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Volume Author/Editor: Frederick R. Macaulay

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Chapter Author: Frederick R. Macaulay

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APPENDIX B

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PROFESSOR IRVING FISHER'S STATISTICAL MEASURES OF 'PRICE CHANGE'

IN DISCUSSING the relations between commodity prices and interest rates, Professor Irving Fisher designates *level* of commodity prices by the letter P, and *changes in that level* by P'. He uses a third symbol $\overline{P'}$ to represent various moving averages of P'. In the course of merely verbal discussion, as distinct from mathematical or statistical presentation, P is usually referred to as 'commodity price level', and P' as 'change in commodity price level'. The third symbol $\overline{P'}$ is usually defined in a semi-mathematical fashion either as 'the distributed lag of P'' or as 'the weighted average of sundry successive P''s'.[1]

The symbol P, or the *level* of commodity prices, is defined mathematically as the successive values of an 'index number' of commodity prices. It may represent almost any index number that is readily available and that seems adequate for the particular purpose at hand. When discussing interest rates and commodity prices in the United States, Professor Fisher lets P represent one of the index numbers of American wholesale prices constructed by the United States Bureau of Labor Statistics.

The second symbol, P', is repeatedly referred to merely as 'price change'.[2] But that expression seems always to be used as a contraction or abbreviation of the longer expression '*rate* of price change'. Even when Professor Fisher seems to have deliberately defined P' as 'price change' rather than 'rate of price change', the context will usually show that the latter is really meant.[3] The emphasized characteristics of P' are always those of a ratio or a function of a ratio,

[1] Cf. Irving Fisher, *The Theory of Interest*, p. 419.

[2] '. . . i stands for interest rate and P' for price change . . .' *The Theory of Interest*, p. 412.

'In Chart 44, price change (P') is represented . . .' *Ibid.*, p. 414.

[3] For example, '. . . the price change (P') is assumed to be at the rate of 5 per cent per annum' (*The Theory of Interest*, p. 413). If the author had intended P' to mean '*rate* of

rather than those of an arithmetic 'difference'. The various mathematical definitions are all such that the value of P' would be unaffected by *multiplying* by a constant the successive terms of the price index number from which it is calculated. But its value would be definitely changed by *adding* a constant to, or *subtracting* a constant from, those terms.

The third symbol, $\overline{P'}$, is mathematically always just what it is stated to be, 'a weighted average of sundry successive P''s'. But this is essentially an operative rather than an explanatory definition. It describes one way in which $\overline{P'}$ may be calculated; but, in the absence of analysis, throws little light on what $\overline{P'}$ means.

Professor Fisher verbally defines P' as the change in prices from one month (or year) to the next. He has proposed two arithmetic methods of measuring this 'change'. These arithmetic statements are, of course, the real definitions. The first of these arithmetic definitions appeared in an article published in 1923.[4] It was given as a measure of the rise or fall of prices 'for each month'. It was calculated for any particular month by dividing the price index for the succeeding month by the price index for the preceding month and subtracting unity. In other words, if the index numbers for three consecutive months be designated a, b, c, the value of P' for the month whose price index was b would be $\frac{c}{a} - 1$. This is one one-hundredth or one per cent of the percentage change in the price level from the first month to the third month. For example, if $a = 100$ and $c = 125$, P' would equal $\frac{25}{100}$.

It is not worth while to examine critically this 1923 definition of P'. Professor Fisher himself discarded it within eighteen months. Indeed, it is strange that, in December 1923, a year after the publication (December 1922) of his book *The Making of Index Numbers* with its detailed and exhaustive analysis of the problem of 'bias', he could

price change', we would expect him to have written '. . . the rate of price change (P') is assumed to be 5 per cent per annum'. However, on the chart (43) to which the discussion refers, we find such notations as $P' = 5\%$, $P' = 10\%$, etc.

