CHAPTER 12

Reliability of the Estimates

For reasons indicated in Chapter 11, the reliability of the estimates for the numerous cells that make up the income total year by year cannot be revealed clearly in a classification based on the character of the underlying data and procedures. To get a quantitative measure the margins of error must be evaluated by those who, being familiar with the estimates, dare to surmise how far wrong they may be. In the first two sections of this chapter we describe in detail the procedure by which the investigators who participated actively in the preparation of the estimates and hence know them best evaluated their margins of error. The evaluation concerns the margin of error that could be assigned to each total for the various cells making up national income in each year and cannot be applied *directly* to the changes from year to year or to differences among estimates for various cells. We discuss in the third section the margins of error that could be assigned to the estimated *changes* from year to year, or to *differences* among totals for the various important industrial divisions or types of income within national income.

1 *Reliability of the Totals—The Procedure*

In general, the procedure consists in classifying, by the size of the error margins, the estimates within cells formed by the combination of the allocation by industrial source with the classification by type of income. The industrial divisions were:

1 Agriculture
2 Mining, total
3 Anthracite coal
4 Bituminous coal
5 Metal mining
6 Oil and gas
7 Other mining
8 Manufacturing, total
9 Food & tobacco
10 Textile & leather
11 Construction materials & furniture
12 Paper
13 Printing
14 Metal mfg.
15 Chemical
16 Miscellaneous & rubber
17 Construction
18 Electric light & power
19 Manufactured gas
20 Steam railroads, Pullman, & express
21 Street railways
22 Water transportation
23 Pipe lines
24 Telephone
25 Telegraph
26 Trade
27 Banking
28 Insurance
29 Real estate
30 Service, total
31 Professional
32 Personal
33 Domestic
34 Miscellaneous service
35 Government, total
36 Federal
37 State
38 County
39 City incl. public education
40 Miscellaneous

The type of income and employment categories were:

a Wages
b Salaries
c Employee compensation
d Entrepreneurial withdrawals
e Dividends
f Interest
g Net savings, corporate & government
h Net savings, entrepreneurial
i Net income originating
j Wage earners
k Salaried employees
l Total employees
m Entrepreneurs

The 40 industry and 13 type of income or employment categories make up a total of 520 cells for each year. Of course, for no year in the period were estimates available for all 520 cells; and some cells are interdependent in the sense that the estimate for one is a sum of estimates for several others. But all the available estimates, both components and totals, were classified, since the margin of error for the total is not necessarily the sum of the margins of error for the components.

The various estimates in these industry type of income or employment cells were then classified under one of four categories by the size of the probable maximum error: [1]

1 The margin classes used originally in these classifications were from 3 to 7, 8 to 12, 13 to 27, and 28 to 52 (with central values of 5, 10, 20, and 40). A subse-

I An error of 5 to 10 per cent, with 7.5 as the average. If the margin appeared to be less than 5 per cent, the estimate was put in this category.

II An error of 11 to 20 per cent, with 15 as the average.

III An error of 21 to 40 per cent, with 30 as the average.

IV An error of 41 to 80 per cent, with 60 as the average. If the margin appeared to be greater than 80 per cent, the estimate was put in this category.

The margin of error was judged on the basis of what the estimate was, rather than what it should have been conceptually. For example, our estimate of net dividends originating in each industrial division was classified by its maximum error as a measure of net dividends, not of dividends paid directly to individuals by enterprises in the industrial division. But we did assign a larger margin of error to estimates of dividends, interest, and business savings originating in the various industrial divisions because of the distortion in the industrial allocation caused by consolidated returns.

Our classification was based upon maximum errors, not minimum or average errors; i.e., we were concerned with how large the error could be. An error of 5 per cent meant that this was the maximum error to which the estimate was likely to be subject. The minimum error for each estimate is zero and the average error too indefinite to estimate.

The setting up of error classes, with fairly wide class limits, might be interpreted in two ways: it may be thought: (1) that we know precisely what the probable maximum error for each estimate is but, for the sake of economy in presentation, forbear to give the exact figures, grouping them into four classes; or (2) that we do *not* know precisely what the probable maximum error is, but can approximate it within certain limits. If we put an estimate in Class I, we mean that its probable maximum error is between 1 and 10 per cent.

Of these two interpretations of our procedure the second

quent upward revaluation of error margins resulted in the classes above; see below.

is valid. We do *not* know the precise percentage at which to set the probable maximum error for each estimate; and we established error classes in order to allow a range for the maximum error. In subsequent calculations, it is true, we assume a single central value for each class; but this is a simplification, needed to arrive at compact results. It does not mean that for every estimate in Class I the most probable value of the maximum error is 7.5 per cent. On the contrary, there are appreciable differences in the relative accuracy of estimates in Class I (from a maximum error of 1 or 2 to 10 per cent), and especially in the upper error classes with their wide range between class limits. But for purposes of calculation a single central value is assigned to each class, and the probable maximum error of the combined estimates in the class is assumed to be this central value.

Since the ranges indicated above represent both positive and negative errors, estimates included in Class I, for example, may be from 1 to 10 per cent greater or less than the true value. This would seem to indicate that the maximum range of Class I is from — 10 to + 10. As a matter of fact the maximum errors are in one direction for a majority of the estimates. Thus the error in our estimate of employee compensation for manufacturing, in Census years, is likely to be negative, i.e., our estimate is probably always somewhat less than the true one. But it would bring us upon too uncertain ground were we to distinguish positive and negative errors, and try to set up classes accordingly.

The estimates were classified by the four error classes independently by each of the three investigators who participated most directly in the study and were most familiar with sources and methods.[2] It was thought that a combination of independent classifications would reduce any bias that might result from pessimism or optimism of the individual investigator. The three evaluations of estimates by classes of probable

[2] Lillian Epstein, Elizabeth Jenks, and the author.

maximum errors—all for identical estimates in identical industry type of income or employment cells—were used in all further calculations of error margins. No attempt was made to reconcile them or change them in any way: it was thought best to use them in the form that corresponds most closely to their real meaning—appraisals by individual investigators familiar with the estimates.

The procedure was followed with one significant exception: we raised all margins of error by one-half [3] because we found that all three investigators tended to underestimate the errors attaching to the results of their labors. The original sets of evaluations were based on the working tables used in preparing the estimates and the detailed description of methods and data in Part Four, but without consulting the comparisons in Chapter 10 or using the tests of interpolation and extrapolation procedures discussed in Chapter 11. Reference to these materials indicated that the margins of error, as originally calculated, were, on the whole, too low, but the relative differences were not affected by the additional information in Chapters 10 and 11.

The estimates were classified by each investigator for every year in the period and, for purposes of analysis, averages, unweighted geometric means of the central values of error classes, were computed for 1919–28, 1929–35, and 1919–35.[4] These average margins [5] for net income originating, for

[3] This uniform adjustment may have resulted in an overestimate of the margins of error for some cells. It is likely that the margins as now presented, even for some of the more comprehensive totals, are somewhat too high, since the estimates placed in Class I may have a mean margin of error somewhat lower than the mid-value used in further computations (i.e., 7.5) and the resulting exaggeration of the margin of error may not be offset by an excess over 60 per cent of the average margin of error of estimates in Class IV.

[4] The computation was made before the estimates for 1936–38 were completed. It was felt that the additional labor necessitated by the inclusion of data for these recent years was not warranted since the margins of error would be affected only slightly.

[5] In subsequent discussion the central values of error classes and the averages derived from them are referred to as margins of error.

number engaged by industrial divisions, and for the countrywide totals of the various types of income and employment are shown in Tables 98–100.

Margins of error were calculated not only for the estimates within each industry type of income cell, but also for certain composite totals; e.g., for wages and salaries in total manufacturing as well as for those in its subdivisions. As mentioned above, most of the composite totals were classified by error classes directly, and are assigned a margin of error that is not derived from the errors of their components. But this procedure was subject to significant exceptions. First, the maximum error of total net income originating in each industrial branch was evaluated both directly and by weighting error margins assigned to the various type of income components. Second, a weighted margin of error of the countrywide totals of each type of income and each category of number engaged was derived from the error margins for the several industrial components, in two variants, one using the ten major industrial divisions, the other, the minor industrial divisions.

