Volume Title: Orders, Production, and Investment: A Cyclical and Structural Analysis

Volume Author/Editor: Victor Zarnowitz

Volume Publisher: NBER

Volume ISBN: 0-870-14215-1

Volume URL: http://www.nber.org/books/zarn73-1

Publication Date: 1973

Chapter Title: Regression Analysis

Chapter Author: Victor Zarnowitz

Chapter URL: http://www.nber.org/chapters/c3552

Chapter pages in book: (p. 175 - 242)

5

REGRESSION ANALYSIS

IN WHAT FOLLOWS, regression models are first employed to answer these questions: How close are the relations between shipments and orders received currently and in the past? What lags characterize these relations? How sensitive are the results to differences in the methods used? The analysis proceeds from simple correlations for varying lags through multiple regressions with several lagged terms, to assumed forms of lag structure such as geometric and second-order distributions; a two-stage procedure using a linear combination of past orders as an "instrumental variable" is also applied. The lag estimates thus obtained are then compared with each other and with the results of the timing comparisons at turning points.

In the second part of the chapter, predictive equations are examined that use only past values of new orders, not the values concurrent with shipments. Here a model is estimated in which the lag coefficients of new orders are made to depend on the ratios of unfilled orders to shipments, so that when these ratios increase, indicating a rise in the rates of capacity utilization, the influence of recent orders decreases relative to that of orders of the more distant past. The performance of this model is compared with the results of regressions with fixed lag coefficients applied to the same series on shipments and new orders. This section also presents some estimates based on transformed variables that take account of observed autocorrelations in the residuals from a few selected distributed-lag equations.

The regressions are fitted to both monthly and quarterly data, seasonally adjusted, for total manufacturing, its durable and nondurable goods sectors, and seven major durable goods industries. The periods covered are 1947–65 for the sectoral aggregates and 1953–65 for the major-industry series. These are all the published data classified by industry groups that were available for such computations from the current statistics of the Bureau of the Census. Earlier, some rather fragmentary work with distributed-lag regressions was done on the pre-1963 major-industry series of new orders and shipments compiled by the Office of Business Economics (see Appendix G on the results of this exploration).

Because of aggregation problems, the variability of the lags, and the importance of production to stock, net new orders received by an industry must inevitably leave much to be desired as predictors of the industry's shipments. Nevertheless, new orders presumably provide the best available tool for prediction of shipments or sales in a large area of manufacturing.

The analysis does not cover regressions in which production rather than shipments appears as the dependent variable. While the values or volumes of production and shipments are highly correlated for industries working to order, the relations for production are interesting in their own right. Such relations, involving changes in output and in orders received or on hand, would provide tools for a study of short-term production scheduling. They receive some attention in Chapter 6.

Estimates of Average Distributed-Lag Relations Between New Orders and Shipments of Major Manufacturing Industries

Simple Correlations for Varying Lags

As the first step in analyzing the relations between new orders (N) and shipments (S) by means of regression and correlation measures, simple correlations between seasonally adjusted values of N_t and S_{t+i} were examined for different values of the lag i. The lags of shipments (= leads of new orders) are assumed to vary from zero to six months in this analysis. This range is in a broad sense consistent with the results of our earlier comparisons of turning-point dates in the aggregate current-value series for N and S (Chapter 4).[1]

The decision to use seasonally adjusted data in this regression analysis is motivated by our general intention to concentrate on nonseasonal and particularly on cyclical movements and the underlying relations.

[1] These comparisons occasionally yield long leads of orders, of nine to twelve months, or even more; but most of the leads are much shorter, and the postwar averages for all manufacturing and the total durable goods and nondurable goods sectors are −3.1, −5.1, and −2.0 months, respectively (see Table 4-7, column 9).

But the procedure could lead to considerable errors if the seasonal adjustments involved were seriously deficient and especially if a consistent seasonal pattern existed in the discrepancies between new orders and shipments. According to our tests, however, this is not the case. Regressions based on seasonally unadjusted data are not greatly improved when dummy variables are added to represent the seasonal components of the relations between N and S; the coefficients of these dummy terms are, for the most part, small relative to their standard errors. The results of these regressions are similar to those obtained by using the seasonally adjusted series for N and S (see below, p. 184).

