Volume Title: Short-Term Macroeconomic Policy in Latin America

Volume Author/Editor: Jere R. Behrman and James Hanson, eds.

Volume Publisher: NBER

Volume ISBN: 0-88410-489-3

Volume URL: http://www.nber.org/books/behr79-1

Publication Date: 1979

Chapter Title: The Short-Run Output-Inflation Tradeoff in Argentina and Brazil

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Chapter URL: http://www.nber.org/chapters/c3886

Chapter pages in book: (p. 133 - 168)

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The Short-Run Output-Inflation Tradeoff in Argentina and Brazil

1. INTRODUCTION

The purpose of this chapter is to study the short-run relationship between output and inflation in the context of a macroeconomic model. Although a considerable number of economists have studied this subject, mainly from the point of view of the Phillips curve theory, most of them have used ad hoc hypotheses regarding the process through which expectations are formed. Other economists (e.g., Lucas, and Sargent and Wallace) have studied the same subject and have postulated a rational expectations hypothesis for analyzing the short-run tradeoff between inflation and output and for testing the "natural rate" hypothesis.

The analysis performed in this chapter is similar to that of Lucas and Sargent and Wallace. In fact, the analysis in Section 2 starts with the assumption that the model previously postulated by Sargent and Wallace (1975) is an appropriate theoretical framework for analyzing the short-run relationship between prices and output. In that section the model to be used is presented, as well as some of its main limitations and implications.

In Section 3 a summary of the results of the structural analysis of the model is presented as well as the estimation procedure followed in order to obtain the estimates for the structural equations of the system. These estimates, based on available information for Argentina and Brazil, are presented in Section 4.

In Section 5 of this chapter an attempt is made in order to analyze the short-run dynamics of price and output based on the empirical findings of Section 4. This obviously implies that the parameters of the model have to be assumed constant over the period of analysis. As argued below, the duration of this period is of particular importance given the assumption of rational expectations that is incorporated in the model.

2. THE MACROECONOMETRIC MODEL

As mentioned above, the model analyzed in this section is a standard macroeconomic model in which expectations will be assumed to be "rational" in the sense of Muth (1961). This assumption was incorporated in similar models by Sargent (1973) and Sargent and Wallace (1975). In this chapter some modifications are introduced in order to arrive at a direct, estimable relationship for a short-run output-inflation tradeoff.

The model consists of the following three equations:

(a) Aggregate supply

$$y_t = y_{n,t} + a(P_t - {}_tP^*_{t-1}) + k(y_{t-1} - y_{n,t-1}) + u_{1t},\ a > 0 \qquad (5\text{-}1)$$

(b) Aggregate demand

$$y_t = y_{n,t} + g + c(r_t - ({}_{t+1}p^*_{t-1} - {}_tp^*_{t-1})) + u_{2t},\ c < 0, g > 0 \qquad (5\text{-}2)$$

(c) Portfolio balance

$$\phi m_t = p_t + y_t + br_t + u_{3t} - \infty < b < 0 \qquad (5\text{-}3)$$

In these equations y_t, p_t and m_t are the natural logarithms of real income, the price level, and the nominal stock of money, g is a constant, and the $u_{i,t}$s, $i = 1, 2, 3$, are disturbance terms. The variable $y_{n,t}$ is a measure of normal productive capacity that will be represented by the trend in real output in the empirical application of the model. Therefore, $y_t - y_{n,t} = y_{c,t}$ represents cyclical or "detrended" output. The variable ${}_{t+1}p^*_t$ represents the public's expectation at time t of the logarithm of the price level expected to prevail at $t + 1$. The variable r_t is the nominal rate of interest.

The u_{it}s, $i = 1, 2, 3$, are random disturbances with zero means that may be serially and contemporaneously correlated.

Equation (5-1) is an aggregate supply equation relating detrended output to the gap between current price level and the public's prior expectation of the current price level. In this equation lagged detrended output indicates that deviations of aggregate supply from normal capacity may display some persistence. A formal derivation of Equation (5-1) can be found in the work of Lucas (1973), and the next few paragraphs will outline some major aspects of his work. However, similar reduced forms could be generated by other models, for example, those Phillips curve models based on an aggregation of multiple markets of the economy such as Lipsey and Hansen, among others, have proposed.

Lucas derives his equation under the assumptions that suppliers are located in a large number of scattered competitive markets and that demand for goods in each period is distributed unevenly over markets, leading to relative as well as general price movements. This means that the situation as perceived by individual suppliers may be different from the aggregate situation. Following Lucas and letting z index markets, supply in market z is:

$$y_t(z) = y_{n,t} + y_{c,t}(z) \qquad \text{(5-1a)}$$

The secular component $y_{n,t}$ is assumed to be common to all markets. The cyclical component varies with perceived relative prices and with its own lagged value:

$$y_{c,t}(z) = S\,[p_t(z) - E(p_t|I(z)] + ky_{c,t-1} \qquad \text{(5-1b)}$$

where $p_t(z)$ is the log of the actual price in z at t and $E(p_t|I[z])$ is the mean current general price level, conditioned on information available in z at the end of $t - 1$, $I(z)$. While this information does not permit exact inference of p_t, it does determine a "prior" distribution on p_t, common to traders in all markets. We assume that this distribution is known to be normal with mean $\bar{p}_t$.

Now let z be the percentage deviation of the price in market z from the average p_t (so that markets are indexed by their price deviation from average). z is assumed normally distributed, independent of p_t with zero mean. Then

$$p_t(z) = p_t + z \qquad \text{(5-1c)}$$

This last expression is used by suppliers to calculate the distribution of p_t conditional on $p_t(z)$ and $\bar{p}_t$. By straightforward calculation it can be proved that the distribution of p_t is normal with mean

$$E(p_t|I(z)) = E(p_t|p_t(z), \bar{p}_t) = (1 - \theta)\, p_t(z) + \theta \bar{p}_t \tag{5-1d}$$

where θ is a ratio between the "relative" price variance and total price variance, that is, $V(z)/(V(\bar{p}_t) + V(z)$, where $V(\bar{p}_t)$ is the variance of the "prior" distribution.

Combining (5-1a), (5-1b), and (5-1c) and averaging over markets (integrating with respect to the distribution of z) gives Equation (5-1) where $a = \theta S$. This is a very important point because the *slope* of the aggregate supply function (1) varies with the fraction, θ, of *total* individual price variance that is due to *relative* price variation.

Equation (5-2) is an aggregate demand equation that relates the deviation of aggregate demand to the real rate of interest, which in turn is represented by the nominal rate of interest minus the expected rate of inflation. This equation resembles the Hicksian "IS" schedule utilized to represent the income expenditure sector in the model of Keynes. Of course, some major arguments of the Keynesian aggregate demand function have been neglected for analytical simplicity. A possible extension of the model could include some variables representing the fiscal action of the government (e.g., see, Sargent and Wallace [1975]).

Some limitations of Equation (5-2) are stated in Sargent (1973) as follows:

> An important thing about equation (2) is that it excludes as arguments both the money supply and the price level. . . . This amounts to ruling out direct real balance effects on aggregate demand. It also amounts to ignoring the expected rate of real capital gains on cash holdings as a component of the disposable income terms that belong in the expenditure schedules that underlie equation (2). Ignoring these things is usual in macroeconometric work.

Another aspect of this model is the lack of symmetry between Equation (5-1) and (5-2), that is, only suppliers have explicit misperceptions of prices and only demanders have an explicit responses to changes in the real rate of interest. Implicitly, the effect of the neglected variables in each equation could be captured if they induced some stable stochastic process in the error terms.

Equation (5-3) is a demand for money relationship with unit real income elasticity (this assumption is not crucial and will be relaxed later on) that summarizes the condition for portfolio equilibrium. In other words, when Equation (5-3) is satisfied owners of bonds and equities are satisfied with the division of their portfolio between money (assumed to be exogenous), on the one hand, and bonds and equities, on the other hand. ϕ is a polynomial in the lag operator (i.e., $\phi = \phi_0 + \phi_1 L^2 + \ldots$, where $\phi_0 + \phi_1 + \ldots = 1$) introduced in an effort to capture the effects of lagged changes of m_t on nominal income. The degree of this polynomial will be determined empirically.[1]

The u_{it}s, $i = 1, 2, 3$, are random disturbances with zero means that may be serially and contemporaneously correlated.

On pure theoretical ground there is not a strong justification for the existence of a lagged response of nominal income to changes in the quantity of money. However, the existence of lags is confirmed in many empirical works that relate money and prices.

The working of the model can easily be illustrated leaving aside the problem of how expectations are formed. This rules out some implications of the dynamics of the model as is usual in comparative statics. It will help understand the problem if we use a geometrical interpretation of the model along the familiar framework of the *IS-LM* analysis. Then Equation (5-2) has the form of a *IS* schedule in the (r, y) plane, Equation (5-3) has the form of a *LM* schedule in the (r, y) plane, and Equation (5-1) is a vertical line in the (r, y) plane at the full employment or natural rate level of real income. Equations (5-2) and (5-3) are drawn in the (r, y) plane under the assumption that the expected rate of inflation is zero (so the nominal and real rate of interest are the same). Then in Figure 5-1, y_o^s, y^d, and m_o will denote Equations (5-1), (5-2), and (5-3), respectively.

Our starting point will be a nongrowth economy in full equilibrium with real output at its natural level (y_n), zero rate of inflation, and consequently nominal and real rate of interest equal. This situation is represented in Figure 5-1 by point *A*.

