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THE TEMPORAL AND SECTORAL AGGREGATION OF SEASONALLY ADJUSTED TIME SERIES

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ABSTRACT

Procedures for the optimal seasonal adjustment of economic time series and their aggregation are derived, given a criterion suitable for the adjustment of data used in political or journalistic contexts. It is shown that data should be adjusted jointly and then temporally or sectorally aggregated, as desired, a procedure that preserves linear aggregation identities. Examination of actual economic time series indicates that the optimal seasonal adjustment and aggregation of data provide a substantial improvement in the quality of sectorally disaggregated, adjusted data and considerably reduces the required subsequent revision of current adjusted series.

INTRODUCTION

How seasonal adjustment should proceed depends on the information available and the context within which adjustment takes place. Since information sets and the purposes of investigators are many and widely varied, different methods of seasonal adjustment have been proposed in the literature, each with desirable properties in certain contexts. If one's final purpose is minimum mean square error estimation of the nonseasonal component of a single series and the seasonal component is assumed to be deterministic, then fairly simple methods can have desirable properties, as Lovell [4] and Jorgenson [3] have illustrated. If one's final purpose is estimation of the parameters of an equation or a system of equations and seasonal disturbances are allowed to have indeterministic components, then more sophisticated procedures are required. (See, e.g., Wallis [7] or Sims [6].) From the proliferation of literature on this topic, it is evident that which procedure is desirable very much depends on the model used and the information available, and no single seasonal adjustment procedure is suitable for all users of economic time series. Those who approach the econometrics of model construction seriously are perhaps least likely to be satisfied with any official method of seasonal adjustment, but these investigators are also probably best equipped to cope formally with seasonality problems on a case-by-case basis. In this paper, it is assumed that the final purpose of seasonal adjustment is extraction of the nonseasonal components of a set of economic time series and that the loss function to be minimized in so doing is mean square error. The purpose of this assumption is to provide an analytically tractable formalization of the objectives of the seasonal adjustment of data for use by policymakers or journalists.

The following section begins with a very general statement of the seasonal adjustment problem that provides an indication of the many ways in which composition of the information set will affect the deseasonalization procedure. Optimal seasonal adjustment for a special case in which the joint distribution of the time series in question and their seasonal and nonseasonal components are known, is illustrated. The main theoretical result of the paper is developed in the section on aggregation and optimal adjustment, where it is shown that, for virtually every conceivable time series (including many processes that have deterministic components, are nonstationary, or even explosive) and a reasonably inclusive class of potential adjustment procedures, minimum mean-square-error adjustment implies that seasonal adjustment should always precede temporal or sectoral aggregation.

The practicality of optimal adjustment and the magnitude of the increase in mean square error that results when the aggregation problem is handled inappropriately are examined in the sections on temporal and sectoral aggregation. The widely used assumptions that the spectral density of the nonseasonal component of any economic time series should be smooth and not show peaks at seasonal frequencies and that seasonal and nonseasonal components are additive and independent are used to achieve identification of the covariance structure of seasonal and nonseasonal components of a vector of time series. It is shown how this assumption leads to optimal seasonal adjustment and revision of recently adjusted data as time proceeds. The optimal adjustment procedures for an artificial series, suggested by Grether and Nerlove [2], and two actual series, U.S. housing starts and the quit rate in U.S. manufacturing, are computed.

In the case of the housing starts and quit-rate data, it is found, in the section on temporal aggregation, that virtual elimination of the seasonal component is possible and that, if monthly data are adjusted optimally, then revisions of data after initial publication are minute. In the seasonal adjustment of historical data (that pertaining to a period 5 or more years ago), it makes virtually no difference, in practice, whether one adjusts monthly data and then aggregates to form a quarterly series or adjusts the quarterly aggregate directly, in spite of the theoretical superiority of the former method. In the seasonal adjustment of more recent data, however, there are substantial gains to be realized from seasonally adjusting the data at the most frequent sampling interval available (in this case, monthly) and then aggregating to the desired frequency. The more recent is the data, the greater is the gain; the procedure reduces, by about half, the total required revision of recently adjusted data as time proceeds.

The housing starts data are used to study three alternative methods of seasonally adjusting data for which cross-section disaggregation is possible. In the first procedure, the sectoral components are adjusted jointly using the estimated covariance matrices for their seasonal and nonseasonal components and then aggregated to yield an estimate of the nonseasonal component of the aggregate series; this is the procedure shown to be optimal in the section on aggregation and optimal adjustment. In the second method, each sectoral component is adjusted optimally, but individually, and the seasonally adjusted aggregate is taken to be the sum of the individually adjusted series. In the third procedure, the aggregate itself is adjusted optimally. Theoretically, all one can say is that the second two methods, in general, yield a larger mean square error than the first; the Grether-Nerlove variable is used to construct examples in which the third method is superior to the second, and vice versa. In the case of housing starts, where data disaggregated by geographical region are available, the mean square error of the first method is about half that of the second or third. While this one example can only be suggestive, it seems plausible that substantial improvement in the quality of seasonally adjusted data can be obtained if joint adjustment of sectoral components is employed.

OPTIMAL ADJUSTMENT

We shall be concerned with a $k \times 1$ vector stochastic process $x(t)$ defined on the usual probability space. This process is a function of two components,

$$x(t)=g(x^N(t),\ x^S(t)) \tag{1}$$

The $k \times 1$ vector processes $x^N(t)$ and $x^S(t)$ are each unobservable and are termed the "nonseasonal" and "seasonal" components of $x(t)$, respectively. The process $x(t)$ is observable, and the function g and the joint distribution of $x^N(t)$ and $x^S(t)$ are assumed to be known. Further assumptions about $x(t)$ will be introduced in some specific examples.

The optimal adjustment problem may be stated formally as follows: Given a realization $\ldots\ x(t-1),\ x(t),\ x(t+1),\ \ldots$ of $x(t)$ and given a class C of functions, each having a domain that has a countably infinite set of real numbers,

$$\min_{f_{jt} \in C} E[f_{jt}(x(t),\ x(t+1),\ x(t-1),\ \ldots,\ x(t+m),\ x(t-m),\ x(t-m-1),\ x(t-m-2),\ x(t-m-3),\ \ldots)-x_j^N(t)]^2 \tag{2}$$

where j is some number between 1 and k. The problem presumes that there exists at least one f_{jt} for which (2) is finite, a presumption that is satisfied by most processes and reasonably inclusive C. We shall denote the solution of (2) by f_{jt}^* and define

$$\hat{x}_j^N(t)=f_{jt}^*(x(t),\ x(t+1),\ x(t-1),\ \ldots,\ x(t+m),\ x(t-m),\ x(t-m-1),\ \ldots) \tag{3}$$

The function f_{jt}^* depends not only on the nature of the processes $x_j(t), x_j^N(t)$, and $x_j^S(t)$ but also on the five following aspects of the problem that must be chosen by the investigator:

1. The solution depends on C. The choice of C may be affected by the need for analytical simplicity (e.g., it may be some subset of linear functions) or practical constraints that arise in the publication and revision of adjusted data for a wide variety of users.
2. The constituents of $x(t)$ affect the solution. As the composition of $x(t)$ is expanded, the magnitude of (2) evaluated at f_{jt}^*, in general, decreases, but the assumption that g and the joint distribution of $x^N(t)$ and $x^S(t)$ are known becomes less tenable; we shall return to this difficulty in the section on sectoral aggregation. In many seasonal adjustment problems, $k=1$.
3. In general, the solution depends on j. Unless the joint distributions of $x_j^N(t)$ conditioned on $x(t)$ are the same for all j—a condition which seems unlikely—f_{jt}^* depends on the variable being adjusted. If $k=1$, this amounts to saying that the relation of seasonal and nonseasonal components is not the same for all variables. This fact is implicit in seasonal adjustment procedures that use dummy variables, the decision not to adjust certain time series while adjusting others and the existence of a certain amount of flexibility within official adjustment procedures.
4. The solution is allowed to depend on t. If $x(t)$ is stationary, f_{jt}^* will be the same for all t, but when $x(t)$ has nonstationary components, such as a trend or a deterministic seasonal, the adjustment of $x_j(t)$ will, in general, change with t. In simple dummy variable adjustment procedures, e.g., t and the joint distribution of $x_j^N(t)$ and $x_j^S(t)$, this is all that matters.

5. The value of m is important in the adjustment. Because of the desire to obtain seasonally adjusted series that are up to date, the problem (2), with m equal to zero or some small positive integer, is especially relevant. The fact that f_{jt}^{*} changes as m increases from zero is responsible for the revision of recent, seasonally adjusted data.

Note that (1) and the assumptions about $x(t)$ are quite general: We leave open the question of whether $x^N(t)$ and $x^S(t)$ are related additively, multiplicatively, or in some other fashion, and the vector $x(t)$ is not required to be a stationary process. Formally, there is no need to require the loss function to be quadratic, and the domain of the functions in C could be limited to a finite number of realizations of $x(t)$. We shall not pursue either of these cases in any detail, since both are straightforward extensions of the problem posed here. The case $m=\infty$ is often instructive, since it is usually analytically more tractable than $m<\infty$ and provides a paradigm for the seasonal adjustment of historical (as opposed to recent) data.

Solution of (2) may be illustrated in two special cases. Perhaps the most widely studied situation is $x^S(t)=S(t)$, where $S(t)$ is a deterministic function of time. Assuming that linear functions are included in C and the seasonal enters additively, the solution of (2) is simply

$$\hat{x}_j^N(t)=x_j(t)-S_j(t)$$

the loss function then having the value zero. This model underlies the application of simple dummy variable procedures.

