Volume Title: Seasonal Analysis of Economic Time Series

Volume Author/Editor: Arnold Zellner

Volume Publisher: NBER

Volume URL: http://www.nber.org/books/zell78-1

Publication Date: 1978

Chapter Title: Seasonality: Causation, Interpretation, and Implications

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Chapter URL: http://www.nber.org/chapters/c4321

Chapter pages in book: (p. 33 - 56)

SEASONALITY: CAUSATION, INTERPRETATION, AND IMPLICATIONS

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CAUSES OF SEASONALITY

It is a very well-known fact that many economic series display seasonality; that is, they have an observable component consisting of a fairly constant shape repeated every 12 months. This component is often treated as being so easily explained that neither an exact definition nor an explanation of its origins is required. It is the objective of this paper to suggest that ignoring consideration of causation can lead to imprecise or improper definitions of seasonality and consequently to misunderstanding of why series require seasonal adjustment, to improper criteria for a good method of adjustment and to have implications for the evaluation of the effects of adjustment both on a single series and when relating two or more series. These considerations do not necessarily lead to better practical methods of adjustment, but they should lead to a better understanding of how to interpret time series and econometric analysis involving seasonal components and seasonally adjusted series. The only other author, prior to this conference, who emphasizes causation of seasonals appears to be BarOn [1].

There are at least four, not totally distinct, classes of causes of seasonal fluctuations in economic data. These classes are discussed in the following sections.

Calendar

The timing of certain public holidays, such as Christmas and Easter, clearly affects some series, particularly those related to production. Many series are recorded over calendar months, and, as the number of working days varies considerably from one month to another in a predetermined way, this will cause a seasonal movement in flow variables, such as imports or production. This working-day problem could also lead to spurious correlations between otherwise unrelated series, as I have discussed elsewhere [4].

Timing decisions

The timing of school vacations, ending of university sessions, payment of company dividends, and choice of the end of a tax year or accounting period are all examples of decisions made by individuals or institutions that cause important seasonal effects, since these events are inclined to occur at similar times each year. They are generally deterministic or preannounced and are decisions that produce very pronounced seasonal components in series such as employment rates. These timing decisions are generally not necessarily tied to any particular time in the year but, by tradition, have become so.

Weather

Actual changes in temperature, rainfall, and other weather variables have direct effects on various economic series, such as those concerned with agricultural production, construction, and transportation, and consequent indirect effects on other series. It could be argued that this cause is the true seasonal, being itself a consequence of the annual movement of the earth's axis which leads to the seasons. Other natural causes can be important, such as the seasonal fluctuations in the abundance of fish, as discussed by Crutchfield and Zellner in their book *Economic Aspects of the Pacific Halibut Fishery* (U.S. Govt. Printing Office, 1963).

Expectation

The expectation of a seasonal pattern in a variable can cause an actual seasonal in that or some other variable, since expectations can lead to plans that then ensure seasonality. Examples are toy production in expectation of a sales peak during the Christmas period, the closing down of certain vacation facilities immediately after Labor Day in expectation of a decline in demand for these facilities, and the expectation of bad weather in New Jersey in January may mean that few plans are made for new house construction during that month. People choose their vacation destinations

Research supported, in part, by NSF Grant Soc. 74/12243.

on the expectation of weather conditions rather than on the actual situation. Without the expectation-planning aspect, the seasonal pattern may still occur but might be of a different shape or nature. An extreme example is that of British egg prices in the early sixties. The eggs were produced almost entirely by battery hens, who had no idea of the seasons as they existed in a closely controlled, stable environment, and, thus, production could be made steady throughout the year. The egg prices were fixed by the Egg Marketing Board who, on being asked why the prices contained a strong seasonal element, replied that "the housewives expect it." The seasonal in egg prices vanished soon after the enquiry was made. Expectations may arise, because it has been noted that the series being considered has, in the past, contained a seasonal, or because it is observed that acknowledged causal series have a seasonal component.

These four groups may be thought of as basic causes. They are not always easily distinguishable, may often merge together, and the list of basic causes may not be complete. Some series may have seasonal components which are only indirectly due to these basic causes. Weather may cause a seasonal in grape production that then causes a seasonal in grape prices, for example. For many series, the actual causation of a seasonal may be due to a complicated mix of many factors or reasons, due to the direct impact of basic causes and many indirect impacts via other economic variables. Even if only a single basic cause is operating, the causal function need not be a simple one and could involve both a variety of lags and nonlinear terms. Two fairly obvious examples follow.

The first example is the impact on a production series of a public holiday, such as Christmas, might be simply modelled as production = $g_t h_t$, where g_t is a stochastic production series on working-day t, and h_t is a dummy variable, taking the value 1 on nonholidays and 0 on holidays. Thus, the initial impact of the advent of Christmas involves a multiplicative seasonal. However, this model is clearly too simple to be an acceptable approximation of the true situation. If there is spare capacity, the occurrence of the Christmas vacation can be allowed for in the production scheduling by increasing production in working days around the vacation, giving both expectations and a delayed effect. The extent to which this planning occurs will depend partly on the state of the economy, or the order book, and on current production levels, or unused capacity of the factory. Thus, the actual seasonal effect may depend on the level of the economic variable being considered and possibly also on other variables.

A second example is the effect of rainfall on a crop, such as outdoor tomatoes grown in California. Early, plentiful rainfall could bring on a good crop, provided it is not followed by further heavy rainfall in the next 2 months to the exclusion of sufficient sun. Thus, the rain provides both a distributed lag effect and also an accumulation effect on the quality, quantity, and timing of the actual crop.

Two important conclusions can be reached from such considerations: (1) The causes of the seasonal will vary greatly from one series to another, and, therefore, the seasonal components can be expected to have differing properties, and (2) the seasonal components cannot be assumed to be deterministic, i.e., perfectly predictable. Although it would be interesting and perhaps worthwhile to perform a causal analysis of the seasonal component for every major economic series, this task would be both difficult and expensive. Nevertheless, it would be unreasonable to assume that all seasonal components are generated by the same type of simple model, and this must be acknowledged when attempting to seasonally adjust a series. Even though some of the basic causes can be thought of as deterministic series (the calendar and timing decisions, for example), there is certainly no reason to suppose that they will lead to deterministic seasonal components, since the reaction to these causes need not be deterministic. The other basic causes, weather and expectations, are not deterministic and cannot lead to deterministic seasonals. Although an assumption of a deterministic seasonal component may have some value, this value is usually very limited and leads to techniques that are capable of improvement. Implications of these conclusions for seasonal models will be discussed in the section "Seasonal Models."

The consideration of causation also throws doubt on the idea of the seasonal being simply either an additive or a multiplicative component, as will also be discussed in the section "Seasonal Models."

Before turning to the problem of how to define seasonality, it is worthwhile considering briefly the types of economic series that are clearly seasonal and those that are not. For purposes of illustration, consider just those series that the U.S. Department of Commerce decides are in need of seasonal adjustment and those that apparently have no such need. The types of series that are adjusted are generally those concerned with production, sales, inventories, personal income and consumption, government receipts and expenditures, profits, unemployment rates, and imports and exports. Series not seasonally adjusted include prices (other than farm and food prices), interest rates, exchange rates, index of consumer sentiment, new orders (manufacturing), liquid liabilities to foreigners, and U.S. official reserve assets. If it is possible to generalize about such a wide range of variables, it seems that those needing adjustment are usually variables requiring planning or long-range decisionmaking, whereas the nonadjusted series are typically those that can quickly change in value and, thus, require only a stream of short-run decisions.

If one tries to write down the main causes of seasonal components in the first group of variables, I think that it is easily seen that the proper specification of these causes is not a simple task and that this problem needs to be tackled by empirical analysis as much as by introspection.

DEFINITION

It is impossible to proceed further without a reasonably precise definition of seasonality, although it is remarkable how many papers discuss the topic without consideration of definition. The belief is, presumably, that the seasonal component is so simple and obvious that it hardly needs a formal definition. Nevertheless, to sensibly discuss such topics, as the objectives of seasonal adjustment or the evaluation of actual methods of adjustment, a formal definition is required. It is obvious that this definition should not be based on a specific model, since this model may not properly reflect reality, nor should it rely on the outcome of a particular method of adjustment, since the method may not be ideal, and it also becomes difficult to evaluate that particular method. These limitations, together with the fact that the most obvious feature of a seasonal component is its repetitiveness over a 12-month period, strongly suggest that a definition can be most naturally stated in the frequency domain, since spectral methods investigate particular frequencies and are essentially model-free.

Let X_t be a stochastic generating process and x_t, $t=1, \ldots, n$ be a time series generated by this process. X_t and x_t might be considered to correspond to a random variable and a sample respectively in classical statistical terminology. For the moment, X_t will be assumed to be stationary, although this assumption will later be relaxed. Let $f(\omega)$ be the power spectrum of X_t and $\hat{f}(\omega)$, the estimated spectrum derived from the observed x_t. Define the seasonal frequencies to be $\omega_s k$, $k=1, 2, \ldots, [N/2]$, where

$$\omega_s=\frac{2\pi}{N}$$

N is the number of observations of the series taken in a 12-month period, and $[N/2]$ is the largest integer less than $N/2$. For ease of exposition, the case of monthly recorded data will be considered almost exclusively in what follows, so that the seasonal frequencies are just $2\pi k/12$; $k=1, 2, \ldots, 6$. Further, define the set of seasonal frequency bands to be

$$\omega_s(\delta)=\{\omega \text{ in } (\omega_s k-\delta, \omega_s k+\delta),\ k=1, \ldots, 5,\ (\omega_s 6-\delta, \pi)\}$$

and so consists of all frequencies within δ of the seasonal frequencies.

