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4 Dynamic Duopoly with Output Adjustment Costs in International Markets: Taking the Conjecture out of Conjectural Variations

Robert Driskill and Stephen McCafferty

Microeconomics in general and trade economists in particular have made wide use of the conjectural variations approach to modeling oligopolistic behavior. Most users of this approach acknowledge its well-known shortcomings but defend its use as a "poor man's" dynamics, capable of capturing dynamic considerations in a static framework. As one example, Eaton and Grossman (1986) organize discussion about optimal trade policy in international oligopolistic markets around the question of whether conjectural variations are Nash-Cournot, Bertrand, or consistent in the sense of Bresnahan (1981). Their primary finding is that the optimal policy might be a tax, a subsidy, or free trade, depending on whether the exogenous conjectural variation is Nash-Cournot, Bertrand, or consistent.

In this paper, we construct a dynamic differential game of duopolistic trade in an international market. We show that the steady state of the closed-loop, subgame perfect equilibrium of our game can be replicated by a conjectural variations equilibrium of an analogous static game. The difference, though, is that the term in our steady-state equilibrium that corresponds to the conjectural variations term in the static game is itself a function of structural parameters in the model. The endogeneity of the conjectural variation allows us to pin down the optimal policy in terms of tax, subsidy, or laissez faire, depending on the structural aspects of our model. The optimal policy no longer depends on an assumed, exogenous value of a conjectural variation. In our particular model, we find that the optimal policy is an export subsidy that credibly shifts profits to the domestic firm.

Our work also has implications for the empirical study on optimal trade and industrial policies of Dixit (1988). Dixit employs a conjectural variations

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model to analyze the U.S. automobile industry. In his analysis, Dixit treats these conjectural variations as "parameters that measure the degree of competition or collusion in market conduct" (p. 142). He is most interested in the equilibrium values of the conjectural variation terms as implied by the historical data. However, he does express some concern that such equilibrium conjectural variations might be functions of tariffs and other policy variables. The term in our dynamic model corresponding to the conjectural variations term in the analogous static model is a function of such taste, technology, and policy parameters. Hence, Dixit's concerns about the validity of using a constant conjectural variation term in the face of policy changes seems well founded.

While our analysis is amenable to easy comparison with works that adopt the conjectural variations framework, we do not claim to rebut those critics who find conjectural variations a flawed behavioral concept. Rather, we view current users of conjectural variations as believing that the concept captures in a static framework the long-run behavior of some unspecified dynamic game.

For our analysis, we develop a duopoly model in which firms incur costs associated with how fast they change their level of output. By positing these adjustment costs, we create what James Friedman (1974) has called a "time-dependent" or "structurally linked" dynamic game and cast the duopoly problem as a differential game. In this game, firms take levels of output as state variables and choose how fast they adjust output.

We think our approach is a natural extension of traditional duopoly theory and especially of the conjectural variations approach. Even though not explicit, dynamics lurks just offstage in these static theories. Both Cournot's discussion of move and countermove and the naming of static first-order conditions as "reaction curves" reflect a concern with dynamics not captured in the formal models. By explicitly introducing a time-dependent structure into a model, we can naturally address these dynamic considerations. An interesting characteristic of such a game is that, in the steady state, the closed-loop, subgame perfect equilibrium differs from the equilibrium of a static, one-shot Nash game. This makes the steady state of our game amenable to comparison with conjectural variations equilibria. We also believe that our approach provides a justification for the reasonableness of the conjectural variations approach.

Our main result is that output in the steady state of our game is greater than it would be in an analogous static Nash-Cournot equilibrium. This holds true even in the limiting case where the adjustment cost term that gives rise to the intrinsic dynamics of the model shrinks to zero. Our steady-state equilibrium can also be replicated by a static game whose players have negative conjectural variations of a particular magnitude. While we do not obtain analytic results concerning optimal taxes or subsidies, we do compute the optimal policy for a number of numerical examples. We find in all cases that the optimal policy is a subsidy on exports. Intuition that suggests this finding is robust is gleaned from comparing our steady state with the consistent conjectures equilibrium of the analogous static game. We find that output in our steady state is always less than that in the consistent conjectures equilibrium. Results from Eaton and Grossman (1986) tells us that, in a static conjectural variations framework, when output is below the consistent conjectures level, the optimal policy is a subsidy.

We should note that in a companion paper (Driskill and McCafferty 1988) we model a dynamic game where the intrinsic dynamics arise from dynamic demand. In that model, steady-state output is above that of the associated consistent conjectures equilibrium, and the optimal policy is a tax on exports. The general lesson seems to be that, while the outcome of a dynamic game can be replicated by a conjectural variations equilibrium, this outcome is dependent on the specific features of the dynamic game, including values of policy parameters.

4.1 The Model

Following Eaton and Grossman (1986), among others, we consider the case of two duopolists, each from a different country, competing in a third country. The government of each duopolist's respective country is assumed to commit, prior to the start of the game, to an ad valorem tax or subsidy on the exports of the domestic firm. Throughout the game, each firm and government takes as given the tax or subsidy imposed by each government. Each firm's objective is the maximization of the present discounted value of profits.

Both firms face a common linear demand curve, given by

$$p = a - u_1 - u_2, \tag{1}$$

where a is a positive constant and u_i is the output of the ith firm.

