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Forecasting in its Relation to Government Policy-Making

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Private firms, especially large ones, usually have sizable central planning departments. Western governments, although their economic problems are more weighty and complex, do not generally employ such assistance. One finds when traveling through the administrative centers of Western countries only small staffs, at scattered locations. One reason may be the idea that looking into the future in a comprehensive, consistent, and centralized fashion is close to socialist planning. Another may be that our knowledge of the mechanism of economic systems is still so primitive that the economist's advice is not always of value.

The situation is slightly different in the Netherlands. After World War II the Central Planning Bureau was established. It now has some ninety employees, half of them graduate economists.[1] One of its tasks is to supply a system of coherent forecasts of the development of certain macroeconomic variables in the next year (the annual "Central Economic Plan"), including predictions based on alternative government policies.

The tool employed is an econometric equation system. The Plan 1955, for example, was based on a system of 27 linear (or linearized) relations of which 11 were behavior relations; 4, institutional relations; and 12, definitions. There were two consumption functions, one for wage income and one for nonwage income. The endogenous variables included price indexes as well as volumes and values. The price indexes (of consumption goods, investment goods, etc.) were described as functions of the wage level and the import price level, the latter two variables being considered as exogenous.[2]

In the following section I shall evaluate briefly the accuracy of recent Dutch predictions.[3] I shall then proceed to the problem of decision-making by means of econometric models.

[1] For a description of the activities of the Bureau in comparison with similar operations carried out in the United Kingdom, see R. L. Marris, "The Position of Economics and Economists in the Government Machine: A Comparative Critique of the United Kingdom and the Netherlands," *Economic Journal*, December 1954, pp. 759-783.

[2] The equation system in its complete form has been published in the *Centraal Economisch Plan 1955*, The Hague, 1955, pp. 110-119.

[3] See also my "A Statistical Appraisal of Postwar Macroeconomic Forecasts in the Netherlands and Scandinavia," a paper read at the Rio de Janeiro Meeting of the International Statistical Institute (1955). This paper will be published in the Proceedings of the Meeting.

Measuring the Accuracy of the Dutch Model Predictions

INTRODUCTION

The analysis of the accuracy of forecasts must be distinguished carefully from a verification of the forecasting procedure. In point predictions (in contrast with interval predictions), a forecasting procedure is a *method* of deriving predictions with the following properties: the forecasting errors have a zero mean, or a zero median, or a zero upper quartile, and so forth. Verification amounts then to a statistical test of the null hypothesis that the observed prediction errors are drawings from a parent characterized by such properties, against some specified alternative hypothesis. Clearly, verification is possible only if the forecaster states explicitly what kind of probability properties his errors are supposed to have. For the Dutch macroeconomic forecasts this is not the case. Nor are probability properties of the disturbances of the equation system, like variances and covariances, specified explicitly. Accuracy analysis, on the other hand, does not require probability assumptions since it is concerned with the empirical variation of the forecasts around the "actual" quantities which they serve to predict.

This section is devoted to an accuracy analysis of the Dutch forecasts. Such an analysis can be carried out at three distinct levels:

1. The forecast values are taken as given and are compared with the corresponding actual values. This is a straightforward approach, which, unlike 2 and 3 below, can be applied irrespective of the existence of an equation system.
2. The exogenous variables are separated from the endogenous ones, and the observed values of the former are inserted in the equation system. The corresponding values of the endogenous variables are then derived and compared with the observed endogenous values. (The forecast here is conditional upon the exogenous variables being as observed.)
3. For each (structural) equation of the model, the variable explained is separated from the explanatory variables, and the observed values taken by the latter are inserted in the equation. The corresponding computed value of the variable explained is then compared with the observed value. For example, if the equation system contains a function describing consumption as dependent on income, prices, and other exogenous and endogenous variables, then the values taken by the latter variables determine a certain consumption figure, which can be compared with the observed amount in order to judge the accuracy of the consumption function.

My analysis here will be confined to approaches 1 and 2.

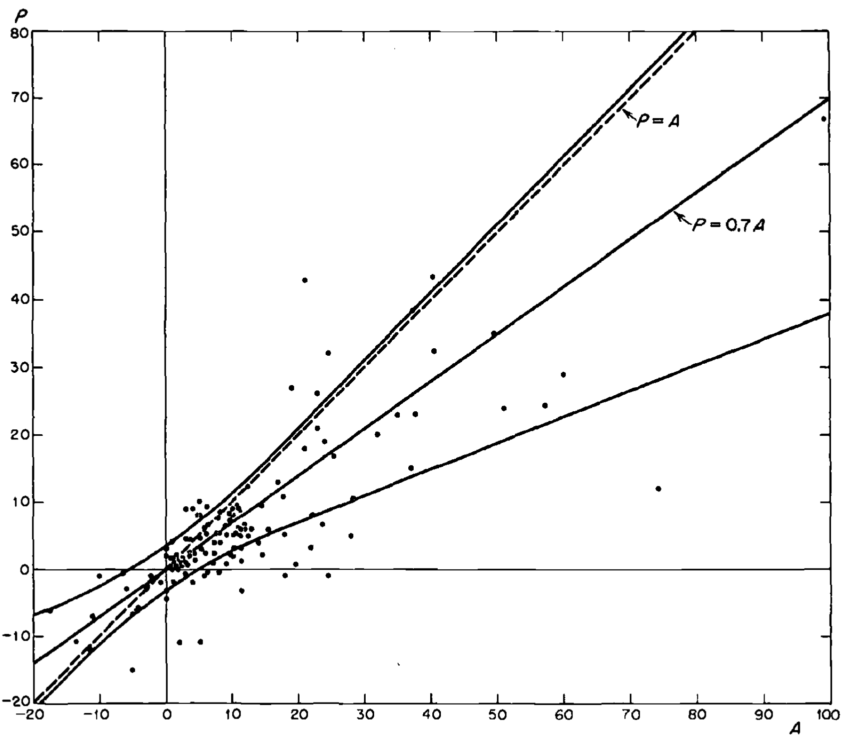
ON THE ACCURACY OF THE UNCONDITIONAL FORECASTS

The analysis is based on the development of twenty-three variables from 1949 through 1955, 1952 being excluded.[4] A summarizing picture of all pairs of forecasts and corresponding observed changes is given in Chart 1. Predicted (percentage) changes are defined as

$$P = \frac{p_t - a'_{t-1}}{a'_{t-1}} 100 \tag{1}$$

CHART 1

Comparison of Actual and Predicted Percentage Changes, 23 Variables, 1949–1955, Excluding 1952
(predicted values taken as given)



Source: Appendix Table A-1.

where p_t is the predicted level of some variable in year t and a'_{t-1}, the actual level of the preceding year as it was known when the forecast was made. The latter value is to be distinguished from a_{t-1}, which is the actual

[4] For 1952, alternative forecasts were prepared, a fact which hampers a satisfactory appraisal. For a survey of the relevant figures, see Table 1, p. 43.

level in $t-1$ according to later statistical data. This level is used in the definition of the actual change:

$$A = \frac{a_t - a_{t-1}}{a_{t-1}} 100 \tag{2}$$

The distinction between a'_{t-1} and a_{t-1} removes the disturbing effect of imperfect statistical knowledge of the preceding year, which is desirable because this imperfection has nothing to do with the quality of predictions for the next year.