Definition 1

The process X_t is said to have property S if $f(\omega)$ has peaks within $\omega_s(\delta)$ for some small $\delta>0$.

Definition 2

The series x_t is said to apparently have property S if $\hat{f}(\omega)$ has peaks in $\omega_s(\delta)$ for some small $\delta>0$.

A process with property S will be called a process with seasonal component. This definition closely resembles that proposed by Nerlove [14] in an important paper on seasonal adjustment.

Definition 3

A process S_t is said to be strongly seasonal if the power contained in $\omega_s(\delta)$ almost equals the total power, for some appropriate, small δ. This can be stated more formally as

$$\int_{\omega_s(\delta)} f(\omega)\,d\omega \Big/ \int_0^{\pi} f(\omega)\,d\omega=\lambda(\delta)$$

where $\lambda(\delta)$ is near 1. Thus, the variance due to the seasonal band frequencies is nearly equal to the total variance of the process S_t. It follows that $f(\omega)$ is relatively small for ω in the region not $-\omega_s(\delta)$, compared to ω in the region $\omega_s(\delta)$. The choice of δ is unfortunately arbitrary and has to be left to the individual analyst. It can be strongly argued that the need for allowing the seasonal component to be nondeterministic implies that it is not correct to take $\delta=0$. If $\lambda(0)$ is some positive quantity, then the seasonal does contain a deterministic component, but, given just a finite amount of data, this hypothesis cannot be tested against the alternative $\lambda(\delta)>0$ for some small positive δ, which allows also a nondeterministic seasonal component.

The assumption of stationarity in these definitions is too restrictive for our needs. Although the spectrum is strictly based on this assumption, the problem can be removed in the case of actual data analysis if in the definitions one replaces the estimated spectrum by the pseudospectrum [10]. The pseudospectrum is essentially the spectrum estimated by the computer as though the data were stationary. It can also be loosely thought of as the average of a time-changing spectrum. If this way out is taken, peaks in the pseudospectrum at the seasonal frequency bands will indicate that the series did have property S for at least some of the time span considered.

SEASONAL MODELS

There are many time series models that generate data with property S. Some examples follow, using the notation that X_t is a process with property S, Y_t is a process without property S, and S_t is a strongly seasonal process. The additive seasonal models then take the form

$$X_t=Y_t+S_t$$

where Y_t is a unrestricted nonseasonal series. Various forms for S_t have been suggested.

Model 1

S_t is perfectly periodic so that $S_t = S_{t-12}$. Thus, S_t can always be represented by

$$S_t = \sum_{j=1}^{6} a_j \cos(\omega_s jt + \Theta_j)$$

or by

$$S_t = \sum_{j=1}^{12} a'_j d_j$$

where $d_j = 1$ is j^{th} month of the year
$= 0$ in all other months.

In this model, S_t is deterministic.

Model 2

S_t is almost periodic, so that

$$S_t = \sum_{j=1}^{6} a_{j,t} \cos(\omega_s jt - \Theta_{j,t})$$

where a_{jt}, Θ_{jt} are slowly time-varying and can either be assumed to be deterministic functions of time, such as $a_{jt} = exp(\alpha_j t)$, or they can be considered to be stochastic processes with spectra dominated by low-frequency components. If S_t is perfectly periodic, as in model 1, its theoretical spectrum will be zero, except at the seasonal frequencies, whereas, if S_t is almost periodic, its spectrum will be almost zero outside of the frequency band $\omega_s(\delta)$ where the size of δ will depend on the rate at which a_{jt}, Θ_{jt} change in value.

Model 3

S_t is a strongly seasonal process. For example, S_t could be a multiple of a filtered version of an observed strongly seasonal causal series, such as a weather series. Equally, S_t may be generated by a simple ARMA model with coefficients such that the resulting process is strongly seasonal. An example S_t generated by

$$S_t = 0.9 S_{t-12} + \eta_t + 0.6 \eta_{t-1}$$

where η_t is white noise, as considered by Grether and Nerlove [9]. The weather series just considered might also be considered to be generated by such a model, but presumably the η_t could then be estimated by analysis of causal series. If the causal series has not been identified, the S_t component might be thought of as unobservable, meaning that the η_t cannot be directly observed or estimated from data. These problems are further considered in the next section.

Model 4

In multiplicative models, where

$$X_t = Y_t \cdot S_t$$

and Y_t is constrained to be positive, S_t can be taken to be generated by any of the previous models, plus a constant to ensure that it is positive. These models seem to be suggested to allow for the apparently observed fact that the amplitude of the seasonal component increases in size as does the level of X_t. An assumption of a multiplicative model is an attractive one as the application of a logarithmic transformation to the data produces an additive seasonal model. However, although attractive, this assumption is not necessarily realistic, since the amplitude of S_t may be trending in the same direction as is X_t but not proportionately. Other transformations may be appropriate or much more general classes of models should perhaps be considered.

Model 5

Harmonic processes is a very general class of the nonstationary processes, which allow the amplitude of one frequency component, such as the seasonal, to be correlated with that of another component, such as the low-frequency component corresponding to business cycles. In the frequency domain, such processes have the representation.

$$X_t = \int_{-\pi}^{\pi} e^{it}\, dz(\omega)$$

where

$$E[dz(\omega)\overline{dz(\lambda)}] = d^2F(\omega, \lambda), \text{ all } \omega, \lambda$$

$d^2F(\omega, \lambda)$ is the bivariate spectral function, and its values are in a sense dominated by values along the main diagonal $\omega = \lambda$. If $d^2F(\omega, \omega) = f(\omega)\, d\omega$ and $f(\omega)$ has peaks at the seasonal frequency bands, the harmonic process will be said to have property S. Stationary processes, the almost-periodic model 2 and the smoothly changing class of nonstationary processes considered by Priestley [14], are usually members of the class of harmonic processes. Unfortunately, much of this class has not been studied empirically and no specific set of time-domain models have been identified that represent the majority of the class.

Model 6

Adaptive models are any models that take a white noise series and, by the choice of an appropriate filter, produce a series where property S is clearly an appropriate model. The class of such models, suggested by Box and Jenkins [2], has received the most attention recently and can be represented by

$$\frac{\alpha_s(B^s)(1-B^s)^{d_s} X_t}{b_s(B^s)} = Y_t$$

where B is the backward operator. $s = 12$ for monthly data, a, b are polynomials in B^s and Y_t does not have property S and is usually taken to be ARIMA. The only values of d_s required for economic data seem to be 0 or 1, and, typically, a series with property S is assumed to have $d_s=1$. They are called adaptive models, as the seasonal can change shape in an unspecified way and still belong to the class. Other adaptive models have also been suggested, but they are generally special cases of the Box-Jenkins model. It has been suggested that a very simple form of the model, such as

$$(1-\lambda B^{12})Y_t=\epsilon_t$$

where ϵ_t is white noise and λ is near one, cannot be used to represent real data. The reason given is that the estimated spectrum of real series, with property S, has peaks of very unequal heights, whereas the theoretical spectrum, generated by the simple model, has peaks of almost equal heights at the seasonal frequencies. Although the theory is correct, in practice, a series generated by the simple model can produce almost any seasonal spectral shape depending on the starting values used. Thus, the full model consists of the generating equation, plus the 12 starting values. It is clearly important to fit the model to a series whose first year is in some way typical in terms of its season shape.

This list of models, producing series with property S, is merely an illustrative one and does not pretend to be complete. However, it is sufficient to show the wide variety of models available and how inappropriate it is to just assume that some single model is the correct one and then to base subsequent analysis on this chosen model, without performing any confirmatory analysis or diagnostic checking. In practice, given a limited amount of data from a single series, it may be impossible to distinguish between various models. This suggests that a good method of seasonal adjustment must be fairly robust against various types of generating models. However, it does not follow that it is sufficient to assume the correctness of a simple model, such as a deterministic Model 1, and to adjust using a method designed to deal with such a model. Although a complicated, stochastic model might be well approximated by a deterministic model, in some sense over time periods, to use the simple model can lead to important problems when a sophisticated analysis is undertaken. It is similar to saying that a random walk with drift can be approximated by a linear-trend function and then using this function to forecast future values of economic variables (or their logarithms). Although such forecasts may not be disastrously bad in the short run, they can be easily improved upon.

DECOMPOSITION

A great deal of the academic literature dealing with seasonal problems is based on the idea that a seasonal series can always be represented by

$$X_t=Y_t+S_t$$

this is possibly after a logarithmic transformation, where Y_t does not have property S, and S_t is strongly seasonal. It is often further assumed that the two components Y_t and S_t are uncorrelated. This idea is so basic that it needs very careful consideration. At one level, it might be thought to be clearly true, given the assumption that Y_t is stationary. Let $X_t(\omega_s(\delta))$ be the summation of all of the frequency components of X_t over the frequency set $w_s(\delta)$ and let $X_t^e=X_t-X_t(\omega_s(\delta))$; then S_t can be associated with $X_t(\omega_s(\delta))$ and Y_t with X_t^e. S_t will necessarily be strongly seasonal, Y_t will not have property S, and S_t, Y_t are uncorrelated.