[4] "The Business Cycle largely a 'Dance of the Dollar'," *Journal of the American Statistical Association*, December 1923, pp. 1024-28.

have proposed defining P' as $\frac{c}{a} - 1$. For the function $\frac{c}{a} - 1$ has a pronounced upward 'bias'.[5]

The second arithmetic definition of P' appeared in an article published in 1925.[6] In that article Professor Fisher proposed that the value of P' for the month whose price index is b be defined (if a, b, c be, as before, the index numbers for three consecutive months) as $\frac{c-a}{2b}$, or, 'on an annual basis', as $\frac{6(c-a)}{b}$[note 7]. The value of $\frac{c-a}{2b}$ $\left(\text{or } \frac{6\,(c-a)}{b}\right)$ is, of course, the same when a, b, c, are 0, 100, 10 as when they are 95, 100, 105 or 99, 10, 100.

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[5] Professor Fisher speaks of P', both when defined as $\frac{c}{a} - 1$ and when defined (in his later work) as $\frac{c-a}{2b}$, as the 'slope' of P and as the 'derivative' of P. But the reader must be warned that, for purposes of analogical comparison, he uses these strictly mathematical terms in a loose and colloquial manner.
Strictly speaking, only if a 'slope' is constant over a period can it be termed *the* 'slope' of a curve *during* the period. *And*, of course, only if there be a 'curve' in the strict mathematical sense of that term can there be a 'slope'. The 'slope' of a curve at a specified point on the curve is the trigonometric 'tangent' of the straight line (geometrically) tangent to the curve at the point. Only if the curve itself be a straight line can its 'slope' be constant.

If the time scales and the price scales were assumed to be so related that the price figures were expressed in the time scale (the time interval from the abscissa of a to the abscissa of c being taken as unity), and if the points whose ordinates were a and c were joined by a straight line, the slope of that line (throughout its length) would be $c - a$.

Of course, if a new curve were now constructed each of whose infinite number of ordinates was the natural logarithm (to the base e) of an ordinate of the original straight line having the same abscissa, $\frac{c}{a} - 1$, which equals $\frac{c-a}{a}$, would be the 'slope' of the tangent to this new curve at the point whose ordinate was $\log a$. But, as this new curve is *not* a straight line, only at this one point would $\frac{c}{a} - 1$ be its 'slope'.

[6] "Our Unstable Dollar and the so-called Business Cycle", *Journal of the American Statistical Association*, Vol. XX, New Series No. 150, June 1925, pp. 179-202 incl.

[7] " . . . a derivative of P, namely, the curve P' . . . such that the *height* of P' is expressed by the same figure as the *slope* of P" (1925 article, p. 182). At the bottom of the page there is a note on the word *slope* which Professor Fisher had italicized. The note reads: "The slope for any given month is measured by subtracting the index for the *preceding* month from that for the *succeeding* month and reducing the result to a percentage of the given or intervening month. This percentage, being for two months, is multiplied by six to give a *per annum* rate."

The function $\frac{c-a}{2b}$ contains a totally irrelevant variable. The value of b has no more logical place in a measure of price change from the level a to the level c than has the altitude of a balloon an hour ago to its change in altitude during the past two hours. The change in altitude of the balloon may be calculated from the change in altitude during the first hour and the change during the second hour; but the altitude at the end of the first hour will not affect the change in altitude during the two hours.[8]

It may, of course, be argued that $\frac{c-a}{2b}$ is not intended to measure price change from the middle of the first month to the middle of the third month but from the beginning to the end of the second month; that we are not measuring the change in the altitude of the balloon during the past two hours but during the single hour that ended half an hour ago; that, for the determination (by interpolation) of the probable price level at the *beginning* of the second month, b is as important as a; and, similarly, as important as c for the determination of the price level at the *end* of the second month.

But, after the acceptance of this condition, the difficulties reappear in full force as soon as any attempt is made to develop a rational and systematic scheme of interpolation that will yield $\frac{c-a}{2b}$ as a not-absurd function of the interpolated values, without introducing b otherwise than as it occurs in those interpolated values. For example, if the level of prices at the beginning of the second month be taken as the arithmetic average of a and b and the level at the end of the month as the arithmetic average of b and c, we are faced with the difficulty that a knowledge of the values of only these averages is insufficient to determine $\frac{c-a}{2b}$. The necessity of introducing b remains.[9]

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[8] Of course, b is only technically irrelevant if it is introduced in such a manner that it eventually cancels out. For example, $(\log c - \log b) + (\log b - \log a) = \log c - \log a$; and $\frac{c}{b} \times \frac{b}{a} = \frac{c}{a}$.

