

This PDF is a selection from an out-of-print volume from the National Bureau of Economic Research

Volume Title: The Behavior of Prices

Volume Author/Editor: Frederick C. Mills

Volume Publisher: NBER

Volume ISBN: 0-87014-010-8

Volume URL: <http://www.nber.org/books/mill27-1>

Publication Date: 1927

Chapter Title: Price Dispersion

Chapter Author: Frederick C. Mills

Chapter URL: <http://www.nber.org/chapters/c5330>

Chapter pages in book: (p. 251 - 286)

twelve-month index of this sort would be to increase the number of entries above 100, intensifying and lengthening the swings of the index above the base line. A downward trend would have the reverse effect.

An index of this type cannot, of course, replace those of the familiar fixed base type, but it is useful in presenting price fluctuations in a somewhat different light. In its construction it accords with the current practice of comparing prices at a given date with prices prevailing at a date twelve months earlier. And, as will appear later, it is a useful companion measure to certain measures of dispersion and displacement which appear to be most significant on a twelve-month basis.

The measures discussed in the preceding pages relate to a first and extremely important aspect of price instability—instability of the general level of prices. There have been presented different types of index numbers which measure, with varying degrees of accuracy, the changes through which the level of wholesale prices has passed since 1890. Certain points of some technical interest, relating to weighting and to the reliability of different types of index numbers, have been noted in passing. It has not been the purpose of this section, however, to discuss the technique of index number construction, and no attempt has been made to deal with the various "crossed" formulas derived by Professor Irving Fisher.

But it is an inadequate survey of the price problem which contents itself with the information concerning price changes which is yielded by index numbers of the type given above. These are merely averages of diverse distributions of price relatives, and they relate only to one aspect of price behavior. Other important aspects are still to be described. When this has been done, and appropriate measures have been computed, the relation of instability in the price level to other types of price instability may be considered.

#### IV Price Dispersion<sup>1</sup>

Of those aspects of price behavior which are not reflected in the movements of an index number of the orthodox type, probably

<sup>1</sup>I regret that I was not able to include in this section the results secured by Dr. Maurice Olivier in his study of price dispersion (*Les Nombres Indices de la Variation des Prix*, Paris, Giard, 1927, pp. 90-98.) His book came into my hands after the text of this volume had gone to the printer. This study of price dispersion, based upon the movements of the price series used in the construction of the Federal Reserve Board's index of wholesale prices for France, covers the years 1920-1924, by months. Arithmetic and logarithmic measures of dispersion are employed. Dr. Olivier finds, during this period, a tendency toward a positive relationship between changes in the price level in France and the dispersion of price relatives in natural form.

the most obvious is the variation, or dispersion, of price relatives about their average values. It is this factor of dispersion which now calls for consideration.

### 1. NATURE AND SIGNIFICANCE OF PRICE DISPERSION

The nature of price dispersion is indicated by the column diagrams plotted above in Figure 21. In some years there is a fairly close concentration of the various price relatives about their mean values. The figure for the year 1926, showing unweighted link relatives on the natural scale, exemplifies this situation. In other years there is a wide scatter, the individual price relatives deviating materially from the average. The column diagram of unweighted fixed base relatives on the natural scale for the year 1918 illustrates dispersion of this type.

The dispersion of price relatives has practical significance from two different viewpoints. If our purpose be to measure the degree of change in the price level between two different dates we may treat each price relative as a single observation, a single attempt to measure the change in the purchasing power of the dollar. When the price relatives are closely concentrated about their average value we have a situation corresponding to that which prevails when a very accurate field piece is directed upon a target, or when a group of experienced surveyors measure a certain distance. Each individual observation is marked by a very small margin of error. The average of these individual observations carries, therefore, considerable weight as a close approximation to the true value. When, on the other hand, the price relatives are widely dispersed each one gives a measure of the change in the purchasing power of the dollar which is marked by a wide margin of error, and the average has less significance as a representative figure. This must be taken to mean that the force which is affecting the price level (or the purchasing power of the dollar) is less direct in its incidence, that it is obscured and complicated by the action of a diversity of factors which affect individual commodity prices. Each price relative constitutes a less accurate observation upon the change in the purchasing power of the monetary unit than is the case when the relatives are closely concentrated.

An index number of prices of the usual type may be looked upon, therefore, as a measure of the intensity of the force, or combination of forces, which is affecting the general price level. An

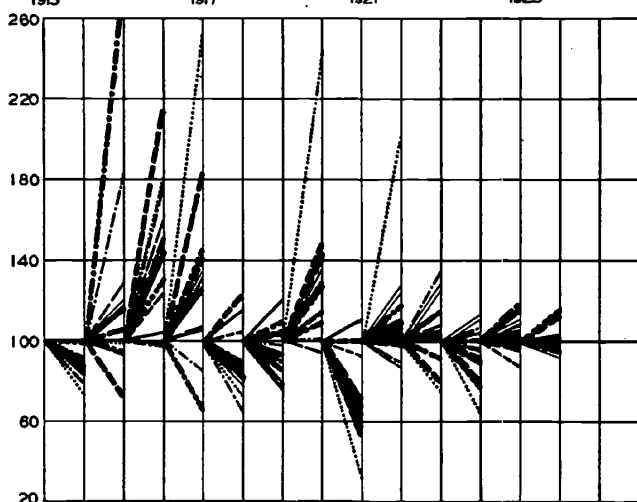
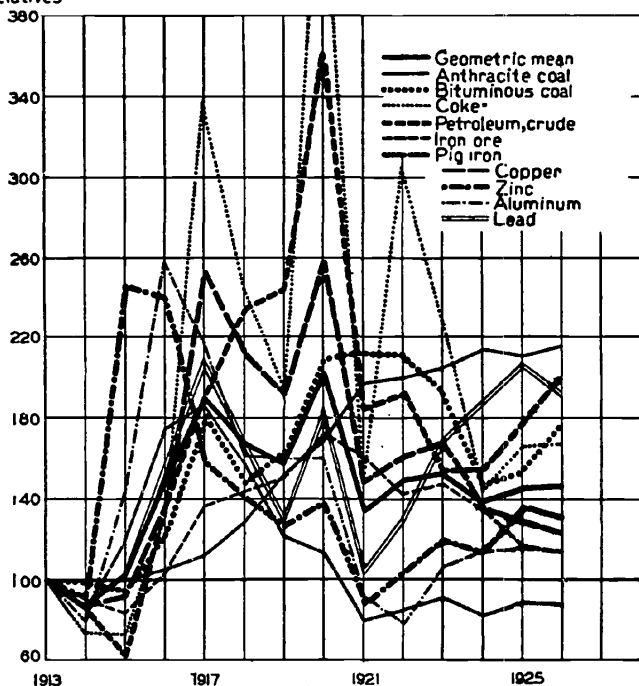
# ILLUSTRATIONS OF PRICE DISPERSION

## FIGURE 25

Relative Prices of Ten Minerals, with their Geometric Mean, 1913-1926.

(1913 = 100)

Scale of  
relatives



## FIGURE 26

Link Relatives of Ten Minerals, with their Geometric Mean, 1913-1926.

(The x-scale and the legend of Figure 25 apply also to Figure 26.)

index of dispersion is a measure of the intensity of the disruptive forces, the forces which are operating not to change the price level but to change individual commodity prices in unequal degrees. The less direct the incidence of the force which is acting upon the price level, and the greater the relative importance of the host of specific price-making factors which affect individual commodities, the more widely dispersed will the price relatives be. The validity and the significance of any index number depend, therefore, upon the dispersion of the price relatives upon which it is based, and a measure of dispersion is a necessary complement to such an index number.

The disruptive forces possess interest and significance in their own right. For every inequality of movement affects the buying and selling relations upon which the movement of goods depends. Every inequality of movement introduces some element of instability into the price system. This point may perhaps be made clearer in terms of specific commodity prices. Figure 25 shows the movements from 1913 to 1926 of the relative prices of ten commodities, on the 1913 base. Only ten are plotted, for the sake of simplicity. A truer picture of the actual situation would be given if 400 or more series were represented. In Figure 26 link relatives of the same commodities are plotted. In both charts the movements of the geometric means of the individual relatives are indicated. The point need not be argued that the differences in the degree of price change shown in these charts involve material readjustments in trade relations. The importance of these relations is obvious when the commodities considered are those entering into a connected series of trading operations, such as cattle, hides, leather and shoes. Movements of relatives derived from wholesale prices of these commodities are shown in Figure 27.

Individual commodity prices are constantly changing, and since most price-making factors are localized and specific in their incidence, any large group of commodities will show many inequalities in the direction and degree of change. Such inequalities are more pronounced at certain times than at others. Interest attaches, therefore, to variations in the degree of dispersion, and to the relation of such variations to fluctuations in the price level and to changes in business conditions.

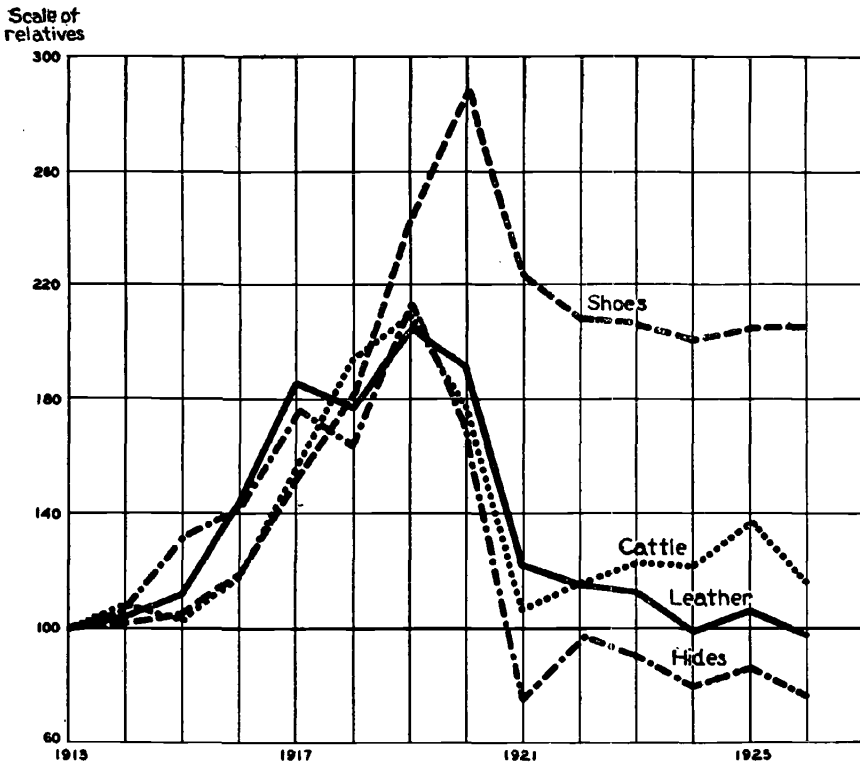
These matters have received some attention from students of prices in the past. F. Y. Edgeworth touched upon this subject in his classic *Memoranda*.<sup>1</sup> Wesley C. Mitchell has made use of an in-

<sup>1</sup>Reprinted in *Papers Relating to Political Economy*, MacMillan, London, 1925, Vol. I.

