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where depreciation is ignored and human capital is excluded from its own production. Using the notation of the previous section, Pontryagin necessary conditions for a maximum for lifetime utility are

$$\alpha a X^{\alpha-1} = \lambda P \quad (\text{G-4a})$$

$$\phi f L^{\phi-1} = \lambda w \quad (\text{G-4b})$$

$$c \gamma \mu I^{\gamma-1} = \lambda w \quad (\text{G-4c})$$

$$\mu(T)w(T) = 0 \quad (\text{G-4d})$$

$$\lambda(T)A(T) = 0 \quad (\text{G-4e})$$

$$\dot{\lambda} = -r\lambda \quad (\text{G-4f})$$

$$\dot{\mu} = -\lambda(D - L - I) \quad (\text{G-4g})$$

Since

$$\lim_{X \rightarrow 0} X^{\alpha-1} \rightarrow \infty, \quad \lim_{L \rightarrow 0} L^{\phi-1} \rightarrow \infty, \quad \lim_{I \rightarrow 0} I^{\gamma-1} \rightarrow \infty,$$

the only possibility of a corner solution comes from the constraint  $D \geq L + I$ . If this is binding, equations G-4b and G-4c condense to

$$\beta b L^{\beta-1} = c \gamma \mu I^{\gamma-1} \quad (\text{G-4b}')$$

and equation G-4g' becomes

$$\dot{\mu} = 0 \quad (\text{G-4g}')$$

Existence of optimal controls is assured by a theorem of Cesari (1965) since the Hamiltonian is concave in the control variables.

From equation G-4f, since  $A(T) = 0$  (because  $U(t)$  is unbounded),

$$\lambda(t) = \lambda(0)e^{-rt}$$

Since  $w(T) = 0$

$$\mu(t) = \int_t^T e^{-r\tau} [D - L(\tau) - I(\tau)] d\tau$$

From equations G-4a, G-4b, and G-4c, if  $L + I < D$

$$X = \left( \frac{\lambda(0)e^{-rt}P}{\alpha a} \right)^{\frac{1}{\alpha-1}} \quad (\text{G-5a})$$

$$L = \left( \frac{\lambda(0)e^{-rt}w}{\phi f} \right)^{\frac{1}{\phi-1}} \quad (\text{G-5b})$$

$$I = \left( \frac{\lambda(0)e^{-rtw}}{c\gamma\mu} \right)^{\frac{1}{\gamma-1}} \tag{G-5c}$$

During periods of specialization,  $L + I = D$ . From equation G-4b', since  $\dot{\mu} = 0$ ,  $L$  and  $I$  are constant during a period of specialization, and the rate of growth of wages must decrease throughout such a period. During such a period

$$c\gamma\mu(t)I(t)^{\gamma-1} > \lambda(0)e^{-rtw(t)}$$

while at the end of the period, this inequality becomes a strict equality. Since  $\mu(t)$  is a continuous function of time, and since  $w(t)$  is also continuous, no jump occurs in  $I$  at the end of a period of specialization. For the interval to terminate, the *average* rate of growth of wages must exceed the interest rate. Otherwise, a strict equality would never hold, and there would be no end to the period of specialization. But it is clearly never optimal to invest without ever working. Thus, if investment is never sufficiently productive, no period of specialization need arise.<sup>4</sup>

Suppose that time is sufficiently productive so that a period of specialization occurs. Will there be more than one period of specialization? If not, when does it occur? I will show that, at most, one period of specialization occurs, and that if it occurs, it comes at the earliest stage of the life cycle.

Before demonstrating these propositions, assume they are correct, and consider the behavior of the variables at the end of the period of specialization. Since  $\mu(t)$  is continuous and  $w(t)$  is continuous,  $I(t)$  and  $L(t)$  are also continuous functions of time. If, coming out of the period of specialization, wages grow at a rate exceeding the interest rate  $r$ , leisure decreases as does investment time. Thus, time in the market increases, and hours of productive work must increase. Some care must be taken in interpreting this result. If time at school is not counted as work or investment time, the period following the period of specialization will appear to have a sharp jump in working hours. In fact, a large portion of initial market time is, in reality, a continuation of investment time to an alternative institutional arrangement.

To establish these propositions, logarithmically differentiate equations G-5b and G-5c with respect to time to reach

<sup>4</sup> For example, if  $cD^\gamma < r w(0)$  (i.e., if all available time were devoted to investment and the rate of growth of wages is less than the rate of interest), no specialization would ever occur.



$$\left(\frac{\dot{L}}{L}\right) = \frac{1}{1-\beta} \left(r - \frac{\dot{w}}{w}\right)$$

$$\left(\frac{\dot{I}}{I}\right) = \frac{1}{1-\gamma} \left(r - \frac{\dot{w}}{w} + \frac{\dot{\mu}}{\mu}\right)^5$$

Since  $1 > \phi > 0$ , and  $1 > \gamma > 0$ , if  $\dot{w}/w > r$ , leisure declines and, a fortiori, investment declines ( $I < 0$ ) since  $\dot{\mu} < 0$ . Accordingly, the proportion of market time spent investing declines monotonically and earnings rise. Wage growth rates decelerate, since investment is decreasing and the base of the rate is expanding. Sometime before, or at the end of life,  $\dot{w}/w < r$ , and leisure begins to increase.

As investment decreases, the growth rate in wages falls below the rate of interest, so that the consumption of leisure begins to increase while investment hours and hours of work both decrease. As investment continues to decrease, the wage rate grows at an ever slower rate, so that it is possible (but by no means necessary) that investment time will begin to increase, and total hours of productive work will fall if wage growth is low enough so that

$$\frac{\dot{w}}{w} < r + \frac{\dot{\mu}}{\mu}$$

Intuitively, at this stage in the life cycle, the discounted marginal cost of investment is low. Eventually, investment time must decrease again, since at the end of the horizon, investment terminates  $I(T) = 0$ , and  $I$  is a continuous function of time. (Equivalently,  $\dot{\mu}/\mu$  becomes increasingly more negative near the end of the horizon.)

Note that the "position" of wage growth rates defined by

$$\frac{\dot{w}}{w} = r + \frac{\dot{\mu}}{\mu}$$

has a stability property. If the rate of wage growth falls below  $r + (\dot{\mu}/\mu)$ , investment begins to increase, tending to raise  $\dot{w}/w$ , and to shut off the growth in investment time. If the wage growth exceeds  $r + (\dot{\mu}/\mu)$ , investment decreases, and wage growth is slowed.

To gain further insight into this case, it is of some interest to interpret  $\dot{\mu}/\mu$ . From equation G-4g, it is seen that

<sup>5</sup> Note that although  $w(t)$  and  $\dot{w}(t)$  are continuous functions of time,  $\dot{\mu}$  need not be continuous. Accordingly, right derivatives are used where appropriate.

$$\frac{\dot{\mu}}{\mu} = - \frac{e^{-rt}H(t)}{\int_t^T e^{-r\tau}H(\tau)d\tau}$$

The term in the denominator is that portion of the discounted (from  $t = 0$ ) stock of lifetime hours of work from  $t$  to the end of life. Accordingly,  $\dot{\mu}/\mu$  is the percentage decline of discounted remaining lifetime hours of work. Near the end of life, this is large in absolute value. Moreover, if hours of work are constant from  $t$  on

$$\frac{\dot{\mu}}{\mu} = - \frac{r}{1 - e^{-r(T-t)}} < -r$$

so that if future hours of work are constant or decreasing,  $r + (\dot{\mu}/\mu) < 0$ . Since  $\dot{w} \geq 0$ , investment can never increase if future hours of work at each instant are less than, or equal to, current hours of work.

