision until section 1.4.

We now construct a log-linear model of aggregate demand from the above conditions. While we use equation (10) in our calculations of the implied steady state level of investment, $\bar{I}$, we do not use an approximation of equation (10) to describe quarter-to-quarter fluctuations in capital or investment. Instead, we treat capital as exogenous and, for tractability, let the movements of log investment around its steady state value be a random walk. Thus we have

$$i_t = g_k + i_{t-1} + e_{it}, \tag{13}$$

where $g_k \geq 0$ is the average growth rate of capital, $E_{t-1}e_{it} = 0$, and $E(e_{it}^2) = \sigma_{ei}^2$. In equation (13) and below, lowercase letters denote logarithms of variables.

It would be standard practice to complete our specification of technology with the usual log-linear law for capital accumulation,

$$k_{t+1} = \frac{1 - \delta}{1 + g_k} k_t + \frac{\delta + g_k}{1 + g_k} i_t, \tag{13a}$$

along with a law of motion for the (log) technology shock $a_t$. But since we are treating capital movements as exogenous, and since leisure does not appear in the household's utility function, the "flexible-price" or "capacity" level of log output, $\bar{y}_t = a_t + \alpha k_t$, is exogenous in our setup. It makes sense therefore to make assumptions directly about the $\bar{y}_t$ process, instead of its two components. By doing so we lose the connection between investment and capacity output implied by equation (13a), but this does not seem a serious omission for purposes of business cycle analysis because of the minor contribution that investment makes to the existing capital stock during a typical business cycle. We assume that $\bar{y}_t$ follows an AR(1) process:

$$\bar{y}_t = \varsigma + \rho_{\bar{y}}\bar{y}_{t-1} + e_{yt}, \tag{14}$$

where $|\rho_{\bar{y}}| \leq 1$ and $e_{yt} \sim N(0,\sigma^2_{ey})$, $E_{t-1}e_{yt} = 0$.[11]

Define the nominal interest rate as $R_t = r_t + E_t\Delta p_{t+1}$, where $\Delta p_{t+1} \equiv \log(P^A_{t+1}/P^A_t)$. Then equations (8), (11), and (14) and the economy's resource constraint imply (after log-linearization)

$$y_t = E_t y_{t+1} - \sigma(C^{ss}/Y^{ss})(R_t - E_t\Delta p_{t+1} - \bar{r}) + \sigma(C^{ss}/Y^{ss})(\omega_t - E_t\omega_{t+1}), \tag{15}$$

where the superscript ss denotes steady state value. We assume that the preference shock $\omega_t$ is an AR(1) process with AR parameter $|\rho_v| < 1$. Then if we define $v_t \equiv \sigma(1 - \rho_v)\omega_t$, it is the case that

$$v_t = \rho_v v_{t-1} + e_{vt}, \tag{16}$$

and so equation (15) becomes

$$y_t = E_t y_{t+1} - \sigma(C^{ss}/Y^{ss})(R_t - E_t\Delta p_{t+1} - \bar{r}) + (C^{ss}/Y^{ss})v_t, \tag{17}$$

which is like the optimizing IS functions of Kerr and King (1996), Woodford (1996), and McCallum and Nelson (forthcoming).

Let $m_t - p_t$ denote the logarithm of $M_t/P^A_t$. Then log-linearizing equation (9), we have (up to a constant)

$$\begin{aligned} m_t - p_t = {} & (\sigma\gamma)^{-1}(Y^{ss}/C^{ss})y_t - (\sigma\gamma)^{-1}(I^{ss}/C^{ss})i_t \\ & - (\gamma R^{ss})^{-1}(R_t - R^{ss}) + \gamma^{-1}(\chi_t - \omega_t), \end{aligned} \tag{18}$$

where $R^{ss} = r^{ss} + (\Delta p)^{ss}$. This money demand function has scale (consumption) elasticity $(\sigma\gamma)^{-1}$ and (annualized) interest semielasticity $-0.25(\gamma R^{ss})^{-1}$. We permit the shocks $\omega_t$ and $\chi_t$ to be arbitrarily correlated; hence, it is simpler to define the composite disturbance $\eta_t = \gamma^{-1}(\chi_t - \omega_t)$ and make assumptions directly about $\eta_t$. Then equation (18) may be written

$$\begin{aligned} m_t - p_t = {} & (\sigma\gamma)^{-1}(Y^{ss}/C^{ss})y_t - (\sigma\gamma)^{-1}(I^{ss}/C^{ss})i_t \\ & - (\gamma R^{ss})^{-1}(R_t - R^{ss}) + \eta_t, \end{aligned} \tag{19}$$

11. In our empirical work we use a measure of $\bar{y}_t$ (described in section 1.4) that grows over time, but in stochastic simulations we adopt the standard practice of abstracting from this growth.

and we assume $\eta_t$ is AR(1):

(20) $$\eta_t = \rho_\eta \eta_{t-1} + u_t,$$

where $|\rho_\eta| < 1$ and $u_t \sim N(0,\sigma_u^2)$, $E_{t-1}u_t = 0$. Since we have allowed $u_t$ and $e_{vt}$ to be correlated, we may write the latter as

(21) $$e_{vt} = \psi_u u_t + \varepsilon_{vt},$$

where $\varepsilon_{vt} \sim N(0,\sigma_{\varepsilon v}^2)$, $E_{t-1}\varepsilon_{vt} = 0$, and $E_t(u_t \varepsilon_{vt}) = 0$. Thus the aggregate demand block of our model consists of the behavioral equations (17) and (19), together with (13) and the laws of motion (14), (16), (20), and (21).

## 1.4 Price Level Adjustment

In this section we develop the particular model of individual and aggregate price adjustments that will be utilized below. For a typical producer, let $\bar{p}_t$ represent the value of $p_t$—its output price in log terms—that would be optimal in period $t$ if there were no nominal frictions, and let $\bar{y}_t$ be the corresponding level of (log) output $y_t$, which we will for shorthand refer to as "capacity" output. The producer faces a demand curve of the form

(22) $$y_t = y_t^A - \theta(p_t - p_t^A),$$

where $y_t^A$ and $p_t^A$ are indexes of aggregate values of $y_t$ and $p_t$, these being appropriate averages of the values relevant for the individual producers.[12] From equation (22) we note that

(23) $$y_t - \bar{y}_t = \theta(\bar{p}_t - p_t).$$

Perhaps the most widely used model of gradual price adjustment at present is the Calvo-Rotemberg model, which is justified by Rotemberg (1987) as follows. Although $\bar{p}_t$ would be charged in $t$ by the typical firm if there were no adjustment costs, in the presence of such costs (assumed quadratic) the producer will instead choose $p_t$ to minimize

(24) $$E_t \sum_{j=0}^{\infty} \beta^j [(p_{t+j} - \bar{p}_{t+j})^2 + c_1(p_{t+j} - p_{t+j-1})^2],$$

where $c_1 > 0$ reflects the cost of price changes in relation to the opportunity cost of setting a price different from $\bar{p}_t$. From expression (24) one can find the first-order optimality condition and rearrange to obtain the relation

(25) $$\Delta p_t = \beta E_t \Delta p_{t+1} + (1/c_1)(p_t - \bar{p}_t).$$

12. Thus $p_t^A = [\int_0^1 p_t(i)^{1-\theta}\, di]^{1/(1-\theta)}$ and $y_t^A = [\int_0^1 y_t(i)^{(\theta-1)/\theta}\, di]^{\theta/(\theta-1)}$ with $\theta > 1$, where $p_t(i)$ and $y_t(i)$ pertain to producer $i$, as in Dixit and Stiglitz (1977). In the text, the indices are suppressed for the sake of notational simplicity.

Then using equation (23), we have for the typical producer

$$\Delta p_t = \beta E_t \Delta p_{t+1} + (\theta/c_1)(y_t - \bar{y}_t). \tag{26}$$

Assuming symmetry across firms, equation (26) can be used for aggregative analysis. Both Rotemberg (1987) and Roberts (1995) show that an indistinguishable relation is implied by Calvo's (1983) model that emphasizes staggered setting of "contract" prices to prevail until a new price-change opportunity arrives, with probabilities of these arrivals being constant and exogenous. Also, Roberts (1995) shows that the two-period version of Taylor's (1980) well-known model of staggered wage contracts gives a relation that is basically similar.

In what follows, consequently, we shall utilize a quarterly version of Roberts's formulation of the Calvo-Rotemberg model in one variant of our macroeconomic system. There are, however, two theoretical drawbacks to this model. First, the assumed quadratic cost of changing prices is rather unattractive theoretically. One reason is that one might expect the magnitude of price-change costs to be independent of the size of the change, especially if these are to be interpreted as literal resource costs of preparing new price lists, and so forth. More basically, however, it seems somewhat undesirable to emphasize costs of changing prices, which are rather nebulous, while neglecting the costs of changes in output rates, which are more concrete and arguably quite substantial.[13] Second, as is shown below, the Calvo-Rotemberg model does not satisfy the natural rate hypothesis.[14]

Accordingly, let us consider a reformulated setup in which the producer chooses $p_t$ to minimize expression (27) rather than (24):

$$E_{t-1}\sum_{j=0}^{\infty} \beta^j[(p_{t+j} - \bar{p}_{t+j})^2 + c_2(\tilde{y}_{t+j} - \tilde{y}_{t+j-1})^2]. \tag{27}$$

Here $\tilde{y}_t = y_t - \bar{y}_t$, so we are assuming that it is costly for a producer to alter his output rate, relative to capacity, from its previous value. The reason for using $(\tilde{y}_{t+j} - \tilde{y}_{t+j-1})^2$ rather than $(y_{t+j} - y_{t+j-1})^2$ is that changes in capacity stem primarily from technological improvements or capital installations,[15] neither of which give rise to changes in the labor force needed to produce $\bar{y}_t$—but it is labor force changes that provide the primary rationale for the presumption that output changes are costly.[16] Neither $(\tilde{y}_{t+j} - \tilde{y}_{t+j-1})^2$ nor $(y_{t+j} - y_{t+j-1})^2$ is entirely appropriate, perhaps, but the former seems somewhat preferable theoretically; and it gives rise to a tidy, tractable model, as will be seen shortly. Another feature of expression (27) to be noted is that the presence of $E_{t-1}$ before

13. On this topic see Gordon (1990, 1146).

14. Empirically, it has been suggested that the model does not imply as much persistence of inflation rates as exists in the U.S. data. On this, see Ball (1994), Fuhrer and Moore (1995), and Nelson (1998).

15. There may in actuality be installation costs for new capital goods, but if so, this can in principle be taken account of in the IS portion of the model, not the price-setting portion.

16. Models with quadratic costs of changing employment appear frequently in Sargent (1979).

the summation sign implies that $p_t$ is chosen before the producer knows about demand conditions during $t$; that is, $p_t$ is predetermined in each period.[17] Then on the basis of the prevailing $p_t$, output in $t$ is taken to be demand determined. Labor-leisure trade-offs are assumed relevant for the determination of $\bar{y}_t$, but not for temporary departures of $y_t$ from $\bar{y}_t$. This is in accordance with the "installment payment" nature of current wages, as emphasized by Hall (1980).

Next we can define $\tilde{p}_t = p_t - \bar{p}_t$ and, in light of relation (23), can rewrite expression (27) as

$$(28) \qquad E_{t-1}\sum_{j=0}^{\infty} \beta^j[\tilde{p}_{t+j}^2 + c(\tilde{p}_{t+j} - \tilde{p}_{t+j-1})^2],$$

where now $c > 0$ is the cost of output "gap" changes in relation to departures of $p_t$ from $\bar{p}_t$. It might appear that $c\theta^2$ should appear in expression (28) where $c$ does, but $\theta^2$ can be absorbed into $c$ (and indeed this is entirely consistent with a symmetric treatment of the two terms). To minimize expression (28), the relevant first-order condition is

$$(29) \qquad E_{t-1}[\tilde{p}_t + c(\tilde{p}_t - \tilde{p}_{t-1}) - \beta c(\tilde{p}_{t+1} - \tilde{p}_t)] = 0$$

or

$$(30) \qquad E_{t-1}\tilde{p}_t = \alpha\tilde{p}_{t-1} + \alpha\beta E_{t-1}\tilde{p}_{t+1},$$

where $\alpha = c/(1 + c + c\beta)$. Then since this relation in effect involves only the single variable $\tilde{p}_t$, we can see that its MSV solution will be of the simple form $E_{t-1}\tilde{p}_t = \phi\tilde{p}_{t-1}$, with $E_{t-1}\tilde{p}_{t+1} = E_{t-1}\phi\tilde{p}_t = \phi^2\tilde{p}_{t-1}$.[18] Substitution into equation (30) gives $\phi\tilde{p}_{t-1} = \alpha\tilde{p}_{t-1} + \alpha\beta\phi^2\tilde{p}_{t-1}$, so $\phi$ must satisfy

$$(31) \qquad \alpha\beta\phi^2 - \phi + \alpha = 0.$$

Thus the MSV solution for $\phi$ is

$$(32) \qquad \phi = \left(1 - \sqrt{1 - 4\alpha^2\beta}\right)/2\alpha\beta.$$

From the definition of $\alpha$, we know that $4\alpha^2\beta < 1$, so $\phi$ in equation (32) is real. With $0 < \beta < 1$, we have $\phi > \alpha$, so the forward-looking objective increases the inertia of $\tilde{p}_t$. Also, it is the case that $\phi$ lies in the interval (0, 1).[19]

In any event, we have developed a price adjustment rule of the form $p_t - E_{t-1}\bar{p}_t = \phi(p_{t-1} - \bar{p}_{t-1})$. Thus by simple rearrangement we can write

17. This is our assumption regarding price stickiness per se. Implicitly, it embodies the assumption that sellers' costs of changing prices are prohibitive within periods but negligible between periods.