[9] If, through the three points $(-\frac{1}{2}, \frac{a+b}{2})$, (O, b), $(+\frac{1}{2}, \frac{b+c}{2})$, a second degree parabola $(y = A + Bx + Cx^2)$ be drawn, and another curve be constructed such that each of its ordinates is the natural logarithm of an ordinate of this parabola, $\frac{c-a}{2b}$ will be the slope of this new curve at the point $(O, \log b)$. But, not merely has b been directly introduced, but the slope is at a point only.

By itself, P' or $\frac{c-a}{2b}$ is a highly erratic function. But Professor Fisher makes much more use of $\overline{P'}$ than of P', and $\overline{P'}$ is definitely less erratic than P'. The cumulation of P' tends to iron out some of its irregularities.[10] We discuss the reasons for this fact later in this appendix.

Professor Fisher describes $\overline{P'}$ as 'the distributed lag of P'' or as 'the weighted average of sundry successive P''s'. It was in articles dealing with the relation of changes in commodity price levels to interest rates, bond yields and the activity of business in general that he approached the problem of deciding what particular weights should be assigned to the 'sundry successive P''s'. He presented the hypothesis that any appreciable change in the general level of commodity prices influenced the level of interest rates, etc., for long periods, though the strength of such influence declined with the passage of time. Conversely, that though a present level of interest rates, for example, had been most powerfully affected by recent changes in commodity prices, it had also been influenced, though to a less degree, by changes that had occurred in the distant past.

One of Professor Fisher's early efforts to decide upon what relative weights should be assigned to the influences of recent and remote price changes led him to base those weights on the ordinates of a skew probability curve which had its maximum ordinate in the very near past. The present writer cannot say that he grieves over the fact that this hypothesis was soon thrown overboard. The combination of the $\frac{c-a}{2b}$ method of measuring P' and the skew-probability weights for $\overline{P'}$ would have left that latter function in a position to

[10] Though the erratic nature of P' may be very clearly illustrated by means of cumulations and averages. The function $\frac{c-a}{2b}$ has no *systematic* upward or downward 'bias', but its cumulation can yield strange results. Consider, for example, a hypothetical monthly series running 1, 4, 16, 2, 8, 32, 4, 16, 64 . . . These figures show a pronounced upward trend; each is double that for the third month back. However, a moving three months' simple arithmetic average of the $\frac{c-a}{2b}$ functions is constant and *negative.* Its value is always $-\frac{1}{16}$. If the series were presented in reverse order, it would of course show a pronounced downward trend. But the three months' moving average of the $\frac{c-a}{2b}$ functions would be constant and *positive.*

defy successfully any attempt at a simple presentation of its meaning. A right-angled triangle with the right angle on the x-axis and the maximum ordinate at the most recent month was substituted for the skew probability curve. Moreover, the renunciation was complete. It was no mere sop to the exigencies of computation. Professor Fisher announced that the results obtained with the right-angled triangle were distinctly more acceptable to him than were those that had been obtained through the use of the skew probability curve.[11] With the 'triangular' weighting, the function $\overline{P'}$ is defined as a weighted average of n successive P''s in which the earliest P' has a weight of 1, the next in time a weight of 2, the third a weight of 3, and the *nth* (or most recent) P' a weight of n. The function $\overline{P'}$ therefore equals

$$\frac{P'_1 + 2P'_2 + 3P'_3 + \ldots + nP'_n}{\Sigma n}.$$

Up to this point we have designated data by the letters a, b, and c because single letters are so easy to remember without confusion. But, as the discussion from now on is in terms of hyperbolic functions, a different nomenclature is desirable. For a, b, and c let us substitute the symbols e^{y_1}, e^{y_2}, e^{y_3}, where e stands for the base of the natural system of logarithms,[12] and y_1, y_2, y_3 are log a, log b, and log c. Also, let $z_1 = y_2 - y_1$ and, in general, $z_n = y_{n+1} - y_n$; or in terms of the earlier notation, $z_1 = \log b - \log a$ and $z_2 = \log c - \log b$. In this new notation,

$$P'_n = \frac{e^{y_{n+2}} - e^{y_n}}{2\,e^{y_{n+1}}} = \tfrac{1}{2}(e^{z_{n+1}} - e^{-z_n})$$

$$= \tfrac{1}{2}\left(\sinh z_{n+1} + \sinh z_n + 2\sinh^2\frac{z_{n+1}}{2} - 2\sinh^2\frac{z_n}{2}\right).^{\text{note 13}}$$