Chart 1 shows that the majority of the points are situated in the "correct" first and third quadrants, implying that there are relatively few turning-point errors. But it shows also that most of the points in the first quadrant are below the line of perfect forecast (the broken line through the origin), which implies a bias toward underestimation of changes. More precisely, the percentage distribution of the 134 forecasts over the categories is as follows: turning-point errors[5]—12; correct-sign predictions —88, of which 65 were underestimation of changes[6] and 24 were overestimation of changes.

To compare these results with those obtained by two well-known "naïve" forecasting methods—the no-change extrapolation ($P \equiv 0$) and the extrapolation of last year's change ($P \equiv A_{-1}$)—the inferiority of the $P \equiv 0$ method is well illustrated if all points of Chart 1 are shifted vertically until they reach the horizontal axis. While this reduces the number of turning-point errors to zero, the picture as a whole is considerably worse. The implications of the method $P \equiv A_{-1}$ are illustrated in Chart 2. The distribution over turning-point errors and underestimation and overestimation of changes is as follows:[7] turning-point errors—20; correct-sign predictions—80, of which 39 were underestimation of changes and 41 were overestimation of changes. Although the bias toward underestimation appears to be absent, again the picture as a whole is evidently worse.

The comparisons are favorable to the Dutch forecasts but the forecasts are far from perfect.

1. There is the bias toward underestimation of changes, which can be formalized by the regression:

$$P = 0.7A \tag{3}$$

implying that the predictions are on the average equal to 70 per cent of the corresponding observed changes. This regression is indicated by the solid upward sloping line through the origin of Chart 1.

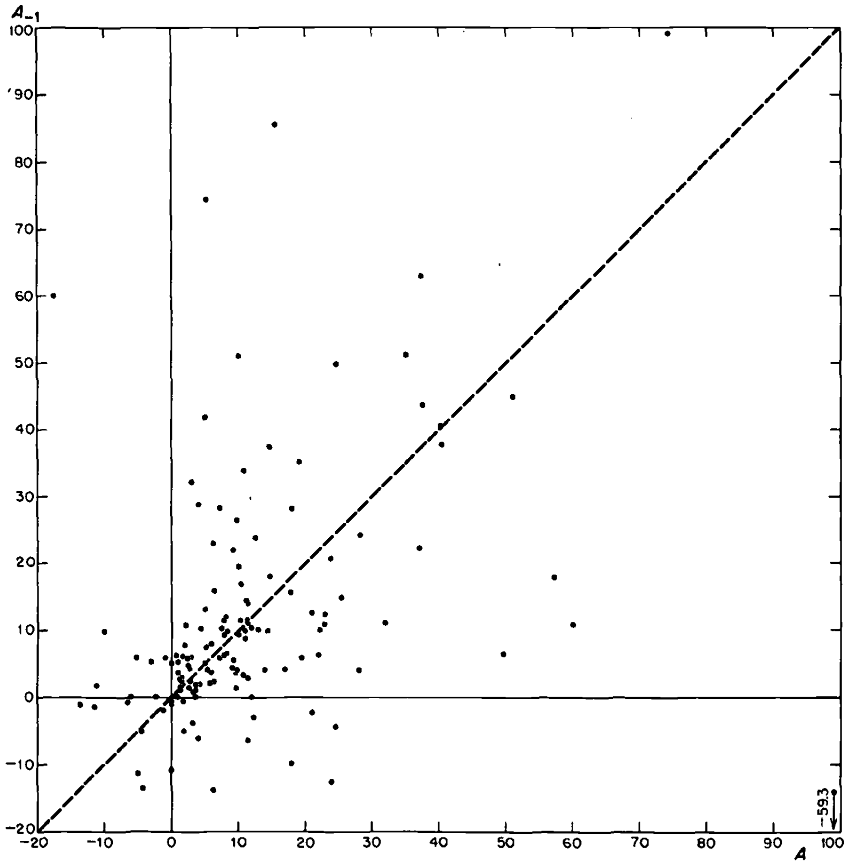
[5] Including nonzero predictions of zero changes and zero predictions of nonzero changes. The frequency of these cases is 4 per cent.

[6] The (rare) cases of perfect forecasts have been equally divided between underestimation and overestimation.

[7] See footnotes 5 and 6.

CHART 2

Comparison of Actual and Predicted Percentage Changes, 23 Variables, 1949–1955, Excluding 1952 (predicted values derived from extrapolation of previous year's change)



2. The residuals around the regression line do not have constant variance. A closer inspection of Chart 1 suggests that the variance is least near the origin and that it increases when we proceed in a northeastern or southwestern direction. This can be formalized by the "scedastic" regression:

$$(P - 0.7A)^2 = 10 + 0.1A^2 \tag{4}$$

The two curvilinear solid lines of Chart 1 are the functions

$$P = 0.7A \pm \sqrt{10 + 0.1A^2} \tag{5}$$

which describes the forecasts according to the regression $(0.7A) \pm$ the standard deviation.[8]

ON THE ACCURACY OF CONDITIONAL FORECASTS

For 1949-54 an analysis of the conditional forecasts, based on approach 2, was carried out by Lips and Schouten.[9] It was based upon the development of 14 variables, of which 5 are price indexes; 5, volumes; and 4, values; so, as a whole, there are 84 conditional forecasts (P_C) and corresponding observed changes (A). Chart 3 shows that the forecasts are substantially better than the unconditional forecasts (Chart 1). The percentage distribution over turning-point errors and underestimation and overestimation of changes is as follows;[10] turning-point errors—19; correct-sign predictions—81, of which 44 were underestimation of changes, and 37 were overestimation of changes.

The bias toward change underestimation is much less important than it was for the unconditional forecasts and may not exist at all. The rather large number of turning-point errors is partly due to the inclusion of 1952, which was quite stable and showed minor changes relative to 1951. It also reflects the inclusion of five price indexes (instead of the three in the analysis of the unconditional forecasts), which were relatively stable in four of the six years. Obviously, when small changes are involved, the danger of turning-point errors is greater than in the case of substantial changes.

COMMENT

The unconditional forecasts, though imperfect, are not bad when compared with naïve methods, and the conditional forecasts are even better. Hence most of the errors in the unconditional predictions must be due to erroneous predictions of future exogenous values. Furthermore, while the unconditional forecasts show a clear tendency toward underestimation of changes, the conditional forecasts do not. Hence there must be a similar bias in the predictions of future exogenous values.