However, this solution to the decomposition problem is not a generally acceptable one, since X_t^e does not have the kind of properties that are usually required, at least implicitly, for the nonseasonal component Y_t. This component is almost inevitably taken to be a typical kind of series, generated, for instance, by a nonseasonal ARIMA process and, thus, to have a smooth spectrum with neither peaks nor dips at seasonal frequencies. On the other hand, X_t^e has a spectrum which takes zero values at the seasonal frequency band $\omega_s(\delta)$. The equivalent requirement, placed on S_t for the decomposition to be acceptable, is that it contributes the peaks to the spectrum at the seasonal frequencies but not the total power at these frequency bands. If this requirement is not imposed, a series without property S, such as a white noise, would have a seasonal decomposition into X_t^e and $X_t(\omega_s(\delta))$.

To illustrate the consequent difficulties that arise concerning seasonal decomposition from these considerations, suppose that Y_t is generated by

$$a(B)Y_t=b(B)\epsilon_t$$

and S_t by

$$a_s(B)S_t=b_s(B)\eta_t$$

where ϵ_t, η_t are two white-noise or innovation series, a and b are chosen so that the spectrum of Y_t has no peaks at seasonal frequencies, and a_s, b_s are such that the spectrum of S_t has virtually no power outside the seasonal frequency band $\omega_s(\delta)$ for some small δ. If ϵ_t and η_t are uncorrelated, the spectrum of X is the sum of the spectra for Y and S. However, if the only data available for analysis are a sample from X, then Y_t and S_t are unobservable components: it follows that there is no unique decomposition of X_t into Y_t plus S_t. Coefficients of a and b can be chosen so that $f_y(\omega)$ has a slight dip at $\omega=2\pi/12$, and the coefficients of a_s, b_s are altered so that $f_x(\omega)$ remains unchanged. Only by imposing very stringent conditions, of a rather arbitrary kind, on the shape of $f_y(\omega)$ around the frequencies in $\omega_s(\delta)$, can a unique decomposition be achieved, and one rarely has a strong a priori knowledge about the series to impose such conditions.

The situation becomes clearer if S_t is a filtered version of a causal series, such as monthly rainfall R_t. Suppose

$$S_t = c(B) R_t$$

and

$$R_t = \frac{b_s^*(B)}{a_s(B)} \eta_t$$

where a_s, b_s^*, and c are all polynomials in B. It then follows that $b_s(B) = b_s^*(B) \cdot c(B)$. By analysis of R_t, the η_t series can, in principle, be estimated, and, by the joint analysis of R_t and X_t, the seasonal component S_t can be isolated, hence also Y_t. It is then seen that, at least intrinsically, the use of causal knowledge can allow a unique seasonal decomposition that cannot be achieved without the use of this knowledge.

It is interesting to relate the decomposition model with that employed by Box and Jenkins, discussed as Model 6. Using the notation introduced at the start of this section,

$$X_t = Y_t + S_t = \frac{b(B)}{a(B)} \varepsilon_t + \frac{b_s(B)}{a_s(B)} \eta_t$$

so

$$a_s(B) a(B) X_t = b(B) a_s(B) \varepsilon_t + a(B) b_s(B) \eta_t$$

The righthand side is the sum of two uncorrelated moving averages and, consequently, can always be represented by $d(B)\theta_t$, where $d(B)$ is a polynomial in B of limited order, and θ_t is a white noise. Typically, θ_t is a very complicated amalgam of the two component white noises ε_t, η_t. (For proof, see [6].) Thus, a Box-Jenkins-type model is achieved but with the driving series θ_t involving η_t, which is the driving series of the seasonal component S_t. If one analyzes X_t by building a single-series Box-Jenkins model, it will be virtually impossible to pick out Y_t and S_t. By the use of partial fractions, one might be able to obtain a decomposition of the form

$$X_t = A_s(B)\theta_t + \mathrm{A(B)}\theta_t$$

where $A_s(B)\varepsilon_t$ is strongly seasonal, but now both the seasonal and the nonseasonal components are driven by the some innovation series θ_t. The two components clearly will not be independent.

It is probably true to say that the requirement that a series be decomposed into seasonal and nonseasonal parts has the implicit idea that the two parts have their own separate, nonoverlapping sets of causes. It is these different causes that ensure the two parts are uncorrelated and, in fact, independent and also provide one reason for seasonally adjusting. However, using a single series, it is seen that the seasonal decomposition is very difficult and perhaps impossible to achieve, provided the seasonal component is taken to be stochastic, which is essential. The only sure way of achieving the required decomposition is by a full-scale causal analysis of one or another of the components, which may not always be practical. In later sections, a method of adjustment that uses the past values of the series to be adjusted will be called an autoadjustment method, whereas, if the past values of seasonal causal series are also used, the method will be called a causal adjustment.

WHY ADJUST?

The seasonal components of economic series are singled out for very particular attention. Why should this be so, and why is so much effort expended on trying to remove this component? Presumably, the seasonal is treated in this fashion, because it is economically unimportant, being dull, superficially easily explained, and easy to forecast but, at the same time, being statistically important in that it is a major contributor to the total variance of many series. The presence of the seasonal could be said to obscure movements in other components of greater economic significance. Such statements contain a number of value judgments and, thus, should not be accepted uncritically. It can certainly be stated that, when considering the level of an economic variable, the low frequency components, often incorrectly labelled the "trend-cycle components," are usually both statistically and economically important. They are statistically important, because they contribute the major part of the total variance, as the typical spectral results and the usefulness of integrated (ARIMA) models indicates. The economic importance arises from the difficulty found in predicting at least the turning points in the low-frequency components and the continual attempts by central governments to control this component, at least for GNP, employment, price, and similar series. Because of their dual importance, it is desirable to view this component as clearly as possible and, thus, the interference from the season should be removed. This argument can be taken further and leads to the suggestion that only the low-frequency component is of real economic importance and, thus, all other components should be removed. This is easily achieved by applying a low-band pass filter to the series. However, if one's aim is not merely to look up the business cycle component but to analyze the whole series, this viewpoint is rather too extreme.

I think that it is true to say that, for most statistically unsophisticated users of economic data, such as most journalists, politicians, and upper business management, the preference for seasonally adjusted data is so that they can more clearly see the position of local trends or the place on the business cycle. It is certainly true that for any series containing a strong season, it is very difficult to observe these local trends without seasonal adjustment. As these users are an important group, there is clearly a powerful reason for providing seasonally adjusted data.

For rather more sophisticated users who wish to analyze one or more economic series, without using supersophisticated and very costly approaches, it also makes sense to have adjusted data available. If one is forecasting, for instance, it may be a good strategy to build a forecasting model on the adjusted series, possibly using simple causal techniques such as regression, and then to add a forecast of the seasonal component to achieve an overall forecast. Similarly, if the relationship between a pair of economic variables is to be analyzed, it is obviously possible to obtain a spurious relationship if the two series contain important seasonals. By using adjusted series, one possible source of spurious relationships is removed. The kinds of users I am thinking of here are economists or econometicians employed by corporations, government departments or financial institutions.

There are obviously sound reasons for attempting to produce carefully adjusted series, but there are equally good reasons for the unadjusted series to also be made equally available. For very sophisticated analysis, an unadjusted series may well be preferred, but, more importantly, many users need to know the seasonal component. Firms having seasonal fluctuations in demand for their products, for example, may need to make decisions based largely on the seasonal component. The Federal Reserve System is certainly concerned with seasonal monetary matters, and a local government may try to partially control seasonal fluctuations in unemployment. Many other examples are possible. Only by having both the adjusted and the unadjusted data available can these potential users gain the maximum benefit from all of the effort that goes into collecting the information.

It is seen that alternative users may have different reasons for requiring adjusted series. I believe that it is important for econometricians and others who analyze economic data to state clearly why they want their data seasonally adjusted and what kind of properties they expect the adjusted series to possess, since these views may be helpful in deciding how best to adjust.

It is not completely clear why the central government should place most of its control effort on the long swings in the economy and yet make little attempt to control the seasonal. Perhaps, people prefer having seasonal components in the economy rather than not having them because of the generally surprise-free variety of experience provided. One wonders if, when a group of astronauts go on a 20-year trip through space, their enclosed environment will be given an artificial seasonal.

OVERVIEW OF ADJUSTMENT METHODS

There is certainly no lack of suggested methods for seasonal adjustment; dozens already exist, and others are continuously being proposed. It would be inappropriate to attempt to review even the major properties or objectives of all of these methods in this paper. There are, however, certain features of the methods actually used that deserve emphasis. I think that it is fair to say that virtually all of the methods are automatic ones in that essentially the same procedure is used on any series given as an input to the computer rather than being individually redesigned for each series. Secondly, all of the methods are based on the past values of the series being adjusted and not on the values taken by other series. That is, they are autoadjustment methods rather than causal adjustment.

