Volume Title: A Theory of the Consumption Function

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Volume Publisher: Princeton University Press

Volume ISBN: 0-691-04182-2

Volume URL: http://www.nber.org/books/frie57-1

Publication Date: 1957

Chapter Title: The Implications of the Pure Theory of Consumer Behavior

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Chapter URL: http://www.nber.org/chapters/c4404

Chapter pages in book: (p. 7 - 19)

CHAPTER II

The Implications of the Pure Theory of Consumer Behavior

THE relation between the theoretical constructs used in consumption research and the observable magnitudes regarded as approximating them has, I believe, received inadequate attention. It therefore seems desirable to start by setting forth in considerable detail the implications of the pure theory of consumer behavior, even though this involves repetition of some familiar material.

1. *Complete Certainty*

Let us consider first the behavior of a consumer unit under conditions of complete certainty. It knows for certain, we suppose, that it will receive a definite sum in each of a definite number of time periods; it knows the prices that will prevail for consumer goods in each period and the rate of interest at which it can borrow or lend. Under these conditions there are only two motives for spending on consumption less or more than it receives in any time period. The first is to "straighten out" the stream of expenditures—by appropriate timing of borrowing and lending, the unit can keep its expenditures relatively stable even though its receipts vary widely from time period to time period. The second is to earn interest on loans, if the interest rate is positive, or to receive payment for borrowing, if the interest rate is negative. How it will behave under the influence of these motives depends, of course, on its tastes—the relative utility it attaches to consumption at different points of time.[1]

To facilitate graphic presentation, consider the special case of two discrete time periods, say years 1 and 2.[2] The relevant features of a consumer unit's tastes at a point in time, say year 1, can then

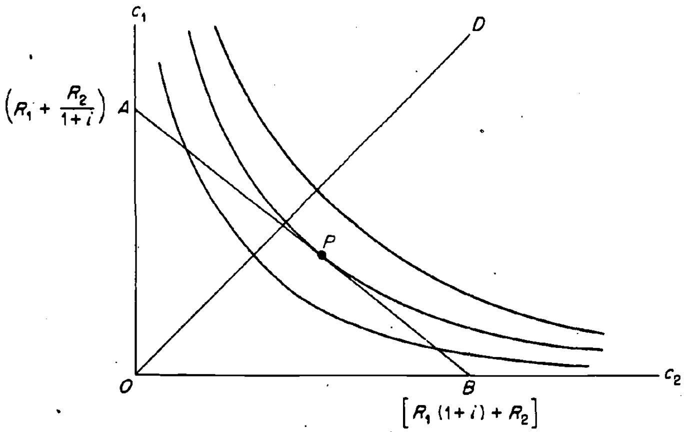
[1] See Irving Fisher, *The Rate of Interest* (New York: Macmillan, 1907), esp. Chap. VI, pp. 87–116; *The Theory of Interest* (New York: Macmillan, 1930), esp. Chaps. X and XI.

[2] The analysis of this special case is essentially identical with that given by Fisher, *The Rate of Interest*, pp. 387–392, and by Kenneth E. Boulding, *Economic Analysis*, Rev. ed. (New York: Harper, 1948), pp. 734–741.

be summarized by a two-dimensional system of indifference curves, as in Figure 1. c_1, measured along the vertical axis, is the money value at year 1 prices of services consumed in year 1; c_2, measured along the horizontal axis, is the money value at year 2 prices of services consumed in year 2. A point on the diagram thus represents a particular combination of consumption in the two years. Underlying each such point is already a prior maximization process: the expenditures represented by the corresponding c_1 and c_2 are supposed optimally distributed among the various consumption services for the given prices. As always, a single indifference curve

FIGURE 1

Hypothetical Indifference Curves and Budget Lines of a Consumer Unit for Consumption in Two Time Units



is the locus of combinations of c_1 and c_2 among which the consumer unit is indifferent—as it views the situation in year 1. The slope of the indifference curve at any point gives the rate at which it is willing to substitute consumption in year 2 for consumption in year 1. For the usual reasons, the indifference curves can be taken to be negatively sloped and convex to the origin.