When $x^N(t)$ and $x^S(t)$ are jointly wide-sense stationary and independent, $x(t)=x^N(t)+x^S(t)$, $m=\infty$, and C is the class of all linear functions of $x(t)$, the problem (2) becomes

$$\min_{a_j} E[x_j^N(t)-a_j{*}x(t)]^2 \qquad (4)$$

In this expression, $a_j{*}x(t)$ denotes the convolution of $a_j(s)$, a $1\times k$ vector of functions defined on the intergers' with $x(t)$

$$a_j{*}x(t)=\sum_{i=1}^{k}\sum_{s=-\infty}^{\infty} a_{ji}(s)x_i(t-s)$$

It is actually convenient to solve the more general problem

$$\min_{A} E[c{*}(x^N(t)-A{*}x(t))]^2 \qquad (5)$$

where $c(s)$ is any $1\times k$ vector of functions for which the expectation exists, and $A(s)$ is the $k\times k$ matrix of functions having a j'th row that is $a_j(s)$. In the frequency domain, this problem becomes the minimization of

$$\int_{-\pi}^{\pi}\tilde{c}(\omega)\{S_x^N(\omega)-S_x^N(\omega)\tilde{A}(\omega)'-\tilde{A}(\omega)S_x^N(\omega)' + \tilde{A}(\omega)S_x(\omega)\tilde{A}(\omega)'\}\tilde{c}(\omega)'d\omega \qquad (6)$$

with respect to

$$\tilde{A}(\omega)=\sum_{s=-\infty}^{\infty}A(s)\exp(-is\omega)$$

the Fourier transform of $A(s)$. The $1\times k$ vector $\tilde{c}(\omega)$ is similarly defined to be the Fourier transform of $c(s)$. The $k\times k$ matrix $S_x^N(\omega)$ is the spectral density matrix of $x^N(t)$, and $S_x(\omega)$ is the spectral density matrix of $x(t)$. So long as $S_x(\omega)^{-1}$ exists, the expression (6) has a regular global minimum at

$$\tilde{A}(\omega)=S_x^N(\omega)S_x(\omega)^{-1} \qquad (7)$$

The inverse Fourier transform of $\tilde{A}(\omega)$,

$$A(s)=\frac{1}{2\pi}\int_{-\pi}^{\pi}\tilde{A}(\omega)\exp(is\omega), \quad s=\ldots-1, 0, 1, \ldots$$

is the matrix of functions having a j'th row that solves (4).

AGGREGATION AND OPTIMAL ADJUSTMENT

In the special case of the preceding example, seasonal adjustment amounts to a signal extraction problem where the solution is the population regression of the unobservable component $x^N(t)$ on the observable vector $x(t)$. The regression interpretation is clear in expressions (6) and (7). Since every type of linear aggregation of $x^N(t)$ and $x(t)$ amounts to a choice of $c(s)$ and since the solution (7) of (5) is the same for all $c(s)$, we should seasonally adjust first and aggregate—across variables or over time—second. In this case, aggregation at the outset amounts to a loss of information that, in general, precludes optimal solution of (4).

The suboptimality of preliminary aggregation is not solely a property of the historical seasonal adjustment of wide-sense stationary processes: It applies in more general and realistic situations as well. Turning our attention to solution of (2) in the general case, the result may be stated formally in the theorem and proof.

Theorem

Let C have the following property: If $f_{j,t}$, $f_{j,t-1}$, $f_{j,t-2}$, ... all belong to C, then so does

$$g_{jt}=\sum_{s=0}^{\infty}b(s)f_{j,t-s}$$

where b is any function defined on $s=0, 1, 2, \ldots$. Let $\hat{x}_j^N(t)$ be the solution of the problem

$$\min E[\hat{x}_j^N(t)-x_j^N(t)]^2$$

for given C, $x(t)$ and m, and denote $x^N(t)=(x_1^N(t), \ldots, x_k^N(t))'$. Let $c(s)$ be any $1\times k$ vector of functions defined on $s=0, 1, 2, \ldots$ such that $E[c^*(\hat{x}^N(t)-x(t))]^2$ is finite. Then, a solution of the problem

$$\min_{\hat{\hat{x}}^N} E[c^*(x^N(t)-\hat{\hat{x}}^N(t))]^2$$

is $\hat{\hat{x}}^N(t)=\hat{x}^N(t)$.

Proof

1. $E[\hat{x}^N(t)-x^N(t)][\hat{x}^N(s)-x^N(s)]'=0$ for all $s<t$. To see this, let $B^*(\hat{x}^N(t)-x^N(t))$ be the linear, least squares projection of $\hat{x}^N(t)-x^N(t)$ on all past values of itself. Define a new process

$$z(t)=\hat{x}^N(t)-B^*(\hat{x}^N(t)-x^N(t))$$

Then $E[z(t)-x^N(t)][z(t)-x^N(t)]'$

$$\begin{aligned}&=E[\hat{x}^N(t)-x^N(t)-B^*(\hat{x}^N(t)-x^N(t))]\\&\quad[\hat{x}^N(t)-x^N(t)-B^*(\hat{x}^N(t)-x^N(t))]'\\&=E[\hat{x}^N(t)-x^N(t)][\hat{x}^N(t)-x^N(t)]'\\&\quad-E[B^*(\hat{x}^N(t)-x^N(t))][B^*(\hat{x}^N(t)-x^N(t))]'\end{aligned}$$

Since $\hat{x}$ is the solution of the problem (2), we can choose $B\equiv 0$.

2. To complete the proof of the theorem, observe that

$$E[c^*(\hat{x}^N(t)-x^N(t))]^2$$

$$=E\left[\sum_{j=1}^{k}\sum_{s=-\infty}^{\infty} c_j(s)(\hat{x}_j^N(t-s)-x_j^N(t-s))\right]^2$$

$$=\sum_{j=1}^{k}\sum_{s=-\infty}^{\infty} c_j^2(s)E[\hat{x}_j(t-s)-x_j^N(t-s)]^2$$

The foregoing theorem is applicable in a wide variety of cases. So long as $x(t)$ has finite variance and one restricts attention to square summable $c(s)$, the conditions on $x(t)$ are met: $x(t)$ need not be a wide-sense stationary process. The theorem also applies to any vector process $x(t)$ that has a multivariate ARIMA representation. The theorem assumes, as we did at the outset, that $m\geq 0$, but with slight rewording could be modified to include the case $m<0$, optimal forecasting of the nonseasonal component of $x(t)$. As it stands, the above result embraces the seasonal adjustment and revision of recent data as well as historical series. For example, it implies that if one's criterion is minimum mean square error and monthly data are available, then the quarterly seasonally adjusted series for 1976: III should be formed as the sum of the adjusted July, August, and September observations. The only restrictions of real consequence in the theorem are on the adjustment process itself. The class C may embrace a wide variety of adjustment functions, including quadratic as well as linear mappings or allowing the exclusion of outliers in $x(t)$, so long as these adjustments may be followed by any one-sided linear adjustment. Put another way, the adjustment procedure must be constructed so that optimal linear adjustment of the adjusted series leaves that series unchanged.[1] This property of the adjustment procedure is necessary for the result, and, without it, the conclusion that seasonal adjustment always should precede temporal or sectoral aggregation no longer follows.

Two further questions remain to be examined before conclusions about the importance of the aggregation problem in seasonal adjustment can be drawn. First, of what order of magnitude is the increase in mean square error that results when temporally or sectorally aggregated series are adjusted? Seasonal adjustment of daily time series or the joint adjustment of data for each of 50 States is sufficiently burdensome that one might be satisfied with a less than optimal solution if the sacrifice were not too great. Second, our assumption that the joint distribution of $x^N(t)$, $x^S(t)$ and $x(t)$ is known is never strictly correct in any applied situation. Do we, in fact, know enough about these distributions that the foregoing theoretical result can even serve as a paradigm for seasonal adjustment? These problems are considered in the next two sections.

TEMPORAL AGGREGATION: SOME EXAMPLES

What loss is incurred if aggregation, over time, precedes optimal seasonal adjustment? In this section, we answer this question, both for the adjustment of historical data (that pertaining to a period infinitely long ago) as well as recent data (that which pertains to a few periods ago, the current time, or even the future) for the case in which loss is measured by mean square error and the process in question is wide-sense stationary. Three series will be examined: A widely used representative process and two actual economic time series that display considerable seasonality. Our examination is limited to the univariate case to isolate the problems of temporal aggregation from those of sectoral aggregation, considered in the next section.

Consider, first, the adjustment of historical data that are the realization of a wide-sense stationary process $x(t)=x^N(t)+x^S(t)$. Suppose data are available monthly, $x^N(t)$ and $x^S(t)$ are independent, the spectral densities $S_x^N(\omega)$ of $x^N(t)$ and $S_x^S(\omega)$ of $x^S(t)$ are known, and C is the

[1] This requirement should not be confused with the suggestion of Lovell [4] that the seasonal adjustment process should be idempotent. Lovell's suggestion is that f_{jt}^* should satisfy

$$f_{jt}^*(x(t), x(t+1), \ldots)=f_{jt}^*(f_{jt}^*(x(t), \ldots), f_{jt+1}^*(x(t+1), \ldots), \ldots)$$

a property that will not, in general, be exhibited by the solution of (2). Our requirement is that f_{jt}^* has the property

$$\min_a E[a^*\hat{x}^N(t)-\hat{x}_j^N(t)]^2=E[\hat{x}_j^N(t)-x_j^N(t)]^2$$

where $\hat{x}_j^N(t)$ is defined in (3) and a is any one-sided linear function.

class of all linear functions of $x(t)$. From (6) and (7), the solution of the optimal adjustment problem is

$$\hat{x}^N(t)=a*x(t)$$

where the function $a(s)$ has Fourier transform

$$\tilde{a}(\omega)=S_x^N(\omega)/S_x(\omega)$$

For monthly data, the mean square error of $\hat{x}^N(t)$ is

$$E[\hat{x}^N(t)-x^N(t)]^2=\int_{-\pi}^{\pi}\left[S_x^N(\omega)-\frac{S_x^N(\omega)^2}{S_x(\omega)}\right]d\omega \qquad (8)$$

Optimal seasonal adjustment of quarterly data is the special case of (5) in which $c(s)=1/3$ if $|s|\leq 1$, and 0, otherwise. The Fourier transform of the function $c(s)$ in this case is $D_3(\omega)=(1+2cos(\omega))3$, and the mean square error of the resulting optimally adjusted quarterly process is

$$E[\hat{x}_Q^N(t)-x_Q^N(t)]^2=\int_{-\pi}^{\pi}D_3(\omega)^2\left[S_x^N(\omega)-\frac{S_x^N(\omega)^2}{S_x(\omega)}\right]d\omega \qquad (9)$$