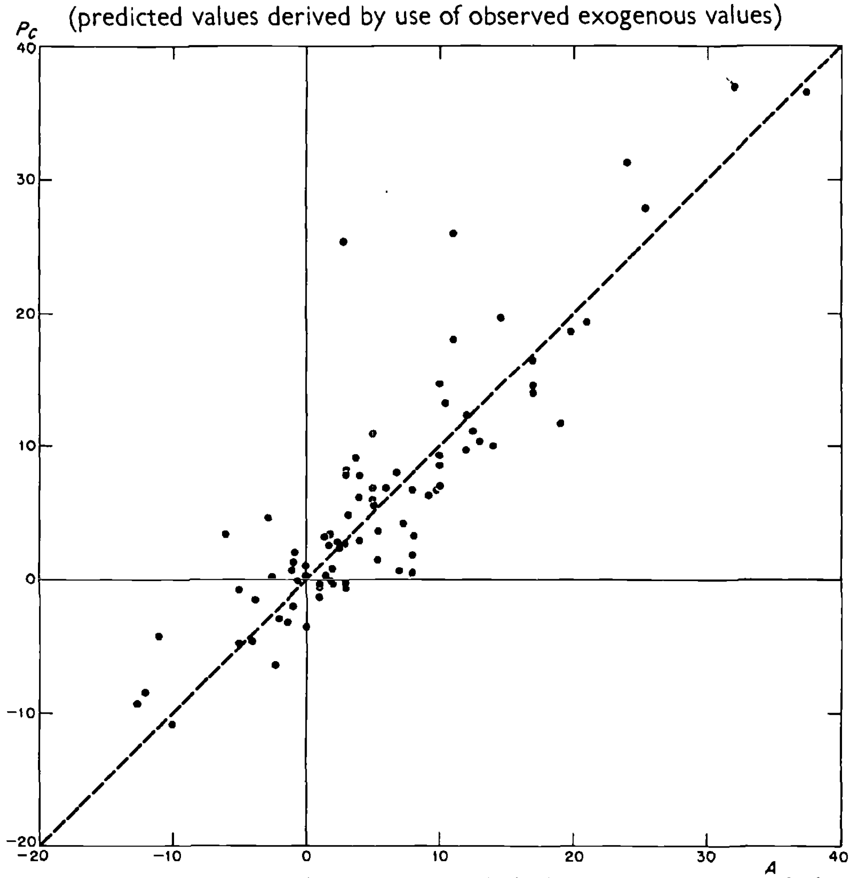
[8] The coefficients 0.7, 10, 0.1 of the two regressions are derived from the values for 1949-51 and 1953 in the following iterative manner. First, the median (m') of the individual ratios P/A are used as a preliminary estimate of the coefficient m in the regression $P=mA$; next, the squares of the deviations $P-m'A$ are grouped according to increasing order of $|A|$ and plotted in a scatter against the square of A. This yields the values 10 and 0.1 mentioned above. The final estimate of m is obtained by taking a weighted average of the ratio P/A, the weights being equal to $A^2/(10+0.1A^2)$. This is in accordance with weighted regression theory. For an alternative approach based on the "second regression line," see my *Economic Forecasts and Policy*, North-Holland, 1950.

[9] J. Lips and D. B. J. Schouten, "The Reliability of the Policy Model of the Central Planning Bureau," a paper read at the Hindsgavl Meeting of the International Association for Research in Income and Wealth, 1955 (*Income and Wealth Series VI*, London, Bowes and Bowes, 1957, pp. 24-51).

[10] See footnotes 5 and 6.

CHART 3

Comparison of Actual and Predicted Percentage Changes
14 Variables, 1949–1954
(predicted values derived by use of observed exogenous values)



But other factors contributed to the relatively good showing made by the Dutch forecasts:

1. The scarcity of turning-point errors owed much to the scarcity of turning points, thanks to strong upward trends for most of the variables in most of the years studied.

2. The Central Economic Plans were sometimes published rather late in the year to which they refer.[11]

3. The model contains many important exogenous variables. The change

[11] This "advantage" should not be overestimated. If we take the preliminary forecasts for 1949-51 and 1953 that were prepared around January (the final predictions for 1954 and 1955 were published rather early), we arrive at forty-four predictions. About 80 per cent of them are closer to the corresponding observed changes than no-change extrapolations, half the remainder consist of ties. Similarly, the extrapolations $P \equiv OA_{-1}$ are worse than about 70 per cent of these forecasts.

in the volume of exports, for instance, is largely exogenous from the Dutch point of view and the total amount of exports is considerable in relation to the national product. Another such variable is the wage level. Although the central government cannot change wages arbitrarily, it must consent to wage increases; and this makes the wage level effectively exogenous. Clearly, the insertion of the observed values of these variables in the model must contribute to the quality of the conditional forecasts of the endogenous changes.

4. The equation system underlying the conditional forecasts (that of Plan 1955) is one of a series of successive systems which are the result of theoretical and empirical experimentation. Although the systematic analysis of Lips and Schouten was the first of its kind, such experimentation, too, must contribute to the quality of the conditional forecasts for the earlier years of 1949-54.

These special features may partly explain the difference between the findings reported here and those of Christ's analyses of the predictive power of Klein's models of the United States economy.[12]

The Use of Forecasts in Decision-Making

The exposition in this section is mainly theoretical; it is presented in general, rather than in numerical terms. A more detailed treatment is presented in my monograph, *Economic Forecasts and Policy* (North-Holland Publishing Company, 1958). I shall confine myself to the static variant of the theory.

Consider a policy-maker (who may or may not be government employed), who has certain *instruments* $(x_1, \ldots, x_m)$ at his disposal and who is interested in certain *noncontrolled variables* $(y_1, \ldots, y_n)$. The relationship between these variables is supposed to be linear, and may be written briefly:

$$y = Rx + s \tag{6}$$

where x and y are column vectors of instruments and noncontrolled variables respectively, R a $n \times m$ matrix of multiplicative coefficients, and s a column vector of additive coefficients. The elements of the multiplicative structure (i.e. the elements of R) describe the effectiveness of the various instruments for the various noncontrolled variables.

Assuming that (6) describes the relationship between instruments and noncontrolled variables during a certain period, and that the policy-maker must choose among alternative actions (i.e. alternative x-vectors),

[12] C. F. Christ, "A Test of an Econometric Model for the United States, 1921-1947," in *Conference on Business Cycles*, Special Conference 2, National Bureau of Economic Research, 1951, pp. 35-107; also his "Aggregate Econometric Models," *American Economic Review*, June 1956, pp. 385-408.

the criterion will be derived by the introduction of a preference function which is supposed to be quadratic:

$$w(x, y) = a'x+b'y+\tfrac{1}{2}(x'Ax+y'By+x'Cy+y'C'x) \tag{7}$$

Hence, in mathematical terms, the policy-maker's problem consists of maximization of (7) subject to (6). This is largely comparable to the consumer's problem of maximizing a utility function subject to a linear budget constraint. However, classical demand theory ignores problems of uncertainty, which are vitally important here, because of the role played by predictions. Most economic models, from which reduced forms of the type (6) are derived, contain exogenous variables *not* controlled by the policy-maker. An example is the import price level in the Dutch equation system. Such variables do not fall under x. Instead their values have to be predicted before they are included in the vector s. Thus s is a rather heterogeneous mixture: it includes values assumed by certain exogenous variables and also disturbances.

The policy-maker will have in general an imperfect knowledge about the parameter matrixes of his constraints, and especially about s. To handle the problem in a probabilistic manner, we must assume that the policy-maker's preferences satisfy the Von Neumann–Morgenstern axioms, and that his preference function (7) is such that, if he is in an uncertain situation, he maximizes expected utility.