[11] Compare *The Theory of Interest*, p. 421.

[12] Unless otherwise stated, the logarithms referred to in this appendix are 'natural' logarithms (to the base e) and not 'common' logarithms (to the base 10).

[13] As sinh z, or the 'hyberbolic sine of z' equals $\frac{e^z - e^{-z}}{2}$, $2\sinh^2\frac{z}{2} + 1 = \frac{e^z + e^{-z}}{2}$.

Hence, $e^{z_{n+1}} = \sinh z_{n+1} + 2\sinh^2\frac{z_{n+1}}{2} + 1$ and

$-e^{-z_n} = \sinh z_n - 2\sinh^2\frac{z_n}{2} - 1.$

Now, it is apparent that, if $e^{y_{n+1}}$ be the geometric mean of e^{y_n} and $e^{y_{n+2}}$, z_n will equal z_{n+1} and hence the $\sinh^2\frac{z}{2}$ terms in the expression for P' will cancel out and disappear. And, even with data that do not constitute a geometric progression, it is apparent that, if a total (or a simple unweighted arithmetic average) of n successive P''s be taken, all but the first and last of the $\sinh^2\frac{z}{2}$ terms will cancel out and disappear. The unweighted arithmetic average of n successive P''s from P'_1 to P'_n equals

$$\frac{1}{2n}\left\{\begin{array}{l}(\sinh z_1 + 2\sinh z_2 + 2\sinh z_3 + \ldots + 2\sinh z_n + \sinh z_{n+1}) \\ \quad + 2\left(\sinh^2\frac{z_{n+1}}{2} - \sinh^2\frac{z_1}{2}\right)\end{array}\right\}.$$

Furthermore, the relative influence on the average exerted by the two $\sinh^2$ terms that remain will tend to decrease as the value of n is increased. There will always be $\frac{n}{2}$ times as many $\sinh z$ terms as there are $\sinh^2\frac{z}{2}$ terms. If we assume (with Professor Fisher) that it is legitimate and proper to average arithmetically 'sundry successive P''s', we must conclude that the $\sinh z$ terms in the expression for P' are, at least for the purpose of analyzing the essential characteristics of such an average of P''s, fundamental and the $\sinh^2\frac{z}{2}$ terms essentially extraneous and irrelevant.

And this conclusion (derived from the fact that the influence of the $\sinh^2\frac{z}{2}$ terms tends to decrease *pari passu* with increases in the number of P''s in the average) is reinforced by considering some of the essential characteristics of these terms and of the complete functions of z_1, and z_{n+1}. In the first place, though the function $\sinh^2\frac{z}{2}$ may be thought of as a measure of *absolute* fluctuation, it cannot properly be thought of as a measure of advance or decline. Its value is not affected by the inherent algebraic sign of z. A decline of the data from 100 to 95 yields the same value for $\sinh^2\frac{z}{2}$ as an advance from 95 to 100. And similarly with the expression

$\sinh^2 \frac{z_{n+1}}{2} = \sinh^2 \frac{z_1}{2}$. Whether this expression is, as a whole, positive or negative depends in no way on the inherent algebraic signs of z_{n+1} and z_1.[14] It depends only on the *absolute* values of z_{n+1} and z_1 and the external algebraic signs that precede $\sinh^2\frac{z_{n+1}}{2}$ and $\sinh^2\frac{z_1}{2}$. And the sign of the expression, even as thus derived, is, as may be seen from an examination of the complete function of z_{n+1},[note 15] and the complete function of z_1,[note 16] essentially arbitrary and accidental.

