Volume Title: Output and Productivity in the Electric and Gas Utilities, 1899-1942

Volume Author/Editor: Jacob Martin Gould

Volume Publisher: NBER

Volume ISBN: 0-87014-046-9

Volume URL: http://www.nber.org/books/goul46-1

Publication Date: 1946

Chapter Title: Appendix to "Output and Productivity in the Electric and Gas Utilities, 1899-1942"

Chapter Author: Jacob Martin Gould

Chapter URL: http://www.nber.org/chapters/c4275

Chapter pages in book: (p. 143 - 185)

APPENDIX A

Electric and Gas Utilities
Construction of Indexes of Gross Output

THE DATA UNDERLYING THE INDEXES of electric and gas output are presented and discussed in the text, together with the indexes themselves and the general method of their construction. In this Appendix we discuss the step by step construction of the several indexes, thereby illustrating some technical problems of index construction, in the course of which we present supplementary data of interest to students of the utility industries.

The output of an industry through the years may be traced by means of an index. Let us consider first an industry that makes a single product. If among the units put out in any one year there are no wide differences in quality that would warrant the sale of some at prices much higher than others, the industry's output is simply the sum of the number of units produced in a base year. But most industries make more than one product. The quantities of such products cannot be added unless they are reduced to a common denominator. When the various products are so different that they do not have a common physical unit of measurement, the output aggregates must be translated into money terms before they can be added. Even when the products do have a common physical unit of measurement, if quality differences are great (reflected in their widely divergent prices) the sum of all units produced will not adequately measure total output.

Physical aggregates may be translated into money values by multiplying the number of units in each product group by the average price at which they exchange during a given period.[1] Adding the results for each product group in the given year yields an output aggregate (in value terms) equal to the total money value of the industry's output. We may represent this sum as $\Sigma q_1 p_1$, where q_1 is the number of units produced in the given year

[1] As the units of a product group are seldom absolutely uniform in quality or price, it must be assumed that price divergences are usually so small as to make the group average price representative.

in a particular product group, and p_1 their average price. This procedure applied to the base year yields $\Sigma q_0 p_0$ as the total output aggregate in value terms. These two value aggregates reflect changes in both quantity components and prices. Since we wish to consider quantity changes only, and to hold the price factors constant, we must use fixed price weights in converting the physical quantity aggregates into value terms. For example, for any particular product group, we multiply the number of units produced in both the given and base years by the same price weight — the arithmetic average of the prices obtaining in both the given and base years. The index formula adapted to this purpose is the Edgeworth: $\frac{\Sigma q_1(p_0 + p_1)}{\Sigma q_0(p_0 + p_1)}$, in which q_1 and q_0 represent the number of units in each product group in the given and base year respectively, and p_1, p_0 the corresponding prices paid for the units of each product group.[2]

The formula has the advantage that the weighting system is different for each comparison, and when computed for successive pairs of years, additional commodities can be included as data become available. The index takes the form of a chain of such comparisons. However, a series of successive year to year comparisons between, say, 1899 and 1942, may yield a different result from that obtained in a single direct comparison between the two years. To minimize this difference, and allow the same index to reflect both year to year changes and long term trends, a compromise method was adopted, involving the following steps. In accordance with the procedure adopted in preceding reports of this series, pairs of years as, say, 1899 and 1909, 1909 and 1919, 1919 and 1929, 1929 and 1937, and 1937 and 1942, were compared directly. A chain index was also computed for the entire period 1899–1942; this annual series was then fitted into the framework provided by the direct comparisons. For example, for 1929–37 we adjusted the annual chain index by distributing the discrepancy between it and the direct comparison 1929–37 evenly over the 8 years. The choice of link intervals depends upon the data. These and other necessary steps leading to the final index of total electric and gas utility output are described below.

1 *Electric Light and Power: General Note to Tables A1–7*

As indicated in Chapter 1, the weighted index of electric light and power was constructed in three segments: 1902–17, 1917–27, and 1927–42. For the first, 1902–17, because the quantity data could not be classified by consumer groups, an unweighted index based on total kilowatt hours generated was constructed (Table 2). It was corrected for shifts in the

[2] For a discussion of the Edgeworth formula as used in this report see Solomon Fabricant's *The Output of Manufacturing Industries, 1899–1937* (National Bureau of Economic Research, 1940), pp. 33–5, 358–62.

composition of total output by averaging the weighted indexes (1917 based on 1902) yielded by making two assumptions (A and B in App. Table A2) concerning the degree of price divergence in 1902. Appendix Table A2 presents the computations required by the Edgeworth formula as applied to the data in Appendix Table A1. The rationale underlying this method is discussed in Chapter 1, Section 2.

The computations in Appendix Tables A1 and 2 allowed us to adjust the unweighted index of output for 1902, 1907, 1912, and 1917 as in Table 5 and so obtain an approximation to a 'weighted' index. A weighted index, composed of light, power, and rail kilowatt hour sales, was then constructed for 1922 and 1927 (App. Table A3). Since 1927 a more detailed weighting structure was available (App. Table A4). The final steps consisted of interpolating annual entries in the preceding series of quinquennial indexes (App. Tables A5 and 6). The annual interpolation of the unweighted output index is given in Appendix Table A7. All computations in these tables were carried to five figures, though the figures shown may be rounded.

TABLE A1

Electric Light and Power Output, 1902 and 1917
Based on Two 1902 Price Assumptions

	LIGHT	POWER	RAIL	TOTAL
1902	ASSUMPTION A			
1 q_0: mil. kwh. sold	1,086.0	694.8	315.8	2,096.6
2 p_0: ¢ per kwh.	6.458	1.426	.898	3.953
	ASSUMPTION B			
3 q_0: mil. kwh. sold	1,774.2	250.7	71.8	2,096.6
4 p_0: ¢ per kwh.	3.953	3.953	3.953	3.953
5 p_0q_0: revenue, th. $	70,138	9,910	2,837	82,886
1917				
6 q_1: mil. kwh. sold	5,122.5	13,174.8	2,985.8	21,273.1
7 p_1: ¢ per kwh.	5.542	1.224	0.771	2.198
8 p_1q_1: revenue, th. $	283,334	161,201	23,010	467,545

Data in lines 5, 6, 7, and 8 and all totals for light, power, and rail were taken from or based upon Census reports, as in Table 4 and Ch. 1, note 16. The totals under Assumption A (lines 1 and 2) were based on the assumption that the degree of price divergence among the three consumer groups in 1902 was the same as in 1917. Thus with p_0, p_0', and p_0'' representing the average prices charged for light, power, and rail service in 1902, and p_1, p_1' and p_1'' the corresponding prices in 1917, and assuming that $\frac{p_0}{p_0'} = \frac{p_1}{p_1'}$ and $\frac{p_0'}{p_0''} = \frac{p_1'}{p_1''}$, we get the two equations: $\frac{p_0}{p_0'} = \frac{5.542}{1.224} = 4.528$ and $\frac{p_0'}{p_0''} = \frac{1.224}{0.771} = 1.588$. Since $p_0q_0 = \$70,138$ thousand, $p_0'q_0' = \$9,910$ thousand, $p_0''q_0'' = \$2,837$ thousand, and $q_0 + q_0' + q_0'' = 2,096.6$ mil. kwh, we have six equations in all, enough to determine the six unknowns (lines 1 and 2).

Assumption B is at the other extreme of probability: $\frac{p_0'}{p_0'} = \frac{p_0'}{p_0''} = 1$.

In conjunction with the four other known equations we derive the values of q_0, q_0', and q_0'' in line 3.

Appendix Table A2
Electric Light and Power
Construction of Weighted Output Index in 1917, Assumptions A and B
Edgeworth Formula (1902:100)

	LIGHT	POWER	RAIL	TOTAL	RATIO $\frac{\Sigma q_1(p_0+p_1)}{\Sigma q_0(p_0+p_1)}$	INDEX
				ASSUMPTION A		
p_0+p_1(¢ per kwh.)	12.00	2.65	1.67			
	EDGEWORTH CONSTANT $ TOTALS, MILLIONS					
1902: $q_0(p_0+p_1)$	130	18	5	154	6.582	100.0
1917: $q_1(p_0+p_1)$	615	349	50	1,014		658.2
				ASSUMPTION B		
p_0+p_1(¢ per kwh.)	9.50	5.18	4.72			
	EDGEWORTH CONSTANT $ TOTALS, MILLIONS					
1902: $q_0(p_0+p_1)$	168	13	3	185	7.085	100.0
1917: $q_1(p_0+p_1)$	486	682	141	1,310		708.5

The computations are those required by the application of the Edgeworth formula $\frac{\Sigma q_1(p_0+p_1)}{\Sigma q_0(p_0+p_1)}$ to the data in Appendix Table A1. The weighted index for 1917 (based on 1902) used in subsequent calculations, 683.3, is the average of the indexes based on Assumptions A and B.

Appendix Table A3
Electric Light and Power, 1917, 1922, and 1927
Construction of Weighted Output Index

	LIGHT	POWER	RAIL	TOTAL	RATIO $\frac{\Sigma q_1(p_0+p_1)}{\Sigma q_0(p_0+p_1)}$	INDEXES 1927:100	1917:100	1902:100
1917 based on 1927								
p_0+p_1(¢ per kwh.)	10.46	2.59	1.72					
	EDGEWORTH CONSTANT $ TOTALS, MILLIONS							
1917: $q_1(p_0+p_1)$	593	378	54	1,024	.3002	30.0	100.0	683.3
1927: $q_0(p_0+p_1)$	2,421	874	116	3,412		100.0	333.1	2,276.3
1922 based on 1927								
p_0+p_1(¢ per kwh.)	10.68	3.17	2.01					
	EDGEWORTH CONSTANT $ TOTALS, MILLIONS							
1922: $q_1(p_0+p_1)$	1,126	632	96	1,855	.5044	50.4	168.0	1,148.2
1927: $q_0(p_0+p_1)$	2,472	1,068	136	3,677		100.0	333.1	2,276.3

The computations are those required by the application of the Edgeworth formula to the data in Table 6. The year 1927 rather than 1917 was chosen as the base for the 1922 comparisons since the 1917 data had been largely estimated from fragmentary Census figures.

APPENDIX TABLE A4

Electric Light and Power, 1927, 1932, 1937, and 1942

Construction of Weighted Output Index

	Farm	Domestic	COMMERCIAL Small	COMMERCIAL Large	Municipal st. lighting	St. & interurban rr.	St. & Electrified st. rr.	Other	Total	RATIO $\frac{\Sigma q_1(p_0+p_1)}{\Sigma q_0(p_0+p_1)}$	INDEXES 1927:100	INDEXES 1902:100
1932 based on 1927												
p_0+p_1 (¢ per kwh.)	4.92	12.23	8.05	2.86	8.55	1.82	1.95	4.14				
EDGEWORTH CONSTANT $ TOTALS, MILLIONS												
1927: $q_0(p_0+p_1)$	41.1	1,029.9	978.2	956.8	149.0	114.1	10.1	9.8	3,288.9	1.1340	100.0	2,276.3
1932: $q_1(p_0+p_1)$	74.0	1,442.0	970.5	925.3	191.2	80.1	14.9	31.9	3,729.8		113.4	2,581.4
1937 based on 1927												
p_0+p_1 (¢ per kwh.)	4.74	11.04	7.40	2.51	8.06	1.81	1.93	3.63				
EDGEWORTH CONSTANT $ TOTALS, MILLIONS												
1927: $q_0(p_0+p_1)$	39.6	929.5	899.7	840.2	140.5	113.2	10.0	8.6	2,981.2	1.6670	100.0	2,276.3
1937: $q_1(p_0+p_1)$	126.5	1,856.0	1,283.2	1,326.6	164.1	88.5	30.1	94.8	4,969.8		166.7	3,794.6
1942 based on 1937												
p_0+p_1 (¢ per kwh.)	4.85	7.97	6.23	2.07	7.58	1.67	1.61	3.05				
EDGEWORTH CONSTANT $ TOTALS, MILLIONS												
1937: $q_0(p_0+p_1)$	81.1	1,410.1	1,125.7	1,064.5	141.1	73.8	20.7	73.2	3,990.2	1.5560	166.7	3,794.6
1942: $q_1(p_0+p_1)$	140.3	2,147.1	1,696.1	1,831.8	156.1	70.9	38.2	128.1	6,208.7		259.4	5,904.4

The computations are those required by the application of the Edgeworth formula to the data in Table 7. The indexes were put on a 1902 base by splicing to the indexes in Appendix Table A3.