18. MSV stands for "minimal state variable." Thus we are adopting the bubble-free solution, in the manner outlined by McCallum (1983).

19. To show that $4\alpha^2\beta < 1$, it suffices to show that $(1 + \beta)^2 > 4\beta$. But that is equivalent to $1 + 2\beta + \beta^2 > 4\beta$. Then subtracting $4\beta$ from each side, we have $1 - 2\beta + \beta^2 > 0$, which is certainly true since the left-hand side is $(1 - \beta)^2$. Next, that $\phi > 0$ is clear from inspection of eq. (32), given that $0 < 4\alpha^2\beta < 1$. To see that $\phi < 1$, note that this is the same as $1 - \sqrt{1 - 4\alpha^2\beta} < 2\alpha\beta$, which reduces to $\alpha(1 + \beta) < 1$. Since $1/\alpha = 1/c + 1 + \beta$, the last inequality holds.

$$p_t - p_{t-1} = (1 - \phi)(\bar{p}_{t-1} - p_{t-1}) + E_{t-1}(\bar{p}_t - \bar{p}_{t-1}), \tag{33}$$

which can be seen to be equivalent to the price adjustment formula that was termed the "P-bar model" by McCallum (1994). This model was developed and utilized by Herschel Grossman, Robert Barro, Michael Mussa, and McCallum in the 1970s and early 1980s; for references, see McCallum (1994, 251–52).

An important feature of the model, not noted in previous work, is that equation (23) permits the MSV solution $E_{t-1}\tilde{p}_t = \phi\tilde{p}_{t-1}$ to be alternatively expressible as

$$E_{t-1}\tilde{y}_t = \phi \tilde{y}_{t-1}. \tag{34}$$

Thus in analytical or numerical solutions of a macromodel that includes the P-bar price adjustment theory, equation (34) can be included as the relation that governs price adjustment behavior. From the perspective of an undetermined-coefficients solution procedure, equation (34) fails to provide conditions relating to the coefficients on current shocks in the solution expression for $\tilde{y}_t$ (or for $y_t$ given $\bar{y}_t$). But these are compensated by the restriction that $p_t$ is predetermined and thus the shock coefficients in its solution equation are zeros. Thus, with this approach, the variable $\bar{p}_t$ need not be included in the analysis at all!

To illustrate the solution approach, suppose only for this paragraph that monetary policy was conducted in a manner that leads nominal income, $x_t$ in log terms, to behave as follows:

$$\Delta x_t = \psi \Delta x_{t-1} + \xi_t, \tag{35}$$

where $0 < \psi < 1$ and $\xi_t$ is white noise. Then one could consider the system consisting of equations (34) and (35) and the identity $\Delta x_t = \Delta p_t + y_t - y_{t-1}$, where we temporarily adopt the assumption that $\Delta\bar{y}_t = 0$. How does inflation $\Delta p_t$ behave in this system? By construction, the MSV solution will be of the form

$$\Delta p_t = \phi_{11}\Delta x_{t-1} + \phi_{12} y_{t-1} + \phi_{13}\xi_t, \tag{36}$$

$$y_t = \phi_{21}\Delta x_{t-1} + \phi_{22} y_{t-1} + \phi_{23}\xi_t, \tag{37}$$

in which we know a priori that $\phi_{13} = \phi_{21} = 0$ and $\phi_{22} = \phi$. Substitution into equation (35) gives

$$\phi_{11}\Delta x_{t-1} + \phi y_{t-1} + \phi_{23}\xi_t - y_{t-1} = \psi\Delta x_{t-1} + \xi_t. \tag{38}$$

Thus $\phi_{11} = \psi$, $\phi_{12} + \phi - 1 = 0$, and $\phi_{23} = 1$ are implied by undetermined-coefficients reasoning, which completes the solution.

It may also be noted that equation (34) provides the basis for an extremely simple proof that the P-bar model satisfies the strict version of the natural rate hypothesis. This version states that $E\tilde{y}_t = 0$, for *any* monetary policy, even one with accelerating inflation. But the application of the unconditional expecta-

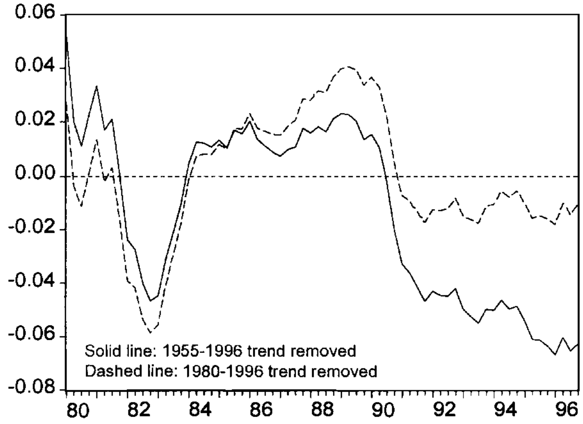


**Fig. 1.1 Measures of detrended output, 1980–96**

tions operator to each side of equation (34) yields $E\tilde{y}_t = \phi E\tilde{y}_t$, which with $\phi > 0$ implies that $E\tilde{y}_t \equiv 0$. With the Calvo-Rotemberg model (26), by contrast, we have $E(y_t - \bar{y}_t) = (c_1/\theta)E(\Delta p_t - \beta E_t \Delta p_{t+1})$. Using Roberts's (1995) approximation of $\beta \approx 1$, we have $E(y_t - \bar{y}_t) = (c_1/\theta)E(\Delta p_t - E_t \Delta p_{t+1})$, so any policy that yields on average an increasing or decreasing inflation rate will keep $E\tilde{y}_t \neq 0$.[20] Indeed, if $\beta < 1$ is retained, then even a constant $E\Delta p_t \neq 0$ will keep $E\tilde{y}_t \neq 0$.

In implementing our model—indeed, any model with gradual price adjustment—a very important issue is how to measure $\bar{y}_t$ and therefore $\tilde{y}_t$. Much of the policy rule literature, including Taylor (1993) and Rotemberg and Woodford (1997), simply uses deviations from a fitted linear time trend for $\tilde{y}_t$, thereby implicitly estimating $\bar{y}_t$ as the fitted trend. This seems unsatisfactory both practically and in principle. Practically, one major difficulty is that the resulting measure can be excessively sensitive to the sample period used in fitting the trend. To illustrate this sensitivity, figure 1.1 plots $\tilde{y}_t$-values for the United States over 1980–96 based on trends fitted (i) to a 1980–96 sample period and (ii) to the 1955–96 period that we use below. Clearly, they give markedly different pictures of the behavior of $\tilde{y}_t$ over the period 1990–96. And neither of them reflects the widely held belief that output has been unusually high relative to capacity in 1995 and 1996.

In principle, the fitted trend method—even if the detrending is done by a polynomial trend or the Hodrick-Prescott filter—seems inappropriate because it does not properly reflect the influence of technology shocks. Suppose that the production function is

20. It is interesting to note that the Calvo-Rotemberg-Taylor model implies that an increasing inflation rate will reduce $\tilde{y}_t$ whereas a typical NAIRU model implies that an increasing inflation rate will raise $\tilde{y}_t$—permanently. Both implications seem theoretically unattractive, although the former is perhaps less implausible (and certainly less dangerous from a policy perspective).

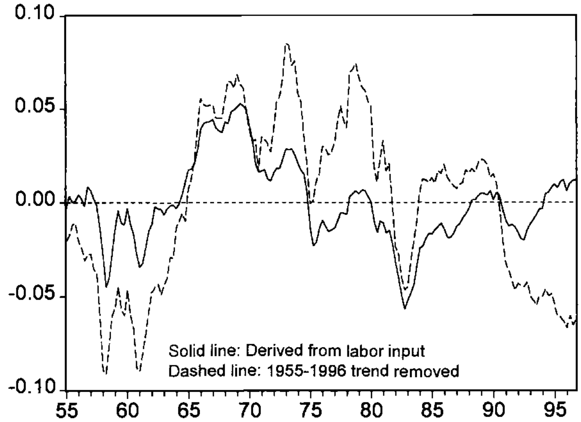


**Fig. 1.2 Output gap measures**

$$y_t = \alpha_0 + \alpha_1 t + \alpha k_t + (1 - \alpha)n_t + a_t, \quad (39)$$

where $k_t$ and $n_t$ are logs of capital and labor input, while $a_t$ is a technology shock. Then if $\bar{n}_t$ is the value of $n_t$ under flexible prices, $\bar{y}_t$ equals $\alpha_0 + \alpha_1 t + \alpha k_t + (1 - \alpha)\bar{n}_t + a_t$ and so reflects the realization of $a_t$. But the fitted trend methods do so either not at all or inadequately.

The approach that we use below relies on the observation that equation (39) implies

$$\tilde{y}_t = y_t - \bar{y}_t = (1 - \alpha)(n_t - \bar{n}_t). \quad (40)$$

Of course, this requires that we have some measure of $\bar{n}_t$. In general, it will depend on households' labor supply behavior as well as producers' demand, but for the present study we are adopting the simplifying assumption that labor supply is inelastic, that is, that $\bar{n}_t$ is a constant. Then variations in $\tilde{y}_t$ will be proportional to variations in $n_t$, the hours worked per household under sticky prices. We assume that this actual employment level is demand determined in each period.[21] The measure that we use for $n_t$ is total man-hours employed in nonagricultural private industry divided by the civilian labor force. A plot of the implied $\tilde{y}_t$, using $\alpha = 0.3$, is shown in figure 1.2, together with the fitted trend value based on the 1955–96 sample period.

## 1.5 Model Estimation

We estimate our model by instrumental variables. Some of the system's equations are estimated on a single-equation basis, but the two aggregate demand relations are estimated jointly:

21. Thus, as stated above, we are assuming that current-period wages are irrelevant for determination of current-period employment.

$$(41)\quad y_t = b_0 + E_t y_{t+1} - \sigma(C^{ss}/Y^{ss})(R_t - E_t \Delta p_{t+1}) + (C^{ss}/Y^{ss})v_t,$$

$$(42)\quad \begin{aligned} m_t - p_t &= c_0 + (\sigma\gamma)^{-1}(Y^{ss}/C^{ss})y_t - (\sigma\gamma)^{-1}(I^{ss}/C^{ss})i_t \\ &\quad - (\gamma R^{ss})^{-1}R_t + \eta_t. \end{aligned}$$

Here equations (41) and (42) are the IS and LM equations (17) and (19), allowing for constant terms. We estimate these equations jointly to take into account the cross-equation restriction (the appearance of the parameter $\sigma$ in both equations), as well as possible cross-correlation between $v_t$ and $\eta_t$ via equation (21).

One advantage of the instrumental variables procedure is that if the orthogonality conditions involving the instruments and the model errors are valid, parameter estimation is consistent under quite general assumptions about the serial correlation of the disturbances, and the precise form of the serial correlation does not have to be specified in estimation. To benefit from this advantage, we do not impose, in our estimation of equations (41) and (42), the AR(1) assumptions about the $v_t$ and $\eta_t$ processes that we make in our general equilibrium model (in eqs. [16] and [20]).

Equations (41) and (42) contain the expectational variables $E_t y_{t+1}$ and $E_t \Delta p_{t+1}$. We proceed with estimation of the system by replacing these expected values with their corresponding realized values, thereby introducing expectational errors such as $y_{t+1} - E_t y_{t+1}$ into the equations' composite disturbances. To obtain consistent estimates, we instrument for all the variables in equations (41) and (42). Because of the likely serial correlation in the error terms of the first two equations, lagged endogenous variables are not admissible instruments; only strictly exogenous variables are legitimate candidates. We therefore use as instruments a constant, a time trend, lags one and two of $\Delta g_t^{def}$ (i.e., the log change in quarterly defense spending), plus the dummy variables $d_{1t}$ and $d_{2t}$, which take the value unity in 1979:4–82:2 and 1979:4–96:4, respectively.