THE LOSS OF WELFARE DUE TO SUBOPTIMAL DECISIONS

First I shall disregard all problems of uncertainty and derive the policy-maker's "best" x (to be denoted by x^0), given preferences and constraints. Then w_x is the utility level attained when the vector x is applied:

$$w_x = w(x, Rx+s) = k_0+k'x+\tfrac{1}{2}x'Kx \tag{8}$$

where $k_0 = b's+\frac{1}{2}s'Bs$

$$k = a+R'b+(C+R'B)s = [I \quad R'] \begin{bmatrix} a & C \\ b & B \end{bmatrix} \begin{bmatrix} 1 \\ s \end{bmatrix} \tag{9}$$

$$K = A+R'BR+CR+R'C' = [I \quad R'] \begin{bmatrix} A & C \\ C' & B \end{bmatrix} \begin{bmatrix} I \\ R \end{bmatrix}$$

K being a square and symmetric matrix independent of s. Maximization of (8) with respect to x gives

$$x^0 = -K^{-1}k \tag{10}$$

Comparing the parameter matrixes R and s with the parameters of the consumer's budget constraint (i.e. with prices and income), and supposing that the elements of these matrixes take alternative values, we see that (10) can be regarded as the policy-maker's analogue of the consumer's demand functions. I shall call them the *optimal reaction functions.* The corresponding functions of the noncontrolled variables (y) are found by substituting (10) into (6). It is immediately apparent that the functions are all linear in s. This, too, is comparable with consumer's demand theory. For, if the utility function is quadratic, the Engel curves are all straight lines; and these curves describe the consumer's optimal purchases as dependent on the additive coefficient of his constraint (i.e. on his income).

If the policy-maker applies the instrument vector x^0, then he arrives at the maximal welfare level attainable:

$$\text{(11)} \qquad \begin{aligned} \hat{w} &= k_0 - \tfrac{1}{2}k'K^{-1}k \\ &= k_0 - \tfrac{1}{2}x^{0\prime}Kx^0 \end{aligned}$$

If he applies any other instrument vector x, then the level attained is given by (8). Subtracting (8) from (11), we find the *loss of welfare* due to the suboptimal decision x:

$$\text{(12)} \qquad \begin{aligned} \hat{w} - w_x &= \tfrac{1}{2}[x^{0\prime}Kx^{0\prime} - 2x^{01}Kx + x'Kx] \\ &= -\tfrac{1}{2}(x-x^0)'K(x-x^0) \end{aligned}$$

which is a quadratic form in the decision error $x - x^0$, the matrix of this form being $-\frac{1}{2}K$.

This result suggests that K should be a negative-definite matrix. I shall not go into detail here, but confine myself to the following remarks. If the generalization of consumer's demand theory is carried out consistently, substitution and complementarity relationships can be defined in a natural manner. In particular, there is a substitution matrix of order $m+n$ and rank m, containing substitution terms for all pairs of variables, instruments as well as noncontrolled variables. Consider then that submatrix of order $m \times m$ of the substitution matrix that corresponds to pairs of instruments only. It can be shown that this submatrix is equal to the inverse of K. Hence the matrix of the loss of welfare is determined by substitution and complementarity relationships among instruments.

THE LOSS OF WELFARE DUE TO IMPERFECT PREDICTIONS

The loss defined above is associated with a certain x, and is entirely independent of how the decision is arrived at. Consider now the loss of welfare due to imperfect forecasts. Suppose that the policy-maker's actual constraint is given by (6), but that he thinks that his constraints are

$$\text{(13)} \qquad y = Rx + s_e$$

If $s_e \neq s$, the additive structure of the constraints in the relevant period is predicted imperfectly. It is also possible to proceed under the more general assumption that the multiplicative structure is known imperfectly, so that the matrix R of (13) must be replaced by $R_e \neq R$. This is, however, beyond the scope of the present paper. The special attention given to the case $R_e = R$ can be justified by reference to the empirical finding that the large difference between the quality of the conditional and the unconditional forecasts is due to deficiencies in the prediction of future exogenous values. The values, as far as they belong to noncontrolled exogenous variables, are components of s, not of R.

If the policy-maker assumes that his constraints are given by (13), he chooses that x which will maximize utility under these conditions. This x is different from x^0. Actually, it is not difficult to see that the resulting decision error $x - x^0$ is only the change in x according to the optimal reaction functions that would take place if the constraints changed from (6) to (13). Given the linearity of these functions in s, we find that the decision error resulting from the prediction error $s_e - s$ equals

$$x - x^0 = H(s_e - s) \tag{14}$$

where (cf. equations (9) and (10))

$$H = -K^{-1}(C + R'B) \tag{15}$$

Combining (14) with (12), we find for the loss of welfare due to the forecasting error $s_e - s$:

$$-\tfrac{1}{2}(s_e - s)'F(s_e - s) \tag{16}$$

where

$$F = H'KH \tag{17}$$

It will prove convenient in the following discussion to use a utility scale (or loss-of-welfare scale) which is fully numerical. Given the assumption that the Von Neumann–Morgenstern axioms are satisfied, this means that we should define a zero and a unit. A zero has already been defined as the loss of welfare which corresponds to a perfect forecast ($s_e = s$). I shall define the unit as the loss of welfare that results from no-change extrapolation; that is, as the loss which occurs if the policy-maker acts under the assumption that s equals the additive structure of the constraints in the preceding period, s_{-1}. Writing the components of the additive structure as deviations from the corresponding components of the preceding period (so that s_{-1} becomes a zero vector), gives what will be called the *failure* of the forecast s_e:

$$\text{fail}\, s_e = \frac{(s_e - s)'F(s_e - s)}{s'Fs} \tag{18}$$

The failure is obviously zero for a perfect forecast,[13] one for a no-change

[13] The failure can also be zero for particular imperfect forecasts since F may be semi-definite, e.g. if $m < n$ (fewer instruments than noncontrolled variables). In that case a class of vectors $s_e \neq s$ exist of which the errors $s_e - s$ "compensate" each other in such a way that they do not affect the choice for x.

extrapolation, and it has no finite upper limit. Similarly, the *success* of a forecast is defined as the excess of the welfare level attained over the level corresponding to the no-change extrapolation, measured in the same unit:

$$\text{suc } s_e = 1 - \text{fail } s_e \tag{19}$$

The success is one if the forecast is perfect, zero if it is a no-change extrapolation, and it can take negative values.

The justification of the definitions is threefold.

1. The policy-maker is always able to keep failure and success between zero and one, provided his information is of sufficient quality and provided that the multiplicative structure of the preceding period is known (for then he can derive s_{-1} as $y_{-1} - R_{-1}x_{-1}$, the -1 indexes being interpreted as referring to the previous period).
2. They are related to the comparison with naïve forecasting methods.
3. They can be easily brought in connection with the bias toward underestimation of changes.

To grasp the last assertion, consider the simple case in which the changes in all components of the additive structure are proportionally underestimated. Since I wrote $s_{-1} = 0$, this assumption implies

$$s_e = (1-\theta)s \tag{20}$$

θ being a scalar in the interval (0, 1). If for θ the average mentioned for the Dutch forecasts (30 per cent) is taken and combined with (20) and (18), the result is a failure of 0.09 and hence a success of 0.91; a result independent of the numerical characteristics of the policy-maker's constraints as well as of those of his preferences.

THE POSSIBILITY OF A NEGATIVE PARENT SUCCESS

The picture of the Dutch forecasts presented here is too favorable. When the underestimation of changes is not proportionate, the success is reduced. I shall discuss in particular the pattern of errors which was observed in the unconditional forecasts and also the success of the forecasts in a statistical population rather than in some specified case.