Now let us assume an unanticipated increase in the stock of money. From Equation (5-3) we see that with prices and income as yet unchanged, portfolio balance can be attained through a reduction in the rate of interest. There, the m_o curve shifts rightwards to m_1. In the goods market a reduction in the rate of interest means an excess demand for goods, exerting an upward pressure on the price level as well as on real output. As has been argued in the derivation of Equation (5-1), individual suppliers have to assess whether a given price change is a "relative" price change or a "general" price change. In this last case suppliers are assumed not to respond because their supply functions are homogeneous of degree zero in absolute prices.

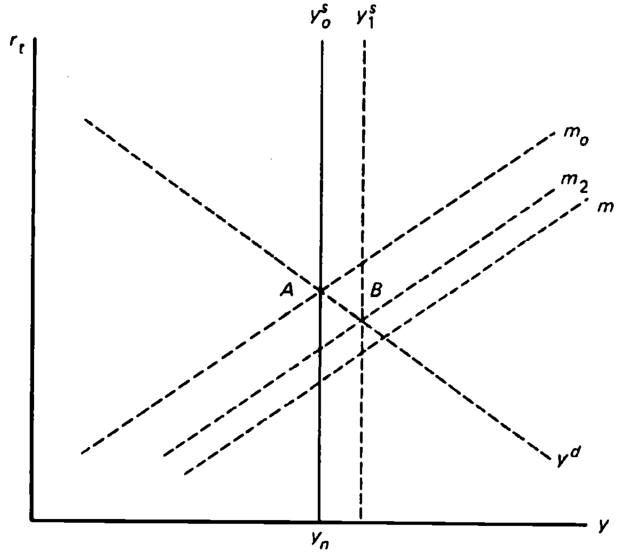

FIGURE 5-1. A Geometrical Interpretation of the Model

Therefore, output changes are associated only with price misperceptions. Then in our case a new, temporary equilibrium with a higher level of output and a lower interest rate is attained through a misperception effect that shifts y_0^s to y_1^s. A point of temporary equilibrium like B can be reached with m_1 shifting leftward to m_2 due to the price increase. The movement from B to A, that is, the long-run equilibrium position, will depend upon the process of adjustment of expectations. The same apparatus can be used to analyze other types of shocks.

To complete the model we should specify how expectations are formed. This is a delicate matter. It has been customary to postulate different ad hoc hypotheses about the formation of expectations. The most popular is Cagan's hypothesis of adaptive expectations although the explanations for its use were confined to the fact that adaptive expectations seemed reasonable and proved useful in explaining data. The hypothesis of rational expectations used in this chapter follows Muth's proposal that expectations are informed predictions of future events based on the available information and the relevant economic theory. This has a strong implication. With this assumption the economist who is modeling an economy does not have a superior knowledge of the "reality." This in turn is confirmed by the fact that actual expectations "are more accurate than naive models and as accurate as elaborate equations systems" (see Muth

[1961], p. 316). Thus, our model is completed with the following equations.

$$ {}_t p^*_{t-1} = Ep_t \tag{5-4} $$

$$ {}_{t+1} p^*_{t-1} = Ep_{t+1} \tag{5-5} $$

where Ep_t is the conditional mathematical expectation of p_t formed using the model and all the information assumed to be available as of the end of period $t - 1$ (hereafter the E operator will always be conditional on the information available as of the end of period $t - 1$).[2]

The algebra of rational expectations is tedious and is confined to Appendix A. The following two equations correspond to Equations (A5-12) and (A5-13) of Appendix A and they show expected prices as a function of the past and expected future behavior of the money supply. Indeed, this property is an essential feature of rational expectations. The equations are.

$$ EP_t = [1/(1-b)] \sum_{j=0}^{\infty} [1/(1-b^{-1})]^j \, [E\phi m_{t+j} - y_{n,t+j}] \tag{5-6} $$
$$ + [J_3/(1-J_0)] \sum_{j=0}^{\infty} [\alpha/(1-b^{-1})]^j \, y_{c,t-1} + c_0 $$

and

$$ Ep_{t+1} = [1/(1-b)] \sum_{j=0}^{\infty} [1/(1-b^{-1})]^{j+1} \, [E\phi m_{t+j+1} \tag{5-7} $$
$$ - y_{n,t+j+1}] + [J_3/(1-J_0)] \sum_{j=0}^{\infty} [\alpha/(1-b^{-1})]^{j+1} \, y_{c,t-1} + c_0 $$

In these equations J_0, J_3, and c_0 are constants that are complicated functions of the structural parameters of the model.

From these equations, it is easy to illustrate the process of formation of expectations; let us assume for a moment that $b = 0$ (i.e., that the interest elasticity of the demand for money is zero). Then, after taking first differences (D operator), Equation (5-6) can be reduced to:

$$EDp_t = \phi_0\ EDm_t - (\beta + kDy_{c,t-1}) + \phi_1 Dm_{t-1} + \phi_2 Dm_{t-2} + \ldots \qquad (5\text{-}8)$$

where β is the trend in real output and $J_3/(1 - J_0) = -k$ when $b = 0$. Equation (5-8) clearly shows that the expected rate of change of prices depends upon the expected rate of change in the money supply in period t, the trend rate of growth in output (β), a term in the cyclical component of output in $t - 1$, and past rates of change of the money supply. If $b \neq 0$, then the results are not far from the quantity theory in expectation form although the algebraic expression representing the expectation formation process is more complicated.

The money supply on the basis of which the public makes its forecasts of the future path of m_{t+j} is of particular relevance. The empirical analysis of Section 4 considers two processes as determining the money supply; an ARIMA process that in its "inverted form" is

$$Dm_t = \pi_1 Dm_{t-1} + \pi_2 Dm_{t-2} + \pi_3 Dm_{t-3} + \ldots + v_{3t} \qquad (5\text{-}9)$$

where the πs are parameters. The second process will be a model of the form

$$Dm_t = \pi_t' Dm_{t-1} + \pi_2' z_t + u_t \qquad (5\text{-}10)$$

where π_1' can be a parameter or a polynomial in the lag operator and π_2' can be a row vector of parameters or a row vector of polynomials in the lag operator while z_t is a column vector of predetermined variables.[3]

The empirical tests will not be carried out directly in the form of Equations (5-9) and (5-10) but indirectly through the transfer functions of the next section.

3. TOWARD AN EMPIRICAL TEST OF THE MODEL

In this section we outline the method followed in order to get the estimates for the structural equations of the model. Thus two points are jointly developed, one is the computation of expected prices and the other is the endogeneity of p_t that precludes the straightforward estimation of Equation (5-1) using ordinary least squares.

At the estimation stage we shall concentrate on Equations (5-1) and (5-3). The main problem with equation (5-2) is the variable r_t for which data do not exist in Argentina and Brazil. This problem is eliminated in Equation (5-3) because it is assumed that variation in r_t is dominated by variation of the expected rate of inflation and the public's forecast of the rate of inflation (based on information available at $t - 1$) is used as a proxy for r_t. It is obvious that this substitution cannot be made in Equation (5-2) because the term $(r_t - ({}_{t+1}p^*_{t-1} - {}_tp^*_{t-1}))$ would vanish if r_t were replaced by ${}_{t+1}p^*_{t-1} - {}_tp^*_{t-1}$. Nevertheless, the system formed by Equations (5-1) and (5-3) is perfectly determined when a proxy is used for r_t, let us say, $r'_t = Dp^*_t = {}_{t+1}p^*_{t-1} - {}_tp^*_{t-1}$.

For convenience, we write Equations (5-1) and (5-3) again:

$$y_{c,t} = a(p_t - {}_tp^*_{t-1}) + ky_{c,t-1} + u_{1t}, \ a > 0 \tag{5-1}$$

$$p_t + y_t = \phi m_t - br'_t - u_{3t}, \qquad -\infty \ b < 0 \tag{5-3}$$

where $y_{c,t} = y_t - y_{n,t}$ (detrended output).

Let us start with Equation (5-1). We know that a direct estimation of this equation is not possible because p_t and y_t are jointly determined and ${}_tp^*_{t-1}$ is not observable. Thus, in this section our objective is to obtain an estimable relationship to replace (5-1), making use of the relationships previously developed.

Let us consider first the case of ${}_tp^*_{t-1}$ that, as we said, is unobservable. Rational expectations imply that the actual log of the price level differs from the expected value by a random component, let us say, u_{4t}, so we can write

$$p_t = Ep_t + u_{4t} \tag{5-14}$$

Now what we need is a process for forming expectations that, using the available information, the model's structure, and suitable coefficients, yields an unbiased forecast of p_t. This is obtained through some algebraic manipulations that are shown in Appendix B. The relevant equation for our ends is one in which the actual change in prices is a function of lagged variables as follows:

$$Dp_t = v(L)\,LDm_t + h_0 Dy_{c,t-1} + c + u_{5t} \tag{5-14a}$$

where $v(L)$ is polynomial in the lag operator and c and h_0 are parameters.

To estimate Equation (5-14a), we have to consider the problem of collinearity, especially in the case of quarterly data, where a reasonable lag of two years would imply that m_t should be lagged eight times. A way of dealing with Equation (5-14a) is to consider it to be a multiple input transfer function. The transfer function form of Equation (5-14a) can be parsimoniously (in terms of the number of parameters) represented by

$$Dp_t = \frac{v_1(L)}{\alpha_1(L)} LDm_t + \frac{v_2(L)}{\alpha_2(L)} LDy_{c,t} + \frac{0(L)}{\theta(L)} u_t + c \qquad (5\text{-}15)$$

The estimation of Equation (5-15) can be done using the Marquardt algorithm, and the forecast made using the estimated version of Equation (5-15) is a minimum mean square error forecast. Thus the problem of ${}_tp^*_{t-1}$ being unobservable is solved using the forecast yielded by the estimated version of Equation (5-15) over the sample period. In other words, from Equation (5-15) we can construct a series of "expected prices." The reader should realize that our Equation (5-15) substitutes for the popular hypothesis of adaptive expectations that constructs forecasts from an exponentially weighted, distributed lag polynomial.

