Chapter Title: Annuity Prices and Saving Behavior in the United States

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2 Annuity Prices and Saving Behavior in the United States

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One of the most puzzling contrasts between observed behavior and the implications of standard economic theory is the fact that, at least in the United States, few elderly individuals purchase life annuities. The conventional life-cycle model, based on the appealing concept that people save so as to smooth their consumption over their lifetimes, suggests that elderly retired individuals would seek to dissave out of their available resources as their remaining life expectancy shortens. Instead, observed age-wealth profiles among the elderly are more nearly flat.[1] Given the uncertainty associated with any individual's life expectancy, this reluctance to dissave would be a natural consequence of risk aversion if individuals could not avoid that risk by buying annuities. Since a well-developed individual life annuity market does exist in the United States, however, the challenge is to explain why so few people actually avail themselves of it.[2]

In an earlier paper, the authors offered an explanation for this phenomenon based on a combination of the cost of annuities and a bequest motive.[3] Annuities are costly, in the first instance, because the insurer must price them to defray ordinary costs of doing business and then earn a competitive profit. In addition, the typical individual in the population finds annuities even more costly because of adverse selec-

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tion—in other words, the tendency of longer-lived people to buy more annuities than people facing shorter life expectancies. Both kinds of costs understandably discourage the purchase of individual life annuities within the context of the familiar life-cycle model. By contrast, if individuals choose not to buy annuities because they have accumulated wealth to leave to their heirs, rather than to finance their own consumption after retirement, then the life-cycle model—and with it, a variety of well-known implications for economic behavior and economic policy—fails to withstand scrutiny. The principal finding of the authors' earlier research was that during the early years of retirement the observed cost of annuities can independently account for the absence of purchases of individual life annuities, while at older ages the combination of the observed cost of annuities and a bequest motive of plausible magnitude can account for this phenomenon.

The object of this paper is to experiment with an alternative form of the authors' earlier analysis by representing the cost of annuities as the (positive) differential between the premium on an annuity and its implicit expected value, rather than as the (negative) differential between the implicit expected yield on an annuity and the available yield on alternative forms of wealth holding. Given the inverse relationship between price and yield for any fixed-income investment vehicle, in principle these two forms of analysis are simply the duals of one another. The difference here stems from the need to compromise with reality in order to investigate the implications of life annuities within the context of a readily tractable model of the consumption-saving and portfolio-allocation decisions. In effect, the analysis both here and in the authors' earlier paper represents these annuities as if they were one-year contracts. Here, however, the analysis represents the cost of annuities as a one-time proportional charge to enter a market in which actuarially fair annuities are available, while in the earlier paper this cost consists of a continual unfairness in the pricing of the (one-year) annuity contracts. One advantage of the approach taken here is that, because large short-sales of annuities are no longer optimal, the non-negativity constraint that was necessary in the earlier paper is no longer required. Because these formulations of the problem imply alternative opportunity sets, the results given by the two approaches differ.

Section 2.1 presents raw data on the prices of individual life annuities sold in the United States during 1968–83, together with transformations of those data that correspond to familiar concepts in economic discussions of consumption-saving behavior. Section 2.2 reviews the model of consumption-saving and portfolio-allocation behavior, for an individual with an uncertain lifetime, developed in the authors' earlier paper. Section 2.3 uses simulations of this model, based on the observed pricing of annuities, to draw inferences about the respective roles of

annuity costs and a bequest motive in accounting for the typical elderly retired individual's preference for maintaining a flat age-wealth profile instead of buying annuities. Section 2.4 briefly summarizes the paper's principal findings and reemphasizes some limitations that apply to the analysis here as well as to the authors' earlier work.

2.1 Prices of Individual Life Annuities

Table 2.1 presents data for 1968–83, compiled from successive annual issues of the *A. M. Best Flitcraft Compend,* on the per-dollar prices of guaranteed single-premium immediate annuities offered in the United States for 65-year-old males. In each case the value shown is the price (premium) charged to purchase a stream of payments equaling $1 per month, to begin in the month immediately following the purchase and to continue for the life of the annuitant.

Column 1 of table 2.1 indicates the mean premium charged on this basic annuity contract by the ten largest insurance companies in the United States. These data are probably the most relevant for analyzing economy-wide individual behavior. The largest insurers usually do business in all regions of the country, so that the typical 65-year-old U.S. male has access to annuities at this mean price with little or no search costs. As would be expected, the average annuity premium has fallen over time, as the effect of rising interest rates has predominated over the effect of increasing life expectancy.

Table 2.1   **Premiums for Immediate $1 Monthly Life Annuities**

| | Ten Largest Insurers | | | Complete Sample |
Year	Mean	High	Low	Low
1968	$132.10	$136.20	$128.60	$127.20
1969	129.90	134.30	125.20	123.90
1970	127.40	133.70	119.30	116.70
1971	124.60	133.70	115.80	115.80
1972	124.70	133.70	117.70	117.70
1973	123.20	131.00	117.70	117.70
1974	121.70	127.60	115.80	115.50
1975	118.70	123.80	113.30	113.30
1976	116.60	123.30	111.40	107.90
1977	116.60	122.10	113.30	109.10
1978	116.60	122.10	113.30	109.10
1979	117.20	122.40	113.30	105.90
1980	113.40	119.90	105.70	101.80
1981	109.00	116.60	103.20	92.30
1982	104.90	116.60	87.60	75.30
1983	103.70	116.60	90.80	81.80

NOTE: Quotations are for 65-year-old males.

The remaining columns of table 2.1 indicate the potential returns to market search by showing the dispersion of premiums charged for this same basic contract by different insurers.[4] Columns 2 and 3 show data for the highest and lowest premiums charged for this contract by any of the ten largest insurers. Presumably most 65-year-old males have access to the lowest premium in this group at only modest search cost. Column 4 shows the lowest premium charged for this contract by any of the fifty-odd insurers in Best's sample. Because the smaller companies in the sample do not necessarily maintain sales forces in all parts of the country, however, there is no presumption that the typical 65-year-old male has ready access to this complete-sample lowest premium.

Table 2.2 presents calculations of the present expected value of an immediate $1 monthly annuity for the life of a 65-year-old male. The two key ingredients in such calculations, of course, are the assumed interest rate and the assumed structure of mortality probabilities.[5] The table reports annual calculations of present expected value based on two different interest rates, the 20-year U.S. government bond yield and the average yield on corporate debt directly placed with major life insurance companies. In the calculations underlying columns 1 and 2, the assumed mortality probabilities are the general population mortality probabilities for 65-year-old males reported in the 1970 and 1980 U.S. Life Tables, adjusted by a factor of .985 to reflect the 1.5 percent annual

Table 2.2   Present Expected Values of $1 Monthly Life Annuities

|	General Population		Annuity Purchasers	
Year	Government Bonds	Direct Placements	Government Bonds	Direct Placements
1968	$104.80	$92.19	$120.81	$104.80
1969	99.49	87.64	113.81	98.92
1970	96.67	80.14	110.04	89.50
1971	101.41	85.84	116.59	96.86
1972	102.74	88.48	118.06	100.00
1973	96.16	88.25	109.46	99.53
1974	91.23	82.23	103.07	91.91
1975	90.93	79.49	102.52	88.41
1976	93.23	83.67	105.20	93.37
1977	94.83	87.88	107.01	98.39
1978	90.66	85.04	101.65	94.75
1979	86.64	80.67	96.56	89.29
1980	77.43	70.21	85.61	76.92
1981	69.31	63.24	75.75	68.57
1982	72.25	65.57	79.15	71.23
1983	78.69	74.38	86.77	81.58

NOTE: Calculations are for 65-year-old males.

improvement in U.S. male mortality probabilities that has occurred over the last two decades, and by a further factor of .9925 to reflect the assumption of a future 0.75 percent annual improvement in male mortality probabilities for all ages.[6]

Which of the two interest rates used in table 2.2 is most relevant depends upon the perspective taken in the analysis. From the standpoint of the actuarial "fair" value to an insurer who has access to (and typically owns) direct placement securities, the associated higher yield is the correct one to choose. Alternatively, from the standpoint of an individual's opportunity cost of funds, the lower yield on U.S. government bonds is relevant if the individual has no better investment vehicle. Because direct placements bear higher yields than do government bonds, the present expected values calculated using this yield are smaller than the corresponding values calculated using the government bond yield. Nevertheless, the present expected values calculated on either basis, including the government bond yield, are always smaller than even the lowest premium charged in the same year by any insurer in Best's sample.