For total manufacturing and total nondurables, simultaneous timing of new orders and shipments yields the highest correlation coefficient (r). The correlations decline steadily but very slowly as shipments are lagged by increasing intervals of from one to six months. This is summarized in Table 5-1, which gives the highest and lowest r coefficients and identifies the lags of shipments that are associated with these correlations (see columns 1–3, which relate to monthly series). For example, for all manufacturing, r varies merely from .984 (for $i = 0$) to .974 (for $i = 6$).

For the total durable goods sector, the coefficients first increase and then decrease steadily, reaching the highest value at $i = 2$ and the lowest at $i = 6$, but again differing little. Although very high, these correlations are lower than those for total nondurables and also somewhat below those for all manufacturing. This would be expected, since N_t and S_t are assumed to be equal for the greater part of the nondurable goods sector which consists of industries that do not report unfilled orders. Thus N_t and S_t have here a large common component, which biases their correlation toward unity.

This source of bias is removed when only the industries reporting unfilled orders are included in the all-manufacturing and all-nondurables aggregates. Such "advance orders" series can be computed from the currently available Census data for the period since 1953, and the correlations based on these estimates are summarized in Table 5-1. They are, of course, always lower than the correlations for the series that include the nondurable "shelf-goods" industries.[2]

[2] For comparability, calculations, limited to the 1953–65 period, were also made for the more comprehensive aggregates. These yield correlations that differ very little from the corresponding statistics for 1947–65 listed on the first six lines of Table 5-1. Thus, for all manufacturing, 1953–65, $\bar{r}^2$ varies from .965 to .928 as i increases from 0 to 6 months; for total nondurables, the coefficient $\bar{r}^2$ descends similarly from .998 to .980.

Table 5-1

Highest and Lowest Simple Correlations Between New Orders and Coincident or Lagged Shipments, Monthly and Quarterly, Major Manufacturing Industries, 1947–65 and 1953–65

	Correlations of Monthly Series [a]			Correlations of Quarterly Series [a]		
Industry	Lag of Shipments Behind New Orders [b] (months) (1)	Corr. Coeff.[c] (2)	Adj. Determination Coeff.[d] (3)	Lag of Shipments Behind New Orders [e] (quarters) (4)	Corr. Coeff.[f] (5)	Adj. Determination Coeff.[d] (6)
			1947–65 [g]			
All manufacturing	0	.984	.968	0	.986	.972
	6	.974	.948	2	.978	.957
Durable goods	2	.959	.919	0	.961	.923
	6	.946	.894	2	.955	.911
Nondurable goods	0	.998	.997	0	.999	.998
	6	.988	.976	2	.991	.980
			1953–65 [h]			
Reporting unfilled orders						
All industries [i]	2	.968	.937	1	.977	.954
	6	.937	.877	2	.952	.905
Nondurables industries [j]	0	.990	.981	1	.996	.991
	6	.980	.960	2	.989	.977

Primary metals	1	.812	.656	0	.809	.648
	6	.571	.322	2	.622	.426
Blast furnaces, steel mills	1	.675	.451	1	.753	.559
	6	.362	.125	2	.466	.201
Fabricated metal prod.	0	.929	.862	0	.960	.921
	5	.906	.820	2	.954	.909
Machinery exc. elect.	3	.969	.938	1	.977	.954
	0	.960	.920	0	.964	.929
Elect. machinery	0	.962	.924	0	.976	.951
	6	.927	.859	2	.948	.897
Transport. equip.	0	.865	.746	0	.925	.852
	6	.772	.593	2	.849	.715
Other durable goods [k]	0	.982	.964	0	.990	.981
	6	.932	.866	2	.952	.905

[a] For each industry, entry on first line is for the highest of the correlations, entry on second line is for the lowest.

[b] Zeros identify correlations between monthly values of new orders and shipments taken in the same month.

[c] Highest and lowest simple correlation coefficients observed when the lag of shipments behind new orders is varied from zero to six months.

[d] Adjusted for numbers of observations and constants according to the formula $\bar{r}^2 = 1 - (1 - r^2)[(n - 1)/(n - m)]$, where n is the number of observations (see notes g and h) and m is the number of the parameter estimates in the equation (here, 2).