Suppose that the change in the additive structure ($s - s_{-1} = s - 0 = s$) consists of a systematic part ($\bar{s}$) and a stochastic part (u):

$$s = \bar{s} + u \tag{21}$$

The stochastic part is supposed to have a zero mean and a finite covariance matrix which is independent of the instruments:

$$Eu = 0 \qquad E(uu') \text{ independent of } x \tag{22}$$

Perfect forecasting of the additive structure of the constraints implies perfection with respect to both $\bar{s}$ and u, but I shall not make this assumption. Instead, I shall write for the predicted additive structure s_e:

$$s_e = \beta_s \bar{s} + \beta_u u + v \tag{23}$$

where β_s and β_u are scalars defined according to

$$\beta_s = \frac{E(\bar{s}'Fs_e)}{\bar{s}'F\bar{s}} \quad \text{and} \quad \beta_u = \frac{E(u'Fs_e)}{E(u'Fu)} \tag{24}$$

Given these β-definitions, the relation (23) is merely a definition of the vector v. I shall call the elements of this vector "forecast disturbances" since (23) can be regarded as a parent regression of s_e on $\bar{s}$ and u, provided that the elements of these three vectors are interpreted as "observations" on three "variables." Another proviso is that we interpret this regression in the sense of Aitken's method of generalized least-squares, the inverse of the residual covariance matrix of this method being replaced by the positive-definite (or semi-definite) matrix $-F$.

The relevance of (23) can be shown as follows. If $0<\beta_s<1$, then the change in the additive structure, as far as its systematic component ($\bar{s}$) is concerned, can be said to be underestimated. If $\beta_u \neq 1$, the forecast is imperfect for the stochastic component. If $\beta_u = 0$, nothing is achieved in this respect—which will be frequently true. As to the forecast disturbances v, they make it possible to take account of the hyperbolic standard deviations of the residuals around the empirical regression developed in the analysis of unconditional forecasts. Assume:

$$E(v'Fv) = h_s \bar{s}'F\bar{s} + h_u E(u'Fu) \tag{25}$$

h_s and h_u being fixed positive scalars. The second term on the right can be regarded as a constant, given the assumption that the covariance matrix of the stochastic component u is independent of the instruments; compare (22). Hence (25) describes the expectation of a quadratic form in the forecast disturbances as a general linear function of the same quadratic form in the elements of the systematic part of the additive structure of the constraints. If we compare the forecast disturbances v with the residuals $P-0.7A$ and recall that their squares can be approximately described as linearly dependent on A^2, we see that assumption (25) is closely related to this empirical result.

Next I shall introduce the concepts of failure and success in a parent population. They are obtained by replacing sample moments by corresponding parent moments, and will be indicated by "Fail" and "Suc," respectively:

$$\text{Fail}\, s_e = \frac{E[(s_e - s)'F(s_e - s)]}{E(s'Fs)}; \quad \text{Suc}\, s_e = 1 - \text{Fail}\, s_e \tag{26}$$

Then, using (21)–(25), we find after some algebraic rearrangements:

$$\text{(27)} \qquad \text{Suc } s_e = p(2\beta_s - \beta_s^2 - h_s) + q(2\beta_u - \beta_u^2 - h_u)$$

where

$$\text{(28)} \qquad p = \frac{\bar{s}'F\bar{s}}{\bar{s}'F\bar{s} + E(u'Fu)} \quad \text{and} \quad q = 1 - p$$

The interpretation of this result is as follows. If the systematic part $\bar{s}$ of the change in the additive structure of the constraints is sufficiently large compared with the stochastic variation of u, then p is close to one and q close to zero. In that case (27) becomes effectively

$$\text{(29)} \qquad \text{Suc } s_e = 2\beta_s - \beta_s^2 - h_s$$

which means that the prediction achievements for u are then irrelevant. The maximum of (29), given h_s, is reached for $\beta_s = 1$; and this maximal value equals $1 - h_s$, which will generally be positive. If, on the other hand, $\bar{s}$ is a vector of sufficiently small elements, (27) becomes approximately

$$\text{(30)} \qquad \text{Suc } s_e = 2\beta_u - \beta_u^2 - h_u$$

Here everything depends on the forecaster's achievements for the random component u. If he is unable to achieve anything in this direction (i.e. if $\beta_u = 0$), then the success takes a *negative* value $(-h_u)$. This means that the policy-maker should not use his forecast s_e at all. He should act under the assumption of a zero change in the additive structure, in which case his success is zero. This does not mean that he should take no measures. Since the adaptation of his instruments at the beginning of the preceding period will in general have been suboptimal owing to prediction errors for that period he should adapt the instruments to the correct additive structure of the previous period. He should, however, disregard his ideas about the additive structure of the new period, for this would lead to a larger loss—not necessarily in a particular sample case, but in the parent population of such cases.

ISOLATED REDUCTIONS OF PREDICTION ERRORS

Quite frequently a forecaster is advised to revise his prediction in a particular direction. For example, he may be advised to change the first element of the predicted additive structure s_e. It might seem that this advice should be adopted if it brings the element closer to the corresponding component of the additive structure s, but this is not necessarily true. To see this, consider the numerator of the failure of s_e: $(s_e - s)'F(s_e - s)$. If the failure matrix F is diagonal, the numerator is a weighted sum of squares of the forecasting error of the additive structure, the weights all having the same sign. Then a numerical reduction of any component of $s_e - s$ is desirable, irrespective of whether the error component obtains a different

TABLE 1

Predicted and Actual Percentage Changes in Dutch Macroeconomic Variables, 1948-1955