If all individuals had identical mortality probabilities, a comparison between the calculated present expected values shown in columns 1 and 2 of table 2.2 and the actual premiums shown in table 2.1 would indicate the "load factor" by which the pricing of these annuities differs from their fair actuarial value. In fact, many individuals have information that leads them to expect either a shorter or a longer life than the population-wide average. Insurers, however, typically charge a uniform premium to all individuals of the same age and sex, presumably because information about individual mortality probabilities is either impossible or too costly to obtain and use. Individuals expecting longer (shorter) than average lifespans will therefore perceive life annuities as more (less) attractively priced, and hence will be more (less) likely to buy them.[7] This adverse selection—adverse from the viewpoint of the insurer, that is—will lead to underwriting losses if the insurer continues to charge a premium that is actuarially fair to the population as a whole.

Columns 3 and 4 of table 2.2 therefore present the results of further annual calculation of the present expected value of the same basic annuity contract for a 65-year-old male, based on the same two interest rates as before, but now based on alternative mortality probabilities compiled from the actual company experience on individual life annuity contracts issued in the United States during 1971–75, again adjusted as indicated above to reflect the improvement in mortality probabilities.[8] Figure 2.1 illustrates the extent to which the subpopulation who choose to buy annuities in fact have a greater survival probability than the general population. Because of this greater life expectancy, the

Survival probability
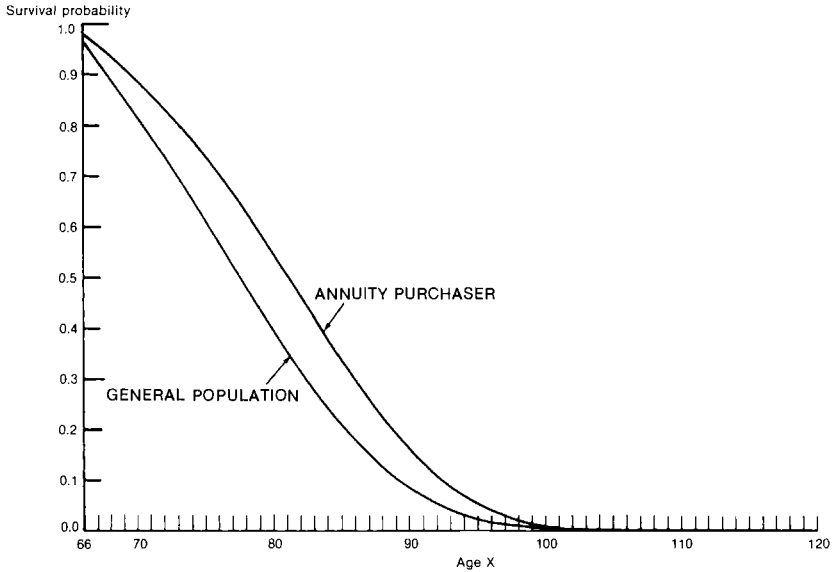
Fig. 2.1   Probability of survival to age X for males at age 65 in 1970.

present expected values shown in columns 3 and 4 of table 2.2 are greater than the corresponding values shown in columns 1 and 2, based on the same two interest rates but on general population mortality probabilities. For each year, however, even the greater values resulting from the actual company experience mortality probabilities are still uniformly smaller than even the lowest corresponding premiums shown for that year in table 2.1. Even within the subpopulation who voluntarily buy annuities, therefore, the price is not actuarially fair.

Table 2.3 summarizes the differences between the actual premiums charged on this basic annuity contract and the corresponding actuarially fair values by showing the 1968–83 average of the ratio of premium charged to present expected value for each of the four premiums re-

Table 2.3   **Average Load Factors on Life Annuity Premiums**

	General Population		Annuity Purchasers	
Premium	Government Bonds	Direct Placements	Government Bonds	Direct Placements
Ten-Largest Mean	1.32	1.48	1.18	1.33
Ten-Largest High	1.39	1.55	1.24	1.40
Ten-Largest Low	1.24	1.39	1.11	1.25
Complete-Sample Low	1.20	1.34	1.06	1.20

NOTE: Calculations are for 65-year-old males.

ported in table 2.1 and each of the four present expected value cal-
culations reported in table 2.2. The resulting average load factors range
from a low of 1.06 for the smallest premium charged by any company
in Best's sample, compared to the present expected value based on
government bond yields and actual company experience mortality
probabilities, to a high of 1.55 for the largest premium charged by any
of the ten largest insurers, compared to the present expected value
based on direct placement yields and general population mortality tables.

The comparisons in table 2.3 that are probably most relevant for
studying economy-wide individual behavior are those shown in row 1
for the mean premium charged by the ten largest insurers versus the
present expected value based on either government bond yields or
direct placement yields, and on either general population or company
experience mortality probabilities. The load factor of 1.32, for the first
case considered, means that a 65-year-old U.S. male, randomly se-
lected from that population, and for whom the government bond yield
represents the opportunity cost of capital, typically pays $1.32 for each
$1.00 of expected present value when he purchases a life annuity.
Among the (on average longer-lived) subpopulation of 65-year-old U.S.
males who actually choose to buy life annuities, the load factor based
on the same opportunity cost is only 1.18. In other words, of the 32¢
per dollar load factor to the general population, 14¢ represents the
effect of adverse selection and the remaining 18¢ the combination of
transaction costs, taxes, and profit to the insurer. If the annuity pur-
chaser's opportunity cost of capital is instead the direct placement
yield—for example, because of ability to buy shares in packages of
intermediated private securities—then the load factor per dollar of ex-
pected present value is 48¢, of which 15¢ represents the effect of ad-
verse selection and the remaining 33¢ the insurer's costs, taxes, and
profit.[9] As the appendix shows, these results for 65-year-old males are
similar to those for females or for males of different ages.

The question for consumption-saving behavior, then, is whether an
average load factor of 1.18 (or even 1.48) is sufficient to account for
the small participation in the individual life annuity market in the United
States. In short, do most elderly retired people choose not to consume
out of their wealth, and therefore leave unintentional bequests, merely
because they are reluctant to pay $1.18 (or $1.48) for every $1.00 of
present expected value of annuities?

It is difficult to answer this question on the basis of casual evidence
only. At first thought, a load factor of this magnitude seems a large
price to pay for pooling risk. Nevertheless, it is not out of line with
loads charged elsewhere in the insurance business, in product lines
that almost everyone buys. For example, data from *Best's Key Rating
Guide: Property-Casualty* indicate that the recent average load factor
in premiums charged for property and casualty insurance written by

large companies has been 1.37—essentially the same as that on individual life annuities.

Moreover, despite the apparently large load factor, individual life annuities are not a "dominated asset" in the sense that the cost per unit of payoff, unadjusted for mortality probabilities, is greater than the analogous cost of alternative investment vehicles. Table 2.4 shows the present value of a 35-year certain $1 monthly annuity, calculated using the two interest rates used in tables 2.2 and 2.3 for each year during 1968–83.[10] As comparison to column 1 of table 2.1 shows, the value of the certain annuity based on the government bond yield exceeded the mean premium on the life annuity charged by the ten largest insurers in all years of the sample except 1980–82, while the value based on the direct placement yield exceeded the mean premium in most years until 1979.

Hence some more formal approach to this issue is necessary. Section 2.2 develops a framework for such an analysis, and section 2.3 applies that framework in the context of the premium and load factor data reported here.