Money is measured by the St. Louis monetary base, new definition, $R_t$ is the Treasury bill rate (measured in quarterly fractional units), and $p_t$ is the log GDP deflator, defined as $x_t - y_t$. The income variables $x_t$ and $y_t$ are logs of nominal and real GDP, with values of GNP spliced on for observations prior to 1959:1. Also, $i_t$ is gross private fixed investment. All data except interest rates are seasonally adjusted. We fix $C^{ss}/Y^{ss}$ at 0.81, $I^{ss}/Y^{ss}$ at 0.19, and $R^{ss}$ at 0.014. The estimates of equations (41) and (42) are then

$$(43)\quad \begin{aligned} \hat{y}_t &= \underset{(0.129)}{-0.973} + E_t y_{t+1} - \underset{(0.017)}{0.203}(C^{ss}/Y^{ss})(R_t - E_t \Delta_t p_{t+1}), \\ \bar{R}^2 &= 0.999,\ \text{SEE} = 0.0098,\ \text{DW} = 1.35 \end{aligned}$$

$$(44)\quad \begin{aligned} \widehat{(m - p_t)} &= -0.007 + \underset{(0.001)}{0.753}(Y^{ss}/C^{ss})[y_t - (I^{ss}/Y^{ss})i_t] \underset{(0.015)}{-0.152}(R^{ss})^{-1}R_t, \\ \bar{R}^2 &= 0.942,\ \text{SEE} = 0.0617,\ \text{DW} = 0.14. \end{aligned}$$

The estimates imply an intertemporal elasticity of substitution of $\sigma = 0.20$ (standard error 0.018) and an interest elasticity of money demand of $-\gamma^{-1} = -0.15$ (standard error 0.015). In turn, these estimates imply a consumption elasticity of money demand of $(\sigma\gamma)^{-1} = 0.75$. The reported standard errors need to be interpreted with caution both because of the residual autocorrelation and because of the trending behavior of the $y_t$ and $m_t - p_t$ series.[22]

For the variant of our model that uses the P-bar price-setting specification, aggregate supply behavior is represented compactly by equation (34). As in section 1.4, we measure $\tilde{y}_t$ by $1 - \alpha$ times $n_t - \bar{n}$, where $\bar{n}$ is the mean of log hours and $\alpha = 0.3$. Equation (34) implies that the expectational error $\tilde{y}_t - \phi\tilde{y}_{t-1}$ should be white noise, but in preliminary estimation of $\phi$ we found substantial serial correlation in the estimated residuals. We therefore decided to correct for first-order serial correlation in our estimation of $\phi$, although such serial correlation is ignored both in our theoretical model and in the stochastic simulations of that model in section 1.7 below.[23] Estimation by instrumental variables, with the instruments being those used for equations (41) and (42) plus lags two to four of $\tilde{y}_t$, produces

$$\widehat{(E_{t-1}\tilde{y}_t)} = \underset{(0.063)}{0.891}\tilde{y}_{t-1}, \tag{45}$$

$$\bar{R}^2 = 0.956, \text{ SEE} = 0.0047, \text{ DW} = 1.95,$$
estimated AR(1) correction parameter = 0.59.[24]

Our measure of (log) potential output $\bar{y}_t$ is obtained by adding our estimated $\tilde{y}_t$ measure to $y_t$. We found that a random walk (with drift) process ($\rho_{\bar{y}} = 1.0$ in eq. [14]) describes the $\bar{y}_t$ series well.[25] Subject to that restriction, the constant (or "drift") term in equation (14) becomes interpretable as the long-run growth rate of capacity output.[26] For the investment-output ratio to be a mean-reverting

22. We assume that $m_t - p_t - (\sigma\gamma)^{-1}(Y^{ss}/C^{ss})[y_t - (I^{ss}/Y^{ss})i_t]$ is a stationary process. It is common in the empirical literature instead to estimate money demand functions such as (19) using cointegration methods, with $m_t - p_t$, $y_t$, and $R_t$ modeled as I(1) series. We do not do so because treating $R_t$ as I(1) is incompatible with our theoretical model unless $\Delta p_t$ is I(1). It is also inconsistent with most estimated policy rules, including our own specification (50) below, which model nominal interest rates as stationary within each policy regime.

We also experimented with a first-differenced money demand function, finding it produced a poorer fit and less plausible parameter estimates than eq. (44).

23. Our need to correct for serial correlation indicates that the first-order dynamics of the output gap implied by eq. (34) are rejected by the data. In future work we hope to generalize the P-bar specification to allow for more realistic dynamics.

24. Eq. (45) is based on the assumption that $\tilde{y}_t = \phi\tilde{y}_{t-1} + \tilde{e}_{yt}$, with $\tilde{e}_{yt}$ following $\tilde{e}_{yt} = \rho_e\tilde{e}_{yt-1} + \varepsilon_t$, with $\varepsilon_t$ white noise. By substitution, $\tilde{y}_t = (\phi + \rho_e)\tilde{y}_{t-1} - \phi\rho_e\tilde{y}_{t-2} + \varepsilon_t$. The parameters $\phi$ and $\rho_e$ appear symmetrically in this expression and thus cannot be individually identified without further information; to identify them, we assume that $\phi$ is the larger of the two parameters.

25. The behavior of our empirical measure of capacity output therefore supports the analytical model of Clarida, Galí, and Gertler (forthcoming), in which it is assumed that $\bar{y}_t$ follows a random walk.

26. And also as the long-run growth rate of actual output, since the output gap is assumed to average zero over our sample period.

series, the drift terms in equations (13) and (14) must be identical, and we therefore estimate those equations jointly subject to that restriction:

(46) $$\widehat{(\Delta i_t)} = \underset{(0.0052)}{0.0073}, \quad \text{SEE} = 0.0250, \text{DW} = 0.99,$$

(47) $$\widehat{(\Delta \bar{y}_t)} = \underset{(0.0052)}{0.0073}, \quad \text{SEE} = 0.0070, \text{DW} = 2.00,$$

implying $g_k = \varsigma = 0.0073$, $\sigma^2_{ei} = (0.0250)^2$, and $\sigma^2_{ey} = (0.0070)^2$. The Durbin-Watson statistic for equation (46) indicates strong serial correlation in the estimated residuals, contrary to the assumptions of our model, and suggests some deficiencies in the dynamic specification of the latter.

To simulate our model, we need to have values for the AR parameters and innovation variances in equations (16) and (20). Fitting an AR(1) model by least squares to the estimated residuals, $\hat{\eta}_t$, of equations (43) and (44) produces

(48) $$\hat{v}_t = \underset{(0.073)}{0.3233}\hat{v}_{t-1}, \quad \text{SEE} = 0.0114,$$

(49) $$\hat{\eta}_t = \underset{(0.028)}{0.9346}\hat{\eta}_{t-1}, \quad \text{SEE} = 0.0225,$$

so that $\rho_v = 0.3233$, $\rho_\eta = 0.9346$, $\sigma^2_{ev} = (0.0114)^2$, and $\sigma^2_u = (0.0225)^2$. The residuals of equation (49) are virtually uncorrelated with those of equation (50), leading us to set $\psi_u = 0$ and $\sigma^2_{\varepsilon v} = (0.0114)^2$ in equation (21).

Finally, we turn to the policy rule. To describe actual policy behavior, we use equation (3), although our simulations in the next section will consider alternative, counterfactual policy rules. Since we specify the error term in equation (3) as an innovation, lagged endogenous variables are legitimate instruments in the estimation of the equation. Our instrument list for this equation consists of a constant, a time trend, $d_{1t}$, $d_{2t}$, $\Delta x_{t-1}$, $\Delta x_{t-2}$, $d_{2t-1} \cdot \Delta x_{t-1}$, $d_{2t-2} \cdot \Delta x_{t-2}$, $\Delta p_{t-1}$, $\Delta p_{t-2}$, and $n_{t-1}$.[27] The resulting estimated rule is

(50) $$\hat{R}_t = \underset{(0.035)}{0.103} + \underset{(0.049)}{0.866}R_{t-1} + \underset{(0.005)}{0.023}E_{t-1}\tilde{y}_t + \underset{(0.034)}{0.117}E_{t-1}\Delta x_t + \underset{(0.001)}{0.002}d_{1t} + \underset{(0.031)}{0.064}d_{2t} \cdot E_{t-1}\Delta x_t,$$

$$\bar{R}^2 = 0.939, \ \text{SEE} = 0.0017, \ \text{DW} = 1.99.$$

The large coefficient on the lagged dependent variable suggests a high degree of interest rate smoothing. The coefficient on the interactive dummy $d_{2t} \cdot E_{t-1}\Delta x_t$ indicates a substantial permanent increase in the restrictiveness of

27. As before, we use $0.7n_t$ to measure (up to a constant) the output gap $\tilde{y}_t$.

monetary policy from 1979. After 1979, a 1 percent increase in expected nominal income growth leads to a steady state increase in the nominal interest rate of 1.35 percentage points, compared to only 0.87 points prior to 1979. This result is similar to the post-1979 increase in the coefficient on expected inflation in Clarida et al.'s (1998) estimates of the Taylor rule. The estimated intercept shift in the 1979–82 period is statistically significant and amounts to an upward shift of 0.8 percentage points when the interest rate is measured in annualized percentage units.

In the variant of our model that includes the Calvo-Rotemberg price-setting specification, the aggregate supply equation (26) appears. As is conventional, we set $\beta = 0.99$. The remaining coefficient in the equation is the ratio $\theta/c_1$. Using annual data, Roberts (1995) estimates this coefficient to be about 0.08. His version of equation (26), however, contained an additive disturbance term. Our equation (26), by contrast, has no explicit shock term; the randomness in inflation comes only from the stochastic behavior of the right-hand-side variables $E_t\Delta p_{t+1}$ and $\tilde{y}_t$. As a result, a much higher value of $\theta/c_1$ than Roberts's estimate, such as 0.30, is required to produce plausible inflation variability, for any of the policy rules that we consider. Thus 0.30 is the value of $\theta/c_1$ that we employ. With $\theta$, which is interpretable as the inverse of the aggregate markup under the aggregation scheme that we have used, set to 6, a value of $\theta/c_1 =$ 0.30 implies $c_1 = 20$.

## 1.6 Simulation Results I

In this section we report simulation results for the variant of our macroeconomic model that uses the Calvo-Rotemberg specification of price adjustment behavior. In calculating these results, as well as those in the next section, we have made one change in the aggregate demand portion of our model, replacing $E_t y_{t+1}$ with $E_{t-1} y_{t+1}$ on the right-hand side of the expectational IS function (43). This change, which represents a modification of the same basic type as those employed by Rotemberg and Woodford (1997), but less severe, produces more plausible values for the variability of inflation in all our simulations (for both the specifications of aggregate supply that we contemplate).[28]

We begin with simulations involving versions of the Taylor rule, some of them suggested by the conference organizer to facilitate comparison across papers by different researchers. In particular, table 1.1 includes results for various values of the policy parameters $\mu_1$, $\mu_2$, and $\mu_3$ in a rule of the form

$$R_t = \mu_0 + \mu_1 \Delta p_t + \mu_2 \tilde{y}_t + \mu_3 R_{t-1}, \tag{51}$$

where $\mu_0$ is in principle set so as to deliver the chosen average inflation rate and where policy responses are unrealistically assumed to reflect contemporaneous

28. This is particularly important in the context of the P-bar variant, where the two forward-looking components of the model interact in an overly sensitive way. In subsequent work, we plan to explore different modifications of our IS function, as suggested by the results of Campbell and Mankiw (1989) and Fuhrer (1997).

responses to the state of the economy. In the original Taylor rule $\mu_3 = 0$, but we have also considered cases with $\mu_3 = 1$ (to reflect interest rate smoothing by the Fed) and $\mu_3 = 1.2$ (to investigate a case recommended by Rotemberg and Woodford, chap. 2 of this volume). The simulation results reported are standard deviations (in annualized percentage units) of inflation $\Delta p_t$, the output gap $\tilde{y}_t$, and the interest rate $R_t$.[29] In these simulations constant terms are not included, so the standard deviation of $\Delta p_t$ is interpretable as the root-mean-square deviation from the inflation target value $\pi^*$, as is also the case for $\tilde{y}_t$. The values reported are mean values over 100 replications, with each simulation being for a sample period of 200 quarters.[30] In solving the model, we use the algorithm of Paul Klein (1997), which builds on that in King and Watson (forthcoming).

Examination of the results in table 1.1 shows that they suggest that for a given value of the smoothing parameter $\mu_3$, stronger responses to $\Delta p_t$ or $\tilde{y}_t$—that is, higher values of $\mu_1$ or $\mu_2$—lead invariably to lower standard deviations of that variable. Indeed, higher values of $\mu_1$ or $\mu_2$ lead in most cases to lower standard deviations of both $\Delta p_t$ and $\tilde{y}_t$ (basically because of the nature of the price adjustment equation). This suggests that if there were no concern for variability of the interest rate, the central bank could achieve extremely good macroeconomic performance merely by responding very strongly to current departures of inflation and output from their target values. In our opinion, however, that would be a highly unrealistic conclusion to draw; the conduct of monetary policy by actual central banks is much more difficult than that. But such a conclusion tends to be obtained from exercises in which the central bank is assumed to possess knowledge of $\Delta p_t$ and $\tilde{y}_t$ when setting its instrument value ($R_t$ in this case) for period $t$. In other words, the policy rule (51) does not represent an operational specification.

