Volume Title: Some Theoretical Problems Suggested by the Movements of Interest Rates, Bond Yields and Stock Prices in the United States since 1856

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Volume Publisher: NBER

Volume ISBN: 0-87014-032-9

Volume URL: http://www.nber.org/books/maca38-1

Publication Date: 1938

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Chapter URL: http://www.nber.org/chapters/c6344

Chapter pages in book: (p. 85 - 127)

CHAPTER IV

THE RELATION OF THE MOVEMENTS OF BOND YIELDS TO THE GRADES OF THE BONDS— ECONOMIC 'DRIFT'

THE drift of the arithmetic and geometric index numbers of railroad bond yields referred to at the close of the preceding chapter is, as we there stated, not a mathematical drift. It is an economic drift. It is not like the upward drift of chained arithmetic averages of relative prices or the downward drift of chained harmonic averages of relatives. During most of the period covered by this study (January 1857 to January 1936) the drift has been *downward* but that *direction* was not mathematically necessary. Indeed, the drift was sometimes upward. For example, while the geometric index figure (see Appendix A, Table 4-1) for January 1857 (9.52) was more than 151 per cent of the yield of the lowest yield bond used in that month (6.27) and the index figure for January 1929 (4.50) was only 104 per cent of the yield of the lowest bond for that month (4.33), the index figure for January 1933 (5.22) was 128 per cent of the lowest yield for the month (4.06), or *larger* than the percentage (127) that the index figure for January 1865 (6.87) was of the lowest yield for that month (5.43). In other words, using such a criterion of drift, we find a pronounced *downward* drift from January 1857 to January 1929, a period of seventy-two years; but, on the other hand, when we compare January 1865 with January 1933, sixty-eight years from the one date to the other, we find for the period as a whole, a slight *upward* drift.

An examination of Chart 15, on which a cumulation (inverted) of the drift[1] is presented, shows that the *upward* drift of the yields of the lower grade bonds was so great in the recent economic collapse that, from November 1928 to June 1932, the cumulated drift recovered all

[1] The method used to measure drift is explained later in this chapter.

the ground it had lost from 1865 to 1929.[2] The picture presented by Chart 15 is one of rapid drift *downward*[3] from 1857 to 1872, an irregular but roughly *horizontal* movement from 1872 to about 1900, a renewal of the rapid drift *downward* from 1900 to 1912, an *upward* drift from 1912 to 1920, a pronounced *down* drift from 1920 to 1929 followed by an *up* movement, so violent that it can hardly be described as a 'drift' at all, that lasted until the middle of 1932.

In 1913 Professor Wesley C. Mitchell published an index number of the yields of ten American railroad bonds,[4] monthly, from January 1890 to December 1911 inclusive. Now, though there seems to have been little or no 'drift' in railroad bond yields from 1890 to 1900, there was a pronounced down drift from 1900 to 1912[5] and hence the total drift from January 1890 to December 1911 was strongly downward. This downward drift showed itself, as was to be expected, in Professor Mitchell's group of bonds. He noticed and explained it. His discussion and comment runs, in part, as follows:[6]

"The differences shown by Table 19 among the net yields of the ten bonds may readily be accounted for by differences in the proportionate value of the underlying properties, by the existence or non-existence of prior liens, by the relative financial strength of the issuing or guaranteeing corporations, etc. Similarly, the frequent changes in rank among the bonds may be ascribed to alterations in these particular conditions, which a well-advised investor considers in estimating the risks he runs in buying securities.

"But there is one fact of more general interest about these differences in yield. The margins between the higher and lower yields have grown narrower in the course of twenty years. In 1890 the maximum difference was 5.24–3.88 per cent=1.36; in 1900 it was 4.40–3.42 per cent=0.98; in 1909, 4.24–3.87 per cent=0.37. The chief cause of this narrowing of the margins has been an improvement since the middle nineties in the credit of the lower grade issues among investors. The risks imputed to the holding of bonds of such railways as, for example, the Chicago and Eastern Illinois have diminished. With one exception—the bonds of the West Shore Railroad—all the bonds gave lower yields in 1911 than in 1890. The West

[2] For the figures, see Table 6, Appendix A.

[3] Note again that in Chart 15 the cumulated drift is *inverted.*

[4] There were no substitutions; the same ten bonds were used throughout the period.

[5] The violent up-drift during 1907 was immediately offset on the cumulated drift by the violent down-drift of 1908 and the first half of 1909 (see Chart 15).

[6] *Business Cycles* (University of California Press, Berkeley, 1913), pp. 156 and 157.

Shore bonds, guaranteed principal and interest by the New York Central and having over 400 years to run, were rated decidedly higher by investors in 1890 than any other security in the present list. But, since then, the improvement in the financial condition and prospects of other railways has gradually brought their obligations closer to the high standard of securities guaranteed by the New York Central. Indeed, in recent years the bonds of the Burlington, the Milwaukee, and the Central of New Jersey have frequently outranked the bonds of the West Shore."

This is all quite as one might expect. Movements of the yields of bonds of the highest grade reflect primarily changes in 'long term interest rates'. On the other hand, movements of common stock prices reflect also the market's estimate of future earnings and dividends. Bonds other than those of the highest grade naturally partake of the nature of both the highest grade bonds and common stocks. The movements of their yields are affected not only by long term interest rate considerations but also by forecasts of earnings. In Chapter V it is shown that there has been a noticeable similarity between the major movements of the cumulated railroad bond yield 'drifts' and the major movements of prices of railroad common stocks. And the secular trend of railroad stock prices was upward from January 1890 to December 1911 while the secular trend of the cumulative bond yield drift was downward.[7]

But, though the drift in the index numbers based on arithmetic and geometric averages of the yields themselves is not a 'mathematical' drift, the reader may possibly wonder to what extent it is a merely technical drift. In the selection of the bonds, those that improved in quality year after year were not discarded because of that fact but those that deteriorated *very rapidly* (as the roads became bankrupt or ran into great financial difficulties) were eliminated. And, as the reader will realize later in this chapter, the retention of such bonds throughout their periods of deterioration and collapse might easily have affected to a greater or less extent the movements of the cumulated 'drift'—by reducing its downward movements and increasing its upward movements.

Indeed, if such deteriorating bonds had been retained, the cumulated drift line of Chart 15 might possibly have been more a shadow of the

[7] In connection with this *inverse* relation, the reader must note that the comparison is not between stock prices and bond prices but between stock *prices* and bond *yields* (see Chart 15).

stock price line than it is.[8] It is intriguing to think so. When an economist presents an interesting statistical relation to his readers' attention, it is pleasant to imagine and he may easily be tempted insidiously to suggest that, if the data were more adequate or more skillfully handled, the relation would stand out even more strikingly than it does. And, in at least one minor movement (from January 1903 to January 1904, during which time stock prices declined violently while the cumulated 'drift' of the bond yields also declined, instead of advancing as it would have been expected to do), the non-conformity is clearly traceable to the difference in the action of medium grade and definitely low grade bonds, as seen in the non-linearity of the scatter.[9]

But, unfortunately, tests with unchanging groups containing bonds of grades ranging all the way from the highest to really low grade suggest strongly that the elimination of bonds when they begin to deteriorate rapidly, as evidenced by the rise in their yields, has usually only a very minor effect on the *long term* movements of the cumulated linear drift. Convexity of the scatter [10] over a short period of falling stock prices tends to be followed by concavity while stock prices are recovering and the *long term* effects on cumulated linear slopes to be thus offset.

That mere changes in the list do not necessarily increase the downward drift is well illustrated (though not proved) by comparing the cumulated drift of our changing list of bonds from January 1890 to December 1911 with the cumulated drift of Dr. Mitchell's unchanging list of ten bonds. The ratio of Dr. Mitchell's index number to our unadjusted geometric index was, in January 1890, 1.04. In January 1900 it was 1.03, and in January 1911, 0.99. The ratio of his index number to our unadjusted arithmetic index was in January 1890, 1.03, in January 1900, 1.02, and in January 1911, 0.99. The downward drift of his index number was therefore slightly *greater* than that of our unadjusted numbers.

Indexes from which drift has not been eliminated are not indexes of the yields of a uniform grade of bonds. They have, moreover, no essential relation to the average yield of all bonds outstanding

[8] The reader must be reminded again that the cumulated drift line of Chart 15 is, on that chart, *inverted*.

[9] See Chart 8 and discussion in the last section of this chapter.

[10] With respect to the x axis.

or to the average rates at which new issues are coming out. They are indexes 'of the net returns which permanent investors have received upon current purchases of bonds' only if the assumption be made that the investors bought only these particular bonds. To have included a few bonds of a lower grade than those used would probably have altered not merely the level of the index numbers, but also their movements. In a period of downward drift, it would increase the downward (or decrease the upward) movement of the index numbers. Though it is true, as Dr. Mitchell has suggested, that index numbers from which drift has not been eliminated tell a different story from those without drift, that story, as we shall see, is more difficult to interpret than he indicated:

"The average yield of all ten bonds is the best available gauge of the changes in the rates which large American corporations have paid for new loans on long time since 1890, and also the best gauge of the net returns which permanent investors have received upon current purchases of bonds. But it is distinctly not the best gauge of changing rates upon long loans of substantially uniform security. For the latter purpose the yield of the West Shore bonds is preferable, since the financial credit of the guarantor was so firmly established in 1890 as to be little shaken by the years of depression and little strengthened by the years of prosperity. In other words, the yields of this issue reflect the changes in the supply of, and the demand for, loan capital for fixed investment with less distortion by the factor of risk than do the yields of the nine other bonds. But, since the yields of the other bonds are more typical of American experience since 1890, the detailed tables have been arranged to show both the net yields of the West Shore bonds, and the average net yields of all ten" (*Business Cycles*, p. 157).

Though Dr. Mitchell was correct when he stated that the yields of West Shore 4's give, during the period he was covering, a better picture of 'changing rates upon loans of substantially uniform security' than does his index based on the arithmetic average of the yields of ten bonds, the fact is that the yields of no single bond are completely adequate for this purpose. There are erratic movements in the yields of any individual bond, even West Shore 4's. Though West Shore bonds acted for long periods remarkably well, there is strong evidence that their grade varied appreciably at times. For example, a comparison of their yields with the yields of the very highest grade railroad bonds, or better, with the '4.50 sigma' index, which we present and explain later, shows that the West Shore bonds were declining in grade from 1890 to 1900 and improving from 1900 to 1911. Since 1930 the bond has been definitely out of the class of bonds of the highest grade.

How well 'changing rates upon loans of substantially uniform security' may be measured by the yields of individual bonds may be seen by examining Chart 2. One of the indexes (line C) of that chart was constructed by chaining together the yields of three extremely high grade bonds:[11] bond no. 10 (Pennsylvania 6's, due December 31, 1880) is used from January 1857 to January 1870; bond no. 33 (Lehigh Valley 6's, due June 1, 1898), from January 1870 to January 1886; and bond no. 70 (West Shore 4's, due January 1, 2361), from January 1886 to January 1936.

If, instead of constructing an index number by chaining together the yields of a few high grade bonds (as is done in Chart 2), we use as the index for each month the yield of the bond showing the lowest yield in that month, we should, theoretically, obtain an even closer approximation to a measure of the yield of long term loans of an extremely high and nearly unchanging degree of security. The difficulties of this procedure are practical rather than theoretical. Because any especially good bond may shoot forward into first place in any particular month, such an index tends to be distinctly more erratic in its minor movements than one constructed by chaining together a few superlative bonds. Furthermore, the necessity of removing entirely from the field of choice a specific high grade bond because it has approached too close to maturity may cause sudden erratic movements in the level of the entire index number. The next best bond available may be selling on an appreciably higher yield. Of the bonds we used in this study, the yields of those having the lowest yield each January, April, July and October, together with various index numbers, are presented on Chart 1.

It might be thought that an improvement on both procedures outlined above could be made by constructing a chain index number, using each month the yield of the bond having the lowest yield in that month and carrying *the same bond* through to the next month. Each of the individual index numbers that would be chained together would be

[11] The West Shore bonds were allowed to remain in the chart after 1930 to illustrate how a bond may suddenly deteriorate even though it has been high grade for years.

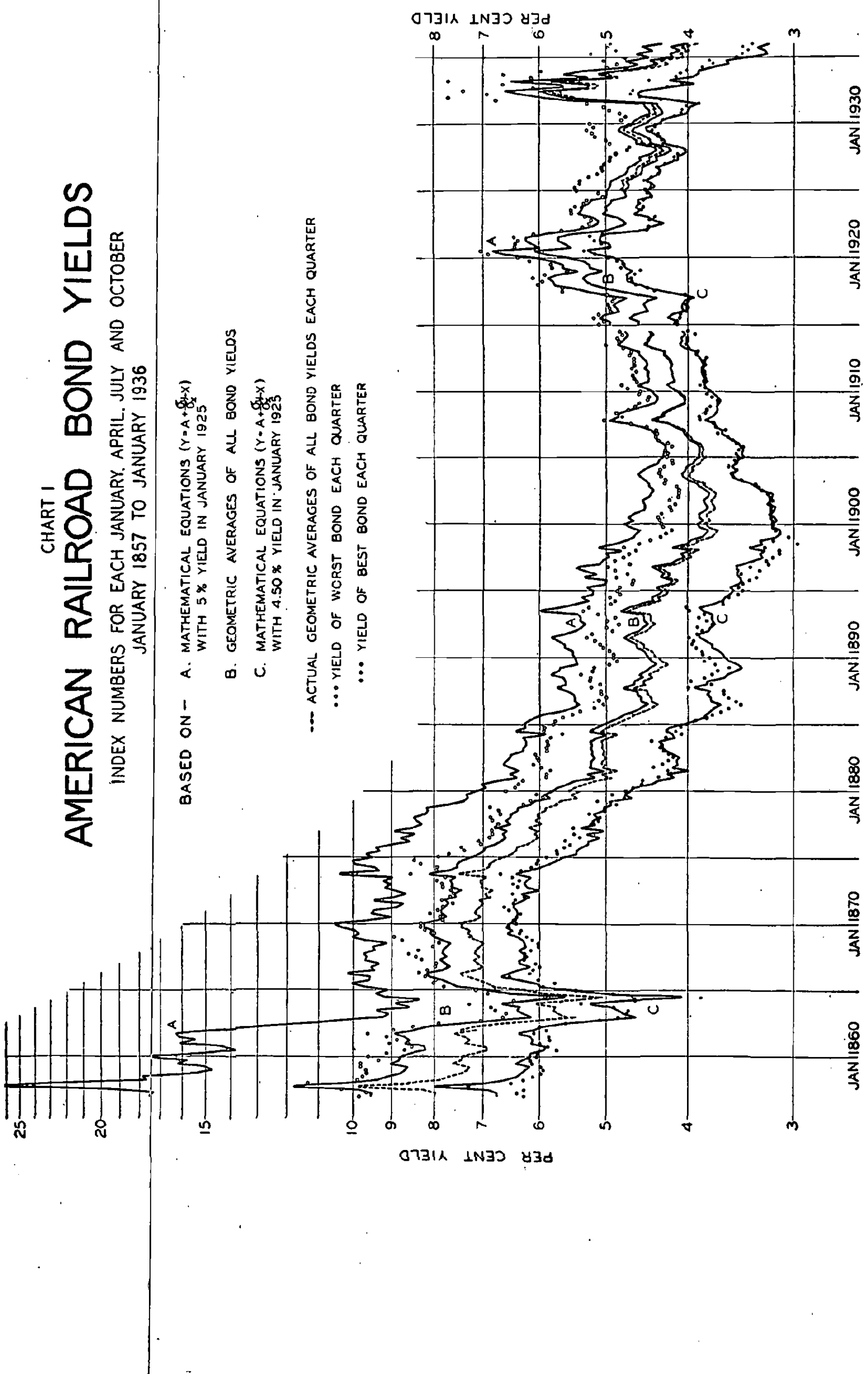


CHART I
AMERICAN RAILROAD BOND YIELDS

only one month long. The first piece would consist of the yield of the bond having the lowest yield in the first month and the yield of *the same bond* in the second month. The second piece would consist of the yield of the bond having the lowest yield in the second month and the yield of this same bond in the third month, etc. Such an index number would be free from the sudden changes in level that result from using each month the unadjusted yield of the bond having the lowest yield in that month. Moreover, during periods when the chaining procedure happened to use the yields of only one specific bond, the results would, of course, be similar to those shown by line C of Chart 2.