[e] Zeros identify correlations between values of new orders and shipments taken in the same quarter.

[f] Highest and lowest simple correlation coefficients observed when the lag of shipments behind new orders is varied from zero to two quarters.

[g] Effective sample size is 222 monthly observations (columns 1 and 2) and 74 quarterly observations (columns 3 and 4).

[h] Effective sample size is 150 monthly observations (columns 1 and 2) and 50 quarterly observations (columns 3 and 4).

[i] Includes all durable goods industries and the four nondurable goods industries reporting unfilled orders (see note j).

[j] Includes textiles, leather, paper, and printing and publishing.

[k] Includes professional and scientific instruments; lumber; furniture; stone, clay, and glass products; and miscellaneous industries.

Among the major components of the durable goods sector, nonelectrical machinery ranks first according to the average correlations between N and S (proceeding from the largest to the smallest). Here r increases steadily from .960 to .969 as i rises from 0 to 3, then decreases steadily to .963 as i is extended further to six months. For electrical machinery, the correlations are just slightly lower, but they decline from the beginning, i.e., r is largest at $i = 0$, and it gets smaller as the length of the lag increases. Similar patterns of declining r prevail also in fabricated metal products, the group of other durable goods, and transportation equipment including motor vehicles (where the range of the r values is relatively large). The lowest correlations and the largest differences among them are found in total primary metals and, particularly, in the blast furnaces and steel mills subdivision (Table 5-1).

The monthly regressions are affected, probably often strongly, by random effects, such as strikes, which would have their main impact upon both N and S at about the same time. This tends to produce a correlation between N_t and S_t even if no orders were filled in the same month as received.[3] It is not clear how this source of simultaneity bias could be removed; it is present in all the lagged regression models to be analyzed, but presumably in different degrees. Some models imply less emphasis upon the very short movements in the series than do other models, and more emphasis upon the longer movements; and the above argument gives a reason for preferring the models that do so.

It should be noted that the primary metals series are particularly vulnerable to strike-related disturbances. While approximately simultaneous, the reactions of N and S to such events often differ substantially in intensity. During the large steel strikes, shipments of primary metals dropped off suddenly and drastically, then recovered just as rapidly, while new orders showed much smaller dips that were occasionally, as in 1956, within the range of their usual month-to-month variations (see Chart 3-2, lines 1 and 2).

The smallness of the differences between the correlation coefficients that result from varying the lags i suggests that the series involved are highly autocorrelated. If S_t is almost as closely associated with N_{t-1} as with N_t, for example, then N_t and N_{t-1} are probably also closely correlated with each other. That this should be so is not surprising since, on this level of aggregation, expansions and contractions of new

[3] I am indebted to Geoffrey Moore for suggesting this point.

orders are cumulative movements, and shipments must reproduce or follow these movements in smoother form. As noted in Chapter 2, there are several reasons why the lags of shipments behind new orders are presumably distributed and variable rather than discrete and constant. In sum, the measures given in Table 5-1 portend major difficulties in properly identifying the structure of these lags. Where relations with several different lags are all very close, discrimination among various combinations of the lags is likely to be a troublesome problem.

Working with quarterly rather than monthly data may help to reduce these problems. First, there are then fewer possible specifications of the lag structure. Second, autocorrelation is often greater as the successive values of the series move closer together in time. Finally, it is likely that the erratic or "random noise" component will be smaller in the quarterly than in the monthly series.

Simple correlations between quarterly (seasonally adjusted) series on new orders and shipments were compared for (1) simultaneous timing of N and S; (2) a one-quarter lag of S; and (3) a two-quarter lag of S. For most of the industries, the highest correlations are obtained under timing assumption (1) and the lowest under (3): that is, the coefficients r decline as the lags in quarters increase (Table 5-1, columns 4–6). In some cases – the groups of industries reporting unfilled orders, and blast furnaces and nonelectrical machinery – the timing relationship that maximizes correlation is the one-quarter lag of shipments rather than simultaneity.[4] However, the differences between the correlations for varying lags tend to be quite small, just as for the monthly data. The largest differences are again those for the primary metals industry and its blast furnaces division.