2.2   A Model of Saving and Annuity Demand

The model used in Friedman and Warshawsky (1985)[11] to analyze the demand for individual life annuities in the context of life-cycle saving and a bequest motive is an annuity analog of Fischer's (1973) model of the demand for life insurance, generalized to incorporate fixed mandatory holdings of socially provided annuities.[12] The individual's

Table 2.4   Present Value of a 35-Year Certain $1 Monthly Annuity

Year	Government Bonds	Direct Placements
1968	$190.41	$153.04
1969	172.26	139.60
1970	162.68	120.14
1971	176.32	133.88
1972	178.51	139.47
1973	158.30	137.77
1974	144.21	122.38
1975	142.28	115.40
1976	146.91	123.96
1977	149.69	132.79
1978	138.42	125.35
1979	128.12	115.03
1980	108.21	93.95
1981	91.80	80.87
1982	96.86	84.62
1983	108.96	100.45

decision problem in this expanded life-cycle context is to maximize expected lifetime utility:

$$(1) \qquad E(U) = \sum_{t=0}^{w-x-1} [p_t U_t(C_t) + p_t q_{t+1} V_{t+1} (G_{t+1})],$$

where w is the assumed maximum length of life, x is the individual's age as of time $t = 0$, p_t is the probability that an individual of age x at $t = 0$ will be alive at any time $t > 0$, q_{t+1} is the (conditional) probability that such an individual who was alive at time t will die at time $t + 1$,[13] $U_t(C_t)$ is utility received from consumption C at time t, and $V_{t+1} (G_{t+1})$ is utility received from (anticipation of) a bequest G at time $t + 1$. Following Fischer (1973), it is convenient to specify the two utility functions in the isoelastic form:

$$(2) \qquad U_t (C_t) = \frac{C_t^{1-\beta}}{1 - \beta} \cdot \alpha^t \quad,$$

$$(3) \qquad V_t(G_t) = \frac{G_t^{1-\beta}}{1 - \beta} \cdot b_t \quad,$$

where β is the Pratt-Arrow coefficient of relative risk aversion, α is the time preference parameter, and b_t (in comparison to α^t) indicates the relative utility attached to bequests left in period t.

The usual life-cycle specification of behavior with no bequest motive is therefore just the special case of this model with $b_t = 0$ for all $t > 0$. In general, however, people may value bequests, and they may value them differently at different times. The application of the model in section 2.3 below follows Yaari's (1965) suggestion that b_t follows a hump-shaped pattern with higher values during the years when family dependency is important, so that b_t is declining during retirement years when children have typically become independent.

The individual's problem is to maximize $E(U)$ subject to a given initial wealth position and to a nonnegativity constraint on wealth in each subsequent time period, given the menu of available investment opportunities (including any mandatory holding of socially provided annuities) and their respective yields.[14] In each period the individual must decide not only how much of current wealth to consume but also how to allocate the remainder among the available investment vehicles. The specific asset menu considered here includes a riskless one-period bond bearing gross rate of return R_t, a one-period social annuity bearing gross rate of return Q_t^S to survivors, and a one-period market annuity bearing gross rate of return Q_t^A to survivors.[15] Both annuities are actuarially fair—that is, there is no load factor on either—if

$$(4) \qquad (1 - q_{t+1})Q_t^S = (1 - q_{t+1})Q_t^A = R_t.$$

With little relevant loss of generality, it is convenient to set R_t constant at R for all $t > 0$.

The dynamic programming solution to this problem proceeds from the final period $t = w - x - 1$, in which the certainty of death at the end of the period ($q_{w-x} = 1$) simplifies the problem of an individual who has survived to that date to merely choosing C_{w-x-1} to maximize the sum of utility from current consumption $U_{w-x-1}(C_{w-x-1})$ and utility from bequests $V_{w-x}(G_{w-x})$, subject to then-remaining wealth W_{w-x-1} and the constraint

$$(5) \qquad G_{w-x} = R \cdot (W_{w-x-1} - C_{w-x-1}).$$

Given the isoelastic utilities assumed in equations (2) and (3), the solution is just

$$(6) \qquad C_{w-x-1} = k_{w-x-1} \cdot W_{w-x-1} \quad,$$

where

$$(7) \qquad k_{w-x-1} = \frac{R(Rb_{w-x})^{-1/\beta}}{1 + R(Rb_{w-x})^{1/\beta}} \quad,$$

and the corresponding indirect utility function,

$$(8) \qquad J_1[W_{w-x-1}] = \max_{C_{w-x-1}} \; [U_{w-x-1}(C_{x-x-1}) + V_{w-x}(G_{w-x})] \quad,$$

is

$$(9) \qquad J_1[W_{w-x-1}] = \delta_{w-x-1} \cdot \frac{W_{w-x-1}^{1-\beta}}{1 - \beta} \quad,$$

where

$$(10) \qquad \delta_{w-x-1} = k_{w-x-1}^{-\beta}.$$

The consumption decision in equation (5) represents the entire solution for $t = w - x - 1$, since in that period the availability of annuities is irrelevant to the analysis.

The dynamic programming solution next proceeds to the individual's optimal consumption and portfolio decisions for the immediately prior period, given wealth remaining at that time. An individual alive at $t = w - x - 2$ will die at the end of that period with probability q_{w-x-1}. Hence the relevant maximand governing the decisions to be taken as of $t = w - x - 2$ is $U_{w-x-2}(C_{w-x-2})$ plus the bequest motive $V_{w-x-1}(G_{w-x-1})$ with probability q_{w-x-1} and the indirect utility function in equation (9) with probability $(1 - q_{w-x-1})$. The indirect utility function for $t = w - x - 2$ is therefore

$$J_2[W_{w-x-2}] = \max_{C_{w-x-2}, A_{w-x-2}} \left\{ \frac{C_{w-x-2}^{1-\beta}}{1-\beta} + (1 - q_{w-x-1})\alpha\delta_{w-x-1} \right.$$

$$\cdot \frac{(W_{w-x-2} - C_{w-x-2})^{1-\beta}}{1-\beta} \cdot [R(1 - A_{w-x-2} - S_{w-x-2})$$

(11)

$$+ Q_{w-x-2}^A A_{w-x-2} + Q_{w-x-2}^S S_{w-x-2}]^{1-\beta} + q_{w-x-1} b_{w-x-1}$$

$$\cdot \frac{(W_{w-x-2} - C_{w-x-2})^{1-\beta}}{1-\beta} \cdot [R(1 - A_{w-x-2} - S_{w-x-2})]^{1-\beta} \right\} ,$$

where A and S are the proportions of saving $(W - C)$ invested in market annuities and in (mandatory) social annuities, respectively. The usual life-cycle model with no market for annuities is therefore just the special case represented by $A_t = 0$ for all $t \geq 0$ (and, if there are no social annuities either, $S_t = 0$ for all $t \geq 0$ also).[16]

The first order conditions for equation (11) then give the optimal values of consumption and purchases of market annuities at $t = w - x - 2$ as

(12) $$C_{w-x-2} = \delta_{w-x-2}^{-1/\beta} \cdot W_{w-x-2}$$

and

(13)

$$A_{w-x-2} = \frac{R\left[\left(\dfrac{Rb_{w-x-1} q_{w-x-1}}{(1 - q_{w-x-1})\alpha\delta_{w-x-1}(Q_{w-x-2}^A - R)} \right)^{-1/\beta} - 1 \right]}{Q_{w-x-2}^S + R\left[\left(\dfrac{Rb_{w-x-1} q_{w-x-1}}{(1 - q_{w-x-1})\alpha\delta_{w-x-1}(Q_{w-x-2}^A - R)} \right)^{-1/\beta} - 1 \right]} - S_{w-x-2} ,$$

and the corresponding value of the indirect utility function as

(14) $$J_2[W_{w-x-2}] = \delta_{w-x-2} \cdot W_{w-x-2} ,$$

where

(15) $$\delta_{w-x-2} = \left[\frac{k_{w-x-2}}{1 + k_{w-x-2}} \right]^{-\beta} ,$$

and

(16) $$k_{w-x-2} = \{\alpha\delta_{w-x-1} [R(1 - A_{w-x-2} - S_{w-x-2})$$

$$+ Q_{w-x-2}^A A_{w-x-2} + Q_{w-x-2}^S S_{w-x-2}]^{1-\beta}$$

$$+ b_{w-x-1}q_{w-x-1}[R(1 - A_{w-x-2} - S_{w-x-2})]^{1-\beta}\}^{-1/\beta}.$$

The remainder of the dynamic programming solution proceeds backward to the initial period $t = 0$ in an analogous way. The expressions for each period's optimal consumption and purchases of market annuities, and for each period's value of the indirect utility function, are of the same form (but with subscripts adjusted accordingly) as equations (12), (13), and (14), respectively.

2.3   Simulation Results

The model developed in section 2.2 generates lifetime streams of consumption and annuity purchase values that are optimal for given values of parameters describing preferences (β, α, and b), the market environment (R, Q^A, and Q^S), and mortality probabilities (p and q). The principal focus of interest in this paper is on one aspect of preferences and one aspect of the market environment—the bequest motive and the availability of market annuities, respectively.