FIGURE 27

DISPERSION OF RELATED PRICE SERIES.

Relative Prices of Cattle, Hides, Leather and Shoes, 1913-1926.\*  
(1913 = 100)



\*The prices here plotted relate to the commodities numbered 15, 42, 166 and 441.

genious method of picturing price dispersion, and has discussed the relation of price dispersion to changes in the general price level.<sup>1</sup> Norman Crump, in a more recent study,<sup>2</sup> has tested various methods of measuring price dispersion and has suggested that it may be possible to trace some relation between the degree of dispersion and the future course of the price level.

It remains to test these relations, and to inquire as to the

<sup>1</sup>*Business Cycles*, University of California Press, 1913. *The Making and Using of Index Numbers* (Part I of *Bulletin 284*, U. S. Bureau of Labor Statistics), 1921.

<sup>2</sup>"The Interrelation and Distribution of Prices and their Incidence upon Price Stabilization." *Journal of the Royal Statistical Society*, Vol. 87, Part II, 1924.

general economic significance of price dispersion. The first step is the construction of a suitable measure of dispersion.

## 2. THE MEASUREMENT OF PRICE DISPERSION

Most of the customary measures of variation have been applied from time to time in the measurement of price dispersion. The *modulus* ( $\sigma\sqrt{2}$ ) was employed by Edgeworth, though the use to which he put the measure differed somewhat from that in mind at present. Wesley C. Mitchell made effective use of deciles in picturing the dispersion of price relatives. Dr. Silverstolpe has employed the mean deviation.<sup>1</sup> Irving Fisher, in connection with the studies described in *The Making of Index Numbers*, made use of the standard deviation, computed from relative prices and from logarithms of relative prices. Norman Crump experimented with the arithmetic standard deviation and the logarithmic standard deviation. The measures finally employed by Crump were three in number, the arithmetic standard deviation, the arithmetic coefficient of variation, and a measure of the "angle of deviation," derived from the standard deviation and the arithmetic mean.<sup>2</sup> A. L. Bowley has made use of the "mean percentage divergence," a measure similar to the mean deviation, except that the variations which are averaged are the percentage deviations of individual relatives from their geometric mean.<sup>3</sup>

In the present study there have been employed several measures of dispersion, corresponding to the different averages cited above. The basic measure, in every case, is the standard deviation. This is in natural form when the price relatives are combined as natural numbers. If we assume, in handling price relatives, that we are dealing with material which has most significance in logarithmic form, then the standard deviation in logarithmic form, or some derivative of it, is the logical measure to employ. This measure has precisely the same meaning in respect to logarithms that the standard deviation has in dealing with natural numbers. For convenience in interpretation the standard deviation has been modified

<sup>1</sup>Silverstolpe's measures have been published in the *Göteborgs Handels och Sjöfartstidning*.

<sup>2</sup>Crump's measures of dispersion are now published currently in the *Financial Times* of London.

<sup>3</sup>"Relative Changes in Price and other Index Numbers", *London and Cambridge Economic Service*, Special Memorandum No. 5, February, 1924. It may be noted that Professor Bowley uses the term "variability of prices" for what is here termed price dispersion. The term variability is used in the present study in a different sense.

somewhat in securing the measures of dispersion. A fractional part of the standard deviation (.6745σ, the quantity which defines the limits of the 50 per cent zone, in a normal curve) has been employed, and this has been expressed in each case as a percentage of the average to which it relates. The term "index of dispersion," as used in this study, refers to the measure derived from the logarithmic standard deviation.<sup>1</sup>

The measure of dispersion derived from natural numbers gives the percentage limits, measured from the arithmetic mean, within which 50 per cent of the price relatives would fall if the distribution of price relatives were normal. The index of dispersion derived from logarithms gives the approximate percentage limits, measured from the geometric mean, within which 50 per cent of the price relatives would fall if the distribution of logarithms of price relatives were normal. The assumption that the distributions are normal is not true either of natural or logarithmic distributions, except in occasional instances, but the validity of the measure, for comparative purposes, does not rest upon the truth of this assumption. Essentially, the standard deviation in natural and logarithmic form is being used as a measure of dispersion. A fractional part of the

<sup>1</sup>The following formulas indicate the processes employed in computing the measures of dispersion from relatives in natural and in logarithmic form:

$$\text{Measure of dispersion (natural)} = \frac{.6745\sigma}{M} \times 100$$

$$\text{Logarithmic standard deviation} = \sigma_{\log} = \sqrt{\frac{\Sigma(\log m - \log G)^2}{N}}$$

$$\text{Index of dispersion} = \frac{[\text{anti-log}(+.6745 \sigma_{\log}) - 1] + [1 - \text{anti-log}(-.6745 \sigma_{\log})]}{2} \times 100$$

The symbols employed above are

- m: a price relative
- M: the arithmetic mean of price relatives
- G: the geometric mean of price relatives
- σ: the standard deviation of price relatives
- σ<sub>log</sub>: the standard deviation of logarithms of price relatives.

A given value of the standard deviation in logarithmic form represents, of course, a certain percentage deviation above the geometric mean and a different (smaller) percentage deviation below the geometric mean. It seems desirable, in converting to natural numbers, to average these percentage deviations, instead of using the two percentages. A true account of the percentage dispersion which corresponds to a given distribution of logarithms would only be given by the two figures, but the convenience of a single measure outweighs the slight loss of accuracy resulting from the averaging. The formula for the index of dispersion indicates the method of averaging. The following figures illustrate the process:

Arithmetic mean of unweighted logarithms of link relatives, 1892	=	1.97523
Geometric mean	=	94.5
Standard deviation of logarithms (σ <sub>log</sub> )	=	.04870
.6745σ <sub>log</sub> .	=	.03285
Index of dispersion = $\frac{[(1.079 - 1) + (1 - .928)] \times 100}{2}$	=	7.6



standard deviation is cast into percentage form, in order to secure a measure which has immediate significance and which may be interpreted in terms which will be generally understood.

Thus we might describe the price change between 1891 and 1892 in this fashion: The unweighted geometric mean of 195 price relatives in 1892, on the 1891 base, is 94.5. The index of dispersion has a value of 7.6 which indicates that approximately 50 per cent of the individual link relatives differ from the mean, 94.5, by more than 7.6 per cent, while roughly 50 per cent differ from the mean by less than 7.6 per cent. This situation may be compared with that in 1920. In the latter year the geometric mean of relatives on the 1919 base is 111.0; the index of dispersion is 15.7 per cent, more than twice as great as in 1892.

The various measures of dispersion secured in analyzing frequency distributions of annual price relatives are presented in Table 98. Brief comparisons of the different types of dispersion measures may precede the more general discussion of their significance.

The dispersion of fixed base relatives is greater than the dispersion of link relatives in all cases except where the two coincide (i. e. in the first year after the base year, in each of the three periods covered by the fixed base relatives). This is, of course, to be expected. The scatter of price relatives when the base is two years or more distant is naturally greater than the scatter with reference to the preceding year as base.

A detailed comparison of the dispersion of weighted<sup>1</sup> and unweighted price relatives shows that there is no constant relation between the two sets of measures. Under usual price conditions the weighted and unweighted indexes of dispersion follow much the same path. This is particularly marked in the case of logarithms of link relatives, though even here there are certain years when the differences are material. During the war years the unweighted dispersion indexes rose to higher levels than the weighted. For fixed base relatives in natural form the dispersion of the unweighted relatives was very much greater than that of the weighted relatives the difference being accentuated from 1915 to 1918. For all types of distributions, covering the years from 1891 to 1926, the dispersion of the weighted relatives was less than that of the unweighted relatives 74 out of 136 times. (The comparison in each case is between distributions differing only in the matter of weighting.)

<sup>1</sup>The weights employed are given in Appendix Table I.

TABLE 98

DISPERSION OF WHOLESALE PRICES IN THE UNITED STATES, 1891-1926

(1) Year	(2) No. of price series	(3) Measures computed from fixed base relatives <sup>1</sup>				(4) Measures computed from link relatives			
		Unwtd. arith-metic	Wtd. arith-metic	Unwtd. geo-metric	Wtd. geo-metric	Unwtd. arith-metic	Wtd. arith-metic	Untwtd. geo-metric	Wtd. geo-metric
1891	195					8.9	9.7	8.8	10.6
1892	195	7.2	7.2	7.6	7.3	7.2	7.2	7.6	7.3
1893	195	10.9	12.8	10.6	12.2	8.6	8.6	8.3	8.3
1894	195	11.4	12.6	12.3	13.3	7.6	7.8	7.8	7.9
1895	195	12.8	17.0	12.7	14.5	9.2	10.6	9.0	9.8
1896	195	14.0	16.1	15.2	15.8	8.9	9.0	9.8	9.8
1897	195	13.1	11.6	14.4	12.9	10.8	9.5	10.4	9.7
1898	195	14.2	12.8	14.4	13.3	8.1	7.3	7.8	7.7
1899	195	15.2	17.0	15.7	16.1	11.9	11.9	10.3	10.5
1900	195	15.7	16.1	16.5	15.8	8.4	9.4	8.2	8.4
1901	195	16.5	15.5	16.6	15.5	8.8	7.7	8.5	7.9
1902	195	16.8	18.1	17.0	17.6	8.4	8.5	8.7	8.6
1903	205	7.6	9.2	7.7	9.2	7.6	9.2	7.7	9.2
1904	205	11.6	14.0	12.0	14.1	8.2	9.8	8.6	10.2
1905	205	12.6	13.5	11.5	13.5	6.5	7.1	6.8	7.4
1906	205	13.8	12.6	13.4	12.6	7.3	7.9	7.3	7.7
1907	205	14.0	12.3	13.7	15.7	7.1	5.4	7.4	5.7
1908	205	13.8	12.1	13.6	12.8	8.9	8.3	9.1	8.7
1909	205	13.6	13.6	14.0	13.9	7.4	7.8	7.2	7.6
1910	205	16.7	15.9	16.3	15.5	7.6	8.2	7.4	7.9
1911	205	20.4	14.8	17.5	14.6	9.9	9.1	9.3	8.7
1912	205	19.4	13.5	16.5	13.8	7.2	7.3	7.4	7.3
1913	205	16.5	13.9	15.7	14.1	7.6	8.7	8.0	8.4
1914	391	8.8	7.1	8.2	7.4	8.8	7.1	8.2	7.4
1915	391	43.5	21.6	17.9	12.6	25.9	14.0	15.0	10.6
1916	391	62.2	22.7	22.2	14.6	17.0	16.2	13.8	13.7
1917	391	47.9	23.0	23.7	20.1	13.0	12.3	13.8	12.4
1918	389	37.5	19.2	21.8	18.1	13.3	13.5	13.9	14.2
1919	391 <sup>2</sup>	21.8	16.4	18.4	17.8	16.3	12.0	16.6	11.9
1920	391	22.4	20.3	22.2	21.6	16.5	16.2	15.7	15.7
1921	391	22.1	21.2	23.3	23.5	17.3	18.0	18.3	18.3
1922	391	19.4	18.4	20.8	21.0	13.3	12.8	12.4	11.7
1923	390	19.7	18.8	21.2	21.5	9.7	11.0	9.6	11.0
1924	390	20.2	18.5	21.0	21.0	9.7	8.3	8.7	8.3
1925	387	18.1	16.2	18.8	16.3	10.4	13.9	9.0	11.5
1926	385	19.1	18.0	19.0	17.4	7.6	8.5	7.7	8.9

<sup>1</sup>Bases: first period, 1891; second period, 1902; third period, 1913.<sup>2</sup>389 link relatives were employed.