Note that the increase in investment time must come at an age after the total amount of time spent in the market has peaked ( $\dot{L} = 0$ ), and at a time when total productive hours of work are decreasing. Moreover, for investment to increase, subsequent hours of work must increase to a level greater than the level of hours worked at the age at which investment time begins to increase. "Greater" is the appropriate term since the paths of all variables are continuous, and in the initial phase of increasing investment, hours of work must decrease. Note, too, that once the rate of growth of wages falls below the rate of interest, it never again exceeds that rate. This follows from the previously stated stability property that  $\dot{i} \geq 0$  as  $\dot{w}/w \leq r + (\dot{\mu}/\mu)$  since  $\dot{\mu} < 0$ . Accordingly, the lifetime peak in market hours supplied is never reattained, and no further period of specialization arises. Moreover, the period of specialization, if one occurs, must be in the first part of life. For, if investment were to increase until a time when specialization arises,  $(\dot{w}/w) < r + (\dot{\mu}/\mu) \Rightarrow (\dot{w}/w) < r$ , at the beginning of the specialization interval, and so the specialization interval would never terminate, and hence it is nonoptimal. If investment were to decrease or remain constant up to the beginning of the specialization interval level of investment, it must be the case that leisure is increasing, and  $(\dot{w}/w) < r$ , and hence, the previous argument applies. Thus, investment time is specialized in one period in the first part of life if, indeed, specialization of time ever arises.

Thus far, I have pursued the case of wage growth greater than the

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rate of interest immediately after the period of specialization. Suppose that wage growth is less than the rate of interest at the end of the specialization period. Leisure increases (i.e., total time spent in the market is at lifetime peak just after the period of specialization is over), investment decreases and hours of work increase. Precisely the same analysis as before holds for this case of  $(\dot{w}/w) < r$ . Again, investment may fluctuate around the "stability point" for wage growth

$$\frac{\dot{w}}{w} = r + \frac{\dot{\mu}}{\mu}$$

This second case appears to be empirically uninteresting, since it has the very strong, and empirically unacceptable, prediction that the peak in market activity arises just after the completion of schooling.

To summarize this discussion, consider Figure 1, in which the time derivative of investment is plotted against the time derivative of leisure. At any point in time, the graph is broken into three shaded regions. The lower left region (A) is the case of wage growth greater than the rate of interest. In this region, both leisure and investment time decrease. If the rate of growth of wages is less than  $r$ , but greater than  $r + (\dot{\mu}/\mu)$ , leisure increases, the total amount of time spent in the market contracts, and investment time decreases (region B). Only if the rate of growth of wages falls below  $r + (\dot{\mu}/\mu)$  will investment time increase (region C), but the growth in wages will choke off this investment increase, and eventually the path returns to region B. However, the fact that investment is declining (and wage growth falls) keeps the trajectory in region B or region C. It can never reenter region A. One possible path is sketched for a case in which immediately after the period of specialization, the proportional rate of growth in wage rates exceeds the rate of interest.<sup>6</sup>

### II. ESTIMATING A HUMAN CAPITAL PRODUCTION FUNCTION

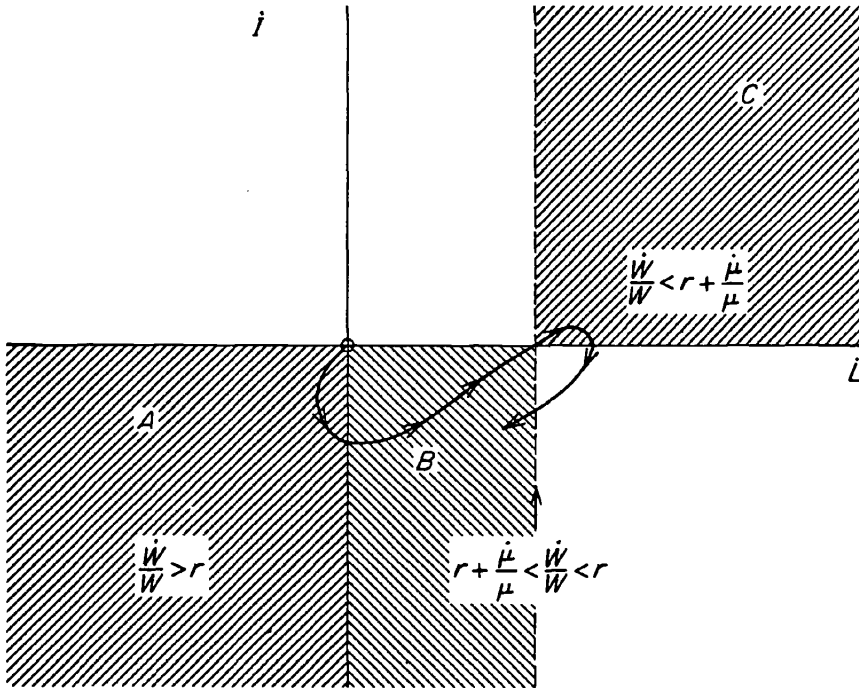
#### *A. Choice of Functional Form and Methodology*

No simple profile characterizes human capital accumulation programs. Depending on initial conditions, preferences, and the nature of the production function, almost any accumulation pattern may be generated. If more information is available about the form of the production function, it might be possible to narrow the band of ignorance about capital accumulation profiles, and through simulations with

<sup>6</sup> Remember that the vertical boundary between B and C will shift with time.

FIGURE 1

The Three Phases of the Investment-Leisure Relationship



different initial wage rates and asset levels, to determine typical profiles.

In this section, a Cobb-Douglas production function for human capital is estimated. En route to the production parameter estimates, provisional estimates of the proportion of market time spent investing at each age are generated. The methodology is quite general, and the approach is applicable to human capital models with exogenous or endogenous labor supply.

The human capital production function is specialized to

$$\dot{w} = cI^\gamma w^\beta - \sigma w, 0 < \gamma < 1 \tag{9}$$

If  $\gamma = \beta$ , the production function satisfies the neutrality hypothesis. If  $\beta = 0$ , and  $\sigma = 0$ , the production function is precisely that analyzed in Section I(G). If  $\gamma = \beta = 1/2$ , the production function is equivalent

to the production functions in the models of Haley (1973) and Rosen (1973).

For postspecialization investment periods, the optimality conditions equivalent to equations 6(e)' and 6(f)' become

$$\dot{g}(t) = (\sigma + r)g(t) - \left[ D - L(t) - I(t) \left( 1 - \frac{\beta}{\gamma} \right) \right] \quad (10)$$

and

$$w(t) = g(t)c\gamma I^{\gamma-1}w^\beta \quad (11)$$

Equation 10 dramatically underscores the very special nature of the "neutrality" hypothesis. Only if  $\gamma = \beta$  will the shadow price of investment  $g(t)$  be independent of the pattern of future investment. Note further that unless leisure is fixed, as in conventional income maximizing models, or as in models in which Michael neutrality is invoked, the shadow price of investment depends on the profile of future time supplied to the market  $D - L(t)$ . For both reasons, Becker's (1967) conjecture that market bias ( $\beta < \gamma$ ) leads to an accelerated decline in investment compared to the neutrality case ( $\gamma = \beta$ ) is seen to apply only to the cost schedule. When neutrality is relaxed, and labor supply is endogenous, the demand price of human capital need not monotonically decline with age, and so investment may increase with age, at least for some age ranges.<sup>7</sup>

The wage rate plays two distinct roles. Looking backward, the current wage rate is a result of previous investment. Integrating equation 9, the current wage rate is the accounting identity<sup>8</sup>

$$w(t) = e^{-\sigma t} \left[ (1 - \beta) \int_0^t c I^\gamma(\tau) e^{(1-\beta)\sigma\tau} d\tau + (1 - \beta)w(0)^{1-\beta} \right]^{\frac{1}{1-\beta}} \quad (12)$$

Looking forward, the wage rate is also the marginal cost of current investment time. Integrating equation 10 and substituting into equation 11,

$$w(t) = c\gamma I^{\gamma-1}w(t)^\beta \int_t^T e^{-(\sigma+r)(\tau-t)} \left[ D - L(\tau) - I(\tau) \left( 1 - \frac{\beta}{\gamma} \right) \right] d\tau \quad (13)$$

Suppose that there are data on a typical consumer's profile of time supplied to the market ( $M(t) = D - L(t) = H(t) + I(t)$ ), but that it is

<sup>7</sup> Even if labor supply is set exogenously, and it increases with time, investment may increase for a period in the life cycle.