However, when the bonds used are not the same bond, a clear-cut *upward* mathematical drift would be introduced. In any particular month the bond selected is the bond having the lowest yield in that month. If the same bond has the lowest yield in the next month, the movement of the index will be the same as the movement of an index that used each month the unadjusted lowest yield. If the bond used in the first month is not the bond having the lowest yield in the second month, it must have a higher yield. One step in an upward drift has been made. That step can never be retraced. We begin all over with the bond having the lowest yield in the second month. In general, the shorter the periods used for the individual index numbers, the greater will be the upward drift. It was clearly not worth while to calculate such an index number on a monthly basis merely for the purpose of showing that it had an upward drift such as to make it totally unusable.

But we did calculate one such index on an annual basis—January to January each year. Beginning in January 1857 with the yield of the bond having the lowest yield in that month (6.27 per cent) and working forward, the index gives a value of 5.63 per cent for January 1879. In that month the yield of the 'best bond' was 4.86 per cent, and even the geometric average of all the bonds used was only 5.71 per cent. By January 1900 the index is 3.83 per cent. This is greater than the geometric average of all the bonds used in that month (3.73 per cent). The yield of the best bond was only 3.18 per cent. In January 1932 the index stood at 7.73 per cent. This is not only much greater than the yield of the best bond in that month (4.57 per cent) and greater than the geometric average (5.68 per cent) but also actually a shade greater than the yield of the bond showing the highest yield of any bond used in that month (7.72 per cent). The results

CHART 2

FOUR INDEX NUMBERS OF THE YIELDS OF AMERICAN RAILROAD BONDS

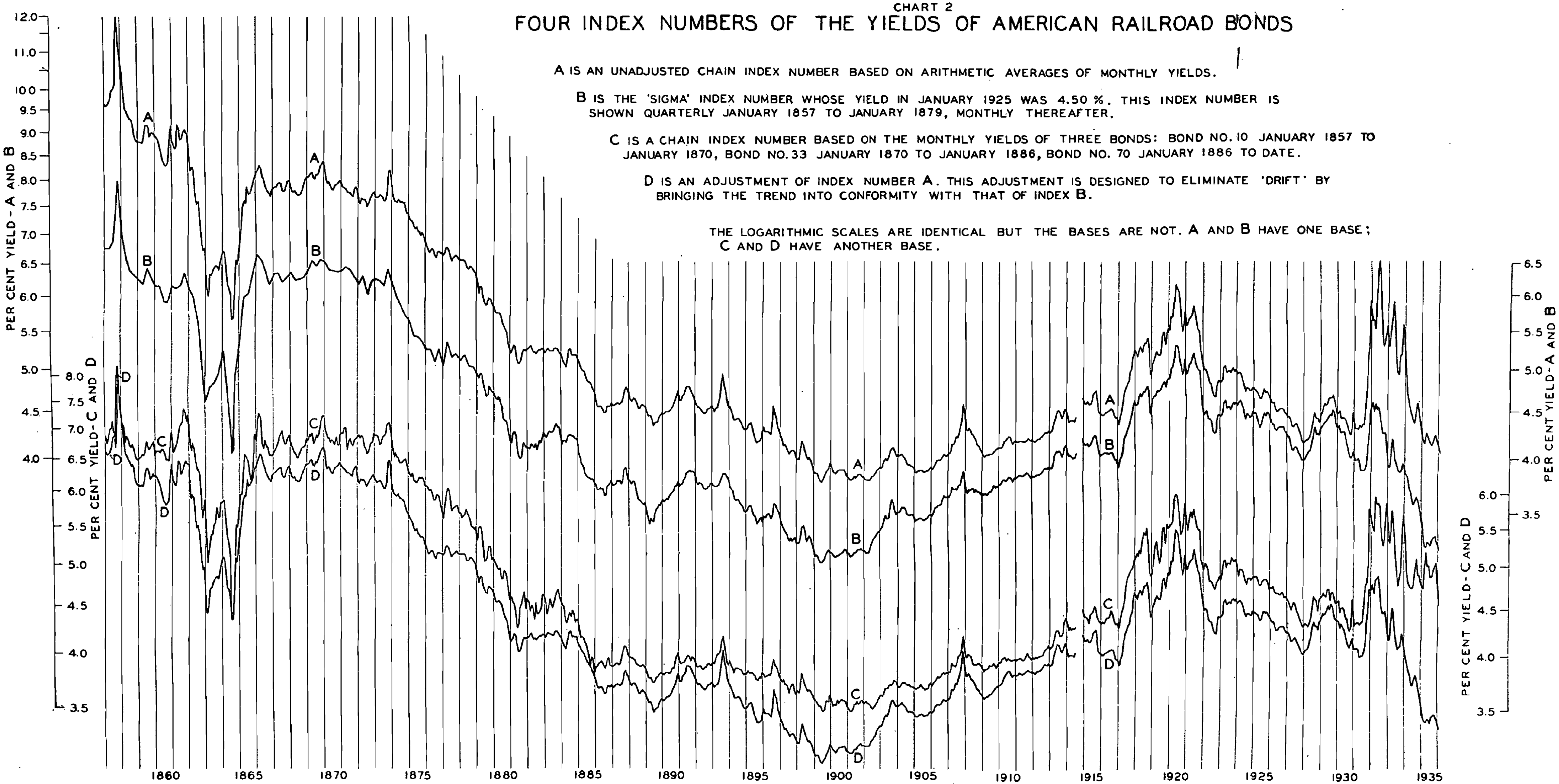
A IS AN UNADJUSTED CHAIN INDEX NUMBER BASED ON ARITHMETIC AVERAGES OF MONTHLY YIELDS.

B IS THE 'SIGMA' INDEX NUMBER WHOSE YIELD IN JANUARY 1925 WAS 4.50 %. THIS INDEX NUMBER IS SHOWN QUARTERLY JANUARY 1857 TO JANUARY 1879, MONTHLY THEREAFTER.

C IS A CHAIN INDEX NUMBER BASED ON THE MONTHLY YIELDS OF THREE BONDS: BOND NO. 10 JANUARY 1857 TO JANUARY 1870, BOND NO. 33 JANUARY 1870 TO JANUARY 1886, BOND NO. 70 JANUARY 1886 TO DATE.

D IS AN ADJUSTMENT OF INDEX NUMBER A. THIS ADJUSTMENT IS DESIGNED TO ELIMINATE 'DRIFT' BY BRINGING THE TREND INTO CONFORMITY WITH THAT OF INDEX B.

THE LOGARITHMIC SCALES ARE IDENTICAL BUT THE BASES ARE NOT. A AND B HAVE ONE BASE; C AND D HAVE ANOTHER BASE.



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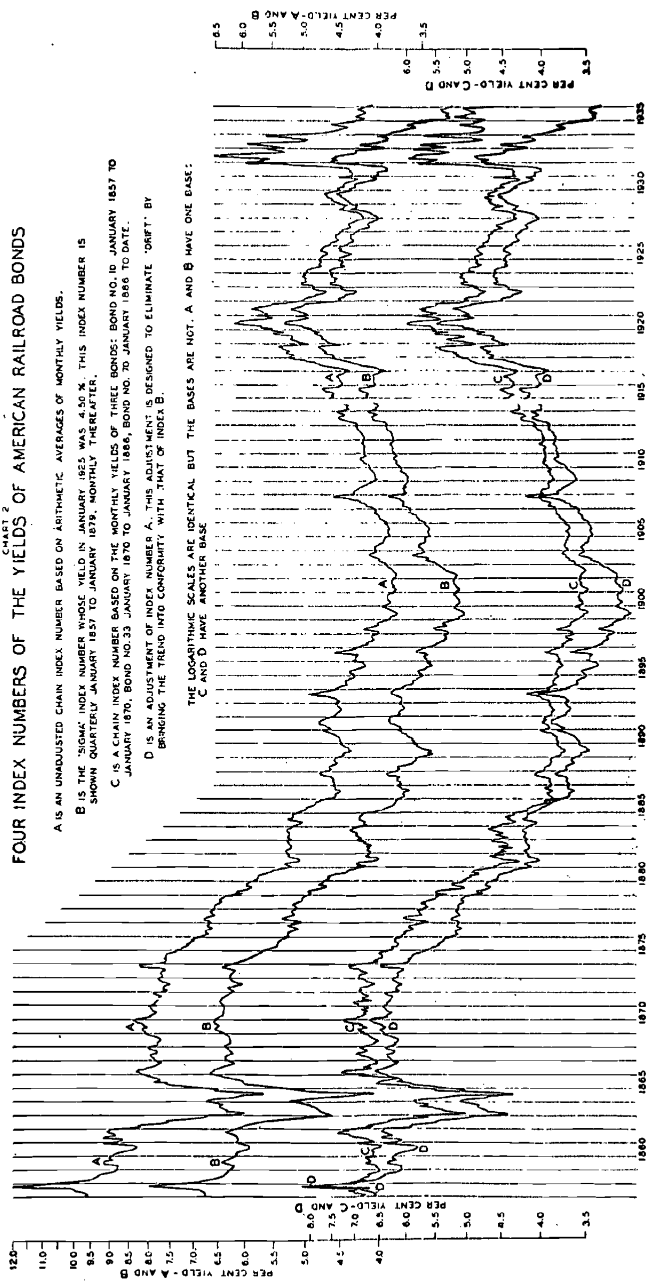
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PER CENT YIELD - A AND B
PER CENT YIELD - C AND D
1860
1865
1870
1875
1880
1885
1890
1895
1900
1905
1910
1915
1920
1925
1930
1935

would have been still more startling if we had constructed the index on a monthly instead of a yearly basis.

An index number constructed by chaining together individual pieces each of which consisted of the yield of the lowest yield bond at a particular date and the yield of the same bond in the *preceding* year or month would show a contrary mathematical drift, downward instead of upward.

More interesting results may be obtained if we do not restrict ourselves to one bond. Index numbers constructed from the best five bonds in each yearly period naturally show much less erratic and individualistic movements than index numbers constructed from the yields of single bonds.

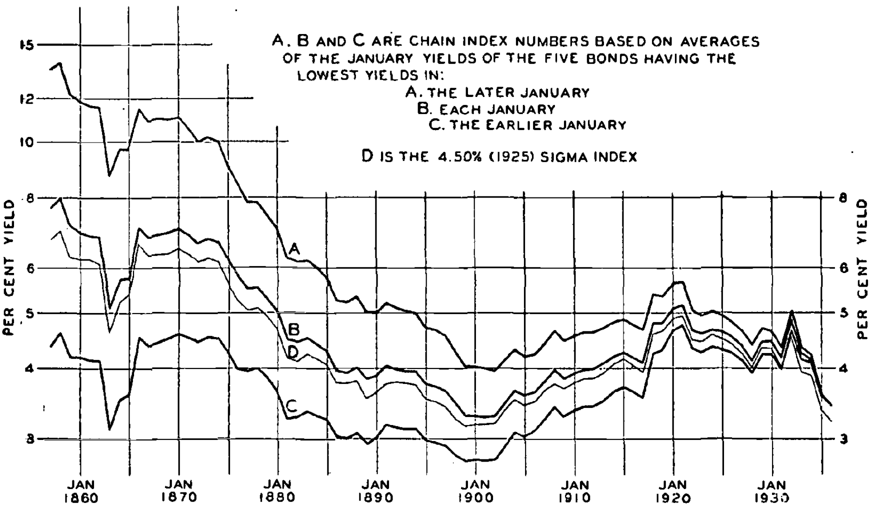
Our bonds are in annual groups. Each group contains quotations for the yields of certain bonds from January to January inclusive. If out of such a group we select the five bonds having the lowest yields in the first January and the five bonds having the lowest yields in the second January—whether or not they be the same—and compare the geometric average of the yields of the five bonds chosen in the first January with the geometric average of the yields of the five bonds chosen in the second January, we obtain a set of 79 index numbers each extending from one January to the next.[12] The result of chaining together these 79 index numbers is presented in Chart 3 (line B). Line D is the '4.50 sigma' index, which we later present as a tentative solution of the problem of drift. It will be seen that the index constructed from the best five bonds, in the manner we have just described, is very similar to the '4.50 sigma' index. The downward drift is slightly greater but the year-to-year movements are almost identical. This particular five-bond index has, of course, no mathematical drift. For example, the same results would be obtained if the index were cal-

[12] Though the 'best' five (or lowest yield) bonds chosen in a particular January in order to construct the index number from the preceding January to that January are usually the same as the best five bonds chosen in that January to construct the index number from that January to the succeeding January, they are not always so. The dropping of old bonds and the introduction of new bonds sometimes prevents such a condition. For example the 'best' five bonds chosen for January 1925 to construct the index number from January 1924 to January 1925 are not the same bonds as those chosen to construct the index number from January 1925 to January 1926.

culated from month to month as if it were calculated from year to year.