Although, Equation (5-15) can be used to obtain an estimate of the unobservable ${}_tp^*_{t-1}$, Equation (5-1) still cannot be estimated directly because of the simultaneity between p_t and $y_{c,t}$. To solve this problem we follow a "two-stage" method, computing estimates for p_t from a "reduced-form equation" for p_t. A reduced form for p_t is given by Equation (A5-3) of Appendix A. Combining this equation with Equations (5-6), (5-7), and (5-9) and following the same algebraic manipulations to obtain (5-15) we get

$$Dp_t = \frac{\omega'_1(L)}{\alpha'_1(L)} Dm_t + \frac{\omega'_2(L)}{\alpha'_2(L)} LDy_{c,t} + \frac{\theta'(L)}{\phi'(L)} u'_t + c' \qquad (5\text{-}16)$$

where the meaning of the notation is the same as in Equation (5-15). Notice that the main difference between (5-15) and (5-16) is that in (5-15) Dm_t appears lagged one period.

Now let me recall that Equation (5-9) represents the hypothesis that the money supply follows an ARIMA process. If we use the assumption (5-10) for the money supply, then Equations (5-15) and (5-16) should be extended to include terms in the components of z_t.

Although the algebraic analysis is rather long, its intuitive interpretation is quite simple and straightforward. The rational expectation feature of the model implies that the public forms their expectations using the information available as of the end of period $t - 1$. In forming these expectations the money supply expected to prevail in future periods is important, and it is assumed that the public forecasts future values of m_t by considering the history of m_t available at $t - 1$ (as well as other variables if (5-10) is used). But the history of m_t is not only relevant for forecasting future values; the recent past values of m_t also directly affect the price level because of the lagged response of prices to changes in the money supply. This is also considered in the expectations formation process. Equation (5-15) is oriented to capture this process.

Equation (5-16), although very similar to (5-15), is quite different. It is a reduced form for p_t implied by the system (5-1)-(5-5) and the assumption in (5-9) or (5-10) for the money supply. In (5-16), m_t directly affects the price level. The economy as a whole need not forecast m_t; it is an exogenous variable determined by monetary authorities in period t that will have an immediate effect on p_t.

The fitted values for p_t from (5-16) will be introduced in (5-1) in place of p_t and the fitted values of (5-15) will be introduced in (5-1) in place of ${}_tp^*_{t-1}$ in order to estimate Equation (5-1).

Consider now Equation (5-3). This equation assumes that the real income elasticity of demand for money is one. This assumption need not be maintained since all the previous algebraic expressions can be rearranged to include an additional parameter (the real income elasticity of the demand for money). Hereafter we will relax this assumption writing (5-3) as

$$y_t = \phi m_t - br'_t - u_{3t}$$

where $y_t = p_t + iy_t$, i representing the real income elasticity of demand for money. It should be noticed that if $i = 1$, then y_t is the log of nominal income. Then the system (5-1) - (5-3) can be interpreted as follows. Equation (5-3) determines nominal income and Equation (5-1) determines the division of nominal income between changes in prices and changes in output.

For analyzing the cases in which $i \neq 1$, we will evaluate the results for three possibilities: $i = 0.5$, 1.5, and 2. An attempt was made to estimate i using an instrumental variable for Dy_t and an ARIMA process; however, the results were not reliable because y_t behaves almost like a random walk. At the estimation stage, Equation (5-3) will be expressed in the form of a transfer function with all variables in first differences. The transfer function form will allow us to estimate the lag operator ϕ.

4. EMPIRICAL RESULTS

In this section we proceed to test and estimate the model presented in Sections 2 and 3 with the available data for Argentina and Brazil. First, we construct a series of expected prices on the basis of the results obtained in fitting Equation (5-15). Secondly, we construct a series of actual prices from the reduced form for prices, that is, Equation (5-16) (recall that this step is necessary in order to avoid the problem of simultaneity in estimating Equation (5-1). "Actual prices" minus expected prices give us the misperceptions of prices that are needed to estimate Equation (5-1). Finally, we estimate Equation (5-3) under different assumptions with respect to the real income elasticity of the demand for money and using a proxy for the interest rate.

4.1 The Data

All the data for Argentina were obtained from International Financial Statistics (International Monetary Fund). They include quarterly data for the index of industrial production, wholesale prices, currency and demand deposits, wages set in collective bargaining, and the balance of trade (all seasonally adjusted by the method of moving averages). The observations relate to the period 1956-I to 1973-II (this period was chosen in order to base the analysis on the maximum number of observations available for the index of industrial production).[5]

The log of the index of industrial production for Argentina was detrended splitting the data into two parts: from 1956-I to 1962-IV and from 1963-I to 1973-II. This was done because in the first period there is no apparent trend in real output and if a single trend line were fitted to the whole period we would lose most of the cyclical fluctuations.[6]

The data for Brazil were obtained from two sources: International Financial Statistics (IMF) and Goncalves (1974). From International Financial Statistics we obtained the series of wholesale prices (excluding coffee) and currency and demand deposits. From Goncalves we obtained a series of real output. All the observations relate to the period 1955-I to 1971-IV (this period was chosen in order to base the analysis on the maximum number of observations available for real output). The data were seasonally adjusted by the method of moving averages.

All the variables were expressed in first differences of logs prior to estimation except in the case of balance of trade. This variable was computed as the log of exports minus the log of imports (this because of the impossibility of taking the log of a negative number in the case of trade deficits).

The estimation of transfer functions was carried out using Marquardt's (1963) algorithm.

4.2 Estimates of the Transfer Function for Expected Prices

In Table 5-1 we present the estimates obtained for Equation (5-15), which is the expression that determines expected prices.[7] These models have been selected from a larger number of models with different lag structures and different error terms. The selection has been carried out using the likelihood ratio test proposed by Zellner and Palm (1974) (see Fernandez [1975] for a description and application of this test).

Models (1) and (2) for Argentina assume that the money supply follows a process as represented by Equation (5-9) and model (3) considers the assumption implied by Equation (5-10). In model (3) we have computed the transfer function with wages and balance of trade as input variables. We notice from Table 5-1 that in the case of Argentina there is a slight reduction in the *RSS/DF* and a small increase in the adjusted R^2 when passing from model (1) or (2) to model (3).[8]

At the bottom of the table we present the results obtained for Brazil, where an insignificant reduction in the *RSS/DF* occurs when we go from the simple lag structure of model (1) to the more complex lag structure of model (2).

4.3 Estimates of the Reduced Form for Prices

Table 5-2 shows the estimates of the transfer functions for prices (i.e., Equation (5-16). Here again, for the case of Argentina models (1) and (2) incorporate assumption (5-9) for the money supply while model (3) incorporates assumption (5-10). The models (1) and (2) are the best results obtained for each hypothesis regarding the money supply. In both Table 5-1 and Table 5-2, the coefficient of the balance of trade variable is significantly different from zero at the 5 percent level. Only in Table 5-2 does the balance of trade variable have a coefficient estimate with an algebraic sign that the theory predicts (i.e., positive sign).

For Brazil we observe again that no appreciable reduction in the *RSS/DF* is obtained in going from the simple lag structure of model (1) to the more complex lag structure of model (2).

In both Table 5-1 and Table 5-2, the estimates for the variables $DY_{c,\ t-1}$ and dummy or constant are small numbers not significantly

Table 5-1. Estimated Transfer Functions for Expected Prices

Model	Residual Sum of Squares (RSS)	Degree of Freedom (DF)	RSS/DF	Estimates of the AR and MA Parts of Dm_{t-1}	Estimates of $Dy_{c,\ t-1}$	Dummy or constant	Wages	Bal. of Trade	Estimates of the AR and MA Parts of u_t	Adjusted R^2
					Argentina 1956-I-1973-II					
(1)	0.122293	59	0.00207	$\dfrac{\underset{(0.129)}{0.492}}{1 - \underset{(0.082)}{1.041L} + \underset{(0.087)}{0.785L^2}}$	-0.006 (0.094)	0.011 (0.012)			$\dfrac{1}{1 - \underset{(0.133)}{0.402L} - \underset{(0.130)}{0.252L^2}}$	0.47
(2)	0.117639	57	0.00206	$\dfrac{\underset{(0.166)}{0.693} - \underset{(0.221)}{0.317L}}{1 - \underset{(0.135)}{1.112L} + \underset{(0.102)}{0.719L^2}}$	-0.060 (0.104)	0.011 (0.013)			$\dfrac{1}{1 - \underset{(0.135)}{0.376L} - \underset{(0.131)}{0.271L^2}}$	0.48
(3)	0.116154	59	0.00197	$\dfrac{\underset{(0.146)}{0.613}}{1 - \underset{(0.099)}{1.009L} + \underset{(0.098)}{0.741L^2}}$	-0.022 (0.091)	0.006 (0.009)	0.044 (0.081)	-0.004 (0.001)	$\dfrac{1}{1 - \underset{(0.124)}{0.436L}}$	0.49
					Brazil 1955-I-1971-IV					
(1)	0.066609	60	0.00111	$\dfrac{\underset{(0.122)}{0.188}}{1 - \underset{(0.174)}{0.723L}}$	-0.009 (0.077)	0.016 (0.024)			$\dfrac{1}{1 - \underset{(0.116)}{0.520L}}$	0.46
(2)	0.065183	59	0.00110	$\dfrac{\underset{(0.099)}{0.119}}{1 - \underset{(0.662)}{1.388L} + \underset{(0.578)}{0.556L^2}}$	-0.012 (0.078)	0.013 (0.023)			$\dfrac{1}{1 - \underset{(0.116)}{0.508L}}$	0.46

Note: This table presents the estimates of Equation (5-15). The terms *AR* and *MA* represent the autoregressive and moving average parts, respectively, of the rational polynomials. Figures in parentheses are large sample standard errors.