Let R_1 and R_2 be the consumer unit's expected receipts in year 1 and 2 respectively, and i the interest rate. The maximum amount the unit can spend in year 1 if it spends nothing in year 2 is $R_1 + [R_2/(1 + i)]$, that is, its receipts in year 1 plus the maximum loan it can repay with its receipts in year 2. The maximum amount it can spend in year 2 if it spends nothing in year 1 is $R_1(1 + i) + R_2$ or its receipts in year 1 plus the interest it would earn if it loaned out the whole of its year 1 receipts, plus its receipts in year 2. A straight line between these two points (AB in Figure 1) then defines the combinations of consumption in the two years that are open to the consumer unit; it can attain any point in the triangle *OAB*. If we suppose that the two years stand for the whole future for which plans are being made, there is nothing that the unit can gain by not spending all it receives, so that the combination chosen will be on the budget line *AB*. The optimum combination is, of course, the point at which the budget line is tangent to an indifference curve, point *P* in Figure 1.

We have introduced three variables to describe the consumer unit's opportunities: R_1, R_2, and i. However, it is clear from the diagram that consumption in year 1 depends in any meaningful way not on three variables but only on two: the slope of the budget line and its position. Changes in R_1 or R_2 affect consumption in year 1 only through their effect on what we may term the consumer unit's wealth in year 1, or

$$W_1 = R_1 + \frac{R_2}{1+i}. \tag{2.1}$$

Changes in R_1 and R_2 that do not affect its wealth do not affect its consumption. To put it differently, it appears at first that we need to know three things to determine c_1, namely, R_1, R_2, and i; in fact, we need to know only two, namely, a particular combination of R_1, R_2, and i; and i itself. There are different combinations of R_1, R_2, and i that we could use; that is, different ways of collapsing the three original variables into two. One way, already suggested, is to take W_1 and i as the two variables[3] and to write the consumption function as

$$c_1 = f(W_1, i). \tag{2.2}$$

This elementary formulation already sheds considerable light on the usual view about the consumption function. What we have been calling receipts in year 1 (R_1) or some slight modification thereof,

[3] This is equivalent to the usual way of writing the demand curve for a particular good as a function of its price, for given money income and other prices. Changes in wealth shift the budget line parallel to itself, and the resulting points of tangency trace out the effect of changes in wealth on consumption. Changes in the interest rate pivot the budget line about the point *A*, and the resulting points of tangency trace out the effect of changes in the interest rate for a given wealth. This procedure has the disadvantage of the usual demand curve procedure that it does not separate substitution effects fully from the effect of a general increase or decrease in available opportunities—income effects in the usual demand analysis. An alternative that would be comparable to the real income demand curve I have discussed elsewhere is to define the wealth variable so that a change in the interest rate pivots the budget line about the initial point of equilibrium *P*. Since our interest here is primarily in the relation of *c* to *W*, rather than to *i*, these issues are neglected in the text. See "The Marshallian Demand Curve," *Journal of Political Economy*, LVII (December 1949), pp. 463–495, reprinted in my *Essays in Positive Economics* (Chicago: University of Chicago Press, 1953), pp. 47–99.

is usually, and particularly in statistical budget studies, called "income" and taken as the variable on which consumption depends. Now in our simple case it is clear that consumption in year 1 does not depend directly on R_1 at all; a change in R_1 affects consumption only through its effect on W_1 and, if accompanied by an appropriate opposite change in R_2, may not affect consumption at all. This is clearly eminently sensible: if a consumer unit knows that its receipts in any one year are unusually high and if it expects lower receipts subsequently, it will surely tend to adjust its consumption to its "normal" receipts rather than to its current receipts. On the other hand, if savings are defined as the difference between current receipts and current consumption, they do depend on current receipts, for, from (2.2), savings are then given by

$$s_1 = R_1 - c_1 = R_1 - f(W_1, i)\,. \qquad (2.3)$$

Equation (2.3) is the formal rationalization for the frequently expressed view that savings are a "residual."