Given $\hat{x}^N(t)$, $\hat{x}_Q^N(t)=(\hat{x}^N(t-1)+\hat{x}^N(t)+\hat{x}^N(t+1))/3$. Assume that, instead, one were to form the unadjusted quarterly series $x_Q(t)=(x(t-1)+x(t)+x(t+1))/3$ and then optimally adjust this process. The process $x_Q(t)$ has spectral density

$$S_x^Q(\omega)=\sum_{j=0}^{2}D_3\left(\omega+\frac{2\pi j}{3}\right)^2 S_x\left(\omega+\frac{2\pi j}{3}\right)=F_3[D_3^2S_x](\omega)$$

where $S_x(\omega)$ is taken to be periodic with period 2π and the truncated folding operator F_3 is implicitly defined by the latter equality.[2] Similarly, $S_x^{QN}(\omega)=F_3[D_3^2S_x^N](\omega)$ and $S_x^{QS}(\omega)=F_3[D_3^2S_x^S](\omega)$. Another application of (6) and (7) shows that the adjusted series is

$$\hat{x}_Q^N(t)=a_Q*x(t)$$

where $a_Q(s)$ has Fourier transform

$$\tilde{a}_Q(\omega)=F_3[D_3^2S_x^N](\omega)/F_3[D_3S_x](\omega)$$

The mean square error of this adjustment procedure is

$$E[\hat{x}_Q^N(t)-x_Q^N(t)]^2$$

$$=\int_{-\pi/3}^{\pi/3}\left\{F_3[D_3^2S_x^N](\omega)-\frac{(F_3[D_3^2S_x^N](\omega))^2}{F_3[D_3^2S_x](\omega)}\right\}d\omega \qquad (10)$$

Since temporal aggregation before seasonal adjustment is suboptimal, the value of expression (10) exceeds that of (9), and, given $S_x^N(\omega)$ and $S_x(\omega)$, the magnitude of this increase in error can be measured. In similar fashion, the increase in mean square error that results when one optimally adjusts annual data rather than forming annual adjusted data as the sum of adjusted monthly data can be computed.

The expressions (8), (9), and (10), as well as the analogues of (9) and (10) for annual aggregation, were evaluated and compared for three processes. The first process is the one considered by Grether and Nerlove [2]—

$$x(t)=\frac{(1+0.8L)\epsilon(t)}{(1-0.95L)(1-0.75L)}+\frac{(1+0.6L)\nu(t)}{(1-0.9L^{12})}+\eta(t)$$

The processes $\epsilon(t)$, $\nu(t)$, and $\eta(t)$ are mutually independent and serially uncorrelated. The first term is the trend-cycle component of $x(t)$, with var$(\epsilon(t))$ such that the trend cycle accounts for 85 percent of the variance of $x(t)$; the second term is the seasonal component, with var$(\nu(t))$ such that the seasonal accounts for 10 percent of the variance of $x(t)$. The term L is the conventional lag operator.

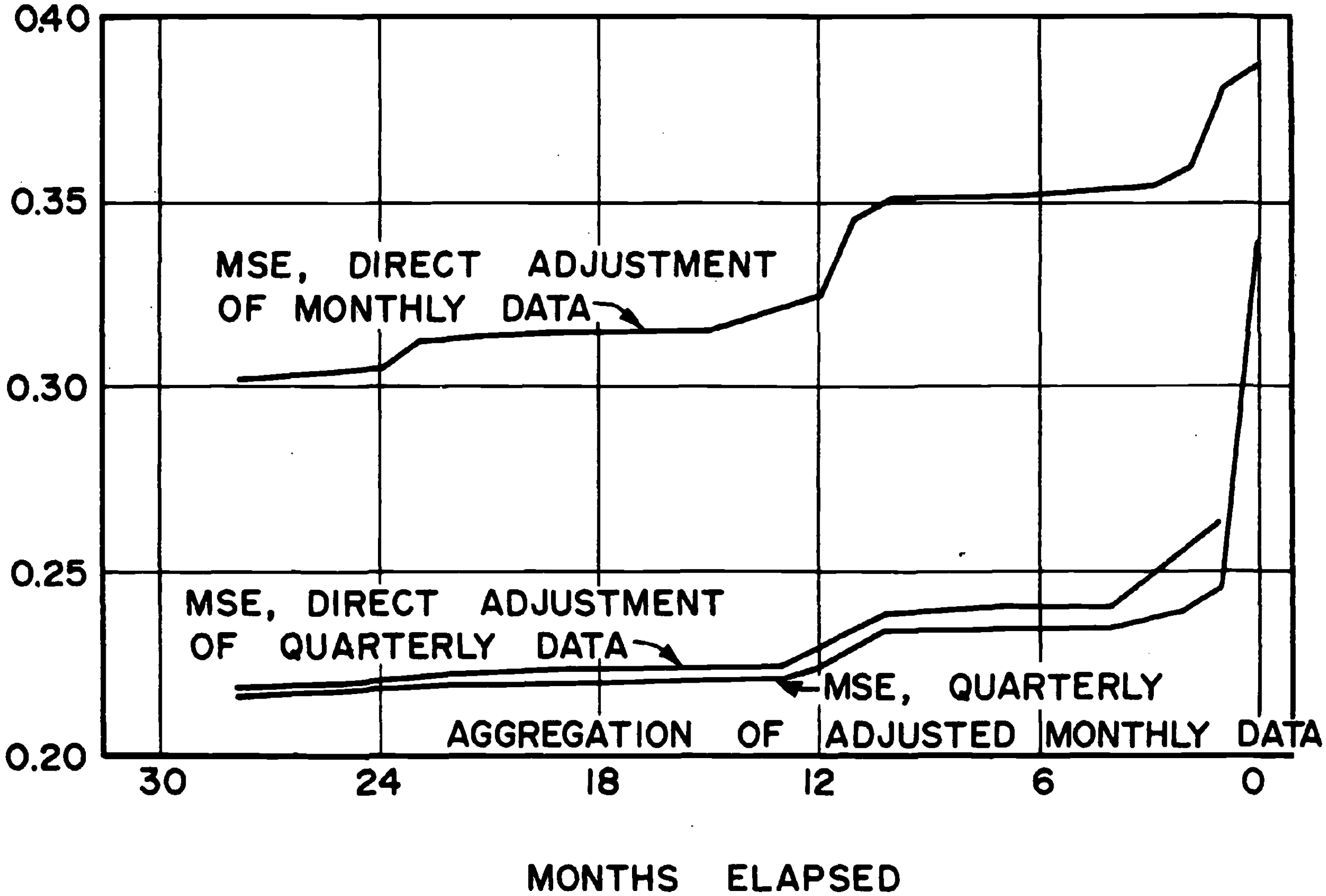
The other two processes are U.S. housing starts and the quit rate in U.S. manufacturing. Both were assumed to be wide-sense stationary, strictly indeterministic processes with additive, independent seasonal, and nonseasonal components. In each case, these components were identified using Nerlove's criterion (see [5]) that $S_x^N(\omega)$ should be smooth and exhibit no peaks at seasonal frequencies. This form of identification is far from exact, but experimentation showed that the results reported here are robust regarding a variety of exact interpretations of Nerlove's criterion. Exact computational procedures are described in the appendix.

The results presented in figures 1 and 2 indicate that, for the adjustment of historical economic time series, it makes essentially no difference whether one seasonally adjusts monthly data and then aggregates to form quarterly data or adjusts the quarterly data directly. The increase in mean square error from following the latter procedure is greatest in the case of the quit rate, and there the increase is only 2.74 percent. For both housing starts and the quit rate, the optimal adjustment process is very effective, leaving an adjusted process having a mean square error that is less than 10 percent of the variance of the seasonal component and a much smaller fraction of the total variance. For the Grether-Nerlove variable, the adjustment process is not nearly as successful, but the increased error that results from the direct adjustment of quarterly data is only 0.3 percent. For annual data, the proportional difference in mean square error is large in the case of housing starts and the quit rate, but the fraction of variance in the annual series accounted for by the seasonal term is so small that the point is moot.

In the optimal adjustment of recent economic time series, the increase in mean square error that results from the direct adjustment of quarterly series, as opposed to the quarterly aggregation of adjusted monthly series, is much greater. Since $x^N(t)$ and $x^S(t)$ are jointly wide-sense stationary processes, the signal extraction problem of

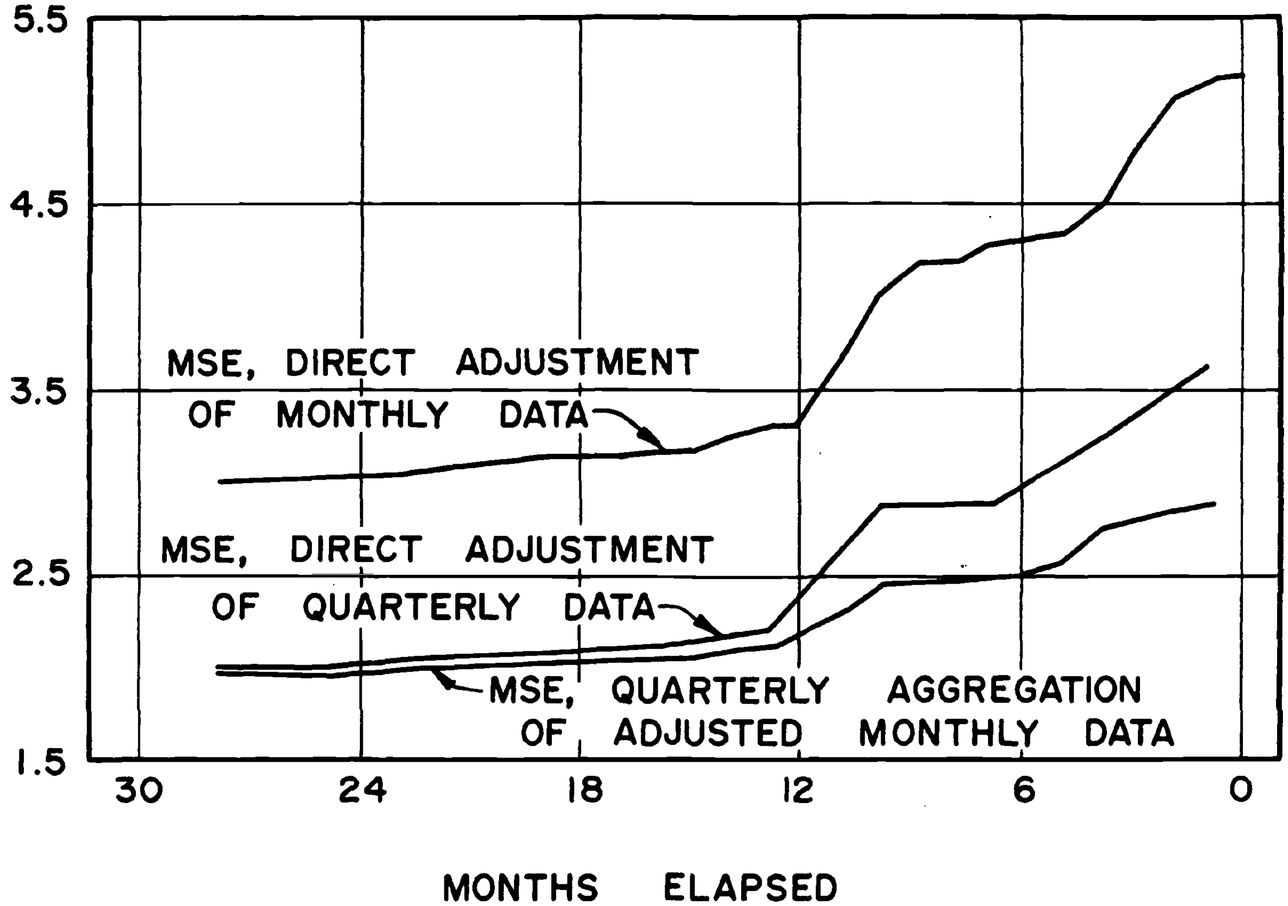
[2] The folding operator reflects the aliasing inherent in the temporal aggregation of a wide-sense stationary process. (See, e.g., [1, pp. 36–38].