<sup>8</sup> This equation is analogous to Becker's income identity equation (Becker 1964, eq. 28, and Mincer 1973, eq. 3.1).

not possible to distinguish work time from investment time. Although the true wage rate is unknown, the consumer's earnings ( $w(t)H(t)$ ) are known. Denote the proportion of market time devoted to investment by  $S(t)$ . Measured wage rates ( $w^*(t)$ ) obtained by dividing earnings by reported hours in the market may be written as

$$w^*(t) = w(t)[1 - S(t)]$$

since only  $1 - S(t)$  of the reported working hours have no investment content.

Conditional on a profile of  $S(t)$ , equations 12 and 13 may be written in terms of observable variables, and upon taking natural logarithms, the following optimality conditions for postschool investment are obtained:

$$\ln w^*(t) = \frac{\ln c\gamma}{1-\beta} + \frac{\gamma-1}{1-\beta} \ln S(t)M(t) + \ln [1 - S(t)] + \frac{1}{1-\beta} \ln \left\{ \int_t^T e^{-(\sigma+r)(\tau-t)} M(\tau) [1 + (1-\beta/\gamma)S(\tau)] d\tau \right\} \quad (14)$$

and

$$\ln w^*(t) = \ln [1 - S(t)] - \sigma t + \frac{1}{1-\beta} \ln \left\{ \int_0^t c[S(\tau)M(\tau)]^\gamma e^{(1-\beta)\sigma\tau} d\tau + w(0)^{1-\beta} \right\} \quad (15)$$

Since actual data come in discrete time intervals, it is necessary to make a discrete approximation to the continuous equations to estimate the parameters.<sup>9</sup>

The consumer is assumed to have  $T$  years in his postspecialization working life so that there are  $T$  observations for  $M(t)$  and  $w^*(t)$ , and  $2T$  approximate equations generated by  $S(t)$ ,  $\sigma$ ,  $\beta$ ,  $\gamma$ ,  $c$ , and  $r$ . If  $S(t)$  is a judiciously parameterized function of time,<sup>10</sup> it is possible to estimate the parameters determining the  $S(t)$ , as well as  $\beta$ ,  $\sigma$ ,  $\gamma$ ,  $c$ , and  $r$ . In this paper,  $r$  is assumed to be 10 per cent.<sup>11</sup>

A measure of concordance of the parameters with the data may be defined as the squared deviation of each equation at each age from a

<sup>9</sup> The discrete approximations are discussed more fully in Section II (B), footnote 14.

<sup>10</sup> In particular, if  $S(t)$  is a polynomial in time, it is necessary that the degree of the polynomial be less than  $T - 4$  so that degrees of freedom are left to determine the remaining parameters  $\beta$ ,  $\sigma$ ,  $\gamma$ , and  $c$ , assuming  $r$  is fixed. To determine these parameters with any precision, the degree of the polynomial should be much less than  $T - 4$ .

<sup>11</sup> Given data on consumption expenditure, earnings, and initial assets, an  $r$  can be determined which sets  $A(t)$  on page 230 to zero. This approach was not pursued in this paper.

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perfect fit. Thus, introducing disturbances in the discrete approximation, and letting  $V(i)$  be the disturbance for the first equation at age  $i$ , and letting  $U(i)$  be the disturbance for the second equation, parameters may be chosen to minimize

$$\begin{vmatrix} \Sigma V_i^2 & \Sigma U_i V_i \\ \Sigma U_i V_i & \Sigma V_i^2 \end{vmatrix} \quad (16)$$

or its logarithm. Note that this criterion allows for inter-equation correlation in disturbances. If there are errors of measurement in the wage series, it is plausible that there is such inter-equation residual correlation. Minimizing this function conditional on a set of realized values for market time ( $M(t)$ ) is equivalent to maximizing the likelihood function if equation errors are normally distributed, and independent of measurement error in the hours of market-time series.<sup>12</sup>

In such a complicated statistical model, identification of parameters is difficult to determine analytically. Recent work by Smallwood (1970) in estimating a somewhat similar model suggests that even when formal identification is secured, the likelihood function may be virtually indeterminate, and a wide variety of parameter estimates may lead to practically the same value for sample likelihood. Moreover, recent work by Rosen (1973) in estimating models of human capital accumulation suggests that determination of even a limited number of parameters may be a difficult task.

With these considerations in mind, the discrete  $S(t)$  series is parameterized in a general logistic form to

$$S(t) = \left( 1 + e^{\sum_{i=0}^R \chi_i t^i} \right)^{-1} \quad (17)$$

where  $R$ , and the  $\chi_i$ ,  $i = 0, \dots, R$  are estimable parameters. One advantage of this parameterization is that it constrains the estimated proportions of market time spent investing to lie inside the interval 0 to 1. The procedure followed in this paper is to begin with a simple model, adding successive polynomial time terms until their contribution to sample likelihood is negligible.<sup>13</sup>

### *B. The Data*

In actual practice, no complete life-cycle data on any "representative consumer" exist, and resort to a synthetic cohort is necessary.

<sup>12</sup> Note, however, that the normality assumption is not crucial for establishing desirable asymptotic properties for nonlinear least squares. See Jennrich (1969).

<sup>13</sup> The actual test is a multiple equation heuristic F which asymptotically becomes a likelihood ratio test. For details, see Goldfeld and Quandt, Chapter 2. Each parameter was assumed to remove one degree of freedom from the data series.

Following the suggestion of Becker and Ghez (1972) and Rosen (1973), data from a cross section of individuals may be used to approximate the time series for a typical individual. This procedure ignores vintage effects and a variety of historical conditions that might influence the optimal paths of human capital accumulation for individuals of different birth cohorts.<sup>14</sup>

The data used in this study are from the 1970 One in a Hundred Census Tape for white college-educated employed males who are not enrolled in school, and who have not undertaken postgraduate education. These data are plotted in Figures 2A and 2B. The postschool work life is assumed to run from age 23 to age 65. A synthetic representative profile is constructed from computing geometric means of estimated annual hours worked, and geometric means of hourly wage rates. A major defect of these data is that the hourly wage data are generated by dividing reported earnings by estimated annual hours worked. Accordingly, the assumption that measurement error in the wage series is uncorrelated with measurement error in the hours-of-work series may be untenable if there is measurement error in constructing the hours-of-work variable. Since the current study is largely exploratory in nature, a more sophisticated analysis was felt to be inappropriate until some experience with simpler techniques was available.<sup>15</sup>

### *C. Empirical Results*

Since identification is a touchy issue, a cautious approach is adopted in forming parameter estimates. The first model estimated is one with the production function of Section I(G), in which human capital is excluded from its own production ( $\beta = 0$ ) and the rate of deprecia-

<sup>14</sup> In fairness to Becker and Ghez (1972) and Rosen (1973), it should be stated that both studies propose methods for eliminating "smooth" vintage effects although only the first study actually implements such methods.