The designation of current receipts as "income" in statistical studies is an expedient enforced by limitations of data. On a theoretical level, income is generally defined as the amount a consumer unit could consume (or believes that it could) while maintaining its wealth intact.[4] On our analysis, consumption is a function of income so defined. In the simple example considered here, W_1 is the consumer unit's wealth in year 1 and iW_1, its income in this sense for year 1. If receipts in year 1 exceed iW_1, the difference must be set aside as a "depreciation allowance" to be added to receipts in year 2 in order that wealth in year 2 be the same as in year 1. If receipts in year 1 fall short of iW_1, the difference is the amount that the unit can borrow to spend in addition to its receipts without reducing wealth in year 2 below its level in year 1.[5]

[4] The well-known problems raised by this definition are not relevant to the analysis that follows. For a discussion of some of them see J. R. Hicks, *Value and Capital* (Oxford, 1939), pp. 171–188.

[5] The use of discrete points of time raises difficulties of timing that disappear if the receipts are considered as continuous. Perhaps the simplest way to show the arithmetic involved in the discrete case is to suppose that R_1 and R_2 are received at the beginning of the respective time periods and that the expenditures are made at the end of the time periods. Then R_1 will have grown to $R_1(1 + i)$ by the end of the first period. The depreciation allowance is $R_1(1 + i) - iW_1$, or

$$R_1(1 + i) - i\left(R_1 + \frac{R_2}{1+i}\right) = R_1 - \frac{iR_2}{1+i}\,.$$

Total wealth at the beginning of the second period is this sum plus receipts at the beginning of the second period, or R_2, which gives

$$W_2 = R_1 - \frac{iR_2}{1+i} + R_2 = R_1 + \frac{R_2}{1+i} = W_1\,.$$

A similar problem arises about the meaning of "consumption." We have been using the term consumption to designate the value of the *services* that is it *planned* to consume during the period in question, which, under conditions of certainty, would also equal the value of the services actually consumed. The term is generally used in statistical studies to designate actual expenditures on goods and services. It therefore differs from the value of services it is planned to consume on two counts: first, because of additions to or subtractions from the stock of consumer goods, second, because of divergencies between plans and their realization.

Let us use the terms "permanent income" and "permanent consumption" to refer to the concepts relevant to the theoretical analysis, so as to avoid confusion with the frequent usage of income as synonymous with current receipts and consumption as synonymous with current expenditures, and let us designate them by y_p and c_p respectively, with an additional numerical subscript to denote the year in question.[6] We can write the consumption function as

$$c_{p1} = f\left(\frac{y_{p1}}{i}, i\right) = g(y_{p1}, i) = g(iW_1, i)\,, \tag{2.4}$$

since $y_{p1} = iW_1$.

This approach seems somewhat forced for the present simple case of a horizon of only two years. Initial wealth is then spent on consumption during the two years, rather than being maintained. It makes much more sense if (2.4) is regarded as a generalization from this special case to a longer horizon.[7]

The only empirical restrictions that have been imposed on the indifference curves up to this point are that they be negatively sloped (to be consistent with the observed absence of a tendency for individuals to give their wealth away indiscriminately) and convex to the origin (to be consistent with the observed absence of a tendency for individuals to spend their entire wealth on

[6] The adjective "planned" would perhaps be more appropriate in the present context than "permanent." The reason for using the latter will appear in Chap. III below.

[7] The transformation used to convert (2.2) into (2.4) raises difficulties for $i = 0$. If there is a finite perpetual income stream, the value of wealth is then infinite for the consumer unit, and it can satisfy its desires for current consumption without limit—it is in the economic *nirvana* where the economic problem disappears. If the income stream is limited in duration, the value of wealth is finite. A finite level of consumption can then be maintained only for a finite period and the implicit generalization of (2.2) to a (2.4) regarded as referring to a perpetual stationary state, in which it is possible to assign stationary flows without specifying their duration, is impossible. These are the usual difficulties that arise in connection with a zero interest rate supposed applicable to *all* sources of services. They may not arise if the interest rate is zero only for some sources of services, for example, only for nonhuman sources of services.

consumption in a single time period). These restrictions on the indifference curves impose only rather mild restrictions on the shape of a consumption function described by (2.2) or (2.4). To get a more specific hypothesis about the shape of the consumption function we shall have to go farther.