Chart 3 shows two other indexes based on the best five bonds each January. One of these (line C) is constructed by chaining together yearly indexes in each of which the geometric average of the yields of the best five bonds in the earlier January is compared with the geometric average of the yields of the *same five bonds* in the later January. In the other index (line A) the five bonds are the best five in the later January and the comparison is made with the yields of the *same five bonds* in the earlier January. Both the upward drift of the index of yields based on choosing the bonds in the earlier January [13] and the downward drift of the index based on choosing the bonds in the later January are, of course, mathematical and not economic drifts. The drift would in each case have been greater if the indexes had been constructed from month to month instead of from year to year. The three

CHART 3
FOUR INDEX NUMBERS OF JANUARY YIELDS OF AMERICAN RAILROAD BONDS



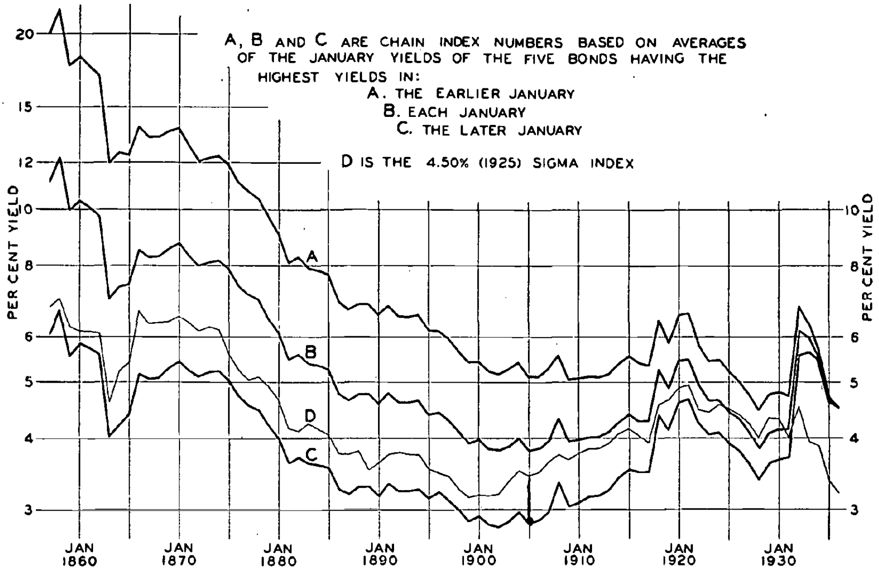
[13] As a digression, it may be interesting to note that the index based on choosing the best five bonds at the beginning of the year gives a rather sad picture of the fate of the investor who would buy nothing but bonds with the very lowest possible yields, on the assumption that such low yields proved the bonds to be of the most desirable type, and insisted upon selling them, whenever their yield rose, and reinvesting in other 'superlative' bonds.

index numbers based on the 5 bonds having the lowest yields each year are presented in Appendix A, Table 5.

Chart 4 is introduced as a further illustration of mathematical rather than economic drift and to show the dangers of attempting to correct for economic drift by means of introducing a mathematical drift that is in the opposite direction. The bonds chosen each January are

CHART 4

FOUR INDEX NUMBERS OF JANUARY YIELDS OF AMERICAN RAILROAD BONDS



the *worst* five bonds instead of the best five bonds, that is, the five bonds having the highest yields instead of the five bonds having the lowest yields. Throughout most of the period 1857–1936 there has been a pronounced downward economic drift in the yields of these 'worst' bonds. However, the mathematical upward drift introduced by picking them in the later January and comparing the geometric average of their yields in that January with the geometric average of the yields of the *same bonds* in the earlier January (see line C) is more than sufficient to overcome the downward economic drift.[14]

On the other hand, the intermediate movements show driftings apart and driftings together. After remaining below D until January 1931,

[14] Indeed, if the chaining had been from month to month instead of from year to year, line C would have shown an almost continuous *upward* drift when compared with the 4.50 sigma line.

the line C suddenly shoots up and remains thereafter much above D. The minor movements of the two lines often have little or no relation to each other. While the adjustment for drift obtained by using the *best five bonds, regardless of whether they were or were not the same bonds in both Januaries,* is an economic adjustment and led to relatively good results throughout, the adjustment based on choosing the *worst five bonds in the later January* is purely mathematical. The goodness of the results in any period is quite accidental.

It is quite illegitimate to attempt to eliminate economic drift by means of a formula giving a mathematical drift in the opposite direction. In the first place, the economic drift is not necessarily in the same direction in one period as it was in the preceding period, whereas the mathematical drift tends to be always in the same direction. In the second place, any mathematical drift that results from 'chaining' varies in degree with the duration between the items chained. Monthly chaining gives a more pronounced drift than yearly chaining.

The disturbing effects of economic 'drift', of course, usually increase with the length of time covered. Over short and comparatively undisturbed periods index numbers from which drift has not been eliminated often have a relatively definite and simple significance. Without any great error, they may be interpreted as picturing the movements of the yields of bonds of the same grade. Over long periods they may, however, have little essential relation to the yields of bonds of any specified grade. Even if the same bonds are used throughout, their grade at the end of the period, as seen from the relation of the average of their yields to the yields of the best bonds, may be entirely different from what it was at the beginning of the period.

Of course, if we had been interested only in *eliminating* drift, we might, as has already been suggested, have done so pretty well by some such procedure as the chaining together of indexes made up from the yields of the five lowest yield bonds whether they were the same bonds at both dates or not. But we were fully as much interested in discovering the characteristics of drift and how it can be measured as we were in eliminating it. We wished not merely to present a picture of the movements of the yields of railroad bonds of the highest grade but also to show how bonds of lower grade acted and to present in as simple a mathematical form as possible the statistical relations between the movements of the yields of bonds of different grades.

And, finally, we hoped to be able to bring the movements of stock prices into the bond yield picture.

We soon came to the conclusion, already presented in the preceding chapter, that no definite solution could be obtained by studying the financial statements of the railroads. 'Margins of safety', etc., are illusory and misleading. The real 'margin of safety' that counts is in the future, not the past. It is better to trust to the opinion of the market. The simple and direct way to decide whether, at a particular time, one bond should be considered as of a higher or lower grade than another bond of the same coupon rate, maturity, marketability, etc., is to compare their yields. The problem is therefore to discover the relation between the movements of the yields and the yields themselves. When this problem is solved it becomes possible to construct, as a by-product, an index number of the yields of the highest grade bonds—even if the grade desired be somewhat higher than the grade of any of the bonds used in discovering the relation.

The railroad bonds we used had as long maturities as were available. Since, as already mentioned, we did not adjust for differences of duration, we formulated our problem as that of discovering, with the data available, what were the relations between the yields of identical bonds at different dates. We began our experiments by considering the matter graphically. We made scatter diagrams in which the yields at a particular date of the various bonds in a group were plotted along the x axis and the yields at a later date along the y axis. We plotted the logarithms of the yields rather than the yields themselves, because a linear relationship seemed more logical on a logarithmic scale than on a natural scale. For many reasons, we desired a linear relationship and the use of a linear relation with a natural scale would tend to lead to absurdities in just the region where we did not wish absurdities—the region of the lower yields. For example, if a straight line were fitted to the yields as such, it might cut the axes and so suggest that a positive yield in one period should be considered as normally associated with a *negative* yield in another period.

After constructing and examining a number of scatter diagrams, we next considered how we ought to fit straight lines to the logarithms of the data. What should be the criterion of fit? We of course realized that the yields in neither the earlier nor the later period—say the earlier or the later January—could logically be considered as independent variables. Both the yields in the earlier and the yields in the later January had to be considered as dependent variables. We did not wish to know what would be the probable yield in the later January of a particular bond having a specified yield in the earlier January any more than we wished to know the probable yield in the earlier January of a particular bond having a specified yield in the later January. For the same sort of reasons that we did not find it desirable to pick the best five bonds in either the earlier or the later January but in both Januaries, we desired a backwards and forwards relation between the yields in the two Januaries. To borrow an expression from the theory of least squares, our problem must be considered one in which both variables are assumed to be 'tinged with error'.[15] The straight line must not be fitted in such a manner that the sum of the squares of either the vertical or the horizontal deviations of the data points from the fitted line be made a minimum but in such a manner that the sum (or, academically, one-quarter of the sum) of the squares of the vertical *and* horizontal deviations be made a minimum,[16] if the size of the errors to which the two variables are subject is the same.

Both variables should be, for the purposes of our problem, considered as 'subject to error'. But we cannot assume that the size of the 'errors' to which each variable is subject is the same. In other words, we cannot assume that the 'errors' of the two variables should have equal weights. Using 'error' in the statistical sense of deviation, we know that the two variables are *not* subject to the same degree of 'error'. Their liability to error is clearly in proportion to their standard deviations. Before fitting the straight line we must therefore weight the variables in inverse proportion to the *squares* of their standard deviations. Such weighting will exactly correct for the fact that their liability to 'error' is in proportion to their standard deviations.

This may all sound rather high-handed. At first glance it might seem simple and proper to give the logarithms of the yields in each January equal weights. However, in a statistical problem in which both variables are considered as subject to 'error', the assumption of equal

[15] Of course the statistician uses the word 'error' in a Pickwickian sense. When he measures the average height of a class of school children and finds that it is so many feet and so many inches, he does not consider the fact that all the children are not of that height as really an 'error' of either God or man.

[16] Cf. Merriman's *The Determination by the Method of Least Squares of the Relation between Two Variables . . . . both Variables being Liable to Errors of Observation* (U. S. Coast and Geodetic Survey, 1890), p. 687.

weights may be very misleading. It tends to prevent any real consideration of the problem of weighting. Only if the variables are measured in absolutely unlike units, such as length and weight or pressure and temperature, will the existence of the problem be even noticed. When, however, as in our problem, both variables are expressed in percentages—or logarithms of percentages—there is a strong tendency to let mere words lull us into the assumption that we have a problem requiring equal weights. However, in our particular problem, a procedure based on such an assumption would have led us to treat a one per cent deviation of observation from theory in one January as of the same importance as a one per cent deviation in the succeeding January, although the scatter of the yields in the later January might be so much greater than the scatter of the yields in the earlier January that their standard deviation was double that of the yields in the earlier January.

The slope of the straight line, fitted in such a manner that the sum of the squares of the vertical and horizontal deviations of the observed from the theoretical values (when weighted in inverse proportions to the squares of the standard deviations of the two variables), will be a minimum, is $\frac{\sigma_y}{\sigma_x}$, where σ_x = the standard deviation of the logarithms of the yields at one date and σ_y = the standard deviation of the logarithms of the yields at the other date. From the mean of the system, the equation of the line is $y = \frac{\sigma_y}{\sigma_x} x$.[note 17]

[17] The weighting is in inverse proportion to the *squares* of the standard deviations, rather than in inverse proportion to the standard deviations unsquared, because the criterion of fit is that the *squares* of the 'deviations' shall be a minimum.

The 'deviations' are not, of course, equal to the perpendicular and horizontal distances of the observational points from the fitted line. For example, a y 'deviation' is not equal to the perpendicular dropped from the observational point to the fitted line, as it would be if the x variable were being considered independent. A y 'deviation' is the difference between the y of an observational point and the y of the corresponding theoretical point on the straight line. And that theoretical point is not perpendicularly above or below the observational point. The slope of the line joining an observational point to its corresponding theoretical point on the straight line is $-\frac{\sigma_y}{\sigma_x}$, that is, the slope of the fitted line with a negative sign. Now the sum of the squares on the two sides of a right angled triangle adjacent to the right angle is equal to the square on the hypotenuse. It is, therefore, the sum of the squares of the *oblique* distances of the observational points from the corresponding theoretical points on the fitted straight line that is made a minimum.

If the straight lines be fitted in this manner, all index numbers based on readings from the fitted lines will fulfill the 'circular test'. If, on the other hand, the observations at the two dates be weighted 'equally', or indeed in any other ratio than in inverse proportion to the squares of their standard deviations, the index numbers obtained from readings on the fitted lines will not fulfill the 'circular test'. Some form or other of purely mathematical drift will be introduced. The results obtained by moving along from one month to the next in constructing the index numbers will not be the same as if we move from one January to the next January.

If ordinary 'regression' lines, in which only one variable is considered dependent, were fitted, the resulting index numbers (of lower than mean yields) would drift violently upward if the earlier dates were taken as the independent variables and violently downwards if the later dates were taken as independent. An index number constructed from our January-to-January scatters by taking the earlier Januaries as the independent variables and showing a yield of four and one-half per cent in January 1927 shows a yield of less than one-billionth of one per cent for January 1857. If the later Januaries be considered independent, a seven per cent yield in January 1857 is associated with less than one-billionth of one per cent in January 1927. The results would be even more extreme if the chaining were from one month to the next instead of from January to January.[18]

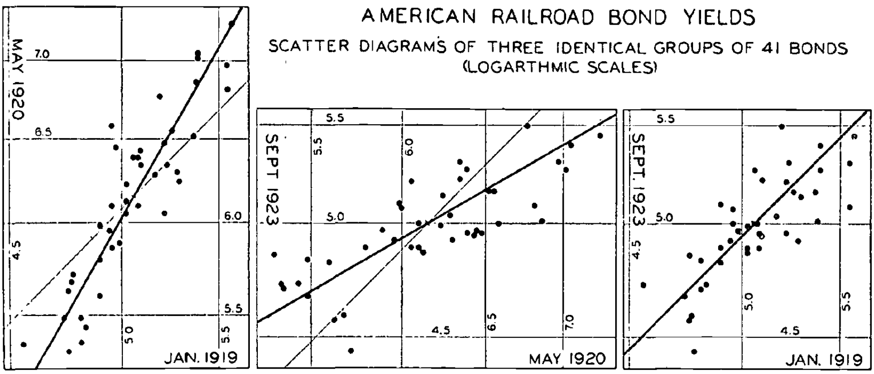
The fulfillment of the circular test may be illustrated by Chart 5. That chart contains three scatter diagrams. The same 41 bonds appear in each diagram. The heavy fitted line in each diagram represents the theoretical relation (weights of the variables in inverse proportion to the squares of their standard deviations) between the yields at an earlier date and those at a later date of bonds of different grades. The

[18] In computing the scatter from one January to the next, the coefficient of correlation (r) might possibly equal +1. If r equaled +1, the fitted lines obtained from assuming one or other of the variables independent or from assuming them both dependent would, of course, be identical. The observations would all lie in the line.

If r is not equal to +1, both the equal weights line and the sigma line will be steeper than the x–as–independent–variable line, and less steep than the y–as–independent–variable line. When $\sigma_y = \sigma_x$, the equal weights line and the sigma line will be identical. Each will be inclined 45°, that is, have a slope of +1. When σ_y is not equal to σ_x, the sigma line will fall between the equal weights line and the 45° line. It will, therefore, be steeper than the equal weights line when σ_y is greater than σ_x and less steep when σ_y is less than σ_x.

light line inclined at an angle of 45 degrees is the line on which all points would fall if the ratio of the yield of each bond at the later date to the yield of the same bond at the earlier date had been, for all bonds regardless of grade, the same as the ratio of the geometric mean of the yields of all the bonds at the later date to the geometric mean of the

CHART 5
AMERICAN RAILROAD BOND YIELDS
SCATTER DIAGRAMS OF THREE IDENTICAL GROUPS OF 41 BONDS
(LOGARTHMIC SCALES)



yields at the earlier date. The first diagram compares the yields in January 1919 and the yields in May 1920, a period during which high grade bonds acted *better* than lower grades. The second diagram compares the yields in May 1920 and September 1923, a period during which high grade bonds acted *worse* than low grade bonds. The third diagram covers both periods. It compares the yields in January 1919 and September 1923, a period during which there is no correlation between the grades of the bonds (as evidenced by their yields) and how they acted. The heavy fitted line in this diagram therefore coincides with the light 45 degree line.