We assume that costs depend both on the level of output and on the time derivative of output, reflecting costs associated with changing output quickly rather than slowly. Furthermore, we assume that the cost of changing output infinitely quickly is infinite: the force of this assumption is to make output levels state variables that do not jump discontinuously but rather evolve smoothly through time. What firms control, then, are rates of change of output. We assume that both firms have identical cost functions given by

$$C_i = cu_i + (A/2)(x_i)^2, \tag{2}$$

where c and A are positive constants and $x_i \equiv \dot{u}_i$, where $(\dot{\ })$ denotes a time derivative. We could add a cost term quadratic in the level of output, but we believe that it would add nothing to the analysis except increased algebraic complexity.

The key assumption about strategic behavior is that each firm's strategy is restricted to be a function only of the current state, that is, a function only of the output levels of both firms. This assumption restricts our equilibria to be "closed loop." We briefly point out the properties of the model when strategies are path strategies, that is, not conditional on the state of the system. Equilibria predicated on path strategies have the undesirable property of not being perfect. The reasonableness and usefulness of the state-space restriction is discussed by Fudenberg and Tirole (1983). Basically, it rules out other perfect equilibria in which strategies depend on "irrelevant" history; in particular, it rules out trigger-strategy equilibria.

Each firm is thus assumed to solve the following problem:

$$\max_{x_i} \int_0^\infty [(a - u_1 - u_2)u_i(1 - t_i) - cu_i - (A/2)x_i^2]e^{-\delta t}dt$$

subject to

$$\dot{u}_i = x_i, \quad \dot{u}_j = x_j(u_i, u_j), \quad i, j = 1, 2, i \neq j,$$

where t_i is the tax or subsidy rate and δ is the common discount rate.

The above maximization problems constitute a differential game. We now state and prove the following theorem.

THEOREM 1: Let

$$x_i^* = K_i + k_{ii}u_i + k_{ij}u_j, \quad i, j = 1, 2, i \neq j, \tag{3}$$

where k_{ii} and k_{ij} solve the following equations:

$$\begin{aligned} k_{ii} = \{&[2(1 - t_i) - Ak_{ij}k_{ji})(k_{jj} - \delta) - Ak_{ii}^2(k_{ii} - \delta) \\ &+ 2k_{ii}(1 - t_i) - Ak_{ii}k_{ij}k_{ji} + A(k_{ii} - \delta)k_{ji}k_{ij} \\ &- Ak_{ij}k_{ji}k_{jj}\}/A(k_{ii} - \delta)(k_{jj} - \delta), \end{aligned} \tag{4}$$

$$\begin{aligned} k_{ij} = \{&[(1 - t_i) - Ak_{ij}k_{jj})(k_{jj} - \delta) - Ak_{ii}k_{jj}(k_{ii} - \delta) \\ &+ 2k_{ij}(1 - t_i) - Ak_{ij}^2k_{ji} - Ak_{ij}k_{jj}(k_{ii} - \delta) \\ &+ k_{jj}(1 - t_i) - Ak_{ij}k_{jj}^2\}/A(k_{ii} - \delta)(k_{jj} - \delta), \end{aligned} \tag{5}$$

$$k_{ii} + k_{jj} < 0, \tag{6}$$

$$\begin{aligned} &k_{ii}k_{jj} - k_{ij}k_{ji} > 0, \\ &i, j, = 1, 2, i \neq j. \end{aligned} \tag{7}$$

If such k_{ij} exist, then the pair x_1^*, x_2^* constitute a stable, closed-loop Nash equilibrium for the dynamic game under consideration.

Proof: We need to show that the stipulated strategies satisfy the Pontryagin necessary conditions for the two players. The first-order conditions for player i are

$$H_{x_i}^i = -Ax_i + \lambda_{ii} = 0, \tag{8}$$

$$\begin{aligned} -H_{u_i}^i + \lambda_{ii}\delta = \dot{\lambda}_{ii} = &-[a(1 - t_i) - c] \\ &+ 2(1 - t_i)u_i + u_j(1 - t_i) - \lambda_{ij}k_{ji} + \lambda_{ii}\delta, \end{aligned} \tag{9}$$

$$-H_{u_j}^i + \lambda_{ij}\delta = \dot{\lambda}_{ij} = u_i(1 - t_i) - \lambda_{ij}k_{jj} + \lambda_{ij}\delta, \tag{10}$$

where H^i is the discounted Hamiltonian, $H^i_{(\cdot)}$ is the partial derivative of H^i with respect to $(\cdot)$, and λ_{ii}, λ_{ij} are the costate variables. Substituting (8) into (9), time-differentiating (8) and substituting into (9), time-differentiating that relation and combining it with (10), and rearranging, we get the following relation between x_i and u_1, u_2:

$$
\begin{aligned}
(11)\qquad u_i = K_i &+ (\{[2(1 - t_i) - Ak_{ij}k_{ji}](k_{jj} - \delta) - Ak_{ii}^2(k_{ii} - \delta) \\
&+ 2k_{ii}(1 - t_i) - Ak_{ii}k_{ij}k_{ji} + A(k_{ii} - \delta)k_{ji}k_{ij} \\
&- Ak_{ij}k_{ji}k_{jj}\}/A(k_{ii} - \delta)(k_{jj} - \delta))u_i + (\{[(1 - t_i) \\
&- Ak_{ij}k_{jj}](k_{jj} - \delta) - Ak_{ii}k_{jj}(k_{ii} - \delta) + 2k_{ij}(1 - t_i) - Ak_{ij}^2k_{ji} \\
&- Ak_{ij}k_{jj}(k_{ii} - \delta) + k_{jj}(1 - t_i) - Ak_{ij}k_{jj}^2\}/A(k_{ii} - \delta)(k_{jj} - \delta))u_j,
\end{aligned}
$$