The calculation of these weighted entries followed a few standard rules. First, the weights throughout were the average absolute values of the estimates whose margins of error were compounded into averages. When these absolute values were either positive or negative, as in the case of savings of enterprises, the average value was computed with signs disregarded. Second, the weighted mean margins of error were computed from the means for the periods, not from entries for each year; e.g., the weighted mean for the countrywide total for 1919–28 was derived from the error margins computed for 1919–28 for wages and salaries in each industrial division, the absolute average value for each wage and salary industry cell for the decade being used as weights. Third, the margins of error thus weighted were averaged by taking an arithmetic mean. The arithmetic mean was used because a geometric mean would underestimate the total absolute and

relative margin of error.[6] Fourth, in obtaining weighted margins of error for the estimates of net income originating in each industry, the most detailed list of components was used, i.e., wages, salaries, entrepreneurial withdrawals, dividends, interest, net savings of corporations and government, and net savings of entrepreneurs. Similar rules were followed in deriving weighted margins of error for the estimates of the number engaged or employed.

Since the evaluations are essentially opinions, they are presented separately in Tables 98–100 to reveal the full extent of agreement or disagreement. But it was thought that some consensus of opinion would be helpful. For this purpose we summarized the evaluations by the three investigators by taking a geometric mean of the margins of error they assigned to the various components and totals over the long periods covered. For each cell we averaged the three error margins; and these geometric means are presented in the tables.

Equal weight was assigned to the three evaluations of error margins. While it cannot be assumed that the three investigators were equally familiar with the estimates of each and every component of national income and of the total employed and engaged, the differences were so vague that it was impossible to assign any weight to them. In addition, each investigator had at hand not only the general rules of classification set forth above, but also the detailed description of sources and methods in Part Four. This description was con-

[6] This can be shown by the following illustration:

Cell	True Value	Estimated Maximum Error	Maximum Possible Value	Minimum Possible Value
a	1,000	15%	1,150	850
b	2,000	30%	2,600	1,400
Total	3,000		3,750	2,250

The maximum possible error is 750/3,000 or 25 per cent. This result would be obtained by weighting 15 per cent by 1 and 30 per cent by 2, and taking the arithmetic mean (dividing by 3). But the geometric mean would be 23.8 per cent, i.e., too low.

tinually referred to as a basis for classification. A greater degree of agreement would have been possible by elucidation of the reasons that led to the differences. But in such a discussion unaccountable personal influences and factors are likely to play a part; and it was thought preferable to avoid the introduction of such elements. The present evaluations have, we believe, a more objective meaning than would evaluations modified after conference.

Finally, we summarized not only the margins of error assigned to each estimate by the three investigators, but also the extent of agreement or divergence. For this purpose, the computation of the geometric means of the error margins was accompanied by a computation of the logarithmic average deviations. The antilogs of these average deviations represent the geometric mean *relative* deviations of the three error margins for each cell from the geometric mean margins of error. These geometric mean relative deviations, expressed as percentages, also appear in the tables. The deviations for the weighted entries were computed directly from the weighted geometric means already established by each investigator.

2 *Reliability of the Totals—The Results*

In interpreting the results of our attempt to evaluate the margins of error, it is imperative to remember that these evaluations are nothing more than informed opinions, since no exact criteria or specific empirical evidence were at hand by which to measure the errors precisely. Consequently, the values in the tables are themselves subject to a considerable margin of error. It is perhaps narrower for the combined judgment of the three investigators than for each separately, but is probably substantial even for the former.

Although the absolute error margins are surrounded by a large zone of doubt and uncertainty, the differences among them are significant. If, for example, the average margin of error for an estimate is 7.5 per cent, the mid-point of Class I,

the true error may well be somewhat smaller or larger. But substantial differences in error margins are significant indicators of substantial differences in the reliability of estimates.

Furthermore, we must warn the reader not to apply margins of error to aspects of the estimates to which they do not refer directly. For example, the weighted average margins are averages of error margins for the components, and since the error margins for the components may partly cancel one another, the weighted average margins may exaggerate the error for the composite total. For this reason a weighted error should often be scaled down, if it is to be interpreted as a margin of error for the composite total; and for such composite totals a directly evaluated margin of error that is appreciably smaller than the weighted mean of the components is the more significant measure of the two.

A INDUSTRIAL DIVISIONS

The margins of error in estimates for various industrial divisions are shown for the income totals in Table 98 and for the number employed or engaged in Table 99. Brief inspection of Table 98 reveals three distinct groupings of industries. In the first, with a margin of error well below 15 per cent, are the basic branches of manufacturing and several public utilities (electric light and power, steam railroads, street railways, telephone, telegraph), industries for which the high reliability of the estimates is largely explained by the frequency of the industrial censuses and availability of comprehensive data from the Interstate Commerce Commission. In the second group, with margins of error of about 15 per cent but well below 30, are agriculture, mining, manufactured gas, pipe lines, trade, banking, insurance, and government—industries for which information is extensive but not complete. In the third group, with margins of error of about 30 per cent and higher, are the dark spots in the statistical picture of the income of the nation—construction.

TABLE 98

Net Income Originating, Margins of Error by Industrial Divisions, 1919–1935

	DIRECTLY ESTIMATED					ESTIMATED BY PARTS				
	INVESTIGATOR				Relative	INVESTIGATOR				Relative
	A	B	C	Mean	deviation	A	B	C	Mean	deviation
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					I 1919–1928					
Agriculture	15.00	15.00	7.50	11.91	36	31.51	51.38	8.43	23.90	100
Mining, total	14.00	27.99	7.50	14.32	56	23.99	28.07	12.65	20.42	38
Mfg., total	7.50	15.00	7.50	9.45	36	11.45	10.44	7.76	9.75	17
Food & tobacco	7.50	15.00	7.50	9.45	36	18.61	11.97	19.98	16.45	24
Textile & leather	7.50	15.00	7.50	9.45	36	13.34	10.63	7.90	10.39	20
Construction materials & furniture	7.50	15.00	7.50	9.45	36	12.69	10.20	7.79	10.03	18
Paper	7.50	15.00	7.50	9.45	36	15.16	17.93	7.64	10.65	27
Printing	7.50	15.00	7.50	9.45	36	14.68	11.24	7.94	10.94	24
Metal	7.50	15.00	7.50	9.45	36	13.69	9.88	7.58	10.08	23
Chemical	15.00	15.00	15.00	15.00	0	29.36	11.06	28.21	20.92	53
Misc. & rubber	7.50	15.00	7.50	9.45	36	17.30	10.77	9.50	12.10	27
Construction	30.00	30.00	30.00	30.00	0	31.60	30.90	29.62	30.70	2
Electric light & power	13.06	15.00	7.50	11.37	32	13.24	18.10	7.53	12.17	38
Mfd. gas	15.00	15.00	30.00	18.90	36	30.21	16.04	18.02	20.59	29
Steam rr., Pullman, & express	7.50	7.50	7.50	7.50	0	7.50	7.50	7.50	7.50	0
Street railways	13.06	13.06	7.50	10.86	28	13.06	13.12	7.50	10.88	28
Water transportation	30.00	30.00	30.00	30.00	0	32.64	30.34	30.00	30.98	4
Pipe lines	22.74	21.21	11.37	17.64	34	18.08	18.29	8.80	14.27	38
Telephone	7.50	7.50	7.50	7.50	0	7.50	7.50	7.50	7.50	0
Telegraph	7.50	7.50	7.50	7.50	0	7.50	8.77	7.50	7.90	7