	1948	1949		1950		1951		1952		1953		1954		1955	
Variables	*A*	*P*	*A*	*P*	*A*	*P*	*A*	*P*	*A*	*P*	*A*	*P*	*A*	*P*	*A*
Price indexes:															
Commodity exports	0	−3.0	−6.0	+9.0	+4.0	+13.0	+17.0	—	+1.7	−7.0	−11.0	−4.5	0	0	+1.0
Commodity imports	+5.3	−3.0	−3.0	+12.4	+12.4	+21.0	+22.9	—	−1.5	−12.0	−11.5	−6.7	−5.0	0	+1.8
Consumption	+4.0	+3.0	+5.4	+5.0	+9.2	+9.0	+10.0	—	−0.8	−3.1	0	+2.0	+3.6	+1.7	+1.0
Quantities and volumes:															
Commodity exports	+44.9	+24.0	+51.0	+23.0	+19.0	+27.0	+35.0	—	+4.0	+4.0	+14.0	+6.0	+11.4	+5.0	+10.3
Commodity imports	+8.7	+9.0	+11.0	+20.0	+32.0	+9.0	+3.0	—	−12.6	+19.0	+24.0	+10.6	+28.2	+0.8	+7.2
Consumption	+5.8	−2.0	−0.9	+2.0	0	−1.0	−2.3	—	+1.9	+4.5	+3.7	+6.2	+5.9	+2.3	+7.3
Industrial production	+19.4	+8.0	+10.0	+6.0	+13.0	+10.0	+5.0	—	0	+6.0	+12.0	—	—	—	—
Construction activity	+41.7	+8.0	+5.0	+8.0	+5.0	+3.0	0	—	−2.4	+18.0	+21.0	—	—	—	—
Available labor force	+4.6	+1.1	+2.5	+0.6	+1.5	+1.0	+1.3	—	+1.1	+1.1	+1.4	+1.7	+2.6	+1.4	+1.3
Labor productivity	+10.1	+4.0	+7.4	+4.7	+5.2	+4.2	+1.0	—	+2.2	+5.2	+6.2	+1.6	+0.7	+0.6	+3.4
Employment in private sector	+5.8	+0.6	+2.4	+1.2	+2.9	+1.5	+1.5	—	−0.6	+1.4	+1.8	+2.3	+4.2	+1.3	+2.7
Employment in public sector	−2.0	−1.8	−1.1	−10.9	−13.5	−5.8	−4.3	—	+6.2	+5.3	+7.9	+2.4	+6.0	+2.1	+1.6
Money values:															
Commodity exports	+43.5	+23.0	+37.6	+32.5	+40.5	+43.4	+40.2	—	+5.9	−0.6	+2.8	+1.2	+11.4	+5.0	+11.4
Commodity imports	+15.7	+6.4	+6.3	+35.1	+49.6	+32.1	+24.6	—	−13.9	+9.3	+6.2	+3.2	+21.9	+0.8	+9.1
Consumption	+10.1	+1.3	+4.4	+6.5	+9.1	+7.7	+7.9	—	+1.1	+2.3	+3.6	+8.3	+9.7	+4.0	+8.3
Net investment (incl. inventories)	+33.8	+9.3	+10.7	+29.0	+60.0	−6.1	−17.5	—	−59.3	+66.9	+99.1	+12.1	+74.3	−10.9	+5.2
Value added in private sector	+16.8	+3.1	+10.4	+6.7	+11.9	+8.5	+8.1	—	+4.0	+2.1	+9.8	+6.1	+11.0	+3.1	+11.4
Government wage bill	−0.8	−0.7	−6.5	−3.2	+11.4	+5.4	+7.9	—	+1.3	+7.3	+9.8	+9.5	+14.4	+8.9	+11.1
Government commodity purchases	+20.5	+6.7	+23.7	+5.0	+12.5	+42.9	+21.0	—	+10.8	+26.1	+22.8	−0.5	+6.3	−0.4	+8.0
Indirect taxes minus subsidies	+62.9	+38.5	+37.3	+14.9	+14.6	+16.9	+25.4	—	−3.8	+4.6	+3.2	+5.3	+10.7	+1.1	+2.1
Exports of services	+50.9	+2.0	+10.0	+8.0	+22.0	+15.0	+37.0	—	+7.6	−11.0	+2.0	−1.0	+5.8	+0.7	+19.5
Imports of services	+28.7	−2.0	+4.0	+5.0	+28.0	−1.0	+18.0	—	+9.7	−1.0	−10.0	+5.1	+17.9	+2.2	+14.7
Surplus on the balance of services	+85.5	+5.9	+15.5	+10.8	+17.7	+24.5	+57.2	—	+5.9	−15.1	−5.1	−6.3	−4.5	−1.0	+24.5

sign. For the alternative case, in which the failure matrix is not diagonal, it is sufficient to consider an example; say, $n=2$ and

$$-F = \begin{bmatrix} 1 & -0.9 \\ -0.9 & 1 \end{bmatrix}$$

Take further the forecasting error $s_e - s = \{1 \quad 1\}$. The numerator of the failure of s_e is then equal to 0.2, apart from sign. Suppose also that it is suggested that the forecast of the second component of s be changed from 1 to a. The numerator of the failure becomes then (apart from sign): $1-1.8a+a^2$, the minimum of which is 0.19, which is reached at $a=0.9$; and it exceeds the level 0.2 mentioned above whenever a is outside the interval (0.8, 1). Clearly, the non-diagonality of F implies that the elimination of the second component of the prediction error is far from desirable. Replacing $s_e - s = \{1 \quad 1\}$ by $\{1 \quad 0\}$ increases the failure fivefold! This holds more generally. If the failure matrix is not of the diagonal type (and there is no reason why it should be), a reduction of one particular component of the prediction error $s_e - s$, the other components remaining the same, does not necessarily lead to a failure reduction. It may be that it is better to raise the component than to reduce it. In the above example, if the original error $s_e - s$ equals $\{1 \quad \frac{1}{2}\}$, the failure is reduced by raising the second component to the 0.9 level, given the value 1 of the first component. The reason is that if F is not diagonal, the components of the error $s_e - s$ may compensate each other *via* their influence on the decision error.

COMMENT

JOHN W. LEHMAN AND JAMES W. KNOWLES, Joint Economic Committee

The opportunity to review Henri Theil's paper is particularly welcome in view of the constant concern of the staff of the Joint Economic Committee with the problems with which he deals. Judging from his paper, work on the day-to-day difficulties of forecasting in relation to government policy-making results in professional thinking along similar lines, regardless of country. Our own experience provides the basis for our comment on Theil's analysis.

ON THE THEIL PAPER

Theil prefaces his discussion of the Dutch forecasts with the observation that most Western governments rely on small, scattered staffs to make forecasts. In the sense that many of the analytical personnel are dispersed among numerous operating agencies, this is true of our own government —though our Committee staff numbers from eleven to thirteen and the

NOTE: The views expressed are those of the authors and do not necessarily represent the views of the Joint Economic Committee or individual members of that Committee.

staff of the Council of Economic Advisers from thirty to fifty, which is not small considering the methods of operation. However, the reasons Theil adduces for decentralization do not apply here. The collection and processing of the basic data, as well as the formulation of detailed operating programs, take place in the various departments and agencies. By being on the spot, individual technicians working on different aspects of the policy problems acquire a more intimate knowledge of the basic procedures.

One method Theil uses to determine the accuracy of the Dutch projections is to compare the forecast values with the corresponding observed values. He defines the former in terms of predicted percentage changes from the actual level of the preceding year as it was known when the forecast was made. The actual values employed are percentage changes computed in the light of later data, both for the preceding year and for the forecast year. He states that this "removes the disturbing effect of imperfect statistical knowledge of the preceding year, which is desirable because this imperfection has nothing to do with the quality of predictions for the next year."

Theil is correct that the forecaster's imperfect knowledge of the base year needs to be taken into account, but his assumption that it has nothing to do with the quality of the forecast is open to serious challenge. For example, there may be several "actual" values as successive revisions are made in the data, so that the result obtained by his method will depend in part on how much time has elapsed between the making of the forecast and the evaluation of its accuracy. More important is the fact that Theil's method of adjustment will yield valid results only under the following rather restricted conditions:

1. The definition of the predicted and actual values as percentage changes must be logically consistent with the structure of the model and with its variables; that is, the predictions must actually have been expressed in this manner. Some complex variables are not readily definable as predicted percentage changes. The predicted figure of government expenditures, for example, is a budgeted sum resting on assumptions that would not necessarily have been altered even if a different estimate of the actual figure for the preceding year had been available. For the net change in investment—especially in inventories—the predicted value is the arithmetical difference between the beginning and ending values of the net stock of investment goods, but it is actually projected in terms of changes in gross values and changes in prices and depreciation or capital consumption.