The two basic approaches involve regression techniques, using seasonal dummy variables or cosine functions and filtering methods, designed to isolate a major part of the seasonal frequency component. These two approaches are not unrelated, and, with an assumption of stationarity, the theoretical properties of these methods can be derived from some well-known theory, the easiest interpretation coming from the effects of linear filters on a spectrum. However, most of the more widely used methods of adjustment are not perfectly equivalent to a linear filter for two reasons that are much emphasized by those applied statisticians, usually in government service, who are most concerned with the mechanics of the adjustments and with the production of adjusted series. These reasons are the strong belief that the seasonal pattern is often time-varying to a significant degree and the concern that an occasional aberrant observation, or outlier, may have an unfortunate effect on the adjusted values over the following few years. In attempting to counteract these apparently observed properties of real data, to which academic writers have generally paid little attention, nonlinear filters or data-specific methods of adjustment have been devised. The properties of these methods cannot usually be determined by currently available theory. As a simple illustration of a method devised to allow for these affects, suppose that the estimate of the seasonal component next January is taken to be the average January figure over the last n years. To allow for the possibility of a changing pattern n has to be kept small, say $n=5$, and, to allow for the possibility of outliers rather than simply averaging over the last five January values, one could reject the smallest and largest of these five values and average the rest. If changing seasonals and outliers are important, as has been suggested, there are clear benefits in adapting methods to take these problems into account, but if they are not really important, the extra costs involved in the performance of the adjustment method may outweigh the benefits, as will be discussed in the section "Effects of

Adjustment in Practice." It would be interesting to see evidence on the frequency of occurrence and importance of evolving seasonal patterns and outliers, since this would be helpful in evaluating methods of adjustment. Some methods would be badly affected by these data properties but others much less so. For example, techniques based on the adaptive models in Model 6., including those associated with Box and Jenkins, cope very well with changing seasonal patterns, if the change is not too rapid, but are very badly thrown out by outliers or extraordinary data values.

The question of how to deal with outliers clearly exemplifies the basic differences between the auto- and causal-adjustment approaches. Just suppose that a series can be decomposed as

$$X_t = Y_t + S_t$$

where Y_t does not have property S, S_t is strongly seasonal, and S_t can be fully explained in terms of known and recorded weather variables. If an exceptionally severe winter occurs, then, since it cannot be explained by past values of X_t, any auto-adjustment technique will have to consider the difference between the exceptional value of X_t and its value predicted from the past as noise. A causal adjustment technique, if based on the correct structural equation will automatically take the exceptional value into account and there will be no residual problems for later years. Now, the outlier is not noise but an integral part of the seasonal and is dealt with accordingly. It is quite clear that the causal-adjustment approach is superior as the exceptional winter is not just noise but correctly considered part of the seasonal, since such winters only occur in the winter months! The other popular cause of outliers, strikes, and particularly dock strikes, have similar properties, since the preponderance of strikes has a seasonal pattern, with few strikes starting in the United States in December and the next few months than at other times in the year. Thus, to consider strike effects as completely noise to the system ignores the fact that this "noise" has property S.

A further problem of considerable practical importance concerns the question of how to adjust up to the present. If one is adjusting historical data, it is generally thought to be very desirable that important components, such as the business cycle are not lagged as a result of the adjustment. If a linear filtering method of adjustment is used, so that

$$x_t^a = \sum_{j=-m}^{m} \alpha_j x_{t-j}$$

with $\Sigma\alpha_j = 1$, the no-lagging property can be achieved by taking $\alpha_j = \alpha_{-j}$. However, as t approaches the present time n, the use of such a symmetric filter is not possible, as the filter involves values of the series that have not yet occurred. One method of proceeding is to alter the parameters of the filter as t gets near n. A simple example is the following filter applied to quarterly data:

$$x_t^a = 1/4[1/2x_{t+2} + x_{t+1} + x_t + x_{t-1} + 1/2x_{t-2}]$$

but, if $t = n-1$

$$x_{n-1}^a = 1/4[3/2x_n + x_{n-1} + x_{n-2} + 1/2x_{n-3}]$$

and, if $t = n$

$$x_n^a = 1/4[5/2x_n + x_{n-1} + 1/2x_{n-1}]$$

It is seen that the filter is here rolled-up, with the weight attached to an unobserved value being given to the latest available figure. The effects of doing this are to remove only part of the seasonal, even if the seasonal is perfectly periodic and deterministic, and to induce lags in nonseasonal components. The seasonal is not properly removed because if in the above example the $n-1$ filter weights were used at all times on a series with property S then the adjusted series will still usually have this property. As an example of the lags induced, if the series to be adjusted contained a cycle of period 40 months, the x_{n-1} will contain this component lagged approximately 0.4 months, and, in x_n, it will be lagged 1.5 months. Thus, if the component peaked at time n, this peak would not be observed until time $n+1$, using quarterly data or n plus 1.5 months if more regularly observed data were available but this particular quarterly filter were used. Methods of adjustment that adapt as the time point approaches the present will generally have this property of inducing varying lags in the most recent data. This is potentially a very annoying feature given that in a policymaking situation this most recent data is by far the most important and if one is model-building it would probably usually be preferred that the model best fits the most recent data. This is certainly true for forecasting purposes. A further unfortunate side effect of using techniques that introduce varying lags in the most recent data is that it then becomes inappropriate to compare the most recent adjusted figure with the adjusted figure for twelve months earlier, to calculate an annual rate of change, for example. The fact that different lags are involved effectively means that the change is being calculated over a period shorter than 1 year.

An alternative approach is to use nonsymmetric filters but with constant coefficients. A simple example would be

$$x_t^a = x_t - x_{t-12}$$

This method always introduces a lag of approximately 6 months to the business-cycle components, but, at least, this is a constant lag and no unnecessary nonstationarity is introduced into the data. Incidentally, any method of adjustment that subtracts from x_t, a measure of seasonal estimated from data at least 1 year old, will introduce a lag of at least 6 months into low-frequency components.

A method of adjustment that has few problems with outliers, adjusting up to the present or changing seasonal shape can be constructed, at least in principle, by identifying causal series for the seasonal component. Suppose that, as before

$$X_t = Y_t + S_t$$

where S_t is strongly seasonal, Y_t does not have property S, and that analysis suggests that

$$S_t = \alpha_0 + \alpha_1 R_t + \alpha_2 R_{t-1} + \eta_t$$

where R_t is a monthly rainfall series, η_t is white noise and α_0, α_1 and α_2 are coefficients that have been carefully estimated. It follows that the seasonal component can be estimated directly from the observed values of the R_t series and up to the most recent time period. Such a method would have to be based on individual analysis of each series to be adjusted and may well prove impractical and too difficult or costly to implement in most cases. A slightly less satisfactory method would be to replace the causal series by a leading indicator of the seasonal. I know of no attempts to construct such leading indicators, although methods devised for the business cycle could be appropriate.

CRITERIA FOR EVALUATION

If, as suggested in the section "Decomposition," it is often difficult to identify completely a seasonal component, it is clearly going to be difficult to evaluate a method of adjustment. It is insufficient to say that the adjusted series should consist just of the nonseasonal component, if this component cannot be identified. I would like to suggest that there are three highly desirable properties that, ideally, one would like a method of adjustment to possess, plus a set of desirable but not totally necessary properties. The first of the highly desirable properties is that there should be no change-of-scale effect. If x_t is the raw series and x_t^a the adjusted series, then refer to property 1.

Property 1

$(cx_t + d)^a = cx_t^a + d$ where c, d are constants. It thus follows that if one is adjusting a temperature series, for example, it is of no consequence whether the temperature is measured in degrees centigrade or fahrenheit. The second highly desirable property described in property 2.

Property 2

x_t^a should not have property S. As property S was defined in terms of spectral shape, it means that property 2 can only be tested by looking at the estimated spectrum of x_t^a; this spectrum should have no peaks at seasonal frequencies. It is almost as important that one has property 2′.

Property 2′

x_t^a should not have property anti-S, which is just an unnecessarily formal way of saying that the estimated spectrum of x_t^a should not have dips at seasonal frequencies, since this would imply that part of the nonseasonal component has also been removed. Property 2 was a criterion used by Nerlove [12] in an early spectral study of adjustment methods.

It is not necessary to emphasize property 2′ further, since it is subsumed in the next property. Assume that $x_t = y_t + S_t$, where S_t is the seasonal component and further suppose that S_t can be fully identified, then it is highly desirable that one has property 3.

Property 3

Coherence $(x_t^a, y_t) = 1$ and phase $(x_t^a, y_t) = 0$, at all frequencies which essentially says that x_t^a and y_t are identical apart from scale effects. This property can only be investigated if S_t or y_t are known, which will be true if x_t is a constructed, simulated series or if S_t has been identified by use of a causal series analysis. Property 3 is stated in the form given, because, by looking at the estimated coherence and phase functions between x_t^a and y_t, the extent to which the property does not hold can be evaluated. Godfrey and Karreman [3] applied this criterion to a wide variety of simulated series and various methods of adjustments. Some of their conclusions will be discussed in the next section. Their results prove why it is easier to evaluate the break-down of property 3, using spectral methods, than it would by using correlation (x_t^a, y_{t-k}) for various k, for example. It is usually pointless to look at the cross-spectrum between x_t and x_t^a as if x_t contains a strong, stochastic seasonal component; the cross-spectrum will not be interpretable at the seasonal frequency bands and leakage will spoil estimates of this function at other frequencies unless a very long series is available. It thus follows that one cannot evaluate a method of adjustment on property 3 given only the raw data of a real economic variable and the adjusted series. Simulation is the only easy method of investigating property 3. A corollary of this property is property 3′.

Property 3′

$(y_t)^a \simeq y_t$ if y_t is nonseasonal, so that adjustment of a series with no seasonal should ideally leave the series unchanged. The combination of properties 2 and 3′ gives us property 3″.

Property 3″

$(x_t^a)^a \simeq x_t^a$ so that a second application of the adjustment procedure should not have any important effect.

An appropriate way of studying this is to use the estimated cross-spectrum between x^a_t and $(x^a_t)^a$ that can be obtained directly from real data. Although not an ideal way to check on property 3, it might well provide a useful test that is quite easy to conduct.