Appendix Table A5
Electric Light and Power, 1926–1942
Annual Interpolation, Weighted Output Index

	Weighted Output Index 1927:100		Interpolated Ratios	Interpolated Census Indexes	
	Census	EEI	(1÷2)	1927:100	1929:100
	(1)	(2)	(3)	(4)	(5)
1926		90.1	1.000	90.1	73.7
1927	100.0	100.0	1.000	100.0	81.7
1928		110.4	.992	109.5	89.5
1929		124.3	.984	122.3	100.0
1930		129.6	.977	126.6	103.5
1931		128.6	.969	124.6	101.9
1932	113.4	118.0	.961	113.4	92.7
1933		118.6	.957	113.6	92.8
1934		127.3	.953	121.4	99.2
1935		139.4	.950	132.4	108.2
1936		159.6	.946	150.9	123.4
1937	166.7	177.0	.942	166.7	136.3
1938		177.7	.942	167.4	136.8
1939		196.9	.942	185.4	151.6
1940		217.9	.942	205.2	167.7
1941		247.5	.942	233.1	190.5
1942		275.4	.942	259.4	212.0

The interpolated index (column 4) is the product of columns 3 and 2. The ratios (column 3) of the Census to the Edison Electric Institute index were interpolated by a straight-line distribution of the quinquennial differences. For 1926 and 1937–42 the corresponding quinquennial ratios were held constant.

The EEI weighted output index was based on Table 11 in accordance with the Edgeworth formula, as already illustrated in Appendix Tables A1–4. Comparisons were based on successive years and a preliminary index, based on 1927, was constructed as the product of a series of chain indexes. To avoid the cumulative bias to which such an index is subject, the annual indexes 1927–37 were adjusted by a straight-line distribution of the difference between the 1937 index obtained by direct comparison with 1927 and the 1937 chain index based on 1927, the chain index for 1926 being left undisturbed. The indexes for 1937–42 were similarly adjusted.

The suitability of the EEI series as an interpolating medium may perhaps be questioned. The companies reporting to the National Electric Light Association and the Edison Electric Institute were described in successive annual statistical bulletins published by the two organizations as making up 90–94 percent of the entire industry, the latter being defined as all "enterprises devoted exclusively to the generation and distribution of electricity, plus the electric departments of all others which maintained electric light and power systems jointly with other public utility services" (NELA, *Statistical Bulletin 7*, June 1931, p. 1; EEI, *Statistical Bulletin 4*, January 1937, p. 1, and *10*, May 1943, p. 1). However, although the reported data were adjusted "to provide 100 percent coverage", certain differences remain between the trends indicated by these sets of data and the trends revealed by Census material, though the differences do not invalidate the use of the former to interpolate annual values in the latter. E.g., the ratios of EEI to Census totals for kwh. sold are .962 for 1927, .967 for 1932, and .987 for 1937, indicating a gradual extension in EEI coverage.

As indicated, the proper medium for interpolating the Census weighted index here is the weighted EEI index of output (1927:100) which reached 118.0 in 1932, when the weighted Census index was 113.4; in 1937 it was 117.0, 6.2 percent above the Census index. Evidently then, the EEI sample emphasizes the more rapid growth of the higher-valued services in 1927–37. However, the general trends indicated by the two weighted indexes are similar; in 1937–42, the EEI data probably reflect adequately the further growth of the industry.

Appendix Table A6
Electric Light and Power, 1902–1927
Annual Interpolation, Weighted Output Index

	WEIGHTED OUTPUT INDEX 1927:100 (1)	OUTPUT, ALL PRIVATE AND PUBLIC AGENCIES (bil. kwh.) (2)	INTERPOLATED RATIOS (1 ÷ 2) (3)	INTERPOLATED INDEXES 1927:100 (4)	1929:100 (5)
1902	4.4			4.4	3.6
1907	9.2			9.2	7.5
1912	15.8	11.6	1.370	15.8	13.0
1913		12.5	1.332	16.6	13.6
1914		14.4	1.294	18.6	15.2
1915		16.2	1.256	20.3	16.6
1916		21.2	1.218	25.9	21.1
1917	30.0	25.4	1.180	30.0	24.5
1918		33.2	1.156	38.4	31.4
1919		38.9	1.133	44.1	36.0
1920		43.3	1.109	48.1	39.3
1921		40.9	1.085	44.4	36.3
1922	50.4	47.5	1.062	50.4	41.2
1923		55.6	1.101	61.2	50.0
1924		58.9	1.141	67.2	54.9
1925		65.8	1.181	77.6	63.5
1926		73.7	1.220	89.9	73.5
1927	100.0	79.4	1.260	100.0	81.7

The data in column 2, used as an interpolating medium, are from *Electric Power Statistics, 1920–1940* (Federal Power Commission, Washington, D. C., 1941) F.P.C.S.–20, App. 4. They include the output of "privately owned electric utilities, that portion of mining and manufacturing plants that is devoted to public use, railways and railroads, municipal electric utilities, Bureau of Reclamation, other Federal projects, cooperatives power districts, state projects and publicly owned noncentral stations." The FPC classification is thus seen to be somewhat broader than the Census.

The data for 1912–19 are from the *Electrical World*, Sept. 9, 1922.

In column 3 the ratios of the annual output series to the Census weighted index are interpolated by a straight-line distribution of the differences between quinquennial ratios. The interpolated index is the product of columns 2 and 3.

Appendix Table A7
Electric Light and Power, 1902–1942
Annual Interpolation, Unweighted Output Index

	UNWEIGHTED OUTPUT INDEX 1927:100 (1)	OUTPUT, ALL PRIVATE AND PUBLIC AGENCIES (bil. kwh.) (2)	INTER-POLATED RATIOS (1 ÷ 2) (3)	INTERPOLATED INDEXES 1927:100 (4)	INTERPOLATED INDEXES 1929:100 (5)
1902	3.6			3.6	3.0
1907	8.5			8.5	7.0
1912	16.7	11.6	1.443	16.7	13.8
1913		12.5	1.443	18.0	14.9
1914		14.4	1.443	20.8	17.2
1915		16.2	1.443	23.3	19.3
1916		21.2	1.443	30.6	25.4
1917	36.7	25.4	1.443	36.7	30.4
1918		33.2	1.388	46.1	38.2
1919		38.9	1.333	51.9	43.0
1920		43.3	1.277	55.4	45.8
1921		40.9	1.222	50.0	41.4
1922	55.4	47.5	1.167	55.4	45.9
1923		55.6	1.185	65.8	54.5
1924		58.9	1.204	70.9	58.7
1925		65.8	1.223	80.4	66.6
1926		73.7	1.241	91.5	75.8
1927	100.0	79.4	1.260	100.0	82.8
1928		86.6	1.259	109.0	90.3
1929		95.9	1.259	120.7	100.0
1930		94.7	1.258	119.1	98.6
1931		90.7	1.257	114.1	94.5
1932	103.5	82.4	1.257	103.5	85.7
1933		84.7	1.265	107.2	88.8
1934		90.8	1.274	115.7	95.8
1935		98.5	1.283	126.3	104.6
1936		112.2	1.291	144.8	120.0
1937	158.4	121.8	1.300	158.4	131.1
1938		116.7	1.308	152.6	126.4
1939		130.3	1.317	171.6	142.1
1940		145.0	1.325	192.1	159.1
1941		168.2	1.333	224.2	185.7
1942	253.8	189.2	1.342	253.8	210.2

Column 1 from Table 10; column 2 from the source indicated for column 2 of Appendix Table A6. The interpolating method follows that of Appendix Table A6.

2 *Manufactured and Natural Gas: General Note to Appendix Tables A8–18*

The weighted index of manufactured gas output was constructed in three segments: 1899–1919, 1919–29, and 1929–42, in which 1909, 1919, 1929, and 1940 were chosen as base years for the index comparisons. Appendix Table A8 presents the computations underlying the weighted index for

1899–1919. The weighted index for the next period, 1919–29 (App. Table A11), was based on greater detail than was available for the preceding, and the adjustments contributing to this greater detail are presented in Appendix Tables A9 and 10. A final adjustment, made in Appendix Table A12, is designed to account for the output of products of the industry not directly represented in the output index.

Appendix Tables A13 and 14 carry forward the annual interpolation of both the weighted and unweighted indexes of manufactured gas output.

Appendix Table A15 presents Census output data and a weighted index based on them for 1929–35. This material is not used in the text but is presented for comparison with the corresponding AGA data, on which our weighted index is based after 1929 (App. Table A16).

In Appendix Table A17 we estimate the annual value of products in the manufactured and natural gas industries in order to provide the unit value weights essential for computing *combined* indexes of output (App. Table A18). Appendix Table A18 also serves as a model for the computation of the combined indexes of electric and gas output in Table 40.

Appendix Table A8
Manufactured Gas, 1899–1919
Construction of Weighted Output Index

	Gas (th. cu. ft.)	Byproduct: Coke (sh. t.)	Byproduct: Tar (gal.)	Total	Ratio $\frac{\Sigma q_1(p_0+p_1)}{\Sigma q_0(p_0+p_1)}$	Indexes: 1919:100	Indexes: 1929:100
1899 based on 1909 p_0+p_1 ($ per unit)	1.95						
	EDGEWORTH CONSTANT $ TOTALS, MILLIONS						
1899: $q_1(p_0+p_1)$	131			131	.4448	22.3	15.7
1909: $q_0(p_0+p_1)$	295			295		50.0	35.3
1904 based on 1909 p_0+p_1 ($ per unit)	1.92	6.40	0.055				
	EDGEWORTH CONSTANT $ TOTALS, MILLIONS						
1904: $q_1(p_0+p_1)$	216	11	4	231	.7577	37.9	26.8
1909: $q_0(p_0+p_1)$	290	10	5	305		50.0	35.3
1909 based on 1919 p_0+p_1 ($ per unit)	1.83	10.74	0.063				
	EDGEWORTH CONSTANT $ TOTALS, MILLIONS						
1909: $q_1(p_0+p_1)$	277	18	6	300	.5004	50.0	35.3
1919: $q_0(p_0+p_1)$	566	26	8	600		100.0	70.6
1914 based on 1919 p_0+p_1 ($ per unit)	1.77	11.07	.064				
	EDGEWORTH CONSTANT $ TOTALS, MILLIONS						
1914: $q_1(p_0+p_1)$	361	25	8	395	.6779	67.8	47.9
1919: $q_0(p_0+p_1)$	548	27	8	582		100.0	70.6

The computations are those required by the application of the Edgeworth formula to the data for 1899–1919 in Table 24. The indexes were put on a 1929 base by splicing to the 1919 index, obtained by direct comparison with 1929 (see Appendix Table A11).

APPENDIX TABLE A9

Manufactured Gas, Census Data, 1919–1935

Total Gas Sales, Adjusted to exclude Gas Purchased within the Industry and Corrected for Distribution Losses

		1919	1921	1923	1925	1927	1929	1931[a]	1931[b]	1933	1935
		BILLIONS OF CUBIC FEET									
1	Sales, incl. purchased gas	308	306	357	359	412	408	374	372	306	312
2	Unaccounted for	28	24	27	29	30	29	21	20		
3	Total (1 + 3)	337	330	384	388	442	438	395	393		
4	Ratio (1 ÷ 3)	.916	.926	.930	.926	.932	.934	.948	.948		
5	Purchased within industry	30	30	36	22	20	5	6	6		
6	Purchased within industry, corrected for distribution losses (4 × 5)	28	28	34	20	18	5	6	6		
7	Sales, excl. gas purchased within industry (1 – 6)	281	278	323	339	394	404	368	366	306	312
		MILLIONS OF DOLLARS									
8	Sales, incl. purchased gas	282	372	394	396	446	438	400	397	322	314
9	Price of gas purchased within industry, $ per th. cu. ft.	.562	.539	.517	.495	.473	.516	.500	.500		
10	Total purchased within industry (9 × 6)	15	15	17	10	9	3	3	3		
11	Sales, excl. gas purchased within industry (8 – 10)	267	357	377	386	438	436	397	394	322	314
12	Price of gas, excl. gas purchased within industry, (11 ÷ 7) $ per th. cu. ft.	0.95	1.28	1.17	1.14	1.11	1.08	1.08	1.08	1.05	1.01

The presence of considerable quantities of 'gas purchased within the industry' indicates that the series 'Gas, for sale', as reported by the Census, is subject to duplication. After 1919 the relative importance of such duplication changed radically. Accordingly, we subtract the quantity and value of the gas purchased within the industry from the industry sales totals. But since the latter are already net with respect to distribution losses, the 'gas purchased' series must, before subtraction, be adjusted downward to account for such losses. The adjustment of the gas sales series, together with the corresponding adjustment in the value data, are shown above.