2. The statistical data system must remain substantially unchanged in all of its detail between the time that the actual value for the preceding year is estimated as a basis for the forecast and the time at which the

evaluation is carried out. Otherwise, a different percentage change may occur simply because of a change in the data system.

3. Possible changes in the data system must not have been allowed for in the preparation of the forecast.

4. The predictions must not be affected by any difference that arises between the estimate of each variable for the preceding year available when the forecast was made and the estimate available when the forecast is appraised. Frequently this will not be true. For example, when stock and flow variables such as gross inventories, change in inventories, sales, and output are explicitly or implicitly incorporated into the system, a difference in the estimate of the initial values might result in a significantly different forecast.

5. The forecast is adjusted for the estimated effects of predicted changes in policies resulting from the forecast.

It is not clear to us that these conditions are met by the forecasts Theil evaluates.

We also question whether the forecasting qualities of an equation system can be tested by comparing its predictions with the predictions yielded by some "naïve" forecasting method. A naïve model is one selected in a specific instance without regard to its adaptability to the task at hand. No method is inherently naïve.[1] Hence, a comparison of the type made by Theil may be quite inconclusive unless the method selected is naïve from the standpoint of the forecasting problems involved.

Perhaps a more important consideration is whether Theil actually confronts his model with a test of its crucial requirements in comparison with the performance of such a naïve model. If the Dutch model is supposed to produce only estimates of the magnitude of change when all the variables are continuing in the same direction, one should ask whether any simple alternative gives equal or better results. This is similar to part of the test Theil made. In approximately 20 per cent of the cases his model is about equal in effectiveness to the method which projects the percentage change of the previous year. The other 80 per cent of the cases are fairly evenly distributed between those in which the complex set of equations gives poorer results and those in which it gives better results. If we eliminate the exogenous variables from consideration, the equation system seems to turn in a slightly better performance.

If, on the other hand, interest is in the turning points, there appears to be no significant difference between the results given by the more and less sophisticated procedures. The naïve model missed all true turning points but never forecast a false one. The multi-equation model predicted about

[1] James W. Knowles, "Relation of Structure and Assumptions to Purpose in Making Economic Projections," *Proceedings of the Business and Economic Statistics Section of the American Statistical Association*, September 1956, pp. 7-23.

two-thirds of the turning points, but called eight which did not occur. About 12 per cent of all errors were at turning points, but if the exogenous variables had been perfectly known, 19 per cent of the errors would be at turning points, compared to 20 per cent for the naïve method. Since the only improvement which the model provides at the turning points is due to errors in forecasting the exogenous variables, we would invert Theil's conclusion and say that the successes in the unconditional predictions must be due to the degree of failure achieved in predicting these values.

These findings lead us to conclude that our own reluctance to invest scarce funds in complex mathematical systems has been justified, just as it has been in previous tests. Our conclusion is reinforced by Theil's observation that the model seems to produce a tendency toward consistent underestimation such that the predicted values, on the average, are about 70 per cent of the actual values. The errors in predicting the exogenous values may be due to a tendency to avoid predicting large changes, or changes in direction, on the theory that this is a more conservative or less "risky" practice. We offer the suggestion because we have observed this psychology in business and government forecasting in this country. If this is true, better results might be achieved by concentrating on improving the prediction of the exogenous variables rather than on formulating an elaborate mathematical procedure for predicting the endogenous variables.

Our reluctance to accept Theil's appraisal of the accuracy of these forecasts can be traced mainly to the belief that each projection should be evaluated in the light of how well it served the purpose for which it was designed rather than by testing the degree to which actual events correspond to the prediction. General economic projections used in government policy-making are not forecasts, strictly speaking, since they are intended to lead to decisions during the forecast period. The decisions may call for program changes which alter the assumptions on which the projections are based. For example, a projection indicating a deflationary tendency in the coming year might result in policies which would check the implied decline. Under these conditions, an evaluation of the forecast in terms of its correspondence to observed events would be irrelevant and misleading. The vital question is whether or not any alternative projection would have provided a better basis for policy-making.

There are three tests of the quality of a prediction:

1. Were the projected quantities the best estimates which could be derived from the assumptions used?

2. Would other assumptions have been more efficient or appropriate for the particular prediction?

3. Since the prediction was stated in terms of and derived from assumptions through the use of an economic model, was the model's

structure the best one for the particular conditions surrounding the prediction?[2]

Only the first of these tests has been applied by Theil. It would be interesting to see what results he would get by applying the other two.

ON THE JOINT ECONOMIC COMMITTEE PROJECTIONS

Perhaps it would be useful to describe some of our experience at the Joint Economic Committee. The first purpose of the staff projections is to set forth for the Committee's evaluation the nature and magnitude of any adjustments necessary to achieve the objectives of the Employment Act, along with the implications for the economy if the adjustments are not forthcoming. A second purpose is to provide a basis for an internally consistent program aimed at achievement of the nation's major economic goals. These purposes have led to the development of a highly aggregative model, one containing none of the detailed industry-by-industry or product-by-product "goals" which might be appropriate where there exists detailed government direction of the day-to-day operations of the economy. The statistical testing of our projections involves three readily identifiable areas.

Evaluation of the Underlying Data. Through its Subcommittee on Economic Statistics, the Joint Economic Committee has brought technical comment and public expression to bear on the accuracy and adequacy of the underlying statistical data. The Subcommittee has held hearings and published reports itself and has also presented material before Congressional committees concerned with appropriations for statistics and assisted in the development of new statistical programs and techniques. Most of these activities have been directed toward those statistics which underlie all national economic projections.

Testing the Hypotheses. The particular purposes and structure of our projections dictate a number of the procedures used and explain our concern with the underlying statistics. The staff prepares a national economic budget, showing projected incomes and expenditures for government, business, and consumers. On the expenditure side, we rely heavily upon budget plans for the government sector. In the business sector, we use such series as the Department of Commerce—Securities and Exchange Commission and McGraw–Hill surveys of business plans for plant and equipment expenditures, the Dun and Bradstreet surveys of businessmen's expectations (first made at the request of the Joint Economic Committee in 1947), and the recently inaugurated *Newsweek*–National Industrial Conference Board survey of capital appropriations. In the

[2] See footnote 1. A projection should be appraised *ex post* in the light of the information, framework of measurement, and analytical tools available *ex ante*. However, innovations can be incorporated to determine whether their use would have produced a better forecast.

consumer sector, we use such materials as the field surveys of consumer intentions and expectations, conducted by the University of Michigan Survey Research Center for the Federal Reserve Board and other groups.

On the supply side, however, the staff has developed production functions as a basis for computing a "potential" output of the economy believed to be consistent with "maximum employment, production, and purchasing power."[3] The published estimates are derived from trends in labor force, employment, hours of work, and output per man-hour—in government, agriculture, and total private nonagricultural activity. Up to now data problems have precluded use of more complex models. We look forward to development of usable production models that take into consideration both capital and labor inputs, with allowance for such influences as the ratio of output to capacity, and changes in product mix, in hours of work, in the age distribution of the capital stock, and in technology.