Trade	30.00	30.00	12.18	22.22	49	33.64	37.79	20.66	29.72	27
Banking	15.00	15.00	24.38	17.64	24	17.36	14.09	21.47	17.38	15
Insurance	15.00	15.00	15.00	15.00	0	18.76	17.79	17.23	17.92	3
Real estate	30.00	30.00	60.00	37.80	36	29.69	34.42	56.90	38.74	29
Service, total	30.00	30.00	30.00	30.00	0	41.54	41.42	59.25	46.71	17
Government, total	16.08	16.08	30.00	19.80	32	26.17	24.90	24.26	25.10	3
Miscellaneous	60.00	60.00	60.00	60.00	0	60.46	57.22	57.52	58.39	2
Total, based on major divisions	20.44	20.48	20.83	20.59	1	26.36	29.02	24.50	26.56	6
Total, based on minor divisions	20.60	20.64	20.98	20.74	1	27.26	29.08	25.16	27.12	5
					II	1929–1935				
Agriculture	20.19	15.00	7.50	13.14	45	33.47	52.57	9.05	25.16	98
Mining, total	13.59	15.00	7.50	11.52	33	25.49	14.73	11.67	16.36	34
Mfg., total	7.50	15.00	7.50	9.45	36	11.43	10.22	7.69	9.65	16
Food & tobacco	7.50	15.00	7.50	9.45	36	16.23	10.75	15.20	13.84	18
Textile & leather	7.50	15.00	7.50	9.45	36	14.36	10.40	7.82	10.53	23
Construction materials & furniture	7.50	15.00	7.50	9.45	36	17.23	10.67	7.74	11.25	33
Paper	7.50	15.00	7.50	9.45	36	13.88	9.69	7.57	10.06	24
Printing	7.50	15.00	7.50	9.45	36	12.80	10.76	7.78	10.23	20
Metal	7.50	15.00	7.50	9.45	36	15.33	9.68	7.55	10.39	30
Chemical	12.30	15.00	12.30	14.64	9	27.56	10.36	21.04	18.18	46
Misc. & rubber	7.50	15.00	7.50	9.45	36	16.59	10.34	8.79	11.47	28
Construction	30.00	15.00	27.16	23.04	33	31.70	20.14	26.54	25.68	18

Table 98 (concl.)

	DIRECTLY ESTIMATED					ESTIMATED BY PARTS				
	INVESTIGATOR				Relative	INVESTIGATOR				Relative
	A	B	C	Mean	deviation	A	B	C	Mean	deviation
	(1)	(2)	(3)	(4)	(5)	(6) (concl.)	(7)	(8)	(9)	(10)
				II 1929–1935						
Electric light & power	13.59	15.00	7.50	11.52	33	18.03	14.61	8.80	13.24	31
Mfd. gas	15.00	15.00	20.19	16.56	14	29.50	14.45	15.82	18.89	35
Steam rr., Pullman, & express	7.50	7.50	7.50	7.50	0	7.50	7.50	7.50	7.50	0
Street railways	13.59	13.59	7.50	11.14	30	13.88	13.94	7.50	11.32	32
Water transportation	30.00	30.00	15.00	23.80	36	34.64	17.63	15.90	21.34	38
Pipe lines	15.00	15.00	7.50	11.91	36	15.00	15.00	7.50	11.90	36
Telephone	7.50	7.50	7.50	7.50	0	7.50	7.50	7.50	7.50	0
Telegraph	7.50	7.50	7.50	7.50	0	7.50	7.96	7.50	7.65	3
Trade	30.00	27.16	7.50	18.28	81	26.32	22.52	11.52	18.98	39
Banking	15.00	12.30	12.30	13.14	9	17.97	11.50	10.02	12.75	26
Insurance	15.00	15.00	13.59	14.52	4	18.74	17.63	15.12	17.09	9
Real estate	30.00	27.16	54.34	35.38	33	29.70	36.92	48.97	37.72	19
Service, total	30.00	30.00	15.00	23.80	36	31.42	31.03	29.30	30.57	3
Government, total	15.00	15.00	15.00	15.00	0	20.21	19.02	13.58	17.35	18
Miscellaneous	60.00	60.00	30.00	47.62	36	58.46	36.50	29.43	39.74	29
Total, based on major divisions	21.22	20.21	14.97	18.58	16	24.27	23.45	17.05	21.33	16
Total, based on minor divisions	21.40	20.25	15.10	18.70	15	25.24	23.52	17.56	21.85	16
				III 1919–1935						
Agriculture	16.95	15.00	7.50	12.40	40	32.14	51.78	8.65	24.32	99
Mining, total	13.83	21.64	7.50	13.10	45	24.04	19.79	12.20	17.97	29

Mfg., total	7.50	15.00	7.50	9.45	36	11.62	10.34	7.73	9.76	17
Food & tobacco	7.50	15.00	7.50	9.45	36	17.72	11.48	17.86	15.38	21
Textile & leather	7.50	15.00	7.50	9.45	36	13.66	10.51	7.88	10.42	20
Construction materials & furniture	7.50	15.00	7.50	9.45	36	14.12	10.36	7.77	10.43	22
Paper	7.50	15.00	7.50	9.45	36	14.71	10.06	7.61	10.41	26
Printing	7.50	15.00	7.50	9.45	36	13.93	11.00	7.88	10.64	22
Metal	7.50	15.00	7.50	9.45	36	14.26	9.79	7.57	10.18	25
Chemical	13.83	15.00	13.83	14.20	4	28.56	10.87	25.47	19.92	50
Misc. & rubber	7.50	15.00	7.50	9.45	36	16.94	10.52	10.54	12.34	24
Construction	30.00	22.54	28.80	26.91	12	31.64	25.42	28.35	28.36	8
Electric light & power	13.28	15.00	7.50	11.43	32	14.66	16.22	7.94	12.36	34
Mfd. gas	15.00	15.00	25.48	17.90	27	30.29	15.78	17.57	20.32	30
Steam rr., Pullman, & express	7.50	7.50	7.50	7.50	0	7.50	7.50	7.50	7.50	0
Street railways	13.28	13.28	7.50	10.98	29	13.39	13.45	7.50	11.06	30
Water transportation	30.00	30.00	22.54	27.27	14	33.34	23.71	22.92	26.26	17
Pipe lines	19.16	18.39	9.58	15.00	35	16.43	16.54	8.07	12.99	37
Telephone	7.50	7.50	7.50	7.50	0	7.50	7.50	7.50	7.50	0
Telegraph	7.50	7.50	7.50	7.50	0	7.50	8.38	7.50	7.78	5
Trade	30.00	28.80	9.98	20.50	62	30.55	30.22	16.58	24.82	31
Banking	15.00	13.83	18.39	15.63	11	17.62	12.87	15.63	15.25	12
Insurance	15.00	15.00	14.40	14.80	2	18.75	17.72	16.30	17.56	5
Real estate	30.00	28.80	57.60	36.78	35	29.69	35.48	53.47	38.33	25
Service, total	30.00	30.00	22.54	27.27	14	36.86	36.65	44.27	39.10	9
Government, total	15.63	15.63	22.54	17.66	18	23.54	22.31	47.94	29.31	39
Miscellaneous	60.00	60.00	45.10	54.56	14	58.73	47.14	43.44	49.36	12
Total, based on major divisions	20.88	21.83	18.16	20.23	7	25.40	26.36	21.12	24.18	9
Total, based on minor divisions	21.04	21.87	18.31	20.35	7	26.34	26.43	22.13	24.88	8