Figure 1. ALTERNATIVE ADJUSTMENT PROCEDURES FOR THE GRETHER-NERLOVE VARIABLE



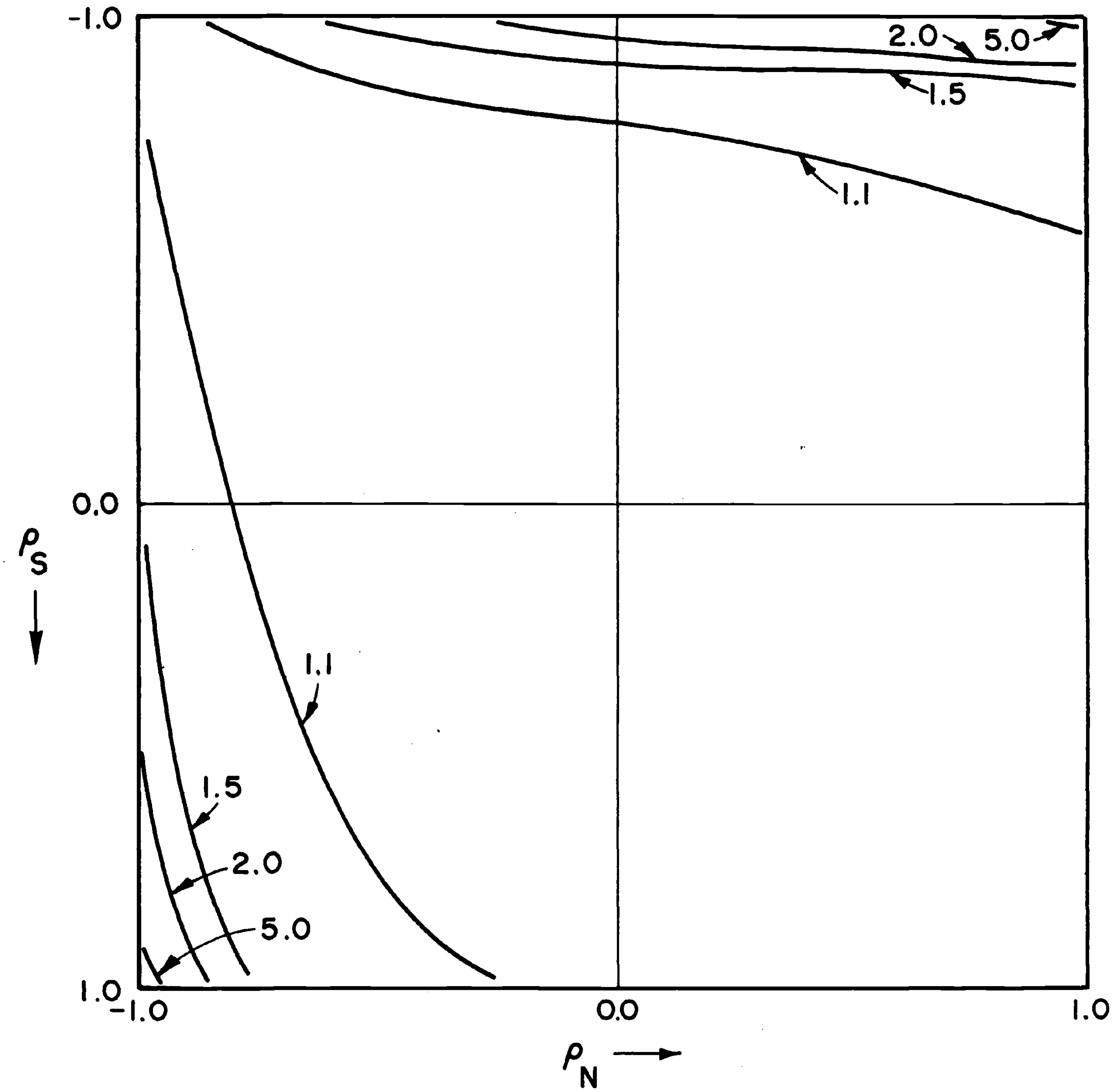
Note: Variable generated by the process $x(t) = \frac{(1+0.8L)\ (t)}{(1\text{-}0.95L)\ (1\text{-}0.75L)} + \frac{(1+0.6L)(t)}{(1\text{-}0.9L^{12})}$. (Mean square errors were calculated as described in the text.)

Figure 2. ALTERNATIVE ADJUSTMENT PROCEDURES FOR HOUSING STARTS



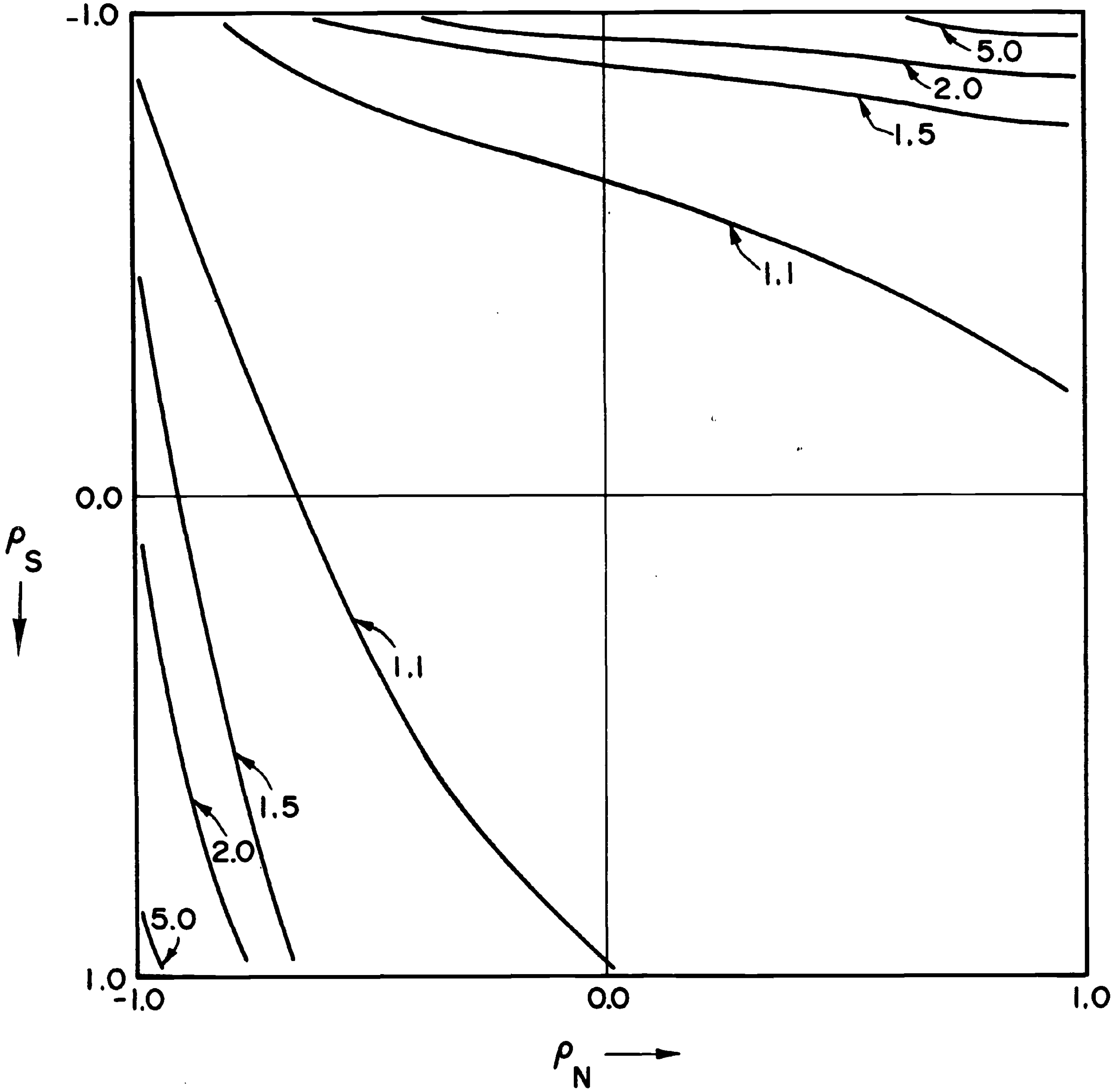
Note: Covariance properties of the variable were estimated from housing starts data, 1959-74, for which var $(x^N(t))$=64.588 and var $(x^S(t))$= 37.403. (Mean square errors were calculated as described in the text.)