where K_i is a constant. Hence, if k_{ii}, k_{ij} equal the coefficients on u_i, u_j, respectively, then the stipulated pair of strategies satisfies the Pontryagin first-order conditions. If they also satisfy the auxiliary conditions that $k_{ii} + k_{jj} < 0$ and $k_{ii}k_{jj} - k_{ij}k_{ji} > 0$ (i.e., if they satisfy the Routh-Hurwicz conditions), then the strategies are also stable; that is, for any arbitrary initial values $u_i(0)$, $u_j(0)$, u_i and u_j converge to finite steady-state values. Q.E.D.

In general, we do not know under what parameter values such strategies exist or, if they do, whether they are unique. In Driskill and McCafferty (in press), we prove existence and uniqueness of a symmetric linear set of strategies for the special case $t_i = t_j = 0$. In this paper, we compute equilibria numerically for a wide variety of parameter values.

4.2 The Steady State

We wish to emphasize two aspects of the steady state of our dynamic game: first, that our equilibrium can be replicated by a conjectural variations equilibrium and, second, that output in our equilibrium steady state is higher than it would be in a static Nash-Cournot game played under the same demand and cost conditions except with no adjustment costs.

Setting all time derivatives to zero, we can derive the following relation from each player's maximization problem:

$$(12)\qquad u_i = \{[a - c(1 - t_i)]/[2 + r_j]\} - u_j/(2 + r_j),$$

where

$$(13)\qquad r_j = -k_{ji}/(k_{jj} - \delta).$$

We will refer to (12) as a *steady-state reaction curve*.

Now consider the static conjectural variations analogue to our dynamic problem, that is, the same demand and cost conditions except for the lack of adjustment costs. The reaction curve for each player is readily derived as

$$(14)\qquad u_i = \{[a - c(1 - t_i)]/[2 + \theta_j]\} - u_j/(2 + \theta_j),$$

where θ_j is firm i's conjecture about firm j's response to a change in u_i.

Note that the steady state of our game and the outcome of the conjectural variations game are identical if $\theta_j = r_j$. This means that there is a conjectural variation that replicates the steady state of our game. In the conjectural variations approach, though, the value of the conjectural variation is taken as exogenous, except in the consistent conjectures approach. The r_j in our dynamic game is, in contrast, a function of demand, cost, and policy parameters.

To compare output between the static Nash-Cournot analogue of our game and the steady state of our game, we need to know the sign of r_j. In Driskill and McCafferty (in press), we prove that, for the special case of $t_i = t_j = 0$, there exist symmetric negative k_{ij}'s that uniquely solve the Pontryagin first-order conditions for each firm's maximization problem and that give rise to r_j's strictly between zero and minus one. For this case, it is straightforward that output in the steady state is greater than output in the static Nash-Cournot game, which corresponds to the static case in which $\theta_1 = \theta_2 = 0$. For the asymmetric case we study in this paper, we are forced to compute solutions to equations (4) and (5) numerically. For a wide variety of parameter values, we always find $r_j \epsilon (-1, 0)$. For $r_j \epsilon (-1, 0)$, a straightforward inequality comparison exercise on equations (12) shows that output for the industry is unambiguously higher than in the static Nash-Cournot case.

What pushes output beyond the static Nash-Cournot level is a purely strategic force associated with the closed-loop aspect of our game. In the closed-loop game, each firm takes account of the effect that the value of the state variable has on its rival's optimal response. Consequently, each firm knows that, if it expands output, its rival's response is to reduce its rate of change of output, leading to a lower level of its output through time. This occurs since the k_{ij}'s are negative. Thus, each firm has an incentive to increase output even more since this shifts out its future residual demand curve.

4.3 Welfare

Following Eaton and Grossman (1986), we measure welfare contributions to each country by national product generated in the steady state by the home firm:

$$w_i = pu_i - cu_i \; . \tag{15}$$

We look only at the steady state in considering welfare effects. Our purpose is to compare conjectural variations results with those derived from the steady state of a completely specified dynamic game; our interpretation of the conjectural variations justification as "poor man's dynamics" is that conjectural variations equilibria can be thought of as just such a steady state. A truly dynamic welfare analysis, while perhaps desirable, is also beyond our computational abilities at this time.

While we have no analytic results to report, we did solve our model numerically for a wide variety of parameter values and compute associated welfare levels. A representative display of our findings is presented in table 4.1, where welfare levels for each country are shown as functions of both countries' tax rates. The table shows that, for a zero foreign-country tax rate, the optimal home-country response is a subsidy of about 34 percent. For all the different parameter values we tried, the optimal response was qualitatively the same: a subsidy.