The computations are based on data in successive Censuses of Manufactures. For 1921, 1923, and 1925 the prices paid for gas purchased within the industry (line 9) were estimated. In 1919, when the price was reported (*Census of Manufactures, 1919*, X, 712), 30,006 mil. cu. ft. of coal gas, water gas, and mixed coal and water gas, valued at $16,852 th., were purchased within the industry. The prices for the intervening years are straight-line interpolations of the 1919 and 1927 prices (*ibid, 1927*, p. 759; *1929*, II, 758). The 1931 price was arbitrarily set at $0.50 per th. cu. ft.

[a] Comparable with 1929.

[b] Comparable with 1933.

Appendix Table A10

Manufactured Gas, 1919–1929

Segregation of Census Gas Sales, Domestic and Other Components (billions of cubic feet)

	1919	1920	1921	1922	1923	1924	1925	1926	1927	1928	1929
AGA	300	320	327	350	385	405	421	456	471	495	524
Domestic	218	240	246	260	279	291	300	323	316	326	336
Househeating									13	18	23
Industrial-commercial	70	69	71	81	92	106	113	126	136	148	163
Miscellaneous	11	11	10	9	14	8	8	6	6	4	2
Percentage of total	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
Domestic	72.8	75.0	75.2	74.3	72.4	71.8	71.3	70.9	67.1	65.8	64.1
Househeating									2.7	3.6	4.4
Industrial-commercial	23.5	21.7	21.6	23.0	24.0	26.2	26.8	27.7	29.0	29.8	31.1
Miscellaneous	3.8	3.3	3.2	2.7	3.6	2.1	1.9	1.3	1.2	0.8	0.4
Census	281	285	278	296	323	333	339	374	394	397	404
Domestic	204	214	209	220	234	239	242	265	265	261	258
Househeating									11	14	18
Industrial-commercial	66	62	60	68	78	87	91	104	114	118	126
Miscellaneous	11	10	9	8	12	7	6	5	5	3	2
Ratio: AGA to Census	1.07	1.12	1.18	1.18	1.19	1.22	1.24	1.22	1.20	1.25	1.30

Before 1927 the Census of Manufactures did not segregate gas sales by type of use. We based such a segregation on data published by the American Gas Association (*Statistical Bulletin 8*, Oct. 1930, p. 29). The percentages of total AGA sales for each consumption group were applied to the corresponding Census total. Census sales totals for even-numbered years were interpolated by dividing the AGA totals by estimated ratios of AGA to Census sales totals; the latter ratios are averages of the successive ratios for odd-numbered years.

APPENDIX TABLE A11

Manufactured Gas, 1919–1929

Construction of Weighted Output Index (1929:100)

	Total (1)	Domestic (2)	GAS* House-heating (3)	Industrial-Commercial (4)	Misc. (5)	Sum (2 + 3 + 4 + 5) (6)	RATIO (6 ÷ 1) (7)	(6) RAISED TO LEVEL OF (1) (6 ÷ 7) (8)	BYPRODUCT COKE* (9)	BYPRODUCT TAR* (10)	(8 + 9 + 10) (11)	RATIO $\frac{\Sigma q_1(p_0 p_1)}{\Sigma q_0(p_0 + p_1)}$ (12)	FINAL INDEX (13)
1919 based on 1929													
$p_0 + p_1$ ($ per unit)	2.03	2.39		1.73	1.73				15.05	0.09			
						EDGEWORTH CONSTANT $ TOTALS, MILLIONS							
1919: $q_1(p_0 + p_1)$	570	488		114	18	620	1.074	578	37	11	625	.706	70.6
1929: $q_0(p_0 + p_1)$	819	659		218	3	880	1.074	819	52	14	885		100.0
1919 based on 1921													
$p_0 + p_1$ ($ per unit)	2.23	2.39		1.73	1.73				15.87	0.086			
						EDGEWORTH CONSTANT $ TOTALS, MILLIONS							
1919: $q_1(p_0 + p_1)$	628	488		114	18	620	.996	623	39	10	672	1.014	70.6
1921: $q_0(p_0 + p_1)$	621	499		104	15	618	.996	621	32	10	663		69.6
1921 based on 1923													
$p_0 + p_1$ ($ per unit)	2.45	2.39		1.73	1.73				18.52	0.094			
						EDGEWORTH CONSTANT $ TOTALS, MILLIONS							
1921: $q_1(p_0 + p_1)$	682	499		104	15	618	.900	687	38	11	736	.873	69.6
1923: $q_0(p_0 + p_1)$	792	558		134	20	712	.900	792	40	10	842		79.7
1923 based on 1925													
$p_0 + p_1$ ($ per unit)	2.31	2.39		1.73	1.73				18.54	0.095			
						EDGEWORTH CONSTANT $ TOTALS, MILLIONS							
1923: $q_1(p_0 + p_1)$	744	558		134	20	712	.954	747	40	10	798	.951	79.7
1925: $q_0(p_0 + p_1)$	782	577		158	11	746	.954	782	44	12	839		83.8
1925 based on 1927													
$p_0 + p_1$ ($ per unit)	2.25	2.39		1.73	1.73				17.21	0.100			
						EDGEWORTH CONSTANT $ TOTALS, MILLIONS							
1925: $q_1(p_0 + p_1)$	763	577		158	11	746	.974	766	41	12	820	.860	83.8
1927: $q_0(p_0 + p_1)$	886	632	25	198	8	863	.974	886	51	16	953		97.4
1927 based on 1929													
$p_0 + p_1$ ($ per unit)	2.19	2.39	2.39	1.73	1.73				16.37	0.101			
						EDEGWORTH CONSTANT $ TOTALS, MILLIONS							
1927: $q_1(p_0 + p_1)$	863	632	25	198	8	863	.995	867	49	16	932	.974	97.4
1929: $q_0(p_0 + p_1)$	884	617	42	218	3	880	.995	884	56	16	957		100.0

The computations are those required by the application of the Edgeworth formula to the data in Table 24. The method applied in Appendix Table A8 had to be modified because unit prices were not available for gas sales to the various consumer groups except for 1929. We therefore first computed the Edgeworth constant dollar totals for gas sales for domestic, house heating, industrial-commercial, and miscellaneous uses, based on constant 1929 prices (columns 2, 3, 4, and 5); then adjusted these totals to the level of the base year Edgeworth constant dollar totals obtained from total gas sales (column 1) for which all the necessary price data were available, as illustrated in columns 7 and 8. The adjusted Edgeworth totals for gas sales were then added to those for byproduct coke and tar (column 11). In this way we are able to weight the various components of gas sales (albeit by means of constant 1929 prices) and still use the information on the changing price weights for the coke and tar byproducts.

The fixed 1929 gas prices are from Table 25.

* The computations were carried out in terms of the following units: gas, th. cu. ft; byproduct coke, th. sh. t; byproduct tar, gal.

Appendix Table A12

Manufactured Gas, Census Data, 1899–1929

Adjustment of Weighted Output Index (1929:100) for Changing Coverage (millions of dollars)

		1899	1904	1909	1914	1919	1921	1923	1925	1927	1929
1	Value of products, entire industry	74	121	160	209	312	399	426	428	487	481
2	Value of gas purchased within industry, corrected for distribution losses					15	15	17	10	9	3
3	Value of products, excl. duplication (1 – 2)					296	384	409	418	478	478
4	Value of products directly represented in index	69		139							
			120	146	187	305	395	421	423	480	473
5	Value of products in index, excl. duplication (4 – 2)					289	380	404	413	471	471
6	Coverage ratios										
	Line 4 ÷ line 1	.942		.868							
	Line 4 ÷ line 1		.992	.917	.894	.977					
	Line 5 ÷ line 3					.976	.990	.987	.987	.985	.984
7	Coverage adj. factor, 1929:100	101.0	100.6	93.1	90.7	99.1	100.6	100.2	100.3	100.1	100.0
8	Weighted index, unadj.	15.7	26.8	35.3	47.9	70.6	69.6	79.7	83.8	97.4	100.0
9	Weighted index, adj. (8 ÷ 7)	15.6	26.6	38.0	52.8	71.2	69.2	79.5	83.6	97.3	100.0

The adjustment of the weighted output index obtained in Appendix Table A11 is designed to account for the output of products of the manufactured gas industry not directly represented in the index — mainly byproducts other than coke and tar (ammonia, oil derivatives, carbon, carbon black, fuel briquettes, etc.). In 1929 they account for less than 2 percent of the total value (excl. duplication).

The value data exclude receipts from rents and sales of lamps and appliances. The method of adjustment, developed by F. C. Mills and Solomon Fabricant, is described in detail in the latter's *Output of Manufacturing Industries, 1899–1937*, pp. 362–9. The coverage ratios (line 6) expressed as an index (line 7) are divided into the unadjusted weighted output index to yield the final index (line 9 and Table 24).

APPENDIX TABLE A13
Manufactured Gas, 1899–1929
Annual Interpolation, Weighted Output Index (1929:100)

	WEIGHTED INDEX (1)	GAS SALES (bil. cu. ft.) AGA (2)	Census (3)	INTERPOLATED RATIOS (1 ÷ 2) (4)	(1 ÷ 3) (5)	INTERPOLATED WEIGHTED INDEX (6)
1899	15.6					15.6
1901		101.6		.234		23.7
1902		92.7		.234		21.6
1903		105.7		.234		24.7
1904	26.6	113.9		.234		26.6
1905		112.4		.240		27.0
1906		122.8		.246		30.2
1907		132.0		.253		33.3
1908		138.6		.259		35.9
1909	38.0	143.1		.265		38.0
1910		149.4		.265		39.6
1911		159.1		.265		42.2
1912		178.2		.265		47.3
1913		188.3		.266		50.0
1914	52.8	198.8		.266		52.8
1915		204.3		.260		53.1
1916		231.4		.254		58.8
1917		264.5		.249		65.8
1918		271.6		.243		66.0
1919	71.2	300.0	280.9	.238	.254	71.2
1920			285.1		.251	71.6
1921	69.2		278.0		.249	69.2
1922			295.7		.248	73.2
1923	79.5		322.9		.246	79.5
1924			333.0		.246	82.0
1925	83.6		339.3		.246	83.6
1926			373.9		.247	92.2
1927	97.3		394.1		.247	97.3
1928			396.9		.247	98.2
1929	100.0		403.5		.248	100.0

The weighted output index (column 1) is the final adjusted index in Appendix Table A12.

The data in column 2, used to interpolate for 1899–1919, are from American Gas Association *Statistical Bulletin 8*, October 1930, p. 30.

The interpolating series for 1919–29 (column 3) is the annual Census gas sales series in Appendix Table A10, based on AGA sales totals. The ratios between the weighted output index and the AGA gas sales series (column 4) were interpolated by a straight-line distribution of the quinquennial differences.

For 1901–03 the ratio obtaining in 1904 were held constant. The ratios between the weighted output index and the Census gas sales series (column 5) were interpolated by a straight-line distribution of the differences between the odd-numbered years. The interpolated index for 1899–1919 is the product of columns 2 and 4; for 1919–29 of columns 3 and 5.

TABLE A14
Manufactured Gas, 1899–1929
Annual Interpolation, Unweighted Output Index (1929:100)

	GAS SALES (bil. cu. ft.) Census (1)	AGA (2)	INTERPOLATED RATIOS (1 ÷ 2) (3)	INTERPOLATED CENSUS GAS SALES, (bil. cu. ft.) (4)	INTERPOLATED UNWEIGHTED INDEX (5)
1899	67.1			67.1	16.4
1901		101.6	.988	100.4	24.6
1902		92.7	.988	91.6	22.4
1903		105.7	.988	104.4	25.6
1904	112.6	113.9	.988	112.6	27.6
1905		112.4	1.001	112.6	27.6
1906		122.8	1.014	124.6	30.5
1907		132.0	1.028	135.6	33.2
1908		138.6	1.041	144.2	35.3
1909	150.8	143.1	1.054	150.8	36.9
1910		149.4	1.048	156.6	38.3
1911		159.1	1.042	165.8	40.6
1912		178.2	1.036	184.7	45.2
1913		188.3	1.030	194.0	47.5
1914	203.6	198.8	1.024	203.6	49.9
1915		204.3	1.025	209.4	51.3
1916		231.4	1.026	237.3	58.1
1917		264.5	1.027	271.5	66.5
1918		271.6	1.027	279.0	68.3
1919	308.4	300.0	1.028	308.4	75.5
1920		319.9	.982	314.2	76.9
1921	306.1	327.0	.936	306.1	74.9
1922		350.0	.931	326.0	79.8
1923	356.6	384.7	.927	356.6	87.3
1924		405.2	.890	360.6	88.3
1925	359.4	421.4	.853	359.4	88.0
1926		455.6	.864	393.7	96.4
1927	412.2	471.0	.875	412.2	100.9
1928		495.0	.827	409.5	100.3
1929	408.4	524.1	.779	408.4	100.0

Column 1, from Table 24, includes gas purchased within the industry.