Figure 3. RATIO OF MSE TO MSE OF OPTIMAL ADJUSTMENT PROCEDURE WHEN x_1 AND x_2 ARE SUMMED AND THEN SEASONALLY ADJUSTED



Note: Both variables have the typical spectral shape of the variable in fig. 1, with var $(x_2^N(t))=2\text{var}(x_1^N(t))$; var $(x_2^S(t))=2\text{var}\ (x_1^S(t))$; corr $(x_1^N(t), x_2^N(t))=\rho_N$; and corr $(x_1^S(t), x_2^S(t))=\rho_S$.

Figure 4. RATIO OF MSE TO MSE OF OPTIMAL ADJUSTMENT PROCEDURE WHEN x_1 AND x_2 ARE SEASONALLY ADJUSTED AND THEN SUMMED



Note: Both variables are the same as those in fig. 3.

estimating $x^N(t)$, given a realization $x(t+m)$, $x(t+m-1)$, $x(t+m-2), \ldots$, can be approached using the Wiener-Hopf technique, an exposition of which is given in [8, ch. 6]. A brief summary of the procedure follows.[3]

Let $R_x(t)$, the autocovariance function of $x(t)$, and $R_x^N(t)$, the autocovariance function of $x^N(t)$, have Laurent expansions

$$g_x(z)=\sum_{s=-\infty}^{\infty} R_x(s)z^s \tag{11}$$

and

$$g_x^N(z)=\sum_{s=-\infty}^{\infty} R_x^N(s)z^s \tag{12}$$

respectively, where z is any complex number. (Note that $g_x(e^{-i\omega})=S_x(\omega)$ and $g_x^N(e^{-i\omega})=S_x^N(\omega)$.) Assume that $g_x(z)$ and $g_x^N(z)$ are analytic in some closed annulus of the unit circle and $S_x(\omega)>0$ for all ω.[4] Under these conditions, $x(t)$ has a moving average representation

$$x(t)=\sum_{s=0}^{\infty} b(s)\epsilon(t-s)$$

where $\epsilon(t)$ is a serially uncorrelated process having a variance σ^2 that is the mean square error in forecasting $x(t)$ one period ahead, and $b(0)=1$. Let $B(z)$ denote the Laurent expansion of b. Then,

$$\hat{x}_m^N(t)=\sum_{s=-m}^{\infty} a_m(s)x(t-s) \tag{13}$$

where $a_m(s)$ has Laurent expansion

$$A_m(z)=\frac{1}{\sigma^2 B(z)}\left[\frac{g_x^N(z)}{B(z^{-1})}\right]_{-m} \tag{14}$$

Following the notation of [8], the last term in expression (14) denotes the Laurent expansion of the first m future, the current, and all past coefficients of the function having a Laurent expansion that is $g_x^N(z)/B(z^{-1})$. The function $B(z)$ may be obtained as the canonical factorization of $g_x(z)$, described in [8, p. 26]. Writing

$$g_x(z)=\exp\left[\sum_{s=-\infty}^{\infty} c(s)z^s\right] \tag{15}$$

we have

$$B(z)=\exp\left[\sum_{s=1}^{\infty} c(s)z^s\right] \tag{16}$$

[3]The contents of the following paragraph are taken directly from [8], making only the obvious substitutions required for application to the problem of seasonal adjustment.

[4]This requirement as well as the stipulation that $ln(g_x(z))$ have a Laurent expansion analytic in some spen annulus of the unit circle are not restrictive. They are almost tantamount to saying that the process in question cannot be perfectly predicted, given its entire past. (See [8, pp. 23–26].)

so long as $ln(g_x(z))$ has a Laurent expansion analytic in some open annulus of the unit circle.

The validity of expressions (11)–(16) on the unit circle and the existence of the fast Fourier transform computational algorithm makes possible extremely rapid computation of the optimal deseasonalizing filter $a_m(s)$ for any m and any autocovariance functions $R_x(t)$ and $R_x^N(t)$. From (15),

$$ln\,(S_x(\omega))=\sum_{s=-\infty}^{\infty} c(s)e^{-is\omega}$$

so that $c(s)$ may be formed as the inverse Fourier transform of $ln(S_x(\omega))$. Having obtained $c(s)$, use (16) to form

$$\tilde{b}(\omega)=\exp\left[\sum_{s=1}^{\infty} c(s)e^{-i\omega s}\right]$$

Form the ratio $S_x^N(\omega)/\tilde{b}(-\omega)$, inverse Fourier transform, and set to zero all terms beyond the m'th future of the resulting function; Fourier transform to obtain $[g_x^N(z)/B(z^{-1})]_{-m}$ evaluated at $z=e^{-i\omega}$. From (14), division by $\tilde{b}(\omega)$ yields $\tilde{a}_m(\omega)$ having an inverse Fourier transform that is the function $a_m(s)$ in (13). (The further computational details needed to reproduce the results presented here are provided in the app.)

Once $\tilde{a}_m(\omega)$ is known, the mean square error of the estimate $x_m(t)$ may be evaluated using the relation

$$E[\hat{x}_m^N(t)-x^N(t)]^2=E\left[\sum_{s=-m}^{\infty} a_m(s)(x^N(t-s)+x^S(t-s))-x^N(t)\right]^2$$

$$=\int_{-\pi}^{\pi}\{|1-\tilde{a}_m(\omega)|^2S_x^N(\omega)+|\tilde{a}_m(\omega)|^2S_x^S(\omega)\}\,d\omega \tag{17}$$

From the result on aggregation in the section on aggregation and optimal adjustment, optimally adjusted recent quarterly data for the quarter centered at month t is

$$\hat{x}_{Qm}^N(t)=\left[\sum_{s=-m+1}^{\infty} a_{m-1}(s)x(t+1-s)+\sum_{s=-m}^{\infty} a_m(s)x(t-s)\right.$$

$$\left.+\sum_{s=-m-1}^{\infty} a_{m+1}(s)x(t-1-s)\right]/3$$

$$=\sum_{s=-\infty}^{\infty}(a_{m-1}(s+1)+a_m(s)+a_{m+1}(s-1))x(t-s))/3$$

$$=\sum_{s=-\infty}^{\infty} a_m^Q(s)x(t-s)$$

where it is understood that $a_m(s)=0$ for $s<-m$ and the last equality is defining for $a_m^Q(s)$. The mean square error for this quarterly adjusted series is

$$\int_{-\pi}^{\pi}|D_3(\omega)-\tilde{a}_m^Q(\omega)|^2S_x^N(\omega)d\omega+\int_{-\pi}^{\pi}|\tilde{a}_m^Q(\omega)|^2S_x^S(\omega)d\omega \tag{18}$$

where $\tilde{a}_m^Q(\omega)$ denotes the Fourier transform of $a_m^Q(s)$. The mean square error of the adjustment process that works directly from unadjusted quarterly data is obtained exactly as that for the monthly data, except that one begins with $F_3[D_3S_x](\omega)$ and $F_3[D_3S_x^N](\omega)$ in lieu of $S_x(\omega)$ and $S_x^N(\omega)$, respectively, and the integration in the analogue of expression (17) is over the interval $[-\pi/3, \pi/3]$.

Mean square errors for the optimal adjustment of recent monthly data (17), for the quarterly aggregation of adjusted monthly data (18), and for the adjustment of quarterly data (the quarterly analogue of (17), were measured over a wide range of values of m, for the artificial variable constructed by Grether and Nerlove, and for U.S. housing starts. $S_x^N(\omega)$ and $S_x^S(\omega)$ for the latter were estimated as described in the appendix. The results are presented in figures 1 and 2, which show that the more information one has (i.e., the larger is m), the better the estimate $\hat{x}_m(t)$ becomes. Revision of recent data as time proceeds improves the quality of the adjusted series, but in neither of the cases reported here is the mean square error reduced by as much as half, once the unadjusted data is available.

Figures 1 and 2 indicate that, for recent data, the improvement which results when one aggregates adjusted monthly data to form an adjusted quarterly series instead of adjusting quarterly data directly, is nontrivial: The improvement is greatest for the current quarter and tapers off thereafter. (It turns out that, if one attempts to forecast future values of $x^N(t)$ from current and past $x(t)$, the reduction in mean square error is proportionately even greater.) In the case of housing starts, directly adjusted quarterly data do not reach the quality of the current quarterly aggregation of adjusted monthly data until half a year has elapsed. Given the wide journalistic and political use of seasonally adjusted data, revision of adjusted data or delays in its publication are undesirable. The results reported in figures 1 and 2 indicate that substantial reduction in both revisions and delays may be achieved by seasonally adjusting data measured at the finest available time intervals, and then aggregating over time to the desired reporting interval.