There is apparent a slight tendency for the dispersion of the un-weighted relatives to be greater than that of the weighted, a tendency which was pronounced during the years of violent price change succeeding the outbreak of the war. The fact that the weighted measure follows a less erratic course during the war years, coupled

with the initial assumption that the weighted measure is more representative of price conditions, has led to its general use in the present study.

The measures of dispersion derived from relatives in logarithmic and in natural form are directly comparable, since both have been expressed as percentages of the mean values. Of 136 pairs of measures the index numbers of dispersion derived from logarithms are found to be smaller in 70 cases. There is practical equality between the two. The natural measures, however, were marked by much wider fluctuations during the war years, this being particularly true of the fixed base relatives.

Before a choice can be made between arithmetic and logarithmic measures of dispersion, it is desirable that their relative reliability be determined. This reduces to the familiar problem of computing the probable or standard errors of these measures, in order to determine their liability to sampling fluctuations. The standard errors of the unweighted standard deviations are given in percentage form in the table on the next page.<sup>1</sup>

The greater reliability of the logarithmic measures of dispersion is manifest, from a survey of this table. In each of the 25 cases in which comparison is possible the standard error of the logarithmic measure is materially less than that of the corresponding arithmetic measure. The standard error of the logarithmic standard deviation of link relatives exceeds 2 per cent only once, and is in the neighborhood of 1 per cent for most of the years covered. The standard error of the arithmetic standard deviation of link relatives rises as high as 23 per cent in 1915. For the fixed base relatives the errors are greater. The largest error for the logarithmic measures is 4.0 per cent of the standard deviation, the largest for the arithmetic measures is 35.7 per cent. The reason for the much greater errors of the arithmetic measures, especially during the war

<sup>1</sup>The standard error of the standard deviation, for any type of distribution, is given by the formula:

$$\sigma_{\sigma} = \sqrt{\frac{\mu_4 - \mu^2_2}{4\mu_2 \cdot N}}$$

Since the moments of the logarithmic distributions are in terms of logarithms, the standard error emerges as a logarithm, while for the arithmetic measures it is in natural form. By taking the antilogarithm of the former, and expressing the latter as a percentage of the standard deviation to which it relates, these measures are cast into comparable form.

In applying the above formula for the standard error of the standard deviation, no allowance has been made for the intercorrelation between prices which was discussed in the preceding section. No correction is necessary for the comparisons which concern us at present. If interest attaches to the absolute values of these measures a correction similar to that explained on p. 247 should be made.

TABLE 99

STANDARD ERRORS OF STANDARD DEVIATIONS COMPUTED FROM PRICE  
RELATIVES, 1891-1926

(Expressed as percentages of the measures to which they relate)

(1) Year	(2) Standard error of standard deviation of unweighted fixed base relatives <sup>1</sup>		(4) Standard error of standard deviation of unweighted link relatives	
	Arithmetic	(3) Logarithmic	Arithmetic	(5) Logarithmic
1891				.81
1892	6.86			.90
1893	9.15			.94
1894	5.63			.78
1895	13.38			1.10
1896	8.58			1.21
1897	5.25			1.50
1898	8.05			.74
1899	7.26			1.26
1900	7.04			.80
1901	7.52			.83
1902	7.75			1.04
1903	5.85			.73
1904	6.22			.94
1905	8.57			1.09
1906	10.62			.83
1907	10.82			1.14
1908	7.45			.81
1909	8.32			1.04
1910	14.79			.87
1911	19.31			1.32
1912	19.84			.84
1913	11.04			1.00
1914	12.00	.94	12.00	.94
1915	29.74	3.86	23.52	2.77
1916	35.70	4.00	4.18	1.32
1917	34.99	3.04	4.84	1.28
1918	30.76	2.64	4.95	1.09
1919	20.73	1.71	8.75	1.69
1920	6.43	1.78	5.89	1.02
1921	5.00	1.84	3.48	1.04
1922	4.13	1.84	6.05	.84
1923	3.90	1.51	6.78	.72
1924	5.81	1.80	15.45	1.09
1925	5.12	1.23	13.74	1.01
1926	5.35	1.43	6.60	.76

<sup>1</sup>Bases: first period, 1891; second period, 1902; third period, 1913.

years, is found in the large second and fourth moments of the arithmetic distributions. The use of logarithms serves to reduce the dispersion and the lack of symmetry during times of pronounced price change.

These differences are so pronounced as to leave no room for

doubt as to which is the more reliable of these two types of measures. Index numbers of dispersion computed from different samples of a given population, using logarithms of price relatives, may be expected to fluctuate within relatively narrow limits. The reverse is true of arithmetic measures of dispersion. So large are the errors to which these measures are subject that the results secured from different samples may be expected to fluctuate within very wide limits, and little confidence may be attached to a given value. Because the index of dispersion computed from logarithms of price relatives seems, on the whole, to be much more reliable and more significant than the arithmetic measure, it has been employed generally in the present study.<sup>1</sup> The labor of computation, by the method suggested above (page 220), is no greater than that involved in computing the arithmetic standard deviation.

<sup>1</sup>This conclusion is not in accord with that reached by Mr. Norman Crump, who stated that "the logarithmic standard deviation. . . . simply leads back to the same result as is attained directly from the arithmetic standard deviation." ("The Interrelation and Distribution of Prices and Their Incidence upon Price Stabilization," p. 179.) The conditions under which Mr. Crump's statement would be true appear to be seldom realized in dealing with distributions of price relatives. As a relationship which holds approximately Crump gives  $\sigma_{\log} = \log G - \log(A - \sigma) = \log(A + \sigma) - \log G$ , where  $\sigma$  and  $\sigma_{\log}$  represent, respectively, the arithmetic and logarithmic standard deviations, and  $A$  and  $G$  represent, respectively, the arithmetic and geometric means. When the logarithmic standard deviation as thus derived is compared with that computed directly from the logarithms, material differences are found. For the year 1915 the error is equal to 86 per cent of the true value of the standard deviation; for the year 1916 the error amounts to 26 per cent of the true value. In 26 out of 36 years for which the relationship was tested the discrepancy exceeds 2.58 times the standard error of the logarithmic standard deviation. (These results relate to unweighted link relatives of average annual prices for the period 1891-1926.)

Another method of approximating the measure of dispersion derived from the standard deviation of logarithms has been suggested by Professor Irving Fisher. Using the symbol  $d$  for the antilogarithm of the logarithmic standard deviation, less unity, Fisher gives

$$1 + d = \frac{\sqrt{A^2 - AH} + A}{\sqrt{AH}}$$

where  $A$  and  $H$  represent, respectively, the arithmetic and harmonic means of the data. (*The Making of Index Numbers*, p. 392.) This approximation appears to be much closer than that suggested by Crump. For 391 unweighted link relatives in 1915 Crump's formula involved an error equal to 86 per cent of the true value; the result secured from Fisher's method differs by but 3.6 per cent from the true value. For 1916 the error in Crump's method amounted to 26 per cent of the true value; Fisher's method involved an error of 1.8 per cent. These were the two years of greatest disturbance, when the errors would be expected to be at their maximum.

Since the true value may be readily derived, and since some error is involved in using even Fisher's approximation, the value secured directly from the logarithms is probably to be preferred as a measure of dispersion.

Discussions of the logarithmic measure of dispersion are to be found in the *Journal of the Royal Statistical Society*, Vol. 86 (1923) pp. 428-430 (a review by Professor Yule of Fisher's *Making of Index Numbers*), and in the *Nordisk Statistik Tidskrift*, Band 2 (1923) pp. 402-408 ("Zweck und Struktur einer Preisindexzahl," by L. v. Bortkiewicz).

3. ANNUAL INDEXES OF PRICE DISPERSION, 1890-1926

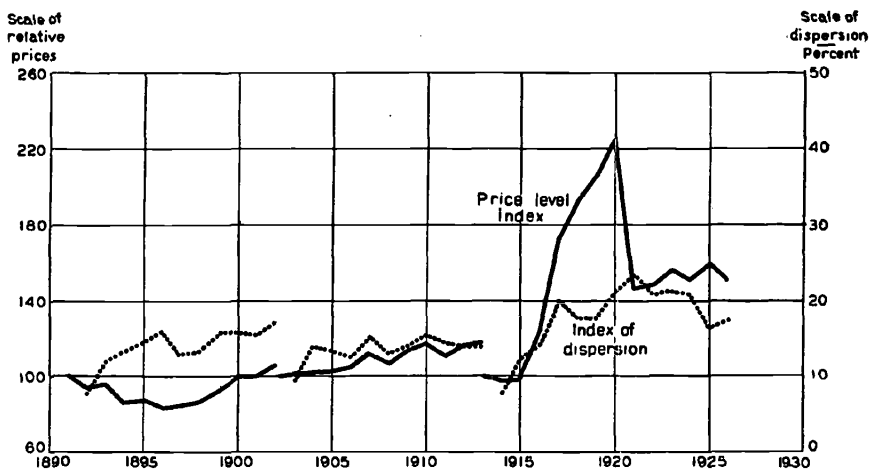
In Figure 28 the dispersion indexes based on weighted logarithms of fixed base relatives are plotted, together with the weighted geometric means of the same data. To facilitate comparison of the measures for different periods the dispersion indexes for the three periods are superimposed, in Figure 29. (The numerical values are given in Table 98.)

FIGURE 28

INDEX NUMBERS MEASURING CHANGES IN THE LEVEL OF WHOLESALE PRICES AND IN PRICE DISPERSION, 1892-1926.

Weighted Geometric Means and Indexes of Dispersion Computed from Fixed Base Relatives for the Periods 1891-1902, 1902-1913, 1913-1926.

(The first year in each period is the base for that period.)