<sup>15</sup> In order to facilitate duplication of the reported results, it is necessary to describe the nature of the discrete approximations. Data on a given age group, e.g., 25 year olds, refers to information on people who just turned 25 as well as to information on people almost 26. Since the relevant census data refer to events in the previous calendar year, and since the census is taken one-quarter of a year away from the previous year, the census 25 year old is actually, on average,  $24 \frac{3}{4}$  years old, so far as the relevant data are concerned. In order to generate discrete approximations, this age was used for 25 year olds, as is a similar displaced age for other age groups.

An initial wage rate is needed in equation 12. Age 23 was selected as the initial date. This age is felt to be sufficiently far removed from college graduation and is used only as an initial value. The residuals for the likelihood function were generated from age 24 to the end of life. Note that inclusion of age 65 implies that the retirement age is actually assumed to be  $65 \frac{3}{4}$  on average. The integrals are approximated by finite sums with one year increments in each step, and midpoint values for  $S(t)$  assigned.



FIGURE 2A

Hourly Wage Rates by Age, Derived from 1970 U.S. Census Data to Construct a Synthetic Cohort Used in the Empirical Analysis  
 Data are for White College-Educated Employed Males  
 Not Currently Enrolled in School

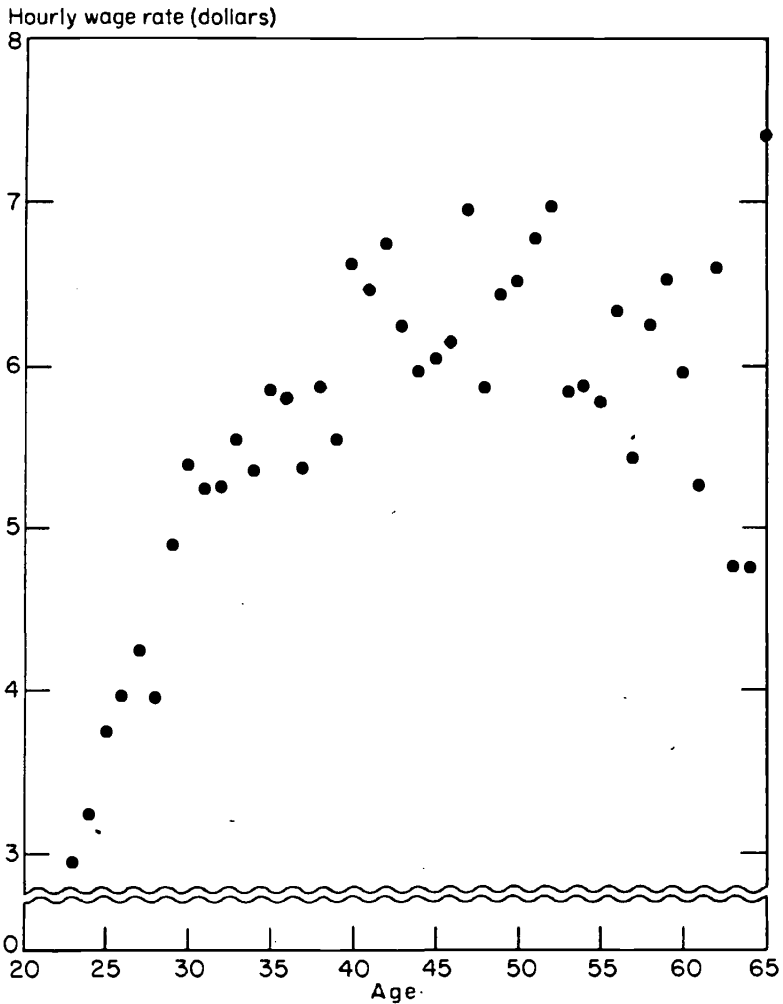
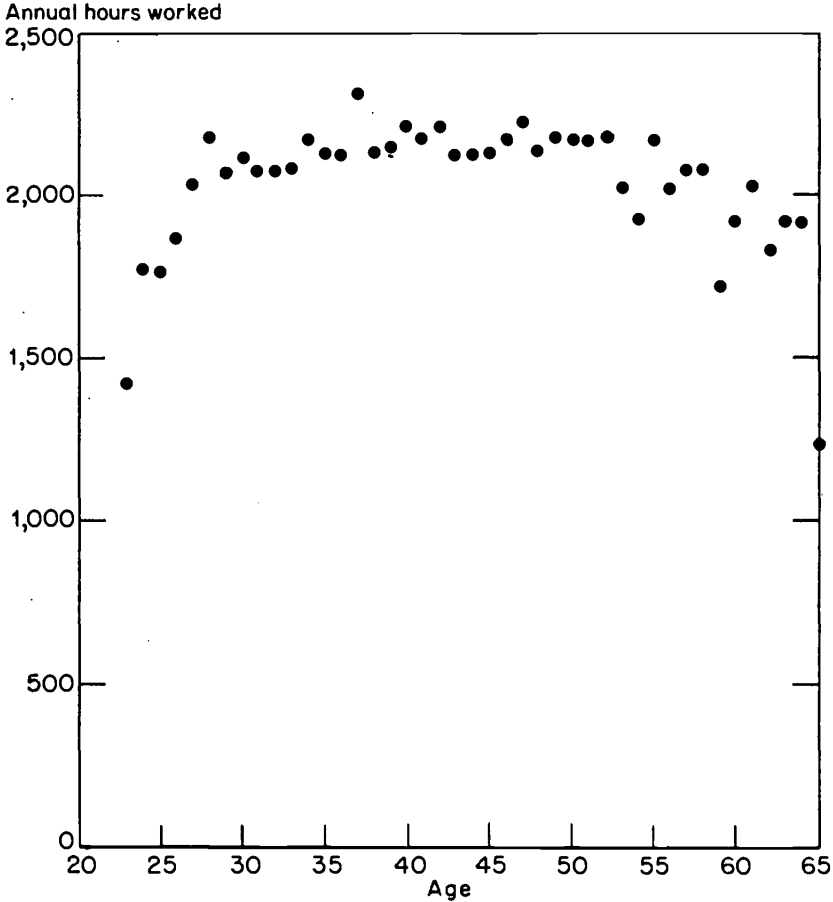


FIGURE 2B

Annual Hours Worked by Age, Derived from 1970 U.S. Census Data to Construct a Synthetic Cohort Used in the Empirical Analysis. Data are for White College-Educated Employed Males Not Currently Enrolled in School



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TABLE 1  
(asymptotic standard errors in parentheses)

	Model I (G)		General Model <sup>a,b</sup>	
<i>c</i>	.14 × 10 <sup>-2</sup>	(.04 × 10 <sup>-2</sup> )	45.49	(3.034)
<i>γ</i>	.67	(.052)	.99	(.003)
<i>β</i>	-	-	-6.69	(.043)
<i>σ</i>	-	-	.0016	(.00025)
<i>X</i> <sub>0</sub>	.99	(.16)	.6073	(.065)
<i>X</i> <sub>1</sub>	.035	(.010)	.0906	(.0081)
<i>X</i> <sub>2</sub>	.39 × 10 <sup>-2</sup>	(.12 × 10 <sup>-2</sup> )	-.00537	(.0005)
<i>X</i> <sub>3</sub>	-	-	.00240	(.0007)
Value of log likelihood	-12.89		4.69	

<sup>a</sup> Test for  $\gamma = 1$  rejects that hypothesis.  $\left(\frac{\gamma - 1}{.0031}\right) = 3.33$ .