The number of quarterly observations for any industry is, of course, only one-third of that of monthly observations. Table 5-1 presents, for both sets of series, coefficients of determination adjusted for numbers of observations and constants in the estimating equations, $\bar{r}^2$ (columns 3 and 6). Comparisons of these statistics suggest that the quarterly correlations are somewhat closer than the monthly ones for similar lags (that is, when coincident timing is used in both cases, or lags of three months and one quarter, or lags of six months and two quarters). Reflecting this rule, the coefficients in column 6 of the table all exceed their counterparts in column 3.

[4] Indeed, for nonelectrical machinery (Table 5-1, "machinery, except electrical"), coincident timing produces a lower correlation than either of the alternative assumptions about the delivery lag.

Multiple Regressions with Several Lagged Terms

Shipments in any unit period (month or quarter) conclude the process of filling orders received either in the same or in some previous period. Assume that a firm gets new orders for its *x*th product in each month and always produces and delivers exactly one-half of the number of units ordered in the same month (t), three-tenths in the next month ($t+1$), and two-tenths in the following month ($t+2$). The relation between new orders and shipments for X would then be expressed by an exact functional form:

$$s_t = 0.5n_t + 0.3n_{t-1} + 0.2n_{t-2}, \qquad (1)$$

using the notation introduced in Chapter 2. Referring to equation (1) there, $\alpha_0 = 0.5, \alpha_1 = 0.3$, and $\alpha_2 = 0.2$; in this case then, $\Sigma\alpha_i = 1$, where the summation is over $i = 0, 1, 2$. The "relevant past" here includes only the two previous months. The lag is distributed, not discrete, but its structure is given and constant.

It is clear that this hypothetical case does not represent the way in which orders are actually translated into output and shipments. If it did, the time path of output and shipments would be completely determined by, and hence perfectly predictable from, the recent course of new orders. In the real world, except in the trivial case where new orders and shipments coincide so as to be for all practical purposes identical, the relations between these variables are not exact but stochastic. The coefficients α_i are not fixed but changing, and they are as a rule unknown. The changes in them may (though need not) be themselves partly systematic. Some of the reasons why this should be so have already been discussed.

Nevertheless, the unrealistic case exemplified by (1) above has some instructive aspects. It incorporates the valid concept of a basic distributed-lag relationship between shipments and new orders: in any reasonably defined (not excessively long) time period, the former may be taken to be a weighted sum of the past (and perhaps current) values of the latter. Since *all* of new orders received in any period must sooner or later result in output and shipments, except for cancellations, it is likewise true that *all* shipments made in any period must be traceable to orders that had at some time been accepted. In other words, the coefficients in an equation relating observed S_t to the terms N_{t-i} ($i = 0, 1, \ldots, m$) should add up to unity, given that new orders are taken net of cancellations and for all relevant past periods up to the present.

Multiple regressions with S_t as the dependent variable and several terms N_{t-i} as the independent variables offer the most direct method of exploring these relationships. This approach, however, faces a major difficulty if the new-order series are highly autocorrelated. Close correlation among independent variables may preclude a reliable estimation of their separate influences upon the dependent variable.[5] For example, if N_t and N_{t-1} are highly correlated, then their joint use in a regression equation designed to "explain" S_t would result in very large standard errors of their regression coefficients. This so-called multicollinearity problem is, of course, quite familiar in relations among economic time series.

We proceed with the following experiment using monthly series: Regress S_t first on N_t and N_{t-1}, next on these two terms plus N_{t-2}, and so on, until seven terms are included (N_{t-i}; $i = 0, \ldots, 6$). This results, for each industry, in six successive least-square estimates of shipments, which have the form $(S_{jt})_{est} = a_j + \Sigma b_{ij} N_{t-1} = S_t - u_{jt}$, where the summation is from $i = 0$ to j ($j = 1, \ldots, 6$). The addition of another term typically reduces the regression coefficient of the preceding term but increases the sum of the coefficients (Σb_{ij}), as shown, for example, by the estimates for the total durable goods sector in the accompanying table.