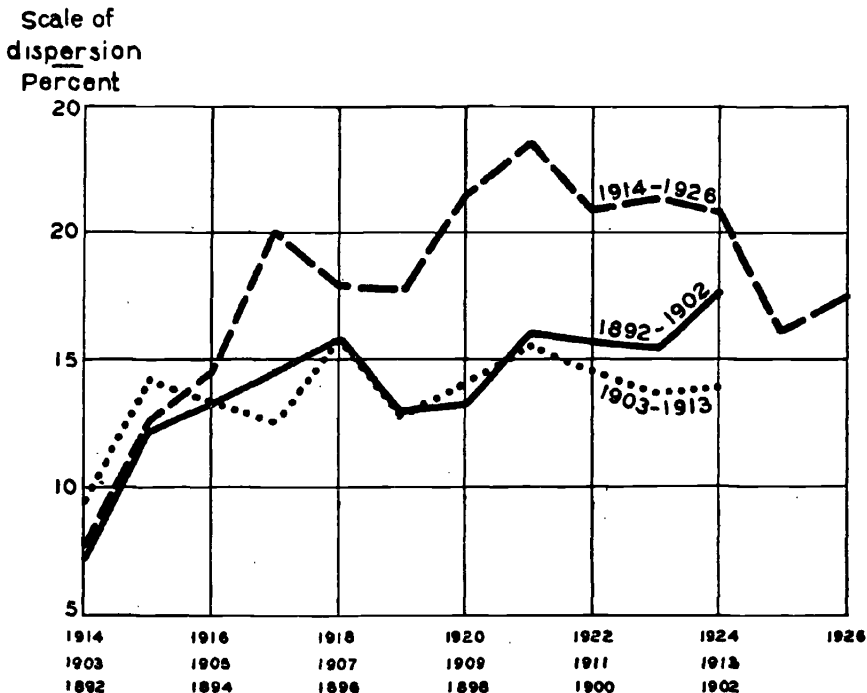
The general story which the charts tell is a fairly simple one. In the case of the fixed base relatives the indexes measuring the dispersion of fixed base relatives follow much the same course during the first two periods. There is a sharp initial increase followed by a much slower increase which, in the second period, becomes virtually oscillation about a constant level. During the last period the increase continues until 1921, with a sharp break, however, in 1918 and 1919.<sup>1</sup> After 1921 there is a pronounced decline to 1925,

<sup>1</sup>It is probable that the decline in dispersion in 1918 and 1919 was in part due to price regulation during the war.

with a minor increase in 1926. Comparing the three periods, we find that the index of dispersion was smallest (14.1) at the end of the second period (1902-1913), somewhat larger at the end of the third period (17.4) and greatest at the end of the first period (17.6). The disturbance of price relations between 1891 and 1902 was slightly greater, as measured by this index, than between 1913 and 1926, a somewhat surprising result in view of the violence of the price movements since 1913. In this comparison the reference is, of course, to the situations at the ends of the periods mentioned. If the intervening years be considered, the third period was marked by much greater disturbance than the first. By 1925, however, the dispersion of fixed base relatives had declined to a level that may be viewed as approximately normal, considering the time interval between the given year and the base year.

FIGURE 29

COMPARISON OF INDEXES OF PRICE DISPERSION COMPUTED FROM FIXED BASE RELATIVES FOR THE PERIODS 1891-1902, 1902-1913, 1913-1926.



During each of the three periods covered by the fixed base measures described above, the initial movements of the indexes of dispersion have been upward. This upward trend continued, as has been noted, throughout the whole of the first period, but in the second and third periods it was checked. The maximum value in the second period (1902-1913) was recorded in 1907, while the maximum value in the third period (1913-1926) came in 1921. These results bear upon a question of some importance: Does the dispersion of fixed base relatives tend to increase indefinitely, the further removed the base becomes, or is there a "ceiling" to the movements of dispersion indexes of this type? Is there a critical value beyond which the index of dispersion cannot, or does not, rise? The evidence of the indexes for the separate periods defined above is conflicting on this point, and the periods covered are too short to permit of generalization. In order to secure an index covering a longer period the series of price relatives on the 1891 base were carried forward through 1926.<sup>1</sup> Measures computed from these relatives are given in Table 100, on the next page. These measures are plotted, on the ratio scale, in Figure 30.

This index shows that the dispersion of relatives on the 1891 base increased, with minor interruptions, between 1892 and 1921, and that there was a slight decline between 1922 and 1925, with an upturn in 1926. We may, however, secure a truer account of the behavior of this index by breaking the entire period into two parts, the first including the years 1892 to 1914, the second the years from 1915 to date. During the first period we find the rapid initial increase in dispersion and the subsequent slowing up which has been described by Wesley C. Mitchell, Irving Fisher and others. The flattening out process which marks the course of the index on the ratio chart was interrupted by the price disturbances of the war period. Between 1915 and 1921 the upward movement of the index was accelerated. The slightness of this acceleration gives evidence of the stability which the index of dispersion had attained by 1915. By that year the relatives on the 1891 base were widely dispersed, and although the violent price disturbances of the next several years increased the dispersion somewhat, the effect was not pronounced. It may be noted that the index of dispersion in 1917, on the 1891 base, with a value of 26.8, exceeded by only a slight margin the index for the same year (1917) on the 1913 base. The

<sup>1</sup>The number of price series used in these computations was 195, except for the years 1918 and 1925, when 194 series were used, and 1926, for which 193 series were available.



TABLE 100

INDEXES OF PRICES AND OF PRICE DISPERSION, 1891-1926

Unweighted geometric averages and indexes of dispersion computed from relatives on the 1891 base.

(1) Year	(2) Geometric mean	(3) Index of dispersion
1891	100.0	
1892	94.5	7.6
1893	93.9	10.6
1894	84.3	12.3
1895	82.8	12.7
1896	79.6	15.2
1897	79.3	14.4
1898	83.0	14.4
1899	90.2	15.7
1900	97.3	16.5
1901	96.3	16.6
1902	99.6	17.0
1903	100.3	17.6
1904	99.5	16.6
1905	101.6	16.8
1906	106.7	17.6
1907	113.0	19.3
1908	104.7	18.8
1909	107.6	19.6
1910	111.5	21.6
1911	109.3	21.3
1912	113.2	21.7
1913	113.6	22.8
1914	113.0	22.6
1915	115.6	24.8
1916	145.4	24.9
1917	199.9	26.8
1918	232.8	29.1
1919	239.9	27.3
1920	269.7	30.4
1921	176.9	30.7
1922	176.0	28.8
1923	187.6	29.2
1924	184.9	29.1
1925	191.5	28.3
1926	183.6	29.6

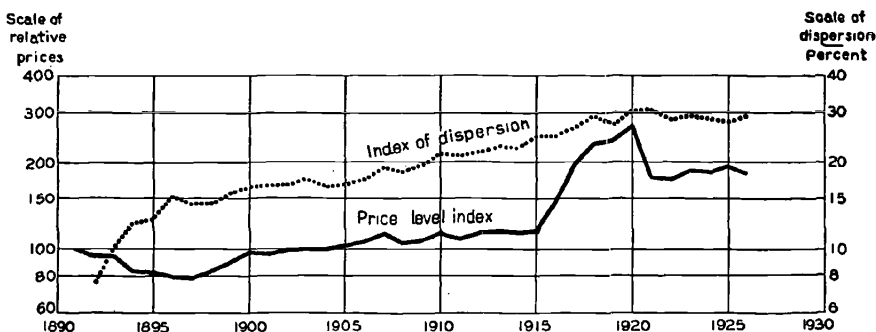
value of the latter, as computed from unweighted relatives, was 23.7.

The record for the years 1892 to 1921 indicates that the dispersion of fixed base relatives may increase over a long period of years. There may be an ultimate check to this movement, but we cannot at present say whether the 1921 value represents a true upper limit or only a temporary high. There are *a priori* reasons for expecting that, in the absence of such exceptional disturbances as the war years brought, the dispersion of fixed base relatives would increase at a decreasing rate, tending finally toward approximate

FIGURE 30

INDEX NUMBERS MEASURING CHANGES IN THE LEVEL OF WHOLESALE PRICES AND IN PRICE DISPERSION, 1891-1926.

Unweighted Geometric Means and Indexes of Dispersion  
Computed from Fixed Base Relatives.  
(1891 = 100)



stability at a relatively high level. This means that after a certain degree of dispersion has developed there may be material changes in the relative positions of fixed base relatives, but that the movement is confined within fairly definite percentage limits of the mean.<sup>1</sup> The behavior, during the years preceding 1914 and the years following 1921, of the index of dispersion on the 1891 base indicates that such a tendency is present.<sup>2</sup>

When we deal with link relatives the increase in dispersion due to the constantly increasing time interval between the given years

<sup>1</sup>The length of time which elapses before the stage of approximate stability is attained probably varies, depending in part upon the price situation in the year chosen as base for the relatives, in part upon developments during particular periods of time. The results secured from the study of the three sets of fixed base relatives employed in the present study point to such variation. Professor A. L. Bowley's figures, relating to the dispersion of 60 price relatives on the 1901 base ("Relative Changes in Price and other Index Numbers," p. 7), show an initial period of increase to 1906, with no regular tendency to increase between 1905 and 1913. It will be recalled that the relatives for the second period (1902-1913, on the 1902 base) show a similar tendency, maximum dispersion being recorded in 1907.

<sup>2</sup>There is, perhaps, an analogy between fixed base price relatives and biological characteristics of the human race, in respect to dispersion. Concerning the latter Galton has written: "If family variability had been the only process in simple descent that affected the characteristics of a sample, the dispersion of the race from its mean ideal type would indefinitely increase with the number of generations, but reversion checks this increase, and brings it to a standstill." (*Typical Laws of Heredity in Man*, p. 10.) The existence of an apparent "ceiling" to the dispersion of price relatives indicates that there is a check, also, to the indefinite increase in the dispersion of prices. Certain economic factors which would tend to check the indefinite scatter of prices come to mind at once, but we may not speculate here on the character of the forces which bring about regression or reversion in the movements of price relatives.

and the base year is not present as a complicating factor. The dispersion of link relatives may be studied by comparing the successive frequency distributions of link relatives shown in Figure 21. The modifications during the price revolution of the war and post-war years are particularly marked. The peaked distributions of the earlier years are flattened and spread out, to contract again in the years following 1921.