Suppose money prices are the same in the two years so that a point on the 45 degree line *OD* in Figure 1 represents equal opportunities in the two years. Suppose also that the unit is regarded as the same in the two years (thus abstracting from "aging" and similar phenomena). It then seems reasonable to suppose that if, in year 1, the consumer unit correctly assesses the relative value of consumption in the two years, the indifference curves will be symmetrical around *OD* so that c_1 and c_2 could be interchanged without altering the curves—alternatively, this can be taken as the definition of the absence of "time preference proper."[8]

This type of symmetry implies that all indifference curves have a common slope of -1 where they intersect *OD*—that is, that the consumer unit is willing to substitute one dollar of consumption this year for one dollar of consumption next year when both dollars will buy the same things and when it is consuming the same real amount in the two years, and that this is true regardless of the level of consumption. The convexity of the indifference curves then implies that when the unit is consuming more in year 1 than in year 2, it is willing to give up more than one dollar of consumption in year 1 for a dollar of consumption in year 2; when it is consuming less in year 1 than in year 2, it requires more than a dollar of additional consumption in year 2 to compensate it for giving up one dollar in year 1. It follows that if $R_1 = R_2$ so that the initial position is on the 45 degree line, the consumer unit consumes more than its receipts in year 1 if the interest rate is negative; exactly its receipts, if the interest rate is zero; and less than its receipts, if the interest rate is positive, as it is for the hypothetical budget line *AB* in Figure 1.

For a zero interest rate, the conditions so far imposed make consumption the same fraction of wealth (1/2 in our special case) at all levels of wealth. It seems reasonable to generalize this relation to other interest rates; that is, to suppose that, just as the indifference curves all have a common slope where they intersect the 45 degree line through the origin, so also they have a common slope where

[8] It should be noted that our special assumptions eliminate some of the usual reasons assigned for time preference, in particular, the possibility that the consumer unit will not live to engage in consumption in subsequent years, or that equally satisfactory consumer goods will not for one reason or another be available then as now.

they intersect any other straight line through the origin—mathematically, to suppose that the utility function is not only symmetrical but also homogeneous in c_1 and c_2. For our special case, this means that the rate at which the individual is willing to substitute consumption in year 2 for consumption in year 1 depends only on the ratio of consumption in the two years, not on the absolute level of consumption. Doubling, let us say, the level of consumption in year 1 may diminish in some sense the urgency of additional consumption in year 1 relative to consumption in year 2, which, by itself, would tend to lower the additional year 2 consumption required to compensate the consumer unit for giving up one dollar of year 1 consumption; however, if the level of consumption in year 2 is simultaneously doubled, this would have the opposite effect, diminishing the urgency of additional consumption in year 2 relative to consumption in year 1, and so, by itself, tending to raise the amount of year 2 consumption required to compensate the consumer unit for giving up one dollar of year 1 consumption. These two effects need not exactly offset one another; but there seems no a priori reason why the first should systematically or generally tend to exceed the second or conversely; the things being compared are of the same stuff, differing only in dating; it is hard to see any reason why this difference in dating should have an asymmetrical effect.[9] There seems nothing unreasonable, therefore, in supposing the two effects exactly to offset one another, and this is surely the simplest hypothesis. We shall, therefore, tentatively accept it, subject as always, of course, to the possibility that empirical evidence will be discovered that turns out to be inconsistent with it and that will therefore require complicating the hypothesis.

[9] This simple argument is the basic reason for questioning the initially plausible conjecture that the ratio of consumption to income decreases with income, if income is appropriately defined as a flow that can be permanently maintained. To put it differently, the ratio of consumption to permanent income is dimensionally free from any absolute units, even if the numerator and denominator are regarded as physical quantities of goods, rather than as value sums, for the physical units in numerator and denominator are the same. One would expect this ratio to depend on dimensionally similar variables or at least on variables that are free of the physical units common to numerator and denominator (like the rate of interest, the reciprocal of which has the dimension of time units). Why should it depend in any obvious way on such a dimensionally different variable as the absolute level of income? Note that this argument does not justify the conclusion that the ratio of one kind of consumption to another can be expected also to be independent of the absolute level of income. Regarded in terms of physical quantities, such a ratio has, for example, the dimensions, pounds of sowbelly per pound of steak, and might readily depend on such dimensionally comparable variables as the ratio of absolute prices of steak to sowbelly (also having the dimensions, pounds of sowbelly per pound of steak) or absolute income (capable of being regarded as the total number of pounds of sowbelly or of steak that could be consumed and so having the dimensions of pounds of sowbelly or pounds of steak).