Table 5-2. Estimated Transfer Functions for Prices

Model	Residual Sum of Squares (RSS)	Degree of Freedom (DF)	RSS/DF	Estimates of the AR and MA Parts of Dm_t	Estimates of $Dy_{c,\,t-1}$	Dummy or constant	Wages	Bal. of Trade	Estimates of the AR and MA Parts of u_t	Adjusted R^2
				Argentina 1956-I-1973-II						
(1)	0.125758	59	0.00213	$\dfrac{\underset{(0.112)}{0.422}}{1 - \underset{(0.102)}{1.216L} + \underset{(0.103)}{0.773L^2}}$	0.049 (0.095)	0.004 (0.012)			$\dfrac{1}{1 - \underset{(0.133)}{0.400L} - \underset{(0.132)}{0.216L^2}}$	0.45
(2)	0.116639	57	0.00205	$\dfrac{\underset{(0.160)}{0.224} + \underset{(0.202)}{0.346L}}{1 - \underset{(0.104)}{1.073L} + \underset{(0.091)}{0.763L^2}}$	0.030 (0.105)	0.004 (0.012)			$\dfrac{1}{1 - \underset{(0.135)}{0.365L} - \underset{(0.132)}{0.280L^2}}$	0.48
(3)	0.113562	57	0.00199	$\dfrac{\underset{(0.180)}{0.207} + \underset{(0.229)}{0.435L}}{1 - \underset{(0.116)}{1.054L} + \underset{(0.098)}{0.749L^2}}$	0.010 (0.095)	-0.003 (0.010)	0.018 (0.084)	0.003 (0.001)	$\dfrac{1}{1 - \underset{(0.124)}{0.438L}}$	0.49
				Brazil 1955-I-1971-IV						
(1)	0.064908	61	0.00106	$\dfrac{\underset{(0.117)}{0.212}}{1 - \underset{(0.155)}{0.716L}}$	-0.004	0.010 (0.023)			$\dfrac{1}{1 - \underset{(0.113)}{0.511L}}$	0.47
(2)	0.064130	60	0.00107	$\dfrac{\begin{matrix}(0.119)\\(0.190)\end{matrix}}{1 - \underset{(0.630)}{1.389L} + \underset{(0.611)}{0.546L^2}}$	-0.005	0.008 (0.022)			$\dfrac{1}{1 - \underset{(0.115)}{0.498}}$	0.47

Note: This table presents the estimates of Equation (5-16). The terms *AR* and *MA* represent the autoregressive and moving average parts, respectively, of the rational polynomials.

different from zero. This is not in contrast with the theoretical model because it is shown in Appendix A that these parameters can indeed be close to zero (See Equations (A5-12) and (A5-13) in Appendix A.)

4.4 Estimates of the Aggregate Supply

Recall that Table 5-1 provides the estimates of Equation (5-15), which in turn allow us to obtain a series of "expected prices" needed to estimate Equation (5-1) of our original model. By the same token Equation (5-16), whose estimates are given in Table 5-2, provides us with a series of "actual prices" to estimate Equation (5-1). Then the next step is to compute a one step ahead forecast from (5-15) that would give us a proxy variable for ${}_tp^*{}_{t-1}$. Similarly a one step ahead forecast from (5-16) would give us a proxy for p_t. The difference between the proxy of p_t and the proxy for ${}_tp^*_{t-1}$ is introduced in Equation (5-1) in place of $(p_t - {}_tp^*_{t-1})$, and the estimation of this equation provides us with an estimate of the slope coefficient of our short-run Phillips equation. Table 5-3 shows the results obtained by this procedure and indicates the different models used for forecasting prices and reduced forms used for prices.

On testing the significance of the Phillips parameter for Argentina using a two-tailed test we notice that at the 5 percent level only the last regression shows an estimate significantly different from zero. Using a one-tailed test (i.e., the alternative hypothesis is that the parameter is greater than zero), estimates of the last three regressions provide evidence for rejecting the null hypothesis at the 5 percent level of significance. In all five cases the Box and Pierce Q statistic favors rejecting the hypothesis of autocorrelated residuals.[9]

Perhaps it is convenient at this stage to take a closer look at the estimates of Table 5-3. Recall that the estimate of parameter "a" is an estimate of the slope of the Phillips curve. Our results for Argentina indicate that there is some evidence in favor of a short-run tradeoff between inflation and output given by the 95 percent confidence intervals for the estimates of the third through fifth regressions. These are (-0.089, 0.991), (0.0, 1.384), and (0.001, 2.268), respectively. This short-run tradeoff is not in contrast to the natural rate hypothesis of Friedman because, as Equation (5-1) indicates, if prices are anticipated correctly output will remain in its long-run trend (or "natural" level).

These results cannot provide evidence either in favor of the naive Phillips curve approach or in favor of the Solow-Tobin analysis. The naive Phillips curve approach says that there is only one Phillips curve indicating a positive tradeoff between inflation and output regardless

Table 5-3. Estimates of the Aggregate Supply Equation

Model for Reduced Form	*Model for Expected Prices*	*a*	*k*	*Adjusted* R^2	*Q-Statistics*
		Argentina 1956-I - 1973-II			
(1) M_2	M_2	0.877 (0.594)	0.564 (0.102)	0.35	15.3
(2) M_2	M_1	0.647 (0.578)	0.574 (0.102)	0.34	16.5
(3) M_1	M_1	0.401 (0.295)	0.575 (0.102)	0.35	15.7
(4) M_1	M_2	0.692 (0.346)	0.569 (0.101)	0.36	14.2
(5) M_3	M_3	1.140 (0.564)	0.778 (0.102)	0.35	15.1
		Brazil 1955-I - 1971-IV			
(1) M_1	M_1	-0.272 (0.919)	0.664 (0.095)	0.430	15.1
(2) M_1	M_2	-0.492 (0.750)	0.660 (0.095)	0.433	16.0
(3) M_2	M_1	0.842 (1.328)	0.657 (0.095)	0.433	14.5
(4) M_2	M_2	-0.140 (1.390)	0.665 (0.095)	0.429	14.9
(5) Actual prices	M_1	0.168 (0.178)	0.634 (0.099)	0.437	15.8
(6) Actual prices	M_2	0.153 (0.172)	0.638 (0.099)	0.436	15.5

Note: Chi-square values from table:

$\chi^2(24) = 33.2$ 0.10 level of significance

$\chi^2(24) = 36.4$ 0.05 level of significance

The models used to represent prices and expected prices are symbolized in this table with the letter M and a subindex. Thus, M_2 in the column headed "Model for Reduced Form" means that model (2) of Table 5-2 is being used to represent actual prices in the aggregate supply equation.

of expectations. The Solow-Tobin analysis says that people adjust to changes in prices, but they are subject to some money illusion that allows for a permanent tradeoff between inflation and output. Both of these hypotheses are ruled out by the specification that when actual prices are equal to expected prices output will remain at its long-run natural level.

For Brazil we notice that in the first, second, and fourth equations the estimate of "a" is negative although not significantly different from zero at the 0.05 level in a two-tailed test. The third regression presents the right sign but its a estimate has a large standard error that makes it not significantly different from zero. In all cases the value of the Q statistics favor rejection of the hypothesis of autocorrelation in the residuals.

In order to compare our results with other results obtained for Brazil by Goncalves (1974) we estimated the last two models of Table 5-3 using actual prices instead of the forecast of the reduced form for prices. Goncalves performed a similar estimation under the assumption that the price level was exogenously determined (mainly due to strongly enforced price controls in most of his period of analysis). He worked with the period 1959-1969 and used another hypothesis for expectations formation. His results provide an estimate of a equal to 0.41 (standard errors are not reported in his work). Another of his results shows a equal to 0.27 when a dummy variable is included with a value of unity from 1961-I to 1963-II and zero elsewhere (this dummy variable is supposed to capture the effect of price controls). It should be noted that this last result, obtained by Goncalves, is close to the last two models of Table 5-3. From our results for Brazil we must conclude that the empirical evidence does not favor a stable short-run tradeoff between output and inflation even in the short run.[10]

4.5 Estimates of the Transfer Function for Nominal Income

Now we proceed to the estimation of Equation (5-3) of our original model. Recall that in this equation we are using nominal income as the dependent variable when the real income elasticity of the demand for money is assumed equal to one, and we are using as dependent variable the term $p + iy$ where i is the real income elasticity for all the cases in which it is assumed that $i \neq 1$.