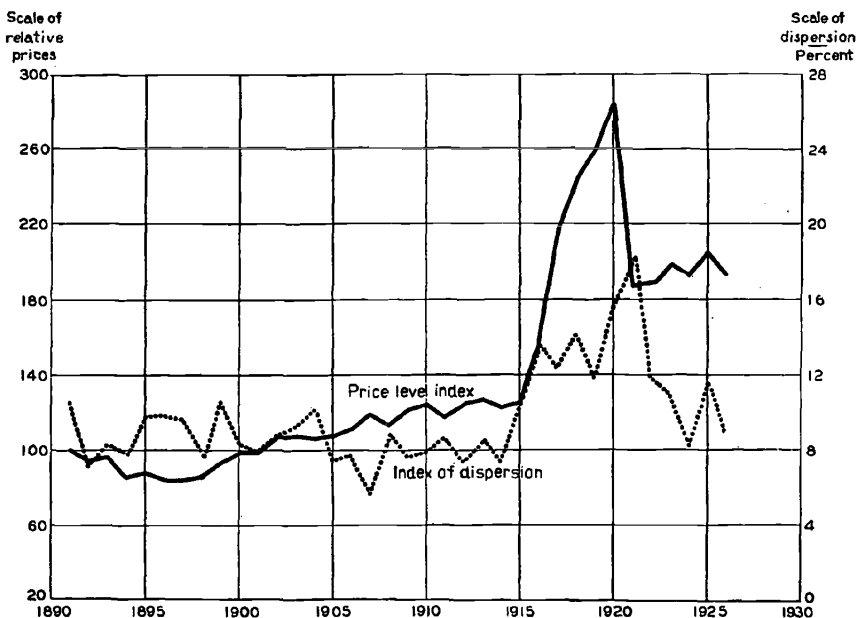
This comparison of frequency distributions is suggestive, but lacks precision, in so far as the measurement of dispersion is concerned. A more accurate representation of the changes in the dispersion of link relatives from year to year is afforded by Figure 31, in which is plotted an index of dispersion computed from weighted logarithms of annual link relatives for the period 1891-1926. (The figures appear in Table 98). The geometric means of link relatives, chained on the 1891 base, are plotted in the same figure.

FIGURE 31

INDEX NUMBERS MEASURING CHANGES IN THE LEVEL OF WHOLESALE PRICES AND IN PRICE DISPERSION, 1891-1926.

Weighted Geometric Means and Indexes of Dispersion  
Computed from Annual Link Relatives.

(The geometric means are chained on the 1891 base.)



The general movement of this index of dispersion was gradually downward during the period prior to 1914. Relatively high values were obtained in 1891, from 1895 to 1897, in 1899 and in 1904. During the years of prosperity between 1904 and 1907 the degree of dispersion declined. The dispersion of prices in 1907 was the lowest recorded in the entire study. There was a sharp increase with the readjustments between 1907 and 1908, followed by an irregular downward movement to 1914. From 1905 to 1914 the average value of the index of dispersion was 7.7.

Between 1914 and 1916 there was a further increase in dispersion, the figure for the latter year being materially greater than any recorded during the 25 years preceding. The cause is found, of course, in the readjustments due to the European war. The index of year-to-year dispersion remained at approximately the 1916 level in 1917 and 1918, with a minor decline in 1919, following the end of the war. The downward movement lasted but the one year, however. In 1920 and 1921 there were upward movements comparable to those of 1915 and 1916, but on a much higher level. The disturbance of price relations, as judged from the index of dispersion, was greater between 1920 and 1921 than it was in any period of equal length since 1890. One may go further. The disturbance between 1920 and 1921, as measured by a dispersion index of 18.3, was greater than the disturbance between 1891 and 1902 (index of dispersion, relatives on 1891 base, = 17.6), and exceeded the disturbance between 1902 and 1913 (index of dispersion, relatives on 1902 base, = 14.1). In this one interval of twelve months there was a more violent "scattering" of prices than during either of the two eleven year periods named.

Following the extreme dispersion recorded between 1920 and 1921 the index declined materially in 1922 and 1923, and in 1924 fell to 8.3. This represents an approximately normal degree of dispersion, judged by pre-war standards. Not since 1914 had a comparable value been recorded. There was a considerable rise (compared with pre-war changes) in 1925, followed by a decline in 1926.

### §An Index of Dispersion Based on Group Index Numbers

An index of dispersion may be derived from the 28 sub-group index numbers which the Bureau of Labor Statistics now publishes in its annual volumes on *Wholesale Prices*. The absolute values thus secured differ, of course, from those obtained when all the individual price relatives are employed, but general changes in price dispersion are reflected

in this index. The labor of computation is much lighter when the sub-group index numbers are employed.

The process is identical with that followed when the data are individual price relatives. The geometric mean of the 28 relative numbers and the logarithmic standard deviation are computed. (No weights were used in these calculations, although weights had been employed, of course, in the computation of the original group index numbers.) From the latter measure is derived an index of dispersion of the type previously described. This index is shown below, together with the index of dispersion computed from the individual price relatives, weighted. Results secured from both link and fixed base relatives are given.

TABLE 101

COMPARISON OF INDEXES OF DISPERSION DERIVED FROM INDIVIDUAL PRICE RELATIVES AND FROM SUB-GROUP INDEX NUMBERS OF THE UNITED STATES BUREAU OF LABOR STATISTICS

(1) Year	(2) Measures of dispersion of fixed base relatives (1913=100)		(4) Measures of dispersion of link relatives	
	Derived from individual price relatives, weighted	Derived from 28 sub-group index numbers	Derived from individual price relatives, weighted	Derived from 28 sub-group index numbers
1914	7.4	5.3	7.4	5.3
1915	12.6	11.2	10.6	9.3
1916	14.6	12.2	13.7	11.5
1917	20.1	13.7	12.4	9.0
1918	18.1	12.2	14.2	10.7
1919	17.8	12.9	11.9	11.4
1920	21.6	15.6	15.7	9.9
1921	23.5	17.1	18.3	12.9
1922	21.0	17.9	11.7	7.6
1923	21.5	17.3	11.0	7.1
1924	21.0	16.4	8.3	5.3
1925	16.3	13.5	11.5	5.9

The index of dispersion computed from the group index numbers fluctuates on a lower plane than that based upon individual relatives, but it traces much the same general course. The most pronounced difference between the two measures relating to fixed base relatives is that the index based on groups shows the dispersion to have been at its maximum in 1922, whereas the other index reached its peak in 1921. In the case of link relatives, the two index numbers move in opposite directions between 1918 and 1919 and between 1919 and 1920, but there is agreement at all other dates. More confidence must attach to the indexes based on individual relatives, but the group index numbers provide simple and fairly accurate measures of the general changes in the degree of dispersion. For certain purposes the differences between the various sub-group index numbers might be of more interest than differences between individual price relatives.

#### 4. MONTHLY INDEXES OF PRICE DISPERSION, 1906-1908, 1920-1927

In the computation of monthly measures of dispersion the same method has been employed as in handling annual data. The monthly index of dispersion is thus a measure, in percentage form,

of the approximate limits of the zone within which half of the price relatives in a given month will be found. Each relative is secured by expressing the price of a commodity in a given month as a percentage of the price of the same commodity in the preceding month.

In Table 102, below, are given index numbers of dispersion computed from weighted logarithms of monthly link relatives for the years 1906-1908, 1920-1926. The relatives were derived from the price quotations for 100 important commodities, selected from the general list given in Appendix Table I.<sup>1</sup>

TABLE 102

INDEXES OF DISPERSION COMPUTED FROM MONTHLY LINK RELATIVES OF  
COMMODITY PRICES, 1906-08, 1920-26

(Based upon the weighted relatives of 100 commodities)

(1) Month	(2) 1906	(3) 1907	(4) 1908	(5) 1920	(6) 1921	(7) 1922	(8) 1923	(9) 1924	(10) 1925	(11) 1926
Jan.	3.7	2.9	4.2	5.7	6.5	5.6	4.1	4.0	4.7	4.4
Feb.	3.4	2.3	2.4	5.3	7.5	4.9	5.0	4.6	5.0	3.6
Mar.	2.4	3.3	4.7	4.9	5.5	4.5	3.7	5.0	5.7	4.6
Apr.	2.8	2.5	5.1	8.2	4.3	2.7	3.4	3.0	4.2	2.4
May	2.8	4.6	4.0	5.2	6.3	4.8	3.8	3.4	4.0	3.6
June	3.2	3.9	4.4	6.0	4.4	4.6	3.1	4.1	4.0	2.6
July	2.3	3.0	3.7	4.6	6.1	4.6	3.7	4.0	3.2	2.6
Aug.	3.1	3.1	3.7	5.9	4.3	6.9	3.6	4.6	2.8	2.6
Sept.	2.6	3.2	2.9	7.2	8.2	5.2	4.5	4.8	3.8	4.0
Oct.	3.0	4.5	4.2	6.8	6.2	3.9	4.9	4.2	4.7	6.0
Nov.	2.6	5.1	2.9	6.6	5.6	5.0	5.5	4.0	4.1	4.4
Dec.	2.2	3.9	2.4	6.7	2.6	3.3	3.7	3.5	5.3	5.4

<sup>1</sup>Following are the numbers of the commodities employed. These may be identified by reference to Table I. Commodities which were substituted for others in some of the calculations are indicated by the numbers in parentheses.

Farm products (14 commodities): nos. 1, 2, 4, 6, 14, 15, 18 (1906-08:19), 25, 31, 37, 42, 47, 51, 56.

Foods (18 commodities): nos. 64, 69, 70 (1906-08:71), 74 (1906-08:76), 78, 84 (1925:83, 1906-08:89), 106, 109, 116, 118 (1906-08:119), 120, 132 (1906-08:128), 135 (1906-08:130), 138, 146, 149, 150, 158.

Cloths (18 commodities): nos. 166, 178 (1906-08:177), 180, 183, 185, 187, 191, 195, 196, 199, 203, 206, 210, 213, 216, 217, 221, 226.

Fuel (8 commodities): nos. 233, 235, 237, 238, 239, 243 (1906-08:244), 246 (1906-08:247), 249 (1925:248).

Metals (12 commodities): nos. 259, 260, 266, 269, 275, 279 (1906-08:282), 284 (1906-08:280), 293, 295, 296, 300, 302.

Building materials (9 commodities); nos. 303, 310 (1906-08:306; 1925:311), 317 (1925:316), 324, 327 (1925:328), 330, 338, 343, 348.

Chemicals (5 commodities): nos. 360, 361 (1906-08:362), 376 (1906-08:377), 382, 393, 396.

House-furnishings (7 commodities): nos. 406, 412, 414, 415, 418, 424, 428.

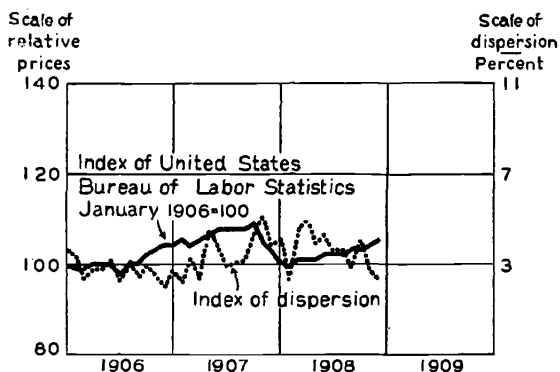
Miscellaneous (8 commodities): nos. 437 (1906-08:436), 441, 444, 449 (1906-08:448), 450, 451, 454 (1906-08:455), 457.

The index for the years 1906-1908 is plotted in Figure 32, together with an index of changes in the general level of prices. January, 1906, is taken as the base of the price index.