The strategy adopted here for representing the bequest motive (confronting a 65-year-old male) follows Fischer (1973) by assuming that b_t in equation (3) varies according to

(17) $b_t = (1.04 - .01t) \cdot \theta$, $t = 0, \ldots, 35$,

where θ is a non-age-specific parameter indicating the individual's lifelong preference for bequests relative to current consumption, given the other parameters of the model, including in particular the interest rate (R), the curvature of the utility function (β), and—because θ implicitly gives the relative weight of a stock (the bequest) versus a flow (consumption)—the assumed time unit of analysis. For any given value of θ, however, b_t declines linearly with t.[17] Given θ and b_t, the bequest amount is larger as R is higher, and smaller as β is higher. For example, from equations (5)–(7) and (17), θ takes the value $(G_{w-x}/C_{w-x-1})^\beta \cdot (.69)/R$, where (G_{w-x}/C_{w-x-1}) is just the ratio of the final-period bequest to the prior-period consumption. The normally limiting case for altruistic bequests, in which an individual provides for his heirs' consumption at the same level as his own, indicates $(1/R - 1)^\beta \cdot (.69)/R$ as the logical upper bound on θ.[18] In the simulations reported below, the strength of the bequest motive is indicated initially by θ, and subsequently by the corresponding bequest/saving ratio (G_{w-x}/C_{w-x-1}), given the other assumed parameters.

The strategy used here to represent the market for private annuities follows Kotlikoff and Spivak (1981) by assuming that private annuities are either available at the actuarially fair price $Q^A = R/(1 - q)$, or, alternatively, not available at all ($A = 0$). The object of the analysis, therefore, is to determine what load factor an individual would be willing to pay in order to have access to market annuities under the

assumed values of all of the model's other parameters. By comparing this critical load factor with the typical load factors summarized in table 2.3, in light of the observation that in fact very few individuals actually purchase life annuities, it is then possible to assess the reasonableness of the assumed values of the model's other parameters—including, in particular, the strength of the bequest motive.

This treatment of the cost of annuities captures the chief implications of the fact that, although the model in principle refers to one-period annuities, in fact the annuities available for purchase are life annuities. Even when an individual makes monthly annuity purchases over time, as in many defined contribution retirement plans, what he is buying each month is an additional life annuity. It is therefore plausible to treat the load factors on annuity premiums shown in table 2.3 in a lump-sum fashion, not as a load to be repeated in every period.

The simplest place to begin is the special case of the model developed in section 2.2 corresponding to the standard life-cycle model with neither bequest motive nor Social Security (which is equivalent to the model in Kotlikoff and Spivak [1981]). Table 2.5 summarizes the results of two simulations of the model, both based on the assumptions that $b_t = 0$ and $S_t = 0$ for all t. As in Kotlikoff and Spivak (1981), the assumed time preference parameter is $\alpha = .99$, and the assumed market interest rate is constant at $R = 1.01$. The assumed coefficient of relative risk aversion is $\beta = 4$.[19] The assumed mortality probabilities are those for a 65-year-old male reported in the 1980 U.S. Life Tables, adjusted as described in section 2.1. For each simulation, table 2.5 shows the relevant solution values for the initial year (age 65) and every fifth year thereafter until the assumed maximum life span (age 110).

Table 2.5   **Simulation Results for the Standard Life-Cycle Model**

	No Annuities		Annuities Available		
Age	Consumption	Bonds	Consumption	Bonds	Annuities
65	4.08%	100.00%	7.20%	.00%	100.00%
70	3.90	84.43	7.20	.00	83.36
75	3.67	69.06	7.20	.00	68.49
80	3.37	54.26	7.19	.00	55.58
85	2.98	40.44	7.19	.00	44.55
90	2.49	28.13	7.19	.00	36.00
95	1.95	17.81	7.19	.00	29.70
100	1.41	9.78	7.19	.00	25.23
105	.96	3.96	7.19	.00	19.61
110	.00	.00	.00	.00	.00

NOTES: Assumed values are $\alpha = .99$, $R = 1.01$, and $\beta = 4$. Calculations are for 65-year-old males. Values are percentages of initial wealth. Annuity values are present expected values.

The first simulation considered within this traditional life-cycle context represents the case in which market annuities (like social annuities here) are unavailable. The individual's only choice is therefore how much to consume in each period, since the unconsumed portion of initial wealth is automatically invested in one-year bonds. Column 1 of table 2.5 shows the optimal age-consumption profile, while column 2 shows the corresponding implied profile of remaining wealth (consisting entirely of bonds), with both sets of values stated as percentages of initial wealth. These simulated values immediately indicate the important contrast between reality and this set of assumptions, in that they show the optimality of a declining age-consumption profile and a sharply declining age-wealth profile—phenomena not observed in available data.

The second simulation within this traditional life-cycle context, summarized in columns 3–5 of table 2.5, shows that simply relaxing the assumption that market annuities are unavailable avoids this strikingly counterfactual result only at the expense of leading to another. This simulation differs from the first one in assuming that individual life annuities are available in the private market at an actuarially fair price $[Q^A = R/(1 - q)]$, and the table reports values for optimal consumption as well as optimal wealth holdings in bonds and annuities, respectively.[20] In this case, the individual's optimal course of action is to hold no bonds at all but to stabilize the age-consumption profile almost completely by investing all wealth in annuities. The implied flat age-consumption profile is roughly consistent with the available evidence, but the implied large demand for market annuities is sharply counterfactual.

Following Kotlikoff and Spivak (1981), it is possible to infer the lump-sum value to the individual, under the conditions assumed in the simulations reported in table 2.5, of having access to a market for actuarially fair life annuities. From equation (14), the initial value of the indirect utility function in each simulation is

$$(18) \qquad J_{w-x}[W_0] = \delta_0 \frac{W_0^{1-\beta}}{1 - \beta} \quad ,$$

for given initial wealth W_0. The proportional increment in the individual's initial wealth required to render the individual as well off, in the sense of an equal initial value of the indirect utility function, in the absence of an annuity market as with such a market is therefore just

$$(19) \qquad M = \left[\frac{\delta_0 | Q^A = R/(1 - q)}{\delta_0 | A = 0} \right]^{\frac{1}{1-\beta}} - 1 \quad ,$$

where $\delta_0|Q^A = R/(1 - q)$ is the value of δ_0 in equation (18) in the simulation with a market for fair annuities, and $\delta_0|A = 0$ is the analogous value in the simulation with no annuity market.

For the pair of simulations reported in table 2.5, the calculation in equation (19) yields $M = 1.13$. Under the conditions assumed in these simulations, therefore, it would still be preferable to put all initial wealth into annuities than to buy none at all, as long as the load factor did not exceed $L^* = 2.13$.[21] Because $L^* = 2.13$ far exceeds the load factors in actual annuity prices calculated on any of the bases reported in table 2.3, and yet in fact there is little individual demand for life annuities, some other assumption common to the two simulations shown in table 2.5 must be importantly counterfactual.

One possibility, of course, is that $\beta = 4$ overstates the coefficient of relative risk aversion. Alternative simulations with a smaller risk-aversion parameter show that this is not the source of the problem, however. Table 2.6 summarizes a pair of simulations that are identical to those reported in table 2.5 except for the new assumption $\beta = 2$.[22] Although the specific age-consumption and age-wealth profiles shown in table 2.6 differ somewhat from those in table 2.5, the same counterfactual implications are again readily apparent. Indeed, because of the lower risk-aversion, the optimal age-consumption and age-wealth profiles when annuities are unavailable decline even more sharply. When actuarially fair annuities are available, it is again optimal to invest all of initial wealth in them. Most importantly, even with lower risk aversion the proportional increment in initial wealth required to render the

Table 2.6   **Simulation Results for the Standard Life-Cycle Model (Low Risk Aversion)**

	No Annuities		Annuities Available		
Age	Consumption	Bonds	Consumption	Bonds	Annuities
65	5.25%	100.00%	7.20%	.00%	100.00%
70	4.81	78.91	7.20	.00	83.36
75	4.26	59.21	7.20	.00	68.49
80	3.58	41.63	7.19	.00	55.58
85	2.79	26.89	7.19	.00	44.55
90	1.96	15.57	7.19	.00	36.00
95	1.20	7.86	7.19	.00	29.70
100	.63	3.33	7.19	.00	25.23
105	.29	1.03	7.19	.00	19.61
110	.00	.00	.00	.00	.00

NOTES: Assumed values are $\alpha = .99$, $R = 1.01$, and $\beta = 2$. Calculations are for 65-year-old males. Values are percentages of initial wealth. Annuity values are present expected values.

individual as well off in the absence of an annuity market as with such a market is still $M = .88$. Even with lower risk aversion, therefore, it would still be preferable to put all of initial wealth into annuities rather than to buy none at all, as long as the load factor did not exceed $L^* = 1.88$—a critical value again well in excess of any of the observed load factors reported in table 2.3.