TABLE 99

Number Employed, Margins of Error by Industrial Divisions, 1919–1935

	EMPLOYEES					ENTREPRENEURS				
	INVESTIGATOR				Relative	INVESTIGATOR				Relative
	A	B	C	Mean	deviation	A	B	C	Mean	deviation
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					I 1919–1928					
Agriculture	26.12	15.00	7.50	14.32	54	7.50	13.06	7.50	9.03	28
Mining, total	14.00	14.00	7.50	11.37	32	52.23	14.00	7.50	17.64	106
Anthracite coal	7.50	7.50	7.50	7.50	0					
Bituminous coal	14.00	14.00	7.50	11.37	32					
Metal	14.00	14.00	7.50	11.37	32					
Oil & gas	26.12	26.12	14.00	21.21	32					
Other	14.00	15.00	7.50	11.64	34					
Mfg., total	7.50	7.50	7.50	7.50	0	10.60	10.60	7.50	9.45	17
Food & tobacco	7.50	7.50	7.50	7.50	0	10.60	10.60	7.50	9.45	17
Textile & leather	7.50	7.50	7.50	7.50	0	10.60	10.60	7.50	9.45	17
Construction materials & furniture	7.50	7.50	7.50	7.50	0	10.60	10.60	7.50	9.45	17
Paper	7.50	7.50	7.50	7.50	0	10.60	10.60	7.50	9.45	17
Printing	7.50	7.50	7.50	7.50	0	10.60	10.60	7.50	9.45	17
Metal	7.50	7.50	7.50	7.50	0	10.60	10.60	7.50	9.45	17
Chemical	7.50	7.50	7.50	7.50	0	10.60	10.60	7.50	9.45	17
Misc. & rubber	7.50	7.50	7.50	7.50	0	10.60	10.60	7.50	9.45	17
Construction	30.00	30.00	30.00	30.00	0	30.00	27.99	26.12	27.99	5

Electric light & power	13.06	22.74	7.50	13.06	45	52.23	26.12	15.00	27.34	54
Mfd. gas	10.60	10.60	7.50	9.45	17					
Steam rr., Pullman, & express	7.50	7.50	7.50	7.50	0					
Street railways	13.06	13.06	7.50	10.86	28					
Water transportation	30.00	30.00	30.00	30.00	0	60.00	60.00	15.00	37.80	85
Pipe lines	15.00	22.74	11.37	15.70	28					
Telephone	7.50	7.50	7.50	7.50	0					
Telegraph	7.50	7.50	7.50	7.50	0					
Trade	27.99	30.00	12.18	21.70	47	27.99	30.00	15.00	23.26	34
Banking	15.00	30.00	30.00	23.80	36					
Insurance	15.00	30.00	30.00	23.80	36					
Real estate	30.00	30.00	26.12	28.65	6					
Service, total	30.00	30.00	30.00	30.00	0	30.00	30.00	30.00	30.00	0
Professional	30.00	30.00	60.00	37.80	36	30.00	30.00	60.00	37.80	36
Personal	30.00	30.00	15.00	23.80	36	30.00	30.00	15.00	23.80	36
Domestic	30.00	30.00	30.00	30.00	0					
Misc.	60.00	30.00	30.00	37.80	36	60.00	60.00	30.00	47.62	36
Government, total	15.00	16.08	15.00	15.34	3					
Federal	16.08	16.08	15.00	15.70	3					
State	15.00	14.00	39.58	20.25	56					
County	30.00	26.12	39.58	31.42	17					
City, incl. public education	15.00	14.00	24.38	17.24	26					
Miscellaneous	60.00	60.00	60.00	60.00	0	60.00	60.00	60.00	60.00	0
Total, based on major divisions	19.78	19.85	16.31	18.58	9	15.50	19.34	13.52	15.94	14
Total, based on minor divisions	20.58	19.84	17.34	19.21	7	16.58	20.41	14.42	16.96	13

Table 99 (cont.)

	EMPLOYEES					ENTREPRENEURS				
	INVESTIGATOR				Relative	INVESTIGATOR				Relative
	A	B	C	Mean	deviation	A	B	C	Mean	deviation
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					II 1929–1935					
Agriculture	24.62	15.00	7.50	14.04	52	12.30	12.30	7.50	10.44	25
Mining, total	13.59	12.30	7.50	10.78	27					
Anthracite coal	7.50	7.50	7.50	7.50	0					
Bituminous coal	13.59	12.30	7.50	10.78	27					
Metal	13.59	12.30	7.50	10.78	27					
Oil & gas	27.16	13.59	7.50	14.04	55					
Other	13.59	12.30	7.50	10.78	27					
Mfg., total	7.50	7.50	7.50	7.50	0	10.10	12.30	7.50	9.76	19
Food & tobacco	7.50	7.50	7.50	7.50	0	10.10	12.30	7.50	9.76	19
Textile & leather	7.50	7.50	7.50	7.50	0	10.10	12.30	7.50	9.76	19
Construction materials & furniture	7.50	7.50	7.50	7.50	0	10.10	12.30	7.50	9.76	19
Paper	7.50	7.50	7.50	7.50	0	10.10	12.30	7.50	9.76	19
Printing	7.50	7.50	7.50	7.50	0	10.10	12.30	7.50	9.76	19
Metal	7.50	7.50	7.50	7.50	0	10.10	12.30	7.50	9.76	19
Chemical	7.50	7.50	7.50	7.50	0	10.10	12.30	7.50	9.76	19
Misc. & rubber	7.50	7.50	7.50	7.50	0	10.10	12.30	7.50	9.76	19
Construction	30.00	15.00	15.00	18.90	36	30.00	27.16	13.59	22.29	39

Electric light & power	13.59	13.59	7.50	11.14	30	54.34	27.16	15.00	28.08	55
Mfd. gas	10.10	13.59	7.50	10.10	22					
Steam rr., Pullman, & express	7.50	7.50	7.50	7.50	0					
Street railways	13.59	13.59	7.50	11.14	30					
Water transportation	*30.00*	*15.00*	*15.00*	*18.90*	*36*	*60.00*	*49.22*	*15.00*	*35.38*	*77*
Pipe lines	15.00	15.00	7.50	11.91	36					
Telephone	7.50	7.50	7.50	7.50	0					
Telegraph	7.50	7.50	7.50	7.50	0					
Trade	22.29	11.14	7.50	12.30	49	22.29	22.29	7.50	15.51	62
Banking	15.00	20.19	12.30	15.51	19					
Insurance	15.00	27.16	13.59	17.68	33					
Real estate	30.00	30.00	15.00	23.80	36					
Service, total	30.00	30.00	15.00	23.80	36	30.00	30.00	15.00	23.80	36
Professional	30.00	30.00	30.00	30.00	0	30.00	30.00	30.00	30.00	0
Personal	22.29	22.29	7.50	15.51	62	30.00	30.00	7.50	18.90	85
Domestic	30.00	30.00	27.16	29.02	4					
Misc.	30.00	30.00	15.00	23.80	36	60.00	60.00	15.00	37.80	85
Government, total	15.00	15.00	7.50	11.91	36					
Federal	15.00	15.00	7.50	11.91	36					
State	15.00	15.00	30.00	18.90	36					
County	30.00	30.00	60.00	37.80	36					
City, incl. public education	15.00	15.00	15.00	15.00	0					
Miscellaneous	60.00	33.12	30.00	39.06	33	33.12	33.12	30.00	32.04	4
Total, based on major divisions	19.83	15.81	10.54	14.90	26	17.48	17.46	9.54	14.28	31
Total, based on minor divisions	19.72	15.62	12.59	15.71	16	18.35	18.33	10.08	15.02	30

Table 99 (concl.)