$(S_{jt})_{est}$	Constant	Regression Coefficients of N_t	N_{t-1}	N_{t-2}	N_{t-3}	N_{t-4}	N_{t-5}	N_{t-6}	$\Sigma b'_{ij}$	$\bar{R}^2$
$j = 1$	.738	.432	.513						.936	.923
	(.262)	(.090)	(.090)							
$j = 2$	.657	.341	.159	.445					.944	.931
	(.249)	(.086)	(.110)	(.087)						
$j = 3$	.614	.292	.137	.193	.328				.950	.936
	(.241)	(.085)	(.106)	(.107)	(.085)					
$j = 4$	.561	.349	.047	.170	.086	.304			.956	.939
	(.235)	(.084)	(.107)	(.104)	(.107)	(.084)				
$j = 5$	.520	.327	.117	.085	.070	.067	.291		.958	.942
	(.230)	(.082)	(.106)	(.104)	(.104)	(.106)	(.082)			
$j = 6$	.470	.350	.082	.153	−.020	.056	.055	.290	.966	.945
	(.224)	(.080)	(.104)	(.103)	(.104)	(.103)	(.104)	(.081)		

[5] This is a matter of degree. It is immediately clear that in the extreme case of perfect correlation between any two explanatory variables either one of them could be used just as well as the other, and there would be no reason to use both. In the much more likely case of high but not perfect correlation, the estimated parameters may or may not have an unsatisfactorily low level of accuracy.

In this case, the terms that definitely retain significance, as indicated by the size of their coefficients relative to the corresponding standard errors (which are given in parentheses), are the first and the last of the independent variables in each equation, that is N_t and N_{t-j}. Apparently because of the autocorrelation of new orders, the effects on S_t of the intermediate terms cannot be neatly separated but are largely absorbed in the coefficients b_{tj} and b_{jj} of each jth equation ($j = 1, \ldots, 6$). The last coefficient may also include the influence of the omitted terms with lags greater than j. These results are consistent with the plausible condition that the correlation between N_t and N_{t-i} weakens as i increases.

The sum of the regression coefficients, already high in the first equation with two terms only, increases by small steps but steadily as the number of the N_{t-i} variables is increased. The constant terms a_j decline greatly, in absolute size and relative to their standard errors, but even in the last equation ($j = 6$), a_j is still more than twice as large as its error and hence probably significant.[6] The ratio of a_6 to the average level of new orders in the sample period is $0.470/13.705 = 0.0343$, which, when added to 0.9659, totals approximately 1.000.

Although some of the N_{t-i} terms have coefficients that appear to lack statistical significance, the determination coefficients adjusted for the number of degrees of freedom increase steadily as the variables representing longer lags are added. There seems to be no plausible reason why the degree of intercorrelation among the explanatory variables should have any systematic or biasing effects on the estimates of the over-all multiple correlation.[7]

When seasonally unadjusted data (denoted by the superscript u) are used, the following estimate is obtained with three new-order terms:

$$(S^u_{2t})_{\text{est}} = .919 + \underset{(.060)}{.593N^u_t} + \underset{(.071)}{.033N^u_{t-1}} + \underset{(.060)}{.301N^u_{t-2}};\ \Sigma b_i = .926;\ \bar{R}^2 = .938$$

where the summation of the b's is over $i = 0, 1, 2$. This equation may be compared with that for $(S_{2t})_{\text{est}}$ in seasonally adjusted values (see data, p. 183, above). Some similarities will be noted; e.g., in each case, the coefficient of the second N term (for $t - 1$) is not significant. The

[6] With as many as 222 observations, the ratio $t = 0.4696/0.2240 = 2.096$ would be exceeded by chance only about once in thirty trials (3–4 per cent significance level).