Now, if we find, from an examination of the first diagram, that a yield of x in January 1919 should theoretically, as determined by a point on the heavy fitted line, be associated with a yield of y in May 1920 and, from an examination of the second diagram, that a yield of y in May 1920 should theoretically be associated with a yield of z in September 1923, we may know that, if from the third diagram we ask what yield in September 1923 should theoretically be associated with a yield of x in January 1919, the answer will be z. Index numbers constructed from readings of straight lines fitted by the method of least squares in such a manner that the sum of the squares of the vertical and horizontal deviations, weighted in inverse proportion to the squares of their respective standard deviations, is a minimum, fulfill the 'circular' test. To avoid the continual use of lengthy phrases we shall from now on call a straight line fitted in such a manner a sigma line. We coined this term because the slope of such a line is $\frac{\sigma_y}{\sigma_x}$.

Chart 6 contains 79 scatter diagrams, one for each pair of Januaries from January 1857 and January 1858 to January 1935 and January 1936. Sigma lines are fitted to each scatter. From the nature of the slopes $\frac{\sigma_y}{\sigma_x}$ of a sigma line, the slope of a line fitted for two widely separated dates can easily be computed from the slopes of the lines for the intervening sections. Thus, if the standard deviation of the yields of a group of bonds be σ_x, in January 1920, σ_y in January 1921 and σ_z in January 1922, the slope of the sigma line for January 1920 and January 1921 will be $\frac{\sigma_y}{\sigma_x}$, for January 1921 and January 1922 $\frac{\sigma_z}{\sigma_y}$, and for January 1920 and January 1922 $\frac{\sigma_z}{\sigma_x}$. From the upper line of Chart 15 the reader can estimate the slope of the sigma line for any two dates. This upper line of Chart 15 represents the cumulated product of the slopes of the successive sigma lines fitted to the scatters of the yields in successive adjacent pairs of months. As the chart is drawn on a logarithmic scale, a difference in height of the line for any two different dates represents the logarithm of the slope of the sigma line applicable to those two dates.[19] The annual and quarterly equations of the sigma lines are presented in Appendix A, Tables 7 and 8. The slopes of the monthly equations are given in Table 9.

The sigma lines may be used for two purposes and are so used in this study: (1) to construct index numbers, (2) to illustrate, measure and study the differences in the movements of the yields of bonds of different grades in different periods.

Beginning with any given yield at any particular date, we may construct a complete index number from January 1857 to January 1936. But the reader must not assume that such an index number would necessarily give a picture of the action of bonds of the same grade throughout the period. If the original yield with which the computa-

[19] The 'difference in height' on the chart may be exactly determined from column 2 of Table 6 in Appendix A.

CHART 6

AMERICAN RAILROAD BOND YIELDS

SCATTER DIAGRAMS

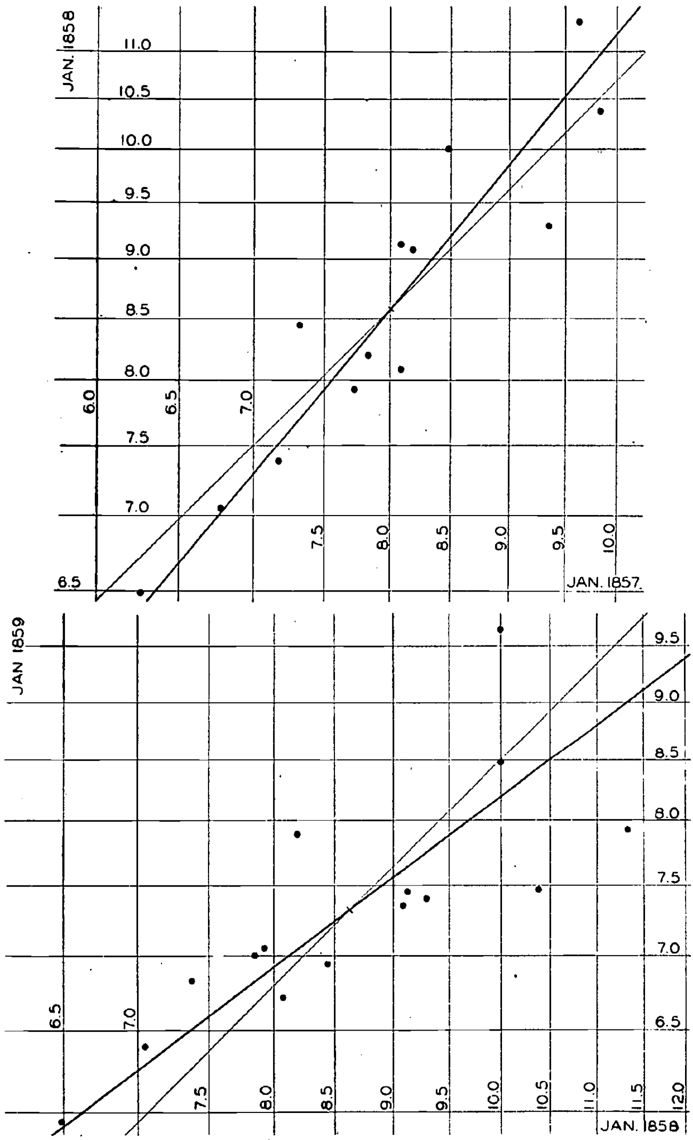
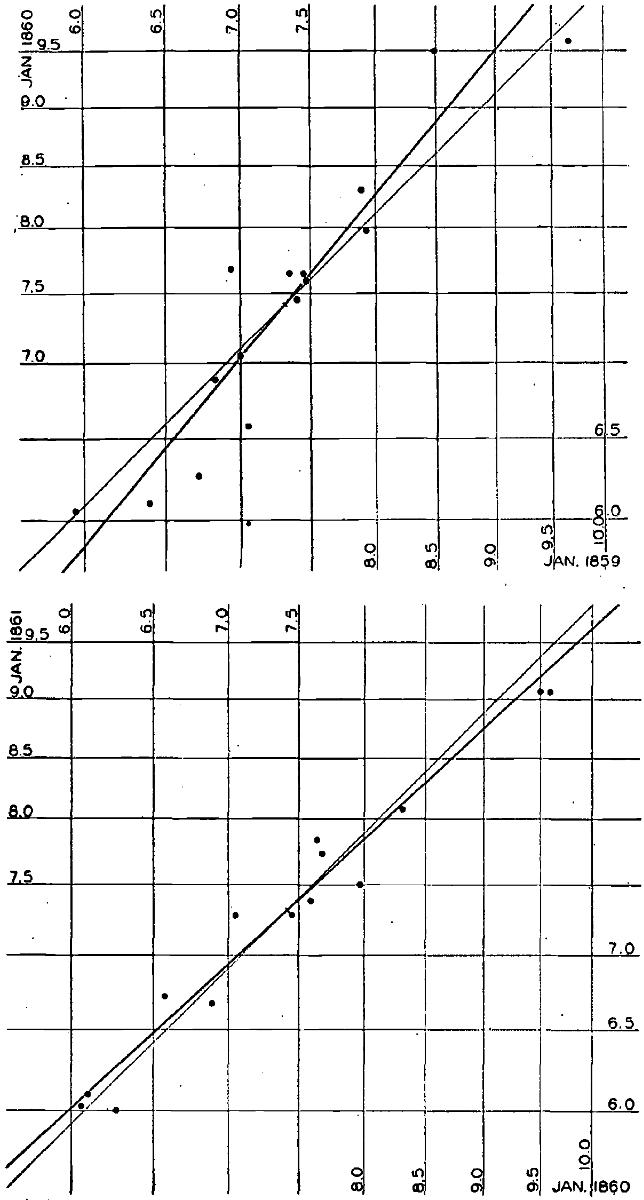


CHART 6 (CONT.)

RAILROAD BOND YIELDS



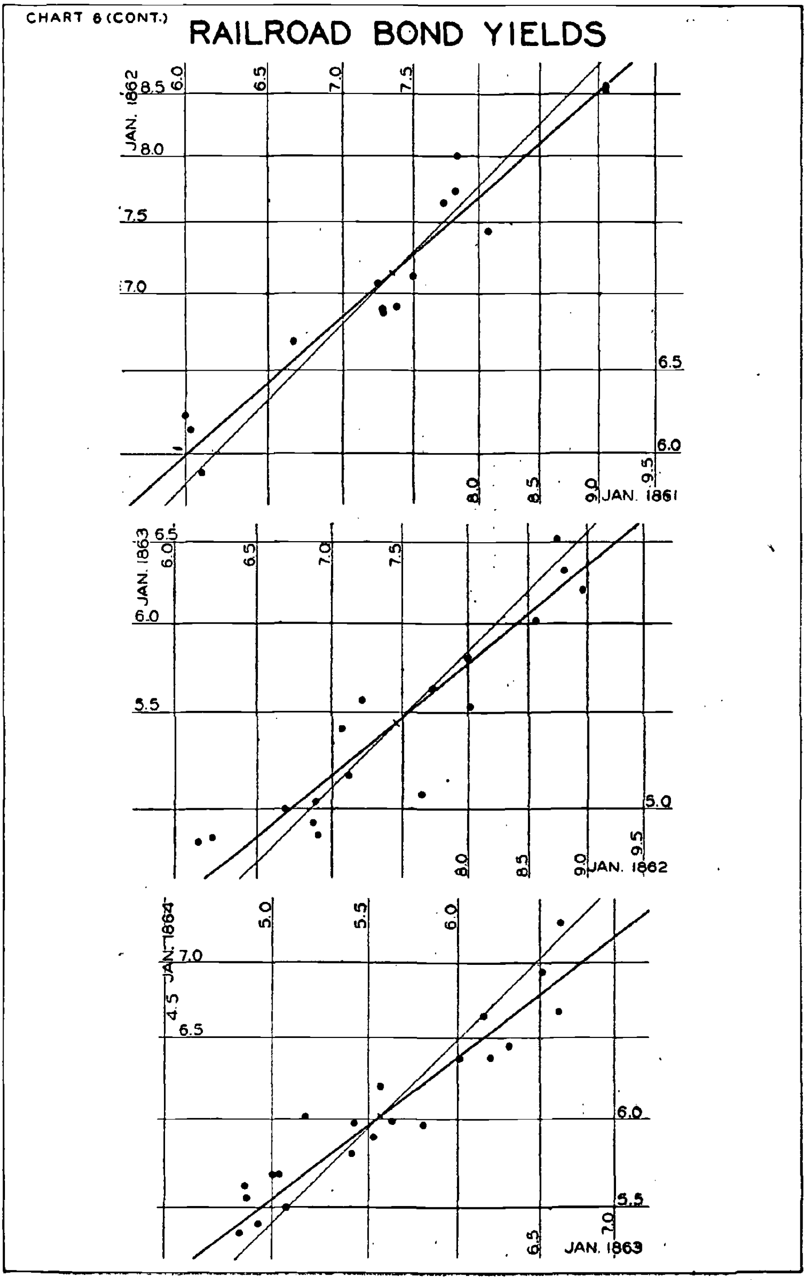


CHART 6 (CONT.)
RAILROAD BOND YIELDS
JAN. 1862
JAN. 1861
JAN. 1863
JAN. 1862
JAN. 1864
JAN. 1863

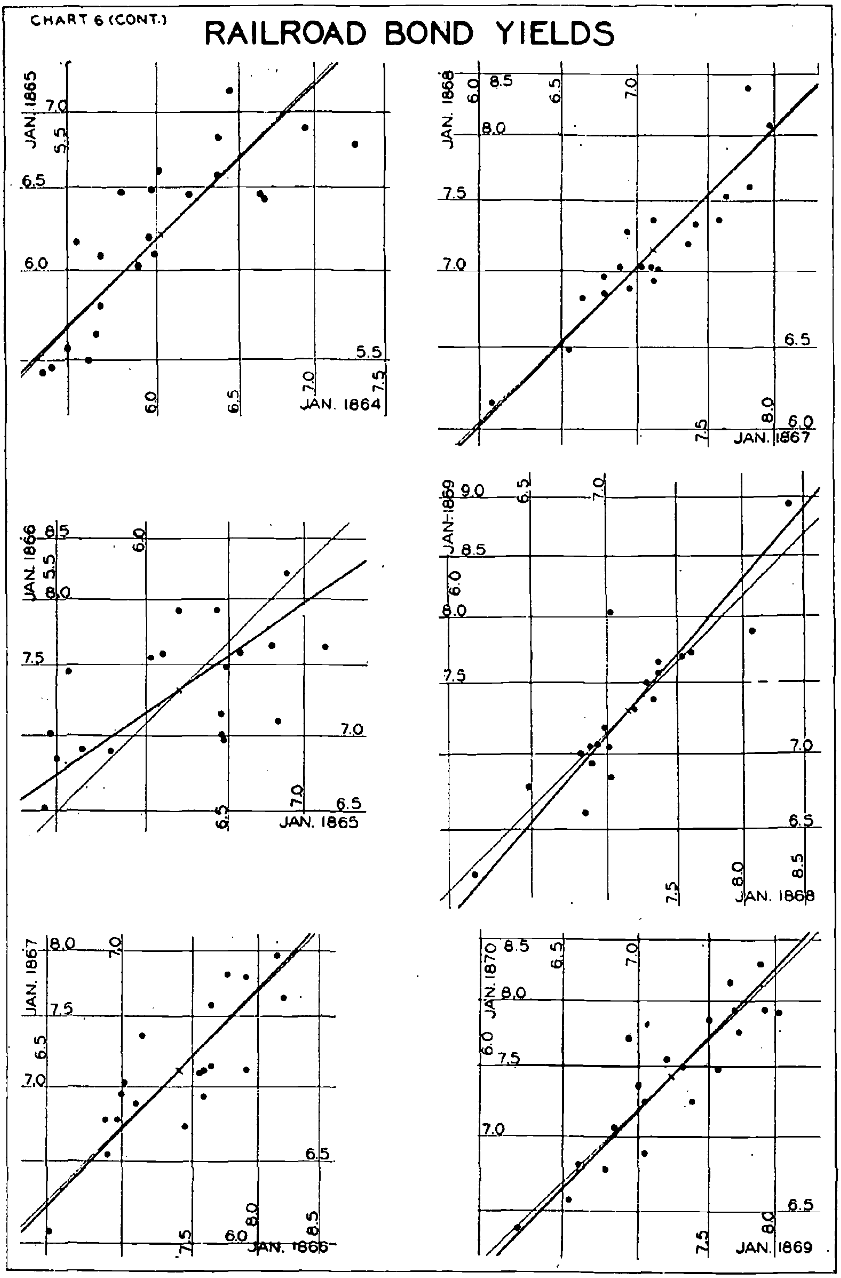
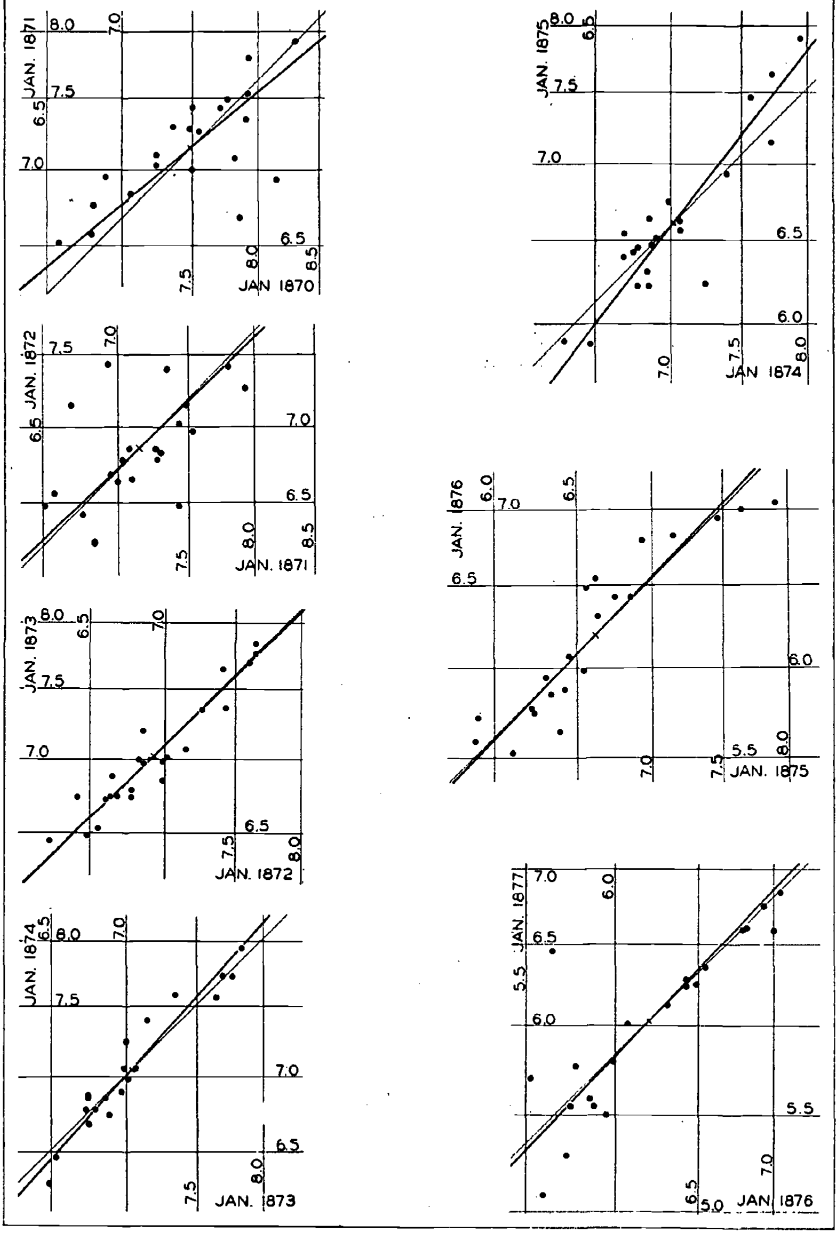