If we remember that a z is not a raw datum (*e.g.*, a price) but a function (the logarithm) of the ratio of a datum to the preceding datum, it would seem reasonable to assume that the time order of the z's should not affect an arithmetic average of a function such as P' that is intended to measure data changes. In fact, the unweighted arithmetic average of successive P''s is unaffected by the time order of any of the z's except the earliest z and the latest z. But, unless z_{n+1} equals plus or minus z_1, the average is affected by the time order of those two z's. If the earliest z be substituted for the latest z and vice versa, the value of the average is altered. Though the sinh z terms in the average remain unchanged, *the sign of the* $\sinh^2\frac{z}{2}$ *factor* ($\sinh^2\frac{z_{n+1}}{2} - \sinh^2\frac{z_1}{n}$) *is reversed.* If arithmetic averaging be considered legitimate, the case for treating as mere erratic elements the sinh² terms in the average would seem complete.

The function $\overline{P'}$ is a weighted and not a simple arithmetic average of 'sundry successive P''s'. But the conclusions we have arrived at concerning the essential irrelevancy of the sinh² elements hold with respect to the weighted average as definitely as they hold with respect to the unweighted average. After the collection and cancellation of terms, $\overline{P'}$[note 17] appears as

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[14] The expression will vanish if z_{n+1} equals *plus or minus* z_1.

[15] $\sinh z_{n+1} + 2\sinh^2\frac{z_{n+1}}{2}$.

[16] $\sinh z_1 - 2\sinh^2\frac{z_1}{2}$.

[17] $\overline{P'} = \frac{P'_1 + 2P'_2 + 3P'_3 + \ldots + nP'_n}{\Sigma n}$.

$$+ \sinh z_1 - 2 \sinh^2 \frac{z_1}{2}$$

$$+ 3 \sinh z_2 - 2 \sinh^2 \frac{z_2}{2}$$

$$+ 5 \sinh z_3 - 2 \sinh^2 \frac{z_3}{2}$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$+ (2n - 1) \sinh z_n - 2 \sinh^2 \frac{z_n}{2}$$

$$+ n \sinh z_{n+1} + 2 n \sinh^2 \frac{z_{n+1}}{2}$$

all divided by $2\Sigma\, n$.

If the data constitute a geometrical progression,[18] the $\sinh^2 \frac{z}{2}$ terms cancel out and disappear as they do in the *unweighted* average. It is true that, when the data are not so related to one another, no such wholesale cancellation of $\sinh^2 \frac{z}{2}$ terms occurs as occurs in the case of the unweighted average. Indeed, each z is, in the weighted average, represented by a $\sinh^2 \frac{z}{2}$ item. But the absence of any relation of the sign of these terms to advance or decline of the data is as complete as it was with the unweighted average. The $\sinh^2 \frac{z}{2}$ terms are indices of mere absolute fluctuation. Their algebraic sum equals the deviation of the most recent $\sinh^2 \frac{z}{2}$ (*i.e.*, $\sinh^2 \frac{z_{n+1}}{2}$) from the arithmetic average of the n preceding $\sinh^2 \frac{z}{2}$'s, divided by $\frac{n+1}{2}$. note 19 It is no more than a comparison of how the size of a particular function of the *extent* of the most recent fluctuation (*up or down*) compares with the average size of the function in the past n periods. As such, it is for our purposes, an erratic and meaningless expression.

[18] Or, more generally, if the ratio of the larger of each pair of adjacent data points to the smaller be constant, as would be the case, for example, if the data ran 2, 4, 8, 4, 8, 16, 8, 4.

[19] $\frac{2\,\Sigma\, n}{2n} = \frac{n+1}{2}$.

But, if n be made large enough, this $\sinh^2 \frac{z}{2}$ expression virtually vanishes from the picture. For example, if $n = 120$ (as it does in one of Professor Fisher's quarterly commodity price illustrations), the deviation of $\sinh^2 \frac{z_{121}}{2}$ from the average of the 120 preceding $\sinh^2 \frac{z}{2}$'s is divided by $\frac{121}{2}$. With data that fluctuate no more violently than do quarterly commodity price index numbers, the largeness of this divisor reduces the $\sinh^2$ elements in the formula to complete negligibility.