In addition, we are using the first difference in the one step ahead forecast for prices from model (1) of Table 5-1 as a proxy for the nominal rate of interest. The estimates for these transfer functions are presented in Table 5-4. In this table it is shown that for Argentina when i is greater than one both the degrees of polynomials estimated and the RSS/DF, are higher than when i is equal to or lower than one. I have no explanation for this except, as mentioned above, Dy_t is a very noisy series and as i becomes large it magnifies the noise of the series of "nominal income." The last transfer function reported for Argentina in Table 5-4 includes a second-order autoregres-

Table 5-4. Estimated Transfer Functions for Nominal Income

Model	*Residual Sum of Squares (RSS)*	*Degree of Freedom (DF)*	*RSS/DF*	*Estimates of the AR and AM Parts of* Dm_{t-1}	*Estimates of the AR and MA Parts of* $Dr^1{}_t$	*Estimates of the AR and MA Parts of* u_t	*Adjusted* R^2
				Argentina 1956 I - 1973 II			
(1) (i = 1.5)	0.249863	56	0.00446	$\dfrac{\underset{(0.269)}{1.021} - \underset{(0.311)}{0.264L} + \underset{(0.247)}{1.283L^2}}{1 + \underset{(0.208)}{1.739L} + \underset{(0.234)}{0.596L^2}}$	0.565 (0.311)		0.27
(2) (i = 1)	0.140220	58	0.00242	$\dfrac{\underset{(0.149)}{0.406} - \underset{(0.182)}{0.147L}}{1 - \underset{(0.120)}{1.496L} + \underset{(0.106)}{0.737L^2}}$	0.504 (0.213)		0.38
(3) (i = 0.5)	0.139771	58	0.00241	$\dfrac{\underset{(0.165)}{0.424} - \underset{(0.212)}{0.130L}}{1 - \underset{(0.120)}{1.436L} + \underset{(0.106)}{0.737L^2}}$	0.475 (0.215)		0.38
(4) (i = 0.5)	0.121832	54	0.00225	$\dfrac{\underset{(0.194)}{0.442} - \underset{(0.260)}{0.141L}}{1 - \underset{(0.176)}{1.407L} + \underset{(0.141)}{0.722L^2}}$	0.266 (0.216)	$\dfrac{1}{1 - \underset{(0.141)}{0.276L} - \underset{(0.150)}{0.171L^2}}$	0.41

Table 5-4 continued

Model	*Residual Sum of Squares (RSS)*	*Degree of Freedom (DF)*	*RRS/DF*	*Estimates of the AR and AM Parts of Dm_{t-1}*	*Estimates of the AR and MA Parts of Dr^1_t*	*Estimates of the AR and MA Parts of u_t*	*Adjusted R^2*
				Brazil 1955 I – 1971 IV			
(1) (i = 1.5)	0.377770	57	0.00593	$\dfrac{\underset{(0.348)}{0.212} + \underset{(0.526)}{0.103L} + \underset{(0.521)}{0.348L^2}}{1 - \underset{(0.422)}{0.383L}}$	$\underset{(0.516)}{-0.093}$		0.093
(2) (i = 1)	0.182688	57	0.00320	$\dfrac{\underset{(0.255)}{0.253} - \underset{(0.389)}{0.272L} + \underset{(0.359)}{0.356L^2}}{1 - \underset{(0.306)}{0.419L}}$	$\underset{(0.380)}{0.115}$		0.171
(3) (i = 0.5)	0.095822	57	0.00168	$\dfrac{\underset{(0.184)}{0.292} - \underset{(0.285)}{0.161L} + \underset{(0.243)}{0.366L^2}}{1 - \underset{(0.213)}{0.466L}}$	$\underset{(0.375)}{0.723}$		0.293
(4) (i = 0.5)	0.086180	53	0.00162	$\dfrac{\underset{(0.182)}{0.197} - \underset{(0.272)}{0.061L} + \underset{(0.251)}{0.221L^2}}{1 - \underset{(0.226)}{0.613L}}$	0.085	$\dfrac{1}{1 - \underset{(0.167)}{0.223L} - \underset{(0.169)}{0.169L^2}}$	0.342

sive process for the error term that yields an appreciable reduction in the *RSS*.

For Brazil when the real income elasticity is relatively large (1 or 1.5) the adjusted R^2s are low. The best explanation is obtained with $i = 0.5$ and with a second-order polynomial in the disturbance term.

It should be noticed that if the estimate for the parameter "a" is assumed to be zero and if the real income elasticity of the demand for money is assumed to be one, our system is reduced to a special formulation of Friedman's theory of nominal income.

This can be explained as follows: If "a" is assumed equal to zero Equation (5-1) can no longer be used to break down the changes in nominal income obtained from Equation (5-3) into changes in prices and output. Thus the system explains only nominal income.

In this case, Equation (5-15) determines the price expectations (still under the hypothesis of rational expectations) that would dominate the changes in the nominal rate of interest in Equation (5-3). Let me recall that from Equation (5-15) we obtain the proxy Dr_t' for the nominal rate of interest.

Section 5, which analyzes the short-run dynamics of prices and output, makes use of the estimates of this section. In choosing the estimates that will represent our model we make use of models (1) of Table 5-1 and Table 5-2 for Argentina and models (1) of Table 5-1 and models (2) of Table 5-2 for Brazil—the models that most appropriately represent the process for expected prices and prices, respectively, under the hypothesis of Equation (5-9) for the money supply. This implies that the third equation of Table 5-3 (for both Argentina and Brazil) was used to represent the Phillips equation. Finally the second model for Argentina ($i = 0.5$, second order regressive error) and the first model for Brazil ($i = 0.5$) of Table 5-4 were used as transfer functions for nominal income.

5. The Short-Run Dynamics of Output and Prices

In this final section we analyze the short-run behavior of a system like that developed in Section 2. In doing so we will perform a deterministic simulation of a change in the rate of growth of the money supply in order to observe the short-run adjustment of the endogenous variables of the system.

By the adjustment process we mean the path followed for the variables from one long-run equilibrium position to another long-run equilibrium position when an exogenous force shocks the system. In our simulations the shock will be a shift in the rule governing the money supply. These long-run equilibrium positions have been already stated in the literature for models of this kind. So, for example, if the real income elasticity of the demand for money is unity and if we change the rate of growth of the money supply from 3 percent to 10 percent, then the long-run equilibrium position of nominal income will shift from 3 percent to 10 percent (this is no more than the quantity theory; for a discussion of this proposition see Friedman [1971], pp. 56–58). The question to be answered by the analysis of the short-run dynamics of the system is how the endogenous variables, for example, nominal income, moves from the 3 percent position to the 10 percent position.

In order to illustrate the short-run dynamics of prices and output implied by this model we use three different simulations concerning the behavior of the money supply. However, before proceeding, it is necessary to emphasize the circumstances under which these simulations can cast some light on the short-run dynamics of prices and output. Our model, under the hypothesis implied by equation (5–9) about the money supply, states that all the parameters of equation (5–15) are stable as long as the process followed by the money supply is the same. That is, if the money supply has followed an ARIMA (3, 1, 2) process, then the forecasts will be accurate as long as this process stays the same, mainly because the people compute their expectations as if they knew the process ARIMA (3, 1, 2) governing the money supply. If we change this process then the parameters of equation (5–9) will eventually change and consequently the parameters of equation (5–15). This can be proved mathematically. However, there also is a clear intuitive explanation. We cannot expect that people will continue indefinitely to form their expectations on the basis of one process ARIMA (3, 1, 2) for the money supply, when the monetary authorities have changed the rule governing the money supply to, let us say, a rate of growth of m_t at 10 percent per period. If this second rule has been in operation for a long enough time period, then in computing their expectations people will use the process $Dm_t = 0.10$ instead of the previous ARIMA process. From this discussion it should be clear that the implicit assumption that all the parameters of the model are constant while we change the rule governing the money supply, is a strong assumption, particularly if we want to analyze long periods of time.[11] Nevertheless, we assume that there will be a transition period during which people will utilize something approximating the old process. That is, perhaps the past is full of promises and attempts from the government or monetary authorities that prices will remain stable or that the rate of inflation will be lower or that monetary emission will slow down. Moreover, there are probably costs in changing the rule. Therefore, people take some time in assuming that any change in the rule governing the money supply is permanent. It is precisely during this period that our simulations will be relevant.

Figures 5-2 to 5-4 refer to Argentina. The first simulation, illustrated in Figure 5-2, shows the paths followed by the rate of change in nominal income, the rate of inflation, and the rate of change in detrended output when the money supply is shifted from a rate of growth of 0 percent to a rate of growth of 10 percent per period. The convergence to the new steady state is oscillatory for the three variables. Inflation accelerates during the first year, reaching a peak at the end of the fourth quarter; during the second year inflation slows down and then accelerates again reaching a second peak at the thirteenth quarter. The rate of change in detrended output also accelerates during the first year, but it peaks one period later than inflation; therefore during the first quarter of the second year output increases while the rate of inflation decreases. Similarly, the first trough of output is two periods later than the trough in the rate of