<sup>b</sup> Test for neutrality hypothesis  $\gamma - \beta = 0$ ,  $\frac{\gamma - \beta}{[\text{Var}(\gamma - \beta)]^{1/2}} = \frac{7.68}{.229} = 33$ .

tion ( $\sigma$ ) is set at zero.<sup>16</sup> The interest rate is fixed at 10 per cent. The error sum of squares function (16) was minimized using both the Powell conjugate gradient method and Grad X.<sup>17</sup> Experimentation with polynomial time-trend terms beyond the second power failed to produce any significant improvement in likelihood. The results with this model are reported in the first column of Table 1. The estimated proportion of time spent investing is plotted in Figure 3. The initial proportion of time spent investing is quite high but, by age 47, becomes negligible.

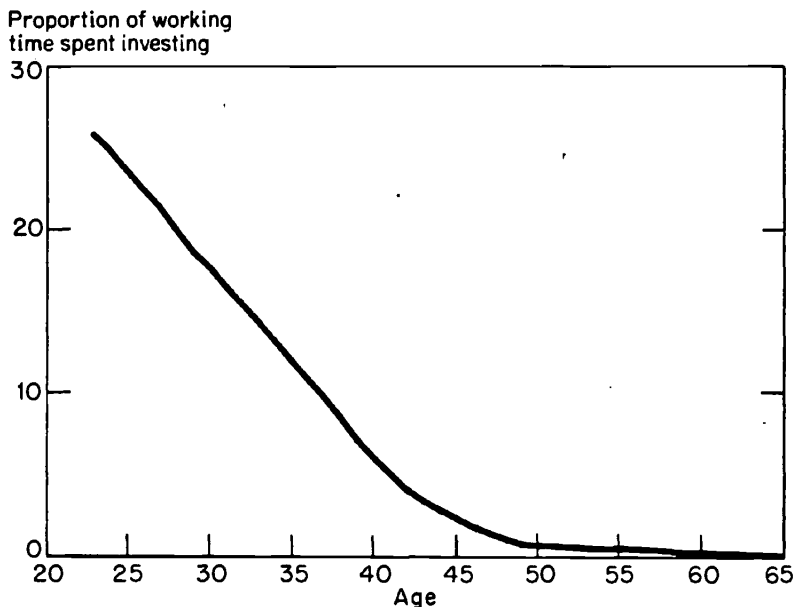
Figure 4 graphs the amount of time invested against age. Note that the *initial* increase in investment time is in apparent contradiction with the pattern predicted in Section I(G). The dollar costs of investment plotted in Figure 5 show a similar initial increase followed by a decline. With the precision afforded by these data, the logically possible

<sup>16</sup> Note that no special assumption is made about preferences. The effect of preferences is embodied in the observed hours-of-work series. However, given an assumption about preferences such as that made in Section I(G), it is possible to make some refutable statements about the time profile of investments.

<sup>17</sup> For a discussion of these techniques, see Goldfeld and Quandt (1972), Chapter 1. Both sets of optima reported in this paper were tested by using "substantial" displacements from the reported optimum parameter estimates. Both optima were stable. Of course, these experiments do not prove that a truly global optimum has been located.

FIGURE 3

Estimate of the Age Profile of the Proportion of Measured Working Hours Devoted to Learning and Investment Activities, Derived from a Model That Assumes Human Capital Is Not Self-Productive



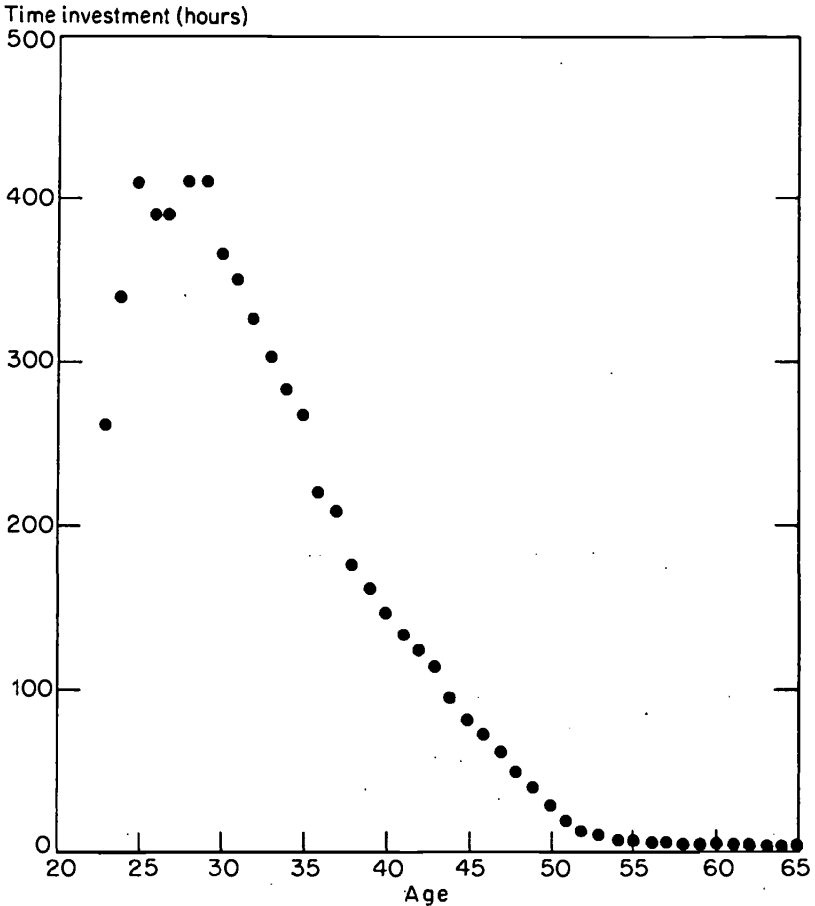
phenomenon of rising investment time following an initial decline in postschool investment appears to be empirically irrelevant.

The increase in postschool investment time immediately after the completion of schooling may be interpreted as a refutation of one of the predictions of model I(G). Either the assumption about preferences or the assumption about technology may be incorrect. With this result in mind, a more general model is also estimated. In this model, both  $\beta$  and  $\sigma$  are estimated, rather than assumed to be zero. Thus, it becomes possible to make a direct parametric test of the neutrality hypothesis ( $\gamma = \beta$ ).

Empirical results with the revised model are reported in the right-hand column of Table 1. The heuristic F test indicates that a cubic time-trend term should be included. The most dramatic result with the more general model is that the returns-to-scale parameter ( $\gamma$ ) is near