For indifference curves satisfying these assumptions, the consumption function defined by (2.4) assumes a particularly simple form, namely,

$$c_{p1} = k(i, u) \cdot y_{p1} = k(i, u) \cdot iW_1 , \qquad (2.5)$$

where the function has been written so that it can be regarded as applying to an indefinitely long horizon, and not merely to two years. While k does not depend on the level of wealth or permanent income; it obviously does depend on the interest rate. It also depends on any factors that determine the shape of the indifference curves, symbolized in (2.5) by the variable u (for utility factors).[10]

If u is regarded as including such factors as age, family composition, and the like, we can drop the earlier assumption that the consumer unit is the same in the two (or more) years considered, and along with it the special assumption that if $i = 0$, $c_1 = c_2$ in the two year case. That is, the preceding analysis can be interpreted as referring to consumer units of a given kind in a particular year.

The simple function (2.5), though derived from such elementary and abstract considerations, is a cornerstone of the theory of the consumption function presented in this monograph. We shall see that the introduction of uncertainty gives no reason to alter it fundamentally, and that it is not inconsistent with existing empirical evidence on consumption behavior, provided that its variables are appropriately identified with observable magnitudes.

2. *The Effect of Uncertainty*

Uncertainty about the future has effects of two kinds on the preceding analysis: first, it complicates the interpretation of the indifference curve diagram; second, it introduces an additional reason for saving that requires distinguishing among different kinds of wealth.

a. THE INDIFFERENCE CURVE DIAGRAM

Under conditions of certainty, the alternatives open to the consumer unit in year 2 for each level of consumption in year 1 can be described completely by a single number, namely, the maximum level of real consumption attainable in year 2, or the abscissa of the budget line AB in Figure 1. Under conditions of uncertainty, such a simple description is impossible; it must be replaced by a probability distribution of possible maximum levels of real

[10] Duesenberry reaches the same conclusion, that consumption is proportionate to income in long-run comparative statics, on the basis of a somewhat different line of reasoning. *Income, Saving, and the Theory of Consumer Behavior* (Cambridge, Mass.: Harvard University Press, 1949), pp. 32–37.

consumption in year 2, the dispersion among the possible levels reflecting both the direct effect of uncertainty about future receipts and future prices and the indirect effect of this uncertainty on the possibility of lending or borrowing.

Suppose that there is no uncertainty about future tastes (as viewed from the present), and that the c_2 axis continues to be interpreted as showing actual consumption. The indifference curves are then unaffected by the introduction of uncertainty. However, the budget line is significantly altered in meaning. The probability distribution of possible future consumption associated with each level of consumption in year 1 has some utility to the consumer unit, and there is in general some single value of consumption that has the same utility. The locus of such "certainty equivalents" traces out a curve comparable to the budget line in the sense that its point of tangency with an indifference curve is the optimum position. But there is no reason for this curve to be a straight line, and it cannot be computed solely from knowledge of the opportunities open to the unit; it depends also on its tastes. The sharp dichotomy between tastes and opportunities that is the central attraction of the indifference analysis under certainty is shattered.

An alternative is to interpret the c_2 axis of Figure 1 as referring to expected consumption in year 2, where "expected" is used in the sense of "mean value" rather than of "anticipated." If there were no disagreement about probability distributions, so that expected receipts could be borrowed or loaned at a fixed rate of interest, the budget line would be unaffected and would remain a straight line. The indifference curves would now, however, be significantly altered in meaning. The utility attached to a given expected value depends on the probability distribution yielding that expected value. The indifference curves can therefore be drawn only if the probability distribution yielding each expected value is specified; once again, any sharp separation between opportunities and tastes is destroyed.