[7] J. Johnston, *Econometric Methods*, New York, 1963, pp. 204–206. A related point is that "if forecasting is a primary objective, then intercorrelation of explanatory variables may not be too serious, provided it may reasonably be expected to continue in the future" (*ibid.*, p. 207).

sum of the b coefficients is here somewhat lower, the value of $\bar{R}^2$ slightly higher. When eleven monthly dummy variables (D_k) are included along with the N^u_{t-2} terms, $\bar{R}^2$ is raised just a little. The coefficients (d_k) of five of the eleven dummy terms are lower than their standard errors, and two of the others are of doubtful significance. It is well to observe that the results of the regression with the dummy seasonals (with the summation of dD taken over $k = 1, 2, 3, \ldots, 11$; and of b over $i = 0, 1, 2$)

$$(S^{ud}_{2t})_{est} = .916 + .398N^u_t + .142N^u_{t-1} + .384N^u_{t-2} + \Sigma d_k D_k;$$

$$\Sigma b_i = .909; \bar{R}^2 = .946$$

are quite similar to those reported for the seasonally adjusted data in the table above. These tests suggest that working with series corrected for seasonal variations gives satisfactory results, at least for our present purposes.

Applications to Monthly Series

The results of fitting the six estimating equations to the monthly Census data for each of the major manufacturing industries are summarized in Table 5-2. The sums of the regression coefficients, Σb_{ij}, taken from $i = 0$ to j, typically continue to increase with i until all seven terms N_{t-i} ($i = 0, \ldots, 6$) are included. That is, the lowest of these estimates is for $j = 1$ and the highest for $j = 6$ (column 1). These cumulative increases are interrupted only in a few isolated instances as all but four of the coefficients in the range considered are positive (see column 2 and note i). The corresponding $\bar{R}^2$ coefficients similarly increase in this range, with two exceptions (see column 3 and note l).

The outcome for all manufacturing (1947–65) is much like that for the durable goods sector: again, the most significant terms are N_t and N_{t-j}, where j denotes the largest lag, associated with the last term in each of the six equations (column 4). The Σb and $\bar{R}^2$ estimates are somewhat higher. The fits for the nondurable goods sector are, to be sure, still closer; the constant terms are not significant here, and the shorter lag terms are dominant (column 4). The totals in column 2, the sums of the b's, approximate unity for all manufacturing and total nondurables and are only slightly smaller for total durables (first six lines of Table 5-2).

Table 5-2

Relations Between Shipments and New Orders Received in the Current Month and in Each of Six Preceding Months, Multiple Correlation and Regression Measures, Major Manufacturing Industries, 1947–65 and 1953–65

	Range of Estimates[a]				Regression[c] That Maximizes $\bar{R}^2$						
							Regression Coefficients[e]				
Industry	j (1)	Sum of b's (2)	$\bar{R}^2$ (3)	Most Significant Lags[b] (4)	Constant Term[d] (5)	i (6)	Largest (7)	Second Largest (8)	Third Largest (9)	Sum of Three Largest (10)	Av. Lag[f] (mos.) (11)
					1947–65 (222)[g]						
All manufacturing	1	0.976	.973	0, j	0.262 (0.255)	0, 6, 2	.438 (.069)	.245 (.070)	.125 (.094)	.809	2.3
	6	0.996	.981								
Durable goods	1	0.936	.923	0, j	0.470 (0.224)	0, 6, 2	.350 (.080)	.290 (.081)	.153 (.103)	.792	2.6
	6	0.966[h]	.945								
Nondurable goods	1	0.996	.9976	0, 1, 3	0.039 (0.039)	0, 1, 3	.635 (.029)	.148 (.038)	.111 (.038)	.894	1.0
	6	1.002	.9982								
					1953–65 (150)[g]						
Reporting unfilled orders All industries[i]	1	0.839	.941	0, j	3.146 (0.317)	0, 6, 1	.311 (.073)	.192 (.075)	.129 (.093)	.631	2.0
	6	0.886[h]	.959								