In the spirit of Eaton and Grossman, we can also use table 4.1 to analyze optimal foreign policy response. That is, we can think of both governments setting tax rates before the start of the dynamic game between the two competing firms so as to maximize their own steady-state welfare. More precise numerical calculations than those presented in the table demonstrate that the resulting Nash equilibrium would be a 25 percent subsidy granted by

Table 4.1 **Welfare Levels for Various Tax/Subsidy Rates**

	T1							
T2	−.40	−.34	−.28	−.22	−.16	−.10	.0	.10
−.60:								
W1	.0446	.0462	.0475	.0482	.0483	.0474	.0430	.0325
W2	.0546	.0612	.0686	.0770	.0867	.0980	.1211	.1519
−.50:								
W1	.0519	.0534	.0545	.0551	.0550	.0538	.0490	.0380
W2	.0579	.0643	.0715	.0798	.0893	.1004	.1231	.1533
−.40:								
W1	.0607	.0620	.0630	.0633	.0630	.0617	.0563	.0447
W2	.0607	.0669	.0740	.0821	.0914	.1021	.1244	.1541
−.30:								
W1	.0715	.0727	.0734	.0735	.0729	.0713	.0654	.0531
W2	.0627	.0687	.0756	.0834	.0924	.1029	.1246	.1537
−.20:								
W1	.0850	.0859	.0864	.0862	.0853	.0834	.0768	.0638
W1	.0633	.0691	.0757	.0833	.0920	.1022	.1232	.1514
−.10:								
W1	.1021	.1028	.1029	.1025	.1012	.0989	.0915	.0775
W1	.0617	.0672	.0735	.0807	.0891	.0989	.1192	.1465
.0:								
W1	.1244	.1247	.1245	.1237	.1220	.1192	.1109	.0958
W2	.0563	.0615	.0674	.0743	.0822	.0915	.1109	.1371
.10:								
W1	.1541	.1540	.1534	.1521	.1499	.1465	.1375	.1206
W2	.0447	.0495	.0550	.0614	.0689	.0775	.0958	.1206

Note: W1 = country 1 welfare, W2 = country 2 welfare, T1 = tax rate for country 1, T2 = tax rate for country 2, A = 1.0, a = 2.0, c = 1.0, and δ = .05.

both governments. Much as in the Eaton and Grossman analysis, allowing for foreign response leaves the basic results unchanged.

In Driskill and McCafferty (1988), we modeled competing international duopolists facing slow price adjustment along the lines developed by Fershtman and Kamien (1987). In that paper, we found that the optimal policy was about a 5 percent tax imposed by the government. We argued that this finding could be understood with the help of Eaton and Grossman's results about optimal policy under different conjectural variations. Basically, what they found was that, for consistent conjectures, the optimal policy was no tax or subsidy; for conjectures smaller in absolute value than the consistent conjecture, the optimal response was a subsidy; and for conjectures greater in absolute value than the consistent conjecture, the optimal policy was a tax (at least for the case of linear demand and quadratic marginal cost). In our model with sticky price adjustment, we showed that the conjectural variations equilibrium that replicated our steady-state equilibrium had a conjecture greater in absolute value than the consistent conjecture. Hence, Eaton and Grossman's analysis suggests that the optimal policy would be a tax. Of course, the Eaton and Grossman results did not apply exactly since in our model the terms corresponding to the conjectural variations term in the static game were themselves not exogenous but functions of the tax rates.

In this paper, with output adjustment costs instead of sticky price adjustment, analogous reasoning can be used to gain some insight. We find it useful to recast the Eaton and Grossman results in our special case of linear demand and linear costs. Consider the welfare function (15), $w_i = w_i(u_i, u_j)$. Graphically, we depict w_1 in figure 4.1 as a family of isowelfare curves in the (u_1, u_2) plane. The salient characteristics of this graph are that each isowelfare curve is concave, with a maximum at

$$u_1 = \frac{a - c - u_2}{2} \tag{16}$$

and with partial derivatives

$$\partial w/\partial u_2 < 0, \quad \partial u_2/\partial u_1 > -2, \tag{17}$$

for $\partial u_2/\partial u_1 < 0$. Note that the Nash-Cournot one-shot game reaction curve for the country 1 firm is that line along which $\theta_2 = \partial u_2/\partial u_1 = 0$.

Maximization of country 1's welfare calls for picking the pair (u_1, u_2) along country 2's reaction curve that is tangent to an isowelfare locus. This point is illustrated in figure 4.1 as point A. Since the isowelfare curve is concave with a slope that decreases from zero along the Nash-Cournot reaction curve for the country 1 firm, this constrained optimum is necessarily to the right of the Nash-Cournot reaction curve. The question answered by Brander and Spencer is how a country can obtain point A when its firm's reaction curve is the Nash-Cournot one. The answer is that a subsidy will twist out the home firm's

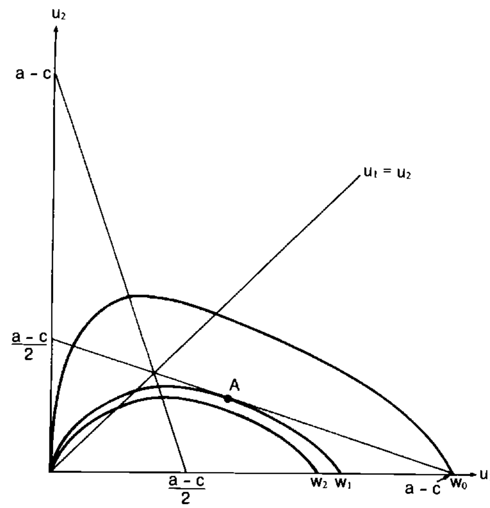

Fig. 4.1 The welfare maximizing output level for country 1

reaction curve so as to intersect the foreign reaction curve at point A. To make the point more clear, we write the home firm's reaction curve when the firm is subsidized at rate t_1 as

$$(18)\quad u_1 = [a(1 - t_1) - c]/[2(1 - t_1)] - (u_2)(1 - t_1)/[2(1 - t_1)].$$

As t_1 decreases from zero, the intercept moves up, and the slope grows flatter. An appropriate subsidy can twist the curve through point A.