If we remove these erratic (and commonly negligible) $\sinh^2$ terms from the expression for $\overline{P'}$, we have

$$\overline{P'} = \frac{\sinh z_1 + 3 \sinh z_2 + 5 \sinh z_3 + \ldots + (2n-1) \sinh z_n + n \sinh z_{n+1}}{2 \Sigma n}.$$

But, as $\sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \ldots$,[20] this expression equals

$$+ z_1 + \frac{z^3{}_1}{3!} + \frac{z^5{}_1}{5!} + \ldots$$
$$+ 3z_2 + \frac{3z^3{}_2}{3!} + \frac{3z^5{}_2}{5!} + \ldots$$
$$+ 5z_3 + \frac{5z^3{}_3}{3!} + \frac{5z^5{}_3}{5!} + \ldots$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$+ (2n - 1)\, z_n + \frac{(2n-1)\, z^3{}_n}{3!} + \frac{(2n-1)\, \Sigma\, z^5{}_n}{5!} + \ldots.$$
$$+ nz_{n+1} + \frac{nz^3{}_{n+1}}{3!} + \frac{nz^5{}_{n+1}}{5!} + \ldots$$

all divided by $2 \Sigma n$.

If the values assumed by z be small ('absolutely' and not algebraically), this function will approximate

$$\frac{z_1 + 3z_2 + 5z_3 + \ldots + (2n-1)\, z_n + nz_{n+1}}{2 \Sigma n}.$$

This is the value obtained by neglecting all powers of z greater than unity. Such a treatment of the function, in which cubes are the next

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[20] $e^z = 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \ldots.$

and $\sinh z = \frac{e^z - e^{-z}}{2}$

higher power after unity, is, for the purposes of our present analysis, quite warranted.[21]

But, as $z_n = y_{n+1} - y_n$, this expression equals $(y_2 - y_1) + (3y_3 - 3y_2) + (5y_4 - 5y_3) + \ldots + \left\{ (2n-3)y_n - (2n-3)y_{n-1} \right\} + \left\{ (2n-1)y_{n+1} - (2n-1)y_n \right\} + (ny_{n+2} - ny_{n+1})$, divided by $2\,\Sigma\, n$; which equals $- y_1 - 2y_2 - 2y_3 - \ldots - 2y_n - y_{n+1} + n\,(y_{n+1} + y_{n+2})$, divided by $2\,\Sigma\, n$. If $Y_n = \frac{y_n + y_{n+1}}{2}$, this expression becomes

$$- Y_1, - Y_2, - Y_3 - \ldots - Y_n + nY_{n+1}, \text{ divided by } \Sigma\, n, \text{ or}$$

$$\frac{2}{n+1}\left(Y_{n+1} - \frac{Y_1 + Y_2 + \ldots + Y_n}{n}\right).$$

In other words, the value to which $\overline{P'}$ approximates (if the month to month—or year to year—fluctuations of the data are not too violent—and they are not with commodity price index numbers) may be described as follows. Take a two-months' moving average of the logarithms of the data (or take the logarithms of the geometric means of adjacent values). With this average as new data, $\overline{P'}$ will approximate $\frac{2}{n+1}$ times the deviation of a datum value from the arithmetic average of the n preceding data values.[22] It is the deviation of the logarithm of present price from an average of the logarithms of past prices.

The function $\frac{2}{n+1}\left(Y_{n+1} - \frac{Y_1 + \ldots + Y_n}{n}\right)$ is the *slope* of the straight line joining Y_{n+1} to the mid point of the moving average $\frac{Y_1 + \ldots + Y_n}{n}$. It is therefore *technically* correct to describe it as a measure of rate of price change. For example, if the Y's all fell on the straight line $Y = A + Bx$ (as would be the case if the original

[21] If z be 'absolutely' small, sinh z does not differ greatly from z. For example, if prices one month are even as much as 125 per cent of what they were in the preceding month, the difference will be extremely small. If $e^z = 1.25$ then $z = .2231\ldots$ and $\sinh z = .2250\ldots$

[22] If common, and not natural, logarithms are used, the result will, of course, be 0.434 . . . times $\overline{P'}$ instead of $\overline{P'}$.

data fell on a compound interest curve), the function would always equal B. No matter what the value of n the arithmetic average of the Y's would advance *pari passu* with the value of the most recent Y from which the average was to be subtracted.