Nondurables industries[j]	1	0.983	.985	0, 3, 5, *j*	0.075 (0.038)	0, 3, 5	.402 (.044)	.167 (.052)	.142 (.052)	.711	2.3
	6	1.022	.992								
Primary metals	1	0.660	.657	1, *j*	0.784 (0.107)	3, 1, 0	.268 (.080)	.258 (.111)	.117 (.079)	.643	1.45[l]
	3, 6[k]	0.747	.712								
Blast furnaces, steel mills	1	0.555	.451	1, 3	0.571 (0.083)	3, 1, 0	.258 (.084)	.243 (.114)	.096 (.084)	.597	1.21[l]
	3, 6[k]	0.660	.515								
Fabricated metal products	1	0.797	.902	0, 1, 2, *j*	0.274 (0.029)	0, 6, 1	.251 (.044)	.204 (.044)	.140 (.046)	.594	2.0
	6	0.846	.941								
Machinery exc. elect.	1	0.812	.934	0, *j*	0.292 (0.031)	6, 0, 1	.270 (.068)	.167 (.071)	.141 (.083)	.578	2.9
	6	0.876	.963								
Elect. machinery	1	0.834	.943	0, 1, *j*	0.249 (0.030)	0, 1, 6	.297 (.050)	.158 (.056)	.128 (.050)	.582	1.9
	6	0.882	.961								
Transport. equip.	1	0.683	.800	0, 1, *j*	1.024 (0.117)	0, 1, 6	.256 (.048)	.148 (.052)	.111 (.049)	.516	1.65
	6	0.765	.839								
Other durable goods[m]	1	0.941	.970	1, 2, *j*	0.078 (0.037)	0, 2, 6	.503 (.047)	.127 (.054)	.111 (.047)	.741	1.69
	6	0.982	.981								

Notes to Table 5-2

[a] The form of the estimating equation is

$$S_t = a_j + \Sigma_{i=0}^{j} b_{ij} N_{t-i} + u_{jt}.$$

The index j identifies the number of terms of N_{t-i} ($i = 0, \ldots, j$) included in the given regression. The number varies from two ($i = 0, 1$) to seven ($i = 0, \ldots, 6$). See text.

The values of j in column 1 refer to the equations with the lowest (first line of entries for the industry) and highest (second line) estimated sums of regression coefficients ($\Sigma_{i=0}^{j} b_{ij}$) shown in column 2, and adjusted coefficients of multiple determination ($\bar{R}^2$), in column 3.

[b] Identifies the time subscripts i of those terms of N_{t-i} ($i = 0, \ldots, 6$) with t ratios of substantial statistical significance (generally, t ratios of 2.0 and more, values which always exceed those at the 0.05 probability level and often exceed the 0.01 level).

[c] Selected for each industry from the six estimating equations $(S_{jt})_{est} = a_j + \Sigma_{i=0}^{j} b_{ij} N_{t-i}$ obtained by varying j from 1 to 6 (see columns 1–4 and notes above). For all but two industries, these regressions include all seven terms N_{t-i} ($i = 0, 1, \ldots, 6$), that is, $j = 6$. For primary metals and blast furnaces, etc., the highest $\bar{R}^2$ is observed when four terms of N_{t-i} ($i = 0, \ldots, 3$) are used as independent variables, that is, $j = 3$. The values of j that maximize $\bar{R}^2$ are listed in column 1; the corresponding values of $\bar{R}^2$, in column 3.

[d] The standard error of the constant term (a_j) is given underneath in parentheses.

[e] In each regression, there are $j + 1$ regression coefficients b_i ($i = 0, 1, \ldots, j$). Column 6 identifies the subscripts of i of the largest, second largest, and third largest of the regression coefficients, in that order. The values of these coefficients are given in columns 7, 8, and 9; their standard errors are given underneath in parentheses. Column 10 shows the sums of the three largest coefficients (i.e., the totals of the corresponding entries in columns 7–9, except for slight differences due to rounding of these entries). These sums may be compared with the second line of entries for each industry in column 2 (see text).

[f] Computed by multiplying the lags i by the corresponding regression coefficients b_i and adding the product. Based on regressions of S_t on N_{t-i} ($i = 0, 1, \ldots, 6$). See text and note 7.

[g] The number of monthly observations, given in parentheses, represents the effective sample size for each of the industries covered in the section that follows.

[h] Includes one coefficient with negative sign.

[i] Includes all durable goods industries and the four nondurable goods industries reporting unfilled orders (see note j).

[j] Includes textiles, leather, paper, and printing and publishing.

[k] The first of the j values identifies the equation with the highest observed $\bar{R}^2$; the second, the equation with the largest observed $\Sigma_{i=0}^{j} b_i$. (Elsewhere, only one entry for j is given, indicating that the same equation produces the minimum or maximum observed values of both Σb_i and $\bar{R}^2$.)