	EMPLOYEES					ENTREPRENEURS				
	INVESTIGATOR				Relative	INVESTIGATOR				Relative
	A	B	C	Mean	deviation	A	B	C	Mean	deviation
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					III 1919–1935					
Agriculture	25.48	15.00	7.50	14.20	53	9.20	12.75	7.50	9.58	21
Mining, total	13.83	13.28	7.50	11.13	30					
Anthracite coal	7.50	7.50	7.50	7.50	0					
Bituminous coal	13.83	13.28	7.50	11.13	30					
Metal	13.83	13.28	7.50	11.13	30					
Oil & gas	26.55	19.95	10.83	17.90	40					
Other	13.83	13.83	7.50	11.28	31					
Mfg., total	7.50	7.50	7.50	7.50	0	10.40	11.28	7.50	9.58	18
Food & tobacco	7.50	7.50	7.50	7.50	0	10.40	11.28	7.50	9.58	18
Textile & leather	7.50	7.50	7.50	7.50	0	10.40	11.28	7.50	9.58	18
Construction materials & furniture	7.50	7.50	7.50	7.50	0	10.40	11.28	7.50	9.58	18
Paper	7.50	7.50	7.50	7.50	0	10.40	11.28	7.50	9.58	18
Printing	7.50	7.50	7.50	7.50	0	10.40	11.28	7.50	9.58	18
Metal	7.50	7.50	7.50	7.50	0	10.40	11.28	7.50	9.58	18
Chemical	7.50	7.50	7.50	7.50	0	10.40	11.28	7.50	9.58	18
Misc. & rubber	7.50	7.50	7.50	7.50	0	10.40	11.28	7.50	9.58	18
Construction	30.00	22.54	22.54	24.80	14	30.00	27.64	19.95	25.48	18

Electric light & power	13.28	18.39	7.50	12.24	39	53.08	26.55	15.00	27.64	54
Mfd. gas	10.40	11.74	7.50	9.70	19					
Steam rr., Pullman, & express	7.50	7.50	7.50	7.50	0					
Street railways	13.28	13.28	7.50	10.98	29					
Water transportation	30.00	22.54	22.54	24.80	14	60.00	55.30	15.00	36.78	82
Pipe lines	15.00	19.16	9.58	14.01	29					
Telephone	7.50	7.50	7.50	7.50	0					
Telegraph	7.50	7.50	7.50	7.50	0					
Trade	25.48	19.95	9.98	17.19	44	25.48	26.62	11.28	19.68	45
Banking	15.00	25.48	20.79	19.95	21					
Insurance	15.00	28.80	21.64	21.08	25					
Real estate	30.00	30.00	20.79	26.55	18					
Service, total	30.00	30.00	22.54	27.27	14	30.00	30.00	22.54	27.27	14
Professional	30.00	30.00	45.10	34.36	20	30.00	30.00	45.10	34.36	20
Personal	26.55	26.55	11.28	19.95	46	30.00	30.00	11.28	21.64	54
Domestic	30.00	30.00	28.80	29.60	2					
Misc.	45.10	30.00	22.54	31.24	28	60.00	60.00	22.54	43.30	54
Government, total	15.00	15.63	11.28	13.83	15					
Federal	15.63	15.63	11.28	14.01	16					
State	15.00	14.40	35.31	19.68	48					
County	30.00	27.64	46.98	33.90	24					
City, incl. public education	15.00	14.40	19.95	16.28	15					
Miscellaneous	60.00	46.98	45.10	50.28	13	46.98	46.98	45.10	46.35	2
Total, based on major divisions	19.77	17.73	13.57	16.82	15	16.07	18.52	11.62	15.13	19
Total, based on minor divisions	20.18	17.65	15.10	17.52	10	17.06	19.51	12.36	16.02	19

water transportation, real estate, direct service industries, and, of course, the miscellaneous division.

The margins of error in the estimates of the number employed or engaged (Table 99) naturally reveal the same grouping of industries. The basic data for service income, the preponderant part of total income originating in each industry, are also a source of information on the number employed and engaged. In fact, information on number is in some instances somewhat more complete than on income flows, e.g., the number of entrepreneurs as compared with their total income in manufacturing. For this reason, some of the income estimates are derived by applying to the number of entrepreneurs (or employees) a per capita income figure. It is, therefore, not surprising that the margins of error in the estimates of the number employed or engaged are often lower than those in the estimates of the corresponding income flows. This is especially true of the number of entrepreneurs as compared with their income, but can be observed also for some of the estimates of the number and income of employees (mining, construction, real estate, service). On the other hand, for some industries information on income flows is more complete than on number, with consequent effects on the respective margins of error (banking, insurance).

Comparison of the directly evaluated average margins of error for estimates of net income by industrial divisions with those obtained by weighting the errors of the components shows that in several industries (agriculture, mining, trade, service, government) the former are significantly lower. This difference arises largely because in the direct evaluation the separate errors attaching to the estimates of entrepreneurial withdrawals and of entrepreneurial net savings partly cancel one another, whereas they are added without any cancellation in the weighted mean of the errors in the components. On the other hand, in pipe lines and miscellaneous industries the directly evaluated margins of error are somewhat

greater than those obtained by weighting parts, even when averaged for the error margins calculated by the three investigators. But these differences are of doubtful significance, as are also the exceedingly minor differences between the two sets of error margins observed for other industries not mentioned. The weighted margins of error for national income, whether on the basis of the ten major industrial divisions or of the more numerous minor divisions, are distinctly smaller when derived from the margins for each industrial division evaluated directly than when derived from the margins for each industrial division obtained by weighting the errors attached to the estimates of the components.

To show changes in the margins of error from the first part of the period to the second (Tables 98 and 99, Sec. I and II) we compare the weighted error margins for net income originating in each industry, because these are more sensitive than those based on direct evaluation. In mining, construction, water transportation, pipe lines, trade, various branches of finance, total service, government, and miscellaneous, the margin of error declined markedly from the average for 1919–28 to that for 1929–35, partly because of the extension of Census coverage in these industries since 1929, partly because of the greater detail in the presentation of data in *Statistics of Income* and employment and payroll samples of the Bureau of Labor Statistics, partly because of special studies, conducted primarily by the Department of Commerce in connection with its estimates of national income. For similar reasons, the margins of error for the number employed in the various industries declined.

Agreement among the three investigators in their evaluation of the margins of error was high for some estimates and low for others. The relative deviation from the geometric mean margin of error in net income ranges from zero for the steam railroad and telephone industries to 99 per cent for agriculture. These variations in the degree of agreement reveal several general features. First, there seems to be a posi-

tive association between the geometric mean margin of error and the size of the *relative* deviation which measures the extent of divergence in the evaluations of the three investigators. This association is suggested by the coefficient of rank correlation computed for the weighted margins of error for the net income totals and for the margins of error directly evaluated for the number of employees. When the underly-

Coefficient of Rank Correlation between Geometric Mean Margins of Error and Mean Relative Deviations

	NO. OF PAIRS OF ITEMS	*1919–1935*	*1919–1928*	*1929–1935*
1 Net income estimates, by industrial division, weighted by parts *	27	+0.28	+0.13	+0.40
2 Estimates of number of employees, by industrial division *	27	+0.22	+0.16	+0.43

* The subdivisions of mining and manufacturing were excluded from line 2 to reduce the number of entries of identical magnitude. The two countrywide totals were excluded from both lines to prevent duplication.

ing data are not sufficiently comprehensive and exact to yield an estimate with a narrow margin of error, there is apparently a greater possibility of disagreement among the investigators as to what margin of error to assign. This does not mean that for weak estimates there cannot be perfect agreement on margins of error. But the estimates whose weakness is so apparent as to compel all three investigators to assign uniformly large margins of error are few. And when unreliability is not obvious, significant disagreement in the evaluation of error arises more easily.

A corollary result is revealed when the relative deviations about the geometric mean error are compared for the two parts of the period. The divergence in the three evaluations is larger for 1929–35 than for 1919–28, and this increase in the relative deviations is concentrated in a few industrial divisions: construction, water transportation, and the miscellaneous (for both net income and number employed), trade (for

net income), real estate and service (for number employed). The margin of error for these industries declined markedly from 1919–28 to 1929–35. What obviously happened is that, with the accretion of new data, the estimates passed out of the definitely weak category to a somewhat superior standing with respect to reliability; and concurrently with this change appeared a greater opportunity for divergence in the evaluation of the error margins.[7]

B TYPES OF INCOME AND EMPLOYMENT

For the countrywide totals of income by type and of the number employed and engaged it was not feasible to evaluate the margins of error directly. Hence, the evaluations of these estimates in Table 100 are all weighted means of error margins assigned to income or number for the various industrial divisions. Since it is likely that errors in the estimates for a given type of income or of employment in the various industries may partly cancel one another, the margins of error in Table 100 exaggerate the errors for the countrywide totals.[8] This is especially likely to be true for property income, for which differentiation among various industries is more difficult than for service income. It is quite possible that the margins of error in the estimates of dividends and interest are appreciably below 15 to 18 per cent for the former and 25 per cent for the latter.