SECTORAL AGGREGATION: SOME EXAMPLES

In this section, we consider the optimal seasonal adjustment of historical data that are a realization of a series $x(t)$ with

$$x(t)=\sum_{i=1}^{n} x_i(t) \tag{19}$$

Each $x_i(t)$ is observed and is, in turn, the sum of unobservable nonseasonal and seasonal components

$$x_i(t)=x_i^N(t)+x_i^S(t),\ i=1,\ \ldots,\ n \tag{20}$$

The $x_i^N(t)$ and $x_i^S(t)$ are assumed to be jointly wide-sense stationary; the components $x_i^N(t)$ and $x_j^S(s)$ are independent for all i, j, t, and s, and C is again the class of all linear functions. To isolate the problem of sectoral aggregation, we shall assume that $x(t)=(x_1(t), \ldots, x_n(t))'$ is observed monthly, but the examples considered here easily could be reworked using weekly or quarterly data. If $S_x^N(\omega)$ and $S_x(\omega)$, the spectral density matrices of $x^N(t)$ and $x(t)$, respectively, are known, then, from (7), the optimally adjusted series is

$$\hat{x}^N(t)=a*x(t) \tag{21}$$

where $a(s)$ is a $1\times n$ vector of functions with Fourier transform

$$e\,S_x^N(\omega)S_x(\omega)^{-1} \tag{22}$$

where the $1\times n$ vector $e=(1, \ldots, 1)$. We shall call this method of adjustment method A. Application of (6) shows that

$$E[\hat{x}^N(t)-x^N(t)]^2 = \int_{-\pi}^{\pi} e\{S_x^N(\omega)-S_x^N(\omega)S_x(\omega)^{-1}S_x^N(\omega)'\}e'd\omega \tag{23}$$

Two practical questions about this procedure arise immediately. How much do we gain, as a matter of practice, in using the multivariate form (21)? When n is large, adjustment may become computationally burdensome. Unless (23) is substantially smaller than

$$\int_{-\pi}^{\pi}\sum_{i=1}^{n}\sum_{j=1}^{n}\left\{\left(1-\frac{S_{x_{ii}}^N(\omega)}{S_{x_{ii}}(\omega)}\right)S_{x_{ij}}(\omega)\left(1-\frac{S_{x_{jj}}^N(\omega)}{S_{x_{jj}}(\omega)}\right) + \frac{S_{x_{ii}}^N(\omega)S_{x_{ij}}^S(\omega)S_{x_{jj}}^N(\omega)}{S_{x_{ii}}(\omega)S_{x_{jj}}(\omega)}\right\}d\omega \tag{24}$$

which is the mean square error when the n series are individually adjusted and then summed (method B) and

$$\int_{-\pi}^{\pi}\left\{e\,S_x^N(\omega)e'-\frac{(e\,S_x^N(\omega)e')^2}{e\,S_x(\omega)e'}\right\}d\omega \tag{25}$$

which is the mean square error when $x(t)$ is adjusted directly (method C), recourse to (21) and (22) may not be worthwhile. We shall adduce evidence that in applied situations the value of (24) and (25) may exceed (23) by a factor of more than 2. The second question is how one might go about estimating the matrices $S_x^N(\omega)$ and $S_x^S(\omega)$, a problem which must be solved before optimal joint adjustment can proceed but is not required in the other two methods. We shall argue that $S_x^N(\omega)$ and $S_x^S(\omega)$ are, indeed, identified by Nerlove's criterion.

The relation between (23), (24), and (25) depends on the covariance structure among $x^N(t)$ and $x^S(t)$: Without restricting this structure, one can say nothing about the relative sizes of (24) and (25) or compare them with (23), except to say that they are, in general, larger. A few examples will illustrate the considerations involved. Consider the case $n=2$ with

$$S_x^N(\omega)=\begin{bmatrix} S_{x_1}^N(\omega) & \rho_N c S_{x_1}^N(\omega) \\ \rho_N c S_{x_1}^N(\omega) & c^2 S_{x_1}^N(\omega) \end{bmatrix},$$

$$S_x^S(\omega)=\begin{bmatrix} S_{x_1}^S(\omega) & \rho_S d S_{x_1}^S(\omega) \\ \rho_S d S_{x_1}^S(\omega) & d^2 S_{x_1}^s(\omega) \end{bmatrix} \qquad (26)$$

When $\rho_N=\rho_S=0$, mean square error for both method A and method B is

$$\int_{-\pi}^{\pi}\left\{(1+c^2)S_{x_1}^N(\omega)-\frac{S_{x_1}^N(\omega)^2[c^2(1+c^2)S_{x_1}^N(\omega)+(d^2+c^4)S_{x_1}^S(\omega)]}{S_{x_1}(\omega)[c^2S_{x_1}^N(\omega)+d^2S_{x_1}^S(\omega)]}\right\}d\omega$$

Since there is no interaction between $x_1(t)$ and $x_2(t)$, both methods adjust each series individually. Unless $d=c$, the adjustment of $x_1(t)$ will not be the same as that of $x_2(t)$, and method C (which forces the same adjustment on both variables) will have a larger mean square error. When $d=c=1$, mean square error for both method A and method C is

$$\int_{-\pi}^{\pi}\left\{2(1+\rho_N)S_{x_1}^N(\omega)-\frac{2S_{x_1}^N(\omega)^2(1+\rho_N)^2}{(S_{x_1}(\omega)+\rho_N S_{x_1}^N(\omega)+\rho_S S_{x_1}^S(\omega))}\right\}d\omega$$

The joint adjustment process in method A treats the two variables symmetrically, using their interaction to help sort out $x^N(t)$ and $x^S(t)$, so long as $\rho_N \neq \rho_S$. Hence, the fact that method C forces a symmetric treatment of $x_1(t)$ and $x_2(t)$ does not increase mean square error, but method B has a higher mean square error unless $\rho_N=\rho_S$. When $d=c$ and $\rho_N=\rho_S=\rho$, all three methods have mean square error

$$(1+2c\rho+c^2)\int_{-\pi}^{\pi} S_{x_1}^N(\omega)\left(1-\frac{S_{x_1}^N(\omega)}{S_{x_1}(\omega)}\right)d\omega$$

In this case, the relative spectral shapes of the pairs $x_1^N(t)$ and $x_1^S(t)$, $x_2^N(t)$ and $x_2^S(t)$, and $(x_1+x_2)^N(t)$ and $(x_1+x_2)^S(t)$ are all the same: The method B and method C adjustment procedures, therefore, coincide. In fact, for any square summable linear functions $a(s)$ and $b(s)$, the ratio of the spectral density of $a*x_1^N(t)+b*x_2^N(t)$ to $a*x_1(t)+b*x_2(t)$ is $S_{x_1}^N(\omega)/S_{x_1}(\omega)$, and this accounts for the inability of method A to improve on method B or method C.

Some further examples are provided in figures 3 and 4. $S_{x_1}^N$ is taken to be the spectral density of the nonseasonal component of the Grether-Nerlove variable, described in the previous section, while $S_{x_1}^S(\omega)$ is the spectral density of its seasonal component that accounts for 10 percent of the total variance in $x_1(t)$. $S_x^N(\omega)$ and $S_x^S(\omega)$ are constructed from (26), with $c=d=2.0$, and the mean square errors are computed using (23), (24), and (25). The correlation ρ_N of the nonseasonal components is assumed to be high, as is the case for the components of many economic aggregates. In general, the smaller is the product $\rho_S\rho_N$ the more method B and method C suffer by comparison with method A, which is able to use the information on interactions in an optimal fashion to eliminate as much of the seasonal noise in $x_1(t)+x_2(t)$ as possible. In most cases, including many not reported here, method C produces better results than method B. A notable exception is the situation in which one series is contaminated and the other is not, and the spectral shapes of $x_1(t)$ and $x_2(t)$ are different enough that there is a real advantage in separately adjusting $x_1(t)$ and $x_2(t)$ before aggregation takes place. Even then, the results achieved are not nearly as good as those obtained by using the optimal method A, which has comparative advantage increases as correlation between the nonseasonal components approaches 1. This case has an obvious empirical analogue in the adjustment of data for which disaggregation by geographical regions with different seasonal factors is possible.

In order to examine the gains likely to be realized for the joint seasonal adjustment of sectoral components, in practice, estimated spectral densities of the seasonal and nonseasonal components of a disaggregated economic time series were used in (23), (24), and (25). The variable chosen was housing starts, available for 1959–74. This series shows large seasonal fluctuations, increasing anywhere between 75 percent and 150 percent from its early winter low to its spring high. Data are disaggregated geographically into the northeast, north-central, southern, and western regions. Seasonality is even more pronounced in the first two components than in the aggregate series, while seasonal variance in the South and West is much smaller. A priori, the housing starts series appears to be one in which substantial gains from joint seasonal adjustment might be realized.

Unfortunately, the problem of estimating the matrices $S_x^N(\omega)$ and $S_x^S(\omega)$ employing Nerlove's criterion, which must be considered before joint seasonal adjustment of time series can proceed, has not (to the author's knowledge) been resolved satisfactorily. In principle, the problem may be attacked using the fact that if the seasonal and nonseasonal vector components are additive and independent, then the spectral density of a linear combination of the nonseasonal components is smooth and exhibits no peaks at seasonal frequencies.[5] This implication is restrictive and, with a suitable parameterization of peaks, might be testable; as a practical matter, use of just enough linear combinations to identify $S_x^N(\omega)$ and $S_x^S(\omega)$ can easily produce estimates of these matrices that are not positive semidefinite. In studying the housing starts data, $S_x^N(\omega)$ was estimated by smoothing the matrix of periodogram ordinates after eliminating those ordinates near the seasonal frequencies. This procedure guarantees that estimated $S_x^N(\omega)$ and $S_x^S(\omega)$ will be positive semidefinite but results in a systematic downward bias of $S_x^N(\omega)$ near

[5] One would, of course, have to exclude linear combinations that have transfer functions which exhibit seasonal peaks.

Figure 5. **OPTIMAL SEASONAL ADJUSTMENT AND SECTORAL AGGREGATION OF HOUSING STARTS VARIABLE**

Component	Estimated covariance matrix				Estimated correlation matrix			
Nonseasonal.	1.3098	0.9579	2.2977	1.4096	1.000	0.522	0.486	0.563
	.9579	2.5639	4.8619	1.9556	.522	1.000	.735	.558
	2.2977	4.8619	17.0810	6.5652	.486	.735	1.000	.726
	1.4096	1.9556	6.5652	4.7902	.563	.558	.726	1.000
Seasonal	2.6567	3.6459	2.2995	1.0650	1.000	.956	.741	.651
	3.6459	5.4743	3.6683	1.7251	.956	1.000	.824	.735
	2.2995	3.6683	3.6179	3.6179	.741	.824	1.000	.886
	1.0650	1.7251	1.6932	1.0076	.651	.735	.886	1.000

Note: Housing starts were disaggregated geographically into northeast, north-central, southern, and western regions, in that order.

Variance of aggregate series		Mean square errors	
Component	Variance	Method	Error
Nonseasonal component	61.8407	A	0.70033
Seasonal component	40.9505	B	1.3900
		C	1.3836

seasonal frequencies and, consequently, a lower estimated variance of the nonseasonal components than was the case in the univariate methods employed in arriving at the results reported in figures 1 and 2. (For details, the reader is referred to the app.) Formal resolution of this problem appears to be a topic, in its own right, that would take us far afield.