CHART 6 (CONT.)
RAILROAD BOND YIELDS
JAN. 1865
JAN. 1864
JAN. 1868
JAN. 1867
JAN. 1866
JAN. 1865
JAN. 1869
JAN. 1868
JAN. 1867
JAN. 1866
JAN. 1870
JAN. 1869

CHART 6 (CONT.)

RAILROAD BOND YIELDS



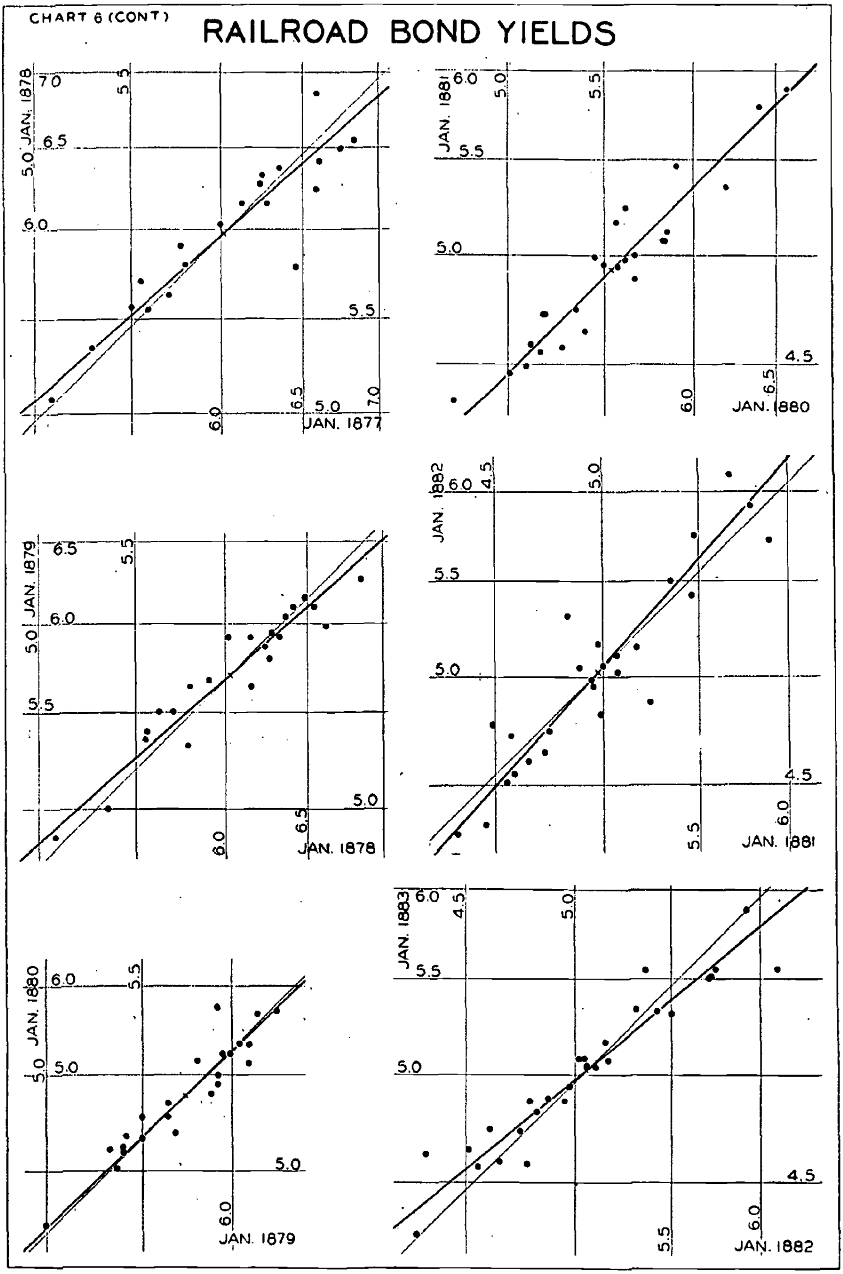
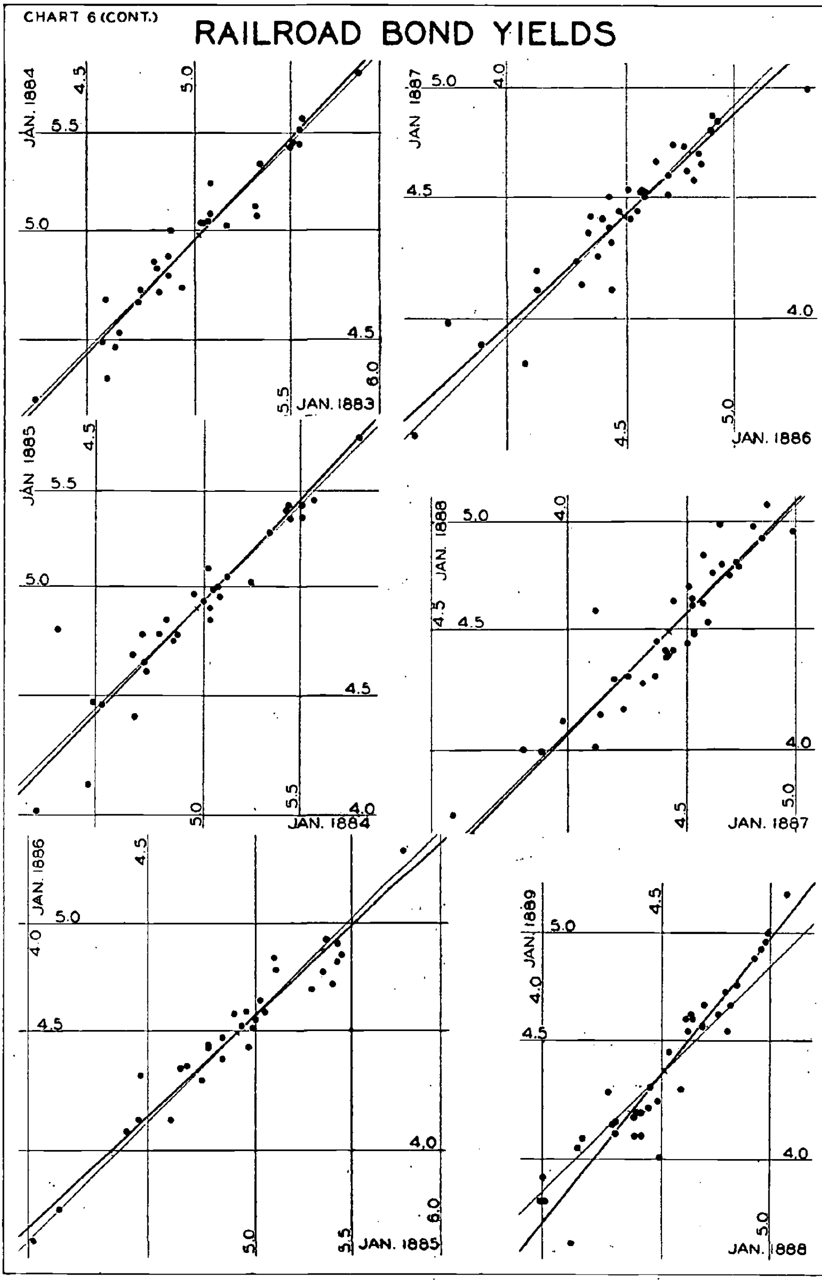


CHART 6 (CONT)
RAILROAD BOND YIELDS
JAN. 1878
JAN. 1877
JAN. 1881
JAN. 1880
JAN. 1879
JAN. 1878
JAN. 1882
JAN. 1881
JAN. 1880
JAN. 1879
JAN. 1883
JAN. 1882

CHART 6 (CONT.)


RAILROAD BOND YIELDS
JAN. 1884
JAN. 1883
JAN 1887
JAN. 1886
JAN 1885
JAN. 1884
JAN. 1888
JAN. 1887
JAN. 1886
JAN. 1885
JAN. 1889
JAN. 1888

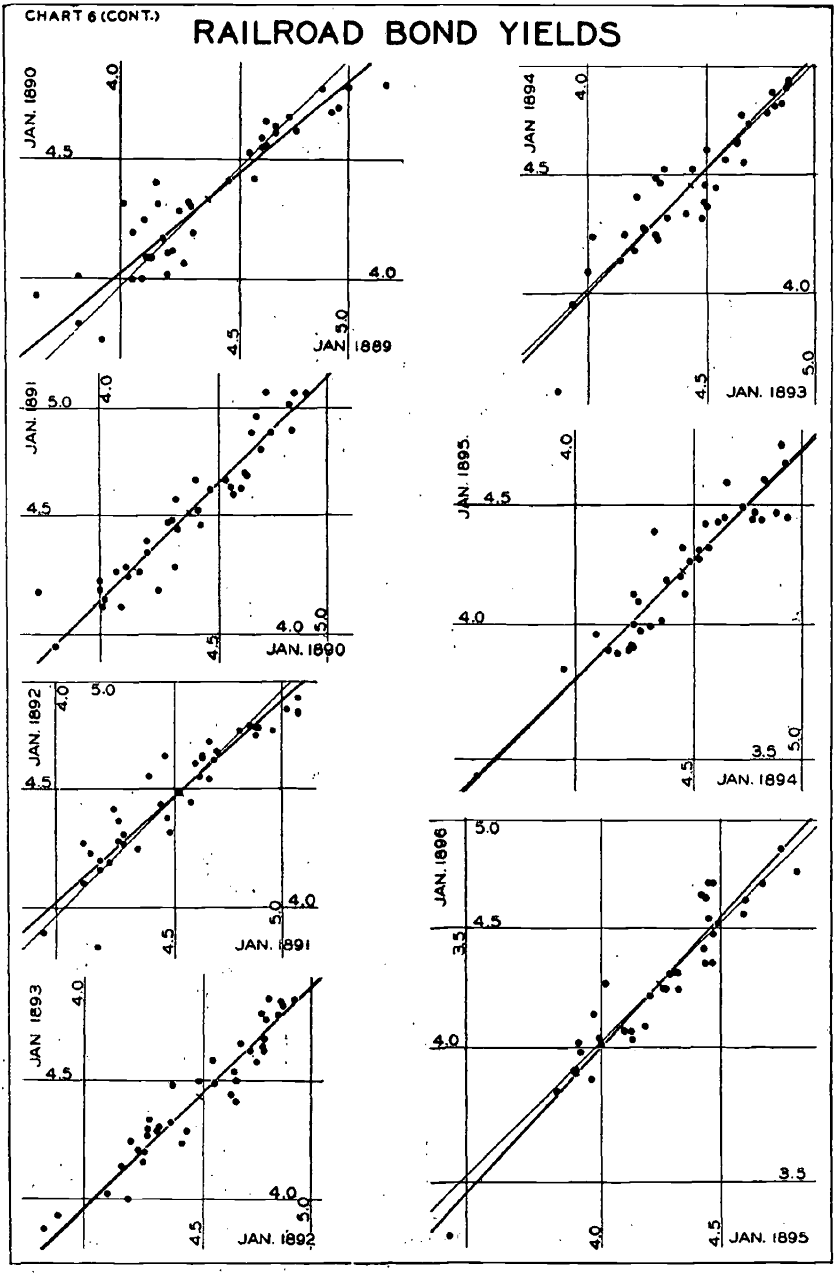
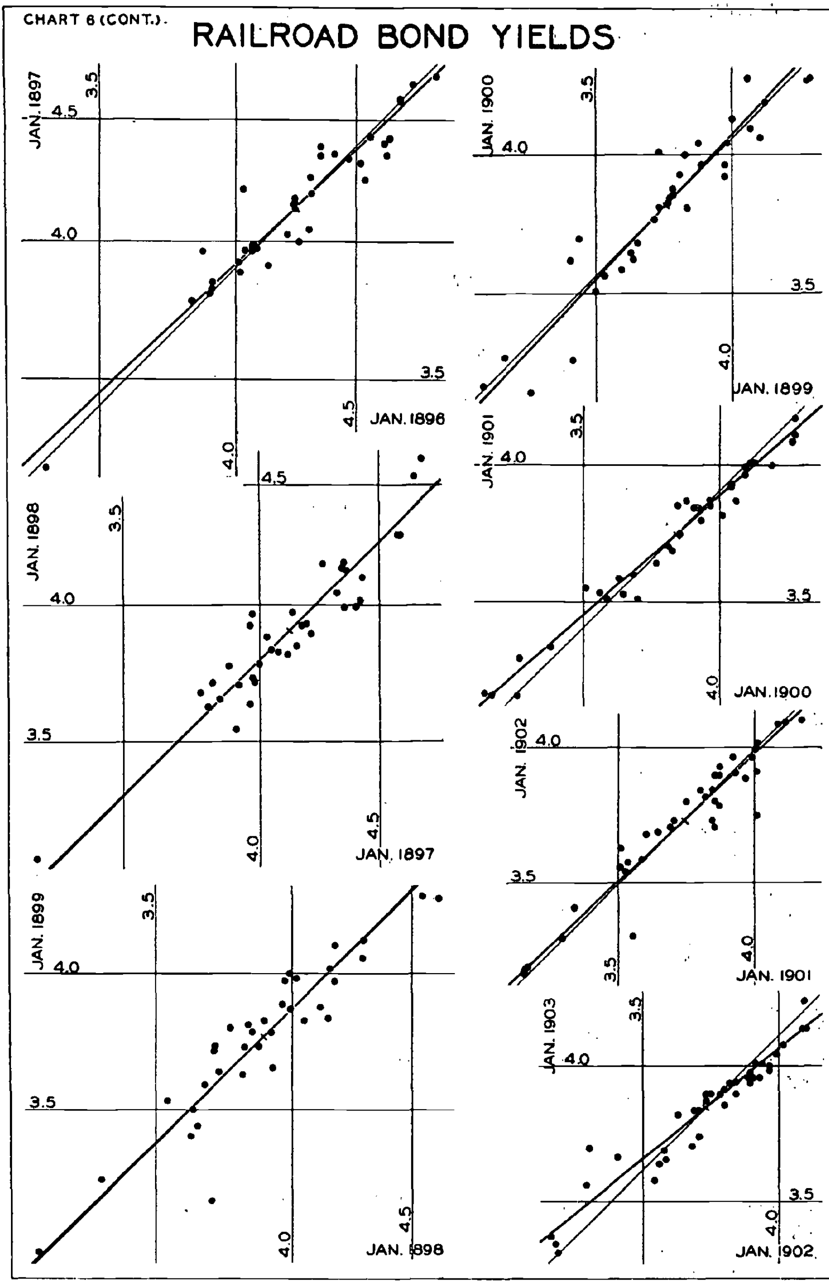