This qualification may be sufficiently great to affect the difference in the margins of error between the estimates of employee compensation on the one hand and of dividends and interest on the other. As the weighted means stand, the margins of error for the estimates of employee compensation, 1919–35, are slightly wider than those for dividends and signifi-

[7] It is obviously this factor, i.e., the unanimity of opinion for especially weak estimates that lowers the association established just above between margins of error and mean relative deviations.

[8] This inference is supported by the fact that the weighted means based on ten major industrial divisions are, by and large, smaller than those derived from the more numerous minor divisions.

TABLE 100

Income Types and Categories of Employment, Margins of Error, 1919–1935

	DERIVED FROM ESTIMATES FOR MAJOR INDUSTRIAL DIVISIONS					DERIVED FROM ESTIMATES FOR MINOR INDUSTRIAL DIVISIONS				
	INVESTIGATOR				Relative	INVESTIGATOR				Relative
	A	B	C	Mean	deviation	A	B	C	Mean	deviation
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
					I 1919–1928					
Wages	12.20	12.41	10.20	11.56	9	12.22	12.15	10.27	11.51	8
Salaries	11.68	12.18	9.56	11.08	10	11.92	12.20	9.66	11.20	10
Employee compensation	19.66	19.85	20.13	19.88	1	20.52	19.77	20.06	20.12	1
Entrepreneurial withdrawals	35.90	50.80	31.93	38.76	20	35.90	50.80	31.93	38.76	20
Dividends	18.56	14.79	16.90	16.68	8	24.02	14.79	24.20	20.48	19
Interest	23.90	26.12	24.02	24.66	4	22.99	27.10	25.43	25.12	6
Net savings, corporate & government	45.42	36.32	36.02	39.02	11	53.74	35.96	38.72	42.14	18
Net savings, entrepreneurial	60.00	60.00	44.54	54.33	14	60.00	60.00	44.32	54.24	14
National income estimated directly	20.44	20.48	20.83	20.59	1	20.60	20.64	20.98	20.74	1
National income estimated by parts	26.36	29.02	24.50	26.56	6	27.26	29.08	25.16	27.12	5
Wage earners	12.70	11.07	9.50	11.01	10	12.74	11.11	9.56	11.06	10
Salaried employees	13.11	12.17	9.16	11.36	15	13.32	12.18	9.22	11.44	15
Total employees	19.78	19.85	16.31	18.58	9	20.58	19.84	17.34	19.21	7
Entrepreneurs	15.50	19.34	13.52	15.94	14	16.58	20.41	14.42	16.96	13

					II 1929–1935					
Wages	11.40	9.13	9.52	9.97	9	11.50	9.08	9.52	9.71	10
Salaries	11.23	12.68	8.92	10.83	14	11.53	12.74	9.16	11.04	13
Employee compensation	18.97	14.94	13.59	15.68	14	19.00	14.88	14.40	15.97	12
Entrepreneurial withdrawals	31.34	44.65	22.76	31.70	26	31.34	44.65	22.76	31.70	26
Dividends	14.72	11.21	11.60	12.41	12	18.27	11.21	16.60	15.04	22
Interest	23.82	27.51	24.43	25.20	6	23.46	29.45	25.70	26.09	8
Net savings, corporate & government	41.68	25.18	23.30	29.02	27	49.99	25.02	25.59	31.75	35
Net savings, entrepreneurial	60.00	60.00	26.55	45.72	44	60.00	60.00	26.45	45.67	44
National income estimated directly	21.22	20.21	14.97	18.58	16	21.40	20.25	15.10	18.70	15
National income estimated by parts	24.27	23.45	17.05	21.33	16	25.24	23.52	17.56	21.85	16
Wage earners	12.25	9.55	8.08	9.82	16	12.32	9.51	8.08	9.82	16
Salaried employees	12.97	12.88	8.02	11.02	24	13.39	12.96	8.14	11.22	24
Total employees	19.83	15.81	10.54	14.90	26	19.72	15.62	12.59	15.71	16
Entrepreneurs	17.48	17.46	9.54	14.28	31	18.35	18.33	10.08	15.02	30
					III 1919–1935					
Wages	11.92	10.75	9.94	10.84	7	11.98	10.59	9.98	10.82	7
Salaries	11.50	12.31	9.26	10.94	12	11.76	12.36	9.43	11.11	12
Employee compensation	19.34	17.23	17.09	17.86	5	19.82	17.14	17.40	18.08	6
Entrepreneurial withdrawals	34.10	48.40	28.16	35.96	22	34.10	48.40	28.16	35.96	22
Dividends	16.64	13.01	14.24	14.56	9	21.27	13.01	20.67	17.89	24
Interest	23.09	26.75	24.01	24.57	6	22.46	28.11	25.36	25.21	8
Net savings, corporate & government	43.72	31.10	30.66	34.67	17	52.09	30.83	32.98	37.56	24
Net savings, entrepreneurial	60.00	60.00	36.69	50.92	24	60.00	60.00	36.52	50.85	25
National income estimated directly	20.88	21.83	18.16	20.23	8	21.04	21.87	18.31	20.35	7
National income estimated by parts	25.40	26.36	21.12	24.18	9	26.34	26.43	22.13	24.88	8
Wage earners	12.52	10.30	8.78	10.42	13	12.56	10.30	8.80	10.45	13
Salaried employees	13.05	12.41	8.61	11.18	19	13.34	12.46	8.71	11.31	19
Total employees	19.77	17.73	13.57	16.82	15	20.18	17.65	15.10	17.52	10
Entrepreneurs	16.07	18.52	11.62	15.13	19	17.06	19.51	12.36	16.02	19

cantly narrower than those for interest. Since the difficulty of proper industrial apportionment is greater for property income than for employee compensation, it is quite possible that a correct direct evaluation of the countrywide totals would yield significantly lower margins of error for the estimates of dividends and of interest. But there is little doubt that the margins of error in the estimates of entrepreneurial withdrawals and net savings of enterprises are significantly greater than in those of payments to employees and property income; and that for industries in which wages and salaries can be estimated separately, the latter are subject to wider margins of error than the former.

For the countrywide total of employees the weighted margin of error is about 17 per cent, surprisingly greater than for the countrywide total of entrepreneurs. But this may be due to the large weight in total entrepreneurs of the agricultural group, for which adequate data are available throughout the period.

Changes in the error margins over time for the various type of income and employment groups were similar to those for net income originating in the several industrial divisions. In most groups the margins of error declined from 1919–28 to 1929–35, the decline being especially great in estimates of entrepreneurial withdrawals, dividends, and net savings of enterprises. In the estimate of total interest the margin of error increased from the first part of the period to the second, but in most branches it declined. The rise in the weighted mean for the country is obviously due to a shift in the distribution of interest toward industries for which the margin of error is greater than the average for the country.

In the association between the error margins and the extent of divergence in the evaluations by the three investigators, the results for various types of income and categories of employment also confirm those for the industrial divisions. The greatest divergence is in the income totals the components of which are subject to the widest margins of

error: entrepreneurial withdrawals, net savings of corporations and government and of unincorporated enterprises. But the association is far from perfect, since the divergence is less for the wider margin of error in the estimate of interest than for the narrower margin of error in the estimate of salaries, and is least for the relatively wide margin of error in the estimate of employee compensation. The geometric mean relative deviation is greater for the narrower error margin in the estimate of entrepreneurs than for the wider error margin for the number of employees.