FIGURE 4

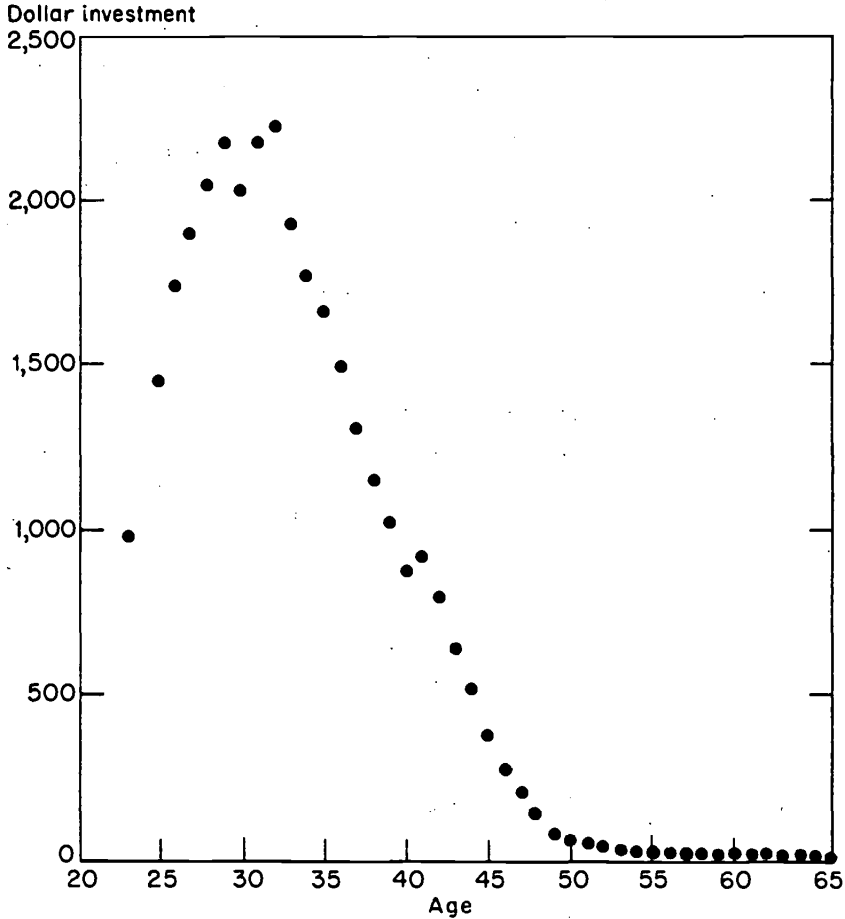
Estimate of the Age Profile of the Number of Hours Devoted to On-the-Job Learning and Investment Activities, Derived from a Model That Assumes Human Capital Is Not Self-Productive



unity, although as noted in the first footnote of the table, it is statistically significantly different from unity. If  $\gamma$  were unity, the model would collapse into a “bang-bang” control problem, and a continuous post-school investment profile would not exist. Although there are few empirical results with which this finding can be compared, estimates produced in two other papers suggest that  $\gamma$  may in fact be near unity.

FIGURE 5

Estimate of the Age Profile of the Dollar Value of the Time Devoted to On-the-Job Learning and Investment Activities, Derived from a Model That Assumes Human Capital Is Not Self-Productive



In his commentary on a paper by Ben-Porath, Mincer (1970) estimates the inverse of  $\gamma$  as approximately 1.01. No confidence interval is stated. A recent paper by Brown (1973) estimates the inverse of  $\gamma$  as 1.15. Again, no confidence interval is available. The comparison between the results in these studies and the results reported here must be qualified since both authors assume neutrality (i.e.,  $\gamma = \beta$ ).

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The rate of depreciation is estimated to be quite small (one-sixth of 1 per cent or so). Human capital is seen to have *negative* self-productivity. The estimate of  $\beta$  is large and negative, and the test in the footnote to the table suggests that the neutrality hypothesis is resoundingly rejected. However, the value of  $\beta$  is too negative to be accepted without some skepticism.<sup>18</sup>

The analogues of Figures 3, 4, and 5 are presented in Figures 6, 7, and 8 respectively. Not surprisingly, in view of the large market bias implicit in the estimate of  $\beta$ , investment time falls off more sharply than in the previous case and, by age 37, becomes negligible.

It is noteworthy that in both models, the profile of the proportion of time spent investing at each age falls off more steeply than the linearly declining profile assumed by Mincer (1972). Moreover, in both models, the curvature of this profile has convexity properties directly opposite to those proposed by Haley (1973).

### CONCLUSION

In this paper, I have discussed life-cycle models of labor supply and human capital accumulation. Methods for estimating the parameters of the human capital production function have been proposed and implemented. By assuming a functional form for the human capital production function, it is possible to utilize both the accounting identity that wage growth is the result of investment, and the optimality condition that the wage rate should equal the marginal benefit of time investment, to estimate the parameters of the underlying production function and to determine the proportion of time spent investing at each age and the empirical relevance of the neutrality hypothesis. The estimates presented in this paper confirm Ben-Porath's (1970) finding that the neutrality hypothesis is not consistent with data on life-cycle profiles, and suggest that the simple time profiles assumed by Mincer (1970) and deduced by Haley (1973) are not empirically relevant for white college-educated males.

<sup>18</sup> One possible reason for the large negative value of  $\beta$  may be that market inputs into postschool investment are omitted, tending to overstate the value of  $\beta$  (in absolute value) if investment time is negatively correlated with the level of the wage rate. A negative correlation is plausible if  $\beta$  is, in fact, negative. For the same reasons,  $\gamma$  will be overstated. Another source of bias may be spurious correlation between the disturbances and the hours of work series. However, preliminary results with the Survey of Economic Opportunity data, in which wage and hours data are independently derived, suggest that estimates of  $\beta$  remain large and negative.

FIGURE 6

Estimate of the Age Profile of the Proportion of Measured Working Hours Devoted to Learning and Investment Activities from a Model That Allows Human Capital to Enter Its Own Production

Proportion of working time spent investing

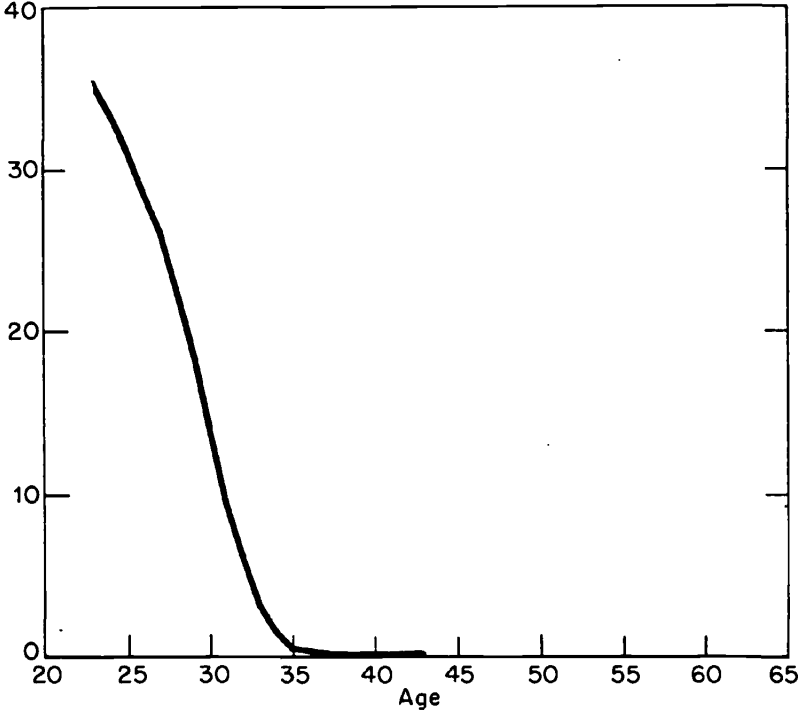




FIGURE 7

Estimate of the Age Profile of the Number of Hours Devoted to On-the-Job Learning and Investment Activities, Derived from a Model That Allows Human Capital to Enter Its Own Production