Turning now to properties that are desirable but not completely necessary, the first is that the adjusted series and the estimated seasonal component are unrelated, which may formally be stated as property 4.

Property 4

$\text{corr}(x_t - x^a_t,\ x^a_{t-k}) = 0$, all k or
cross-spectrum $(x^a_t;\ x_t) =$ spectrum x^a_t

The desirability of this property relates to the idea discussed in the section "Decomposition" that the seasonal and nonseasonal components have separate and distinct causes and, thus, should be unrelated. However, as model 5 suggests, there is no clear-cut reason why the real world should have this property. A number of writers have discussed this property without mentioning the empirical problems that arise when trying to test it. If the adjustment procedure is at all successful, $x_t - x^a_t$ will be highly seasonal and x^a_t, virtually or totally nonseasonal. A method based on a simple correlation or regression between such a pair of series will be biased towards accepting the null hypothesis of no relationship just as a regression between two highly seasonal series or two trending series is biased towards finding significant relationships, as illustrated by Granger and Newbold [7]. It follows that property 4 cannot be effectively tested using estimated correlations when the amount of data available is limited. The correct way to test is to find filters separately for each of x^a_t and $x_t - x^a_t$ that reduce these series to white noises and then to estimate correlations between these residual white-noise series. This is in fact a test of the hypothesis that the two series, x^a_t and $x_t - x^a_t$, have different causes, as shown by Pierce and Haugh [13] and by Granger and Newbold [8].

A further property that has been suggested as being desirable is that of summability in property 5.

Property 5

$(x_{1t} + x_{2t})^a = x^a_{1t} + x^a_{2t}$, but this does restrict the adjustment methods to being linear filters, I suspect, and this is a rather severe restriction. Lovell [11] has a very interesting theorem stating that if property 5 holds and also $(x_{1t}x_{2t})^a = x^a_{1t}x^a_{2t}$, then either $x^a_t = x_t$ or $x^a_t = 0$, so that it is unrealistic to ask that a method of adjustment has both of these properties.

A requirement placed on adjusted series by some government statisticians is a consistency of sums over a calendar year, which may be proposed as property 6.

Property 6

$\Sigma x_t = \Sigma x^a_t$ where the sums are over the months in a calendar year. This property is based on the belief that the sum of S_t, over a 12-month period, should be zero, which follows from the idea that S_t is purely periodic and deterministic, an idea that was earlier suggested should be rejected. The property is politically motivated and is arbitrary in nature, since, if one strictly believed S_t to be purely periodic, then the property should hold for every consecutive 12-month period, but to require this would remove virtually all of the available degrees of freedom and no relevant x^a_t could be found. It might be more reasonable to ask that property 6 holds approximately true, which would, in any case, follow from property 3, and to leave it at that. It is my strong opinion that property 3 is the most important one, although it has been little discussed except by Godfrey and Karreman, due to the difficulty in testing it on actual data.

It will be seen that what is meant in this paper by a good adjustment method is one that removes a seasonal component without seriously altering the nonseasonal component. There also exist various methods which merely try to remove the seasonal but make no claims about not altering the nonseasonal. The application of seasonal filters in the Box-Jenkins approach is such a method. They suggest a method of producing a series without property S and which produces a known effect on the nonseasonal part of the series that can be allowed for in later analysis. A simple example is the use of a twelfth-difference, so that

$$Z_t = x_t - x_{t-12}$$

If x_t contains a strong seasonal, then Z_t will contain, at most, a much weaker seasonal, but the model now to be built on the nonseasonal component has become more complicated. The use of such methods can be considered as one stage in the process of finding a model that reduces the series to white noise, which has been found to be an important technique for building single and multipe series forecasting models. (See Granger and Newbold [8].) However, such topics are too far away from the main theme to be discussed in this paper.

EFFECTS OF ADJUSTMENT IN PRACTICE

It would be inappropriate to try to survey all of the work on the evaluation of actual methods of adjustment, but since spectral methods have been emphasized in the previous section, a brief review of three papers using this approach will be given. Nerlove [12] used an adjustment method, devised by the Bureau of Labor Statistics (BLS), on a variety of economic series and compared the spectra of the raw and the adjusted series. It was found that the spectrum of the adjusted series frequently contained dips at the seasonal frequency bands so that, in a sense, the method was taking too much out of the raw series at these frequencies. In a later paper, Rosenblatt [15] presented spectral evidence that more recent methods of adjustment devised by the BLS and the Census Bureau had overcome this problem. Nevertheless, there do appear to be some occasions even now when the spectral seasonal dips problem arises, as it is sometimes noted when Box-Jenkins model building, that the twelfth autocorrelation coefficient of an adjusted series is signficantly nonzero and negative.

Property 3 was tested by Godfrey and Karreman [3] by adding a constant or time-varying seasonal term to an autoregressive series to form a raw series. This raw series was then adjusted by four different methods. Amongst other quantities, the cross-spectrum between the autoregressive series, being the original nonseasonal component, and the adjusted series, the eventual estimate of this component, was estimated and the coherence and phase diagrams displayed in their figures 6.Om.1, $m=2$, (1), 10. The spectra of the autoregressive and the adjusted series were also compared. It was generally found that the coherence was near one for the low frequencies, that is those up to the lowest seasonal frequency, and the phase was near zero over this band. For other frequencies, the coherence was not near one and was often rather small and the phase was generally near zero but not consistently so. The power spectrum of the adjusted series was generally of a similar shape but lay above that of the autoregressive series. These results suggest that the important business cycle and low-frequency component was generally little affected by the adjustment method, but the higher frequency components were greatly affected, either having an extra high-frequency component added or part of the original high-frequency component being lost and replaced by a component induced by the method of adjustment. Symbolically, one could illustrate these results by

$$x_t = y_t + S_t$$
$$y_t = y_t^h + y_t^H$$
$$x_t^a = (x_t^a)^h + (x_t^a)^H$$

where h indicates a low-frequency component and H, the remaining higher frequency component. Then y_t^h and $(x_t^a)^h$ are virtually identical, but y_t^H and $(x_t^a)^H$ are only imperfectly correlated, and $(x_t)^H$ has a higher variance than y_t^H. Thus, the methods tested by Godfrey and Karreman do not have property 3, except at low frequencies. It seems that little would have been lost by just applying a low-band pass filter to x_t and using that as the adjusted series, particularly since the actual adjusted series are effectively just the original low-frequency component plus an added nonoriginal hash term. The test of property 3 has proved to be both a stringent one and also to point out important failings with adjustment methods.

The zero phase observation, which corresponds to a zero lag, arises partly, because the adjustment used was on historical data, and no attempt was made to adjust to the present.

Both Nerlove and Rosenblatt present the cross-spectra between x_t and x_t^a. It has been argued in the previous section that, with limited amounts of data, these figures are difficult to interpret, but the estimates shown do agree in form with the suggested interpretation of the Godfrey and Karreman results.

RELATING PAIRS OF ADJUSTED SERIES

Suppose that two stationary series, X_{1t}, X_{2t}, are each made up of two components

$$X_{1t} = Y_{1t} + S_{1t}$$
$$X_{2t} = Y_{2t} + S_{2t}$$

where Y_{1t}, Y_{2t} do not have property S, and S_{1t}, S_{2t} are stochastic, strongly seasonal series. There are numerous possible interrelationships between the two X series, for example, Y_{1t} may be causing Y_{2t}, but S_{1t}, S_{2t} are interrelated in a feedback (two-way causal) manner. The effect of seasonal adjustment on the analysis of such relationships have not been studied thoroughly, although both Sims [16] and Wallis [17] have recently considered in some detail the case where Y_{1t} causes Y_{2t}, and S_{1t}, S_{2t} are possibly interrelated.

If S_{1t}, S_{2t} are important components, then it is clear that even if they are not strictly related, so that they do not have any causes in common, it is virtually impossible to properly analyze the relationship between X_{1t}, X_{2t} without using a seasonal adjustment procedure. This is because S_{1t} and S_{2t} will certainly appear to be correlated, with the maximum correlation between S_{1t} and $S_{2,t-k}$ where k is the average distance between the seasonal peaks of the two series. Such spurious relationships are disturbing, and thus an adjustment is required. There are three ways that the seasonal can be allowed for, either by using auto-adjustments on both observed series X_{1t} and X_{2t} or by using individual causal-adjustments on each of these series or by building a bivariate model interrelating the X's but including in the model relevant seasonal-causal series. The third of these procedures is probably preferable and the use of seasonal dummy variables in a model is an inefficient attempt to use this approach. The method that is almost invariantly used is the first, involving auto-adjustment. Unfortunately, this can lead to difficulties in finding the correct relationship between the Y series, as Wallis [17] and Sims [16] show, particularly if an insufficiently sophisticated method of analysis is used, such as a simple distributed lag.

One aspect not apparently previously emphasized is that spurious relations may be found if autoadjustment is used in the case where the Y series are unrelated, but the S series are related. Suppose that the economically important components are Y_{1t} and Y_{2t}, but, in fact, these series are independent. The economic analyst would presumably want the adjusted series to be unrelated in any analysis that he performs. However, in theory, this will not occur if S_{1t}, S_{2t} are correlated, and an autoadjustment is used. This is easily seen in spectral terms by noting that the coherence function is the correct one to measure relatedness between a pair of series, that the coherence between X_{1t} and X_{2t} will be the same as that between S_{1t} and S_{2t}, which is assumed to be nonzero, that all autoadjustment techniques correspond exactly or approximately to a linear filter, and that the coherence function is unaltered by using different linear filters on the pair of series involved.