Because of this type of concern, the conference organizer suggested that results also be obtained for a specification like equation (51) but with inflation and the output gap lagged one quarter. Thus we next conduct simulations with

$$R_t = \mu_0 + \mu_1 \Delta p_{t-1} + \mu_2 \tilde{y}_{t-1} + \mu_3 R_{t-1} \tag{52}$$

as the policy rule and report the results in table 1.2.

For the cases where $\mu_1 = 1.5$ and there is no interest rate smoothing ($\mu_3 = 0$), the standard deviation of inflation is virtually identical in table 1.2 to the corresponding rules in table 1.1. As in table 1.1, rules with smoothing ($\mu_3 = 1.0$) deliver better results with respect to both inflation and output gap variability than the corresponding rules without smoothing. However, while table 1.1 indicated that with smoothing the standard deviation of inflation could be reduced to values as low as 0.65, the lowest standard deviation of inflation in

29. For the purpose of comparison, the actual historical values over 1955–96 are 2.41, 2.23, and 2.80.

30. We ran simulations of 253 periods and ignored the initial 53, so as to abstract from start-up departures from stochastic steady state conditions.

**Table 1.1** **Simulation Results with Calvo-Rotemberg Variant: Taylor Rule, Contemporaneous Response**

| | Value of $\mu_2$ | | | | |
|---|---|---|---|---|---|
| Values of $\mu_1$, $\mu_3$ | 0.0 | 0.5 | 1.0 | 3.0 | 10.0 |
| 1.5, 0.0 | 2.01 | 1.96 | 1.93 | 1.78 | 1.40 |
| | 1.15 | 1.12 | 1.10 | 1.03 | 0.82 |
| | 3.02 | 3.94 | 3.98 | 5.72 | 10.31 |
| 3.0, 0.0 | 1.78 | 1.78 | 1.72 | 1.60 | 1.29 |
| | 1.03 | 1.03 | 1.00 | 0.94 | 0.77 |
| | 5.34 | 5.84 | 6.13 | 7.59 | 11.53 |
| 10.0, 0.0 | 1.24 | 1.20 | 1.19 | 1.14 | 0.98 |
| | 0.75 | 0.73 | 0.72 | 0.69 | 0.60 |
| | 12.35 | 12.33 | 12.63 | 13.49 | 15.74 |
| 1.2, 1.0 | 1.32 | 1.25 | 1.19 | 1.10 | 0.97 |
| | 1.13 | 1.11 | 1.08 | 1.02 | 0.85 |
| | 2.38 | 2.94 | 3.41 | 5.42 | 10.63 |
| 3.0, 1.0 | 1.14 | 1.11 | 1.09[a] | 0.98 | 0.82 |
| | 1.04 | 1.03 | 1.03[a] | 0.97 | 0.81 |
| | 4.51 | 4.95 | 5.14[a] | 6.80 | 10.00 |
| 10.0, 1.0 | 0.85 | 0.83 | 0.83 | 0.78 | 0.65 |
| | 0.86 | 0.85 | 0.85 | 0.82 | 0.71 |
| | 9.32 | 9.51 | 9.90 | 10.71 | 13.40 |
| 1.2, 1.3 | 1.31[b] | 1.32 | 1.36 | 1.54 | 1.64 |
| | 1.12[b] | 1.11 | 1.11 | 1.05 | 0.94 |
| | 2.10[b] | 1.64 | 2.03 | 5.01 | 10.51 |

*Note:* Table reports standard deviations of $\Delta p_t$, $\tilde{y}_t$, and $R_t$, respectively (percent per annum).
[a]$\mu_2 = 0.8$, not 1.0.
[b]$\mu_2 = 0.06$, not 0.0.

table 1.2 is 1.00. It is also clear from table 1.2 that responding to lagged instead of contemporaneous data reduces policymakers' ability to stabilize output: the output gap standard deviation ranges from 0.60 to 1.15 in table 1.1, while in table 1.2 it ranges from 1.16 to 1.34.

Table 1.1 suggested that there were benefits in terms of both inflation and output gap variability from high values of $\mu_1$ or $\mu_2$, such as 10.0. In table 1.2, on the other hand, these benefits are less clear. Whereas in table 1.1, changing the output gap response coefficient $\mu_2$ from 3.0 to 10.0 unambiguously improved performance with respect to both inflation and the output gap, in table 1.2 this increase in $\mu_2$ delivers poorer performance on output gap variability and, in most cases with interest rate smoothing, on inflation variability too. Raising $\mu_1$ from 3.0 to 10.0 does improve inflation performance, just as it did in table 1.1, but in contrast to table 1.1, it fails to improve output gap performance appreciably.

While the results in table 1.2 indicate that there is some deterioration in policy performance with rule (52) instead of (51), the deterioration is not particularly drastic, and the rules still deliver dynamically stable results with large

**Table 1.2** **Simulation Results with Calvo-Rotemberg Variant: Taylor Rule, Lagged Response**

| | Value of $\mu_2$ | | | | |
|---|---|---|---|---|---|
| Values of $\mu_1$, $\mu_3$ | 0.0 | 0.5 | 1.0 | 3.0 | 10.0 |
| 1.5, 0.0 | 1.91 | 1.88 | 1.84 | 1.68 | 1.37 |
| | 1.21 | 1.22 | 1.21 | 1.19 | 1.19 |
| | 2.87 | 3.43 | 3.94 | 6.06 | 13.86 |
| 3.0, 0.0 | 1.71 | 1.68 | 1.66 | 1.54 | 1.31 |
| | 1.19 | 1.19 | 1.19 | 1.17 | 1.19 |
| | 5.14 | 5.62 | 6.17 | 8.12 | 15.79 |
| 10.0, 0.0 | 1.33 | 1.31 | 1.29 | 1.26 | 1.18 |
| | 1.19 | 1.18 | 1.18 | 1.20 | 1.26 |
| | 13.26 | 13.66 | 14.10 | 16.20 | 24.28 |
| 1.2, 1.0 | 1.33 | 1.27 | 1.23 | 1.20 | 1.25 |
| | 1.18 | 1.18 | 1.17 | 1.18 | 1.26 |
| | 2.27 | 2.78 | 3.29 | 5.51 | 12.76 |
| 3.0, 1.0 | 1.19 | 1.17 | 1.15[a] | 1.10 | 1.11 |
| | 1.16 | 1.17 | 1.17[a] | 1.17 | 1.25 |
| | 4.24 | 4.65 | 4.96[a] | 6.95 | 13.50 |
| 10.0, 1.0 | 1.03 | 1.04 | 1.03 | 1.01 | 1.00 |
| | 1.17 | 1.18 | 1.19 | 1.20 | 1.27 |
| | 9.50 | 9.98 | 10.23 | 11.78 | 17.53 |
| 1.2, 1.3 | 1.34[b] | 1.34 | 1.40 | 1.62 | 1.88 |
| | 1.18[b] | 1.16 | 1.18 | 1.19 | 1.34 |
| | 2.03[b] | 2.49 | 3.04 | 5.19 | 12.61 |

*Note:* Table reports standard deviations of $\Delta p_t$, $\tilde{y}_t$, and $R_t$, respectively (percent per annum).
[a] $\mu_2 = 0.8$, not 1.0.
[b] $\mu_2 = 0.06$, not 0.0.

values of $\mu_1$ or $\mu_2$. That finding comes as a surprise to us, but having obtained it we believe that it can be understood as follows. There are two properties of the model at hand that defuse the tendency, mentioned in McCallum (1997, sec. 6), for explosive instrument instability to arise when strong feedback responses are based on lagged variables. First, the values of two parameters crucial for the transmission of policy actions to $\Delta p_t$ are quite small; these are the slope of the "IS function" with respect to the real interest rate ($\sigma \cdot C^{ss}/Y^{ss}$ in eq. [15])[31] and the slope of the price adjustment relation ($\theta/c_1$ in eq. [26]). The smallness of the former implies that aggregate demand responses to changes in $R_t$ are small, and the latter makes aggregate demand changes have small effects on inflation. Second, the Calvo-Rotemberg version of our model is one in which there is no autoregressive structure apart from what is contained in the disturbance terms and the policy rule. The model, that is, is entirely forward looking. We conjecture that models with backward looking IS and price adjust-

31. Our estimated value is less than 1/20 of the value used by, e.g., Rotemberg and Woodford (1997; chap. 2 of this volume).

ment specifications would possess much more of a tendency to generate dynamic instability for large values of $\mu_1$ and $\mu_2$.[32]

Another operationality concern expressed by McCallum (1997) involves a lack of knowledge about $\bar{y}_t$, the market-clearing value of $y_t$. Suppose, then, that the central bank believes that a fitted linear trend line represents $\bar{y}_t$, while in fact our measure is correct. Then the central bank would use detrended $y_t$ instead of $\tilde{y}_t$ in its policy rule and would measure output gap fluctuations in relation to this fitted trend. To get an idea of the implications, we redo the table 1.1 case with $\mu_1 = 1.2$, $\mu_2 = 1.0$, and $\mu_3 = 1.0$ under this assumption. Then the standard deviation of $\Delta p_t$ turns out to be 3.41 instead of 1.19, according to our model, and the central bank would believe that the standard deviation of $\tilde{y}_t$ was 3.91 (although it would actually be 1.09—almost the same as in table 1.1). Also, the standard deviation of $R_t$ would rise from 3.41 to 4.77.[33]

One issue mentioned in our introduction is the stability and desirability of nominal income targeting. To determine whether effects on $\Delta p_t$ and $\tilde{y}_t$ would be much different if targets were set for $\Delta x_t = \Delta p_t + \Delta y_t$, we have conducted simulations using the rule

$$R_t = \mu_0 + \mu_1 \Delta x_t + \mu_3 R_{t-1}, \tag{53}$$

and also with $\Delta x_{t-1}$ replacing $\Delta x_t$, for $\mu_3 = 0$ and $\mu_3 = 1.0$. These results are reported in table 1.3. There we see that nominal income targeting with an interest instrument performs reasonably well. It permits considerably more variability of inflation than does the Taylor rule but tends to stabilize output (in relation to $\bar{y}_t$) almost as well. It should be noted that the good performance in terms of $\tilde{y}_t$ occurs despite the absence of that variable or $\bar{y}_t$ in the policy rule. An advantage of nominal income (growth rate) targeting is that it does not require the central bank to measure capacity output. More interest rate variability occurs for most parameter values, but such variability is quite low (and the $\Delta p_t$ and $\tilde{y}_t$ standard deviations are reasonably small) when $\mu_1$ is assigned the small value of 0.1 with $\mu_3 = 1.0$.

As in table 1.2, for moderate values of the feedback coefficient there is a deterioration in performance with respect to $\tilde{y}_t$ variability, but little deterioration in $\Delta p_t$ variability, when feedback is applied with a one-period lag, that is, to the value of $\Delta x_{t-1}$ rather than $\Delta x_t$. Another similarity with tables 1.1 and 1.2 is that making the feedback coefficient large (in this case, increasing $\mu_1$ in eq. [53] from 3.0 to 10.0) delivers an improvement in performance with respect to $\tilde{y}_t$, $\Delta p_t$, and $\Delta x_t$ variability (at the cost of increased $R_t$ volatility) when policy responds to contemporaneous data, but not when policy responds to lagged information. In the latter case, raising $\mu_1$ from 3.0 to 10.0 actually delivers

32. Even in the present model we found instability to prevail if $\mu_1$ was raised to 1,000 (!) and to prevail at lower values of $\mu_1$ if $\sigma$ was increased sharply. With contemporaneous feedback, there is no instability even in these cases.

33. These results are generated by replacing $\tilde{y}_t$ with $y_t$ in eq. (51), re-solving the model, and then looking at simulation results for $\Delta p_t$, $y_t$, and $R_t$.