Once the seasonal and nonseasonal spectral density matrices have been estimated, the mean square error of each of the three adjustment procedures may be computed. The results of these calculations are shown in figure 5, which reports the estimated variance and correlation matrices for the nonseasonal and seasonal components and the mean square errors of the three methods. Several features of the housing starts data reported in figure 5 are important. First, the four series are contaminated in different ways by seasonality: While about two-thirds of the variance in housing starts in the northeast and north-central States is ascribable to seasonal factors, less than 20 percent of variance in the South and West is accounted for in this way. Of the variance in housing starts in the northeast-north-central region, 72.7 percent is accounted for by seasonality, while only 18.6 percent of the variance in this series can be so described for the rest of the United States. Hence, application of the same deseasonalizing filter to all four series is inappropriate. Second, the series are exceptionally contaminated by seasonality, by conventional standards: Recall that in the typical Grether-Nerlove variable only 10 percent of the variance is due to seasonality. Finally, correlation among the seasonal factors is higher than correlation among nonseasonal factors, and variances differ widely for the four series, conditions under which a substantial gain from an explicitly multivariate treatment is likely.

As the results show, mean square error of the optimal method A is about half that of the other two procedures. In addition, method A produces deseasonalized sectoral series with mean square errors smaller than those from method B. The sum of the four series adjusted by method A equals the aggregated method A deseasonalized series, while the sum of the four series adjusted by method B is not the same series produced by method C. These attractive features of the optimal, joint-adjustment procedure indicate that it may be an important way of improving the quality and internal consistency of officially adjusted data. Until other series are examined in a fashion similar to our study of housing starts and the problem of joint estimation of seasonal and nonseasonal components in the multivariate case is resolved more satisfactorily, this conclusion can only be tentative.

CONCLUSIONS

The results presented in the sections on temporal and sectoral aggregation indicate that optimal seasonal adjustment of economic time series followed by aggregation, as described in the sections on optimal adjustment and aggregation and optimal adjustment, results in seasonally adjusted data that are of substantially higher quality than if data are aggregated first and then optimally adjusted. Mean square error is reduced by about half in the adjustment of current, univariate series and in the historical adjustment of at least one series for which disaggregation by sectors is possible. The fully optimal adjustment procedure has the further advantages of reducing the required revision of current, seasonally adjusted data and producing sectorally disaggregated, seasonally adjusted series that are consistent with the aggregate adjusted series. In interpreting these results, some qualifications are necessary.

First, in the estimation of mean square errors in the sections on temporal and sectoral aggregation, it was assumed that the covariance structure of the seasonal and nonseasonal components was known exactly: No allowance was made for the estimation error that must result in the actual application of these methods. The latter consideration, of course, increases all mean square errors for actual economic time series reported here, but it is not evident how it would affect the relative errors reported in figures 1 and 2 and table 1. If the covariance structure changes over time, a further source of error is introduced.

Second, our comparisons in the last two sections were limited to alternative methods which were all the outcome of seasonal adjustment procedures that were already optimal in some sense. The class of adjustment procedures allowed in each case was much wider than that permitted in official adjustment procedures. It is quite possible that the improvements realized by handling the aggregation problem properly are small compared with the gain achieved by allowing official adjustment procedures to be more flexible than they are now.

Finally, while the examples in the sections on temporal and sectoral aggregation are intended to be both realistic and illustrative, they reflect few of the complications that arise in practice. These include the treatment of nonstationarity, the combination of multiplicative seasonal and nonseasonal components with additive sectoral components, and the use of more specific knowledge about seasonality, the treatment of which is nontrivial.

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APPENDIX

COMPUTATIONAL PROCEDURES

Figures 1 and 2

To compute the spectral density of the nonseasonal portion of the Grether-Nerlove variable, the following sequences were formed:

$$a(1)=1.0,\ a(2)=0.8,\ a(3)=\ldots=a(384)=0.0$$

$$b(1)=1.0,\ b(2)=-0.95,\ b(3)=\ldots b(384)=0.0$$

$$c(1)=1.0,\ c(2)=0.75,\ c(3)=\ldots=c(384)=0.0$$

Using the fast Fourier-transform algorithm the finite Fourier transform of each series was computed, e.g.,

$$\tilde{a}(j)=\sum_{s=1}^{384} a(s)\exp(-i2\pi(s-1)(j-1)/384, \quad j=1,\ldots,384$$

The sequences $\tilde{b}(j)$ and $\tilde{c}(j)$ were computed in the same way. The inner products $|\tilde{a}(j)|^2$, $|\tilde{b}(j)|^2$, and $|\tilde{c}(j)|^2$ were formed, and the nonseasonal spectral density was taken to be $|\tilde{a}(j)|^2/(|\tilde{b}(j)|^2+|\tilde{c}(j)|^2)$+constant, $j=1, \ldots, 384$. Similarly, the seasonal component was taken to be $|d(j)|^2/|\tilde{e}(j)|^2$, $j=1, \ldots, 384$, where $d(1)=1.0$, $d(2)=0.6$, $d(3)=\ldots=d(384)=0.0$ and $e(1)=1.0$, $e(2)=-0.9$, $e(3)=\ldots e(384)=0.0$. The nonseasonal, seasonal, and constant were weighted to achieve the decomposition by variance, noted in the section on temporal aggregation, and a total variance of 1.

The spectral density of housing starts was estimated by removing the mean of the 192 observations in the 1959–74 monthly sample and computing the periodogram at 384 equally spaced ordinates. The periodograms at each of the 11 seasonal ordinates were removed, and the remaining ordinates were smoothed using an inverted V-window that has a base of 9 ordinates, a correction in the smoothing weights being made for the absent seasonal ordinates. The seasonal ordinates of the smoothed periodogram were then removed and replaced by the original periodogram ordinates at those frequencies. The result is the estimated spectral density, allowing for a spike at seasonal frequencies. The spectral density of the nonseasonal component was estimated by removing the ordinates at the seasonal frequencies and three adjacent ordinates in each direction. The remaining portion of the periodogram was smoothed with the inverted V-window, correcting for the downward bias across seasonal frequencies. The spectral density of the seasonal component was estimated by subtracting estimated $S_x^N(\omega)$ from estimated $S_x(\omega)$, setting $S_x^S(\omega)$ to zero and $S_x^N(\omega)$ to $S_x(\omega)$ in those few cases in which the difference was negative. The procedure for the manufacturing quit rate was the same, except that 384 monthly observations for 1943–74 were used.

In all three cases, the mean square errors were computed according to (8), (9), and (10) and the analogues of (9) and (10) for annual aggregation, summation across 384 equally spaced ordinates replacing integration.

For the study of current seasonal adjustment, spectral densities of nonseasonal and seasonal components were formed as previously described. The sum of the two components was taken to be (11), evaluated at 384 equally spaced ordinates on the unit circle, while the nonseasonal component is (12), evaluated in the same way. The function $\tilde{a}_m(\omega)$ was computed as described in the text, evaluation always being at 384 equally spaced ordinates. The inverse Fourier transform, where required, is formed by computing the Fourier transform of the complex conjugate of the sequence in question and dividing by 384. The first entry of the resulting series is the current term in the time domain, coefficients with positive arguments being the successive entries, up through the 192d term. Terms with negative argument begin at entry 384, coefficient with argument $-j$ being in entry $385-j$, $j=1, \ldots, 191$. Entry 193 was split between the 192d and –192d arguments, a division of no real consequence, since entry 193 was always small. The mean square errors (17) and (18) and the quarterly analogue of (17) were computed as the sums across these ordinates.

Figures 3 and 4

S_{x_1} for the Grether-Nerlove variable was estimated as described for figure 5. Equations (23), (24), and (25) were evaluated directly using (26) and replacing integration by summation over the 384 ordinates.

Figure 5

The 4×4 matrix of 384 equally spaced periodogram ordinates was computed. For each of the 10 distinct entries in this matrix, the nonseasonal component of the spectral density was constructed as described for figures 1 and 2, except that correction for the downward bias in the estimate of $S_x^N(\omega)$ at seasonal and nearby ordinates was not made. The matrix $S_x(\omega)$ was estimated as described for figures 1 and 2, allowing for a spike at seasonal frequencies. The matrix $S^S_x(\omega)$ was taken to be the difference between $S_x(\omega)$ and $S^N_x(\omega)$. Mean square error was computed from equations (23), (24), and (25), summation over 384 equally spaced ordinates replacing integration.

Data

The quit rate in U.S. manufacturing was taken from *Employment and Earnings, United States 1909–72,* Bureau of Labor Statistics Bulletin 1312–9, pages 37–38, for January 1943–June 1972 and from various issues of *Employment and Earnings,* table D–1, for July 1972–December 1974.

Data for U.S. housing starts, disaggregated by geographical region, were supplied directly by the Bureau of the Census.

COMMENTS ON "THE TEMPORAL AND SECTORAL AGGREGATION OF SEASONALLY ADJUSTED TIME SERIES" BY JOHN GEWEKE

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John Geweke is to be congratulated on presenting us with a very neat result on a troublesome problem. He shows that "for virtually every conceivable time series . . . and a reasonably inclusive class of potential adjustment procedures, minimum mean-square-error adjustment implies that seasonal adjustment should always precede temporal or sectoral aggregation" [4]. He illustrates the advantage of prior adjustment with artificial data, regional data on housing starts, and data on the quit rate in U.S. manufacturing. He finds that the gain from adjustment prior to aggregation is particularly strong in terms of revision stability; i.e., when initially released data is deseasonalized in advance of aggregation, it is subject to much smaller revision as additional observations accumulate.