Although this proof is completely general, it is interesting to illustrate this result, using a particular Box-Jenkins modelling approach. Suppose that Y_{1t}, Y_{2t} are independent white-noise series, denoted by

$$Y_{1t}=\varepsilon_{1t},\ Y_{1t}=\varepsilon_{2t}$$

and that S_{1t}, S_{2t} are generated by

$$S_{1t}=\alpha S_{1,t-12}+\theta_t$$
$$S_{2t}=\beta S_{2,t-12}+\phi_t$$

where monthly data is considered, θ_t and ϕ_t are zero-mean white-noise series with

$$E[\phi_t\theta_{t-k}]=C \quad k=0$$
$$=0 \quad k\neq 0$$

It follows that

$$(1-\alpha B^{12})X_{1t}=(1-\alpha B^{12})\varepsilon_{1t}+\theta_t$$

and from consideration of the autocovariance sequence of the lefthand side, this becomes

$$(1-\alpha B^{12})X_{1t}=(1-\alpha' B^{12})\varepsilon'_{1t}$$

where ε'_{1t} is a white-noise series and α' is given by

$$\frac{\alpha'}{1+(\alpha')^2}=\frac{\alpha}{(1+\alpha^2)\ \sigma_1^2+\sigma_\theta^2}$$

where σ_1^2 = variance. (ε_{1t}). Thus, applying the filter $(1-\alpha B^{12})/(1-a'B^{12})$ to X_{1t} results in a series without property S, in fact, to a white-noise series. There is a similar filter which reduces X_{21} to the white-noise series ε'_{2t}.

A little further algebra shows that

$$E[\varepsilon'_{1t}\varepsilon'_{2,t-12k}]=\frac{C}{1-\alpha'\beta'}(\alpha')^k \quad \text{if} \quad k\geqq 0$$

$$=\frac{C}{1-\alpha'\beta'}(\beta')^k \quad \text{if} \quad k\leqq 0$$

and $E[\varepsilon'_{1t}\,\varepsilon_{2,t-}S]=0$ if $S\neq 12k$, k an integer. Thus, the cross-correlogram between the series with seasonal removed, by a Box-Jenkins modelling approach, is not zero but is seasonal in nature due to the relationship between the original components. In fact, a feedback relationship is indicated.

It should be clear that autoadjustment of series can lead to results that are not interpretable in terms of the usual view of a decomposition of a series into seasonal and nonseasonal, with the first part being removed by the adjustment procedure.

CONCLUSIONS

By considering the causation of seasonal components, one reaches the conclusions that it is incorrect to believe that the seasonal component is deterministic, that a complete decomposition into seasonal and nonseasonal components is possible by analyzing only the past of the series and that autoadjustment methods do remove the seasonal part, and this can lead to relationships being found between series that are in some sense spurious. Because of these conclusions, most autoadjustment methods cannot be completely evaluated when applied to actual data rather than to simulated or constructed data. An alternate technique is to identify seasonal causal series and to build a structural model using these series so that the seasonal component of the series to be adjusted is estimated from the past of this series and past and present terms of the causal series. Potential advantages of this approach are that the same method of adjusting is used on historical data and also up to the most recently available piece of data, the seasonal might be totally removed so the relationship between a pair of adjusted series is more easily analyzed, and the question of how to deal with outliers becomes of much less importance. In practice, a complete causal analysis is not easy to perform, and the inherent costs may not allow this approach to be used very frequently. It is also not completely clear how a causal analysis would be conducted, although possibilities include demodulation techniques, band-pass spectral analysis,[1] or the causal filtering method suggested by Box and Jenkins [2], used, and generalized by Granger and Newbold [8]. In this latter approach, a filter is found which reduces the causal white noise, the same filter is then applied to the series being adjusted, and this filtered series regressed on the causal white-noise series, both lagged and unlagged. Some of these approaches will be investigated at a later time.

Many problems have not here been considered, including such practical ones as how to adjust a ratio of

[1] See, for example, the paper by Engle included in this working paper.

two series that is to appear in some model—does one take the ratio and then adjust, or adjust and then take the ratio? The latter seems to be recommended, but this would depend on the underlying theory being invoked that suggests the use of a ratio. A particularly important problem that has not been discussed is how to distinguish between additive and multiplicative seasonal effects, the use of instantaneously transformed series and the causes of nonadditive effects. These questions will be examined at another time.

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COMMENTS ON "SEASONALITY: CAUSATION, INTERPRETATION, AND IMPLICATIONS" BY CLIVE W. J. GRANGER

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Clive Granger's paper "Seasonality: Causation, Interpretation, and Implications" takes up a wide range of issues related to seasonality, ties them together in interesting ways, and contains many stimulating observations on both technical and philosophical points. If I were to expand on every remark in the paper, these comments would be at least as long as the original paper. My remarks will bear only on certain portions of the paper, not necessarily the best or even the most interesting parts.

In the section "Criteria for Evaluation," Granger provides us with his version of an old pastime. Just as sailors may amuse themselves by comparing lists of properties for the ideal girl, time series analysts seem attracted to composing and discussing lists of properties for the ideal seasonal adjustment procedure. At best, these lists are only slightly useful. When, as has been customary in the past, the list of properties for a seasonal adjustment procedure is prepared without explicit attention to a model of how the seasonal is generated or an objective function for adjustment, the lists can be seriously misleading, even pernicious.

I think other sections of this paper already provide a context in which we can see that the properties listed in the section "Criteria for Evaluation" should not be taken very seriously. Granger properly emphasizes the wide variety of plausible mechanisms that may generate seasonality and the importance of adapting adjustment procedures to the nature of the likely generating mechanism. He has also emphasized the importance of distinguishing the case where causal variables generating the seasonal are observable from the case where they are not and autoadjustment is necessary. There is a no useful way to prepare a list of ideal properties for a seasonal adjustment procedure that ignores these distinctions.

Granger's list and its ordering appear to be generated by a consideration of how an adjustment procedure would behave if it succeeded perfectly in separating a seasonal from a nonseasonal component. When comparing two adjustment procedures, we know to be imperfect, however, it can easily happen that the better procedure is, in some senses, less similar to an ideal error-free procedure than is the poorer procedure. A similar problem arises in evaluating forecasts. It is by now, I hope, well understood that forecasts that are optimal, in the sense of giving the minimum variance of forecast error given available information, necessarily have smaller variance than the true values that they are meant to be forecasting. It is, therefore, a mistake to pay attention to the closeness of the match of forecast variance to variance of the true values in evaluating forecasts. But properties 2 and 2′ in Granger's list embody just this sort of fallacy. Of course, if adjustment succeeded perfectly, and the y, in Granger's notation, we wish to estimate does not have peaks or dips at seasonals in its spectral density, then the adjusted series x^a will not have peaks or dips at seasonals in its spectral density. But, it is, in fact, generally true that the best imperfect adjustment procedures will systematically produce large dips in the spectral density, for the same reason that the best forecasting procedures systematically give forecasts with lower variance than the actual values.

Consider Grether and Nerlove's framework, in which we assume $x_t = y_t + S_t$ in Granger's notation and we attempt to adjust x_t, using only observations on x_t in order to minimize $E[(y_t - x^a_t)^2]$. The projection of y_t on the x process, which gives the ideal linear x^a_t, is $g^*x(t)$, where g is defined in the frequency domain by $\tilde{g} = S_{yx}/S_x$.[1] If S is orthogonal to y, $S_{xy} = S_y$. With this choice for x^a, it is then easy to compute the spectral density of the adjusted series, $S_{xa} = |\tilde{g}|^2 S_x = S^2_y/S_x$. Taking the logarithm, we see that $\log S_{xa} = 2 \log S_y = \log S_x$. Obviously, if S_y has no peaks or dips at seasonal frequencies, $\log S_{xa}$ has dips at the seasonal frequencies of exactly the same height and shape as the peaks in the original $\log S_x$.

This result is not dependent narrowly on the Grether-Nerlove framework. It is robust and intuitive after some reflection. By transformation into the frequency domain, we convert the adjustment problem to a separate prediction problem at each frequency. It is well-known that if $a = b + c$, with c independent of b,

[1] S_x, S_y, S_{yx} are, respectively, spectral densities of x and y and the cross-spectral density.

and we wish to predict b from observations on a, then the coefficient on a in our prediction will be less than one and closer to 0, the larger is variance in c, relative to variance in b. When the signal-to-noise ratio in a gets very low, we are very cautious in using a to predict b. In the seasonal adjustment problem the signal-to-noise ratio is by assumption especially low in the neighborhood of seasonal frequencies, and this accounts for the reasonableness of dips in x^a. In fact, since the low signal-to-noise ratio at seasonals appears to be a generic characteristic of seasonal adjustment problems, I would guess that nearly any explicit formulation of the problem, with or without observability of causal variables, will imply that when we fail to have $x^a = y$, we should expect S_{xa} to have dips at seasonal frequencies.

Granger's property 3″ is idempotency, $(x^a)^a = x^a$. This is another property that, while clearly satisfied by a perfect adjustment procedure, may be a bad criterion for ranking imperfect procedures. Grether-Nerlove optimal adjustment certainly leads to nonidempotent linear filters. One might attempt to rationalize idemptotency by defining the adjustment procedure as beginning with the estimation of S_y and S_x to determine g. However, since x^a will, in general, have dips in its spectral density at seasonals, it is probably unreasonable to require that any procedure for estimating S_y on the assumption that $S_x = S_y + S_S$ gives sensible results when presented with S_{xa} as input.