**Table 1.3 Simulation Results with Calvo-Rotemberg Variant: Nominal Income Target, Interest Rate Instrument**

| | Contemporaneous Response Value of $\mu_3$ | | Lagged Response Value of $\mu_3$ | |
|---|---|---|---|---|
| Value of $\mu_1$ | 0.0 | 1.0 | 0.0 | 1.0 |
| 0.10 | | 1.96 | | 1.96 |
| | | 1.22 | | 1.23 |
| | | 6.18 | | 6.29 |
| | | 0.65 | | 0.63 |
| 0.50 | | 1.85 | | 1.84 |
| | | 1.13 | | 1.19 |
| | | 5.60 | | 6.22 |
| | | 2.80 | | 2.78 |
| 1.00 | | 1.76 | | 1.79 |
| | | 1.05 | | 1.18 |
| | | 5.11 | | 6.35 |
| | | 5.11 | | 5.17 |
| 1.50 | 2.44 | 1.71 | 2.48 | 1.75 |
| | 1.11 | 0.99 | 1.25 | 1.18 |
| | 5.14 | 4.70 | 5.23 | 6.60 |
| | 7.71 | 7.06 | 10.13 | 7.55 |
| 3.00 | 2.50 | 1.63 | 2.53 | 1.70 |
| | 1.00 | 0.87 | 1.31 | 1.19 |
| | 4.17 | 3.82 | 7.69 | 7.17 |
| | 12.52 | 10.18 | 23.07 | 14.33 |
| 10.00 | 1.92 | 1.49 | All variables | 2.00 |
| | 0.69 | 0.65 | explosive | 2.25 |
| | 2.00 | 2.05 | | 18.16 |
| | 19.96 | 17.33 | | 93.84 |

*Note:* Table reports standard deviations of $\Delta p_t$, $\tilde{y}_t$, $\Delta x_t$, and $R_t$, respectively (percent per annum).

instrument instability when there is no interest rate smoothing. With smoothing, dynamic stability prevails for all variables, but the standard deviations of $\tilde{y}_t$, $\Delta p_t$, $\Delta x_t$, and $R_t$ are all decidedly increased. Thus tables 1.2 and 1.3 are both supportive of the notion that assigning very high values to response coefficients is counterproductive when policy can only respond to lagged information.

Next we retain nominal income as the target variable but consider the use of $\Delta b_t$—the growth rate of the monetary base—as the instrument. In particular, we consider two versions of McCallum's rule (1), one with a "levels" target path $x_t^{*1} = x_{t-1}^{*1} + \Delta x^*$ and the other with a "growth rate" target $x_t^{*2} = x_{t-1} + \Delta x^*$. Stochastic simulation results analogous to those discussed above are presented in table 1.4. There it will be seen that performance is quite close to that in table 1.3, where nominal income targeting is attempted with $R_t$ as the instrument variable. Throughout table 1.4, the variability of nominal income growth is about the same as it is with the best of the lagged response rules in table 1.3; moreover, the variability of inflation is lower than it is table 1.3 and

**Table 1.4** **Simulation Results with Calvo-Rotemberg Variant: Nominal Income Target, Monetary Base Instrument**

| Value of λ in Rule (1) | Levels Target, $x^{*1}$ | Growth Rate Target, $x^{*2}$ |
|---|---|---|
| 0.25 | 1.05 | 1.05 |
| | 1.29 | 1.30 |
| | 5.25 | 5.27 |
| | 1.88 | 1.80 |
| 0.50 | 1.07 | 1.00 |
| | 1.29 | 1.27 |
| | 5.25 | 5.19 |
| | 1.96 | 1.72 |
| 1.00 | 1.03 | 1.01 |
| | 1.27 | 1.28 |
| | 5.25 | 5.20 |
| | 1.02 | 1.78 |
| 3.00 | 1.04 | 1.01 |
| | 1.28 | 1.26 |
| | 5.39 | 5.16 |
| | 3.06 | 2.11 |
| 10.00 | 1.04 | 0.99 |
| | 1.28 | 1.25 |
| | 5.89 | 5.31 |
| | 3.73 | 4.30 |

*Note:* Table reports standard deviations of $\Delta p_t$, $\tilde{y}_t$, $\Delta x_t$, and $R_t$, respectively (percent per annum).

is comparable to the values obtained in table 1.2 with the operational Taylor rule (52). In addition, there is no apparent tendency for interest rate variability to increase sharply when the base is used.

A comparison of the levels target and growth rate target rule performances in table 1.4 shows, somewhat surprisingly, that the results are little different and, in particular, that $\tilde{y}_t$ variability is not lower with the growth rate specification. For both rule types, another striking feature is how insensitive the variability of the nominal income growth rate is to changes in the value of the response coefficient λ.[34] Presumably this is the case because the parameter values estimated in section 1.5 imply an extremely small response of aggregate demand to real money balances $(b_t - p_t)$.

It should be emphasized that the stochastic simulation exercises underlying tables 1.1 through 1.4 do not serve to bring out one aspect of operationality claimed by McCallum (1988) for rule (1), namely, its nondependence on the long-run average growth rate of base velocity. That nondependence, which is

34. The levels target results suggest that nominal income growth $\Delta x_t$ variability is *increasing* in λ; this reflects the fact that in the simulations, the levels target is a constant, so successful nominal income targeting implies that $x_t$ is I(0). $\Delta x_t$ is therefore I(−1), and hence will tend to be highly variable, the more so when nominal income targeting is pursued vigorously (i.e., with high values of λ). The standard deviation of the *level* of nominal income in the simulations underlying the first column of table 1.1 *is* decreasing in λ, taking the values 1.44, 1.35, 1.26, 1.15, and 1.10 for λ = 0.25, 0.50, 1.00, 3.00, and 10.00, respectively.

not possessed by most rules with base or reserve aggregate instruments, is basically irrelevant for the stochastic simulations in which constant terms are omitted. Thus the velocity correction term in rule (1) could be omitted without any appreciable effect on the results of table 1.4, which is most definitely not the case for the counterfactual historical simulations reported in, for example, McCallum (1988, 1993). Accordingly, we plan to include some simulations of this latter type in subsequent work.

## 1.7 Simulation Results II

In this section we report stochastic simulation results analogous to those of tables 1.1 through 1.4 but now using the P-bar price adjustment relation. Table 1.5 gives standard deviations of $\Delta p_t$, $\tilde{y}_t$, and $R_t$ for the same values of $\mu_1$, $\mu_2$, and $\mu_3$ as those considered in table 1.1, under the assumption of contemporaneous feedback responses to $\Delta p_t$ and $\tilde{y}_t$. Again it is the case that an increase in $\mu_1$ ($\mu_2$) reduces the variability of $\Delta p_t$ ($\tilde{y}_t$), but it is not now the case that increasing either $\mu_1$ or $\mu_2$ tends to reduce the variability of both $\Delta p_t$ and $\tilde{y}_t$. Instead, there

**Table 1.5 Simulation Results with P-Bar Variant: Taylor Rule, Contemporaneous Response**

| | Value of $\mu_2$ | | | | |
|---|---|---|---|---|---|
| Values of $\mu_1$, $\mu_3$ | 0.0 | 0.5 | 1.0 | 3.0 | 10.0 |
| 1.5, 0.0 | 8.53 | 9.67 | 10.68 | 14.20 | 31.38 |
| | 2.48 | 2.39 | 2.31 | 1.98 | 0.55 |
| | 12.80 | 13.70 | 14.42 | 16.93 | 30.24 |
| 3.0, 0.0 | 2.88 | 3.13 | 3.37 | 4.51 | 12.74 |
| | 2.51 | 2.43 | 2.34 | 2.15 | 0.78 |
| | 8.64 | 8.83 | 9.01 | 10.01 | 19.75 |
| 10.0, 0.0 | 0.76 | 0.81 | 1.18 | 2.00 | 3.39 |
| | 2.49 | 2.43 | 2.04 | 1.57 | 0.89 |
| | 7.63 | 7.63 | 8.34 | 11.78 | 18.46 |
| 1.2, 1.0 | 3.61 | 3.71 | 4.14 | 6.72 | 19.00 |
| | 2.51 | 2.42 | 2.24 | 1.88 | 0.44 |
| | 5.32 | 5.68 | 5.86 | 7.17 | 13.86 |
| 3.0, 1.0 | 1.95 | 1.96 | 1.98[a] | 2.97 | 6.11 |
| | 2.53 | 2.42 | 2.36[a] | 2.06 | 1.49 |
| | 6.26 | 6.33 | 6.27[a] | 7.14 | 10.10 |
| 10.0, 1.0 | 0.70 | 0.69 | 0.71 | 0.95 | 1.99 |
| | 2.42 | 2.32 | 2.34 | 2.12 | 1.66 |
| | 6.88 | 6.83 | 6.95 | 7.52 | 10.44 |
| 1.2, 1.3 | 3.76[b] | 3.74 | 4.10 | 6.51 | 12.61 |
| | 2.51[b] | 2.35 | 2.27 | 1.82 | 1.15 |
| | 4.49[b] | 4.65 | 4.98 | 6.02 | 8.72 |

*Note:* Table reports standard deviations of $\Delta p_t$, $\tilde{y}_t$, and $R_t$, respectively (percent per annum).

[a] $\mu_2 = 0.8$, not 1.0.

[b] $\mu_2 = 0.06$, not 0.0.

**Table 1.6 Simulation Results with P-Bar Variant: Taylor Rule, Lagged Response to $\tilde{y}$, Contemporaneous to $\Delta p$**

| | Value of $\mu_2$ | | | | |
|---|---|---|---|---|---|
| Values of $\mu_1$, $\mu_3$ | 0.0 | 0.5 | 1.0 | 3.0 | 10.0 |
| 1.5, 0.0 | 8.53 | 9.92 | 11.23 | 16.15 | 29.51 |
| | 2.48 | 2.42 | 2.35 | 2.09 | 1.59 |
| | 12.80 | 13.92 | 14.88 | 18.52 | 28.78 |
| 3.0, 0.0 | 2.88 | 3.25 | 3.54 | 5.23 | 11.08 |
| | 2.51 | 2.53 | 2.44 | 2.35 | 2.08 |
| | 8.64 | 9.04 | 9.13 | 10.18 | 13.78 |
| 10.0, 0.0 | 0.76 | 0.80 | 0.88 | 1.24 | 2.86 |
| | 2.49 | 2.42 | 2.45 | 2.39 | 2.32 |
| | 7.63 | 7.57 | 7.55 | 7.67 | 8.33 |
| 1.2, 1.0 | 3.60 | 3.79 | 4.19 | 7.00 | 15.30 |
| | 2.56 | 2.43 | 2.39 | 2.19 | 1.62 |
| | 5.32 | 5.74 | 6.12 | 7.99 | 12.75 |
| 3.0, 1.0 | 1.95 | 1.95 | 2.07[a] | 3.17 | 7.50 |
| | 2.47 | 2.48 | 2.47[a] | 2.39 | 2.09 |
| | 6.16 | 6.31 | 6.43[a] | 7.17 | 9.51 |
| 10.0, 1.0 | 0.69 | 0.69 | 0.75 | 1.01 | 2.45 |
| | 2.44 | 2.44 | 2.51 | 2.39 | 2.30 |
| | 6.83 | 6.80 | 7.01 | 6.95 | 7.50 |
| 1.2, 1.3 | All variables explosive[b] | 3.75 | All variables explosive | All variables explosive | 13.68 |
| | | 2.50 | | | 1.68 |
| | | 4.81 | | | 10.79 |

*Note:* Table reports standard deviations of $\Delta p_t$, $\tilde{y}_t$, and $R_t$, respectively (percent per annum).

[a] $\mu_2 = 0.8$, not 1.0.

[b] $\mu_2 = 0.06$, not 0.0.

is a variability trade-off at work, with increases in $\mu_2$ often increasing the variability of $\Delta p_t$. The existence of interest rate smoothing, with $\mu_3 = 1$, is helpful in most cases and is so to a greater extent than in table 1.1. Overall, the variability of $\Delta p_t$, $\tilde{y}_t$, and $R_t$ is considerably greater than in table 1.1. For $\tilde{y}_t$, its magnitude is much more realistic, but for $\Delta p_t$ or $R_t$ it is somewhat excessive.

Table 1.6 is partly but not entirely analogous to table 1.2, in which lagged values of $\Delta p_t$ and $\tilde{y}_t$ are used in rule (52). When such values are utilized, dynamically explosive results are obtained for most parameter configurations. Consequently, table 1.6 reports values for feedback responses to the lagged value of $\tilde{y}_t$ but to the current value of $\Delta p_t$. This modification seems justifiable from an operationality perspective because $\Delta p_t$ is a predetermined variable in the P-bar variant of our model, so $\Delta p_t$ is in principle observable at the end of period $t - 1$. The resulting standard deviations are quite close to those of table 1.5 for small and moderate values of $\mu_1$ and $\mu_2$ but are larger for high values of these feedback parameters. There is no evident tendency toward dynamic instability, however, except in the "Rotemberg-Woodford" cases with $\mu_3 = 1.3$.