Finally, as already observed for the industrial divisions, the mean relative deviations increase from the first part of the period to the second. The increase in extent of disagreement is greatest in the relative deviations for entrepreneurial withdrawals, dividends, interest, and savings of enterprises—all categories of estimates whose margin of error declined significantly from 1919–28 to 1929–35. In this respect also, the results for types of income confirm the conclusions drawn for the movement of evaluations of income estimates by industrial divisions.

C NATIONAL INCOME AND NUMBER ENGAGED

The weighted margin of error for the estimate of national income (Table 100, Sec. III) is about 20 per cent (10 major divisions estimated directly). A similar margin for the number employed and engaged, if computed, would approximate 16 per cent. But it is obvious from the preceding discussion that both figures exaggerate the margins of error that would be assigned by the investigators directly to the two comprehensive totals. The merging of the estimates of dividends and of interest each into a countrywide total would appreciably reduce the margins of error assigned to the estimates by industrial divisions, with the result that the error margins for these two countrywide totals would be much lower than the percentages now appearing in Table 100, Section III. Similarly, the merging of entrepreneurial withdrawals with

entrepreneurial net savings would cancel a substantial part of the error margins assigned to these two totals taken separately. Even for employee compensation, the weighted mean of errors attached to the estimates by industrial divisions exaggerates the margin of error that would be applied directly to the countrywide total. Similar considerations bear upon the margins of error in the countrywide total of number employed and engaged. It is reasonable to infer that for the estimates of national income and number employed and engaged the average margin of error is not much above 10 per cent, and perhaps somewhat less.

For both totals the weighted margin of error declines from 1919–28 to 1929–35. The decline has been reduced by the shift in the industrial composition of national income and of the number employed and engaged in favor of industries the data for which are least comprehensive and the estimates for which are therefore assigned rather wide margins of error. As noted in Chapter 5, the relative share of commodity producing industries in national income declined from 1919–28 to 1929–38, and that of industries engaged in service activities increased; and the movements would be even more pronounced in a comparison of 1919–28 with 1929–35. This shift meant a decline in the importance of industries for which estimates were fairly exact and reliable (mining and manufacturing) and a rise in the importance of industries for which information was less adequate and error margins wider (service and government). The decline from 1919–28 to 1929–35 in the weighted error margins for national income and number employed and engaged is all the more significant in that it occurred in the face of this adverse shift in the industrial composition of the two totals.

The mean relative deviations of the weighted error margins in the estimates of national income are moderate; but, as already suggested, they increase from 1919–28 to 1929–35. However, here again the weighted measures exaggerate considerably the true magnitudes that would result from direct

evaluation. They reflect the average divergence in the evaluation of error margins in the estimates for the many components, and, of course, for each component the range and grounds for disagreement are more extensive than in a direct evaluation of comprehensive totals. It is reasonable to assume that direct evaluations of the error margins in the estimates of national income and the number employed and engaged would disagree even less than the weighted means in Table 100, Section III. The increase in disagreement from 1919–28 to 1929–35 now shown for the weighted means in Sections I and II might not occur with direct evaluation.

3 Reliability of Changes and Differences Revealed by the Estimates

The margins of error apply to the totals for the various cells. In evaluating these margins the investigators asked what maximum percentage error could be assumed for each amount of income or employment in a given year in a given industry. But for many uses to which these estimates are put totals may be less important than *differences* among them for industries or types of income, or *changes over time*. The relation of the error margins established for the totals to the error margins of differences among them or of the changes in them over time can best be elucidated by a symbolic exposition.

Let x_1 and x_2 be the totals and e_1 and e_2 the relative error margins. We can then calculate the *relative* error in the relative difference between the two totals as follows:

1) Observed relative difference (i.e., including error) will be:

$$\frac{x_2 + e_2x_2}{x_1 + e_1x_1} - 1$$

2) True relative difference:

$$\frac{x_2}{x_1} - 1$$

3) *Absolute* error in the relative difference:

$$\left(\frac{x_2 + e_2x_2}{x_1 + e_1x_1} - 1\right) - \left(\frac{x_2}{x_1} - 1\right) = \frac{x_2\,(e_2 - e_1)}{x_1\,(1 + e_1)}$$

4) *Relative* error in the relative difference:

$$\left(\frac{x_2\,(e_2 - e_1)}{x_1\,(1 + e_1)}\right) : \left(\frac{x_2}{x_1} - 1\right) = \frac{x_2\,(e_2 - e_1)}{(x_2 - x_1)\,(1 + e_1)}$$

Expression (4) defines the relative error in which we are interested, viz., the relative error of a percentage difference between two totals whose relative errors are known and given. From expression (4) it may be seen that this relative error of a percentage difference depends upon four factors:

a) First and foremost is the correlation in sign between the errors in the totals compared. If the given error of one is negative and that of the other positive, then the numerator of the equation is a sum rather than a difference.

b) If the correlation in sign between e_1 and e_2 is positive, the relative size of the errors is also important. If the two errors are close in size, then obviously the difference between them will be small. If e_1 and e_2 are equal, the numerator becomes zero and there is no relative error in the percentage difference between the two totals compared.

c) The larger the ratio of x_2 to $(x_2 - x_1)$ the larger the relative error of the relative difference; in other words, the smaller the relative difference revealed by the two totals the larger the relative error in the observed difference (other conditions being equal).

d) All other conditions being equal, the relative error will be smaller if e_1 is positive (i.e., if the absolute value in the base year is exaggerated) and larger if e_1 is negative (i.e., if the absolute value in the base year is underestimated).

One could easily set up cases in which the derived relative error of the difference is small when the relative errors in two totals themselves are substantial; as well as cases in which

the errors of the difference would be many times as large as the relative errors in two totals.[9]

We now consider the characteristics of our estimates with respect to the factors that determine how one can pass from the relative errors in the totals to the relative errors of relative differences. Since base years may be reversed without impairing the value of the comparison, the sign of the error in the base year magnitude is of little interest. We must, therefore, consider only (1) the correlation of the signs of the relative errors of totals compared, (2) the size of the relative errors, and (3) the size of the relative difference itself.

1) In dealing with changes from one year to the next, it can be assumed that the errors in totals tend to be of the same sign. If the data lead to an over- or underestimate in a given year and if the error assigned is substantial, it is unlikely that any new error, resulting from the use of an extrapolation or interpolation index, will produce in the next year's estimate an error with the opposite sign. Since most estimates are derived by applying interpolation and extrapolation indexes to some basic, comprehensive value, there is a natural tendency for the error implicit in the basic quantities to persist through the period covered. This makes for a positive correlation of the signs of errors for adjacent time units when the error margins in the totals are at all substantial.

For differences between two totals for the same year the case for the correlation of signs of errors is not quite so clear. When totals of employee compensation, entrepreneurial withdrawals, and number employed or engaged are compared

9 E.g., if we assume that both e_1 and e_2 are 0.2, x_1 is 10, and x_2 is 5, and that the correlation in sign between the e's is positive, the relative error of the relative difference would be zero. If under the same conditions, e_1 is 0.1 and e_2 is 0.2, the relative error of the relative difference would become 0.09. If we assume that both e_1 and e_2 are 0.2 and the correlation in sign is negative, the relative error would become (with e_1 negative) 0.5. Finally, if we assume e_1 is -0.2, e_2 is $+0.2$, and the corresponding x's are 10 and 9 respectively, the relative error of the relative difference becomes 4.5, or 22.5 times as great as the relative error in either of the two totals.

by industrial divisions, we may expect identity in the signs of the errors, since there is a general tendency toward an underestimate arising from the bias of Census data toward undercoverage. Similarly, there is a bias in some of the property income estimates toward negative errors, since their common source is income tax returns, in which there is a tendency to minimize taxable items. But these tendencies toward identity in the signs of errors assigned to various totals for a given year are subject to numerous exceptions. It is not at all unlikely that in many industries entrepreneurial withdrawals or income are overestimated whereas employee compensation is underestimated; or that in some years income originating in the service industries is overestimated whereas income originating in real estate may be underestimated. At any rate, the likelihood of opposite signs of error seems to be greater in comparisons of totals for the various cells for a given year than in comparisons of totals for one and the same cell for successive years.