Column 2, used to interpolate the Census gas sales totals, is from American Gas Association *Statistical Bulletin 8*, October 1930, p. 30. The interpolating method follows that of Appendix Table A13.

Appendix Table A15
Manufactured Gas, Census Data, 1929–1935
Sales to Domestic and Other Consumer Groups

	1929	*1931* [a]	*1931* [b]	*1933*	*1935*
Total gas sales, bil. cu. ft.[c]	404	368	366	306	312
Total gas revenue, $ mil.[c]	436	397	394	322	314
Av. price, $ per th. cu. ft.	1.08	1.08	1.08	1.05	1.01
Domestic & househeating					
Bil. cu. ft.	299	275	274	227	219
Revenue, $ mil.	345	320	317	259	248
$ per th. cu. ft.	1.15	1.16	1.16	1.14	1.13
Industrial					
Bil. cu. ft.	44	34	34	28	38
Revenue, $ mil.	36	28	28	20	25
$ per th. cu. ft.	.81	.82	.82	.69	.66
Commercial					
Bil. cu. ft.	48	44	43	35	32
Revenue, $ mil.	48	42	41	36	30
$ per th. cu. ft.	1.00	.96	.95	1.01	.94
Government agencies [c]					
Bil. cu. ft.	13	15	15	16	23
Revenue, $ mil.	8	7	7	8	11
$ per th. cu. ft.	.61	.48	.48	.51	.50
Byproducts					
Coke, mil. sh. t.	3.44	3.60		3.18	3.33
Revenue, $ mil.	27	27		21	23
$ per sh. t.	7.80	7.41		6.50	6.99
Tar, mil. gal.	164	150		114	118
Revenue, $ mil.	8	7		4	5
¢ per gal.	5.0	4.4		4.0	4.0
Output Indexes (1929:100)					
Unweighted	100.0	91.3		76.1	77.4
Weighted	100.0	92.0		76.5	76.6

The Census of Manufactures data on gas manufacturing are presented for comparison with the American Gas Association data in Table 25. The output indexes, based on Census data for 1929, 1931, 1933, and 1935, were computed by the Edgeworth formula.

[a] Comparable with 1929.

[b] Comparable with 1933.

[c] Excludes the duplicating quantities and value of gas purchased within the industry, as estimated in Appendix Table A9.

APPENDIX TABLE A16

Manufactured Gas, American Gas Association Data, 1929–1942
Construction of Weighted Output Index (1929:100)

YEARS COMPARED	DOMESTIC SALES * $p_0 + p_1$ (1)	DOMESTIC SALES * $q(p_0 + p_1)$ (2)	HOUSEHEATING * $p_0 + p_1$ (3)	HOUSEHEATING * $q(p_0 + p_1)$ (4)	INDUSTRIAL-COMMERCIAL * $p_0 + p_1$ (5)	INDUSTRIAL-COMMERCIAL * $q(p_0 + p_1)$ (6)	MISCELLANEOUS * $p_0 + p_1$ (7)	MISCELLANEOUS * $q(p_0 + p_1)$ (8)	SUM (2 + 4 + 6 + 8) (9)	RATIO $\frac{\Sigma q_1(p_0 + p_1)}{\Sigma q_0(p_0 + p_1)}$ (10)	CHAIN INDEX (11)	CORRECTION (12)	FINAL INDEX (13)
1929	2.42	680	1.68	24	1.75	181	1.68	4	889	1.005	100.0		100.0
1930		684		31		174		4	893		100.5	.2	100.6
1930	2.42	684	1.64	30	1.77	177	1.59	4	895	.972			
1931		670		33		164		3	870		97.7	.3	98.0
1931	2.45	678	1.58	31	1.83	169	1.51	3	881	.923			
1932		644		31		135		3	813		90.2	.5	90.6
1932	2.46	648	1.47	29	1.80	133	1.48	3	813	.941			
1933		599		30		133		3	765		84.8	.6	85.4
1933	2.46	599	1.37	28	1.65	122	1.46	3	752	1.003			
1934		574		39		139		3	755		85.1	.8	85.9
1934	2.49	580	1.31	38	1.51	126	1.42	3	747	.999			
1935		555		46		142		3	746		85.0	.9	85.9
1935	2.56	570	1.28	45	1.42	134	1.39	3	752	.941			
1936		507		53		144		3	708		80.0	1.1	81.0
1936	2.62	520	1.26	52	1.38	140	1.38	3	714	1.008			
1937		512		58		148		3	721		80.6	1.2	81.9
1937	2.63	513	1.27	58	1.37	147	1.38	3	721	.999			
1938		517		61		139		3	720		80.5	1.4	81.9
1938	2.63	518	1.29	62	1.34	136	1.34	3	719	.019			
1939		509		72		149		3	732		82.0	1.6	83.6
1939	2.62	506	1.27	71	1.29	143	1.28	3	723	1.058			
1940		520		87		155		3	765		86.8	1.7	88.5
1929	2.51	706	1.48	21	1.50	155	1.49	3	885	.885			
1940		499		101		180		3	784				88.5
1940	2.61	518	1.28	88	1.24	149	1.23	3	758	1.027			88.9
1941		518		86		170		3	778				90.5
1940	2.60	518	1.28	88	1.23	147	1.20	3	755	1.094			88.5
1942		533		101		188		4	826				96.8

The computations are those required by the application of the Edgeworth index to the data in Table 25. For 1929–40 the index is computed by comparing pairs of successive years. The resulting chain index is corrected for the bias to which chain indexes are subject, by distributing evenly over the period the divergence between the direct 1929–40 comparison and the 1929–40 chain index (column 12). As the data for 1941 and 1942 appeared after these computations had been performed, the indexes for these two years were derived by direct comparison with 1940.

* The price weights, $p_0 + p_1$, are in terms of \$ per th. cu. ft.; and $q(p_0 + p_1)$, the Edgeworth constant \$ totals, are in millions.

Appendix Table A17
Manufactured and Natural Gas, 1899–1929
Annual Interpolation of Value of Product

	MANUFACTURED GAS						NATURAL GAS
	Value of product Census $ mil.	Gas sales AGA bil. cu. ft.	Revenue from gas sales AGA $ mil.	Interpolated ratios (1 ÷ 2)	Interpolated ratios (1 ÷ 3)	Interpolated value of product $ mil.	Value of product $ mil.
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1899	74	68		1.085		74	20
1900							
1901		102		1.076		109	27
1902		93		1.071		99	31
1903		106		1.066		113	36
1904	121	114		1.061		121	38
1905		112		1.072		121	42
1906		123		1.083		133	47
1907		132		1.094		144	54
1908		139		1.105		153	55
1909	160	143		1.116		160	63
1910		149		1.104		165	71
1911		159		1.091		174	75
1912		178		1.078		192	85
1913		188		1.065		201	88
1914	209	199		1.052		209	94
1915		204		1.050		214	101
1916		231		1.047		242	120
1917		264		1.045		276	142
1918		272		1.042		283	154
1919	{312 / 296}	300	285	1.040	1.040	{312 / 296}	{161 / 143}
1920			333		1.026	342	171
1921	384		379		1.012	384	151
1922			394		.989	390	192
1923	409		423		.967	409	209
1924			438		.946	414	222
1925	418		452		.925	418	230
1926			486		.940	457	260
1927	478		501		.955	478	275
1928			518		.926	480	301
1929	478		533		.897	478	340

Column 1 from Appendix Table A12. For the years before 1919 the data are for total value of product including duplications; after 1919 they exclude duplications and an overlap is provided in that year.

The interpolating series (column 2) is from American Gas Association *Statistical Bulletin 8*, p. 30, except for the 1899 sales figure, which is an estimate based on the 67.8 percent increase registered by total gas sales as reported to the Census 1899–1904 (see Table 24). The method of interpolation is that used in preceding tables, i.e., a straight-line distribution of the quinquennial and biennial differences in the interpolating ratios.

Column 7, from Table 29 and successive issues of *Mineral Resources*, are for total sales 1899–1919, including all industrial consumption, but for 1919–29 exclude revenue from sales for field use and carbon black manufacture.

APPENDIX TABLE A18

Manufactured and Natural Gas, 1929–1940

Construction of Combined Weighted Output Index (1929:100)

Value in millions of dollars

		1930 g	1929 b	1931 g	1930 b	1932 g	1931 b	1933 g	1932 b	1934 g	1933 b
1	Mfd. gas index	100.6	100.0	98.0	100.6	90.6	98.0	85.4	90.6	85.9	85.4
2	Natural gas index	102.2	100.0	99.9	102.3	94.9	99.9	92.1	94.9	100.0	92.1
3	Q: Output index ratios, given to base year										
4	Mfd. gas	1.006		.974		.925		.943		1.005	
5	Natural gas	1.022		.977		.949		.971		1.086	
6	V: Value, mfd. gas	447	444	435	447	411	435	378	411	375	378
7	V: Value, natural gas	353	342	335	353	315	335	302	315	328	302
8	QV_0, mfd. gas	447		435		403		388		380	
9	QV_0, natural gas	350		345		318		306		328	
10	V_1/Q, mfd. gas		444		447		445		401		373
11	V_1/Q, natural gas		345		343		332		311		302
12	$\Sigma(V_1 + QV_0)$: (5 + 6 + 7 + 8)	1,597		1,550		1,447		1,373		1,410	
13	$\Sigma(V_0 + V_1/Q)$: (5 + 6 + 9 + 10)		1,576		1,590		1,547		1,438		1,355
14	Ratio: (12 ÷ 13)	1.013		.975		.936		.955		1.041	
15	Chain index	101.3		98.8		92.4		88.3		91.9	
16	Correction	+.03		+.06		+.08		+.11		+.14	
17	Final index (12 ÷ 13)	101.4	100.0	98.9		92.5		88.4		92.0	

		1935 g	1934 b	1936 g	1935 b	1937 g	1936 b	1938 g	1937 b	1939 g	1938 b	1940 g	1939 b	1940 g	1929 b
1	Mfd. gas index	85.9	85.9	81.0	85.9	81.9	81.0	81.9	81.9	83.6	81.9	88.5	83.6	88.5	100.0
2	Natural gas index	107.9	100.0	124.8	107.9	132.1	124.8	126.0	132.1	135.8	126.0	150.0	135.8	150.0	100.0
3	Q: Output index ratios, given to base year														
4	Mfd. gas	1.001		.943		1.010		1.001		1.020		1.059		.885	
5	Natural gas	1.080		1.156		1.058		.954		1.078		1.105		1.500	
6	V: Value, mfd. gas	372	375	358	372	360	358	360	360	365	360	379	365	379	444
7	V: Value, natural gas	356	328	413	356	441	413	417	441	449	417	493	449	493	342
8	QV_0, mfd. gas	375		350		361		360		368		387		393	
9	QV_0, natural gas	354		411		437		421		449		496		514	
10	V_1/Q, mfd. gas		371		379		356		360		358		358		428
11	V_1/Q, natural gas		329		357		417		437		417		446		328
12	$\Sigma(V_1 + QV_0)$: (5 + 6 + 7 + 8)	1,456		1,532		1,599		1,558		1,631		1,754		1,779	
13	$\Sigma(V_0 + V_1/Q)$: (5 + 6 + 9 + 10)		1,403		1,463		1,544		1,598		1,552		1,618		1,543
14	Ratio: (12 ÷ 13)	1.038		1.047		1.036		.975		1.051		1.084		1,153	
15	Chain index	95.4		99.8		103.4		100.9		106.0		115.0			
16	Correction	+.17		+.20		+.22		+.25		+.28		+.31			
17	Final index (12 ÷ 13)	95.5		100.0		103.7		101.1		106.3		115.3		115.3	100.0

How the weighted manufactured and natural gas indexes were combined is shown for 1929–40 in a sample worksheet; the computations for 1899–1909, 1909–19, and 1919–29 were made in exactly the same manner. As data for 1941 and 1942 appeared after the basic computations had been made, the indexes for these two years were based on 1940. The method of combination follows the formula $\frac{\Sigma(V_1 + QV_0)}{(V_0 + V_1/Q)}$, where V_0 and V_1 are the total value of products for each industry in the base (b) and given (g) years respectively, and Q is the ratio of the given year quantity index to the base year quantity index for each industry. This formula, developed by Solomon Fabricant, yields results identical with those obtained by the usual form of the Edgeworth formula, and facilitates the combining of indexes when the latter are based on link-periods that are not identical for the two indexes (see Fabricant, *op. cit.*, p. 370).