Now consider the same problem but assume no taxes. Instead, consider how different conjectural variations twist the Nash-Cournot reaction curve. The conjectural variation reaction curve for country 1 is

$$(19)\quad u_1 = (a - c)/(2 + \theta_2) - (u_2)/(2 + \theta_2).$$

As θ_2 varies from zero (Cournot) to minus one, the reaction curve twists out. Clearly, there is some θ_2 that will make the home-country firm's reaction curve go through point A, the welfare optimum. Eaton and Grossman have proved what this conjectural variation is: the "consistent" one in the sense of Bresnahan (1981) and Perry (1982). With the consistent conjecture, Eaton and Grossman prove that the optimal subsidy is zero. Hence, the consistent conjecture reaction curve must pass through point A. For conjectural variations greater in absolute value than the consistent one, the optimal policy would be a tax; this, for example, is the case in our linear example with Bertrand conjectures. That is, the Bertrand conjecture in terms of quantities is greater in absolute value than the consistent conjecture. For conjectures smaller in absolute value than the consistent one, the optimal policy is a subsidy.

Since for a wide variety of parameter values we compute that welfare is optimized with the imposition of a subsidy, one might guess that r_2, the term in our steady-state reaction curve that occupies the same spot that the conjectural variations term occupies in the one-shot reaction curve, is in fact less in absolute value than the consistent conjecture of the one-shot game. If this is true, it is of interest not only because it helps us understand the welfare simulations but also because it means that the steady-state output of this dynamic game is always less than the output of the one-shot game with consistent conjectures. To show that this may in fact be the case, at least in the neighborhood where $t_i = t_j = 0$, we first calculate r_j (which also equals r_i in this symmetric case) in the limiting case when $A \rightarrow 0$.

The solution to our game obtained when $A \rightarrow 0$ is called the limit game solution. Recall that A is the cost-of-adjustment parameter. If we were to simply set $A = 0$ at the start of the problem and solve the game, we would find that the steady-state output of that game is identical to static Nash-Cournot. In contrast, as we take the limit as $A \rightarrow 0$ of our closed-loop Nash equilibrium, steady-state output tends not to static Nash-Cournot but rather to some level strictly between perfect competition and static Nash-Cournot. This is a standard feature of closed-loop equilibria of differential games (see Fershtman and Kamien 1987; Reynolds 1987; and Driskill and McCafferty 1988, in press).

We now state and prove the following theorem.

THEOREM 2: For $t_i = t_j = 0$, the steady-state output level for each firm in the limit game of $A \rightarrow 0$ is strictly greater than static Nash-Cournot but strictly less than the output level in the static consistent conjectures equilibrium.

Proof: First, we set t_i and t_j to zero in equations (4) and (5) and restrict ourselves to symmetric solutions where $k_{ii} = k_1$ and $k_{ij} = k_{ji} = k_2$. Equations (4) and (5) then become

$$k_1 = \{-Ak_2^2[3k_1 - 2\delta] + 2(k_1 - \delta)\}/ [A(k_1 - \delta)(2k_1 - \delta) + Ak_2^2 - 2], \tag{20}$$

$$k_2 = \{-Ak_1k_2[3k_1 - 2\delta] + 2k_1 - \delta\}/ [A(k_1 - \delta)(2k_1 - \delta) + Ak_2^2 - 2]. \tag{21}$$

Rearranging (20), we can write

$$k_2 = \phi(k_1) = [(Ak_1^2 - A\delta k_1 - 2)/(-2A)]^{1/2}. \tag{22}$$

Consider the value of k_1 where $\phi(k)_1 = k_1$. This value, call it $\bar{k}_1$, is always greater than any equilibrium value k_1^* because of stability condition (6). Now $\bar{k}_1$ is given by

$$\bar{k}_1 = \{(\delta/3) \pm (1/2)[(\delta/3)^2 + (8/3A)]^{1/2}\}. \tag{23}$$

Hence, as $A \to 0$, both $\bar{k}_1$ and k_1^* go to $-\infty$. Now, rearranging (22), we get

$$(24) \qquad A = 2/[2k_2^2 + k_1(k_1 - \delta)].$$

Dividing (21) by (20), and substituting (24) for A in the resulting expression, we get:

$$(25) \; (k_2/k_1) = \{-2[3 - (2\delta/k_1)](k_2/k_1) + [2 - (\delta/k_1)][1 - (\delta/k_1)] + [2(k_2/k_1)^2]\} + \{-2[(k_2/k_1)^2 - 1 + (2\delta/k_1) - (\delta/k_1)^2]\}.$$