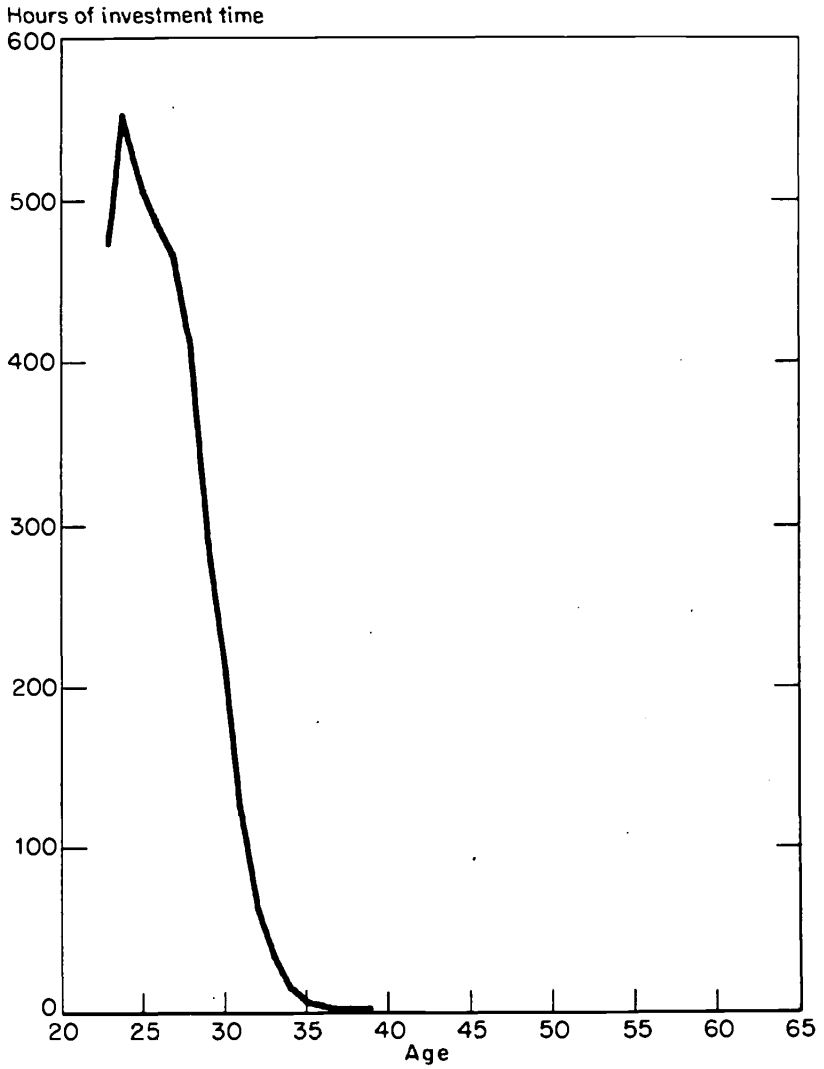
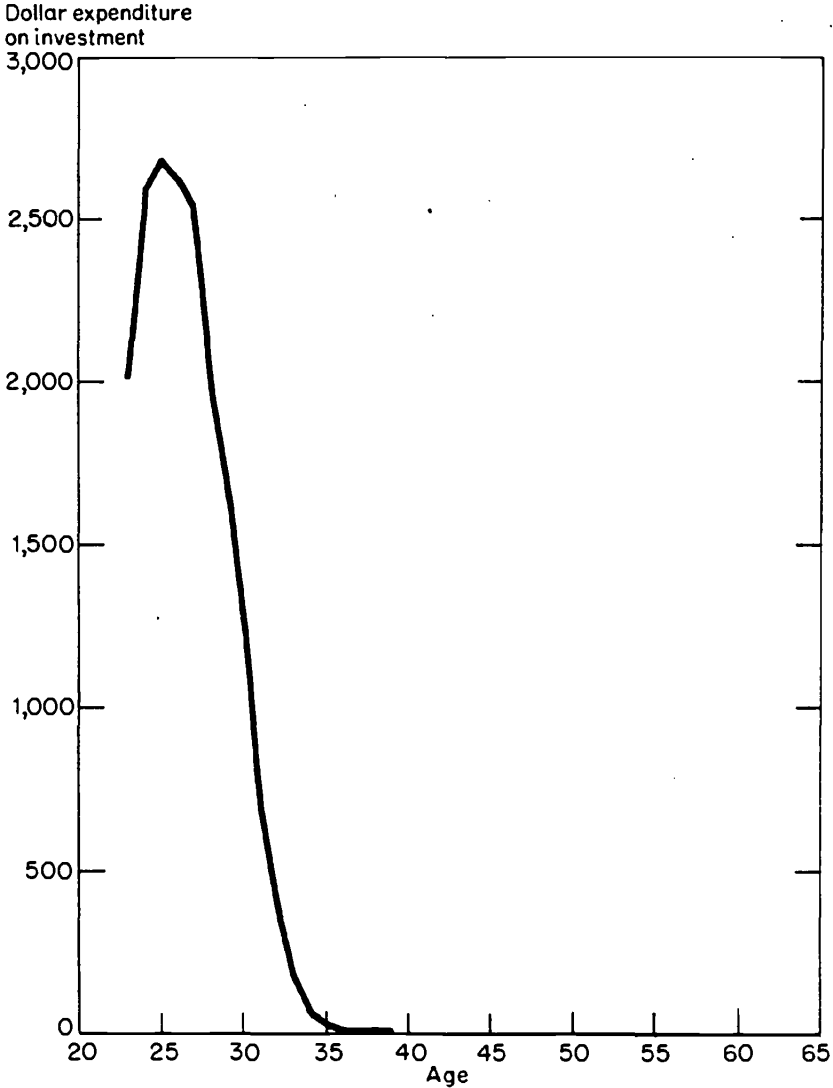


FIGURE 8

Age Profile of the Dollar Value of Time Devoted to On-the-Job Learning, Derived from a Model in Which Human Capital Enters Its Own Production



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# Comments on "Estimates of a Human Capital Production Function Embedded in a Life-Cycle Model of Labor Supply"

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JAMES HECKMAN'S paper presents a dynamic framework for analyzing the life-cycle allocation of time among three ends: activities that augment the individual's market wage (i.e., human capital), market production, and leisure. The process producing human capital is specified such that the optimal path of investment determines the current wage by past investments and equates this wage to the marginal value of future investments of time. The future pattern of investment is relevant because the Ben-Porath neutrality of human capital assumption is relaxed and the labor supply decision is made endogenous. The exposition of the model is clear and thoughtful, and in my view, it represents a unified generalization and extension of the Ben-Porath (1967, 1970) formulation. First, I will restate the problem of under-identification that appears to limit what we can learn about the human capital production function, and describe how Heckman has approached this problem. Then I shall indicate why the econometric techniques and data used by Heckman may not be the most appropriate for estimating the parameters to his formulation of the model.

To understand why market wage rates vary over an individual's life cycle, the human capital framework postulates that investments occur that augment the productive capacity of the human agent, while other processes, such as depreciation and obsolescence, may work to diminish that capacity over time. In the postschooling period of the life cycle, there is, regrettably, no obvious way to measure directly the proportion of time an individual allocates only to increase his future market productivity. Consequently, the general form of the human-capital production function must be restricted if a single time series on market wage rates is to shed much light on the time profile of post-

schooling investment and the many parameters that can be used to characterize the processes that produce and depreciate the stock of human capital.

Ben-Porath's approach to this problem was to assume that non-working time or leisure was fixed, or alternatively that leisure did not enter the utility function. Heckman relaxes this unappealing assumption, making labor supply decisions endogenous and permitting  $\lambda(t)$ , the shadow value of time in leisure, to vary independently of  $\mu(t)$ , the shadow cost of time in human capital investments. The "neutrality" assumption of Ben-Porath, that human capital increases equally the productivity of time in the marketplace and in the production of further human capital, is another restriction that Heckman relaxes, that is,  $\gamma$  need not equal  $\beta$ . Finally, Heckman explores in some detail the effects of depreciation  $\sigma$ , the market rate of interest  $r$ , and individual time preference  $\rho$  on the optimal regime of life-cycle investment.

From this dynamic allocative model, the demands for leisure and goods can be obtained, with predictions on the signs of the time derivatives of the marginal value of goods and leisure, and on the difference between the individual time preference and the market rate of interest. The parameters to these derived demand equations have already been estimated by Heckman (1971) from individual (synthetic) life-cycle information on the consumption of goods and leisure (or labor supply) and the pattern of wages. With great clarity (and candor), Heckman states why the general model leads to regrettably few predictions about *investment behavior* over the life cycle. The critical question is, therefore, How should the generality of the model be restricted to yield new insights and possible empirical applications for the framework?