FIGURE 32

MONTHLY INDEX NUMBERS OF WHOLESALE PRICES AND OF PRICE DISPERSION, 1906-1908.

(Changes in the level of prices are measured by the index of the United States Bureau of Labor Statistics, with the base shifted to January, 1906. The index of dispersion is computed from monthly link relatives, weighted, of 100 commodities.)



The story covering the three pre-war years is fairly clear. During the 16 months preceding May, 1907, the index of dispersion of monthly link relatives had an average value of about 3. Between September, 1906, and February, 1907, there was a relatively sharp increase in the level of wholesale prices (an increase of 6 per cent), but there was no material change in the degree of dispersion. In May and June of 1907 there was a sudden increase in dispersion, then a drop back again to a level of about 3 until October. Thereafter, until November, 1908, the index of dispersion remained high, with the exception of sudden declines in February and September, 1908. That is, the disturbance of price relations continued not only during the sharp liquidation from November, 1907, to February, 1908, but also during the ensuing depression. The highest single value, 5.1, was recorded in November, 1907 (the first month of sharp decline in the price level) and in April, 1908. There was a decline in the degree of dispersion beginning in May, 1908, but not until November, when the worst of the depression was over and the general price index had started a definite upward movement, did the index of dispersion return to the 1906 level.

It is worthy of note that the graph of the dispersion index gives a saw-tooth effect, with sharp movements from month to month. In February, 1908, in the middle of the period which was marked by a fairly high degree of price disturbance, there was a single low value, even below the 1906 level. This is probably in part accounted for by the smallness of our sample (100, for these monthly observations) and the relatively high probable error of the measure of dispersion based upon weighted measures. The somewhat erratic movements probably reflect, in addition, definite changes in underlying price conditions. They suggest that during liquidation and the accompanying price disturbance there are relatively calm spells. From January to February, 1908, there seems to have been such a spell, while from September to October, 1908, there was a sharp disturbance, breaking the decline in the index which was then in process. Because of the relative smallness of the sample, however, such a generalization may only be put forward tentatively.

Omitting the 13 months from October, 1907, to October, 1908, we secure 3.0 as the average value of the monthly index of dispersion. This will serve for comparison with the post-war figures.

The process of post-war price readjustment, in so far as it is measured by the dispersion of monthly link relatives, is portrayed in Figure 33. The monthly index of dispersion here plotted is based upon substantially the same commodities (100 in number) as were employed in constructing the index for 1906-08. A monthly index showing changes in the level of wholesale prices, with reference to January, 1921, as base, is also plotted.

The dispersion during the later period is upon a distinctly higher level than during the pre-war years, this being particularly marked during 1920 and 1921, when the month-to-month dispersion was greatest. During the liquidation and depression of 1907-08 the value of the dispersion index fluctuated between 3 and 5. In 1920 and 1921 it fluctuated between 4.5 and 8. (The index, being logarithmic, is not affected by the difference in price levels.) There is evidence here that the degree of disturbance in 1920 and 1921 was materially greater than in 1907-08, and that, during the years 1922-1926, the month-to-month price variations were more pronounced than they were in a relatively stable period before the war.

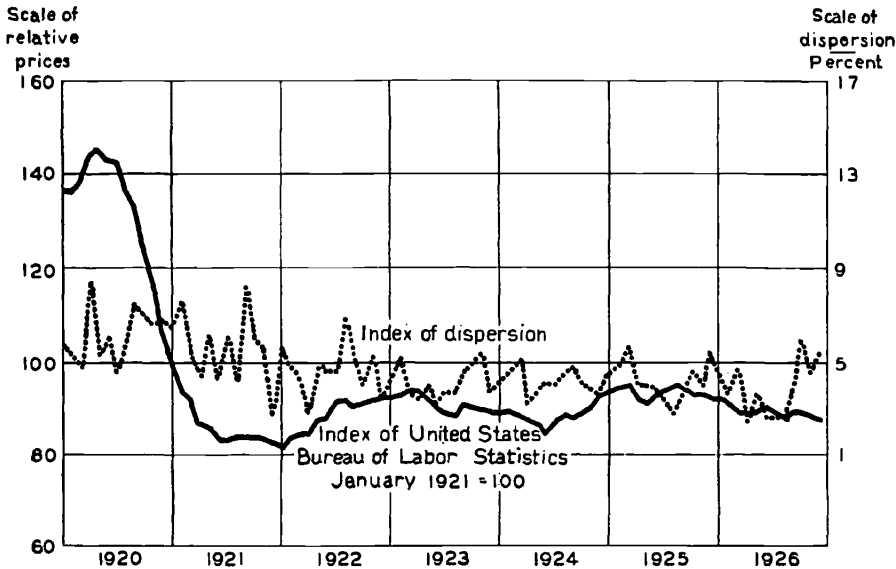
The graph of the index of monthly dispersion from 1920 to 1926 gives a picture of a spasmodic but none the less clearly marked process of stabilization. From an average value of about 7 during the months of most severe readjustment at the end of 1920 and the



FIGURE 33

## MONTHLY INDEX NUMBERS OF WHOLESALE PRICES AND OF PRICE DISPERSION, 1920-1926.

(Changes in the level of prices are measured by the index of the United States Bureau of Labor Statistics, with the base shifted to January, 1921. The index of dispersion is computed from monthly link relatives, weighted, of 100 commodities.)



beginning of 1921, the index declined to an average of 4.1 during the last four years covered. This last figure, it has been noted, is well above the average value (3.0) during the fairly stable months of the period 1906-1908.

The broken and uneven character of the decline from 1920 to 1923 reinforces the conclusion, suggested above, that the economic changes which are reflected in such an index are themselves spasmodic, and that there are months of slight change even during a period of crisis and severe liquidation. July, 1920, April, June, August, and December, 1921, were months of comparative calm. Here again the smallness of the sample (100) and the use of weights which differ materially in value<sup>1</sup> may explain some of these irregu-

<sup>1</sup>An unweighted index of dispersion, based upon the same commodities as were employed in constructing the weighted index, was computed for the months of 1920 and 1921. The weighted and unweighted measures show no difference of any consequence except for September, 1921, when the unweighted index rose to only 6 per cent, as compared with 8 per cent for the weighted. They are very close together at all other points. It is not likely, therefore, that the disparity of weights accounts for the sharp changes from month to month.

larities, but they are probably due in part to an inherent characteristic of price changes from month to month during a period of disturbance. Even in relatively quiet times the changes in dispersion from month to month may vary considerably, but the range of the movements is much less than during the critical period of liquidation.

Another method of measuring price changes has been employed by A. W. Flux and, in the measurement of dispersion, by Norman Crump. This method, based upon twelve-month link relatives, was described in an earlier section. (A twelve-month link relative is derived by expressing the price of a commodity in a given month as a percentage of its price in the same month of the year preceding.) In the following table are given geometric means and indexes of dispersion computed from such twelve-month relatives, weighted, for the 100 commodities listed above.<sup>1</sup> The period covered extends from January, 1920, to September, 1927. The measures for the years 1920-1926 are plotted in Figure 34.

The index of dispersion computed from twelve-month link relatives differs in several respects from the two types already employed, and throws an interesting light upon the price movements of this period. Since the dispersion which is being measured is that which occurs over twelve months, the average value of the index is considerably higher than that of the one-month link relatives. It is also noticeably higher than the dispersion of annual link relatives. The latter, of course, are based upon average annual prices, and the process of averaging would be expected to reduce the dispersion of the individual figures. Finally, the twelve-month index is not marked by the sharp irregularities which were so pronounced in the index based upon monthly links. It follows a much smoother course, and its major fluctuations are clearly apparent. This, again, is to be expected from the nature of the data. The minor month-to-month changes which reflect the play of random forces are smoothed out before twelve months have elapsed. A change which persists after twelve months is, presumably, related to major price and economic changes. The elimination of seasonal influences by the use of a twelve-month period, a point suggested by Crump, is another factor of some consequence in reducing the irregularities found in the index of monthly link relatives.

<sup>1</sup>See footnote, p. 271. In computing the measures for 1926 and 1927 certain minor omissions and substitutions were made.

TABLE 103

GEOMETRIC MEANS AND MEASURES OF DISPERSION COMPUTED FROM TWELVE-MONTH  
LINK RELATIVES OF COMMODITY PRICES, 1920-1927

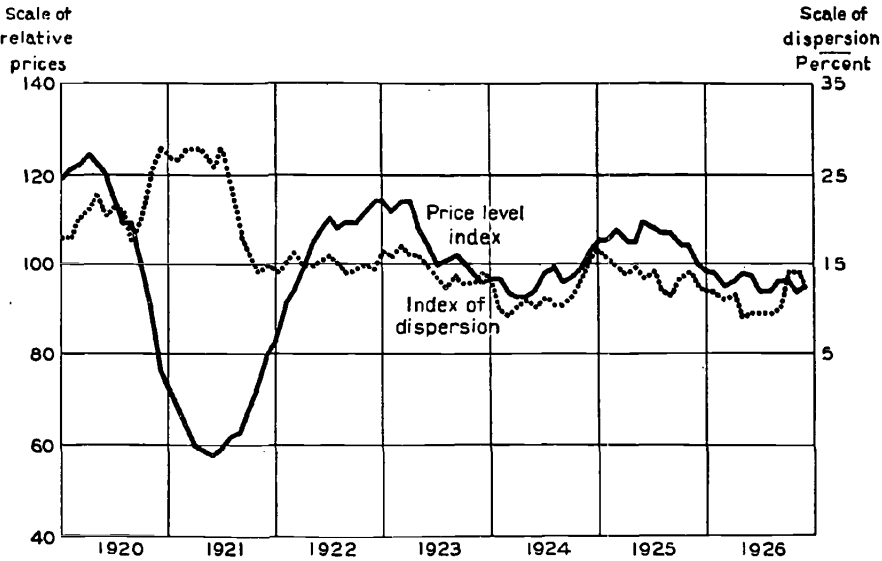
(Based upon the weighted relatives of 100 commodities)