Define $\lim_{A\to 0} (k_2/k_1) \equiv -r$. Using the well-known properties of limits, and remembering that, as $A \to 0$, $k_1 \to -\infty$, we then have

$$(26) \qquad -r = (6r + 2 + 4r^2)/[-2(r^2 - 1)].$$

Rearranging (26), we get

$$(27) \qquad 2r(r^2 - 4) - 2(1 + 2r^2) \equiv \Omega(r) = 0.$$

Over the interval $(-1, 0)$, $\Omega(r)$ is a strictly concave function, with $\Omega(-1) = 0, \Omega(0) = -2$, and $\Omega'(-1) > 0$. Thus, $\Omega(r) = 0$ once and only once over the interval $(-1, 0)$. Now, $r = -1$, which satisfies (27), is inconsistent with stability, as it implies that $k_1 = k_2$, which in turn violates the Routh-Hurwicz condition. Hence, r is strictly greater than -1, and steady-state output is greater than it would be under static Nash-Cournot, which corresponds to $r = 0$. The consistent conjecture for the analogous static problem is given by

$$(28) \qquad \theta_{cc} = -1.$$

Hence, in this symmetric case with $t_1 = t_2 = 0$, our steady-state reaction curve is always steeper than the consistent conjecture equilibrium reaction curve, and steady-state output is less than under consistent conjecture equilibrium. Q.E.D.

At least for the limiting case $A \to 0$, theorem 2 tells us that our steady-state reaction curve with $t_1 = t_2 = 0$ intersects the other firm's reaction curve to the left of point A in figure 4.1. We illustrate this in figure 4.2. Now, for the given values of r_2, we know that a tax will shift our reaction curve out toward point A. What complicates our analysis relative to the static one of Eaton and Grossman, though, is that changing t_i, the tax rate, changes r_1 and r_2. We are unable to determine analytically how these changes in t_i change r_1 and r_2. Such changes in r_1 and r_2 twist and shift both reaction curves, and we have been unable to prove whether these twists and shifts can overwhelm the welfare-improving shift occurring directly from the increase in the tax rate. Our numerical calculations suggest that this in fact does not happen.

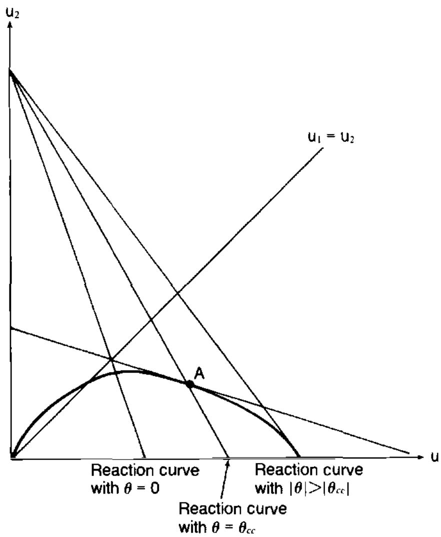

Fig. 4.2 The limit game of $A \rightarrow 0$

4.4 Conclusions

By explicitly solving a dynamic game, we find that steady-state output for two duopolists competing in a third-country market is greater than the output that would be produced under identical cost conditions at a Nash-Cournot equilibrium for a one-shot, static game. We also find that our steady state can be replicated by a static conjectural variations equilibrium. The gain from the explicit solution of the dynamic game is that the conjectural variation that replicates the dynamic outcome is specified as a function of underlying taste and technology parameters and is not simply an assumed, exogenous value. Given Eaton and Grossman's (1986) findings concerning the relation between optimal taxes and conjectural variations, this result has important implications for welfare analysis of the optimal tax. Furthermore, our dynamic analysis points out that welfare analysis is perhaps more complex than the static conjectural variations approach would suggest: for the dynamic analysis, the conjectural variation that replicates the dynamic outcome is itself a function of tax parameters and cannot be assumed to be constant across policy experiments.

For the explicit game we study, we find that the optimal policy under a wide variety of parameter values is a subsidy on exports. This finding seems related to the fact that our steady-state level of output is less than would arise at a consistent conjectures equilibrium. For the linear example we study, an implication of Eaton and Grossman's analysis is that, for conjectural variations equilibria with output less than would arise under consistent conjectures, the optimal policy would be a subsidy. Eaton and Grossman's result thus suggests that the optimal policy in our model should be a subsidy, not a tax. Their result is suggestive, but not definitive, for our model because their analysis takes conjectural variations as exogenous and unchanging in the face of policy changes.

Our results are of course derived from a very specific model. On the basis of other work on dynamic games, we think that some of our results generalize. Work by Driskill and McCafferty (1988) on models with slow price adjustment instead of costs of output adjustment also finds that steady-state output is greater than would occur at a static Nash-Cournot equilibrium. These models can also be thought of as being replicable by a static conjectural variations equilibrium. The optimal policy in such models, though, seems to be a tax instead of a subsidy.