But no such condition would exist if the Y's fell on a periodic curve such as a sine curve (plus a constant). If n equalled the number[23] of data points in one period or 'cycle', the function points would lie on another sine curve of smaller amplitude or 'swing' than the data curve, *but with maxima and minima on the same dates*. With an adjusted scale, it would be an exact reproduction of the data curve, though it would be technically correct to describe it as a measure of rate of change of that curve. But it might easily be more misleading than enlightening to do so.

A sine curve has no long-term trend. In general, if the data show no definite long-term trend, the average of the Y's will, if n be taken sufficiently large, tend to be virtually constant. And $\overline{P'}$ will therefore tend to reproduce the data—minus a constant.

There are innumerable examples of such trendless curves. Some of the most perfect are series that, from their mathematical nature, move within definite limits. The digits of the decimal development of an incommensurable number, such as the 707 calculated digits of π. The number of spots in each successive throw of a pair of dice. Percentages that cannot exceed one hundred, such as the percentage of blast furnaces in blast, etc.

Less perfect examples come from the field of percentages that never approach one hundred; the percentage of the population in receipt of poor relief, the percentage of banks in the hands of receivers, the ratio of bank reserves to bank deposits, etc.

But, even with series that possess unmistakable trends, $\overline{P'}$ may closely approximate P. If the fluctuations of the data (from which the average is to be subtracted) happen to be extremely large as compared with the movements of the average, $\overline{P'}$ will, over short periods, often virtually reproduce P.[note 24]

[23] Or a multiple of that number.

[24] Compare *The Theory of Interest*, Chart 49 (opp. p. 426). In this chart are presented quarterly figures for $\overline{P'}$ and i (short term interest rates in the United States) for the period 1915–1927. In this period occurred the violent war and post-war movements

It is of course true, as Professor Fisher says, that "It certainly stands to reason that *in the long run* a high level of prices due to previous monetary and credit inflation ought not to be associated with any higher rate of interest than the low level before the inflation took place. It is inconceivable that, for instance, the rate of interest in France and Italy should tend to be permanently higher because of the depreciation of the franc and the lira, or that a billion-fold inflation as in Germany or Russia would, after stabilization, permanently elevate interest accordingly." [25]

The function $\overline{P'}$ compares prices with a moving-average base instead of with zero, and therefore cannot remain high indefinitely. But the statistical evidence that the particular base introduced by $\overline{P'}$ is, even empirically, a good base is not strong. Correlations between P and i usually run higher than those between $\overline{P'}$ and i.

And again, if the substitution of $\overline{P'}$ for P, instead of lowering, raised the coefficients of correlation noticeably, it is questionable whether we would be warranted in assuming that it was because $\overline{P'}$ was a 'weighted average of sundry successive P''s' rather than because it was a deviation of P from a moving base. Professor Fisher writes, "It seems fantastic, at first glance, to ascribe to events which occurred last century any influence affecting the rate of interest today".[26] If for the word 'events' we substitute the words 'commodity price changes' (which are the 'events' Professor Fisher is discussing) and for the words 'any influence' the particular measure of that influence Professor Fisher proposes, we might be tempted to counter with, 'Why only at *first* glance?' He immediately continues with, "And yet that is what the correlations with distributed effects of P' show." But is it?

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of prices and interest rates. The value of n is huge, 120. The similarity between $\overline{P'}$ and P, both of which are given in the chart, is striking.

Professor Fisher found that the maximum correlation ($+0.738$) between $\overline{P'}$ and i was obtained when n was made 120. But the correlation between P (the raw data) and i gave $+0.709$ without lagging and, if i was lagged two quarters (half a year), $+0.891$. (*The Theory of Interest*, pp. 427 and 431.)

[25] *The Theory of Interest*, p. 440. The italics are Professor Fisher's.

[26] *The Theory of Interest*, p. 428.