The introduction of uncertainty thus blurs the sharp lines of the above analysis, and suggests additional factors that may produce departures from the shape of the consumption function specified in (2.5). However, on the present level of analysis, there seems no way to judge whether these factors would tend to make consumption a larger or a smaller fraction of wealth the higher the absolute level of wealth. Accordingly, this effect of uncertainty establishes no presumption against the shape assigned to the consumption function, and thus casts no shadow on the "simplicity" that recommends it.

b. MOTIVES FOR HOLDING WEALTH

The introduction of uncertainty adds a new reason for holding wealth to the two motives present under certainty—straightening out the consumption stream and earning interest. This new motive is the availability of a reserve for emergencies—for unexpectedly low receipts, on the one hand, or unexpectedly high levels of consumption on the other. If all forms of wealth were equally satisfactory as a reserve for emergencies, this motive could be regarded as producing simply an alteration in the shape of the indifference curves of Figure 1 and otherwise completely covered by that figure. Any part of wealth not used for current consumption would be available as a reserve for emergencies. Provision for future consumption would therefore be valued not only for its own sake but also because it provided such a reserve. The result would be that the indifference curves would be steeper at each point than otherwise; that is, the consumer unit would be willing to give up a larger amount of current consumption than otherwise to add a dollar to future consumption.

All forms of wealth are not, however, equally satisfactory as a reserve for emergencies. The major general distinction is between human and nonhuman wealth. In a nonslave society, there is no market in human beings comparable to the market for nonhuman capital. It is in general far easier to borrow on the basis of a tangible physical asset, or a claim to one, than on the basis of future earning power. Accordingly, current consumption may be expected to depend not only on total permanent income and the interest rate, but also on the fraction of permanent income derived from nonhuman wealth, or—what is equivalent for a given interest rate—on the ratio of nonhuman wealth to permanent income. The higher this ratio, the less need there is for an additional reserve, and the higher current consumption may be expected to be.[11] The crucial variable is the ratio of nonhuman wealth to permanent income, not the absolute amount of nonhuman wealth. A reserve is needed for protection against unexpected occurrences threatening the realization of a planned level of consumption, or making it urgent to consume at a higher level than that initially planned. A common proportional increase in nonhuman wealth and in permanent income increases both the reserve available and the level of consumption to be protected; it is like a change in scale. In consequence, there seems no a priori reason why such a common proportional increase

[11] To incorporate this effect formally into Figure 1 would require the addition of another axis showing the amount of nonhuman wealth.

in nonhuman wealth and in permanent income should systematically or generally raise the importance attached to increasing the size of the reserve, or conversely.[12] This effect of uncertainty therefore, like the other, establishes no presumption against the form assigned to the consumption function in (2.5). It requires only that the ratio of nonhuman wealth to income be included as a variable determining k, the ratio of consumption to permanent income. This converts (2.5) into

$$c_p = k(i, w, u,)y_p = k(i, w, u)iW, \tag{2.6}$$

where w stands for the ratio of nonhuman wealth to permanent income and, for simplicity, the subscript 1 has been dropped from c, y, w, and W, with the understanding that all variables refer to the same point in time.

The importance attached to a reserve for emergencies depends, of course, on the degree of uncertainty that the consumer unit foresees. The variable u may be taken to include any objective factors that affect its anticipations. For example, the degree of inequality of wealth or income in the community may very well be related to the anticipated degree of uncertainty about receipts and so be a relevant variable.

All forms of nonhuman wealth are not equally satisfactory as a reserve for emergencies; this is the reason why certain kinds of nonhuman wealth, such as so-called "liquid assets," have been singled out for special attention in some empirical studies. But none of the other distinctions among forms of wealth seems as pervasive and fundamental as the distinction between human and nonhuman wealth, or even sufficiently fundamental to justify including it in the consumption function at the present stage.

The distinction among different kinds of wealth implies a corresponding distinction among different rates of interest. The rate of interest at which an individual can borrow on the basis of his future earnings may be different from the rate at which he can borrow on the basis of nonhuman capital; and the rate at which he can borrow may differ from the rate at which he can lend. We shall, however, neglect these complications, letting i stand for the whole complex of rates of interest.

[12] The dimensional argument of footnote 9 applies here.