Month	Geometric mean	Index of dispersion	Geometric mean	Index of dispersion
		1920		1921
Jan.	118.8	17.9	72.6	27.2
Feb.	120.5	18.2	68.5	26.7
Mar.	121.6	19.9	65.0	27.9
Apr.	124.2	21.0	59.7	27.8
May	122.1	23.1	58.9	27.6
June	120.4	20.5	57.7	26.1
July	113.4	21.3	59.1	28.0
Aug.	108.5	20.3	61.0	23.4
Sept.	108.6	17.6	62.7	18.7
Oct.	97.9	21.0	68.4	16.5
Nov.	89.5	25.3	72.7	14.5
Dec.	78.3	28.0	80.0	15.2
		1922		1923
Jan.	83.0	14.7	114.2	16.3
Feb.	91.0	15.2	111.9	16.2
Mar.	93.8	16.5	114.3	17.1
Apr.	98.6	15.2	114.1	16.3
May	103.9	15.1	107.9	15.8
June	108.0	15.7	104.6	15.2
July	109.9	15.9	100.4	13.4
Aug.	108.1	15.2	101.2	12.3
Sept.	108.7	14.1	102.0	13.4
Oct.	109.1	14.3	100.2	13.0
Nov.	112.2	15.0	97.6	13.1
Dec.	113.7	14.4	96.1	14.2
		1924		1925
Jan.	96.7	13.3	104.9	16.4
Feb.	96.5	9.8	104.9	15.3
Mar.	93.9	9.7	106.5	14.5
Apr.	92.9	10.4	104.6	14.0
May	93.4	10.9	104.9	14.7
June	93.5	10.4	108.5	13.7
July	97.8	11.5	107.5	14.1
Aug.	99.2	10.5	106.7	11.9
Sept.	95.9	10.5	107.1	11.5
Oct.	97.2	11.3	104.4	13.4
Nov.	99.4	13.0	104.2	14.0
Dec.	102.8	15.3	99.9	12.5
		1926		1927
Jan.	98.2	12.0	93.5	12.2
Feb.	97.6	12.2	93.9	11.3
Mar.	94.6	11.1	95.2	10.3
Apr.	96.3	11.4	95.0	11.4
May	97.9	8.9	95.1	10.7
June	97.1	9.5	94.5	12.4
July	94.2	9.5	96.5	12.2
Aug.	93.8	9.3	97.5	12.7
Sept.	95.6	10.1	98.3	12.9
Oct.	95.9	13.9		
Nov.	93.9	14.0		
Dec.	94.6	12.6		

FIGURE 34

MONTHLY INDEX NUMBERS OF WHOLESALE PRICES AND OF PRICE DISPERSION, 1920-1926.

Weighted Geometric Means and Indexes of Dispersion Computed from Twelve-Month Link Relatives of 100 Commodities.



The general swings of the dispersion index tell a fairly consistent story of the changes in the degree of price disturbance during this period. From January to May, 1920, this index increased from 18 per cent to 23 per cent, values which are quite high in comparison with the index in more normal years. The first effect of the break in the price level was a sharp drop in dispersion, a decline of 5.5 points in 4 months. There is an indication here, similar to that found in certain of the studies of regional price differences, that the immediate effect of a major price recession is in the direction of equalization. The continuation of the decline brought an abrupt change after September, 1920. Within three months after that date the index had risen by more than ten points. This sharp increase probably reflects the spread of price difficulties and the beginning of drastic and general liquidation in the last quarter of 1920. The index of dispersion continued at a very high level until July, 1921. Then, as the mean values of the twelve-month link relatives started upward again, the degree of dispersion declined sharply. By No-

vember, 1921, the dispersion had declined to a level approximately equal to that maintained in the later years. The disturbance of prices appears, on this basis, to have been most acute between December, 1920, and July, 1921. By November, 1921, the exceptional difficulties were past.

During the rise of prices to a peak early in 1923 the index of dispersion fluctuated about a fairly constant level. The peak for this movement was attained in March, 1923, when the dispersion index had a value of 17.1. Thereafter both price and dispersion indexes declined. The index of dispersion reached a low of 9.7 in March, 1924. The price decline of 1923-24 appears, thus, to have effected a substantial reduction in price inequalities (if we may consider the scatter of prices to be an evidence of inequalities). The decline of 1920-21 had the same final effect, but in the course of this decline there developed inequalities greater than those which had existed at the peak of prices.

Following the low of March, 1924, the index of dispersion rose slightly until September, then more sharply, reaching a peak, in January, 1925. The rise was more pronounced than any that had occurred since 1920, but the level attained was low in comparison with the 1920-21 values. This rise in 1924 accompanied a distinct upward movement in average prices. From January, 1925, to May, 1926, there was an irregular but substantial decline in dispersion, the index falling from 16.4 to 8.9. Succeeding this fall came a minor increase in dispersion, and this was followed by twelve months of fluctuation about a constant and relatively low level.

The index of dispersion based upon twelve-month link relatives is not marked by the irregularities which detract from the utility of the index computed from monthly links. It possesses the smoothness which a fixed base index would have, but is more sensitive to current price movements than is the fixed base measure. It is free, moreover, from the tendency toward a secular increase in dispersion which affects an index derived from fixed base relatives. We have seen that for annual comparisons measures of dispersion computed from annual link relatives seem preferable to other types. When monthly prices are employed, an index of dispersion computed from twelve-month link relatives appears to throw most light upon changes in internal price relations.

## 5. THE DISPERSION OF PRICES AND CHANGES IN THE PRICE LEVEL

In the preceding account of the dispersion of prices passing reference has been made to the relation between price dispersion and movements of the general level of prices. This subject requires somewhat more extended treatment.

Certain previous investigations have touched upon this subject. As a result of these studies the theory has been advanced that with a rising price level the dispersion of price relatives increases, while the degree of dispersion declines with falling prices. In the first edition of *Business Cycles* Wesley C. Mitchell illustrated the dispersion of relative prices by means of a chart showing the movements of the deciles. The period covered extended from 1890 to 1910. The data were relative prices on the base 1890-1899. From a study of the movements of deciles computed from these relatives Mitchell concluded that "concentration around the median becomes denser when prices fall and less dense when they rise," and that "Relative prices are squeezed together by the pressure of business depression, and spring apart when the pressure is relaxed by returning activity."<sup>1</sup> F. Y. Edgeworth reached a somewhat similar conclusion, namely, "when the index number was rising there was much greater disturbance of the standard deviations of price relatives." Although Edgeworth was of the opinion that, in general, the inferential connection ran "from rising index number to greater standard deviation,"<sup>2</sup> he suggested that there might be great changes in dispersion without corresponding changes in the price level. Norman Crump, in the study of price dispersion to which reference has been made, agrees with the general thesis that rising prices mean increasing dispersion, and suggests that changes in the price level may be predicted from this relationship. A rising price level, in Crump's view, is accompanied by an increase of dispersion. Sooner or later the price maladjustments which result from wider dispersion become so acute that trade is thrown out of gear and collapses. "The rise in prices contains the seeds of its own decay." Conversely, he suggests, it may be possible to set a limit to the fall in prices if we can determine the minimum value below which the measure of dispersion cannot fall. Internal evidence, says Crump, may enable us to deduce the future trend of the price level. Finally, Lucien March has observed, in the movements of certain French,

<sup>1</sup>*Business Cycles* (1913), p. 110.

<sup>2</sup>*Journal of the Royal Statistical Society*, Vol. 87, Part II (1924) p. 207.

British and American price series, that the dispersion of price relatives increases as the mean value rises.<sup>1</sup>

The results of the present study show that there is a connection between price dispersion and changes in the price level, but there does not appear to be such a simple and direct relation as that suggested by the authorities quoted. In considering this relationship we may distinguish between price dispersion with reference to a fixed base and dispersion from year to year, measured in terms of link relatives. In respect to fixed base relatives the evidence of Figures 35 and 29 is relevant. Figure 35 shows the actual standard deviations of weighted fixed base relatives, in arithmetic form. The measures for the three periods covered are superimposed in the diagram, to facilitate comparison. The annual values here

<sup>1</sup>"Les modes de mesure du mouvement général des prix" *Metron*, Vol. I, no. 4 (1921) p. 83.

The data upon which M. March bases his statement possess considerable interest, since they are drawn from different countries and different periods. The following extract summarizes the material bearing upon the present point.

	France		Great Britain		United States	
	1913	Dec.1919	1913	Dec.1919	1909	Dec.1919
Number of articles.....	55	54	42	38	203	44
Base of relatives.....	1890-99	1901-10	1890-99	1901-10	1890-99	1901-10
Mean value of relatives.....	113	502.4	113.5	298	123	258
Standard deviation.....	22.5	206.3	20.0	111	31.8	126
Coefficient of variation.....	20	41	18	37	26	49

M. March comments upon these results: "On remarquera d'abord que la dispersion des prix s'accroît quand leur valeur moyenne augmente d'une manière notable. Ainsi l'indice moyen étant égal à 123 aux États-Unis en 1909, tandis qu'il ne dépasse pas 113 ou 113.5 en France et en Angleterre en 1913. L'écarte quadratique moyen est sensiblement plus élevé dans le premier pays que dans les deux autres.

De même si l'on compare les deux époques, avant et après la guerre, on constate qu'à l'accroissement des prix correspond un accroissement plus considérable de leur dispersion.

Le coefficient de variation lui-même est, dans tous les cas, à peu près double en 1919 ou 1920 de ce qu'il était avant la guerre."

The fact to which M. March calls attention, that the rise to a higher post-war price level was accompanied by a general increase in dispersion, is not to be controverted. It is unfortunate that the above figures contain no example of dispersion with a declining price level. It would be of interest to have for the recession of 1920-21 measures of dispersion (absolute and relative) of the price series included in March's calculations for 1919.

The subsequent discussion in the text will bear upon the general relation between price level and dispersion. In connection with the above figures, attention may be called to one exception to the relationship commented upon by M. March. The December, 1919, average for Great Britain is 298, that for the United States is 258. Both the measure of dispersion and the coefficient of variation are appreciably smaller for Great Britain, the country having the greater change in average prices.

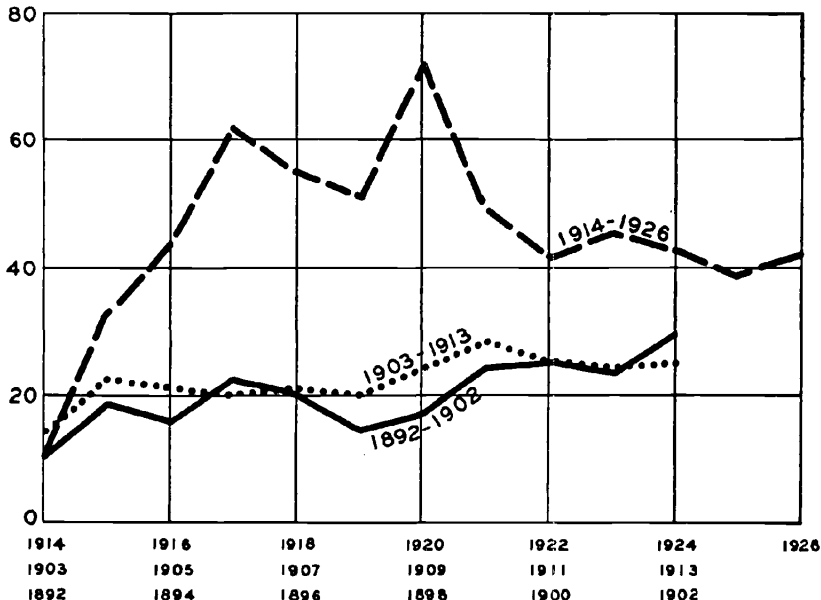
plotted are given in Appendix Table XX, with the corresponding arithmetic averages.

FIGURE 35

MEASURES OF DISPERSION COMPUTED FROM FIXED BASE RELATIVES, BY PERIODS, 1892-1926.