**Table 1.7** **Simulation Results with P-Bar Variant: Nominal Income Target, Interest Rate Instrument**

| | Contemporaneous Response Value of $\mu_3$ | | Lagged Response Value of $\mu_3$ | |
|---|---|---|---|---|
| Value of $\mu_1$ | 0.0 | 1.0 | 0.0 | 1.0 |
| 0.10 | | 7.46 | | 7.51 |
| | | 2.07 | | 2.10 |
| | | 8.50 | | 8.56 |
| | | 1.77 | | 1.72 |
| 0.50 | | 4.36 | | 4.67 |
| | | 2.27 | | 2.19 |
| | | 5.85 | | 6.13 |
| | | 4.07 | | 3.72 |
| 1.00 | 27.65 | 2.97 | 38.87 | 3.56 |
| | 1.77 | 2.24 | 1.73 | 2.21 |
| | 28.62 | 4.58 | 39.42 | 5.16 |
| | 28.62 | 5.67 | 39.28 | 5.12 |
| 1.50 | 6.82 | 2.25 | | 3.41 |
| | 2.31 | 2.09 | All variables | 2.17 |
| | 8.22 | 3.90 | explosive | 4.96 |
| | 12.33 | 6.78 | | 6.26 |
| 3.00 | 2.70 | 1.47 | | |
| | 2.06 | 1.97 | All variables | All variables |
| | 3.91 | 2.95 | explosive | explosive |
| | 11.75 | 9.54 | | |
| 10.00 | 0.98 | 0.96 | | |
| | 1.67 | 1.64 | All variables | All variables |
| | 1.67 | 1.67 | explosive | explosive |
| | 16.71 | 15.77 | | |

*Note:* Table reports standard deviations of $\Delta p_t$, $\tilde{y}_t$, $\Delta x_t$, and $R_t$, respectively (percent per annum).

Next we consider the effect of an incorrect belief by the central bank that a fitted trend line represents $\bar{y}_t$ when in fact our measure is correct. With the P-bar price adjustment relation included, rather than the Calvo-Rotemberg version, this effect is considerably smaller. Thus, in the particular case mentioned in section 1.6—that is, with $\mu_1 = 1.2$, $\mu_2 = 1.0$, and $\mu_3 = 1.0$—the $\Delta p_t$ and $\tilde{y}_t$ standard deviations increase only from 4.14 and 2.24 (respectively) to 4.80 and 2.35. The reduction in this effect obtains, clearly, because the P-bar specification makes $\tilde{y}_t$ very strongly related to $\tilde{y}_{t-1}$. If the central bank responds more vigorously to its (incorrect) beliefs about $\tilde{y}_t$, however, the deleterious effect will be somewhat larger. With $\mu_2 = 3.0$, for example,[35] the standard deviations increase from 6.72 and 1.88 to 10.50 and 2.15.

With nominal income targeting and an interest instrument, the results with the P-bar variant of our model are given in table 1.7. There the results are

35. With $\mu_1$ and $\mu_3$ as before.

Table 1.8 **Simulation Results with P-Bar Variant: Nominal Income Target, Monetary Base Instrument**

| Value of λ in Rule (1) | Levels Target, $x^{*1}$ | Growth Rate Target, $x^{*2}$ |
|---|---|---|
| 0.25 | 6.67 | 7.17 |
| | 2.24 | 2.20 |
| | 7.70 | 8.26 |
| | 2.47 | 2.29 |
| 0.50 | 6.38 | 6.88 |
| | 2.27 | 2.27 |
| | 7.51 | 7.95 |
| | 2.61 | 2.40 |
| 1.00 | 6.08 | 6.38 |
| | 2.28 | 2.27 |
| | 7.20 | 7.49 |
| | 2.83 | 2.56 |
| 3.00 | 5.48 | 5.28 |
| | 2.18 | 2.30 |
| | 6.61 | 6.62 |
| | 3.55 | 3.23 |
| 10.00 | 5.00 | 3.62 |
| | 2.19 | 2.26 |
| | 6.09 | 5.14 |
| | 5.50 | 5.02 |

*Note:* Table reports standard deviations of $\Delta p_t$, $\tilde{y}_t$, $\Delta x_t$, and $R_t$, respectively (percent per annum).

much more favorable with $\mu_3$ equal to 1.0 rather than zero, that is, with interest smoothing. The ability of rule (53) to keep $\Delta x_t$ close to its target value is about the same as with the Calvo-Rotemberg variant, but results in terms of the variability of $\Delta p_t$ (and to a lesser extent $\tilde{y}_t$) are much less desirable. Clearly, the dynamic relationship between $\Delta p_t$ and $\tilde{y}_t$ is very different with these two price adjustment specifications.

Finally, in the table 1.8 case with rule (1), in which $\Delta b_t$ is the instrument variable (and $x_t$ or $\Delta x_t$ the target variable), the performance is about the same as in table 1.7. For a given level of $R_t$ variability, that is, the standard deviations of $\Delta x_t$, $\Delta p_t$, and $\tilde{y}_t$ are about the same. Furthermore, the figures indicate a low degree of responsiveness of nominal income variability to the feedback parameter λ, although the responsiveness is considerably greater than it was with the Calvo-Rotemberg variant of our model (in table 1.4). Again, this low responsiveness is largely a result of the optimizing IS specification that we employ, which implies that aggregate demand is quite insensitive to the quantity of real money balances.

## 1.8 Conclusions

Some conclusions from the simulation results hold for both variants of our model—that is, with both price adjustment relations. The first of these is that the inclusion of the $R_{t-1}$ interest-smoothing term in the Taylor rule is helpful in reducing the variability of $\Delta p_t$ and $\tilde{y}_t$ for given values of the policy response parameters $\mu_1$ and $\mu_2$, while also reducing $R_t$ variability. Second, for moderate values of response coefficients, the use of lagged rather than contemporaneous values of $\tilde{y}_t$ does not bring about any major deterioration in results and does not generate any severe danger of instrument instability.[36] Third, nominal income targeting with an $R_t$ instrument is only mildly effective but shows no noticeable tendency to generate dynamic instability, provided that interest rate smoothing is employed.[37] Fourth, nominal income targeting with a monetary base instrument does not imply drastically greater $R_t$ variability than with an interest instrument. It is, however, only weakly effective—the standard deviation of $\Delta x_t$ is not very responsive to the feedback parameter $\lambda$.[38]

Other conclusions are more sensitive to the model variant. For example, pure inflation targeting ($\mu_1 > 0$, $\mu_2 = 0$) is quite effective in the Calvo-Rotemberg specification but significantly less so with the P-bar relation. More generally, increasing $\mu_1$ or $\mu_2$ tends (for moderate ranges of those parameters) to reduce *both* inflation and output gap variability with the Calvo-Rotemberg variant; by contrast, the P-bar specification generates a trade-off between inflation and output gap variability, so that raising $\mu_2$ for a given $\mu_1$ yields improved output gap performance at the expense of more variable inflation. Furthermore, performance deteriorates sharply if the central bank responds to an incorrect measure of capacity output ($\bar{y}_t$) when the Calvo-Rotemberg relation is used but does so only moderately with the P-bar specification. And nominal income targeting holds down inflation variability much better with the Calvo-Rotemberg version of the model. Finally, when policy responds to lagged rather than contemporaneous output gap data, increasing the value of the Taylor rule response coefficient on the output gap to a very high level (say 10) tends to be counterproductive—in the sense of increasing rather than decreasing output gap variability—when the Calvo-Rotemberg specification of aggregate supply is used. This result does not carry over when the P-bar specification is employed.

These last-mentioned conclusions illustrate the importance, mentioned in our introduction, of the robustness of proposed rules to model specification. In future work, we hope to conduct a small robustness study of our own while also investigating several issues that we have not yet been able to explore.

36. This is not true, as mentioned, for lagged $\Delta p_t$-values in the P-bar variant, in which case $\Delta p_t$ is itself a predetermined variable.

37. With strong feedback or with $\mu_3 = 0$ in the lagged response cases, dynamic instability obtains. It is not, however, of the type mentioned by Ball (1997), which involves instability of $\Delta p_t$ and $\tilde{y}_t$ even though $\Delta x_t$ is stabilized.

38. This conclusion might be changed by alternative specifications of relations analogous to our eqs. (17) and (19).

# References


Ball, Laurence. 1994. Credible disinflation with staggered price setting. *American Economic Review* 84:282–89.

———. 1997. Efficient rules for monetary policy. NBER Working Paper no. 5952. Cambridge, Mass.: National Bureau of Economic Research.

Bank for International Settlements. 1997. Implementation and tactics of monetary policy. Conference Paper no. 3. Basel: Bank for International Settlements.

Brunner, Karl, and Allan H. Meltzer. 1983. Strategy and tactics for monetary control. *Carnegie-Rochester Conference Series on Public Policy* 18:59–103.

Calvo, Guillermo A. 1983. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* 12:383–98.

Campbell, John Y., and N. Gregory Mankiw. 1989. Consumption, income and interest rates: Reinterpreting the time series evidence. In *NBER macroeconomics annual 1989,* ed. O. J. Blanchard and S. Fischer, 185–216. Cambridge, Mass.: MIT Press.

Clarida, Richard, Jordi Galí, and Mark Gertler. 1998. Monetary policy rules and macroeconomic stability: Evidence and some theory. NBER Working Paper no. 6442. Cambridge, Mass.: National Bureau of Economic Research.

———. Forthcoming. The science of monetary policy. *Journal of Economic Literature.*

Dixit, A. K., and Joseph E. Stiglitz. 1977. Monopolistic competition and optimum product diversity. *American Economic Review* 67:297–308.

Friedman, Milton. 1982. Monetary policy: Theory and practice. *Journal of Money, Credit and Banking* 14:98–118.

Fuhrer, Jeffrey C. 1997. Towards a compact, empirically-verified rational expectations model for monetary policy analysis. *Carnegie-Rochester Conference Series on Public Policy* 47:197–230.

Fuhrer, Jeffrey C., and George R. Moore. 1995. Inflation persistence. *Quarterly Journal of Economics* 109:127–59.

Goodhart, Charles A. E. 1994. What should central banks do? What should be their macroeconomic objectives and operations? *Economic Journal* 104:1424–36.

———. 1997. Why do the monetary authorities smooth interest rates? In *European monetary policy,* ed. S. Collignon, 119–74. London: Pinter.

Gordon, Robert J. 1990. What is new Keynesian economics? *Journal of Economic Literature* 28:1115–71.

Hall, Robert E. 1980. Labor supply and aggregate fluctuations. *Carnegie-Rochester Conference Series on Public Policy* 12:7–33.

Ingenito, Robert, and Bharat Trehan. 1997. Using monthly data to predict quarterly output. *Federal Reserve Bank of San Francisco Economic Review,* no. 3:3–11.

Ireland, Peter N. 1997. A small, structural, quarterly model for monetary policy evaluation. *Carnegie-Rochester Conference Series on Public Policy* 47:83–108.

Kerr, William, and Robert G. King. 1996. Limits on interest rate rules in the *IS* model. *Federal Reserve Bank of Richmond Economic Quarterly* 82:47–75.

King, Robert G., and Mark W. Watson. Forthcoming. System reduction and solution algorithms for singular linear difference systems under rational expectations. *International Economic Review.*

Klein, Paul. 1997. Using the generalized Schur form to solve a system of linear expectational difference equations. Stockholm: Institute for International Economic Studies. Working paper.

McCallum, Bennett T. 1983. On non-uniqueness in rational expectations models: An attempt at perspective. *Journal of Monetary Economics* 11:139–68.

———. 1988. Robustness properties of a rule for monetary policy. *Carnegie-Rochester Conference Series on Public Policy* 29:173–203.

———. 1990. Targets, indicators, and instruments of monetary policy. In *Monetary policy for a changing financial environment*, ed. W. S. Haraf and P. Cagan, 44–70. Washington, D.C.: AEI Press.

———. 1993. Specification and analysis of a monetary policy rule for Japan. *Bank of Japan Monetary and Economic Studies* 11:1–45.

———. 1994. A semi-classical model of price adjustment. *Carnegie-Rochester Conference Series on Public Policy* 41:251–84.

———. 1995. Monetary policy rules and financial stability. In *Financial stability in a changing financial environment*, ed. K. Sawamoto, Z. Nakajima, and H. Taguchi, 389–421. London: Macmillan.

———. 1997. Issues in the design of monetary policy rules. NBER Working Paper no. 6016. Cambridge, Mass.: National Bureau of Economic Research. (In *Handbook of macroeconomics*, ed. John B. Taylor and Michael Woodford. Amsterdam: North Holland, forthcoming.)

McCallum, Bennett T., and Edward Nelson. Forthcoming. An optimizing *IS-LM* specification for monetary policy and business cycle analysis. *Journal of Money, Credit and Banking* 31.

Nelson, Edward. 1998. Sluggish inflation and optimizing models of the business cycle. *Journal of Monetary Economics* 42:303–22.

Poole, William. 1982. Federal Reserve operating procedures: A survey and evaluation of the historical record since October 1979. *Journal of Money, Credit and Banking* 14:575–96.

Roberts, John M. 1995. New Keynesian economics and the Phillips curve. *Journal of Money, Credit, and Banking* 27:975–84.

Rotemberg, Julio J. 1982. Sticky prices in the United States. *Journal of Political Economy* 90:1187–1211.

———. 1987. The new Keynesian microfoundations. In *NBER macroeconomics annual 1987*, ed. Stanley Fischer, 69–104. Cambridge, Mass.: MIT Press.