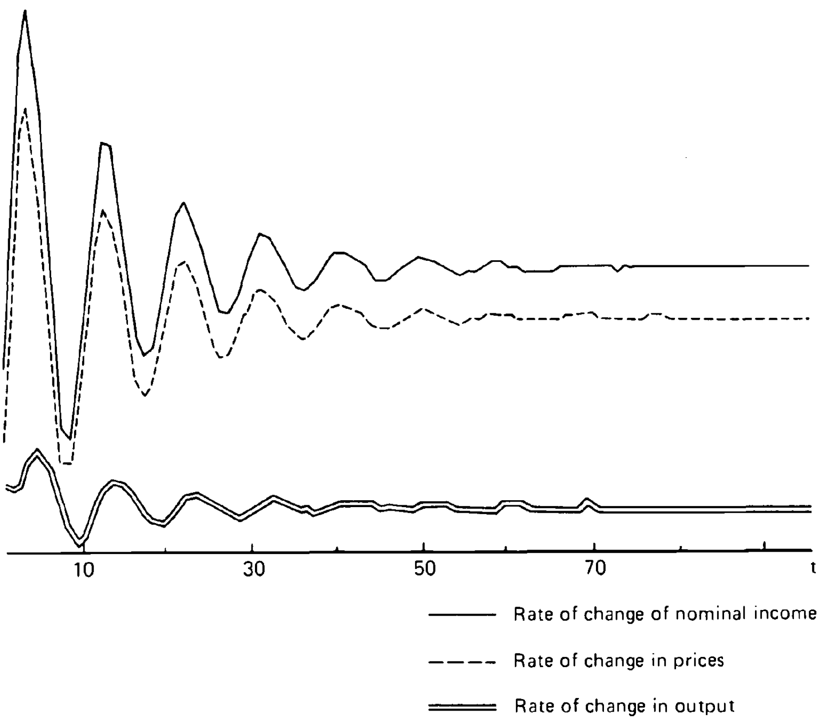

FIGURE 5-2. Simulation 1 for Argentina

inflation, and thus we observe inflation accelerating and output probably decreasing, a phenomenon known as stagflation. It should be noticed that from quarter 8 to 12 the rate of change in detrended output is negative so output will tend to be below the trend and consequently the unemployment rate above its natural level, while the rate of inflation is accelerating; this would be an illustration of a lower part of a counterclockwise loop in the conventional Phillips curve analysis. The other parts of the loop are readily observed in the following quarters as well as in the previous quarters.[12]

The second simulation, that is illustrated in Figure 5-3, assumes the rate of growth of the money supply shifts from 0 to 10 percent from period 1 to 30, and then it is shifted back to zero in period 31 and kept at that level thereafter. In this case we observe that the paths toward the final equilibrium levels of the variables oscillate and that a deep trough in the rate of inflation is reached four quarters after the reduction in the money supply while the trough in output is reached five periods after. One interesting aspect is illustrated in the four quarters between period 34 to period 37; during these quar-

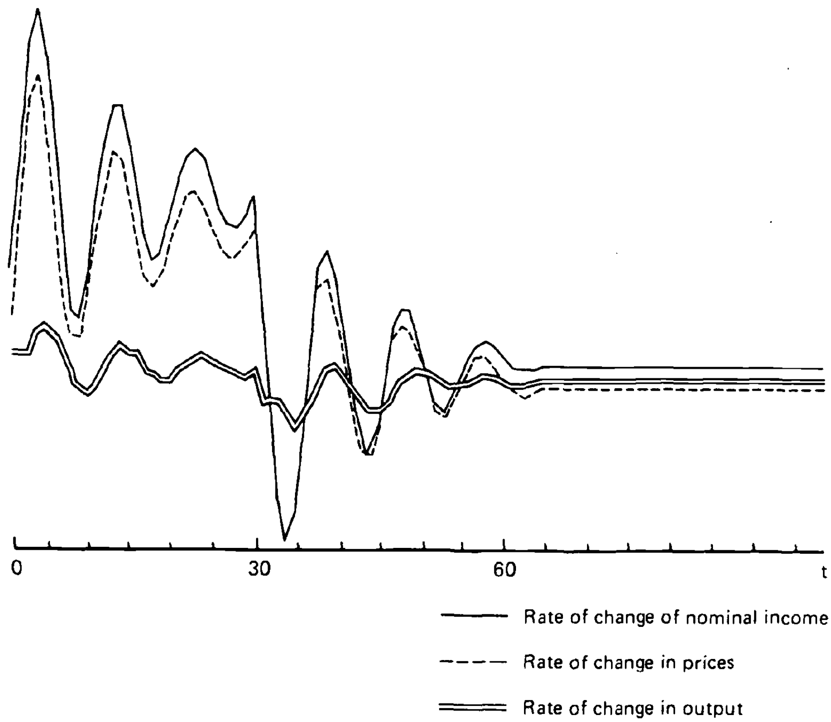

FIGURE 5-3. Simulation 2 for Argentina

ters output is certainly below the trend and consequently we should expect a relatively high unemployment rate. At the same time inflation is accelerating; this is a time when many people could think that the "old remedy to cure inflation does not work" because the reduction in the money supply not only has increased unemployment but also the rate of inflation is accelerating.

The third simulation is similar to the second but in place of an abrupt reduction in the money supply in period 31 we reduce the money supply to 8 percent in the first year, to 6 percent in the second year, and so on.

We observe that the fall in output is not as abrupt as it was in the previous case. In Figure 5-3 the trough in period 34 reached the value -3.4 percent while in Figure 5-4 the trough in period 47 reached the value 1.8 percent. In addition, it should be noticed that the convergence to the new steady state does not exhibit the large

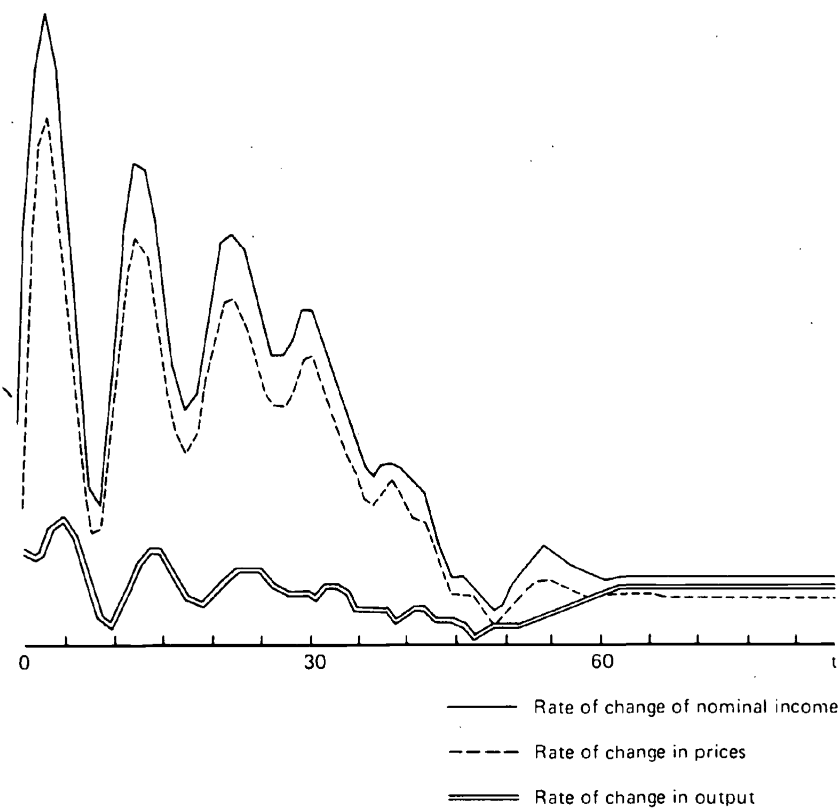

FIGURE 5-4. Simulation 3 for Argentina

oscillations of the previous case; that is, in this case convergence is smoother.

Figures 5-5, 5-6, and 5-7 illustrate the same types of simulations for the case of Brazil. In the first simulation (see Figure 5-5) we observe that the shift of the rate of growth of the money supply from 0 percent to 10 percent produces an initial overshoot of nominal income, prices, and output but after a few oscillations they converge to their long-run equilibrium values. We observe that nominal income and output reach a peak a quarter before prices; then during the fifth quarter we observe prices accelerating and output slowing down.[13]

The second simulation for Brazil (see Figure 5-6) shows a large fall in output produced by an abrupt shift of monetary policy from a rate of growth of the money supply of 10 percent to a rate of growth of 0 percent.

Finally, the third simulation for Brazil illustrates the advantages of gradualism in stabilizing the economy (this same conclusion is reached