In his specific model, Heckman assumes that time preference and depreciation can be eliminated altogether ( $\rho, \sigma = 0$ ), and the polar case from that proposed by Ben-Porath is adopted, the assumption being that human capital has *no effect* on the efficiency with which subsequent human capital is produced. Also, an additive utility function is posited that implies strong restrictions. Even with these specific assumptions, testable implications for investment behavior are scarce. Several interesting interrelationships are, nonetheless, explored and diagramed. It is not clear in my mind, however, that this exercise in generalization has not collapsed to a less realistic model than that used by Ben-Porath, and in neither instance are the empirical implications of the restricted model particularly powerful in accounting for anomalous empirical evidence.

In his empirical estimation of the human capital production function, Heckman has set the interest rate to 10 per cent, and initially assumed that depreciation is zero ( $\sigma = 0$ ), and human capital is excluded from its own production ( $\beta = 0$ ). Average information on single age groups of white college-educated males for "hours worked" and "hourly wage rates" from the 1970 Census cross section are used to estimate the model's remaining five parameters: the market productivity of human capital  $c$ , the returns to scale in human capital investment  $\gamma$ , an intercept parameter and a quadratic function in age:  $\chi_0, \chi_1, \chi_2$ .

The initial restrictions, namely that  $\beta$  and  $\sigma$  are zero, are then relaxed, and a second set of estimates reported for the general model. These unrestricted estimates might provide one with a better basis for choosing between Ben-Porath's "neutrality" hypothesis and Heckman's strong "market-bias" hypothesis. Getting a better sense of the confidence ellipses obtained around these two parameters,  $\gamma$  and  $\beta$ , might provide a further opportunity to explore the sensitivity (or lack thereof) of the model to these analytically convenient restrictions.

Heckman's approach is conceptually attractive because he incorporates into the human capital investment model additional relevant information about life-cycle labor supply decisions to facilitate identification of his parameterization of the human capital production function. Unfortunately, the expected value of labor supply for United States college-educated white males does not vary much from age 27 to age 52 (see Figure 2). And as Sherwin Rosen pointed out, the variation in labor supply from age 22 to age 26 among this group may be in large part a reflection of frictional lags in labor force entry or part-time and part-year employment by persons still in school. Thus, this critical source of variability in the labor supply series that helps to identify the model may be largely spurious from the point of view of the behavioral process Heckman is trying to study. Reestimating the model without these few younger observations would therefore seem reasonable.

There is also the question of how empirical materials can best be used to discriminate among hypotheses and to estimate more confidently the several interesting parameters to the human capital production function. In my judgment, cross-sectional information from age-specific groups is no longer a satisfactory data base for estimating dynamic models of life-cycle behavior. The synthetic cohort was a tolerable first approximation when the questions asked of earnings data were relatively simple. This is no longer true. Meanwhile, longitudinal data for individuals are becoming publicly available. Inter-

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polating wage rate and labor supply series for all ages from the several jobs reported in the Parnes Longitudinal Labor Force Survey (which Heckman [1972] has already used to great advantage) might permit one to estimate Heckman's general model or more restricted variants thereof. Coleman's life-history survey might also be a useful data base. New problems would also arise, of course, by the nature of these time-series data sources. If such longitudinal data are still unable to identify confidently the parameters to the underlying human capital production function, it may be possible to incorporate information on variation in both cross sections and cohort series, specifying with care the stochastic structure of the resulting composite disturbances. Error-component models can also be formulated to derive information about unobserved variables from common effects across related behavioral equations (Griliches). A variety of such new approaches applied to more appropriate longitudinal individual data may clarify the relevant dimensions of the human capital production function.

Two aspects of the maximum likelihood estimation procedure employed by Heckman deserve note. First, in defining the criterion for estimating the parameters to equations 14 and 15, equal weight is attached to the mean square error of both the past and future investment-wage relationships, plus inter-equation residual correlations. As Zvi Griliches suggested at the Conference, there may be reason to think that people do somewhat better optimizing their current behavior as a function of past outcomes than they do in adjusting their behavior to their (uncertain) knowledge of future outcomes. Perhaps past consistency should be weighted more heavily than future consistency in obtaining the "best" parameter estimates. At least some sensitivity analysis might be warranted, given the arbitrary nature of this criterion.

Second, and much more important, the estimates to equations 14 and 15 are obtained conditional on the realized values of market time ( $M(t)$ ). For this labor supply series to be independent of the disturbances in the investment equations, the investment decision-making process must *not* be jointly and simultaneously determined with life-cycle labor supply. Admittedly, Heckman's assumption of independence from stochastic simultaneity and joint errors of measurement<sup>1</sup>

<sup>1</sup> As is widely recognized in the labor supply literature, the Census wage-rate series is obtained by dividing annual earnings by annual hours worked. Hence there is a negative spurious correlation introduced into the wage and hours series by definition if there are errors in measurement of labor supply. The importance of this bias, of course, is much reduced through aggregation by age, but it must still be present to some degree in Heckman's data.

eases the job of estimation. But, in my view, it also contradicts the essential analytical advance proposed in the paper. I have difficulty imagining how the "Human Capital Production Function" can be "Embedded in a Life-Cycle Model of Labor Supply" and yet permit one to maintain the assumption that these two behavioral processes are not part of a jointly and simultaneously determined system of equations. If the life-cycle labor supply decision has an important bearing on human capital investment behavior, the analytical approach of this paper is interesting. But, in that case, the empirical estimates obtained by taking labor supply as predetermined in estimating the parameters to the human capital production function are subject to simultaneous-equations bias. Can one have it both ways? In my opinion, the analytical approach makes sense but the estimation procedure does not.

What is one to conclude from the empirical estimates? Investment rates decline over the life cycle according to the estimated polynomial time trend, in much the same manner as estimated by Mincer's simpler procedures. Diminishing returns to scale in the production of human capital, i.e.,  $\gamma < 1$ , is implied by the restricted model, whereas constant returns to scale is implied for the generalized model,  $\gamma = 1$ . The large negative value of  $\beta$ , in the latter model, suggests that the accumulation of human capital *decreases* substantially the efficiency with which human capital can be subsequently produced. Despite the relatively small asymptotic standard errors, I cannot be confident that the parameter values of the generalized model are precise or even plausible.

One unusual empirical finding is the implied *increase* in the dollar and time costs of investment for the first several years after college, followed by the anticipated monotonic decline. Although dismissed as "empirically irrelevant," this anomaly may deserve further attention, for it is also noted in an empirical investigation of a population of engineering college graduates in Sweden (Klevmarken and Quigley 1973).

One interpretation of these results is that trying to estimate the general parameters of a human capital production function is not now a promising avenue for empirical research. Without additional sources of information that can aid discrimination among the several relevant parameters, the presumption that postschooling investment causes life-cycle variation in wage rates is plausible but also virtually tautological. As a heuristic device for discounting lifetime wage streams, the on-the-job investment hypothesis is a useful accounting mechanism



that has not yet been rejected by any empirical tests.<sup>2</sup> What seems clear to me is that we are seeking a great deal of information about several complex processes from very little observable data. Have we any alternative research strategies? How are we more likely to improve our understanding of the related processes of individual time allocation, investment, and savings, which would seem to be responsible for both differences in individual wage rates at one point in time, and differences in wage rates for specific individuals over time?

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<sup>2</sup> Approximately the view expressed in a moment of perverseness by colleague John Hause.