Lines 1 and 2 are from Table 30; lines 6 and 7 from Appendix Table A17.

APPENDIX B

Electric and Gas Utilities
Construction of Indexes of Fuel Input

FOR INDUSTRIES consuming various types of fuel, indexes of fuel input may be constructed precisely as were the indexes of output in Appendix A, i.e., by an index number formula combining the various fuels weighted by prices. Such price-weighted indexes of fuel input reflect changes in the consumption of high- and low-valued fuels. Here we have used another system of weighting, better suited for our present purpose, i.e., to measure gains in the efficiency of fuel consumption. Each fuel was weighted by its bituminous equivalent or its heat value in British thermal units. In the case of manufactured gas, both methods were used and two indexes of fuel input computed.

The notes to Appendix Tables B1–8 are self-explanatory. All computations were carried to five figure accuracy, though many of the figures shown are rounded.

Appendix Table B1

Electric Light and Power, 1902–1942

Computation of Fuel Consumption, Bituminous Equivalent (mil. sh. t.)

		1902	*1907*	*1912*	*1917*	*1922*	*1927*	*1932*	*1937* [a]	*1937* [a]	*1942* [a]
1	Anthracite				2.44	1.84	2.00	1.56	2.10	1.90	2.79
2	Bituminous				19.38	24.49	35.68	26.00	40.47	42.87	65.64
3	Coke				.06	.03	.03	.13	.09		
4	Fuel oil				1.54	2.96	1.79	1.97	3.18	3.34 [b]	3.66
5	Gas, mfd.				.62	.88	.14	.04	.11	7.04	9.37
6	Gas, natural						2.58	4.19	6.54		
7	Total	6.41	12.34	17.90	24.04	30.20	42.22	33.89	52.49	55.14	81.44
8	Cost of fuel, $ mil. [c]	11.6	23.1	34.9	87.3	162.6	164.2	112.4	181.7	...	...
9	Unit fuel cost, $ per sh. t. [c]	1.82	1.87	1.95	3.63	5.38	3.89	3.32	3.46	...	...
10	Fuel input index, 1927:100 (based on line 8)	15.2	29.2	42.4	56.9	71.5	100.0	80.3	124.3	124.3	183.6

The fuel consumption totals, expressed in terms of th. sh. t. of bituminous equivalent, were estimated separately for 1902–17, 1917–37, and 1937–42. For the mid-period, i.e., 1917–37, the computations were performed by the Bureau of the Census as follows: The fuel consumption totals in commercial units (Table 15) were converted into short tons of bituminous equivalent by the application of conversion ratios supplied by the Bureau of Mines: oil, 4 bbl. per sh. t.; natural gas, 23.0 th. cu. ft. per sh. t.; manufactured gas, 45.0 th. cu. ft. per sh. t.; coke, 1.15 tons per sh. t. In 1917 and 1922 the Bureau of the Census applied the natural gas conversion ratio to the combined gas totals reported for those years, possibly suggesting that natural gas constituted the bulk of the gas total, as in later years (*Census of Electric Light and Power, 1927*, p. 26.)

[a] For 1937 and 1942 the computations were made by the Federal Power Commission by a somewhat different method, described as follows: "In converting fuels other than coal to equivalent tons of coal, the anthracite, lignite and coke was first added to the bituminous coal consumption. This total, expressed in pounds, was then divided by the total production by all of three types of fuel to obtain what is called a 'coal rate'. It is important to remember that although this procedure is consistent with previous practice, the rate obtained is not on the basis of bituminous coal but on the basis of all coal consumed. Since the bituminous coal which is used has a higher Btu. content and thermal efficiency than the lignite, anthracite and coke, it follows that the pounds of coal required on the basis of bituminous coal will be lower than the coal rate obtained by this method. After obtaining the coal rate, the oil and gas consumed are converted to equivalent tons of coal on the basis of the energy produced by each as compared to that produced by all types of coal". *Electric Power Statistics, 1939* (Washington, D. C., 1940, p. 28).

By assuming that the 1937 ratio of kwh. generated per short ton of bituminous equivalent was identical for plants reporting to both the Census and the FPC it is possible to convert the Commission's fuel consumption totals to the 'bituminous equivalent' base used by the Census (see Table 15). This assumption yields a correction factor of .95691 which we may apply to the FPC fuel totals to get 52,766 th. and 77,935 th. respectively as the 1937 and 1942 bituminous equivalent totals.

Other differences between the FPC and Census methods of conversion arise from the different fuels and different areas of coverage involved, all of which contribute to the difference in the fuel consumption totals as estimated by the FPC and the Census in 1937. However, the movement of the fuel consumption totals is probably only moderately affected by the discontinuity in 1937.

[b] Includes negligible amounts of gasoline.

[c] The figures for 1902, 1907, and 1912 were derived by dividing the figures for fuel cost as reported by the Census (line 9) by estimates of unit fuel costs (line 10) as estimated in Appendix Table B2.

The Census cost data for 1917–37 are included here to complete the record, though they are not required for computing the fuel consumption index. Some slight adjustment of the 1927 Census cost figure was required, for in that year the Census reported a fuel cost total of $157.9 mil. for commercial plants only; by means of a correction factor of 1.040, based on the proportions of fuel-generated current reported by the two types of plant, the fuel costs of municipal plants in 1927 were estimated to be $6.3 million.

APPENDIX TABLE B2
Electric Light and Power, 1902–1917
Index of Fuel Costs (1917:100)

	1902	1907	1912	1917
Anthracite, $ per sh. t.	1.84	1.91	2.11	2.85
Constant $ totals, mil.	4.49	4.66	5.15	6.96
Bituminous, $ per sh. t.	1.12	1.14	1.15	2.26
Constant $ totals, mil.	21.71	22.10	22.29	43.81
Petroleum, other than Penn. grade, $ per bbl.	0.54	0.54	0.62	1.44
Constant $ totals, mil.	3.34	3.36	3.81	8.87
Natural gas, $ per th. cu. ft.	0.11	0.13	0.15	0.18
Constant $ totals, mil.	1.56	1.89	2.13	2.54
Total, mil. constant $	31.10	32.01	33.38	62.18
Unit fuel costs index	50.0	51.5	53.7	100.0
Estimated price, $ per sh. ton, bit. equiv.	1.82	1.87	1.95	3.63

The unit price data are from Barger and Schurr, *The Mining Industries, 1899–1939*, pp. 284–5.

An index of fuel costs for the electric light and power industry may be constructed by applying as weights to the unit fuel costs the quantities of the several fuels consumed by the industry in 1917, the earliest year for which such data are available. The quantity weights were: anthracite, 2,442 th. sh. t.; bituminous, 19,385 th. sh. t.; fuel oil, 6,158 th. bbl.; and natural gas, 14,199 mil. cu. ft. The constant dollar totals, the product of the quantity weights and the unit prices, were then added. The fuel cost index (1917:100), based on the aggregate constant dollar totals, was then used to extrapolate backwards the 1917 figure for cost per sh. t. of bituminous equivalent, as computed by the Census. The extrapolated unit cost series is used in Appendix Table B1 to estimate the quantities of bituminous equivalent consumed by the industry before 1917.

Appendix Table B3

Electric Light and Power, Fuel-Burning Equipment Only, 1902–1942

Annual Interpolation, Unweighted Indexes of Output, Fuel Input, and Output per Unit of Fuel (1929:100)

	Bil. kwh. generated by fuel: Census	Bil. kwh. generated by fuel: FPC	Interpolated ratios (1 ÷ 2)	Current generated by fuel (Census) interpolated (bil. kwh.)	Unweighted fuel-generated output index	Fuel consumption (mil. sh. tons of bit. equiv.): Census	Fuel consumption (mil. sh. tons of bit. equiv.): FPC	Interpolated ratios (6 ÷ 7)	Fuel consumption (Census) interpolated (mil. sh. tons of bit. equiv.)	Fuel input index	Kw. hr. generated per sh. ton of bit. equiv.	Unweighted output per unit of fuel index
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
1902	1.8			1.8	3.0	6.4			6.4	13.2	275	23.0
1907	3.4			3.4	5.9	12.3			12.3	25.5	279	23.3
1912	6.8			6.8	11.7	17.9			17.9	37.0	380	31.8
1919	14.1			14.1	24.3	24.0			24.0	49.6	586	49.0
1920		27.4	.823	22.6	38.9		46.2	.800	36.9	76.2	611	51.1
1921		26.0	.823	21.4	37.0		34.9	.800	27.9	57.9	767	64.1
1922	25.1	30.4	.823	25.1	43.2	30.2	37.8	.800	30.2	62.4	830	69.3
1923		36.3	.841	30.6	52.7		43.3	.823	35.6	73.6	857	71.6
1924		39.0	.859	33.6	57.9		42.7	.847	36.1	74.6	928	77.6
1925		43.5	.877	38.2	65.9		45.4	.870	39.5	81.7	965	80.7
1926		47.6	.895	42.6	73.6		46.1	.894	41.2	85.1	1034	86.4
1927	46.0	50.3	.913	46.0	79.3	42.2	46.0	.918	42.2	87.2	1089	91.0
1928		53.2	.918	48.8	84.3		46.5	.919	42.7	88.2	1144	95.6
1929		62.7	.924	58.0	100.0		52.6	.920	48.4	100.0	1197	100.0
1930		62.9	.929	58.4	100.9		50.6	.921	46.6	96.3	1253	104.7
1931		61.1	.934	57.1	98.6		47.1	.922	43.5	89.7	1315	109.9
1932	46.1	49.1	.940	46.1	79.5	33.9	36.7	.923	33.9	70.0	1360	113.7
1933		50.7	.946	48.0	82.8		37.2	.929	34.5	71.3	1389	116.1
1934		57.1	.953	54.4	93.9		41.8	.935	39.1	80.7	1392	116.3
1935		59.4	.960	57.1	98.5		43.2	.940	40.6	83.9	1404	117.4
1936		72.7	.967	70.3	121.2		52.0	.946	49.2	101.6	1428	119.4
1937	75.3	77.3	.974	75.3	130.0	52.5	55.1	.952	52.5	108.4	1435	119.9
1938		71.8	.974	70.0	120.7		50.6	.952	48.1	99.4	1454	121.5
1939		86.3	.947	84.1	145.0		59.5	.952	56.6	117.0	1484	124.0
1940		97.2	.974	94.7	163.4		65.1	.952	62.0	128.0	1527	127.6
1941		116.9	.974	113.8	196.4		77.8	.952	74.1	153.0	1537	128.4
1942		125.0	.947	121.7	210.0		81.4	.952	77.5	160.1	1570	131.2

The indexes cover the industry as defined by the Census of Electric Light and Power, i.e., exclude electric light and power departments of electric railway companies.

The interpolation of Census data is based on data for companies reporting to the FPC. The relation of the FPC data to those of the Census, and the interpolating methods are discussed in notes to Appendix Table A6.

Columns 1 and 6 are from Tables 15 and 16; columns 2 and 7, from *Electric Power Statistics, 1940* and *Production of Electric Energy and Capacity of Generating Plants, 1942*, both published by the Federal Power Commission.