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Comment Elias Dinopoulos

The paper by Driskill and McCafferty develops a dynamic differential game with two firms competing in a third market and facing output adjustment costs. The main finding is that the steady-state equilibrium of the dynamic game can be replicated by a static game with conjectural variations. Comparing the steady-state equilibrium to that of the one-shot static game, the authors find several differences: the term that corresponds to conjectural variations of the static game is a function of the parameters of the dynamic game; when output adjustment costs approach zero, the steady-state output of each firm is higher than that of the static Cournot game and lower than that of the static consistent conjectures game; and, when firms face output adjustment costs, an export subsidy maximizes steady-state welfare, whereas, in the case of price adjustment costs, steady-state welfare maximization requires a tax. In the following discussion, I would like to interpret the results of the paper in the context of the existing literature on conjectural variations and trade policy and offer some remarks on comparative dynamics, steady-state welfare, and adjustment processes.

Bresnahan (1981) introduced the concept of consistent conjectures in an attempt to provide the "right" alternative to Cournot and Bertrand equilibria in a static framework. Eaton and Grossman (1986) used Cournot, Bertrand, and consistent conjectures to show that the optimal trade policy depends crucially on the nature of conjectural variations. They found that Cournot conjectures require an export subsidy, that Bertrand conjectures are associated with an export tax, and that under consistent conjectures free trade is the optimal policy.[1] Conjectural variations were criticized by Stanford (1986). He showed that, in an infinitely repeated game with discounting and discrete time, the only reaction function equilibria that are subgame perfect are those with static Cournot or Bertrand conjectures.

The present paper models the dynamic game using continuous time and obtains results that are similar to those of Stanford with respect to consistent conjectures. It shows that the steady-state output of each duopolist is *different* than the output of the analogous one-shot static game with consistent conjectures. In this sense, it implies some form of trade intervention that is discussed in the welfare section. One of the virtues of the paper is that the authors do an excellent job of indicating the formal connections and the economic intuition that relate the results to particular assumptions. The discussion of why the steady-state output is higher than the static Cournot output is extremely useful. The use of numerical simulations to investigate the nature of the steady-state solution is common practice in problems involving differential games.

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Most of the analysis of the paper focuses on comparing a steady-state equilibrium to the analogous static game. One of the weaknesses of the paper is that it does not deal with issues of existence and uniqueness of the steady-state equilibrium. The lack of a formal proof of the existence of steady-state equilibrium does not allow the reader to compare the results of the present paper to those of Stanford (1986). Moreover, Fisher and Wilson (1988) show that a static Bertrand duopoly game with a homogeneous product and differential tariffs does not possess pure strategy equilibria. Consequently, it is possible that the class of steady-state solutions for the differential game with nonzero tariffs might not exist. The issue of uniqueness is equally important because the existence of multiple steady-state equilibria would question the relevance of comparative dynamics exercises. These exercises are used implicitly in the maximization of steady-state welfare.

Personally, I found the discussion of optimal policies and welfare somewhat unsatisfactory. The paper concentrates on policies (in the form of export taxes or subsidies) that maximize the value of steady-state welfare (sec. 4.3). Given the explicit dynamic framework of the game, optimum policies should be intertemporally efficient in the sense that they maximize the present discounted profits. Starting at a steady-state equilibrium, any change of a tax rate is associated with a time path that drives the system from the old to the new steady state. The transition from one steady state to another involves welfare changes that should be taken into account when comparing the two original steady-state equilibria. Samuelson (1975) and Srinivasan and Bhagwati (1983) show that, in general, a path that maximizes steady-state welfare is not necessarily intertemporally efficient in the sense that it maximizes the present discounted value of welfare. The proper way of calculating optimal taxes or subsidies in the present context is to have each government maximizing the present discounted profits of its firm taking the tax rate of the other government as given and acting as a Stackelberg leader vis-à-vis the game of the two firms. The optimal tax path of each government will be intertemporally efficient, and it could converge into a steady-state value. This value could then be examined in terms of its positive or negative sign. Indeed, it is peculiar to have firms engaged in intertemporal optimization and governments in steady-state optimization. If firms maximize steady-state instead of present discounted profits, I suspect that there will be no difference between the steady-state and the analogous static Cournot game output. However, if the proper methodology is followed, I have no reason to expect that the steady-state values and signs of intertemporally efficient taxes or subsidies are the same as those that maximize steady-state welfare. I realize that the computation of intertemporally efficient policy instruments is analytically difficult, if not impossible. Perhaps an appropriate variant of Diamond's (1980) methodology[2] or even an additional numerical simulation exercise will illuminate the nature of the proper optimal policy instruments.

My final remark concerns the symmetric structure of output adjustment costs. In the present model, each firm faces costs that are quadratic in the rate of output change. I feel slightly uncomfortable with this specification because it implies that a reduction in the rate of output increases costs by the same amount as a raise in the rate of output. Models that focus on capacity constraints assume that increasing output beyond a certain range is not possible in the short-run but that output reduction does not affect variable costs. I would expect more economic intuition on the choice and the role of the output adjustment structure.

To conclude, I think that the paper makes an important contribution to the literature of conjectural variations and trade policy. It suggests that consistent conjectures cannot be rationalized by a dynamic differential game with adjustment costs. However, its normative conclusion, which advocates some form of trade intervention, is based on steady-state welfare maximization and not intertemporally efficient policy paths. I hope that future work in this area will clarify the normative issues that were raised by the present paper.

Notes

1. Cheng (1988) has provided a similar analysis in the context of a home market that is supplied by a domestic and a foreign firm.
2. Diamond has proposed a simple expression for the present discounted value of a change in welfare from one steady state to another along a convergent path.