CHART 6 (CONT.)
RAILROAD BOND YIELDS
JAN. 1890
JAN. 1889
JAN. 1891
JAN. 1890
JAN. 1892
JAN. 1891
JAN 1893
JAN. 1892
JAN 1894
JAN. 1893
JAN. 1895
JAN. 1894
JAN. 1896
JAN. 1895
3.5
4.0
4.5
5.0

CHART 8 (CONT.).


RAILROAD BOND YIELDS
JAN. 1897
3.5
4.5
4.0
3.5
4.5
JAN. 1896
4.0
JAN. 1900
3.5
4.0
3.5
4.0
JAN. 1899
JAN. 1898
3.5
4.5
4.0
3.5
4.0
4.5
JAN. 1897
JAN. 1901
3.5
4.0
3.5
4.0
JAN. 1900
JAN. 1899
3.5
4.0
3.5
4.0
4.5
JAN. 1898
JAN. 1902
4.0
3.5
3.5
4.0
JAN. 1901
JAN. 1903
3.5
4.0
3.5
4.0
JAN. 1902

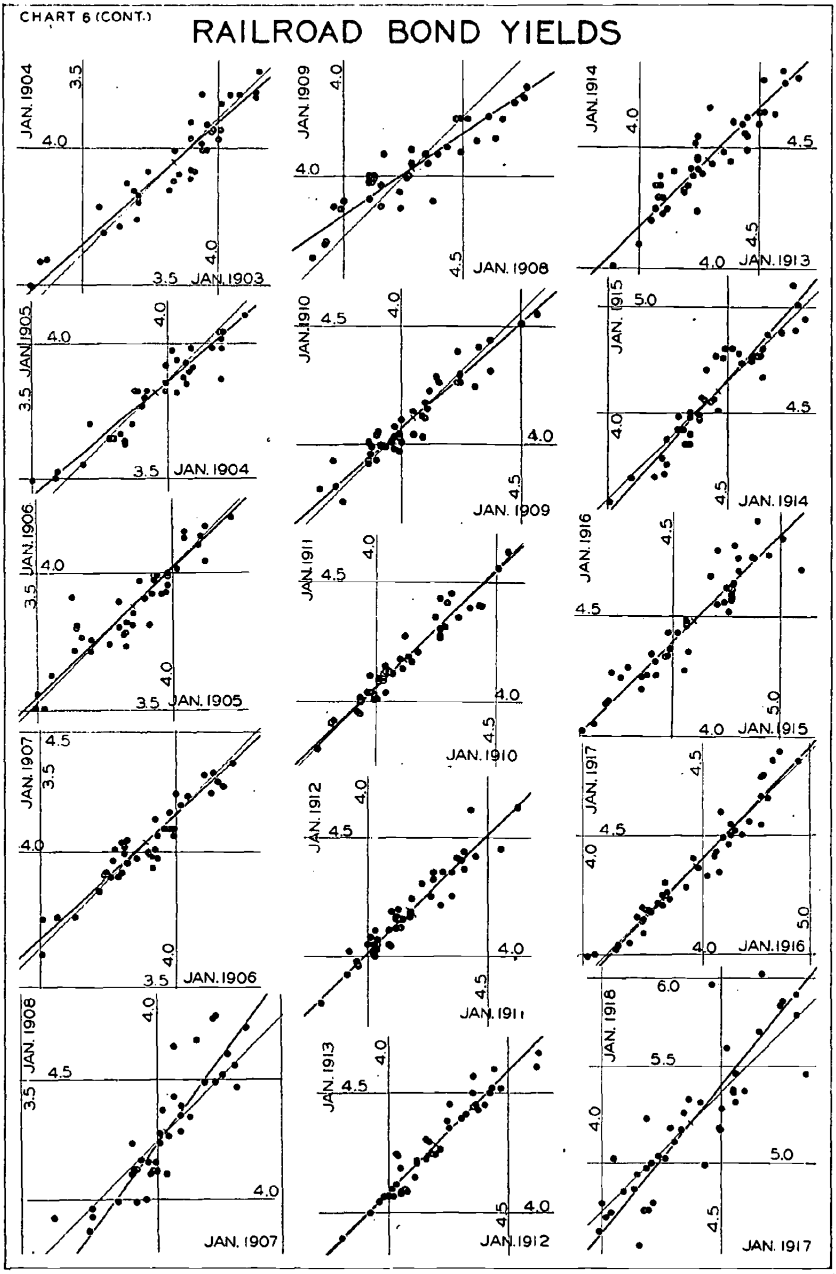


CHART 6 (CONT.)
RAILROAD BOND YIELDS
JAN. 1904
JAN. 1903
JAN. 1909
JAN. 1908
JAN. 1914
JAN. 1913
JAN. 1905
JAN. 1910
JAN. 1915
JAN. 1906
JAN. 1911
JAN. 1916
JAN. 1907
JAN. 1912
JAN. 1917
JAN. 1918

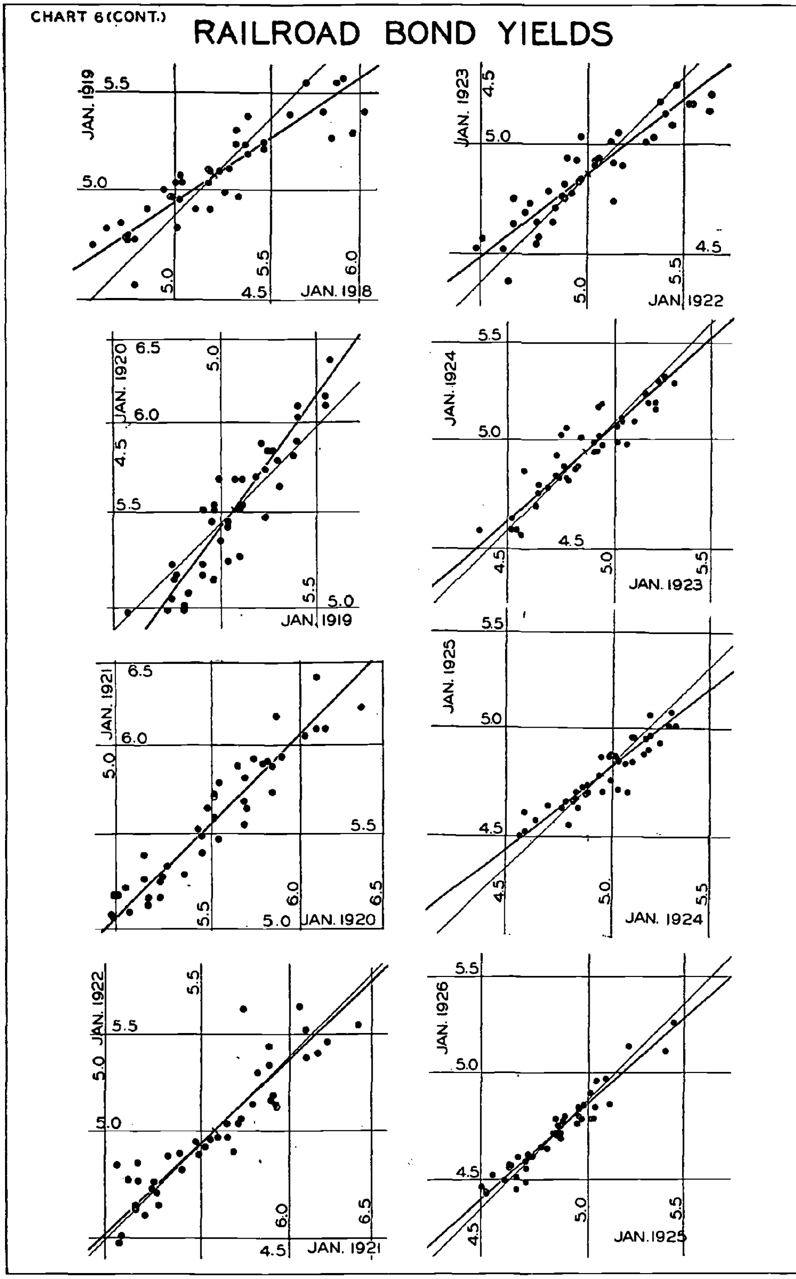
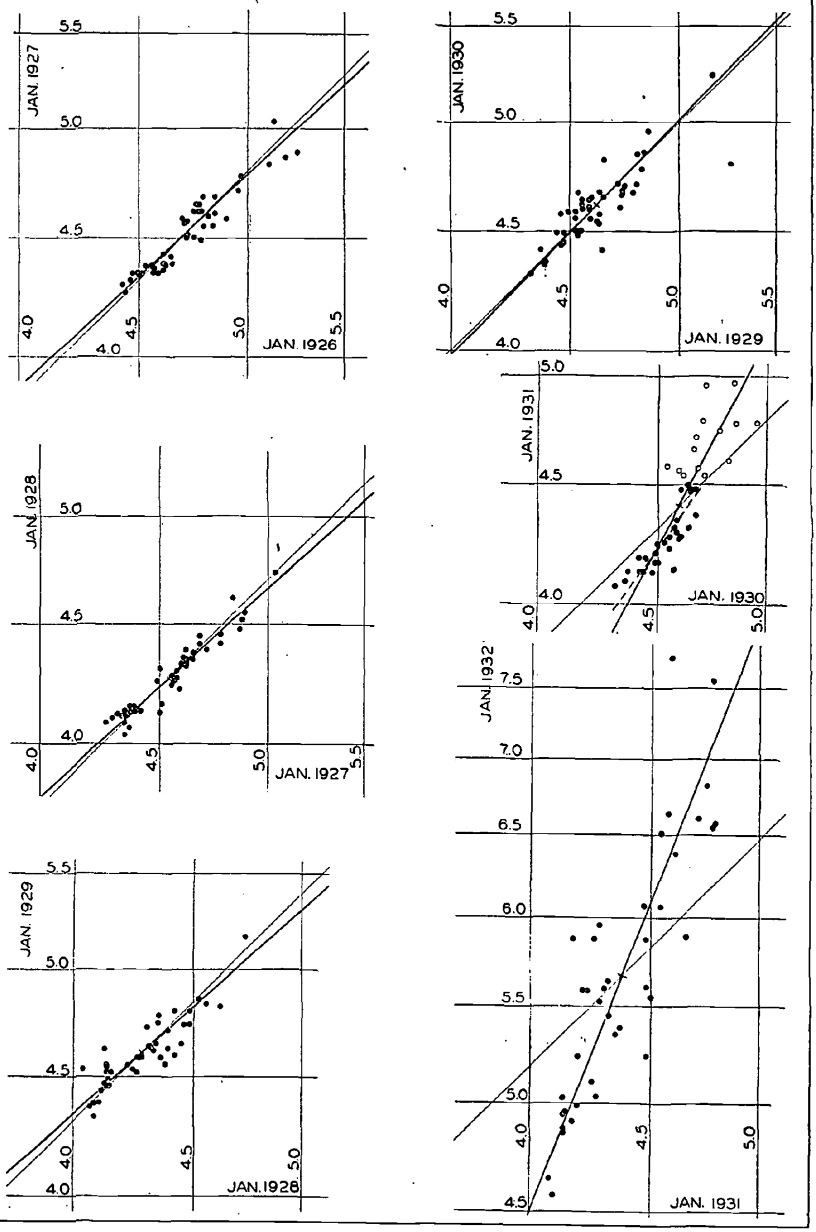


CHART 6 (CONT.)
RAILROAD BOND YIELDS
JAN. 1919
JAN. 1918
JAN. 1923
JAN. 1922
JAN. 1920
JAN. 1919
JAN. 1924
JAN. 1923
JAN. 1921
JAN. 1920
JAN. 1925
JAN. 1924
JAN. 1922
JAN. 1921
JAN. 1926
JAN. 1925

CHART 6 (CONT.)