A further possible explanation for the counterfactual results in both tables 2.5 and 2.6 is that Social Security not only exists but is a large part of wealth for most individuals. Table 2.7 presents a pair of simulations of the more general model developed in section 2.2—first without, and then with, a market for actuarially fair annuities—based on the assumption that actuarially fair Social Security constitutes half of total wealth ($S = .5$).[23] In all other respects, including the absence of a bequest motive, these simulations are analogous to those reported in table 2.5.

The results shown in table 2.7 again exhibit largely the same counterfactual patterns as in table 2.5 and, therefore, suggest that merely allowing for Social Security cannot account for the observed behavior. In the absence of an annuities market, the optimal age-consumption and especially age-wealth profiles decline fairly sharply, although not so much as in table 2.5. If actuarially fair annuities are available, it is optimal to invest all of total wealth other than Social Security in purchasing them. Most importantly, even with a sizeable role for Social Security, the proportional initial wealth increment required to render the individual as well off without as with a private annuity market is $M = .33$. Hence it would still be preferable to put all of total wealth other than Social Security (that is, one-half of total wealth) into private

Table 2.7   Simulation Results for the Model Including Social Security

	No Annuities			Annuities Available			
Age	Consumption	Soc. Sec.	Bonds	Consumption	Soc. Sec.	Bonds	Annuities
65	5.81%	50.00%	50.00%	7.20%	50.00%	.00%	50.00%
70	5.68	41.62	41.62	7.20	41.68	.00	41.68
75	5.51	33.83	33.83	7.20	34.25	.00	34.25
80	5.28	26.79	26.79	7.20	27.79	.00	27.79
85	4.97	20.57	20.57	7.19	22.27	.00	22.27
90	4.57	15.42	15.42	7.19	18.00	.00	18.06
95	4.06	11.30	11.30	7.19	14.85	.00	14.85
100	3.50	7.99	7.99	7.19	12.61	.00	12.61
105	2.92	4.73	4.73	7.19	9.80	.00	9.80
110	.00	.00	.00	.00	.00	.00	.00

NOTES: Assumed values are $\alpha = .99$, $R = 1.01$, $\beta = 4$, and $S = .5$. Calculations are for 65-year-old males. Values are percentages of initial wealth. Annuity values are present expected values.

annuities rather than buy none at all, as long as the annuity load factor did not exceed $L^* = 1.66 = 1 + (.33/.5)$—again far greater than the load factors actually observed. An analogous simulation based on the lower risk-aversion value of $\beta = 2$ (not shown in the table) produces almost identical results, with $M = .31$ and a critical load factor $L^* = 1.62$.

The potential explanation for the observed behavior that is of greatest interest in the context of this paper is that, in general, people may value not just their own consumption but also bequests. Table 2.8 presents three further pairs of simulations of the fully general model developed in section 2.2, in each case based on the same assumptions as in table 2.7 (including the prominent role for Social Security) and, in addition, a positive bequest motive. The first of these three pairs of simulations assumes the bequest motive $\theta = 2$, which is quite modest given the stock-flow dimension of θ and the model's use of an annual time unit, and $\beta = 4$. The second pair of simulations assumes $\theta = 8$, the third $\theta = 24$.

The simulation results reported in table 2.8 show that even a very modest bequest motive is sufficient to eliminate one of the importantly counterfactual aspects of the earlier simulations. In particular, because of the bequest motive it is no longer optimal to invest all of total wealth other than Social Security in private annuities. The fraction of wealth invested in bonds varies positively with the strength of the bequest motive, but even $\theta = 2$ is sufficient to make optimal bond holdings neither zero nor trivially small. Moreover, optimal bond holdings do not decrease (until the final year they actually increase slightly) with age. Hence the general model, with even a modest positive bequest motive, is consistent with observed behavior in implying an approximately flat age-wealth profile for the part of wealth held in nonannuity form.

By contrast, the results for all three pairs of simulations shown in table 2.8 continue to be counterfactual in implying that, when private annuities are available, it is optimal to use a large fraction of total wealth other than Social Security to purchase them. Further analysis, however, indicates that here the load factor in annuity pricing is potentially very important.

For the weak bequest motive $\theta = 2$, the proportional initial wealth increment required to render the individual as well off without as with a private annuity market is $M = .22$. The critical load factor necessary to make buying no annuities at all preferable to investing 41 percent of initial wealth in annuities is there $L^* = 1.53 = 1 + (.22/.41)$, again above the observed load factors reported in table 2.3 for the mean premiums charged by the ten largest insurers, regardless of the mortality probabilities and the interest rates used in the calculations.

Table 2.8   Simulation Results for the General Model

Age	No Annuities			Annuities Available			
	Consumption	Soc. Sec.	Bonds	Consumption	Soc. Sec.	Bonds	Annuities
$\theta = 2$							
65	5.80%	50.00%	50.00%	6.72%	50.00%	8.56%	41.44%
70	5.68	41.64	41.64	6.72	42.31	8.57	33.74
75	5.51	33.87	33.87	6.72	35.42	8.60	26.83
80	5.28	26.86	26.86	6.71	29.44	8.66	20.78
85	4.97	20.69	20.69	6.71	24.31	8.78	15.54
90	4.57	15.61	15.61	6.71	20.33	8.92	11.41
95	4.08	11.61	11.61	6.71	17.37	9.08	8.29
100	3.54	8.54	8.54	6.71	15.25	9.22	6.03
105	3.04	5.79	5.79	6.71	12.60	9.62	2.98
110	.00	.00	3.18	.00	.00	7.01	.00
$\theta = 8$							
65	5.78%	50.00%	50.00%	6.54%	50.00%	11.76%	38.24%
70	5.66	41.67	41.67	6.53	42.54	11.76	30.79
75	5.49	33.95	33.95	6.53	35.86	11.78	24.08
80	5.27	26.99	26.99	6.53	30.06	11.85	18.21
85	4.96	20.90	20.90	6.53	25.08	11.97	13.11
90	4.58	15.93	15.93	6.53	21.20	12.11	9.09
95	4.12	12.10	12.10	6.53	18.31	12.26	6.05
100	3.66	9.30	9.30	6.53	16.24	12.39	3.85
105	3.29	6.94	6.94	6.53	13.64	12.76	.88
110	.00	.00	5.57	.00	.00	9.65	.00
$\theta = 24$							
65	5.76%	50.00%	50.00%	6.35%	50.00%	15.01%	34.99%
70	5.64	41.75	41.75	6.35	42.78	14.99	27.79
75	5.47	34.12	34.12	6.35	36.31	15.01	21.31
80	5.25	27.27	27.27	6.35	30.69	15.06	15.62
85	4.96	21.31	21.31	6.35	25.85	15.17	10.68
90	4.60	16.55	16.55	6.35	22.09	15.30	6.79
95	4.22	13.02	13.02	6.35	19.28	15.42	3.85
100	3.89	10.65	10.65	6.34	17.24	15.51	1.73
105	3.76	8.90	8.90	6.34	14.71	15.83	−1.12
110	.00	.00	9.54	.00	.00	12.33	.00

NOTES: Assumed values (other than θ as shown) are $\alpha = .99$, $R = 1.01$, $\beta = 4$, and $S = .5$. Calculations are for 65-year-old males. Values are percentages of initial wealth. Annuity values are present expected values.

For $\theta = 8$, the initial wealth increment required to render the individual as well off without as with a private annuity market is only $M = .18$, so that the critical load factor that would make buying no annuities at all preferable to investing 38 percent of initial wealth in annuities is $L^* = 1.47$. As table 2.3 shows, this load factor is approximately equal to that charged on average by the largest ten insurers if the underlying present expected value calculation relies on general population mor-

tality probabilities and the interest rate on corporate direct placements. Nevertheless, it still exceeds the implied load factor confronting an individual who knows that his mortality probabilities are characteristic of other annuity purchasers, or whose opportunity cost of funds is the government bond yield (or who searches for the lowest available premium).

Finally, for $\theta = 24$, the initial wealth increment required to render the individual as well off without as with a private annuity market is $M = .14$. Hence the critical load factor that would make buying no annuities at all preferable to investing 35 percent of initial wealth in annuities is $L^* = 1.40$, about in the middle between the actual load factor based on general population mortality probabilities and the direct placement yield and the actual load factor based on alternative assumptions.