Property 4 is presented by Granger as desirable only when we have good reason to suppose S and y uncorrelated. However, even in this case, it will seldom be reasonable to test for lack of correlation between x^a and $x - x^a$. In the case where we know S is a linear distributed lag on certain observable variables (rainfall, date, holiday dummies, etc.), the seasonal adjustment problem, under the assumption that S and y are orthothogonal, becomes a time series regression problem, and the estimated S and y, $x - x^a$ and x^a, are likely, by construction, to emerge as uncorrelated. Of course, if the form of the relation of S to observable variables is subjected to a priori restriction, then it is possible for the observable variables determining S to be correlated with x^a, and tests for lagged correlations between $x - x^a$ and x^a are one type of test for a relation between x^a and the observable determinants of S. Perhaps, it is needless to say that, in executing such tests, it is important to use a multivariate regression framework and not to treat sample correlations of the regression residuals x^a, with the observable determinants of S, as if they were sample correlations of y itself with those observable determinants of S.

In the opposite extreme case where only x itself is observable, we know some natural models and adjustment criteria, including the Grether-Nerlove setup, lead to linear filters as optimal procedures. But, the procedure Granger recommends for checking property 4 is asymptotically valid only as a test for no relation or zero coherence, at all frequencies, between x^a and $x - x^a$. This is perhaps the most extreme example of this section's shopping list of properties leading us in a direction contradictory to the spirit of the rest of the paper. Granger's section "Decomposition" has already pointed out that autoadjustment makes it virtually impossible 1.
to have x^a and $x - x^a$ independent, and the section "Relating Pairs of Adjusted Series" points out that linear filters cannot affect coherence. Any autoadjustment procedure that is a linear filter automatically produces x^a and $x - x^a$ with a coherence of 1.0 at all frequencies, by construction, though the estimation difficulties that Granger emphasizes have sometimes led researchers, mistakenly, to estimate coherencies between such series of much less than 1.0. While nonlinear methods of autoadjustment might produce coherencies of less than 1.0, it is not reasonable to hope that they will pass a careful test, like the one Granger proposes, of orthogonality between x^a and $x - x^a$.

Good criteria for selecting an adjustment procedure are, I think, already implicit in the other sections of Granger's paper. As he suggests, it is important that the objectives of seasonal adjustment should be made explicit. The degree of dependence of the adjustment method on the objective also should be made clear to users of the adjusted data. As this paper also suggests, there is a substantial range of possible models for seasonality. The model relating to a well-behaved adjustment procedure should be specified, and its robustness against variations in that model tested and spelled out. A good example of this kind of analysis appears in the paper, by Cleveland and Tiao [1], that studies the Census X–11 procedure from this point of view.

Granger points to the need of allowing for evolving seasonals in most economic time series. In my own applied work, I was somewhat surprised to find that the reverse point is also true—it is important to allow for deterministic seasonals. Methods of analysis assuming that a process has a spectral density can fail if the process included a strictly periodic component. Many, maybe most, economic time series behave as if they include a component that is, for practical purposes, a strictly periodic seasonal. They may also contain an evolving seasonal, but analysis of the latter component should begin after removal, by regression, of strictly periodic components.

Let me make it clear that I do not regard the critical remarks that I have aimed at the section "Criteria for Evaluation" in Granger's paper as hostile to the main thrust of the paper. Granger gives us good reasons, with which I heartily agree, for approaching any seasonal adjustment problem with a reconsideration of fundamentals. What are the properties of the seasonal? Why have we set out to adjust this series? How should we expect the seasonal and our adjustment for it to affect later stages of our analysis?

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COMMENTS ON "SEASONALITY: CAUSATION, INTERPRETATION, AND IMPLICATIONS" BY CLIVE W. J. GRANGER

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INTRODUCTION

After ponting at a few issues directly concerned with Professor Granger's careful review, I will shift my attention to broader issues of approaching seasonality, including erratic values, nonstatistical inputs, and one-sided fitting.

SPECIFICS

The Finite and the Infinite

While Granger has done a workmanlike job of showing why the spectrum approach does not have the whole answer, it is desirable to go further. After urging the essential character of a definition of seasonality, he defines having and apparently having property S for a process and a (presumably finite) series, respectively. These definitions depend upon whether or not $f(\omega)$ or $f(\omega t)$ has peaks in small intervals around the harmonics of the annual frequency. He then, wisely, calls attention to the difficulties of stationarity and the advantages of the pseudospectrum saying—

> If this way out is taken, peaks in the pseudospectrum at the seasonal frequency bands will indicate that the series did have property S for at least some of the time span considered.

While I do not believe in the essentiality of a reasonably precise definition of seasonality (see [3; 4] on the importance of vague concepts), I am afraid that "he who appeals to Caesar must go to Caesar." Note first that whether or not a series has (as opposed to apparently has) property S, it has not been defined. (We will soon see that this could not be defined.) Note next that there has been no discussion of the connection, if any, between a process having property S and a finite piece of a realization appearing to have it. (We will soon see that such a discussion is at best very difficult.) I suggest these omissions and the difficulty of filling them illustrate how an expressed need for a reasonably precise definition can be a liability rather than an asset.

Any finite-extent function can arise, to an arbitrarily close approximation, as a sample from a process with any spectrum. (A special case of this is dealt with in this paper.) Thus, any attempt to connect our observations to a spectrum must be deeply statistical in a way not removed by having, for instance, 1,400 years of monthly data. I wish I understood this connection well enough to explain it simply and clearly to you—I regret that I do not yet understand it well enough to explain it satisfactorily to myself.

The existence of such a difficult connection between observables and infinite-duration processes is, for me, a good reason to doubt the adequacy of a logical structure focussed on infinite-duration processes to guide the analysis of data.

Suppose, to go beyond the feasible, that we know not just a single $x_1, x_2 \ldots, x_T$ for some T definitely $< \infty$, but that we completely know the joint distribution of $(x_1, x_2, \ldots, x_T)$. We need not then know the spectrum of the process. If the joint distribution is Gaussian, for example, so that the values of the lag covariances

$$C_{ts} = \text{ave}\{(x_t - \bar{x}_t)(x_s - \bar{x}_s)\}$$

complete the description of the finite-length process, then only a finite number of expressions

$$\int_0^\pi \cos[(t-s)\omega]\, dS(\omega)$$

are known, where $S(\omega)$ is the cumulative spectrum, and the spectrum can be seriously modified without changing the finite-length process. We cannot know precisely what the spectrum is if we know only the finite-length process, even exactly. Our fate in the real world is worse, of course, since we cannot know even the finite-length process exactly.

A Lemma

Let us next show that any function defined over a finite length can be realized by a spectrum concentrated as close to the annual cycle as we wish. Since either

Prepared, in part, in connection with research at Princeton University, supported by ERDA.

$\cos(t/2\pi)$ or $\sin(t/2\pi)$ (or both) is bounded away from zero in any short interval, if $P_1(t)$ and $P_2(t)$ are arbitrary polynomials, we can approximate any given $f(t)$ uniformly over a given finite interval by

$$P_1(t)\cos(\omega t)+P_2(t)\sin(\omega t)$$

If we can show that we can approximate, over this same interval, this latter function, by a function with an obviously concentrated spectrum, our lemma will be proved. (Note that restricting $P_2(t)$ to even powers of t does not matter, since $t=0$ can be shifted outside the region of approximation.)

Let Δ be defined as operating on functions $f(t,\omega,\ldots)$ of t, ω, and perhaps other parameters by

$$\Delta f(t,\omega,\ \ldots)=f(t,\omega+\varepsilon,\ \ldots)-f(t,\omega-\varepsilon,\ldots)$$

where the other parameters are left unchanged. Then, if A and B may involve t or these other parameters, but not ω,

$$\Delta(A\cos\omega t)=-2A\sin\epsilon t\sin\omega t$$
$$\Delta(A\sin\omega t)=+2A\sin\epsilon t\cos\omega t$$

and since $-2A\sin\epsilon t)$ and $2A\cos\epsilon t$ are also A's, we have

$$\Delta^k(A\cos\omega t)=\pm(2\sin\epsilon t)^k A\ c\&s\ \omega t$$
$$\Delta^k(A\sin\omega t)=\pm(2\sin\epsilon t)^k A\ s\&c\ \omega t$$

(where s&c is sin or cos according as k is even or odd) showing that a linear combination of Δ^k $(\cos\omega t)$, for $0\leq k\leq K$, which consists of a superposition of cos $(\omega+k\epsilon)t^s$ with $|k\epsilon|\leq K\epsilon$, can product any

$$P_K\left(\frac{\sin\epsilon t}{\epsilon}\right)\cos\omega t$$

which, for $\epsilon\to 0$, tends to the corresponding

$$P_K(t)\cos\omega t$$

The corresponding sine series has its $P_K(t)$ even, but, as noted above, this need not bother us.

As $\epsilon\to 0$, for K fixed, the frequencies $\omega+k\epsilon$ all come as close as we like to ω. To obtain the specific result announced, we need only take $\omega=2\pi/12$, in $(\text{months})^{-1}$.

The Desire for No Dips

Granger suggests, as one of three highly desirable properties, that the seasonally adjusted series should not have dips at seasonal frequencies. It is too easy to accept such a property without a clear understanding of its parallels.