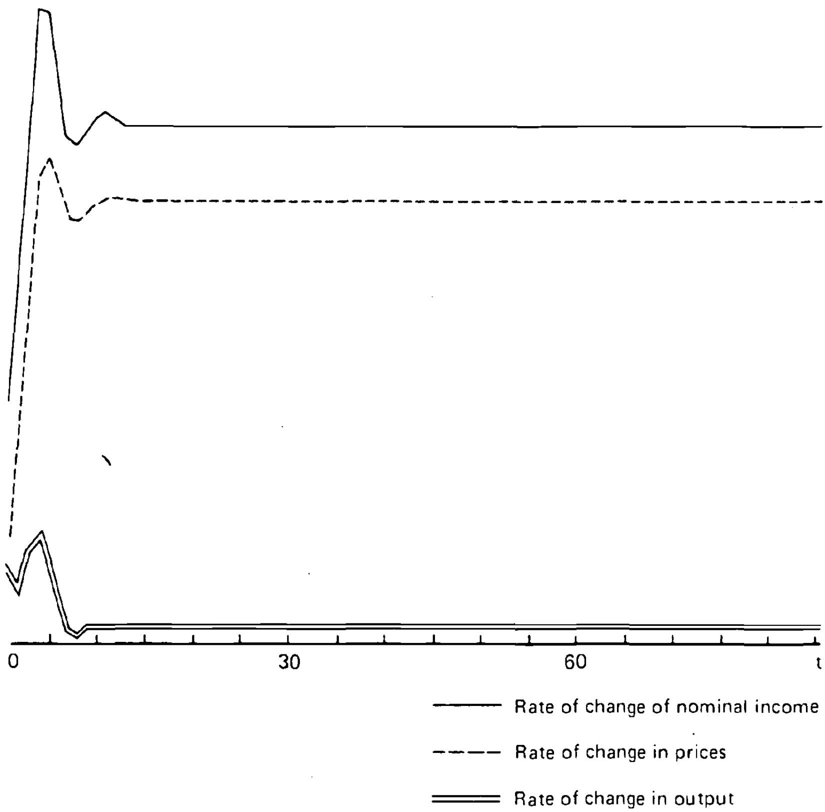

FIGURE 5-5. Simulation 1 for Brazil

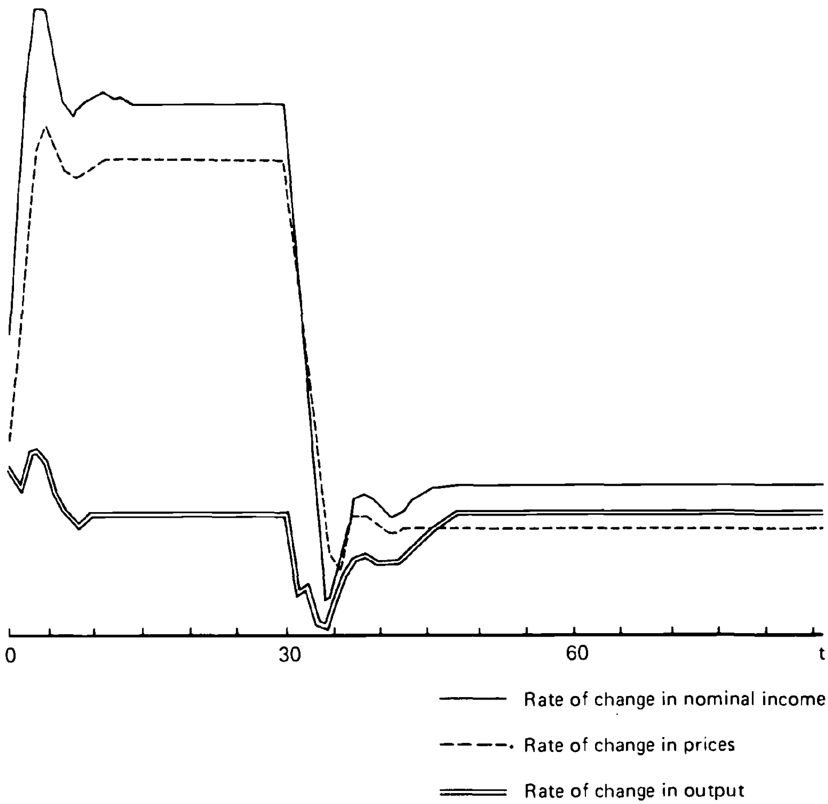

FIGURE 5-6. Simulation 2 for Brazil

by Goncalves [1974] for Brazil although in the context of a different model). We observe from Figure 5-7 that during the stabilization period the fall in output is smaller than in the previous simulation that assumed an abrupt change of monetary policy.

6. CONCLUSIONS

As indicated in the title of this paper we have tried to explain the short-run dynamics of prices and output. An indicator of the degree to which this objective has been achieved could be the part of the variance in prices and output that has been explained by the model. In other words, we could look at the R^2s obtained in our transfer functions or regressions. For the case of Argentina, the R^2s for prices and nominal income have been close to 0.50 while for detrended output the R^2s have been on the order of 0.35.

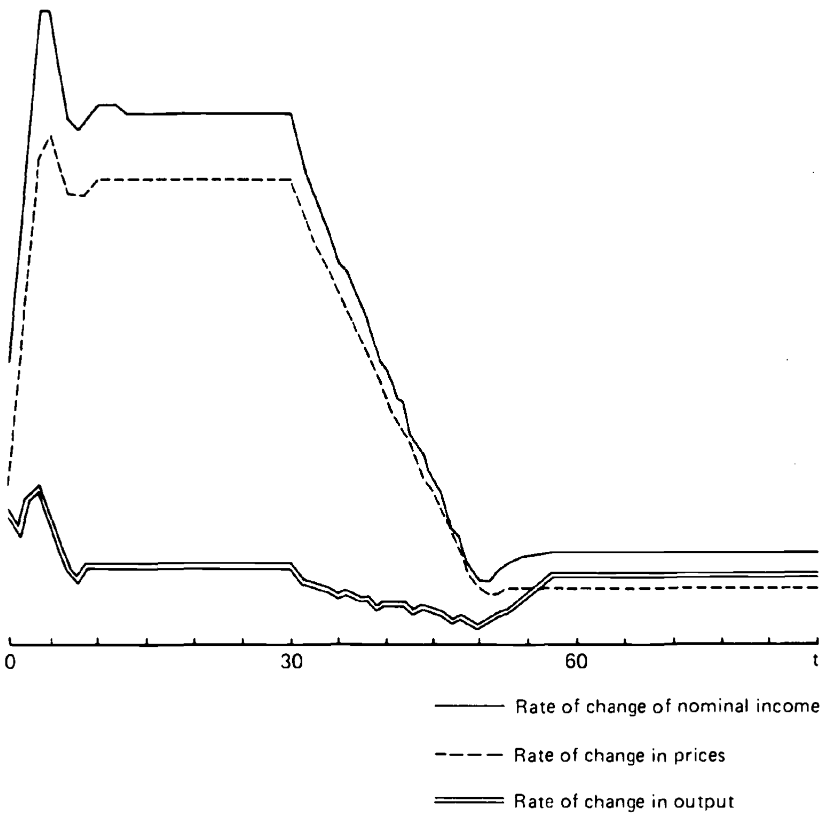

FIGURE 5-7. Simulation 3 for Brazil

For Brazil we obtained R^2s around 0.45 for prices, 0.30 for nominal income, and 0.43 for detrended real income.

Other indicators are the standard error of the estimates and the t values. Standard errors have been reported in the tables of Section 4. Not all the estimates of the parameters are significantly different from zero at the 0.05 level but many of them are indeed significantly different from zero at the 0.05 level in a two-tailed test. Other estimates are small in absolute value and not significantly different from zero—for example, in the case of the estimates of $Dy_{c,t-1}$ and c in the transfer functions for prices and expected prices—however, this does not contradict the theoretical model. As mentioned above, these parameters can be close to zero. Finally, there are other parameters that have large standard errors, in particular, the slope coefficient of our short-run Phillips curve, indicating that this relationship is empirically unstable.

In general, the estimates for Argentina are more precise than the estimates obtained for Brazil. In both countries better fits were obtained for the rate of change in prices than for detrended income. The good performance of the model in explaining the rate of inflation can be illustrated by plotting the actual and fitted values from the reduced form for prices. This is shown in Figure 5-8 for Argentina. Here we observe that the model behaves well in explaining inflation, and for only two observations—one near the beginning and one near the end of the period—do the observed rates of inflation differ substantially from the fitted values.

Figure 5-9 illustrates the case of Brazil. Here we also observe the good performance of the model in explaining the large oscillations of the rate of inflation. Only for a few observations near the middle of the period do actual values differ substantially from the fitted values.

Although our results seem to be good relative to many other empirical studies working with highly noisy quarterly series, we still are not sure that we have really separated the true signal from the

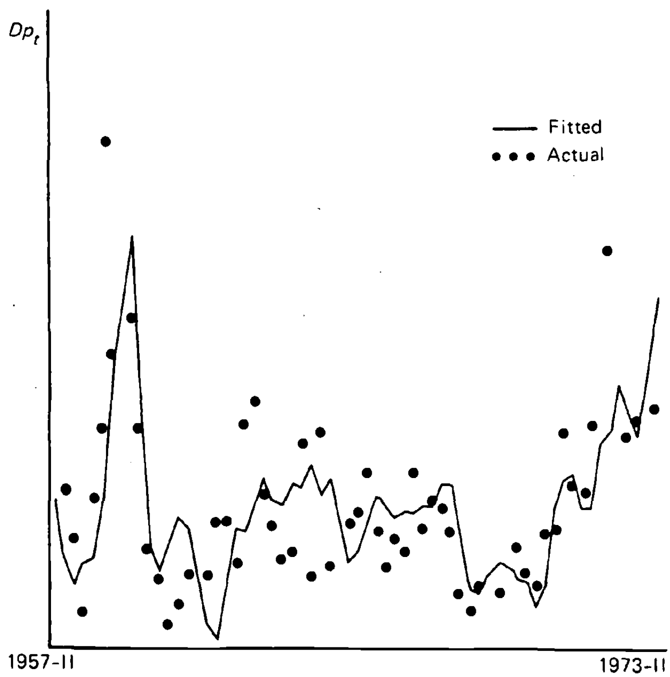

FIGURE 5-8. Actual and Fitted Values for the Rate of Change in Prices (Argentina)

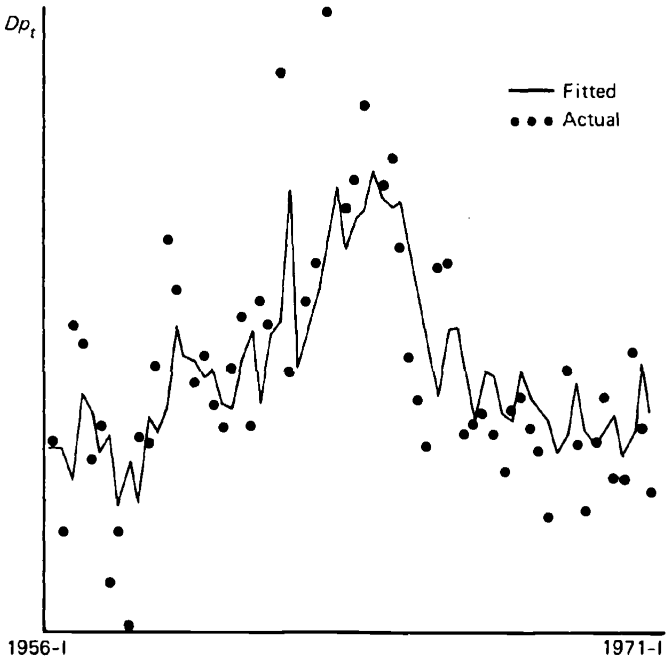

FIGURE 5-9. Actual and Fitted Values for the Rate of Change in Prices (Brazil)

noise. That is, in explaining the movements of output away from its long-run trend we have only used monetary shocks that impeded a correct anticipation of prices, and in this sense people were surprised (or fooled) during short periods of time. As long as this is the only cause that produces cyclical fluctuations around the trend, then our model seems to behave well.