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Comment Ronald D. Fischer

In my view, this paper has two main objectives: (i) to provide a foundation for conjectural variations equilibrium and a model of how the conjectures are generated and (ii) to apply the model to international trade in order to determine the optimal tariff or subsidy and compare it to the corresponding results of the static model of Eaton and Grossman (1986), henceforth EG.

Driskill and McCafferty use a dynamic Cournot game with adjustment costs to generate conjectural variations. They show that, in their model, the reaction functions corresponding to linear output changes (the instruments) do not correspond to those of the static Cournot model. These reaction functions can be written in a form reminiscent of the reaction functions of static conjectural variations models. As the term corresponding to the conjectures is written in terms of the fundamentals of their model, they conclude that their model can be seen as endogenizing the conjectures. This provides support for conjectural variations models against the static Cournot model. But is this really true? Since they restrict themselves to linear instruments,

$$x_i = \dot{u} = K_i + k_{ii}u_i + k_{ij}u_j \ ,$$

and consider the steady state $x_i = 0$, the relation (reaction function) between u_i and u_j must be linear. As they have included adjustment costs, it is clear that the reaction functions will normally be different from the ones derived from a static Cournot model. Consider a better analogue of their model: a two-period model with quadratic adjustment costs (the discount factor is assumed to be zero),

$$\Pi^i = \Pi^i_1 + \Pi^i_2, \quad i = 1, 2,$$

where subscripts denote periods and superscripts denote firms. Writing the profit functions explicitly,

$$\Pi^i_1 = (1 - q^1_1 - q^2_1)q^i_1,$$
$$\Pi^i_2 = (1 - q^1_2 - q^2_2)q^i_2 - A(q^i_2 - q^i_1)^2, \quad i = 1, 2.$$

Here q^i_j represents output of firm i in period j and A is a constant, common to both firms. Solving this two-period model recursively, one obtains higher first-period outputs than in the Cournot equilibrium. The reason is that the adjustment cost allows the firms to try to precommit to higher output (as in the Spencer and Brander [1985] models of investment in research and development). Since the firms are symmetric, higher output results in the equilibrium. This replicates the results of the dynamic model, which suggests that it is the

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different specification (i.e., the inclusion of adjustment costs) that leads to the results and not that the Cournot model is inappropriate.

This raises the question, Does this represent support for conjectural variations? I think it does not. What it shows is that conjectural variations can be used as static shorthand to encompass different models that have their own logical foundations. The dynamic game studied by Driskill and McCafferty is interesting in its own right, not because it has a relation to conjectural variations.

There are two big differences between the two-period model and dynamic models. The first is that the solution of the two-period model is nonstationary. Second, in the two-period model, as the adjustment cost A tends to zero, the solution converges to that of a two-period Cournot model. In the dynamic model, output remains larger even in the limit. It would be interesting to know the source of this difference.

Is such a project of supporting conjectural variations desirable? The notion of conjectural variations, though superficially attractive, suffers from internal inconsistencies. Unless conjectures are restricted in some way, any equilibrium is possible. Certain seemingly plausible restrictions have been proposed but have been shown to be irrational. Daugherty (1985) proved that the only "consistent" conjectural variations equilibrium is the Nash equilibrium. Makowski (1987) has shown that so-called rational and reasonable conjectures are neither.

Consider now the second objective, that is, the applications to trade. In EG (see their n. 2, p. 386), the conjectural variations model is used as a convenient framework that includes conjectures ranging from Cournot to Bertrand. It is also used because it highlights the source of the potential benefits from policy intervention: the difference between conjectural and actual responses.

Driskill and McCafferty confirm the results in EG for quantity competition, showing that, for a range of parameter values, a subsidy on exports will be optimal. The authors relate this result to the fact that the conjectures (in the conjectural variations analogue of their model) are smaller than the consistent conjectures. Eaton and Grossman have shown that in their model the choice between a subsidy and a tax on exports depends on the conjectures being smaller or larger than the consistent conjectures.

Finding another model that supports the results of EG is nice, but is it interesting? In my interpretation, the models of international trade that analyze profit shifting without home consumption are examples designed to show that the classical propositions of trade theory may no longer be valid in a world of imperfect competition. But, as Dixit argues, "It is my belief that research will reveal the profit-shifting argument to be of significance in only a small number of selected industries" (Dixit 1986, p. 291). In EG, the real welfare analysis begins when home consumption is included in the model. This type of analysis is difficult to do in the present model. It is in this sense that I find that the real interest of the model lies in industrial organization rather than in its application to international trade.

The authors have characterized the linear closed-loop solutions to their dynamic model (when they exist). Another interesting question is the possibility of cooperation. Since the model is the continuous version of a repeated Cournot game with adjustment costs, can collusive solutions be supported? Suppose that the firms decide on the following trigger strategy: play collusively, and, if the other firm defects, use the above closed-loop solution. If detection is immediate, the gains from deviating are zero, but the use of the closed-loop strategy in the future represents a loss with respect to cooperation. Thus, the collusive outcome may be supported. In fact, given a low enough discount rate, this is probably true even if detection of defection is delayed (for related work, see Benhabib and Radner 1988). It does not seem to me that such equilibria depend on "irrelevant" history as the authors claim.

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