Under some sets of plausible assumptions, therefore, importantly including a positive bequest motive, the actual load factor included in the premiums on individual life annuities sold in the United States is sufficient to make people prefer buying no annuities at all over buying the amount that would be optimal if annuity prices were actuarially fair. Although this finding is hardly without interest, since it indicates a joint role for the bequest motive and for annuity load factors in explaining the observed behavior, it still does not fully explain the fact that almost no one buys *any* individual life annuities. Nothing forces people to choose between buying either the amount of annuities that would be optimal at actuarially fair prices or buying none at all. Hence showing under what conditions people would prefer no annuities at all to the amount they would purchase at actuarially fair prices still does not establish the conditions under which they would not buy some amount that is significant albeit less than the actuarially fair optimum. For the assumed values of α, R, β, and S underlying the simulations reported in table 2.8, for example, and for an assumed annuity load factor of $L = 1.40$ (the critical value for $\theta = 24$), the strength of the bequest motive required to make the individual indifferent between purchasing private annuities equal to 1 percent of initial wealth (including Social Security) and purchasing none at all is $\theta = 343$—far above $\theta = 24$, yet still below the logical upper bound for θ given the assumed parameter values. Table 2.9 summarizes the results of analogous simulations based on various values of S, R, and β, reporting in each case the value of θ that renders the individual just indifferent between investing 1 percent of initial wealth (including Social Security) in private annuities and purchasing none at all when the load factor is $L = 1.40$.[24]

Both because the quantitative importance of bequests in overall saving is a question with major implications for both positive and normative

Table 2.9   **Bequest Motive Strength Needed to Eliminate Annuity Purchases**

	$S = .4$	$S = .5$	$S = .6$
$R = 1.01$			
$\beta = 2$	18	9	4
$\beta = 3$	169	58	18
$\beta = 4$	1488	343	74
$R = 1.04$			
$\beta = 2$	10	5	3
$\beta = 3$	66	24	7
$\beta = 4$	419	105	22

NOTES: Values shown are for θ, just sufficient to eliminate initial annuity purchases equal to 1 percent of initial wealth. Assumed values (other than S, R, and β as shown) are $\alpha = .99$ and $L = 1.40$. Calculations are for 65-year-old males.

issues,[25] and because there is little other way to evaluate the plausibility of values of θ within the logically admissible range, it is interesting to see just how large these results suggest that the typical bequest should be. Table 2.10 shows, for each of the combinations of parameter values considered in table 2.9, and in each case for the value of the bequest motive parameter θ (as shown in table 2.9) needed to render the individual indifferent between investing 1 percent of initial wealth in private annuities and purchasing none at all when the load factor is $L = 1.40$, the corresponding bequest/consumption ratio (G_{w-x}/C_{w-x-1}). For the most part these estimates are closely bunched, despite the wide variation in the underlying parameter values, typically indicating a bequest equal to 2–4 times the final year's consumption. Especially for the lower end of the range, these estimates appear to be empirically plausible.[26]

Table 2.10   **Ratio of Expected Bequest to Final Period Consumption**

	$S = .4$	$S = .5$	$S = .6$
$R = 1.01$			
$\beta = 2$	3.40	2.40	1.60
$\beta = 3$	4.77	3.34	2.25
$\beta = 4$	5.56	3.85	2.63
$R = 1.04$			
$\beta = 2$	2.57	1.82	1.29
$\beta = 3$	3.52	2.51	1.67
$\beta = 4$	4.08	2.89	1.95

NOTES: Values shown are ratios of expected bequest to final period consumption, given θ just large enough to eliminate initial annuity purchases equal to 1 percent of initial wealth. Assumed values (other than S, R, and β as shown) are $\alpha = .99$ and $L = 1.40$. Calculations are for 65-year-old males.

Given the respective roles of S, R, and β in affecting the demand for annuities in the model developed in section 2.2, their corresponding roles here in determining the strength of bequest motive necessary to eliminate that demand (for a given load factor) is straightforward. The demand for individual life annuities is smaller as Social Security is more important, smaller as the rate of return is higher, and greater as people are more risk averse. Hence the bequest motive implied by the fact that few people buy individual life annuities is weaker as Social Security is more important, weaker as the rate of return is higher, and stronger as people are more risk averse.

In sum, the results shown in tables 2.9 and 2.10 indicate that for plausible sets of assumed parameter values the combination of an annuity load factor in the observed range (see again table 2.3) and an empirically plausible positive bequest motive in the theoretically admissible range for altruistic bequests is sufficient to explain the absence of purchases of individual life annuities. This finding lends strength to the view that desired bequests are an important element in consumption-saving behavior. Moreover, when Social Security is less important and people are more risk averse, the bequest motive assumes an especially large role in explaining why so few people buy annuities. Under these circumstances the indicated bequest is at least 4 times final consumption.

2.4 Conclusion and Further Thoughts

The observed reluctance of most individuals in the United States to buy individual life annuities, and the concommitant approximately flat average age-wealth profile, stand in sharp contradiction to the standard life-cycle model of consumption-saving behavior. The analysis in this paper lends support to an explanation for this phenomenon based on the interaction of an intentional bequest motive and annuity prices that are not actuarially fair.

Premiums charged for individual life annuities in the United States do include a load factor of 32¢–48¢ per dollar, or 18¢–33¢ per dollar after allowing for adverse selection, in comparison to actuarially fair annuity values. Load factors of this size are not out of line with those on other familiar (and almost universally purchased) insurance products. Simulations of an extended model of life-cycle saving and portfolio behavior, allowing explicitly for uncertain lifetimes and Social Security, show that the load factor charged would have to be far larger than this to account for the observed behavior in the absence of a bequest motive. By contrast, the combination of a load factor in this range and a positive bequest motive can do so for some plausible values of the assumed underlying parameters. Moreover, if this combination of factors is leading elderly individuals to avoid purchasing life annuities, it implies a typical bequest that is fairly large in comparison to their annual consumption.

As the authors' earlier work has already emphasized, caution is appropriate in relying on these conclusions without further research. Although the model used here does generalize the standard life-cycle model in several potentially important ways, it still excludes *a priori* a variety of further possible explanations for the observed behavior.[27] First, many people at least say that they choose stable age-wealth profiles, rather than either buying annuities or simply consuming out of wealth, not because of mortality considerations but from fear of the consequences of catastrophic illness. Second, while the analysis here follows the recent literature by implicitly working in real terms, the individual life annuities available in U.S. markets guarantee specified nominal payments. Third, the analysis here does not allow for several more complex kinds of possible interactions within families, including, for example, either nonaltruistic ("manipulative") bequests or intra-family risk sharing.[28] Finally, in contrast to the reliance here (and in just about all of the available literature on the subject) on the standard theory of expected utility maximization, there is evidence that, especially when the prospect of rare events is involved, individuals systematically overweight the probability of rare events.[29]

These further possible explanations for the fact that few people purchase individual life annuities are subjects for future research.

Appendix

Average Load Factor for Life Annuity Premiums

Premium	General Population		Annuity Purchasers	
	Government Bonds	Direct Placements	Government Bonds	Direct Placements
65-Year-Old Females:				
Ten-Largest Mean	1.26	1.42	1.18	1.35
Ten-Largest High	1.34	1.52	1.26	1.44
Ten-Largest Low	1.18	1.33	1.11	1.26
Complete-Sample Low	1.15	1.30	1.08	1.23
70-Year-Old Males:				
Ten-Largest Mean	1.30	1.43	1.16	1.29
Ten-Largest High	1.37	1.51	1.22	1.36
Ten-Largest Low	1.24	1.36	1.11	1.23
Complete-Sample Low	1.18	1.30	1.06	1.17
75-Year-Old Males:				
Ten-Largest Mean	1.29	1.40	1.15	1.26
Ten-Largest High	1.35	1.47	1.21	1.33
Ten-Largest Low	1.23	1.34	1.10	1.21
Complete-Sample Low	1.15	1.25	1.03	1.13

Notes

1. See, for example, Mirer (1979) and Hubbard (1983).

2. The Retirement History Survey indicates that only 2 percent of the elderly population own individual annuities of any sort; see, for example, Friedman and Sjogren (1980).

3. See Friedman and Warshawsky (1985).

4. This dispersion probably reflects search costs; see, for example, Pratt et al. (1979). Alternatively, it could reflect different marketing choices by different insurers.