Suppose we are to adjust a series by subtracting a linear trend in time. The property of the adjusted series $x_t^\bullet$ would be that it fails to have an absence of tilt, in the sense that corresponding to no dips,

$$\sum(t-\bar{t})x_t^\bullet$$

is not zero. Most fitters of lines, I believe, would be hesitant to accept, as a desirable property, that the line has not been fitted as well as one can easily do.

I find it very hard to understand why I should like to fit a line or polynomial as well as I can and be careful not to fit a sum of sines and cosines as well as I can. Can anyone offer good reasons to feel otherwise?

0.348 and 0.333

One of the papers cited by Granger [2] points out the difficulties associated with the alias of weekly cycles (in monthly data) at 0.348 cycles/year and the closeness of this to 0.333 cycles per year (it takes about 60 years for one cycle of the resulting beat). I am surprised that more was not made of this in the review, especially since (1) the harmonics of the weekly cycle alias as follows:

$2(0.348)=0.696$ aliases to 0.304 cycles/year
$3(0.348)=1.044$ aliases to 0.044 cycles/year
$4(0.348)=1.392$ aliases to 0.392 cycles/year

and (2) the working days per month correction, which is emphasized, is a combination of correction for irregular holidays with a very specific weekly cycle where Monday and Friday are presumed to be exactly as good as Tuesday, Wednesday, and Thursday, contrary to our general experience.

Here is a place, I believe, where spectrum thinking could aid us in doing good seasonal adjustment.

An anecdote about 20 years old might be apposite here. I held off from serious spectrum analysis of economic series until I could work with a competent economist. In 1957–58, Milton Friedman and I were at the Center for Advanced Study in the Behavioral Sciences. In the spring, we tried simple and cross-spectrum analysis of some of Milton's monetary series. Two phenomena stuck out. One was traced rather easily to the fixed 4–5 month interval between liberty or war bond drives. The other turned out to be the 0.348 cycles/year effect, and Milton took these series back to NBER for better adjustment for the relation of weeks to the month.

Even in series already so adjusted—with tender and loving care—the 0.348 cycles/year effect can be large. Seasonal adjustments that do not recognize this, overtly or implicitly, seem, to me, likely to be in trouble.

GENERALITIES

Realism and the Bounds of Statistics

A few years ago, a foreign visitor—a statistician from a culture far more rigid than that of Princeton—sat in one of my classes in Princeton while I discussed pooling in complex analyses of variance. After class, he made it quite clear that he would not discuss such dirty matters in class at home, but that he would, of course, do it himself in his office with the door shut. I often wonder about the culture of modern econometrics. Is there a comparably great difference between what good econometricians do in their offices and what they write papers about—or is there little difference?

If there is little difference, then my econometric friends may deserve sympathy—but surely not praise.

The world is not a simple place. Models that are simple enough to be completely treated analytically are not usually good guides when pressed to their extremes. Not so many years ago, my statistical colleagues, especially those who were glad to be called mathematical statisticians, were in a similar plight. (Today, recovery is moderately widespread and moderately rapid.)

The paradigm

$$\text{simple model} \xrightarrow{\text{optimize}} \text{procedure}$$

is quite dangerous. Both safety and strength from the iterative paradigm

$$\begin{tikzcd}
\text{simple model} \arrow[dr, "\text{suggest}"] & & \\
& \text{procedureS} \arrow[r, "\text{select}"] & \text{procedure(s)} \\
\text{more general model} \arrow[ur, "\text{examine}"'] & &
\end{tikzcd}$$

especially if we plan to use more and more general models as a basis for examination. Even a physicist, when pressed to the wall, will admit that all the laws of physics are wrong, but most are devilishly good approximations. This is as true today as it was before Einstein or Newton. In most other fields, the approximations are not nearly as good.

My unhappiness about econometrics contains an added strain—uncertain, conceivably unfounded, but for all that, constant and depressing. This is the idea that the data should be able to provide all the answers—probably through some form of multiple regression. (The change in the price of oil should, one would think, through its consequences for models in which oil price was not a variable, to have educated most of us. After all, before Arab action, judgment offered a much better basis for assessing the coefficients related to oil prices than did all the world's time series.)

It is my impression that rather generally, not just in econometrics, it is considered decent to use judgment in choosing a functional form but indecent to use judgment in choosing a coefficient. If judgment about important things is proper, why should it not be used for less important ones as well? Perhaps the real purpose of Bayesian techniques is to let us do the indecent thing while modestly concealed behind a formal apparatus. If so, this would not be a precedent. When Fisher introduced the formalities of the analysis of variance in the early 1920's, its most important function was to conceal the fact that the data was being adjusted for block means, an important step forward which, if openly visible, would have been considered by too many wiseacres of the time to be cooking the data. If so, let us hope the day will soon come when the role of decent concealment can be freely admitteed.

Such questions are relevant to seasonal adjustment. The reasons for seasonality in some series are both well-known and measured. The coefficient may be better estimated from one source or from another, or, even best, estimated by economic judgment.

It seems to me a breach of the statistician's trust not to use judgment when that appears to be enough better than using data.

Some will think this a counsel of perfection, some of imperfection—may they and all others, at least, think about what should be done about when judgment should be used.

Incomplete Use of Causal Variates

What has just been discussed will no doubt be called unhelpful criticism by some. Too bad!

There are, however, more helpful things to consider. Suppose we are seasonally adjusting a monthly measure of economic activity—and suppose that we have available a monthly measure of days lost by strikes (in the relevant sector). How should we use the latter, which is likely to be occasionally and irregularly large—and usually small?

One way to use it is to slip it in a multiple-linear regression with everything else. Another way is to give up trying to fit history in the months where its value is large, considering only the much more frequent undisturbed months. This will test our adjustment techniques that may not be ready for data with so many holes in them.

Another approach would be to introduce a one-sided response in fitting to months where the strike index is high. Instead of zeroing sums of the form

$$\sum(\text{carrier value})\,\psi\left(\frac{\text{obs'd MINUS fit}}{\text{scale}}\right)$$

for a fixed ψ, where the regression, if it be linear, or its differential, if it be nonlinear, takes the form

$$\sum(\text{constant})\,(\text{carrier})$$

we can, for example, let some ψ's grow in the usual way for positive deviations but much more slowly for negative ones. This sort of idea for minimal-sum-of-absolute-deviations fitting has been proposed by Claerbout [1]; it is equally applicable to still better types of fitting—or even to least squares.

If I were to use it, I would notice how conveniently it could be combined with protection against erratic values, as when

$$\psi(u) = \begin{cases} (1-u^2)^2 u & \text{for} -1 \leqq u \leqq 1 \\ 0 & \text{else} \end{cases}$$

and the scale is six to nine times the median absolute deviation.

I am as reluctant to dive, alone and unguided, into seasonally adjusting economic series today as I was 20 years ago to dive, alone and unguided, into their spectrum analysis. I do believe, however, that with the techniques that have grown up in the last decade—including the $\psi-\omega$ technology just hinted at and resistant/robust smoothing techniques, Velleman [5], to name but two—we have a much greater opportunity now than we had at that time.

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RESPONSE TO DISCUSSANTS

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In my paper, I presented a number of suggested criteria for a good adjustment procedure, most of which I thought to be noncontroversial. In particular, if the series to be adjusted, x_t was actually generated in a simulation experiment as the sum of two parts, S_t and y_t, where S_t is strongly seasonal and y_t does not have property S, then, I suggested that the adjusted series x^a should be very close to y_t. This was expressed by properties 2, 2′, and 3 by requiring the spectrum of x^a_t to have neither peaks or dips at seasonal frequencies and also that the coherence between x^a_t and y_t should be one at all frequencies. Both of my discussants, Sims and Tukey, and also Watts (discussing a later paper), have pointed out that these are unrealistic requirements. A method of adjustment that minimizes the expected square of the difference between the true nonseasonal and the adjusted series, y_t and x^a_t, will always produce an adjusted series with dips in the spectrum at seasonal frequencies. Further, any adjustment method that uses just a filter, or can be well approximated by a filter, will produce a coherence with dips at the seasonal frequencies. Although these properties have been proved only for time-invariant linear filters and a least-squares cost function, there is no reason to suppose that they will not hold equally true in more general situations, including causal adjustment methods.

The consequences for evaluation of adjustment methods are, I believe, profound. The criteria I suggested have been shown to be impossible to achieve in practice, and thus, should be replaced by achievable criteria. However, I am at a loss to know what these criteria should be. There seems to be no ideal process of evaluating a method of adjustment, even using simulated data, where the y_t are known, and, therefore, the situation is considerably more difficult when dealing with actual data when y_t is not known, of course. By further considering why seasonal adjustment is required, it is possible that alternative criteria will arise. It seems that it may be impossible to evaluate a single method, and the best we can hope to do is to rank alternative methods, as is the case when considering forecasting procedures.

One further consequence of these results is that one will always know that a series has been adjusted, by looking at its spectrum. Thus, the seasonality of a series has not been removed but merely altered in character—peaks in the spectrum have been replaced by dips, high serial correlations at 12 months may have been replaced by a smaller negative correlation, etc. For many purposes, the adjusted series will be preferable to the raw data, but for modeling purposes, both time series and econometric, at least part of the problem remains and, possibly, in a more difficult form.

Although these results might be thought depressing, at least my understanding of the situation has been greatly increased by the discussants' remarks, for which I am very grateful.

Section II

Description and Analysis of Seasonal Adjustment Procedures in Use