Usefulness of the Projections. The regular examination of the value of the projections for policy-making is another important feature of our testing procedure. In the Committee's report on the 1956 *Economic Report of the President*, the staff presented an estimate of the potential output (supply) for 1956, made at the beginning of the year, of $327.4 billion in 1947 prices. In February 1957, actual GNP for 1956 in 1947 prices was estimated at $330.4 billion. Actual demand, therefore, was $3.0 billion or about 0.9 per cent above the advance estimate of potential output.

How could this difference be accounted for in terms of employment, hours of work, and productivity? First, employment exceeded the assumed long-term trend. The labor force increased 1.5 million compared to an average increase of 800,000 to 900,000 per year, while unemployment was 3.8 per cent rather than the assumed 4 per cent. This could account for an excess of output over potential of about $4.3 billion. Hours of work, slightly longer than previous trends indicated, accounted for an excess of output of $2.7 billion. Finally, output per man-hour in the private nonfarm sector apparently failed to increase in 1956. This resulted in output falling $4.0 billion below the level possible if output per man-hour had reached the trend value as then estimated, demonstrating the difficulty in using year-to-year movements of this ratio—especially when such preliminary data are used as a basis for forecasting.

The staff also estimated that the January 1956 *Economic Report* and the budget implied a demand for gross national product in 1956 of $400 billion, in fourth quarter of 1955 prices. Before being compared with the actual figure for 1956, this figure must be raised by about 1 per cent ($3.7 billion) to allow for revisions in July 1956 of the 1955 basic statistical data from

[3] See *Potential Economic Growth of the United States during the Next Decade*, Materials prepared for the Joint Economic Committee by the Committee Staff, Joint Committee Print, 83d Cong., 2d sess., 1954.

which the projections were made. It must also be raised by about 2.3 per cent ($9.3 billion) to allow for price increases in 1956. Combined, the adjustments increase the assumed GNP for 1956 to $413.0 billion. The figure reported in the fall of 1957 was $412.4 billion, $0.6 billion or less than 0.2 per cent below the estimate. The difference is negligible since it is far smaller than what may result from later revisions of the now available data for 1956.[4]

The shortcoming of the analysis as a basis for policy in 1956 was the failure to place sufficient emphasis on the prospects for continued price rises, although the analysis of the 1956 *Economic Report* pointed out that some experts anticipated these. And about six weeks later, on April 18, 1956, in a memorandum to the Committee, the staff stressed the inflationary aspects of the situation. But this discussion illustrates an advantage of the projections—they are published in a quantitative form with enough detail and statement of assumptions to make possible later testing in the light of events.

Our experience with the use of forecasts of various types for policy decisions in both business and government suggests that the essential theoretical problem is that of specifying the characteristics required for a particular type of decision in a particular set of historical circumstances. Furthermore, in practical circumstances one cannot ignore the probability that the functional relationships between the instruments and the uncontrolled variables may vary according to the specific conditions which require a decision. It is not clear that the existing body of theory, including the interesting contribution by Theil, provides a suitable framework for laying out the specifications for the required predictions and for the analytical structures in which they are to be used in reaching the decisions.

Reply by Mr. Theil

Replying to Lehman's and Knowles' comments, I shall confine myself to the following points:

1. Predicted percentage changes were defined as deviates from the previous year's levels as these were known at the moment of prediction. This was done because the equation system yields estimates of relative changes, not of (absolute) levels. The prediction of next year's level is obtained by adding the predicted change to the available estimate of the previous year's level. Any adjustment of the forecast for estimated effects of predicted changes in policies resulting from the forecast (cf. point 5 of the Comment) is highly uncertain and dubious. But it seems plausible that it would in general ameliorate rather than deteriorate the quality of the forecasts.

[4] Since this was written, the Department of Commerce has revised its estimate of GNP for 1956 to $419.2 billion.

2. It is not correct to say that the model produces a tendency toward consistent underestimation of changes. This tendency is produced, not by the model itself, but by the underestimation of changes in exogenous variables (see pages 34 to 35 of my paper).

3. As to the comparison with "naïve" forecasting models, the passage where Lehman and Knowles stated that in about 20 per cent of all cases the model is as effective as the naïve method, the remaining 80 per cent being fairly evenly distributed over more favorable and less favorable cases, is not fully clear to me. I thought that the picture of Charts 1, 2, and 3 was sufficiently clear, but for the sake of completeness I present a table containing the mean absolute extrapolation error (i.e. the average of $A_{-1} - A$, disregarding signs) and the mean absolute forecasting error, both for the unconditional forecasts and for the conditional ones. The data for the former forecasts are derived from Table 1 of my paper. It is clear that four variables are better predicted by the naïve forecasts than by the unconditional forecasts, that the opposite is true for eighteen variables, and that there is one case of a tie; further, that the mean absolute forecasting error is less than one-half of the mean absolute extrapolation error in seven cases. As is to be expected, the conditional forecasts show a still better picture: the median ratio of the mean absolute forecasting error to the mean absolute extrapolation error over all fourteen variables is about a third. See table on page 52.

Variables	*Mean Absolute Error of* *Extra-polation* (1)	*Fore-casting* (2)	*Comparison of Mean Absolute Errors* (1)<(2) (3)	(1)>(2) (4)	½(1)>(2) (5)
	UNCONDITIONAL FORECASTS[a]				
Price indexes:					
Commodity exports	8.9	3.6		*	*
Commodity imports	9.6	1.0		*	*
Consumption	2.2	2.2	tie	tie	
Quantities and volumes:					
Commodity exports	8.6	11.3	*		
Commodity imports	19.0	8.2		*	*
Consumption	2.5	1.7		*	
Industrial production	8.1	5.0		*	
Construction activity	16.3	3.0		*	*
Available labor force	1.0	0.6		*	
Labor productivity	3.5	2.0		*	
Employment in private sector	1.9	1.2		*	
Employment in public sector	5.1	1.9		*	*
Money values:					
Commodity exports	3.5	7.6	*		
Commodity imports	21.0	8.7		*	*
Consumption	3.6	2.1		*	
Net investment (incl. inventories)	67.0	25.7		*	*
Value added in private sector	3.2	5.6	*		
Government wage bill	7.2	5.4		*	
Government commodity purchases	8.8	10.8	*		
Indirect taxes minus subsidies	13.7	3.0		*	*
Exports of services	15.2	13.8		*	
Imports of services	18.2	13.7		*	
Surplus on the balance of services	25.4	14.4		*	
Number of cases			4½	18½	8
	CONDITIONAL FORECASTS[b]				
Price indexes:					
Commodity exports	11.4	2.9		*	*
Consumption	4.0	3.0		*	
Investment goods	8.6	3.2		*	*
Inventories	13.0	2.0		*	*
Government commodity purchases	8.4	3.1		*	*
Quantities and volumes:					
Commodity imports	21.2	4.3		*	*
Consumption	2.1	2.9	*		
Gross fixed investment	10.4	2.3		*	*
Gross national product	5.4	1.8		*	*
Employment in private sector	1.4	0.7		*	
Money values:					
Indirect taxes minus subsidies	15.8	2.6		*	*
Income taxes paid by wage-earners	4.5	1.6		*	*
Income taxes paid by others	11.7	3.8		*	*
Nonwage income	4.0	3.8		*	
Number of cases			1	13	10

[a] 1949-51 and 1953-55.
[b] 1950-54 (1949 is omitted because some of the actual changes in 1948 are not available).