I must confess that I was particularly naive on my first reading of this paper, and, while I am still enthusiastic about Geweke's result, I was guilty of thinking that he had achieved much more than he claims. I thought that the theorem had something to say regarding the simmering controversy between John Brittain and the Bureau of Labor Statistics (BLS) [1; 3]. The BLS adds seasonally adjusted unemployment to seasonally adjusted employment in order to obtain the seasonally adjusted labor force. Brittain's residual procedure subtracts seasonally adjusted employment from seasonally adjusted labor force in order to obtain a seasonally adjusted figure for unemployment. The difference is quite marked. You may recall that the initial BLS figure for unemployment for May 1975 was 9.2 percent; Brittain's residual figure was 8.9 percent; revisions announced by BLS at the beginning of 1976 dropped the official May 1975 rate to 8.9 percent and the residually adjusted figure to 8.7 percent. There are several reasons why Geweke's theorem does not resolve this controversy. First, the Census Bureau's X–11 procedure is not what Geweke has in mind when he refers to a "reasonably inclusive class" of seasonal adjustment procedures, and I do not think anyone has claimed that Census Bureau's X–11 is a mean-square-error adjustment procedure. Second, Geweke's result requires that the component series to be summed must be adjusted jointly; as figure 3 reveals, there may be a loss, rather than a gain, from adjusting in advance of aggregation if each of the component series is adjusted individually, rather than by pooling the evidence. Moving-average adjustment programs do not pool the evidence in the way required for Geweke's theorem. Third, Geweke's theorem holds for additive, rather than the more popular multiplicative adjustment. I would guess that, if multiplicative adjustment is achieved by simply taking the logs of the original data, his theorem applies to ratios and/or geometric averages of the data, rather than sums obtained in aggregation. Let me emphasize that my own feeling is that these factors do not distract all that much from Geweke's results. For one thing, I suspect that this conference will contribute to modifications of seasonal adjustment practice in a direction that will make Geweke's theorem applicable. None of us will fight optimal seasonal adjustment (although we may quarrel about what we mean by "optimal"), and joint adjustment of related series is undoubtedly the wave of the future.

There is another problem concerning aggregation that can be clarified by considering data on unemployment. The official BLS figures are based on an age-sex breakdown of unemployment and an age-sex, agricultural-nonagricultural breakdown of employment; thus, 12 separate series are adjusted in obtaining the labor force aggregate. This yields an official figure of 8.9 percent for May 1975, but the figure for that month is 8.8 percent if the data are disaggregated by duration of unemployment, 9.0 percent if disaggregated by industry, and 9.1 percent if disaggregated by occupation.[1] I think Geweke's theorem tells us that, if we were using an optimal additive procedure, it might be worth the tabulation effort and computer burden of breaking down the aggregates into very fine detail, rather than choosing between alternative disaggregation procedures; the payoff might be a reduction in the magnitude of revisions, which is currently a serious problem. But, a difficulty with using so many cells would be that many would have so few observations that they would be subject to large sampling error. I am curious as to whether Geweke's results apply when the individual series are subject to sampling and enumerator errors, particularly when the errors are not independently distributed.

I like Geweke's paper, but I must state that I disagree with his underlying seasonal adjustment philosophy. I suspect that the majority at this conference may prefer his approach to mine, but, in good conscience, I must

[1] Each month, the Commissioner of Labor Statistics customarily discusses the latest figures before the Joint Economic Committee. Inserted in the record is a table with 13 alternative estimates of the current seasonally adjusted unemployment rate.

state my piece. I feel that there is much to be said for using a least squares seasonal adjustment strategy. One advantage of my least squares procedure is that it preserves sums; this means that it makes no difference whether you seasonally adjust the components and then sum or seasonally adjust the aggregate [2; 5]. You don't need to worry about whether you should disaggregate by occupation or by industry; you do not have to worry about the danger that the components will be subject to sampling and classification error. A second advantage of my least squares approach is that it is possible to explicitly model seasonal forces. Geweke notes that the housing-start seasonal is much more pronounced in northern than in southern regions. Obviously, this is because the seasonal in construction stems, in large measure, from the seasonal in the weather. One might try to exploit this fact in explicitly modeling the process, using climatic variables in the regression, rather than dummy them out. A useful model would explain differences between regions; because data from different areas could be pooled, tighter estimates could be gained. I think that users of seasonally adjusted construction data would like to have the effect of an unusually severe winter netted out, and the regression approach can accomplish this.

In conclusion, I would like to say that I particularly like Geweke's emphasis on the question of sensitivity to revision. We need standard errors for seasonally adjusted series that will be revealing with regard to the likelihood of revision, as well as sampling error. Currently, there is a tendency for business analysts to overreact to random pips, such as the erroneous 0.3-percent increase in the initially reported unemployment rate for October 1975; it is hard for the White House to resist the temptation to mumble about faulty seasonal adjustment when indicators move in an undesired direction. I hope it will not be too long before press releases of key economic indicators will contain explicit warnings in the form of engineering-style error bands or confidence intervals concerning the sensitivity of seasonally adjusted data to revision and sampling errors.

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COMMENTS ON "THE TEMPORAL AND SECTORAL AGGREGATION OF SEASONALLY ADJUSTED TIME SERIES" BY JOHN GEWEKE

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By treating the problem of seasonal adjustment as a general minimum mean-square signal extraction problem, Geweke derives a procedure for the seasonal adjustment of aggregate time series that optimally utilizes the correlation structure among the components of the aggregate series. His paper also contains a very general proof that the procedure is efficient, relative to procedures that either ignore the information provided by the component series or use it incorrectly. A byproduct of this derivation is a formula for the minimized value of the mean-square loss function that he uses to compute the efficiency of the optimal procedure, relative to several suboptimal procedures. The general finding is that the relative efficiency of the optimal procedure depends on the spectral density of the nonseasonal and seasonal components of the aggregate series and, in some cases, may be quite large.

In discussing this paper, it will be convenient to illustrate the problem in a considerably more simple model than used in Geweke's paper. This will allow abstraction from some technical issues that would otherwise significantly lengthen the discussion. Although most of the main points can be demonstrated in this less general setting, actual applications, of course, require the full model of the Geweke paper.

Consider two observable economic variables, x_1 and x_2, which are both additively composed of unobservable nonseasonal and seasonal parts

$$x_1 = x_1^N + x_1^S \tag{1}$$

$$x_2 = x_2^N + x_2^S \tag{2}$$

It is assumed that the pair of nonseasonal components (x_1^N, x_2^N) has a zero mean and covariance matrix $\sum^N$, and, similarly, the pair of seasonal components has a zero mean and covariance matrix $\sum^S$. Furthermore, the pair (x_1^N, x_2^N) is uncorrelated with (x_1^S, x_2^S).

The seasonal adjustment of the observable variables x_1 and x_2 can be viewed as a problem of extracting the nonseasonal parts x_1^N and x_2^N. However, suppose that the ultimate objective is not extracting the individual nonseasonal components x_1^N and x_2^N but, instead, the aggregate[1] nonseasonal component $y = x_1^N + x_2^N$. If the loss function is quadratic, then this objective can be formally represented as finding a value $\hat{y}$ to minimize

$$E[(y-\hat{y})^2 | x_1, x_2] \tag{3}$$

Clearly, the minimizing value of $\hat{y}$ is

$$\hat{y}_A = E(y | x_1, x_2) = E(x_1^N | x_1, x_2) + E(x_2^N | x_1, x_2) \tag{4}$$

and, in general, the values

$$\hat{y}_B = E(x_1^N | x_1) + E(x_2^N | x_2) \tag{5}$$

or

$$\hat{y}_C = E(x_1^N + x_2^N | x_1 + x_2) \tag{6}$$

will not minimize the criterion. Given the obvious optimality of the solution $\hat{y}_A$ in this simple problem, one might wonder why $\hat{y}_B$ and $\hat{y}_C$ would ever be of interest. The answer is that, in more complex practical problems with important data or computing limitations, $\hat{y}_B$ and $\hat{y}_C$ are frequently used as solutions to seasonal adjustment problems. In equation (5), each individual series is adjusted before aggregation without reference to the other series. The seasonally adjusted aggregate is then the sum of these individually adjusted series. This method is frequently used in practice when the series of interest is an aggregate of several other series. For example, the U.S. money supply is adjusted in this way, with the components of currency and demand deposits each adjusted before aggregation into M1. In equation (6), on the other hand, the observable series are first aggregated and then seasonally adjusted. This approach is frequently used in cases of temporal aggregation.

The optimal solution in equation (4) is the one derived by Geweke for general time series problems, and its superiority over the other two approaches is the reason for his statement that "for virtually every conceivable time series ... and a reasonably inclusive class of potential adjustment procedures, minimum mean-square error adjustment implies that seasonal adjustment should always precede temporal or sectoral aggregation." However, as it stands, this statement is misleading. As a comparison of (4) and (5) shows, it is not the preaggregation adjustment that is crucial for optimality, but rather the utilization of all observable components simultaneously. The solution $\hat{y}_B$ seasonally adjusts before aggregation but does not

[1]This sum may represent either a temporal or a sectoral aggregate.

utilize the joint distribution of the components of x_1 and x_2 properly. On the other hand, looking only at the first equality in (4), it is optimal to project the aggregate variable y on x_1 and x_2 to get the seasonally adjusted value.

In deciding whether the optimal solution $\hat{y}_A$ should be used in practice, the relative efficiency of this solution should be compared with the other methods. Geweke computes the relative efficiency of the optimal solution for several time series using his general formula for the mean square error, and this is a very useful and informative part of his analysis. The results indicate that, in cases where the series that are aggregated are very heterogeneous or where the stochastic structure of the nonseasonal and seasonal components are dissimilar, the relative efficiency of the optimal procedure is quite high. However, in cases where the series that are aggregated are homogeneous, the efficiency gains are likely to be small.

Some of the intuition behind these results comes from examining the simple model previously discussed. Suppose that $x_1^N, x_2^N, x_1^S, x_2^S$ has a joint normal distribution. Then, it is easy to show that

$$\hat{y}_A = a_1 x_1 + a_2 x_2$$

$$\hat{y}_B = b_1 x_1 + b_2 x_2$$

$$\hat{y}_C = c(x_1 + x_2)$$

where the a, b, and c coefficients depend on the elements of $\sum^N$ and $\sum^S$ in such a way that, if $\sum^S = \sum^N$, then $a_1 = a_2 = b_1 = b_2 = c$ so that each procedure is identical. Further, if the covariance between x_1^N and x_2^N, as well as between x_1^S and x_2^S, is zero, then $\hat{y}_A = \hat{y}_B$.

These results provide a useful guide for the selection of seasonality problems that can potentially benefit from the optimal method. However, the results also suggest that there are likely to be relatively few aggregate series in this category. Moreover, those which are in this category should be disaggregated before serious economic analysis. Aggregation theory suggests that one should avoid aggregating heterogeneous components. Consequently, many analyses are likely to avoid aggregate series composed of grossly heterogeneous series. This may limit the potential usefulness of the results presented in this paper, but further experimentation with the methods is required before a definitive answer can be given.