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APPENDIX H

NOTES ON SOME THEORETICAL ASPECTS OF VARIABLE DELIVERY PERIODS

Joint Optimization of Delivery Period and Price

Consider a firm that sets the delivery period (k) as well as the price (p) in its offer to customers, aiming for an optimal (profit-maximizing) combination of p and k. Other things being equal, let prompter delivery indicate improved quality of the product, i.e., let it increase demand (the quantity of product ordered per unit of time, q^d) but also costs (the average production costs, c, of the quantity supplied per unit of time, q^s).[1] This gives the following demand (D) and cost (C) functions, which are of the simple static type and assumed to be continuous and differentiable:

$$q^d = D(p, k), \tag{H-1}$$

where $D_p = \dfrac{\partial D}{\partial p} < 0$ and $D_k = \dfrac{\partial D}{\partial k} < 0$;

$$c = C(q^s, k), \tag{H-2}$$

where $C_k = \dfrac{\partial C}{\partial k} < 0$.

Suppose p and k are changed by small amounts and in such a way as to have equal and opposite effects upon the rate of ordering and sales.

[1] This view of k as an aspect of product quality permits application in the present context of a simple and effective technique used in Robert Dorfman and Peter O. Steiner, "Optimal Advertising and Optimal Quality," *American Economic Review*, December 1954, pp. 826–36.

If the rates of quantities ordered and supplied are thus kept constant,[2] we get

$$\frac{dp}{dk} = -\frac{D_k}{D_p} \tag{H-3}$$

and

$$dc = C_k dk. \tag{H-4}$$

The economic meaning of equation (H-3) is the marginal rate of substitution of price for delivery period, given a certain quantity ordered, $q^d =$ constant. A system of downward sloping indifference curves is thus conceived,[3] each of which is a locus of all combinations of p and k that are associated with a given value of q^d.

The net effect on profit of small changes in price and delivery period, which leave unchanged the quantity the firm sells ($q = q^d = q^s$), is the difference between the effect on the gross revenue of the change in price (= qdp) and the effect on total costs of the change in the delivery period (= qdc). By substitution from equations (H-3) and (H-4), this net effect on profit equals

$$qdp - qdc = -q\left(\frac{D_k}{D_p} - C_k\right) dk. \tag{H-5}$$

The condition for the "joint optimum" (profit-maximizing combination) of p and k is that this whole expression be equal to zero. This will be so necessarily if, and only if, the parenthetical expression in equation (H-5) equals zero. Otherwise, one could always choose dk (with the compensating dp) such that $dp > dc$, i.e., profit could still be increased. Hence it is required that[4]

$$C_k = -\frac{D_k}{D_p}. \tag{H-6}$$

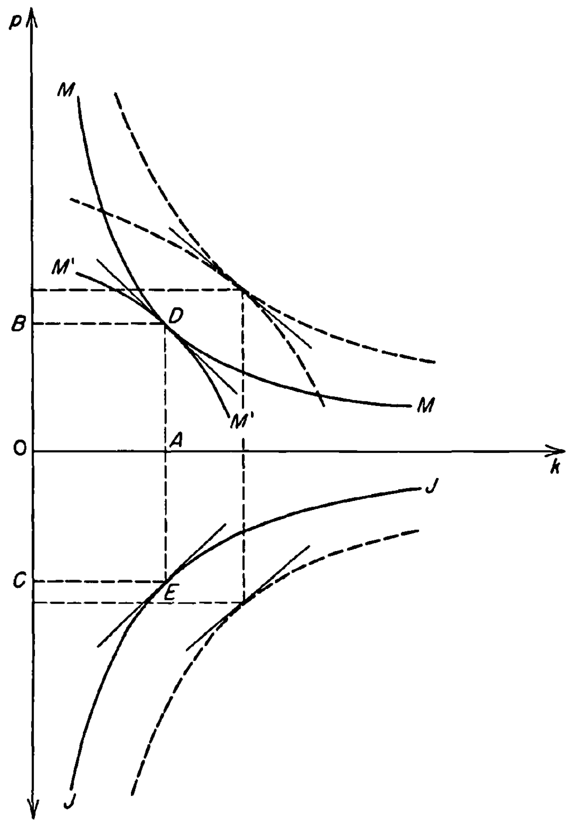
In Figure H-1 this condition is satisfied, for example, at $k = OA$, $p = OB$, and $c = OC$. The "indifference curve" MM represents all the combinations of values of p and k at which the quantity ordered

[2] Equation (H-3) is obtained by differentiating (H-1) totally to get $dq^d = D_p dp + D_k dk$ and setting $dq^d = 0$. Equation (H-4) is the form to which the differential of (H-2) reduces when $dq^s = 0$.

[3] Since $D_p < 0$ and $D_k < 0$, dp/dk must, according to (H-3), be negative.

[4] This is the necessary condition for a maximum profit (if π is net revenue or profit taken as a function of p and k, then $d\pi = 0$, that is, $\partial\pi/\partial p = \partial\pi/\partial k = 0$). To this the sufficient condition should be added, that is, the second-order partial derivatives of the profit function must be assumed to be negative at the point where $\partial\pi = 0$.