Appendix Table B4

Manufactured Gas, Census Data, 1909–1935

Estimation of Unit Fuel Costs * by means of Bureau of Labor Statistics and other Price Indexes (1926:100)

		1909	1914	1919	1921	1923	1925	1927	1929	1931	1933	1935
Anthracite												
1	Unit cost	6.18	6.81	*8.66*	9.91	10.06	9.23	8.21	*7.04*	7.11	6.42	6.22
2	BLS price index	54.1	59.6	75.8	92.5	100.8	99.7	96.3	90.1	91.1	82.2	79.7
3	Interpolated ratios $\frac{(1)}{(2)} \times 100$	.114	.114	.114	.107	.100	.093	.085	.078	.078	.078	.078
Bituminous												
4	Unit cost	2.21	2.28	*5.23*	4.92	6.91	5.65	5.64	*4.92*	4.56	4.46	5.21
5	BLS price index	33.7	34.8	79.8	77.7	113.4	96.5	100.3	91.3	84.6	82.8	96.7
6	Interpolated ratios $\frac{(4)}{(5)} \times 100$	.066	.066	.066	.063	.061	.059	.056	.054	.054	.054	.054
Coke and Breeze, Purchased												
7	Unit cost	*4.51*	*4.66*	*8.82*	10.03	11.01	9.15	8.63	*7.44*	6.17	5.82	6.15
8	Sales price of gashouse coke			7.25	8.62	9.90	8.64	8.57	7.80			
9	Interpolated ratios 7 ÷ 8			1.217	1.164	1.112	1.059	1.007	.954			
Oil, Purchased												
10	Unit cost	*1.24*	*1.45*	*2.42*	1.82	2.22	2.44	2.12	*1.77*	1.00	1.22	1.62
11	BLS price index	61.6	61.6	83.4	63.7	78.9	88.2	77.8	65.6	37.2	45.5	60.1
12	Interpolated ratios $\frac{(10)}{(11)} \times 100$	.020	.024	.029	.029	.028	.028	.027	.027	.027	.027	.027
Natural Gas												
13	Unit cost			.151	.178	.188	.185	.179	*.161*	.168	.175	.174
14	Bureau of Mines cost to commercial consumer			.346	.444	.514	.560	.608	.620		.498	.493
									.457	.478		
15	Interpolated ratios (13 ÷ 14)			.436	.401	.365	.330	.294	.259		.352	.352
									.352	.352		
Coke-oven Gas												
16	Sales, bil. cu. ft.				37.7	58.3	66.5	82.5		108.7	87.0	102.3
17	Value, $ mil.				9.2	17.1	18.8	22.8		28.3	22.6	26.3
18	Unit cost			*.158*	.244	.294	.282	.277	*.272*	.260	.260	.257

Notes to Appendix Table B4

*The price units are; Anthracite, $ per l. t; Bituminous, $ per sh. t; Coke and Breeze, $ per sh. t; Oil, $ per bbl; Natural Gas, $ per th. cu. ft; Coke-oven Gas, $ per th. cu. ft.

The unit cost data for fuels purchased by the manufactured gas industry were reported to the Census in 1919, and in some cases for prior years, and in 1929 (*Census of Manufactures, 1919*, V, 712; *1929*, I, 158, 165). Such Census data are italicized above. The data for remaining Census years were interpolated, with the help of related series, by straight line distribution of the differences between the interpolating ratios, as follows:

Anthracite

The Bureau of Labor Statistics composite wholesale price index for anthracite (line 2) 1914–25 from the BLS Bulletin, *Wholesale Prices, 1930*, pp. 19, 20, and for 1927–35 from *Wholesale Prices, 1935*, p. 9. The 1909 figure was extrapolated by the BLS wholesale price index for anthracite, Chestnut, N. Y. (*Wholesale Prices, 1925*, p. 128) which rose from 90.7 in 1909 to 100.0 in 1914. The ratios (line 3) between Census prices and the BLS composite index were interpolated by a straight line distribution of the decennial differences. For 1909 and 1914 the 1919 ratio was held constant, and for the years after 1929 the 1929 ratio was held constant. The interpolated Census prices in line 1 (not italicized) are the products of lines 2 and 3.

Bituminous

The BLS composite wholesale price index for bituminous coal (line 5) 1914–25 from *Wholesale Prices, 1930*, p. 119; for 1927–35, from *Wholesale Prices, 1935*, p. 9. The composite index for 1909 was extrapolated by means of the BLS wholesale price index for bituminous, Pittsburgh at Cincinnati, (*Wholesale Prices, 1925*, p. 132) which rose from 96.8 in 1909 to 100.0 in 1914. The interpolation process is that indicated for anthracite.

Coke and Breeze

The figures for 1931, 1933, and 1935, from successive issues of the *Minerals Yearbook*, are the average cost of byproduct coke to manufacturers of water gas. For 1921–27 unit coke costs were interpolated by means of a series on the sales price of gas-house coke (line 8) (*Census of Manufactures* surveys of the coke industry), which follow closely the unit costs of coke to the manufactured gas industry.

Oil, Purchased

The interpolating series is the one based on the mean of Oklahoma and Pennsylvania prices for fuel oil at the refinery. The 1909 index was assumed to be the same as the 1914 inasmuch as there were no data on fuel oil in 1909 and the prices of other products in this group remained unchanged (BLS *Wholesale Price* bulletins for 1925 and following years).

Natural Gas

Two interpolating series were used: for 1919–29 line 14 shows the unit costs of natural gas to domestic and commercial consumers; for 1929–35 the unit costs of natural gas to commercial consumers only (see Tables 28 and 29).

Coke-oven Gas

Data for 1921–27 and 1931–35 are for gas sales by byproduct coke-oven plants for ultimate distribution through gas mains, excluding the operations of municipally-owned plants. Collected by the Bureau of Mines and reported in successive issues of the *Minerals Yearbook*, they agree closely with the corresponding Census unit costs.

APPENDIX TABLE B5
Manufactured Gas, Census Data, 1899–1935
Construction of Btu.-Weighted Index of Output (1929:100)

	GAS SALES (bil. cu. ft.)			BTU. EQUIVALENT (trillions)			BTU. OUTPUT INDEX
	Mfd.	Natural	Total	Mfd. gas (1) × 575	Natural gas (2) × 1075	Total (4 + 5)	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1899			67.1			38.7	14.9
1904	112.0	0.5	112.6	64.4	0.6	65.0	25.0
1909	147.6	3.3	150.8	84.8	3.5	88.4	34.0
1914	198.1	5.5	203.6	113.9	6.0	119.9	46.1
1919	{295.3 / 267.8}	13.1	{308.4 / 280.9}	{169.8 / 154.0}	14.1	{183.9 / 168.1}	70.7
1921	277.7	0.3	278.0	159.7	0.3	160.0	67.3
1923	309.4	13.6	322.9	177.9	14.6	192.5	81.0
1925	338.1	1.2	339.3	194.4	1.3	195.7	82.3
1927	392.1	1.9	394.1	225.5	2.1	227.5	95.8
1929	392.2	11.3	403.5	225.5	12.2	237.6	100.0
1931[a]	358.0	10.2	368.2	205.9	10.9	216.8	91.2
1931[b]	356.1	10.2	366.3	204.8	10.9	215.7	
1933	276.8	28.7	305.5	159.2	30.9	190.0	80.4
1935	262.2	50.0	312.2	150.8	53.8	204.5	86.1

Columns 1–3, based on Tables 24 and 31, App. Table A15, exclude gas purchased within the industry after 1919. As data for manufactured gas sales prior to 1919 include such duplicating sales, an overlap is provided in 1919. Total gas sales were separated into the manufactured and natural gas components to allow each the appropriate Btu. weight, as determined by the Bureau of Mines: 575 Btu. per cu. ft. for manufactured gas and 1075 Btu. per cu. ft. for natural gas. The components of total gas sales were separated by assuming that all natural gas purchased by the industry (Table 31) reached ultimate consumers without distribution losses. Before 1919 natural gas sales were estimated by extrapolating the 1919 figure by means of data on total gas purchased by the industry (Table 31), on the assumption that the relative proportions of natural and manufactured gas purchased by the industry remained unchanged before 1919. These assumptions are arbitrary but are believed to yield results adequate for the purpose — to translate gas sales into their Btu. equivalent (columns 4–6).

The 1899 figure for the total Btu. equivalent was obtained by applying the 1899–1904 percentage increase indicated by total gas sales to the 1904 Btu. equivalent total.

[a] Comparable with 1929. [b] Comparable with 1933.

Appendix Table B6

Manufactured Gas, Census Data, 1899–1935: Construction of Btu.-Weighted Index of Fuel Input (1929:100)

		1899	1904	1909	1914	1919	1921	1923	1925	1927	1929	1931 [a]	1931 [b]	1933	1935
	FUELS CONSUMED														
	Commercial Units														
1	Anthracite, mil. sh. t.	.51	.90	1.01	1.06	1.56	1.40	1.40	.92	.30	.20	.13	.12	.12	.12
2	Bit., mil. sh. t.	2.35	4.19	4.67	6.08	7.38	6.96	7.28	7.93	9.04	10.04	9.66	9.59	7.88	7.96
3	Coke & breeze, mil. sh. t.	0.28	0.56	0.76	1.08	1.50	1.62	1.88	2.09	2.15	1.46	1.09	1.07	.84	.92
4	Oil, mil. bbl.	4.6	9.8	13.8	17.0	21.0	20.6	22.2	22.6	22.9	22.0	16.0	10.6	12.7	12.6
5	Natural gas, bil. cu. ft.	...	.5	3.3	5.5	13.1	.3	13.6	1.2	1.9	11.3	10.2	10.2	28.7	50.0
6	Coke-oven gas, bil. cu. ft.	...	1.0	6.0	10.1	24.0	14.2	40.9	41.6	75.2	86.6	79.9	79.9	65.1	50.5
	Btu. Equivalent (trillions)														
7	Anthracite (1) × 27.2 mil.	13.8	24.6	27.4	28.9	42.3	38.1	38.0	25.1	8.1	5.4	3.5	3.4	3.3	3.2
8	Bit. (2) × 26.2 mil.	61.5	109.7	122.3	159.2	193.5	182.5	190.7	207.8	236.7	263.0	253.0	251.3	206.4	208.6
9	Coke & breeze (3) × 24.2 mil.	6.7	13.5	18.4	26.2	36.2	39.2	45.5	50.6	52.1	35.4	26.3	25.8	20.2	22.4
10	Oil, purchased (4) × 6 mil.	27.8	58.5	82.6	102.2	126.0	123.8	132.9	135.3	137.6	131.7	96.0	63.8	76.1	75.7
11	Natural gas (5)(1075)	...	.6	3.5	6.0	14.1	.3	14.6	1.3	2.1	12.2	10.9	10.9	30.9	53.8
12	Coke-oven gas (6) × 575	...	.6	3.4	5.8	13.8	8.1	23.5	23.9	43.2	49.8	45.9	45.9	37.5	29.0
13	Total	110.4	207.5	257.6	328.3	425.8	392.0	445.1	444.0	479.8	497.5	435.7	401.2	374.4	392.7
14	Btu. weighted fuel input index	22.2	41.7	51.8	66.0	85.6	78.8	89.5	89.3	96.5	100.0	87.6		81.7	78.9
15	Btu. output per th. Btu. fuel input	320.7	286.3	313.5	333.7	394.8	408.2	432.4	440.8	474.2	477.7	497.6		469.7	520.9
16	Btu. output per Btu. of input index	67.1	59.9	65.6	69.9	82.6	85.5	90.5	92.3	99.3	100.0	104.2		98.3	109.0

Lines 1–6 taken from, or based on, Table 31. Fuel data for early years were estimated as follows: For anthracite and bituminous coal, the 1899 and 1904 figures are extrapolations of the 1909 figures by the series for "all coal, excluding fuel for boilers and retorts" (Table 31). For coke and breeze, 1899 and 1904 figures are extrapolations of the 1909 figure by "coke and breeze, used for gas-making". The oil, purchased, figures for 1899 and 1904 are extrapolations of the 1909 figure by the series 'oil, incl. fuel oil made on premises'. The natural and coke-oven gas figures before 1919 are extrapolations of the 1919 figures by 'total gas purchased within and outside the industry'. As in Appendix Table B5, such extrapolations are believed to yield results adequate for the present purpose, i.e., to convert commercial units into Btu. equivalents (lines 7–12), with the use of the conversion factors established by the Bureau of Mines (see Ch. 4, note 1).

Line 13 is the sum of lines 7–12; the 1899 total was raised slightly to include the same proportion of gas consumed in 1904.

Line 15 is equal to col. 6 of Appendix Table B5 divided by line 13, multiplied by 1,000.

[a] Comparable with 1929.

[b] Comparable with 1933.

APPENDIX TABLE B7

Manufactured Gas, American Gas Association Data, 1929–1942

Construction of Indexes of Btu.-Weighted Output, Fuel Input, and Output per Unit of Fuel Input (1929:100)

		1929	*1930*	*1931*	*1932*	*1933*	*1934*	*1935*	*1936*	*1937*	*1938*	*1939*	*1940*	*1941*	*1942*
	Btu.-Weighted Output (*trillion Btu.*)														
1	Natural Gas	2	2	8	33	40	44	49	61	54	51	54	63	71	79
2	Mfd. Gas	230	230	221	189	174	176	177	165	173	173	180	190	195	211
3	Total	232	233	228	222	214	220	227	226	227	224	233	253	266	290
4	Index	100	101	99	96	92	95	98	98	98	97	101	110	115	125
	Btu.-Weighted Fuel Input (*trillion Btu.*)														
5	Anthracite	7	6	4	3	4	5	3	4	5	5	5	6	7	8
6	Bit.	254	245	246	212	205	219	206	199	192	176	174	180	186	202
7	Coke & breeze	95	92	85	70	62	63	63	61	58	57	59	63	63	68
8	Oil	104	105	97	86	75	75	73	74	75	84	90	97	99	107
9	Natural gas	2	2	8	33	40	44	49	61	54	51	54	63	71	79
10	All other gas	67	69	64	54	52	54	54	49	53	53	54	56	57	60
11	Total	529	519	504	459	438	460	448	448	438	425	436	465	483	524
12	Index	100	98	95	87	83	87	85	85	83	80	82	88	91	99
13	Btu. output per th. Btu. fuel input $\frac{(3)}{(11)} \times 1000$	437	449	453	483	488	479	506	504	518	527	536	545	551	553
14	Output per Btu. of input index (based on line 13)	100	103	104	110	112	110	116	115	118	120	122	125	126	126

Lines 1, 2, and 5–10 were obtained by application of the Bureau of Mines' conversion ratios to the data in Table 33 (see App. Tables B5 and 6).