RAILROAD BOND YIELDS



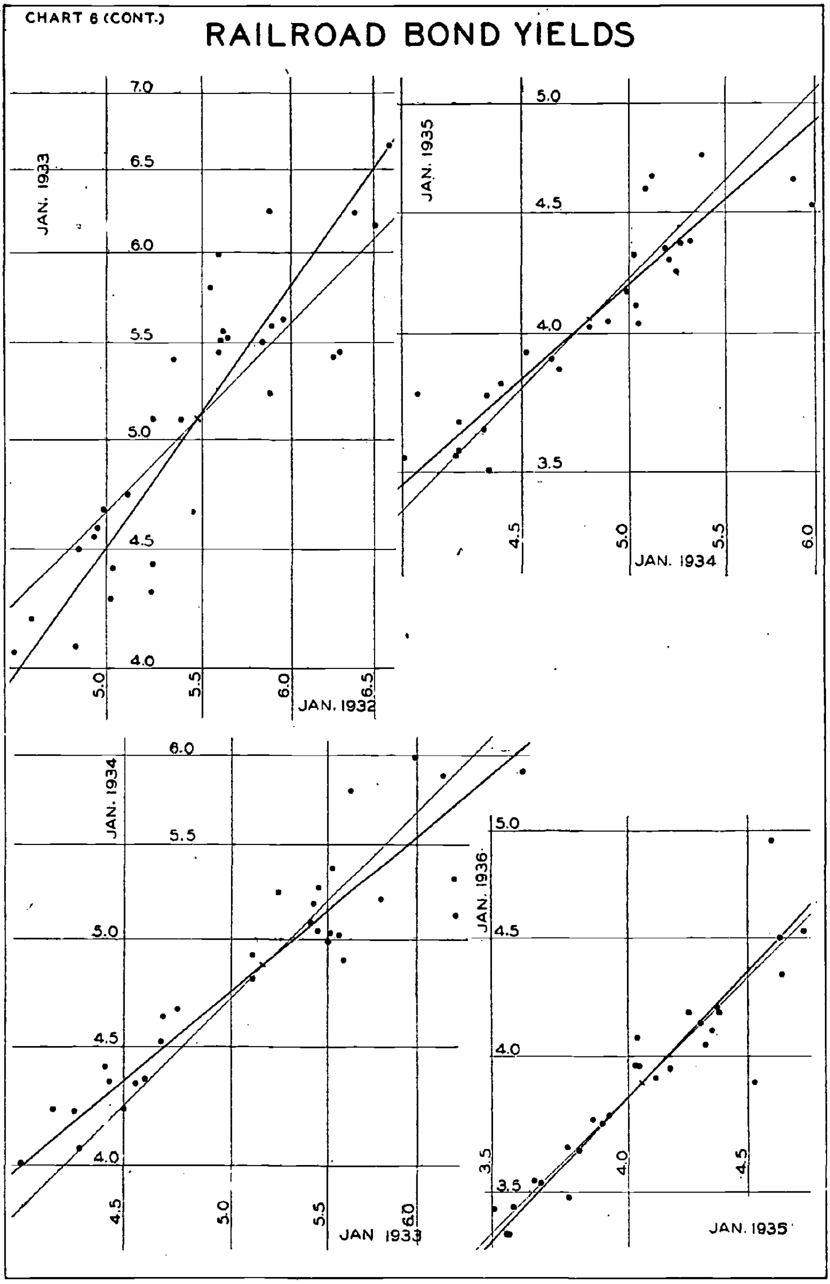


CHART 6 (CONT.)
RAILROAD BOND YIELDS
7.0
6.5
6.0
5.5
5.0
4.5
4.0
JAN. 1933
5.0
5.5
6.0
6.5
JAN. 1932
5.0
4.5
4.0
3.5
JAN. 1935
4.5
5.0
5.5
6.0
JAN. 1934
6.0
5.5
5.0
4.5
4.0
JAN. 1934
4.5
5.0
5.5
6.0
JAN 1933
5.0
4.5
4.0
3.5
JAN. 1936
3.5
4.0
4.5
JAN. 1935

tions begin is a relatively high yield for the date to which it is attached, the index number will be affected by *drift*; and the essence of drift is that it is not a characteristic of the yield of bonds of low but unchanging grade but a movement *that results from change of grade.* The next step in the calculation is, therefore, from a new base, a new grade.

Drift, therefore, remains a disturbing factor unless the original yield from which computations begin is as low as the yields of bonds that at the time are so adequately 'covered' that none but really profound changes in the financial status of the debtor corporations could appreciably lessen the complete confidence of the market that all future payments would be met.

But the original basing point yield cannot safely be chosen appreciably lower than this, in other words appreciably lower than the yields of the best bonds, without introducing an element of unreality which results in an unmeaning pseudo drift in the direction opposite to that to which inferior bonds are subject. Of course, this simply means that extrapolation cannot, without sooner or later introducing palpable absurdities, be carried far into the unreal region of yields so low that no actual bonds can be found having such yields.

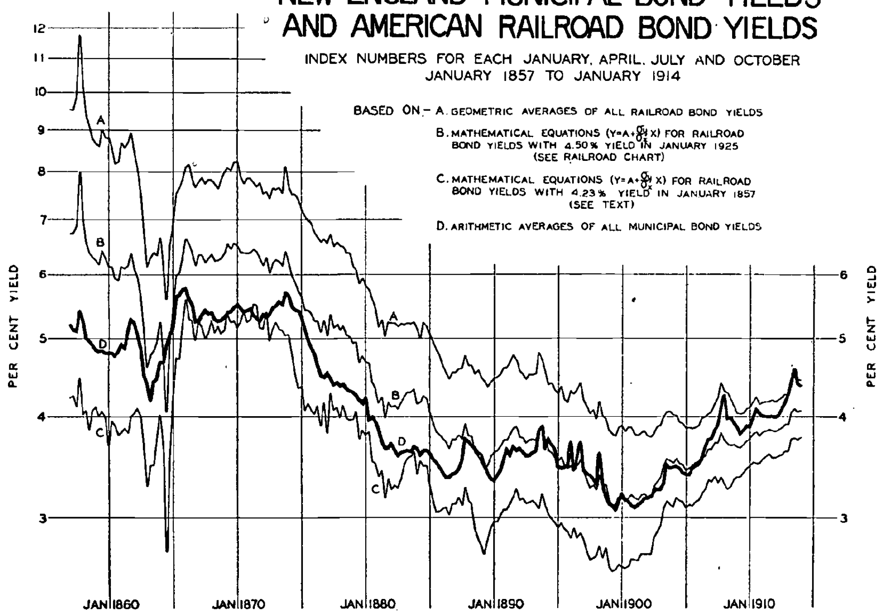
The index number that we calculated from the sigma equations to present a picture of the course of the yields of railroad bonds of the highest possible grade, a grade often slightly higher than that of any actual bond, was constructed by assuming a yield of 4.50 per cent in January 1925 and working backward to January 1857 and forward to January 1936. The lowest yield of any bond we used in January 1925 was 4.50 per cent. However, this is quite accidental. Throughout the entire period from January 1857 to January 1936, the sigma index, though it is more often lower than it is higher, tends to run fairly close to the yields of the lowest yield bonds. But it has no rigid relation to any individual yield. It weaves in and out among the lowest yields only because, in its character as an index from which 'drift' was presumed to be eliminated, it was designed to do so.

However, there is a possibility that some 'drift' remains in even this 4.50 sigma index. The 'best bonds' considered were after all only the best *railroad* bonds. Now an examination of the levels and movements of the highest grade New England municipal bonds in the period before 1914 can hardly fail to suggest that the best railroad bonds of the earlier years were not relatively so high in grade as were those of the later years.[20] In Chart 7 there is presented not only the 4.50 sigma index, which shows a yield of 4.50 per cent in January 1925 and a yield of 6.75 per cent in January 1857, but also a sigma index that shows a yield of only 4.23 per cent *in January 1857.* A comparison of these two railroad sigma indexes with the index of the yields of New England municipal bonds presented on the same chart suggests that

CHART 7

NEW ENGLAND MUNICIPAL BOND YIELDS AND AMERICAN RAILROAD BOND YIELDS

INDEX NUMBERS FOR EACH JANUARY, APRIL, JULY AND OCTOBER JANUARY 1857 TO JANUARY 1914



either the yields of the New England municipal bonds had an upward drift during the period covered by the chart (January 1857 to January 1914) or the 4.50 sigma index had a downward drift. Since about 1875 the *long term* trend of the New England municipal bond index resembles the trend of the sigma index giving 4.23 per cent in Janu-

[20] At the top prices of 1857 the long term 5 per cent (gold) bonds of Boston and Massachusetts both sold at 99, to yield only a small fraction more than 5 per cent; during 1880 Boston 4's of 1899 sold as high as 105; at the top prices of 1897 both Massachusetts 3's of 1923 and Maine 3's of 1921 sold at par. Now the lowest monthly yield of any railroad bond in our 1857 list was 6.17 per cent; in 1880 the lowest monthly yield was 4.31; in 1897 the lowest yield was 3.08. The differences between these railroad yields and the Boston, Massachusetts and Maine yields are 1.1 per cent, 0.7 per cent and 0.1 per cent.

ary 1857 more than it does the trend of the sigma index giving 6.75 per cent in 1857.

The great differences in the movements of the New England municipal bond index and the railroad sigma indexes during the Civil War period are peculiarly difficult to explain. And possibly no great effort to do so is warranted. An examination of Chart 6 will immediately show how very appreciable the probable errors of the slopes of the sigma lines in parts of this period must necessarily be. And the assumption of linearity seems sometimes very difficult to defend.[21] Chart 15 would suggest that, if the relation depicted by that chart is real, the measurement of drift from the wide and possibly non-linear scatters of the Civil War period may be very unreliable. For example, the downward drift must surely be underestimated from January 1862 to the middle of 1864 and overestimated from the middle of 1864 to the end of 1865.

But one of the difficulties of this argument is that great irregularities in the relationship pictured in Chart 15 occur in periods, such as that from the middle of 1877 to the end of 1880, in which the scatters are closely packed around the fitted straight lines and during which there is no great difference in trend between the New England bond index and the railroad sigma indexes. An adequate explanation of the difference in the movements of the municipal and railroad indexes during the Civil War may possibly lie in the different effects of the paper money inflation of the period on the prices and yields of the two types of bonds. However, we failed to develop any very appealing hypothesis offering an explanation on this basis.[22]

[21] Compare the January 1864 and January 1865 scatter in Chart 6.

[22] The index of the yields of New England municipal bonds is a quarterly index. Monthly quotations are not regularly available.

The sources of the original quotations are materials gathered by Joseph G. Martin, a Boston stock broker, and his successors in his firm. For the period from January 1857 to January 1862, Mr. Martin's early book entitled "Twenty-one Years in the Boston Stock Market" was employed. The quotations is this book for 1857, 1858 and 1859 are monthly for the first day of the month. However, in 1860 and 1861 only quarterly quotations appear, namely, for January 2, April 2, July 2 and October 2 of 1860, and for January 1 (sic), April 1, July 1 and October 1 of 1861. The quotations used from January 1862 to January 1914 are from annual pamphlets issued by Mr. Martin at the end of each year. These pamphlets were entitled *"Stock Market Fluctuations"* or *"Stock Quotations from January —— to January ——"* or other closely similar titles that varied somewhat from year to year. Copies of these pamphlets are to be found in the Boston Public Library.

The credit standing of the leading New England municipalities underwent, during the period 1857–1914, no such radical changes as did the credit standing of many American railroads. Indeed, the best of the New England municipal bonds seem to have deserved about the same rating in 1857 as they did in 1914. That, throughout this entire period—aside from the erratic Civil War interruption—the railroad 4.50 sigma index drifts irregularly but continually downward relatively to the New England municipal index would seem therefore to present evidence of some significance. Our holding to the 4.50 index, in spite of this evidence, requires some explanation if not defense. Such an explanation or defense hangs naturally on two considerations: first, the difficulty of deciding how much drift, if any, remains in the 4.50 sigma index and, second, the virtual certainty that any attempt to obtain, from operations restricted to the railroad bond data, an index containing much less drift than the 4.50 sigma index would lead to highly questionable results.

When attempting to decide how much drift, if any, remains in the 4.50 index, we must remember that a difference in the yield of two bonds may result from a difference in the markets for the bonds as well as from a difference in the degrees of public assurance that the promised future payments will be met. The placing of a bond on a list of investments legal for trust funds and savings banks inevitably affects its yield. And mere custom has often almost the force of law. Only gradually did railroad bonds attain the legal status and the popularity with the most conservative investors that they possessed by the early

(Footnote [22] concluded)

The quotations appear to have been at times estimated from the author's knowledge of the municipal bond market. There are occasionally bankers' buying rates when no transactions were actually recorded. The quotations are 'over the counter' rather than open public market quotations such as we used for railroad bonds. There are strong reasons for believing, however, that Martin's quotations, though they were not recorded with the same official exactness as were those of the railroad bonds, and though their accuracy proved, on experiment, to be insufficient to stand the strain of the sigma procedure, reflect with substantial correctness the movements of New England municipal bond prices and yields in the Boston market.

We made no attempt to extend the New England bond index forward beyond January 1914. In so far as we have made comparisons of railroad bond yields and municipal bond yields after January 1914, we have contented ourselves with the Standard Statistics monthly index of the yields of 15 municipal bonds. This index goes back to January 1900 (see Chart 10). Shortly after January 1914 the Federal income tax, with its municipal bond exemptions began to introduce into the picture an extraneous element whose effects are difficult to measure.

years of this century. But, from 1857 to 1930, municipal bonds of the highest grade were 'prime' investments for the ultra conservative. As high grade railroad bonds came to be more and more introduced into the portfolios of such investors, they steadily weakened the monopoly enjoyed by the municipal bonds. With the increase in the composite supply, the yields of the municipal bonds inevitably tended to be greater than they otherwise would have been. The effect was the same as though the volume of the municipal bonds had itself been increased in the same proportion as the total.

Shall we then say that the yield of the railroad bonds was drifting down toward that of the municipal bonds or that the yield of the municipal bonds was drifting up toward that of the railroad bonds, or shall we say that both these things were happening, that the yields were drifting together? From January 1921 to the end of 1929 the interest bearing debt of the United States government declined steadily month after month and year after year. Throughout this period the yields of Federal bonds showed a pronounced long term *downward* trend relatively to the 4.50 sigma index. Shall we, therefore, say that, as these government bonds were throughout the period acknowledged to be of the very highest grade, the 4.50 sigma index had, during this period, an *upward* drift? And that, during the year 1931 for example, in which the interest bearing debt increased by leaps and bounds and in which the yields of Treasury bonds advanced much more rapidly than did the 4.50 sigma index, that index had a *downward* drift?

The 4.50 sigma index gives a picture of the course of the yields of *railroad* bonds of an ultra-superlative grade. It is hard to say just what meaning would have to be given to it if it were adjusted to the movements of high grade bonds in one or more other markets. It certainly would not give us a picture of that economic noumenon 'pure interest'.