Figure H-1



equals a given amount, say, q_1. The curve *JJ* shows the costs per unit (c) of supplying this same quantity at various delivery periods (k). The slope of *MM* at point *D* equals the slope of *JJ* at point *E* (note that p and c are measured vertically from the origin *O*). Hence $dp/dk = dc/dk$, as required by equation (H-6).

Both *MM* and *JJ* are assumed to be convex relative to the origin. However, this need not necessarily be so. The convexity of the *MM* curve means that buyers are ready to pay increasing price premiums for each additional unit reduction in k. Their own production (input) requirements may indeed be such as to make this advisable. But it is also possible that the buyers' willingness to pay for the additional unit decreases in k would gradually decline; the initial speed-up may be needed and valued most, the further ones less and less. The locus of the equivalent $p - k$ combinations (given q_1) would then be a concave curve such as, e.g., $M'M'$ in Figure H-1. The convexity of JJ means that equal additional reductions in k are associated with rising increments in costs. This should be typical, although it is quite possible to conceive situations in which it would not be.[5]

Equation (H-6) can be rewritten as $-D_p = D_k/C_k$, a form convenient to interpret verbally. If the rate of increase in sales attributable to the incremental outlay for delivery-period reduction (D_k/C_k) exceeded the rate of decrease in sales due to the higher price charged to cover the cost increase ($-D_p$), then it would still pay the producer to spend more for a further delivery speed-up. In the opposite case, c should be somewhat decreased, thereby allowing k to lengthen.

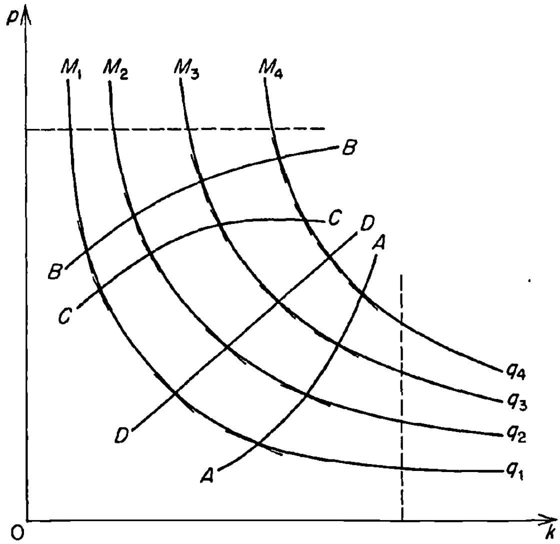
Formally, the above argument can be applied to *any* level of orders received and filled, so that its generality is not unduly restricted by the assumption of a constant q. The broken curves in Figure H-1 suggest an application to a level of orders that is higher than q_1.

Reactions of Price and Delivery Period to Demand Fluctuations

An expansion of demand will in all likelihood be accompanied by increases in both p and k, as illustrated in Figure H-2. Each of the convex curves in this diagram has the same meaning as curve MM in Figure H-1 and corresponds to a given quantity ordered, q_i. The higher and further to the right the curve, the larger the amount of orders per period to which it refers, i.e., $q_2 > q_1$, etc. To simplify presentation, the J-type curves, such as JJ in Figure H-1, are here omitted. Short heavy lines tangential to the M curves are drawn through those points at which the slopes of the paired M and J curves are assumed to be equal. These points are connected by the lines AA, BB, CC, and DD, each of which thus represents one of the many different sequences of

[5] The applicability of the preceding analysis—equations (H-1)–(H-6)—is not affected by whether the curves are convex or concave. For example, in Figure H-1, $M'M'$ is drawn with the same slope as MM. Each of these curves, together with JJ, satisfies equation (H-6). It would also seem sensible to impose certain limits upon the range of variation of p and k, but this again does not prejudge the form of the MM curve. The convex curve, e.g., may have at its ends two segments parallel to the p and k axes, respectively. The concave curve would not reach to either axis.

Figure H-2



the combinations of p and k that may result from an increase of demand from q_1 through q_4. Figure H-2 merely illustrates these various possibilities; it provides no tool for discrimination among them. In one example p increases relatively fast and k relatively slowly (AA). In another, the reverse applies (BB). Each path corresponds to a different combination of the M and J "maps" and depends on the varying slopes and positions of the curves of either set.[6]

It is clear that the diagram simply gives graphical representation to developments that differ essentially with respect to the relative importance of price and backlog adjustments. The broken lines perpendicular to the axes depict the extreme alternatives in which either p or k alone would bear the brunt of the adjustment. For these extremes to be realized, either MM or JJ would have to be nearly horizontal in one case, nearly vertical in the other. That is, there would be no significant substitutability of p and k.

[6] Figure H-2 employs the arbitrary short-cut device of keeping the M map constant, implicitly varying the J map, but one could just as well reverse this procedure. The curves in either set may run parallel or deviate in one direction or the other (as M_3 or M_4). Conceivably, the maps could even show a negative slope for a part of the $p - k$ curve (e.g., CC).