Appendix Table B8

Manufactured Gas, 1919–1929: Construction of Price Weighted Index of Fuel Input (1929:100)

	ANTHRACITE	BITUMINOUS	COKE AND BREEZE PURCHASED	OIL PURCHASED	NATURAL GAS	COKE-OVEN GAS	TOTAL	RATIO $\frac{\Sigma q_1(p_0+p_1)}{\Sigma q_0(p_0+p_1)}$	CHAIN INDEX	CORRECTION	FINAL INDEX
1919 based on 1929											
p_0+p_1,* $ per unit	15.70	10.15	16.26	4.19	0.31	0.43					
			EDGEWORTH CONSTANT $ TOTALS, MILLIONS								
1919: $q_1(p_0+p_1)$	21.8	74.9	24.3	88.0	4.1	10.3	223.4	.8554			185.5
1929: $q_0(p_0+p_1)$	2.8	101.9	23.8	92.0	3.5	37.2	261.2				00.0
1919 based on 1921											
p_0+p_1,* $ per unit	18.57	10.15	18.85	4.24	0.33	0.40					
			EDGEWORTH CONSTANT $ TOTALS, MILLIONS								
1919: $q_1(p_0+p_1)$	25.8	74.9	28.2	89.0	4.3	9.6	231.9	1.0653	83.2	+2.4	85.5
1921: $q_0(p_0+p_1)$	23.2	70.7	30.5	87.5	0.1	5.7	217.7		78.1		
1921 based on 1923											
p_0+p_1,* $ per unit	19.97	11.83	21.04	4.04	0.37	0.54					
			EDGEWORTH CONSTANT $ TOTALS, MILLIONS								
1921: $q_1(p_0+p_1)$	24.9	82.4	34.1	83.3	0.1	7.6	232.5	.8709	78.1	+1.9	80.0
1923: $q_0(p_0+p_1)$	24.9	86.1	39.5	89.5	5.0	22.0	267.0		89.7		
1923 based on 1925											
p_0+p_1,* $ per unit	19.29	12.56	20.16	4.66	0.37	0.58					
			EDGEWORTH CONSTANT $ TOTALS, MILLIONS								
1923: $q_1(p_0+p_1)$	24.0	91.4	37.9	103.2	5.1	23.6	285.2	.9931	89.7	+1.4	91.1
1925: $q_0(p_0+p_1)$	15.9	99.6	42.2	105.1	0.5	24.0	287.1		90.3		
1925 based on 1927											
p_0+p_1,* $ per unit	17.44	11.29	17.78	4.56	0.36	0.56					
			EDGEWORTH CONSTANT $ TOTALS, MILLIONS								
1925: $q_1(p_0+p_1)$	14.3	89.5	37.2	102.9	0.4	23.3	267.6	.9158	90.3	+.9	91.2
1927: $q_0(p_0+p_1)$	4.6	102.0	38.3	104.6	0.7	42.0	292.2		98.6		
1927 based on 1929											
p_0+p_1,* $ per unit	15.25	10.56	16.07	3.89	0.34	0.55					
			EDGEWORTH CONSTANT $ TOTALS, MILLIONS								
1927: $q_1(p_0+p_1)$	4.0	95.4	34.6	89.2	0.7	41.3	265.2	.9858	98.6	+.5	99.1
1929: $q_0(p_0+p_1)$	2.7	106.0	23.5	85.4	3.8	47.6	269.0		100.0		100.0

* The units are: Anthracite, l. t; Bituminous, sh. ton; Coke and breeze, sh. ton; Oil, bbl; Natural gas, th. cu. ft; Coke-oven gas, th. cu. ft. The figures were obtained by applying the Edgeworth formula to data in Table 31. First, a direct comparison for 1919–29 was constructed; then a chain index for 1919–29 computed. The difference between the 1919 chain and direct comparison indexes was then distributed evenly over the 10 years. Similar computations were made for 1899–1909, 1909–19, and 1929–35, and the resulting indexes spliced to yield the price-weighted index of fuel input in Table 32.

APPENDIX C

Electric and Gas Utilities
Construction of Indexes of Net Output

IN CHAPTERS 2 AND 4 we introduced indexes of fuel input in order to illustrate certain aspects of the growing fuel efficiency achieved in producing electricity and gas. The detailed steps in their construction are described in the tables of Appendix B. In this Appendix the fuel input indexes are used in conjunction with the *gross* output indexes, presented in Appendix A, to yield indexes of *net* output.

Indexes of net output are designed to measure that portion of the value of the product added by the industry in the process of manufacture and distribution. Solomon Fabricant defines the *value* of the net output of an industry as the "aggregate value of goods produced (i.e., the value of gross product) less the value of all commodities and services purchased from other business enterprises and consumed in the production process, including periodic allowance for depreciation and depletion and provision for losses by accident." [1]

An index of the net physical output of an industry, following this definition, should take account not only of changes in the physical volume of the industry's final product (corrected for price changes), but also in the quantities of commodities and services contributed by other industries and consumed in the production process. Such an index, when combined with the net output indexes of other industries that exchange goods and services, would yield a measure of aggregate output for the group that would be free from duplication.[2] For instance, if the electric light and

[1] *Output of Manufacturing Industries, 1899–1937* (National Bureau of Economic Research, 1940), p. 25. For a more thorough discussion of the distinction between gross and net output and the conditions under which indexes of net output are preferable to gross output indexes, see pp. 23–33.

[2] Theoretically, the combined net outputs of *all* industries would constitute an estimate of real national income constructed independently of deflated national income totals.

power gross output index were combined with an index of fuel production — the weights based upon the value of product in each industry — the combined output index would give double weight to the fuel consumed by the electric utility industry, for the value of the latter's product would include the industry's fuel costs. The use of value-added (rather than value of product) weights would lessen the effect of the duplication; nevertheless, gross output indexes, such as those appearing here and in preceding reports of this series, are for purposes of combination with other indexes, to be regarded as approximations to net output.[3] To the extent that net and gross output may have diverged in individual industries, group indexes constructed by assigning value-added weights to individual gross output indexes will fall short of depicting the true course of over-all net output. Because data on the cost and quantity of materials consumed are adequate, both the electric and gas utilities afford a unique opportunity to investigate this problem in some detail by allowing us to construct indexes of gross output, input, and net output, and to investigate the degree of the divergence between gross and net output.[4]

Indexes of gross and net output will diverge as the ratio of output to materials and fuels consumed changes. Such changes, at least in the ratio of output to fuel consumed, have been demonstrated in this report for the gas and electric utilities and probably can be shown for many other industries. The trend toward increased efficiency of fuel consumption may be regarded as a type of 'external economy' available to most industries in which the scale of production is rising [5]; and for such industries an index of net output that allows for fuel consumed may be expected to register a greater secular gain than an index of gross output that does not.

An example will illustrate the arithmetic process:

Output	Number of units in base year:	100
	Base year price weight:	$2.00
	Number of units in given year:	200
Fuel Input	Number of units in base year:	100

[3] The index of gross output is $\frac{\Sigma q_1 p_w}{\Sigma q_0 p_w}$; the index of physical input of materials and fuel, $\frac{\Sigma Q_1 P_w}{\Sigma Q_0 P_w}$; and the index of net output, $\frac{\Sigma(q_1 p_w - Q_1 P_w)}{\Sigma(q_0 p_w - Q_0 P_w)}$, where p and q refer to the quantity and unit price of the industry's output, and P and Q to the quantity and unit cost of the materials and fuels bought (Fabricant, *op. cit.*, p. 28).

[4] Ideally, an index of *input* should not only include materials and fuels (which in the gas and electric utilities are indistinguishable) but also allow for capital depreciation. Our inability to include the latter significantly limits our index of net output, as we shall see, in the degree to which it is 'net'.

[5] Cf. Alfred Marshall, *Principles of Economics* (Macmillan, 8th ed.), pp. 278, 279.

Base year price weight: $1.00
Number of units in given year: 150

Gross Output Ratio: $$\frac{\Sigma q_1 p_w}{\Sigma q_0 p_w} = \frac{(200)(\$2)}{(100)(\$2)} = 2.0$$

Fuel Input Ratio: $$\frac{\Sigma Q_1 P_w}{\Sigma Q_1 P_w} = \frac{(150)(\$1)}{(100)(\$1)} = 1.5$$

Net Output Ratio: $$\frac{\Sigma(q_1 p_w - Q_1 P_w)}{\Sigma(q_0 p_w - Q_0 P_w)} = \frac{(200)(\$2) - (150)(\$1)}{(100)(\$2) - (100)(\$1)} = 2.5$$

During a period in which output doubled and output per unit of fuel increased one-third (other things remaining equal), an index of net output rises 25 percent more than the index of gross output, when the fuel cost constitutes one-half of the value of the final product. As noted above, output indexes for groups of industries are usually constructed by combining gross output indexes for individual industries, using value of product or value-added weights. The assumption is that even if the individual gross output indexes are not reasonably good indicators of the movement of net output, the addition of many indexes will cancel individual divergences from net output, and weighting, i.e., value-added weighting, will assign the proper *net* weight to individual industries. Now, divergence from net output may not cancel if, because of a general tendency for output to gain more rapidly than fuel input, a general tendency for gross output indexes to understate the gain registered by net output indexes exists.[6] And if they do not cancel, we must regard group indexes based on combinations of gross output indexes (with or without value-added weights) also as possibly subject to a downward bias.

The electric and gas indexes of net output constructed here give us a good opportunity to investigate the degree of this bias. Fuel consumption in these industries is of more than average importance and the remarkable gains in fuel efficiency might lead us to expect the electric and gas net output indexes to diverge sharply upward from the respective gross output indexes. Actually the divergence, as indicated in Appendix Tables C3 and 5, is not large; e.g., for electric light and power the 1902–42 percentage increase in the net output index is equivalent to an average annual gain of 12.3 percent; in the gross output index, of 11.4 percent. Again, for 1899–1935 the index of net output for the manufactured gas industry rose

[6] Aside from the evidence yielded by the electric and gas utilities, in the majority of the few cases where data for manufacturing industries permitted the construction of indexes of fuel input, and therefore, indexes of net output, the latter outstripped the gross output indexes. The industries were meatpacking, beet sugar, shoes (leather), coke-oven products, cement, steel-mill products, nonferrous-metal products, and automobiles. (Fabricant, *op. cit.*, pp. 130–1, 139, 195, 238–9, 246, 266, 279, 306–10).

at an average annual rate of 4.7 percent; the index of gross output, of 4.5 percent. In the electric light and power industry, despite the marked advance in output per unit of fuel consumed, the effect of this change on net output is qualified by the quantitative relation of 'fuel input' to output. Thus, in 1937 the cost of materials and fuel consumed in the electric light and power industry, on which our index of input is based, was $181.7 million; total sales to ultimate consumers, $2,167.4 million. As the 'input' we have been able to account for constitutes only a small proportion of total output, the effect of even violent changes in the output-fuel input ratio is minimized. In manufactured gas, fuel input is more important than in electric light and power, making up about one-third of the weighted value of final product in recent years (see App. Table C4). The lack of a marked divergence between the gross and net indexes of gas output is a reflection of relatively small gains in the ratio of output to fuel input.

Another pertinent consideration has to do with the composition of input, which by our definition should include not only estimates of materials and fuels consumed but also of capital consumed.[7] In electric light and power, an industry characterized by relatively high capital costs, the depreciation charges (as a measure of the capital consumed in the production process) are greater than the cost of materials and fuels consumed.[8] Conceptually, our index of input for electric light and power should include, together with materials and fuels, a value aggregate designed to measure capital consumption at constant prices. Were such a procedure possible (which it is not, because of the technical difficulty of measuring *physical* changes in capital stock; see Ch. 2, Sec. 2), would the resulting index of net output (i.e., net of fuel and capital input) show more or less divergence from the index of gross output than that noted above?