5. The exact expression used is

$$\sum_{t=1}^{w-x-1} (1 + r)^{-t}p_{xt},$$

where w is the assumed maximum length of life (here taken to be 110 years), x is the age at the date of issue (here 65 years), r is the relevant interest rate, and p_{xt} is the probability that an individual of age x at the time $t = 1$ will survive to any year $t > 1$. These annual calculations are then converted to a monthly basis.

6. The calculations rely on the 1970 tables for years 1968–70, on the 1980 tables for years 1980–83, and on both tables (weighted) for the years 1971–79. See Faber (1982) for a complete discussion of the U.S. Life Tables, and Wetterstrand (1983) for a discussion of improvements in mortality probabilities. In the calculations for females summarized in the appendix, the corresponding adjustment factors are .98 and .99.

7. See Rothschild and Stiglitz (1976) for an analysis of the principles underlying this kind of adverse selection.

8. See Society of Actuaries (1983) for the actual company experience tables.

9. This latter comparison is the relevant one from the perspective of the insurer. Informal discussions with the insurers suggest that, of this 33¢ per dollar, roughly 11¢ reflected transaction costs (narrowly defined), 8¢ taxes, and 14¢ return to capital at risk. This breakdown is at best only suggestive, however.

10. The small probability that a 65-year-old male will survive past age 100 is simply ignored for purposes of this comparison.

11. This section draws heavily on Friedman and Warshawsky (1985); see that paper for additional details and references to relevant literature.

12. In fact, Fischer's model is really an annuity model, despite his application of it to the demand for life insurance.

13. Probabilities p_t and q_{t+1} are, of course, conditional on initial age x. Writing them as $p(x)_t$ and $q(x)_{t+1}$ would be appropriate but would clutter an already cumbersome notation. Conditionally on x is to be understood, here and below.

14. In a more general context it would also be necessary to take account of labor income. The focus of this paper, however, is on the elderly retired population.

15. As in Fischer (1973), the assumption of one-period annuities makes the analysis tractable. The annuities actually available for purchase in the United States are instead life annuities.

16. The model as written here imposes no nonnegativity constraint on choice parameter A—that is, it does not explicitly preclude short sales of annuities. For most reasonable values of the given parameters, however, large short sales are not optimal anyway. If they were, imposing a nonnegativity constraint in solving the model would be straightforward.

17. The time profile in equation (17) is from Fischer's (1973) appendix table A2, extended to age 110. Reasoning analogous to that underlying Abel's (1984) model of life insurance markets suggests that the results could be very sensitive to whether the value of bequests is rising or falling with t. In particular, a sufficiently negatively sloped bequest motive can, under some circumstances, give rise to a negative demand for annuities. The after-age-65 portion of Fischer's time profile, used here, makes b_t decline approximately in step with α^t for $\alpha = .99$ as assumed below.

18. Friedman and Warshawsky (1985) indicate other circumstances under which θ would not be bounded. See also Abel and Warshawsky (1987) for a theoretical analysis in which the "joy of giving" bequest motive assumed in this chapter is interpreted as the reduced form of an altruistic bequest motive to derive a relation between the value of the altruism parameter and the value of the "joy of giving" parameter.

19. Grossman and Shiller (1981) found evidence consistent with a relative risk-aversion coefficient roughly equal to 4. Bodie et al. (1985) also used this value.

20. Wealth held in annuities is valued at the present expected value.

21. In other words, any approximately flat consumption stream exceeding 3.3 percent of initial wealth each year (7.20 percent as in column 3 of table 2.5, divided by 2.13) would be preferable to the declining stream shown in column 1.

22. Friend and Blume (1975) found evidence indicating a relative risk-aversion coefficient roughly equal to 2.

23. One-half is about the fraction of total wealth constituted by Social Security and private pensions for the average retired elderly individual in the United States; see the evidence provided by Kotlikoff and Smith (1983, table 3.7.19, p. 127).

24. The results shown in table 2.9 follow from searching over θ, given the assumed values of the other parameters. For purposes of comparison to the analysis by Kotlikoff and Spivak (1981), the θ values corresponding to $\beta = 1.25$ are (in order, from top to bottom in the table) 3, 2, 1, 2, 1.5, and 1.

25. See, for example, Kotlikoff and Summers (1981).

26. For males, Menchik and David (1982, table 1, p. 193) reported a median bequest equal to 2.1 times annual median labor income (defined as one-fortieth of total lifetime labor earnings), and a mean bequest equal to 4.2 times mean annual labor income.

27. See Friedman and Warshawsky (1985) for a brief discussion of several of these other possible explanations.

28. See, for example, Bernheim et al. (1985) on nonaltruistic bequests and Kotlikoff and Spivak (1981) on intrafamily risk sharing.

29. See, for example, Kahneman and Tversky (1979).

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Comment   R. Glenn Hubbard

It is difficult to think of more important items on the research agendas of public finance and macroeconomics than explaining personal saving behavior (especially the role of intergenerational transfers) and the impacts of social insurance programs on national saving and individual welfare. With respect to both areas, it is now well recognized that, for all its theoretical clarity, the life-cycle model has not tested out very well vis-à-vis predictions about the shape of the wealth-age profile or the impact of pensions on the level of nonpension wealth. While there is evidence that the model, in its most general context of forward-looking intertemporal optimization, is not at variance with the observed saving behavior of most households, many studies have concluded that there exists a nontrivial fraction of households for which the model does not appear to be an adequate description of saving behavior.

One element missing from most of these studies is the wealth allocation choice between assets which can be transferred to heirs upon death (e.g., stocks, bonds, or life insurance proceeds) and those which cannot (e.g., annuities). Analyses of the impact of the introduction of Social Security are typically conducted either under the assumption of a perfect private market in life annuities—in which annuity prices are actuarially fair—or complete market failure in the provision of annuities. Such polar extremes are hardly innocuous.[1]

A key advance of the paper by Benjamin Friedman and Mark Warshawsky is its attempt to quantify the extent to which annuity contracts are "unfairly" priced (in an actuarial sense). After all, the empirical finding that wealth-age profiles do not decline significantly in old age is consistent with the basic life-cycle model (when lifetime is uncertain and there is no bequest motive) only if there are no annuity markets. As Friedman and Warshawsky point out, there are well functioning life annuity markets in the United States, but very few individuals (in the present or the past) have chosen to purchase them.

Consistent with the organization of the Friedman-Warshawsky paper, my comments can be divided into three parts: (i) pricing life annuities and computing "load factors," (ii) modeling the impact of different

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1. For example, if the prices of market annuity contracts were actuarially fair and there were no restrictions on the quantities purchased, then exogenous increases in mandatory Social Security annuities would displace annuity purchases with no effect on bequeathable nonpension wealth. With no market annuities, such increases would displace marketable nonpension wealth. For discussions of implications of these assumptions for models of individual saving, see Bernheim (1984), Hubbard (1987), and Sheshinski and Weiss (1981).

assumptions about annuity pricing on individual saving, and (iii) considering the implications of the modeling approach for more general discussions of individual saving behavior and public policies toward retirement saving.

First, Friedman and Warshawsky have made a real contribution in calculating the load factors associated with the purchase of life annuity contracts in the United States. Their comparison of the present expected values of identical annuities for 65-year-old males for the "general population" and "annuity purchasers" reveals a premium most likely traceable (as they note) to the classic adverse selection problem.[2] That is, annuity prices are not actuarially fair for those individuals who choose to purchase life annuity contracts.[3]

In their table 2.3, Friedman and Warshawsky present the mean load factors under various assumptions. These implied charges are substantial, ranging up to 55 percent for individuals in the "general population." In a particularly interesting calculation, they isolate "the effect of adverse selection" by subtracting the mean load factor for individuals actually purchasing annuities from that for individuals in the general population. This calculation is not conclusive, however, as theoretical treatments of adverse selection have seldom found pure price equilibria to be optimal. That is, quantity rationing may occur. Whether such rationing is in fact important in the market for life annuities is an empirical question, and one worthy of mention in the paper.

Second, the core of the Friedman-Warshawsky paper is an extension of Fischer's (1973) model of the demand for life insurance to examine the extent to which the combination of actuarially unfair market annuities, mandatory social annuities (e.g., Social Security), and bequest motives can explain observed patterns in annuity purchases and consumption in old age. The extension of the dynamic programming model is straightforward, and the authors use the solution expressions as the basis for a simulation model.