2) There is a similar contrast between changes over time and differences among cells in the relative error margins assigned to the two totals compared. In comparisons over time, the error margins change relatively little. Of course, our use of mid-values of wide error classes in deriving Tables 98–100 conceals the changes that may occur in the margin of error in an estimate from one year to the next, changes that we cannot gauge with any degree of accuracy. But it is reasonable to assume that these changes in error margins over time are relatively small, and our tests of interpolations and extrapolations in Chapter 11 tend to support this assumption.

Differences in the relative error in the totals for various cells in the same year are quite substantial (see Tables 98–100). This means that even when the correlation in the *sign* of the relative errors in two totals is positive, the difference in size may yield a substantial numerator in equation (4) above, and a large relative error in the relative difference between the two totals.

3) This undesirable result in comparisons of totals for various cells at a given point of time is offset somewhat by the correlation between differences in the size of relative errors and differences in the size of the totals themselves. The size of the total is one of several factors that guide an estimator in his evaluation of a relative error: the smaller the total the more likely a large percentage error. As a result, large items of $(e_2 - e_1)$ are correlated with fairly high ratios of $(x_2 - x_1)$ to x_2. But in comparisons at a given point of time *small* relative differences may be subject to relative errors much greater than those attached to the totals compared.

The size of the relative difference may seem to be of no importance in considering relative errors in percentage changes over time. This would be true if the hypothesis suggested above, viz., that in such comparisons relative errors are correlated in sign and tend to be approximately equal in size, were always valid. If we could say that in these changes the relative errors in the totals compared are invariably correlated in sign (positively) and are invariable in size, then the size of the difference observed would be of no importance --for the simple reason that under these conditions the relative error of the relative difference would always be zero.

But we cannot assume that the relative error in the totals remains constant from year to year; and the size of the relative change is, therefore, an important factor in the relative error that can be assigned to it. If the error in the total changes even slightly from one year to the next, the estimate of the change may contain a much larger relative error, provided the change itself is merely a small fraction of the total: e.g., if e_1 is 0.20, e_2 is 0.18, and x_1 is 100, then the relative error of the change from x_1 to x_2 will vary much as the change itself varies. Thus if we have a decline to 95, the relative error of that decline of 5 points will be 0.32 (or 32 per cent). But if we have a decline to 50, the relative error of that decline of 50 points will be only 0.02 (or 2 per cent).

Consequently, even for changes over time, it may be said that in general small changes are greatly affected by slight shifts in the error to be attached to the totals compared. In other words, the relative errors in the totals can be applied to changes over time only when the latter constitute substantial proportions of the former. When they are relatively small, their relative errors may be much greater than those in the totals from which they are derived.

The statements above are advanced as tentative conjectures that can serve as a basis for passing from the error margins in totals to errors in changes in or differences among totals.

With respect to changes over time, the following conclusion is suggested. In view of the tendency toward positive correlation in the signs of errors of successive quantities and the minor character of changes that may be assumed to occur in the size of the error in successive years, the error in the totals can be treated, at least provisionally, as a maximum of the relative errors in the changes over time in successive pairs of these totals. When the changes are substantial fractions of the totals themselves there is considerable likelihood that the relative error in the percentage change will be smaller than the relative errors in the totals themselves. But it would be dangerous to assume that a decline or rise of 2 or 3 per cent, or even of 5 per cent, is subject to the same relative error as the totals from which it was derived, or to a smaller relative error.

For differences between totals for a given unit of time there is some basis for a conclusion similar to that made for changes over time, but there is greater likelihood that the errors in the two totals compared may not be the same in sign.

A similar analysis could be carried forward for more complicated comparisons, e.g., for changes over time not between successive time units but between averages for periods separated by a substantial interval; or for changes over time shown by differences, absolute or relative, between two totals. But

the general principles are clear enough. Changes between averages, provided the error margins of the averages are known, will have to be deduced from assumptions concerning the correlation of errors in sign, the relative size of errors, and the relative size of the difference or change. Changes over time in percentages will also have to be tested in the light of the same three considerations. The logic involved is clear enough for any student to follow; and our specific knowledge concerning the applicability of the necessary assumptions is not any greater than that an intelligent reader could himself acquire from perusal of the tables and notes in Parts Four and Five

4 Concluding Comments

To analyze the reliability of data and procedures used to derive national income totals and their components is essentially an insoluble task. Were we able to ascertain the sign and size of error for any given estimate, we could, of course, correct for this error and there would be no need to retain it. Were our procedures or data of such a controlled character that we could make specific assumptions concerning the distribution of errors, if not concerning each single error, it would be possible to apply to our task the full armory of weapons of statistical analysis of sampling errors and limits of inference. But dealing as we do with data that are partly a byproduct of administrative activity, partly a result of direct observation of complex phenomena without controls designed to reduce the variations observed, the best that we can do is to express an opinion in quantitative form.

This we did by setting margins of maximum relative errors for the various cells in the nationwide totals. Even in so doing we have perhaps overstepped the limits to which one should go in assigning a quantitative expression to what are essentially personal judgments, based to a large extent on intuition and guess (although the judgment of more than one investigator and some knowledge of the extent to which esti-

mates of various cells have been revised in the past have possibly produced evaluations that are a bit more than arbitrary opinions). But we did not evaluate margins of errors in differences, percentage shares, changes over time, changes in percentages, etc., partly because of the greater variability of errors attaching to these aspects of measurement and partly because the 'remove' of these aspects from the single absolute amount, which is the first result of our estimating, made evaluation of error difficult. We describe, therefore, in general the theoretical relation between relative errors in totals and relative errors in differences, changes, etc.; and offer a few tentative suggestions as to the character of the factors that govern this relation (correlation in sign among successive errors, similarity in size, etc.) for the estimates that comprise our nationwide totals.

This, perforce inadequate, analysis of the reliability of our estimates has, however, a fairly solid core: the general order of differences in degree of reliability of the various parts of our estimated totals is reasonably reliable. The comparison of various published estimates in Chapter 10; the results of the tests of interpolations and extrapolations in Chapter 11; and the similarity of judgments of the individual investigators in Tables 98–100 support the established differences in reliability among estimates in various industry or type of income or employment categories. While some of this supporting evidence is duplicated, it points to conclusions concerning differences in the reliability of estimates that seem to be significant, much more significant than the results relating to the absolute levels of error margins.

The size of the error margins may raise questions in the minds of students concerning the utility of the estimates and the progress that can be made toward more reliable estimates. As to the possibility that the usefulness of the estimates is fatally impaired by the wide margins of error attributable to them, the only relevant comment is that we believe our estimates to be as good as can be made from available data. Of

course, any individual investigator or group of investigators can commit indiscretions of judgment, overlook possible sources of information, or neglect potentialities of fruitful procedures. But such sins of omission and commission have, we hope, been kept within reasonable bounds; and by far the major source of possible error in our estimates is the inadequacies of existing data. The choice is, therefore, not between present estimates and better estimates: it is largely between present estimates, inadequate as they are, worse estimates, or no estimates at all. For many purposes for which national income estimates are used this set is a significant advance over everyday knowledge which people tend to acquire and use without careful reference to basic data and without any attempt to piece together and collate the evidence.

An extensive and intensive utilization of the data, together with a critical evaluation of their inadequacies, is a step in their better utilization and in furthering the collection of more adequate data. Even more effective than a recital of inadequacies of existing information is an attempt to use it and the resulting tentative revelation of some important finding: this provides an effective stimulus to a quest for further data and for an improvement of procedures. It is thus important to use the data already available, with complete recognition of their faults but without giving way to perfectionist despair. Many of our estimates will prove inadequate in the light of fuller information in the future. But this means only that the present, like all, national income estimates reflect current knowledge, just as they are based upon the current social standards that determine which of the numerous activities in the nation are economic, productive, and hence a source of national income.