(The measures plotted are the standard deviations of weighted fixed base relatives, in natural form. The bases of the relatives are average prices in the years 1891, 1902 and 1913.)

Scale of dispersion



During the first period (1891-1902) a falling price level to 1896 was marked by a sustained increase in dispersion, broken only by two slight declines in 1894 and 1896. By 1896 the degree of dispersion of relatives on the 1891 base was as great as the dispersion in 1907 of relatives on the 1902 base. During the eleven year periods, 1891-1902 and 1902-1913, the courses of the indexes of dispersion were much the same, though the movements of the price level in these periods were distinctly different. The violent price changes of the third period brought a much greater degree of dispersion. Here again, however, we have results which contradict the theory of a direct and positive relation between changes in the price level and



changes in the degree of dispersion. From 1917 to 1919 the price level rose materially; the standard deviation declined ten points. The pronounced drop in the price level between 1920 and 1921 was accompanied by a material decline in the absolute dispersion, a change which accords with the theory mentioned. But from 1921 to 1925, during which the average rose by over 10 points, the standard deviation declined more than 8 points.

These absolute measures of dispersion are misleading, however, and comparison of such measures is hardly justifiable. When the averages of the different distributions differ materially the measures of variation to be compared should be relative rather than absolute. The indexes of dispersion derived from the logarithms of relative prices and expressed as ratios or percentages serve this purpose. These appear in Figure 29. Here the approximate identity of movement during the first two periods is even more apparent than in the preceding chart. The only difference of any moment appears at the ends of the periods. In 1902 the index of dispersion of the 1891 relatives was 17.6 and the geometric mean was 106.0. In 1913 the index of dispersion of the 1902 relatives was 14.1, and the geometric mean was 118.4. Though prices rose considerably more during the second period, the dispersion of relatives at the end of this period was less than at the end of the first period.

The index of dispersion for the third period traces a course even more widely different from that which the theory of a direct relation between price level changes and dispersion would lead one to expect. The great price rise up to and including the year 1919 carried the index of dispersion to exceptional heights only in 1917. From 1914 to 1916 it is no greater than the values of the indexes of dispersion for the corresponding years of the two preceding periods, and in 1918 and 1919 it exceeds the corresponding measures of the earlier periods by amounts which are small in comparison with the price level changes. From 1920 to 1924 the index of dispersion is considerably higher than at corresponding years during the first two periods. From 1920 to 1921, when the price level fell from 225 to 147 (1913=100), the index of dispersion increased from 21.6 to 23.5. In 1925, when the geometric mean was 159.4, the index of dispersion stood at 16.3. This figure, for a year 12 years after the base year, is less than that of 1902 (17.6), a year 11 years removed from the base year and having a price level of 106.0 (compared with 100 in the base year, 1891).

There is no evidence here of a consistent relationship between

changes in the price level and changes in the degree of dispersion, when measures of dispersion are put on a comparable basis. There is a tendency, which is particularly marked during the several years immediately following the base period, for the dispersion of fixed base relatives to increase as the base year becomes further removed. When the movement happens to coincide with a rising price level, as in the years between 1896 and 1920, there is an apparent connection between rising price level and increasing dispersion.<sup>1</sup> The results of the present study indicate that there is no true relationship here. This is clear even when the dispersion is measured in absolute units. It is somewhat more apparent when the measures of dispersion are expressed in relative terms, which permit accurate comparison.

It is relevant to the present point to note that the highest value of the index of dispersion in the second period was attained in 1907, while in the third period the peak came in 1921. The former was a year of prosperity (judged from annual averages, with which we are here concerned), while the latter was a year of depression. This does not accord with Crump's suggestion that there may be a maximum degree of dispersion which precedes price recession, and a minimum degree of dispersion which forecasts a price rise.

A somewhat more accurate impression of the degree of relationship between shifts in the price level and changes in the degree of dispersion may be obtained by computing the coefficient of correlation between the geometric means of price relatives and corresponding indexes of dispersion. In the case of measures derived from

<sup>1</sup>The form of this theory which was advanced some years ago by Wesley C. Mitchell is perhaps attributable to the nature of the price relatives with which he dealt. His prices were expressed as relatives on the base, 1890-99 = 100, and covered the years 1890-1910. There is to be expected an increasing concentration around the median as the middle years of the base period are approached, and a decreasing concentration after these middle years are passed. If the base period were a single year, in the middle of the period covered by the relatives, the measure of dispersion would fall to zero in this year. And if this base year happened to be the year when the price level was at bottom, after a period of falling prices and before a period of rising prices, it would appear that there was a close connection between the direction of price change and the degree of dispersion. This connection would be purely fortuitous, due to the choice of a base year. Some such fortuitous element may be present in Dr. Mitchell's evidence, since the years which were given greatest weight in computing the bases of his price relatives came at the end of a long price fall and at the beginning of a sustained rise.

The use of a natural scale on Mitchell's chart tends, also, to suggest a positive relation between changes in the price level and changes in the degree of dispersion. In a later improvement upon this early graph Dr. Mitchell has given a very effective representation of price changes, plotting upon a logarithmic scale deciles derived from annual link relatives. (See *The Making and Using of Index Numbers*, 1921, p. 15.) The relation between price level changes and dispersion is more accurately pictured in this later chart than in the earlier one.

fixed base relatives, first differences (i. e. absolute changes from year to year) are employed in this calculation, in order to eliminate, in part, the effects of the somewhat similar secular movements in the two variables. The coefficient, based upon annual measures for the years 1892 to 1926, has a value of  $+ .057$ . There is no indication here of a significant relationship.

A similar result is secured when the measures of dispersion and price level changes are derived from link relatives. It is not apparent from Figure 31 that there is any consistent relationship between the degree of dispersion of link relatives and the changes in the price level. The sagging prices of 1892-1896 were accompanied by a steady increase in dispersion. The rising prices from 1904 to 1907, leading up to the panic of 1907, were accompanied by a pronounced decline in dispersion, while the price fall in 1908 was marked by an increase in dispersion. From 1920 to 1921 we have the same sharp difference in direction of movement noticed with the fixed base relatives. The coefficient of correlation between the geometric means of link relatives and the corresponding indexes of dispersion, for the 36 years from 1891 to 1926, has a value of  $+ .019$ . There is no relationship here. High dispersion is equally likely to be found with rising or with falling prices.

We have not, however, exhausted the possibility of discovering a relationship between price level changes and dispersion. It may be that dispersion depends upon the violence of the price change, regardless of direction. Correlating the index of dispersion based on link relatives with the percentage of change in average prices from year to year, taking no account of sign, we secure a coefficient of  $+ .614$ . We may conclude that in so far as the dispersion of prices is affected by changes in the price level it is the degree, or violence, of the change, not the direction of change, which is important.<sup>1</sup> The coefficient is not high enough, however, to indicate that changes in the price level are the only factors affecting dispersion. There appear to be internal movements, unconnected with changes in the price level, which the index of dispersion enables us to follow.

It may develop that one of the most important and useful features of the index of dispersion, as computed from link relatives and expressed in percentage form, is that its value is largely independent of the actual level of prices. In so far as internal stability is related to the price level, it depends upon the violence of changes in that level, not upon the direction of change. Hence the

<sup>1</sup>There is reason to think that the degree of relationship varies from period to period. See the note on p. 364.

index of dispersion may be interpreted in terms of a standard which is not a function of the general price level. (The index of dispersion derived from link relatives may have a trend of its own, but that trend does not appear to be related to the trend of average prices.) If the width of the zone of normal dispersion does not depend upon the price level, we have a means of evaluating price stability which is independent of long-term price trends.

In 1896 this dispersion index had a value of 9.8, the general price index being then 66 with reference to a 1913 base. In 1926 the dispersion index had a value of 8.9, while the index of wholesale prices had a value of 151, on the 1913 base. The price level was more than twice as high, but the zone of year-to-year dispersion, in percentage form, was narrower. We may, perhaps, look upon 10 per cent as the upper limit to the value of this index of dispersion under usual price conditions.

It does not appear from this survey that indexes of dispersion provide a direct means of forecasting changes in the price level, as was suggested by Crump. They possess value not as forecasting devices, but as means of describing one important aspect of a given price situation. Price stability and the normal functioning of the economic system depend as much upon the maintenance of established internal relations (more exactly, perhaps, upon the avoidance of violent changes in these relations) as upon external equilibrium of the kind associated with a stable price level. The index of dispersion is one important measure of the degree of internal change between specific dates. The excessively high level of the index of dispersion between November, 1920, and September, 1921 (as shown in Figure 34) is perhaps more immediately significant of the troubles which afflicted business at that time than is the falling index of general prices. And that the troubles of the business world since 1921 have been relatively mild ones is evidenced by the minor fluctuations of the dispersion index since that year.

Internal instability of the type which an index of dispersion would reveal may appear as a result of a broken and uneven price advance, as an accompaniment of an irregular price decline, or it might possibly develop with no appreciable change in the price level. The price rise which culminated in 1920 brought wide dispersion, but the ensuing decline brought an even greater scatter of prices. The return to internal equilibrium after the development of such a disturbed condition may come as a result of a decline in the level of prices or may be accomplished by the stabilization of prices at a

constant level. The attendant circumstances, considered in connection with the evidence of the dispersion index, may suggest the nature and direction of the change which the restoration of internal equilibrium will involve.

### V Price Displacement

It was suggested at an earlier point that our present problem is essentially that of measuring price instability, and a distinction was made between instability of the price level and internal instability. Internal instability was defined, provisionally, as the condition which develops when a set of established price relations is disturbed. One measure of such instability, the index of dispersion, has already been discussed. We advance in the present section to a further consideration of the problem of measuring those internal disturbances in price relations which are of such profound importance in the everyday processes of buying and selling.

A measure of dispersion, by itself, is inadequate to describe all the alterations in price relations which take place between given dates. This is apparent from a study of Figures 25 and 26, which show the movements of the relative prices of ten commodities from 1913 to 1926. It is clear that the degree of dispersion varies, year by year, and that this dispersion changes the relations between the prices of the individual commodities here presented. But it is also clear that there are other changes in relationship which elude measurement by the index of dispersion. The lines representing fixed base relatives (Figure 25) are constantly crossing and re-crossing. The dispersion of relative prices in two years may be approximately the same (a condition exemplified by unweighted relatives, on the 1913 base, in 1919 and 1924), but the commodities may stand in quite a different order. This shifting of relative position may affect buying and selling relationships just as much as would a change in the degree of dispersion.

The nature of this internal shifting, which is here called *price displacement*, may be made clearer by a hypothetical example. There are sketched in Figure 36 the movements of the relative prices of four commodities, represented by the letters A, B, C and D, from 1913 to 1916. Prices in 1913 furnish the bases of the relatives. If the movements in the prices of these four commodities are to be followed by means of the average alone, no change will be noted between the four years covered. The average is 100 throughout.