Their modeling strategy is clever, namely, to calculate the maximum load factor an individual would be willing to pay to purchase market

2. Recent theoretical contributions to the literature on adverse selection as applied to annuities include Eckstein, Eichenbaum, and Peled (1985) and Judd (1984). The Judd paper in particular points out the potential importance of general equilibrium analysis—since the annuity market equilibrium affects incentives to save, the level of savings and the supply of capital will be affected, altering the supply of goods and the real interest rate. He shows that private insurance markets afflicted by adverse selection generally cannot internalize this spillover into the goods market, the competitive equilibrium will in general not be efficient, and that some sort of compulsory contract will be Pareto-improving.

3. In a companion paper, Friedman and Warshawsky (1985) present information on yield differentials between life annuities and marketable securities; as with the load factors, these differentials are substantial.

annuities and compare that with the actual load factors calculated earlier.[4] Because we do not observe many actual annuity purchases, such a comparison can be used to justify the appropriateness of other assumed parameter values (particularly that for the strength of the "bequest motive"). The results in table 2.4 illustrate (not surprisingly) that the no-annuities-available and market-only-annuities cases yield counterfactual results for consumption-age and wealth-age profiles in old age.

More reasonable results are obtained when Social Security annuities are introduced (based on the assumption that Social Security wealth constitutes half of total wealth). I do, however, have two reservations about the choice of parameter values. First, the values used for the annual rate of time preference and market rate of interest (1 percent for each) seem very low. We are not told how sensitive the simulation results are to this choice of parameter values (in particular with respect to the more likely case wherein the interest rate exceeds the rate of time preference), though an alternative value for the market interest rate is reported later in table 2.9. Second, while the assumption that Social Security wealth and nonpension wealth are roughly equal may be true on average, it is certainly not true for high-income and wealthy
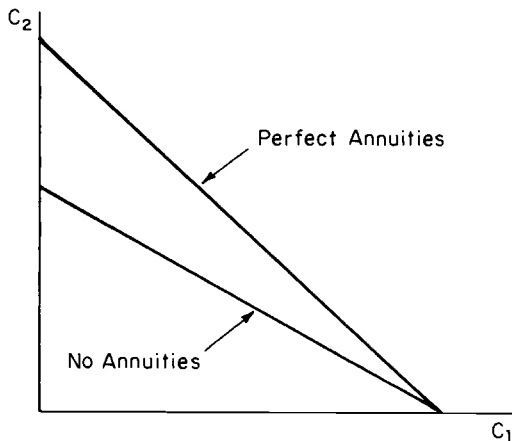
Fig. 2.2   Intertemporal budget constraints implied by no-annuity and perfect-annuity cases.

4. Friedman and Warshawsky consider the load factors on market annuities as fixed, "upfront" fees. An alternative approach would be to consider a continuous rate of return on annuities which is less than the actuarially fair return. Consider for simplicity a two-period model with consumption (C_1, C_2) in the two periods. The intertemporal budget constraints implied by the no-annuity and perfect-annuity cases are illustrated in figure 2.2. The availability of annuities expands the budget set, and compensating or equivalent variation calculations can be made. Actuarially unfair annuities could be considered by drawing an intermediate budget line between the two sketched above. Friedman and Warshawsky (1985) do focus on yield differentials.

individuals, for whom bequest motives are most frequently observed (Menchik and David 1983).[5] I will discuss this point further later.

Since across all cases prior to the introduction of bequests, the "critical load factors"calculated by the model are greater than those actually observed, there remains a puzzle as to why individuals are not more active participants in private annuity markets. The logical next step is to consider the extent to which planned bequests can rationalize the model's predictions with observed annuity purchases. Bequests are indexed by a parameter θ, which represents the ratio of a bequest to current annual consumption.

Table 2.8 is most interesting in this respect, in that the inclusion of a bequest motive, by lowering the difference between critical and observed load factors, removes the feature of earlier simulations that it is optimal to invest all non-Social-Security wealth in market annuities. While this modification does not yet explain the fact that almost no annuities are purchased by anyone in the United States, the authors' table 2.9 summarizes the model's implications very well by calculating the strength of the bequest motive (again indexed by the parameter θ) required to eliminate market annuity purchases.

For parameter values that seem sensible on average—say, an interest rate of 4 percent, a ratio of Social Security wealth to total wealth of 0.5, and a coefficient of relative risk aversion of 2—a relatively modest bequest motive (θ = 5) is required. This still seems troublesome, however, for any claim that a bequest motive per se is all that is needed to explain observed behavior in private annuity markets. Plausible increases in relative risk aversion, reductions in the market interest rate, or declines in Social Security wealth relative to total wealth[6] would require implausibly high bequest motives to eliminate annuity purchases.

These results are even more pronounced in the calculations of the impact of the yield differential between annuities and government bonds. That differential on average is sufficient to eliminate annuity purchases by 65-year-old males (given a coefficient of relative risk aversion of 4 and a ratio of "social annuities" to total wealth of 0.5) irrespective of any bequest motive. For lower values of the coefficient of relative risk aversion (e.g., 2), the combination of even lower differentials and plausible bequest motives can explain the absence of market annuity purchases.

Finally, I would like to devote the remainder of my remarks to three factors that may shed further light on the stimulating findings in the

5. More subtly, the inclusion of Social Security annuities might have been better modeled by using the entire life cycle to capture the effects of Social Security on preretirement consumption and wealth accumulation. For example, these effects would not in general be invariant to the way in which the system is financed.

6. Again, such a decline in Social Security wealth relative to total wealth is perfectly plausible, given that it is wealthy individuals—with comparatively little Social Security wealth—who are most likely to leave a bequest.

Friedman-Warshawsky paper. The first relates to the definition of annuities. While the annuities described in the paper are real annuities, annuities available in the market are subject to inflation risk.[7] Moreover, when a model of the entire life cycle is considered, the lack of fungibility of annuities (e.g., because of problems in intertemporal reallocation of resources and liquidity constraints) may be important. Other "contracts" not considered in the analysis might act as proxies for market annuities, namely, private pension annuities[8] (Hubbard 1987) or implicit intrafamily contracting arrangements (Kotlikoff, Shoven, and Spivak 1987). These added considerations could account for some remaining annuity purchases in the presence of a bequest motive in table 2.8, or for a lower bequest motive required in table 2.9 to "eliminate" market annuity purchases (by augmenting "exogenous" annuities relative to total wealth).

Second, two modeling considerations might increase the robustness of the results. Solving for steady-state values of national saving and individual welfare would permit comparison of the impacts of alternative "social annuity" policies on individual welfare given different assumptions about bequest motives. In addition, it would be useful to know the size of bequests relative to life-cycle resources implied by the values in table 2.9. Such calculations could shed additional light on the importance of intergenerational transfers in accounting for the size of the capital stock.

Finally, and most important, the ultimate relevance of the simulation exercises depends on the extent to which lifetime uncertainty is the dominant form of uncertainty present in individual consumption optimization problems. I am reminded of the cry by Maggie in Tennessee Williams's *Cat on a Hot Tin Roof:* "You can be young without money, but you can't be old without money."

Uncertainty in old age is a general phenomenon. While there is no doubt that uncertain longevity is important, one can easily imagine that uncertainty over health or disability is a significant worry for the elderly.[9] That is, "uncertainty" broadly defined may account for stable wealth-age profiles in retirement. For example, Friedman and Warshawsky point out that market annuities have never been very significant in the United States, yet the past two generations of Americans have witnessed a dramatic expansion in the coverage and generosity of the Social Security and private pension systems. Extrapolating from

7. It is not clear, for example, that individuals prefer real (market) annuities given the existing structure of Social Security benefits (Feldstein 1983).

8. In addition, private pension annuities receive favorable tax treatment relative to market annuities.

9. See the discussion in Diamond and Hausman (1984).

table 2.9, an implausibly high bequest motive would have been required to eliminate life annuity purchases.[10]

Recent papers on the influence of lifetime uncertainty on national saving and this contribution by Friedman and Warshawsky suggest the importance of recognizing and modeling the effects of uncertainty on individual saving decisions and the size of the capital stock. Measuring the size of uncertainty saving may help to reconcile the life-cycle model with observed saving behavior and to reevaluate the importance of intergenerational transfers for the size of the capital stock (as in Kotlikoff and Summers 1981). Such an approach will facilitate welfare analyses of social insurance programs and complementary private programs. Finally, data permitting, cross-country comparisons of the influence of social and private annuities on the capital stock would be useful in empirically disentangling motivations for saving behavior.

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10. Problems remain, however. Empirical studies finding "oversaving" among some groups (e.g., the elderly) have also found a nontrivial fraction of the population whose saving is completely "inadequate" by usual life-cycle standards.

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