From a theoretical point of view we can say that our model uses two relatively new aspects of macroeconomic theory. One is the hypothesis of rational expectations and the other is a sort of Phillips equation to play the role of the "missing equation" that, according to Friedman, explains the difference between the quantity theory of money and the Keynesian income-expenditure theory.

The simulation analysis performed in Section 6 clearly illustrates many of the situations found in practice such as shifting short-run Phillips curves, counterclockwise loops, and stagflation periods. They also illustrate the advantage of gradualism in stabilizing an economy. It was shown that an abrupt fall in the rate of growth of the money supply introduces a big oscillation in the system in the case of Argentina and a deep fall in output in both Argentina and Brazil. On the other hand, a gradual reduction in the rate of growth of the money supply produces a different effect. First, no large oscillations are observed in the endogenous variables of the system. Secondly, the fall in output is not as large as it was in the previous case although the system reaches its new steady state in a longer period of time. It must be emphasized again that this simulation depends upon the hypothesis that the same structure (parameters of the model), including the structure of the expectations formulation, continues to prevail even when policy changes. This assumption is less credible with abrupt policy changes than with gradual ones, and therefore this simulation might overstate the true difference between "gradualism and shock treatment."

NOTES

1. Equation (5-3) can be derived from the simple quantity theory. That is, let

$$Y_t \cdot \frac{1}{V_t} = M_t \qquad (5\text{-}3')$$

Now in order to capture the lagged effect of M_t on the left-hand side we have to specify something like

$$Y_t \cdot \frac{1}{V_t} = f(M_t, M_{t-1}, M_{t-2}, \ldots)$$

and a specific construction is

$$Y_t \cdot \frac{1}{V_t} = \exp\,(\phi\ \ell n\ M_t) \qquad (5\text{-}3'')$$

where ϕ is a polynomial in the lag operator (notice that if $\phi = 1$ we get (5-3′).
Assuming $(1/V_t) = \exp\,(br_t)$ and taking logs on both sides of (5-3″) we get Equation (5-3) of the text.

2. Some testable implications of the model can be derived from a structural analysis of the system. This analysis, following the method suggested in Zellner and Palm (1974), is presented in Fernandez (1975) where the final equations of the system 1 (5-1 to 5) were derived and checked with the data. Also in that work a variant of the system is analyzed in which an adaptive expectation hypothesis was used for prices. This version was *incompatible* with the available information for both Argentina and Brazil while the rational expectations

version of the model (system (5-1 to 5) was compatible under certain conditions. The method of analysis can be described briefly. Given a system of structural equations we can work out the "final equations" for the variables of the system. These are in the form of ARIMA (Autoregressive integrated moving average) processes. On the other hand, we can identify the actual ARIMA processes for the variables using the available information on each variable. If the structural equations of the model are correct, the final equations derived for each endogenous variable should have the same structure as the ARIMA processes identified for those variables from the available information. If this is the case, we say that the model is *compatible* with the available information.

3. In searching for a process determining the money supply we can choose either to postulate a model for the money supply by relating it to a set of "predetermined variables" relative to the model (5-1 to 3) (so m_t still remains as if it were exogenous or determined outside of the system (5-1 to 3) or we can identify a Box-Jenkins ARIMA process. It has been customary in the economics profession to call these models "naive models" because of their rather simple structure by which only past values of a variable are used to predict future values of the same variable. However, it has recently been shown (see Zellner and Palm [1974]) that these models might not be naive at all. Indeed these models (the ARIMA models) represent the "final form" for a variable implied by a highly sophisticated model. I will briefly illustrate this point with a model for the nominal money supply. Let us assume that in a given country the money supply is generated by the following relationship:

$$Dm_t = c_1 + a_1 Dm_{t-1} + b_1 Dg_t + e_1 Dx_t + v_t \tag{5-11}$$

where g_t could be the federal budget relative to lagged GNP, x_t could be the lagged balance of payment surplus relative to GNP, and v_t an error term stochastically independent of the errors in the structural equations. In our case a_1, b_1, and e_1 are assumed to be constants for simplicity, but in a more general analysis we could assume a_1, b_1, and e_1 to be polynomials in the lag operator.

Now we shall show that Equation (5-11) implies a final equation for m_t that is in the form of an ARIMA process. Equation (5-11) can be written as

$$(1 - a_1 L)\, Dm_t = c_1 + b_1\, Dg_t + e_1\, Dx_t + v_t \tag{5-12}$$

Now the predetermined variables g_t and x_t can follow any process over time, that is, both could follow a random walk or one could follow a random walk and the other a given ARIMA process, and so on. To illustrate the problem at hand we will assume that:

$$Dg_t\text{: ARIMA (2, 1) or } \phi\ (2)\ Dg_t = \theta\ (1) v_{1t}$$

$$Dx_t\text{: Random walk or } \phi\ (0)\ Dx_t = \theta\ (0) v_{2t}$$

where the vs are stochastically independent of the disturbances in the structural equations. Multiplying both sides of (5-12) by ϕ (2) ϕ (0), we have:

$$(1 - a_1 L)\,\phi(2)\,\phi(0)\,Dm_t = \phi(2)\,\phi(0)\,c + b\phi(0)\,\theta(1)v_{1t} + e\phi(2)\,\theta(0)v_{2t} + \phi(2)\,\phi(0)v_t \qquad (5\text{-}13)$$

In this last expression we notice that we have obtained an ARIMA (3, 2) process (if no cancellation occurs) for m_t, using Equation (5-11) and the assumption for the predetermined variables g_t and x_t. This clearly illustrates that if we obtain the process ARIMA (3, 2) for m_t this may not be a naive model at all, but on the contrary it could be reflecting the "true" model governing the behavior of the money supply.

Now we go back to our original problem of finding a process for m_t on the basis of which the public makes its forecasts of the future path of the money supply. The above discussion demonstrates that we cannot talk about "alternative models" when we evaluate a model of the sort of Equation (5-11) with respect to a model like (5-13) because (5-13) could be the final form of (5-11). Nevertheless, we have considered it appropriate to check empirically the ARIMA hypothesis for m_t, as well as a model of the sort implied by Equation (5-11). However, no further attention is dedicated to the "theory of the money supply" that underlies our hypothesis of the money supply process, a subject that goes beyond the scope of this chapter.

4. The analysis of transfer functions can be found in Chapters 10 and 11 of Box and Jenkins (1970). The derivation of a transfer function, different from the one presented in this chapter for a simultaneous equation model can be found in Zellner and Palm (1974).

5. The index of industrial production is used for Argentina as a proxy for real income because it is more reliable and complete than existing series of real output. For Brazil the only available information corresponds to real output.

6. As a matter of fact this was exactly the procedure originally followed. The procedure was abandoned because the detrended output obtained in this manner showed an initial period in which output was mostly above the trend, a second period of almost "seven years" in which output was below the trend, and a third period where output was above the trend. A detailed explanation about some institutional aspects that could explain the difference in the trend of real output mentioned above can be found in Fernandez, pp. 36-39.

7. The column headed "dummy" corresponds to the constant c in Equation (5-15). The dummy appears in the empirical results for Argentina because the constant c is a term in the slope coefficient of the trend line for output. As we split the data into two periods and in each period there is a different slope coefficient a dummy with the value of one from 1956-I to 1962-IV and two from 1963-I to 1973-II was incorporated in the transfer function to capture the effect of the change in trend.

8. The adjusted R^2 reported in the tables for transfer functions takes account of the correction for degree of freedom. That is, $1 - R^2$ adj $= n - 1/n - k\ (1 - R^2)$.

9. The Q statistic is calculated from the first K autocorrelations $\hat{r}_k$ ($k = 1,2,\ldots,K$). If the fitted model is appropriate,

$$Q(K) = n \sum_{k=1}^{K} \hat{r}_k^2$$

is approximately distributed as X^2 $(k - p - q)$. If the model is wrong the value of Q will be inflated. For Table 5-3, $p = q = 0$ because there are no autoregressive or moving average parameters in the noise model.

10. It is important to mention here an interesting result obtained by Lucas (1973). He found, in a sample of eighteen countries and working with annual observations, that "in a stable price country like the United States, policies which increase nominal income tend to have a large initial effect on real output, together with a small positive initial effect on the rate of inflation. Thus the apparent short-term trade-off is favorable, as long as it remains unused. In contrast, in a volatile price country like Argentina, nominal income changes are associated with equal, contemporaneous price movements with no discernible effect on real output" (see Lucas [1973] pp. 332-333). Our results for Argentina and Brazil tend to confirm this finding and the underlying theory that specifies that a favorable tradeoff between output and inflation depends upon "fooling" suppliers, which becomes difficult when the variance of the demand shifts becomes large.

11. Of course this might be a problem too for the stability of our estimates. If the ARIMA process is not stable neither can be the parameters of the transfer functions.

12. There are two factors playing an important role in the determination of the loops. One is the lag structure in the transfer functions and the other is the autoregressive term in detrended income.

13. The difference in the oscillatory pattern of nominal income between Brazil and Argentina should be noticed. This is due to the different lag structure in the transfer functions for nominal income.

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