Rotemberg, Julio J., and Michael Woodford. 1997. An optimization-based econometric framework for the evaluation of monetary policy. In *NBER macroeconomics annual 1997*, ed. Ben S. Bernanke and Julio J. Rotemberg, 297–346. Cambridge, Mass.: MIT Press.

Sargent, Thomas J. 1979. *Macroeconomic theory.* San Diego: Academic Press.

Stuart, Alison. 1996. Simple monetary policy rules. *Bank of England Quarterly Bulletin* 36:281–87.

Svensson, Lars E. O. 1997. Inflation targeting: Some extensions. NBER Working Paper no. 5962. Cambridge, Mass.: National Bureau of Economic Research.

Taylor, John B. 1980. Aggregate dynamics and staggered contracts. *Journal of Political Economy* 88:1–23.

———. 1993. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy* 39:195–214.

Woodford, Michael. 1996. Control of the public debt: A requirement for price stability? NBER Working Paper no. 5684. Cambridge, Mass.: National Bureau of Economic Research.

Yun, Tack. 1996. Nominal price rigidity, money supply endogeneity, and business cycles. *Journal of Monetary Economics* 37:345–70.

# Comment Mark Gertler

## Introduction

Ben McCallum has written many papers on the topic of monetary policy rules. His work has heavily influenced my own thinking on the subject. The current paper, coauthored with Ed Nelson, is yet another stimulating effort in this area.

### Overview

The objective here is to study the performance of simple monetary policy rules within a small model of the U.S. economy. Two aspects of the analysis distinguish the approach: First, the authors derive the model from first principles and estimate (most) of the key parameters. The motive is to take the Lucas critique seriously by working with a structural model but at the same take the model seriously enough to make use of its identifying restrictions in the estimation of parameters.

Second, the authors investigate robustness. In particular, they explore the sensitivity of the results to two alternative specifications of price adjustment and several alternative informational scenarios. Here the goal is to address what might appropriately be called "McCallum critique," namely, that the primary obstacle facing policymakers is uncertainty about the exact structure of the economy. For this reason, as McCallum has repeatedly emphasized, it is important when doing policy evaluation to explore how a given rule works across different plausible economic environments.

Much of the analysis proceeds as follows: Let $r_t$ be the net nominal short-term interest rate at time $t$, $\Delta p_t$ the percentage change in the price level from $t - 1$ to $t$, $y_t$ the log of output at $t$, and $\bar{y}_t$ the log of the natural rate of output at $t$ (defined as the level that would arise under perfectly flexible prices). Each variable, further, is expressed as a deviation from its deterministic long-run trend. The authors then consider the family of three-parameter interest rate feedback policy rules given by

$$(1) \qquad r_t = \mu_1 \Delta p_t + \mu_2 (y_t - \bar{y}_t) + \mu_3 r_{t-1},$$

with $\mu_1 > 1$, $\mu_2 > 0$, and $\mu_3 > 0$.

As is consistent with the evidence, the short-term nominal rate is treated as the policy instrument. The target inflation rate defines the steady inflation rate. The target level of output is the natural rate. The rule then has the central bank raise the short rate above trend if either inflation or output is above target. The feedback policy thus has the form of a Taylor rule, but with the addition of the

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lagged interest rate that serves to introduce serial dependence in $r_t$. The authors proceed to explore how different numerical choices for the parameter vector $\{\mu_1, \mu_2, \mu_3\}$ affect the unconditional variance of inflation, the output gap, and the short-term interest rate.

### Punchlines: Hypothetical versus Historical Policy Rules

Importantly, the kind of three-parameter policy rule characterized by equation (1) does a reasonably good job of capturing actual policy for the United States over the Volcker and Greenspan eras. It is thus possible to measure the hypothetical policy rule against historical policy in a reasonably direct manner. With this observation in mind, the authors' main punchlines are as follows: Rules that perform well across a broad range of scenarios have (i) $\mu_1$ and $\mu_3$ large relative to actual policy and (ii) $\mu_2$ small relative to actual policy.

Thus policies that seem to work well are, relative to actual practice, more aggressive in responding to inflation and less aggressive in responding to the output gap. They also allow for more serial dependence in the interest rate. These types of policies work well in the sense that they produce relatively lower volatility in inflation, the output gap, and the interest rate. Further, they perform well not only within the class of policy rules given by equation (1) but also as compared to other types of feedback policies, such as adjusting rates in response to a nominal GDP target or using a narrow money aggregate as the policy instrument.

### Where I'm Headed

In order to understand how these results come about, I will review briefly each of the models that the authors employ. To foreshadow, I conclude that the qualitative conclusions are sensible but that the quantitative conclusions the authors derive appear highly sensitive to model structure. In this vein, neither of the models that the authors consider appears to provide an adequate characterization of the data.

I focus particular attention on the price adjustment equations. The first model the authors study employs the widely used Calvo-Rotemberg formulation of gradual price adjustment. Here I show, based on a very simple and direct test, that this formulation does not capture the apparent inertia in inflation. The second model employs what the authors call "the P-bar formulation" of price adjustment. It is based (in part) on partial adjustment of output. I show in this case, again based on a very simple and direct test, that this version of the model fails to capture the hump-shaped dynamics in output.

On the other hand, it is a great virtue of the authors' approach that the structural equations they derive have directly testable implications. As I show, even though both models are rejected, each is so in a way that provides some guidance for how the respective framework needs to be modified. I conclude with some observations about the problem of model uncertainty.

## Model 1: Calvo-Rotemberg Price Adjustment

The baseline framework is what McCallum and Nelson have referred to in previous work as an "optimizing IS-LM model." It is essentially a dynamic general equilibrium framework modified to allow for money (which enters individual utility functions separably), monopolistic competition, and price stickiness. The authors motivate price stickiness by assuming quadratic costs of changing nominal prices, following Rotemberg (1982). It is also possible, however, to derive the same kind of aggregate supply curve using Calvo's (1983) formulation of Taylor's (1980) time-dependent staggered price-setting model. In addition, the authors assume that investment is exogenous and obeys a random walk about trend.

### The Formal Model

Given this environment, the model may be reduced to three equations: an IS curve, an aggregate supply (AS) curve, and an interest rate feedback policy. The latter is given by equation (1). The IS and AS curves are given by equations (2) and (3), respectively, as follows:

(2) $$y_t = -\sigma(r_t - E_t\Delta p_{t+1}) + E_t y_{t+1} + v_t,$$

(3) $$\Delta p_t = \lambda(y_t - \bar{y}_t) + \beta E_t \Delta p_{t+1},$$

where $v_t$ is an aggregate demand shock (specifically a preference shock). Given that investment is exogenous, the IS curve is essentially a consumption Euler equation, where the coefficient $\sigma$ is the intertemporal elasticity of substitution. The AS curve has the general form of a standard Phillips curve, except that the cost push term depends on expected future inflation, rather than current inflation.

The authors then proceed to explore how varying the parameters of the policy rule affects the unconditional variances of $\Delta p_t$, $y_t - \bar{y}_t$, and $r_t$. To understand the logic behind the results they obtain it is first useful to iterate forward both the IS and AS curves:

(4) $$y_t = -\sigma\sum_{k=0}^{\infty} E_t(r_{t+k} - \Delta p_{t+k+1}) + u_t,$$

(5) $$\Delta p_t = \sum_{k=0}^{\infty} \beta^k \lambda(y_{t+k} - \bar{y}_{t+k}),$$

where $u_t = \sum_{k=0}^{\infty} E_t v_{t+k}$. Importantly, forward-looking expectations drive the behavior of both output and inflation. The IS curve (4) relates output inversely to the long-term real rate (i.e., the expected sum of short-term real rates). The AS curve (5) relates inflation to the discounted sum of the current and expected future output gaps.

Intuition for the Main Results

An important implication of the AS curve is that despite the presence of price inertia, there is no trade-off between stabilization of inflation and stabilization of the output gap. Adjusting the interest rate to stabilize $y_t - \bar{y}_t$ also stabilizes $\Delta p_t$ and vice versa. It is thus apparent why an aggressive interest rate response to inflation (a high value of $\mu_1$) works well, in the sense of producing low volatility of both inflation and output.

If there are no informational frictions, responding aggressively to the output gap (a high value of $\mu_2$) also works well. However, if potential output, $\bar{y}_t$, is not directly observable, it pays to make the interest rate less responsive to output movements and more responsive to inflation (i.e., a high value of $\mu_1$ along with a low value of $\mu_2$). This is particularly true if supply shocks are an important source of variation in output (i.e., if movements in $\bar{y}_t$ are important in the overall movement in $y_t$). In this instance, adjusting rates to stabilize $y_t$ will increase the volatility of both the output gap and inflation, as equation (5) suggests.

Finally, interest rate smoothing (a high value of $\mu_3$) is desirable because it permits the central bank to dampen the volatility of inflation and output with less adjustment in the short-term rate than otherwise. As Rotemberg and Woodford emphasize (chap. 2 in this volume), raising the serial dependence parameter $\mu_3$ increases the sensitivity of long-term rates to movements in current short-term rates (since it implies a larger adjustment of expected future short rates than otherwise). Smoothing thus increases the potency of a given change in the short rate, since it is ultimately the long-term rate that affects the output gap and (indirectly) inflation. The importance of future expectations to current behavior in this framework tends to enhance the gains from smoothing. The gains would be diminished, for example, if some of the interest rate sensitivity of output was due to the short-term rate, independent of the long-term rate.

Brief Assessment

Overall, I find the analysis appealing and think that the qualitative results I discussed above may survive in richer frameworks. The quantitative results (i.e., what values of the policy rule parameter vector $\{\mu_1, \mu_2, \mu_3\}$ produce the most desirable outcomes) do, however, depend on the model structure. Here I have some concerns. To be fair to the authors, so do they. Neither the IS nor the AS curve appears to offer an adequate characterization of the data.

Three issues arise with the IS curve. First, there are no endogenous dynamics: output depends only the long-term rate and an exogenous forcing process. While admittedly it is difficult to say anything with certainty in macroeconomics, my sense is that the hypothesis of no endogenous output dynamics is unlikely to survive careful empirical scrutiny. Nor is it compelling from a theoretical viewpoint (given the likelihood of adjustment costs and so on). Second, the real interest rate affects the economy only by inducing intertemporal substitution in consumption. The evidence (including many recent identified vector autoregression studies) suggests that durable goods, including housing, autos, and producer durable equipment, bear the main brunt of monetary policy.[1] Thus it is problematic as to whether the authors' model really pins down the correct interest sensitivity of output. Finally, the vector autoregression evidence suggests a lag of at least six to nine months in the impact of interest rates on the economy (though some sectors such as housing respond more quickly). In the authors' model, the response is immediate.

I raise these issues not to be picky. Rather, if we are to take seriously the quantitative policy rule that the analysis recommends, it is imperative that we have a (reasonably) correct structural empirical link between short-term rates and the real sector.

The absence of a short-run output-inflation trade-off is a striking implication of the AS curve. It is thus particularly important to assess the reasonableness of this relationship. At issue is whether this pure forward-looking formulation of price dynamics captures the degree of inflation persistence that is present in the data. Others have raised this concern. There is, however, a relatively simple test of this proposition. I turn to this next.

Inflation Persistence in the Calvo-Rotemberg Model: A Simple Test

The aggregate supply curve may be expressed as[2]

$$\Delta p_t = \lambda(y_t - \bar{y}_t) + \beta \Delta p_{t+1} + \varepsilon_{t+1}, \tag{6}$$

where $\varepsilon_{t+1} = -\beta(\Delta p_{t+1} - E_t \Delta p_{t+1})$. Since $E_t \varepsilon_{t+1} = 0$, after controlling for its predictive content for $y_t - \bar{y}_t$ and $\Delta p_{t+1}$, no variable dated $t$ or earlier should help predict $\Delta p_t$. This implication leads to a simple test that is much in the spirit of Hall's (1978) and Campbell and Mankiw's (1989) test of the consumption Euler equation. In particular, consider the instrumental variables estimation of the following equation:

$$\Delta p_t = \pi_1(y_t - \bar{y}_t) + \pi_2 \Delta p_{t+1} + \pi_3 \Delta p_{t-1} + \varepsilon_t \tag{7}$$

under the null of equation (6): $\pi_2 = \beta$ (a number close to one) and $\pi_3 = 0$.

To test the null I proceed as follows: I measure output and inflation using the same data as Rudebusch and Svensson (chap. 5 in this volume). The percentage change in the GDP deflator is the measure of inflation. The output gap is the percentage deviation of output from a quadratic trend. Each variable is expressed as a deviation from a constant mean. The data is quarterly, over the period 1960:1–97:3. Finally, I use as instruments the lagged output gap and

1. For evidence on the responsiveness of the different components of output to monetary policy, see Bernanke and Gertler (1995).