It is at first glance tempting to suppose that the "law of averages" makes the same ratio of nonhuman wealth to permanent income more adequate, the higher the absolute level of both. This does not, however, follow if the increase in both takes the form of a common proportional increase in each possible future receipt, with the probabilities unchanged. In this case, the standard deviation is increased in the same ratio as the mean.

3. *The Relation between the Individual and the Aggregate Consumption Function*

The preceding theoretical analysis has been for an individual consumer unit. Equation (2.6) to which it leads ostensibly describes the behavior of such a unit for different values of its variables. In order to use this equation in interpreting group behavior, we must take the additional step of regarding the same equation as applicable to all members of the group—not merely the same form of equation, but the same functional relation. This is, however, a less drastic step than it may at first appear. The variables in equation (2.6), particularly w and u, are designed precisely to allow for differences among consumer units. If i, w, and some particular specification of u are the same for a number of consumer units and yet the ratio of consumption to permanent income differs among the consumer units by enough to be regarded as significant for the purpose at hand, than either the equation itself must be regarded as deficient, or the particular specification of u as inadequate. The acceptance of (2.6) and a particular specification of u for an individual consumer unit is thus equivalent to its acceptance for all members of a group.

Given that (2.6) applies to every consumer unit in a group, the ratio k of consumption to permanent income will nonetheless vary from consumer unit to consumer unit because of differences among them in the values of i, w, and u; and the absolute amount of consumption will vary because of differences in y_p as well. Aggregate consumption depends therefore not only on the precise form of equation (2.6) but also on the distribution of consumer units by these variables. Let

$$f(i, w, u, y_p)didwdudy_p \tag{2.7}$$

be the number of consumer units for whom the interest rate is between i and $i + di$, the ratio of nonhuman wealth to permanent income is between w and $w + dw$, the taste determining factors are between u and $u + du$, and permanent income is between y_p and $y_p + dy_p$. Then aggregate consumption is

$$c_p^* = \iiiint\limits_{i,w,u,y_p} f(i, w, u, y_p)k(i, w, u)y_p didwdudy_p \,. \tag{2.8}$$

Suppose that the distribution of consumer units by income is independent of their distribution by i, w, and u, so that

$$f(i, w, u, y_p) = g(i, w, u)\cdot h(y_p)\,. \tag{2.9}$$

Equation (2.8) then reduces to

$$c_p^* = k^*(\qquad\qquad) \cdot y_p^*, \tag{2.10}$$

where c_p^* is aggregate permanent consumption; y_p^*, aggregate permanent income; and

$$k^*(\qquad\qquad) = \iiint\limits_{i,w,u} g(i, w, u) k(i, w, u) \, di \, dw \, du. \tag{2.11}$$

k^* depends on the function k, and also on the function g which describes the distribution of individuals by i, w, u. As an approximation, k^* could be expressed as a function of the mean values of i, w, and u, their variances, and the co-variances among them, or other similar parameters describing the distribution. The coefficients of these variables would be determined by the parameters of k. The parenthesis containing the variables has been left blank in (2.10) and (2.11) because there is no way of specifying on the present level of generality a limited number of variables to stand for the functions k and g.

Equation (2.10) is obviously unchanged if both sides are divided by the same number, such as total population or a price index, so in using (2.10), c_p^* and y_p^* can be taken to refer equally to money aggregates, real aggregates, money per capita figures, or real per capita figures.

The assumption used in passing from (2.8) to (2.10), namely, that the distribution of consumer units by income is independent of their distribution by i, w, and u, is obviously false in a descriptive sense. The variable u, for example, covers such factors as age, size of family, perhaps education, and these are all known to be connected systematically with the distribution of income; indeed, we shall have occasion at a later point to use some of these connections to explain certain observed features of consumption behavior. At the same time, although the interdependence between these variables and the distribution of income may be important for some problems, it may not be for this aggregation. The interdependence enters in a rather complex way and equation (2.10) remains an approximation even when interdependence exists. If, as we shall see to be the case, equation (2.10) is a good approximation of the relation among observed magnitudes, this must be interpreted to mean that the interdependence is of only secondary importance.