There is little reason to expect that secular changes in the ratio of output to materials and fuel consumed would parallel changes in the ratio of output to unit of capital consumed. Indeed, if the changes in the former ratio could be attributed to technological advances making greater use of capital equipment, there may be something of an inverse relationship between the two ratios. On this score then, the downward bias associated with gross output indexes due to our inability to measure changes in capital consumption may be of lesser consequence than the downward bias due to ignoring changes in fuel consumption. Nevertheless, there is no reason to expect that capital consumption would increase more rapidly than gross output, even in industries with high capital costs. Thus, we

[7] Our measures of capital consumption should not of course include capital carrying charges; they should be regarded rather as a proper component of net output; i.e., as the return to capital as a factor of production.

[8] In 1937 depreciation charges for electric light and power utilities were $236.9 million; fuel costs, $181.7 million (*ELP, 1937*, p. 14).

found that in the electric utilities, output per unit of deflated capital assets rose 280 percent, 1902–1937 (Ch. 2, Sec. 2). It is reasonable to conclude that output per unit of capital consumption also rose over the years, and that therefore a 'true' index of net output (i.e., net of fuel and capital consumption) would diverge even more rapidly from the gross output index than the net output index adjusted for fuel input alone.

Appendix Table C1

Electric Light and Power, 1902–1942
Generated Output, Fuel Input, and Unit Price and Cost

		1902	1907	1912	1917	1922	1927	1932	1937	1942
1	Mil. kwh. generated	2,507	5,862	11,569	25,438	40,292	74,686	79,657	121,097	188,033
2	Revenue, $ mil.	82.9	163.2	269.1	467.0	944.9	1,667.0	1,821.1	2,167.4	2,864.7
3	Unit price, ¢ per kwh.	3.306	2.784	2.326	1.836	2.345	2.232	2.286	1.790	1.524
4	Fuel input, bit. equiv., th. sh. t.	6.407	12,338	17,895	24,039	30,201	42,215	33,890	52,766	77,935
5	Cost of fuel, $ mil.	11.6	23.1	34.9	87.3	162.6	164.2	112.4	182.7	302.2
6	Unit cost bit. equiv., $ per sh. t.	1.816	1.869	1.949	3.630	5.382	3.890	3.316	3.462	3.878

Line 1 from Table 15; line 2 from Tables 3 and 6; lines 4, 5, and 6 from Appendix Table B1.

The coverage, that of the Census of Electric Light and Power, excludes electric light and power departments of electric railways. The 1942 data, based on FPC and EEI data, were adjusted to the 1937 Census level. The 1942 fuel cost data required special computation; see Appendix Table C2.

Appendix Table C2
Electric Light and Power, 1937 and 1942
Construction of Fuel Costs Index (1937:100)

	Av. consumption 1937 and 1942 $\frac{1}{2}(q_1 + q_0)$ (millions)	Unit price ($) 1937 p_0	Unit price ($) 1942 p_1	Edgeworth constant $ totals, millions 1937: $p_0(q_1 + q_0)$	Edgeworth constant $ totals, millions 1942: $p_1(q_1 + q_0)$
Anthracite (sh. t.)	2.34	9.37	10.31	43.9	48.3
Bituminous (sh. t.)	54.3	4.29	4.78	465.9	518.9
Fuel oil (gal.)	617	.044	.057	54.4	70.4
Natural gas (cu. ft.)	205	.172	.179	70.5	73.4
Total				634.7	711.0
Ratio $\frac{\Sigma p_1(q_1 + q_0)}{\Sigma p_0(q_1 + q_0)}$					1.120
Fuel cost index				100.0	112.0
Av. $ cost per unit of bit. equiv. (Census level)				3.462	3.878

A fuel cost index for the electric light and power industry for 1937 and 1942 may be based (using the Edgeworth formula) upon the four chief fuels consumed, following the method of Appendix Table B2.

The unit prices, from Bureau of Labor Statistics Wholesale Price bulletins for 1937 and 1942 (*Wholesale Prices, December and Year 1937*, pp. 31–2; *Wholesale Prices, January–June 1943*, pp. 26–7) are for chestnut anthracite, mine-run bituminous coal, and Pennsylvania fuel oil.

The natural gas prices, from the American Gas Association *Statistical Bulletin 33* (p. 5) and *53* (p. 5) are for sales to industrial users. The quantity weights (fuel consumed in the electric light and power industry in 1937 and 1942 are those ascertained by the Federal Power Commission) (Table 15). The 12 percent increase revealed for 1937–42 by the fuel cost index was applied to the 1937 Census average price per unit of bituminous equivalent to yield a corresponding 1942 estimate. This figure ($3.878) is used to complete Appendix Table C1.

Appendix Table C3

Electric Light and Power, 1902–1942

Construction of Indexes of Unweighted Gross Output, Fuel Input, and Net Output (1927:100)

	GROSS OUTPUT				FUEL INPUT				NET OUTPUT		
	Price weight (p_0+p_1) ¢ per kwh.	Edgeworth constant $ totals, mil.	Ratio, given to base year	Index	Price weight (p_0+p_1) ¢ per kwh.	Edgeworth constant $ totals, mil.	Ratio, given to base year	Index	Edgeworth constant $ totals, mil. (2–6)	Ratio, given to base year	Index
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
1902 based on 1907											
1902	6.09	153	.428	3.4	3.68	24	.519	15.2	129	.414	2.5
1907		357				46			312		
1907 based on 1917											
1907	4.62	271	.230	7.8	5.50	68	.513	29.2	203	.195	6.1
1917		1,175				132			1,043		
1912 based on 1917											
1912	4.16	482	.455	15.5	5.58	100	.744	42.4	382	.413	13.0
1917		1,059				134			924		
1917 based on 1927											
1917	4.07	1,035	.341	34.1	7.52	181	.569	56.9	854	.314	31.4
1927		3,038				318			2,721		

1922 based on 1927											
1922	4.58	1,844	.540	53.9	9.27	280	.715	71.5	1,564	.517	51.7
1927		3,418		100.0		391		100.0	3,027		
1927 based on 1937											
1927	4.02	3,004	1.621		7.35	310	1.243		2,694	1.665	
1937		4,870				386			4,484		
1932 based on 1937											
1932	4.08	3,247	.658	106.7	6.78	230	.646	80.3	3,017	.659	109.7
1937		4,936		162.1		356		124.3	4,580		166.5
1937 based on 1942											
1937	3.31	4,012	1.553	251.8	7.34	385	1.477	183.6	3,627	1.561	259.9
1942		6,230				569			5,661		

The net output index, based on the formula $\frac{\Sigma(q_1 p_w - Q_1 P_w)}{\Sigma(q_0 p_w - Q_0 P_w)}$ (see Appendix C, note 3), in which the prices weights are the average of given and base year prices, is obtained (columns 9, 10, and 11) from unweighted output and input aggregates. Weighted output aggregates could have been used, with some difficulty, but no correspondingly weighted input aggregates were available. The net output index was constructed for comparison with the gross output index and this comparison would remain unaffected by the use of weighted output and input aggregates.

Appendix Table C4
Manufactured Gas, 1919–1929
Construction of Weighted Net Output Index (1929:100)

	Weighted Gross Output Aggregates (1)	Coverage Ratios (2)	Weighted Gross Output Adjusted (3)	Weighted Fuel Input Aggregates (4)	Net Output (3) – (4) (5)	Ratio, Given to Base Year (6)	Chain Index (7)	Correction (8)	Final Index (9)
	Edgeworth constant $ totals, millions								
1919 based on 1929									
1919: $q_1(p_0+p_1)$	625	.98	641	223	417	.654			65.4
1929: $q_0(p_0+p_1)$	885	.98	899	261	638				100.0
1919 based on 1921									
1919: $q_1(p_0+p_1)$	672	.98	689	232	457	1.012	66.1	–.7	65.4
1921: $q_0(p_0+p_1)$	663	.99	670	218	452		65.3		
1921 based on 1923									
1921: $q_1(p_0+p_1)$	736	.99	743	232	510	.870	65.3	–.6	64.8
1923: $q_0(p_0+p_1)$	842	.99	853	267	586		75.1		
1923 based on 1925									
1923: $q_1(p_0+p_1)$	798	.99	808	285	523	.930	75.1	–.4	74.7
1925: $q_0(p_0+p_1)$	839	.99	850	287	562		80.7		
1925 based on 1927									
1925: $q_1(p_0+p_1)$	820	.99	831	268	563	.834	80.7	–.3	80.5
1927: $q_0(p_0+p_1)$	953	.98	968	292	675		96.8		
1927 based on 1929									
1927: $q_1(p_0+p_1)$	932	.98	946	265	680	.968	96.8	–.1	96.7
1929: $q_0(p_0+p_1)$	957	.98	972	269	703		100.0		100.0

The construction of the weighted index of net output may be illustrated by a sample worksheet for 1919–29. The index for 1899–1909, 1909–19, and 1929–35 was constructed in the same way and the index segments spliced (Appendix Table C5).

Column 1 is from Appendix Table A11; column 2, from Appendix Table A12. The coverage ratios are divided into the gross output aggregates so as to adjust the latter to account for the value of products not directly represented in the gross output index.

The weighted fuel input aggregates are from Appendix Table B8. A final adjustment eliminated the discrepancy between the direct 1919–29 index comparison and that yielded by the chain comparisons, as in preceding Appendix tables.

APPENDIX TABLE C5
Manufactured Gas, 1929–1935
Weighted Indexes of Gross Output, Fuel Input, and Net Output (1929:100)

	GROSS OUTPUT	FUEL INPUT	NET OUTPUT
1899	15.6	17.2	14.9
1904	26.6	33.6	23.8
1909	38.0	47.3	34.2
1914	52.8	61.4	49.4
1919	71.2	85.5	65.4
1921	69.2	80.0	64.8
1923	79.5	91.1	74.7
1925	83.6	91.2	80.5
1927	97.3	99.1	96.7
1929	100.0	100.0	100.0
1931	92.0	86.8	93.7
1933	76.5	79.1	76.0
1935	76.6	72.9	77.9
Average percent rate of change, 1899–1935	4.5	4.1	4.7

The gross output index is taken from Table 24 and Appendix Table A15; the fuel input index from Table 32.

Appendix D

The Use of the Logarithmic Parabola to Depict Growth Trends

The logarithmic parabola is particularly adapted to trace the growth of single industries because the typical pattern indicates rapid growth in the early stages and retardation in the later stages.[1] In the equation $y = ka^x b^{x^2/2}$, where y represents output and x the time variable, the parameter b will serve to determine the existence and degree of retardation of growth. For curves of this type, the ratio of y values for successive annual values of x is $ab^{x+.5}$. This expression represents the ratio of changing values of y between successive discrete intervals of x; consequently b, which may be obtained as the ratio of successive annual values of this 'ratio of change', will indicate the constant factor of decline (or gain) to which the annual ratios of change are subject.

In percentage form the annual rate of retardation may be written as $100(b - 1)$, adopting the usage established by Dr. Burns.[2] In the language of calculus the rate of retardation may be alternatively obtained as the second derivative (with respect to x) of $\log y = \log k + x(\log a) + x^2 \frac{(\log b)}{2}$ (the equation of the growth curve written in log form). This yields the constant $\log b$ as the *instantaneous* rate of change in the rate of change in $\log y$. For our purpose, however, it is more convenient to discuss rates of change in terms of discrete annual intervals, i.e., as percentage rates per annum, as Burns does.

The constants may be determined by fitting a parabola, by the method of least squares, to the logs of the output data; thus the equation obtained in this manner for the electric light and power index, 1902–42, is $y = 69.966\ (1.0898)^x\ (.99705)^{x^2/2}$, x being centered (i.e., set equal to zero) at the year 1927. As shown in Chart 3, the derived curve seems to fit the actual index quite closely. If we accept the fitted curve as an accurate representation

[1] The existence of this pattern has been well established, notably by A. F. Burns in his *Production Trends in the United States since 1870.*

[2] *Ibid.*, pp. 97–8.

of the secular trend of electric light and power output, we may determine how the annual rates of increase fell off at regular intervals (of say ten years) by substituting in the expression $\frac{y_{x+1}}{y_x} = (1.0898)(.99705)^{x+.5}$ the appropriate values of x at ten year intervals and so obtain the decennial ratios of successive y values shown in Chapter 1, Section 5. The annual rate of retardation given by the formula $100\ (b - 1)$, where $b = .99705$, is -0.30 percent per year. The retardation rate is, of course, a negative rate of acceleration.