2. The analysis in this section is based on some work in progress with Jordi Galí.

two lags of inflation, all of which are legitimate instruments under the null hypothesis. Instrumental variables estimation of equation (7) then yields

$$\begin{aligned}(8) \qquad \Delta p_t = \underset{(0.039)}{0.030}(y_t - \bar{y}_t) + \underset{(0.148)}{0.375}\,\Delta p_{t+1} + \underset{(0.125)}{0.585}\,\Delta p_{t-1},\end{aligned}$$

where the numbers in parentheses are standard errors.

Several results stand out. First, the null is clearly rejected.[3] Lagged inflation has a significant and quantitatively important impact on inflation. A 1 percent rise in the lagged inflation rate lifts current inflation 0.585 percent, everything else equal. The effect is significantly different from zero. This evidence is, at least on the surface, inconsistent with the premise of no short-run output-inflation trade-off. The possible implication is that the authors' analysis may overstate the desirability of rules that react to inflation in a very aggressive manner.

On the other hand, expected future inflation also enters significantly: A 1 percent rise in expected future inflation raises current inflation by 0.319 percent; and the effect differs significantly from zero. Thus the forward-looking aspect of inflation that the model emphasizes is clearly present in the data. Thus I believe that the direction one should take is to build on this framework and not to abandon it. Modifying this model to account for the persistence in inflation should be a priority for future research in this area.[4]

## Model 2: "P-Bar" Price Adjustment

The P-bar model begins with the premise that there are quadratic costs of adjusting output relative to capacity. Strictly speaking, however, these costs are in expectation since producers lock in a nominal price ex ante and not output. In response to shocks ex post, the nominal price stays fixed, but output is free to adjust.

3. Some qualification of the test is in order. If the excess demand variable, $y_t - \bar{y}_t$, in eq. (7) is measured with error, then it is possible that rejection could occur even if the null is true. Whether measurement error could explain the degree of rejection I find is problematic, however. I note that the results are robust to using the Congressional Budget Office's measure of potential output to construct the output gap. Note also that, in general, the proper measure of excess demand in these models is (detrended) real marginal cost (see, e.g., Goodfriend and King 1997). In the McCallum-Nelson framework the output gap varies proportionately with real marginal cost, so in this instance it is legitimate to use the output gap as the excess demand measure. In work in progress with Jordi Galí, I am exploring how direct use of marginal cost as the excess demand variable affects the results. Preliminary results suggest that (i) marginal cost works better than output as a gap variable (in the sense that the slope coefficient is statistically significant) and (ii) the forward-looking term becomes more important relative to the backward-looking one, though the null model is still rejected.

4. Larry Ball suggested that in analogy to Campbell and Mankiw, the evidence could be explained by inflation being the product of a convex combination of rational forward-looking price setters and rule-of-thumb price setters (the latter perhaps being the same individuals who are rule-of-thumb consumers in the Campbell-Mankiw setup).

The Formal Model

In any event, let $\bar{p}_t$ be the nominal price that would arise if prices were perfectly flexible. Then the P-bar model leads to an AS curve of the following form (the IS curve remains the same):

$$p_t - E_{t-1}\bar{p}_t = \phi(p_{t-1} - \bar{p}_{t-1}), \quad (9)$$

with $0 < \phi < 1$. The gap between the price level and the market-clearing price level closes monotonically.

I am skeptical of this model of price behavior since the market-clearing price level is likely to drive much of the dynamics of the actual price level. Put differently, the behavior of the price level is likely to closely resemble what would be generated by a real business cycle augmented with money. Since this latter type of framework has difficulty accounting for price dynamics, it is my conjecture that the same is likely to be true of the P-bar model.

A Simple Test

As with the Calvo-Rotemberg framework, a simple test of the P-bar specification is available. As the authors show, the model implies that, in expectation, the output gap closes monotonically. That is, equation (9) implies

$$E_{t-1}(y_t - \bar{y}_t) = \phi(y_{t-1} - \bar{y}_{t-1}). \quad (10)$$

We can then rewrite equation (10) in terms of observables, as follows:

$$y_t - \bar{y}_t = \phi(y_{t-1} - \bar{y}_{t-1}) + \eta_t, \quad (11)$$

where $\eta_t = (y_t - \bar{y}_t) - E_{t-1}(y_t - \bar{y}_t)$ and, accordingly, $E_{t-1}\eta_t = 0$. Then consider the following regression equation:

$$y_t - \bar{y}_t = \psi_1(y_{t-1} - \bar{y}_{t-1}) + \psi_2(y_{t-2} - \bar{y}_{t-2}) + \omega_t \quad (12)$$

under the null $0 < \psi_1 < 1$ and $\psi_2 = 0$.

Since under the null the error term is orthogonal to variables dated time $t$ and earlier, it is possible to estimate equation (12) using least squares. Doing so yields

$$y_t - \bar{y}_t = \underset{(0.078)}{1.233}(y_{t-1} - \bar{y}_{t-1}) + \underset{(0.078)}{0.320}(y_{t-2} - \bar{y}_{t-2}), \quad (13)$$

where the standard errors are in parentheses. The model is clearly rejected since $\psi_1$ is significantly above unity and $\psi_2$ is significantly above zero. Intuitively, the P-bar model implies that the output gap always converges monotonically to trend in expectation. This is inconsistent with the familiar hump-shaped output dynamics that appear to be present in the data.

Again, I stress that it is a great virtue of the authors' approach that the model is testable in a simple but highly informative way.

**Hypothetical versus Historical Policy**

I conclude with some remarks about how the hypothetical policy rules that perform best compare with actual historical policy. Least squares estimation of the policy rule (1) over the Volcker-Greenspan regimes (1979:4–97:3) yields

$$r_t = \underset{(0.085)}{0.245\,\Delta p_t} + \underset{(0.061)}{0.097(y_t - \bar{y}_t)} + \underset{(0.48)}{(0.860)r_{t-1}}, \tag{14}$$

where $r_t$ is the deviation of the federal funds rate from its mean and, as before, $\Delta p_t$ is the percentage change in the GDP deflator expressed as a deviation from its sample mean. Also, as before, the numbers in parentheses are standard errors. Note that if we let $r_t^*$ denote the long-run response of the funds rate, then equation (14) implies

$$r_t^* = 1.75\,\Delta p_t + 0.68(y_t - \bar{y}_t). \tag{15}$$

The two gap coefficients are thus in the same ballpark as those used in the simple Taylor rule (1.75 vs. 1.50 on $\Delta p_t$ and 0.68 vs. 0.50 on $y_t - \bar{y}_t$).

The rules that perform well in the simulations call for a much more aggressive adjustment of rates to inflation than occurs in practice (e.g., 3.000 vs. 0.245) and greater lagged dependence (1.00 vs. 0.86). This conclusion is rather puzzling given that both Volcker and Greenspan have the reputation of being hardnosed about controlling inflation.

The question then arises whether something may be missing from the analysis. As I argued earlier, one possibility is that model misspecification is a factor. For example, given that the aggregate supply curve does not seem to capture the persistence in inflation, the model may understate the output volatility costs of aggressive inflation policies.

Another possibility is that the experiment undertaken does not adequately capture the environment in which policy decisions are made. In particular, in the simulations the model is treated as if it characterizes the way the economy works with certainty, even though it is estimated with error and has a structure that is open to debate. In practice, however, Alan Greenspan does not know how the economy operates with nearly as great confidence as is presumed in the hypothetical experiments. Directly accounting for this uncertainty about the way the world works would seem to be the logical next step in this research agenda.

**References**

Bernanke, Ben, and Mark Gertler. 1995. Inside the black box: The credit channel of monetary policy transmission. *Journal of Economic Perspectives* 9:27–48.

Calvo, Guillermo. 1983. Staggered prices in a utility maximizing framework. *Journal of Monetary Economics* 12:383–98.

Campbell, John, and N. Gregory Mankiw. 1989. Consumption, income and interest rates: Reinterpreting the time-series evidence. In *NBER macroeconomics Annual 1989,* ed. O. J. Blanchard and S. Fischer, 185–216. Cambridge, Mass.: MIT Press.
Goodfriend, Marvin, and Robert King. 1997. The new neoclassical synthesis. In *NBER macroeconomics annual 1997,* ed. Ben S. Bernanke and Julio J. Rotemberg, 221–82. Cambridge, Mass.: MIT Press.
Hall, Robert. 1978. Stochastic implications of the life-cycle/permanent income hypothesis: Theory and evidence. *Journal of Political Economy* 86 (December): 971–87.
Rotemberg, Julio. 1982. Sticky prices in the United States. *Journal of Political Economy* 90:1187–1211.
Taylor, John. 1980. Aggregate dynamics and staggered contracts. *Journal of Political Economy* 88:195–214.

# Discussion Summary

*Bob Hall* pointed out the fact that potential output is measured by hours per worker, a concept used in the cyclical productivity literature, especially by Susanto Basu. Basu interprets hours per worker as the firm's current position on its upward-sloping marginal cost schedule, which is defined, as in this paper, by adjustment costs. Since hours per worker, however, are much less persistent than unemployment, Hall suggested a more traditional measure of the output gap based on inverting and smoothing Okun's law used in Hall and Taylor (1991). In this case, the output gap is inferred from the unemployment gap. *McCallum* replied that the estimated coefficients of a second-order autoregression of the output gap measure used in the paper are similar to those of unemployment indicating that the two series are equally persistent.

*James Stock* emphasized the importance of a stable relationship between inflation and whatever measure of potential output is used. In this sense, unemployment or the Federal Reserve's capacity utilization rate may constitute good measures of potential GDP. Stock also suggested estimating the trend in output with more flexible specifications along the lines of Kuttner (1994) and Stock and Watson (1998).

As to the instrumental variables used in the estimations, Stock noted that identification is mostly achieved by the constant, the dummy variables, and the time trend, with the quarterly growth rate of government spending being only a weak instrument. He also suggested that an approach with nonstochastic instruments might be justified with integrated time series, but this would raise a new set of issues not addressed in the paper. Some estimations show Durbin-Watson statistics of below 0.2, which, in combination with the previous argument, indicate the presence of serious econometric problems that may lead to distributional questions and even biased estimates.

*Laurence Ball* pointed out that Roberts (1997) empirically rejects the model proposed in Roberts (1995) referred to in the paper and constructs a model in which some of the agents have backward-looking expectations. This assumption on expectation formation may be problematic, especially in situations of future policy changes. He then asked whether the coefficients on past and future inflation in the regressions performed in Gertler's discussion could be interpreted as a test of the Roberts (1997) model. *Gertler* cautioned that the coefficients are only consistently estimated under the null, but he agreed with the general intuition.

*Ben Friedman* remarked that the typical policy rules recommended in these papers seem to be more aggressive than the rules actually employed by central banks. This is to be expected when taking into account the central banks' uncertainty about the true model of the economy. The methodological approach in the literature is to ignore such parameter uncertainty in the derivation of policy implications. However, this uncertainty matters for the Fed's decision making. *Donald Kohn* explained that data observations that are at variance with what the central bank expects may be interpreted both as new shocks to the economy and as misspecifications of the economic model. While prudent behavior by central banks may partially be explained by Brainard uncertainty, what decisions are appropriate in such a situation is mostly still an open question.

Friedman then wondered about the robustness of the results derived in the paper with respect to mismeasurement in inflation. *Nelson* replied that as long as the measurement error in inflation was constant over time, the error would be absorbed into the constant terms in the model's equations. However, Nelson expressed doubts regarding robustness when measurement errors are time varying.

*Lars Svensson* liked the microfoundations of the paper but criticized its abuse of the notion of targeting. For example, the paper talks about "inflation targeting" and "nominal GDP targeting" when denoting cases in which the respective variables appear as arguments in the central bank's reaction function. Svensson clarified that the arguments of the reaction function are more appropriately called indicator variables, while the arguments of the loss function are more appropriately called targets.

*Michael Woodford* questioned the microfoundations of the "P-bar" model. Persistence in the effects of a monetary policy shock on output is all explained by the adjustment costs in output. Despite these adjustment costs, output is able to move away from trend because prices are fixed one period in advance. If the adjustment costs are assumed to be high in order to generate a significant persistence in the output response, then it is no longer clear why suppliers match whatever demand is realized without changing prices. Thus large costs in changing prices have to be assumed as well. *McCallum* replied that the P-bar model is the only model presented at the conference that conforms to Lucas's definition of the natural rate hypothesis, which is that monetary policy does not have long-run effects on output relative to capacity.

## References

Hall, Robert, and John Taylor. 1991. *Macroeconomics: Theory, performance, and policy.* New York: Norton.

Kuttner, Kenneth N. 1994. Estimating potential output as a latent variable. *Journal of Business and Economic Statistics* 12 (3): 361–68.

Roberts, John M. 1995. New Keynesian economics and the Phillips curve. *Journal of Money, Credit and Banking* 27:975–84.

———. 1997. Is inflation sticky? *Journal of Monetary Economics* 39 (2): 173–96.

Stock, James, and Mark W. Watson. 1998. Median unbiased estimation of coefficient variance in a time varying parameter model. *Journal of the American Statistical Association* 93:349–58.