A study of the corporations themselves gives us considerable reason to believe that the very best railroad bonds in the early period were inferior to the best bonds in the later period. But, from the yields of the best bonds in the early period or even from the yields of all the bonds in all the periods, it is difficult if not impossible to calculate plausibly what would have been, in the early period, the yield of a railroad bond of apparently as high grade as the very best railroad bonds in the later period. We have already referred to the danger involved in extrapolating the sigma lines far into the region of hypothetical bonds of a higher grade than any actually existing bonds. This danger exists not merely because of the high probable error of the slopes of a few of the sigma lines for some of the January to January scatters and the high probable error which always appears as the slopes are cumulated, but also because of the evidences that the scatters are sometimes and perhaps always non-linear even on the log scale we have used.[23]

The non-linearity of the relation between the yields at two dates, that sometimes appears when extremely high-yield securities are included in the scatter diagram, is illustrated by Chart 8. On that chart are plotted the yields of all bonds and dividend-paying preferred stocks listed on the New York Stock Exchange for which quotations could be obtained for both January 1903 and January 1904. An examination of the chart will show that, though a straight line gives a fairly plausible fit to the yields of the railroad bonds we actually used, it gives an extremely poor fit to the total scatter.[24] An hyperbola is suggested as an empirical curve that might describe more or less adequately the complete scatter. The railroad sigma line lies close to the lower arm of the hyperbola. The non-linearity of this scatter explains the lack of similarity between the cumulated-product-of the-slopes-of-the-sigma-lines and the course of railroad stock prices in this period (see Chart 15). It is, of course, difficult to say to what extent the rather radical short term differences between the (1925) 4.50 sigma line and the (1857) 4.23 sigma line, such as the complete elimination by the 4.23 line of the 1873 peak (see Chart 7), are rational, are the result of high probable errors in the cumulated linear slopes, or are the result of non-linearity of the scatters.[25]

[23] Chart 15 gives some support to the idea that the non-linearity is not of a constant type and that, over long periods, its disturbing effects tend to cancel out every now and then leaving approximately the same result as that obtained from the linear hypothesis. Whether this be true or not, non-linearity has certainly been very disturbing during some short periods, such as January 1903 to January 1905, a period about to be discussed in the text.

[24] In only one year out of the seventy-nine, 1857–1936, did this non-linearity affect the particular group of bonds we used in our index. In 1929–30 only twenty-eight bonds were used to calculate the sigma line. These are the bonds that appear as black dots on Chart 6. The white dots on this chart are the bonds that were omitted in that one year because they affected the linearity of the scatter. Two sigma lines appear on the chart. The solid line is fitted to the entire scatter. The broken line is fitted to the twenty-eight bonds and is the one we actually used in our calculations.

[25] Even the 4.50 sigma index sometimes shows a small amount of erratic irregularity

Chart 9 shows the yields of nearly all the railroad and public utility bonds listed on the New York Stock Exchange or the New York Curb Exchange for which quotations were available for the weeks ending January 7, 1930 and January 6, 1931. The interesting feature of this chart is that, though the railroad scatter appears to be distinctly non-linear, the public utility scatter appears to be comparatively linear, at least outside the range of almost absurdly high yields. There was, in this period, no such collapse of public utility credit as there was of railroad credit.

Because scatter diagrams are so enlightening we have presented a large number—84 in all. The primary reason for having each of the 79 diagrams of Chart 6 cover a period from one January to the next was that we substituted bonds only in January. Each diagram contains all the bonds used in a thirteen-month period. If we had used more critical dates, we would have had to use fewer bonds. Monthly scatter diagrams are seldom very impressive. The drift in a single month is usually so small as hardly to be manifest on a chart. The slopes of the sigma lines are nearly always close to 45°. Of course in a disturbed market even monthly charts are very instructive. Changes in grades are so violent that some of the monthly charts may look like yearly charts of a more normal period. Drift is unmistakable. Interesting as it would be to cover the recent past in much greater detail than the rest of the period, it was decided not to do so in this study. To have presented monthly scatter diagrams for even the last few years would have necessitated the publication of too many charts.

(Footnote [25] concluded)
in its minor month-to-month movements. Line D of Chart 2 presents an index constructed by using a mathematical graduation that follows extremely closely all but the most minor movements of the 4.50 index (see Chart 14 to realize how closely) and on this graduation superimposing, as a substitute for the deviations of the 4.50 index, the deviations of the chain index number constructed from the arithmetic averages of the actual yields from a similar graduation of that index. The resulting composite index follows the 4.50 index in all respects except that it tends to eliminate the irregularities that sometimes appear in the most minor fluctuations of the 4.50 index. The composite index runs from January 1857 to January 1936 monthly. For the period from January 1857 to January 1879 the 4.50 sigma index was calculated only quarterly. Both the extreme similarity of the two indexes and their minor differences may be seen by comparing lines B and D of Chart 2.

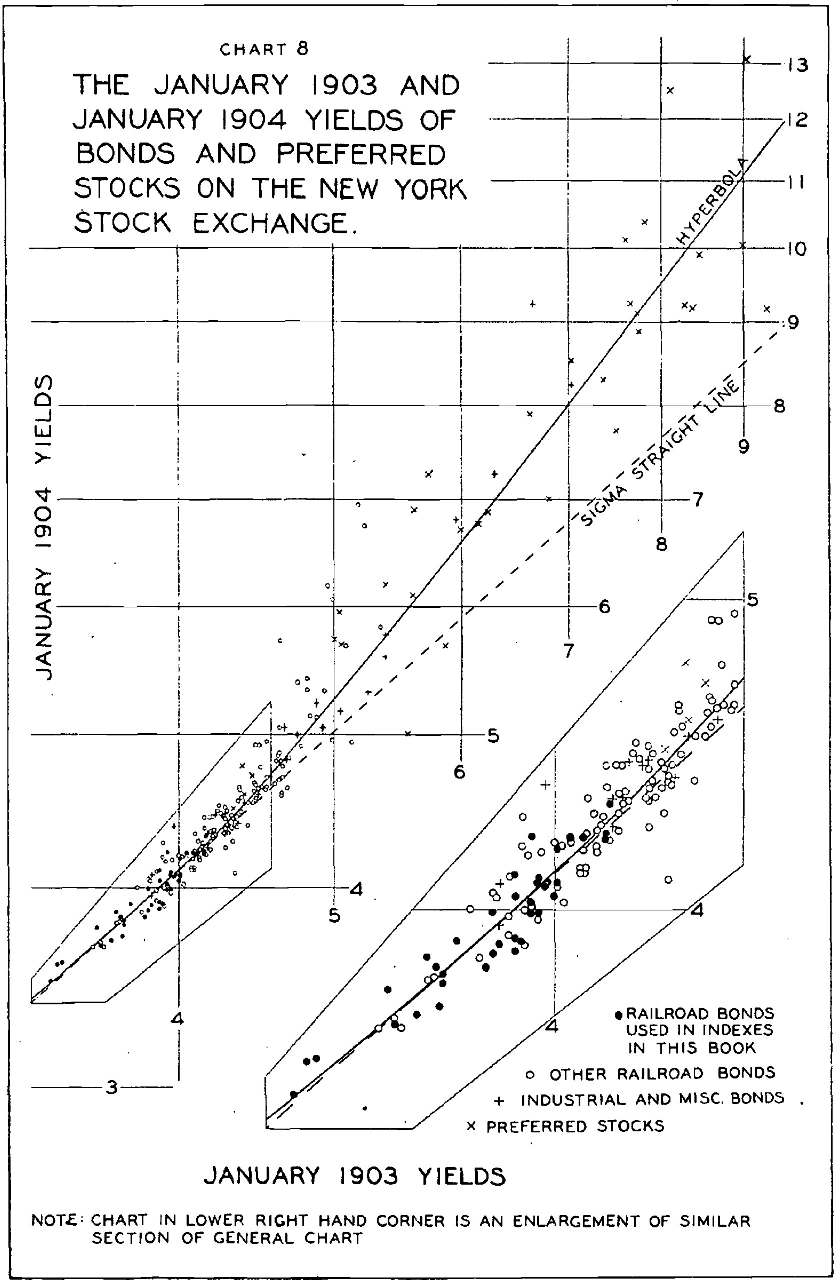


CHART 8
THE JANUARY 1903 AND JANUARY 1904 YIELDS OF BONDS AND PREFERRED STOCKS ON THE NEW YORK STOCK EXCHANGE.
HYPERBOLA
SIGMA STRAIGHT LINE
JANUARY 1904 YIELDS
● RAILROAD BONDS USED IN INDEXES IN THIS BOOK
○ OTHER RAILROAD BONDS
+ INDUSTRIAL AND MISC. BONDS
× PREFERRED STOCKS
JANUARY 1903 YIELDS
NOTE: CHART IN LOWER RIGHT HAND CORNER IS AN ENLARGEMENT OF SIMILAR SECTION OF GENERAL CHART

CHART 9

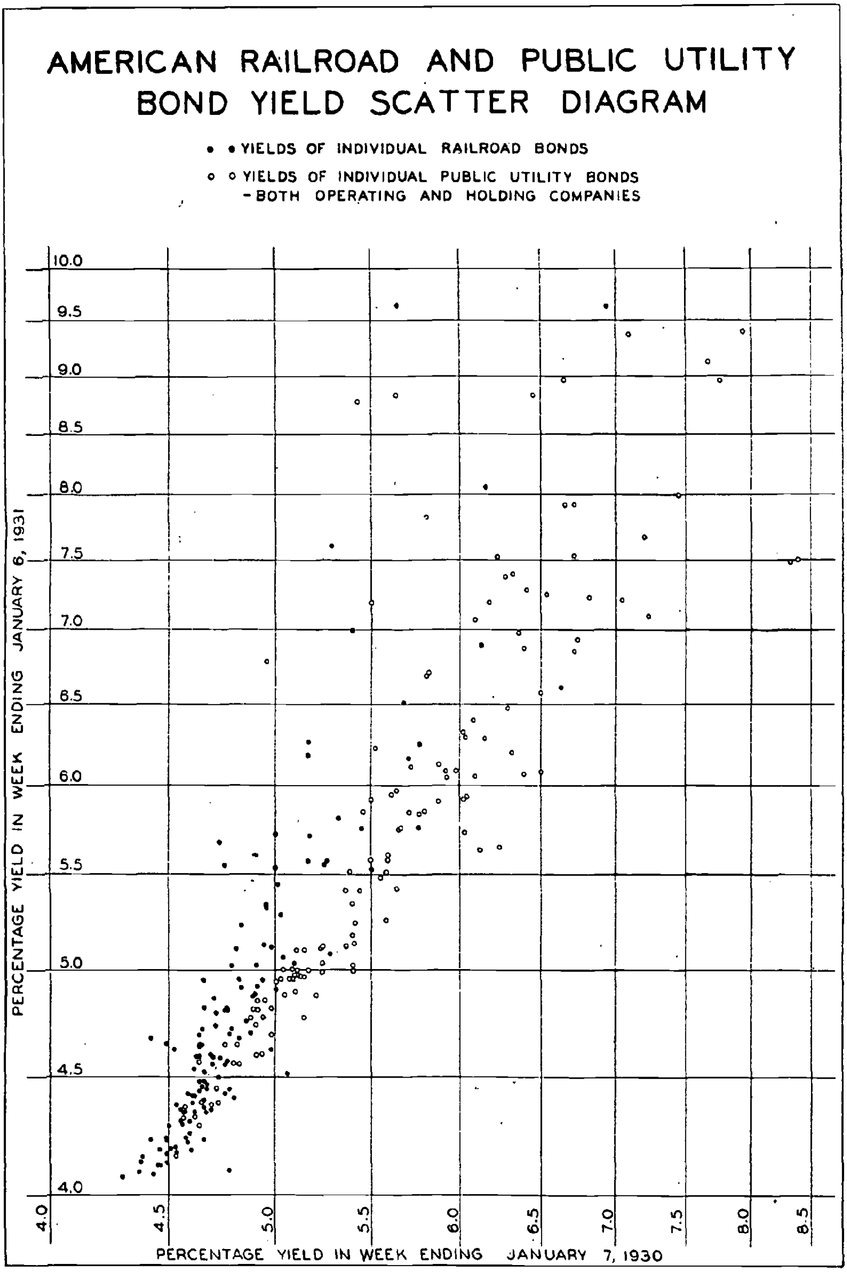

AMERICAN RAILROAD AND PUBLIC UTILITY
BOND YIELD SCATTER DIAGRAM
• YIELDS OF INDIVIDUAL RAILROAD BONDS
○ YIELDS OF INDIVIDUAL PUBLIC UTILITY BONDS
- BOTH OPERATING AND HOLDING COMPANIES
10.0
9.5
9.0
8.5
8.0
7.5
7.0
6.5
6.0
5.5
5.0
4.5
4.0
PERCENTAGE YIELD IN WEEK ENDING JANUARY 6, 1931
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
PERCENTAGE YIELD IN WEEK ENDING JANUARY 7, 1930

CHART 10

AMERICAN MUNICIPAL AND RAILROAD BOND YIELDS

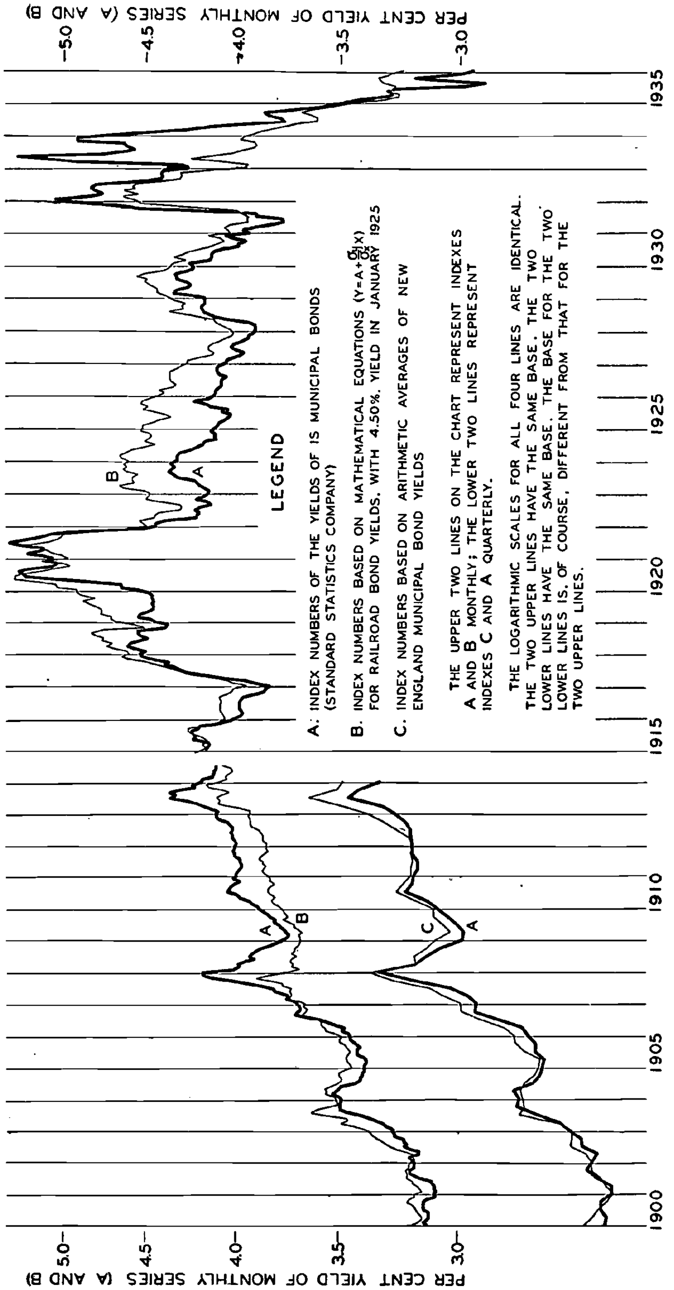


CHART II

DIFFERENT 'DRIFTS' SHOWN BY INDEX NUMBERS OF PUBLIC UTILITY BOND YIELDS, RAILROAD BOND YIELDS, AND MUNICIPAL BOND YIELDS