[l] Based on equations with $j = 6$ (see preceding footnote). The estimated average lags for $j = 3$ are 1.23 for total primary metals and 1.13 months for blast furnaces.

[m] Includes professional and scientific instruments; lumber; furniture; stone, clay, and glass products; and miscellaneous industries.

For the nondurable goods industries reporting unfilled orders (1953–65), the regression coefficients also add up to about 1.000, and $\bar{R}^2$ is very high, but for all industries reporting, the results are less satisfactory. Substantial differences among the major durable goods industries are indicated. Thus the $\Sigma_i^6 b_i$ estimates fall in the ranges 0.84–0.88 (fabricated metals, machinery) and 0.74–0.80 (transportation equipment and total primary metals; for the blast furnaces component of the latter, however, the total is as low as 0.66). In contrast, the corresponding sum for the other durables group comes close to unity (column 2).

In total primary metals and in its blast furnaces and steel mills division, lags of three months and (less so) of one and two months seem to be primarily important; the partial effects of current new orders (N_t) are smaller here than for the other industries, but the influence of new orders received four to six months ago is also smaller (column 4). In fact, the highest $\bar{R}^2$ is observed for these industries when four terms (N_{t-i}, $i = 0, \ldots, 3$) are used as independent variables,[8] whereas in all other cases seven terms ($i = 0, \ldots, 6$) are needed to produce this result. Only about 52 per cent of the variance of shipments can be explained in this way for blast furnaces, etc., and 71 per cent for total primary metals. Elsewhere in the durable goods sector, the $\bar{R}^2$ coefficients all exceed .80, and are as high as .94 to .98 for fabricated metals, the two machinery-producing industries, and the residual group of "other durable goods" (column 3).

New orders received currently and in the last month or two are most significant for fabricated metal products and also for electrical machinery and total transportation equipment (dominated by motor vehicles), though the last terms N_{t-j} tend to be important, too. For nonelectrical machinery, the long lags are of principal significance, in particular N_{t-6} in the last equation, which yields the highest $\bar{R}^2$. In the equations for the "other durable goods," the coefficients of N_t are particularly large, exceeding somewhat the coefficients of all earlier terms N_{t-i} ($i = 1, 2, \ldots, 6$) combined. This presumably reflects the large importance of production to stock in this group.

The three largest regression coefficients in each of the equations that yield the highest observed $\bar{R}^2$ are identified in columns 6–9 of the table.

[8] The addition of N_{t-4}, N_{t-5}, and N_{t-6} makes $\bar{R}^2$ decline gradually from .7119 to .7089 for total primary metals and from .5153 to .5067 for the blast furnaces component.

Their sums represent from about 66 to 96 per cent of $\Sigma_{i=0}^{6} b_i$ (compare the figures in column 10 with the corresponding entries in column 2).

Average lags of S_t behind the terms N_{t-i} can be computed as weighted sums of the lags i, with the corresponding regression coefficients b_i being used as the weights.[9] Since the coefficients are generally positive, the resulting measures tend to increase as additional new orders terms are included. The estimates in column 11 of the table are based on the equations which contain all seven terms N_{t-i}, $i = 0, 1, \ldots, 6$. They seem to be both implausibly small when compared with other evidence and not sufficiently discriminating among the industries: They differ considerably less than those average delivery-lag estimates that are derived either from turning-point comparisons or from regressions that do not arbitrarily limit the number of the N_{t-i} terms.[10]

The values of the Durbin-Watson test statistic, $d = \Sigma(u_t - u_{t-1})/\Sigma u_t^2$, are generally very low for the residuals u from the regressions summarized in Table 5-2: None exceeds 1.3, and most are less than 1.0. This suggests that these residuals or error terms are positively autocorrelated.[11]

Chart 5-1 confirms that the fit of estimates obtained from these regressions leaves much to be desired. Two industries are covered in these illustrations, the total durable goods sector and machinery ex-

[9] For example, applying this procedure to the last equation for total durables as given in the tabulation shown above, one gets $.350(0) + .082(-1) + .153(-2) - .020(-3) + .056(-4) + .055(-5) + .290(-6) = -2.5660$, that is, an average lead of N relative to S of about 2.6 months.

[10] On casual reading, it might seem puzzling that the average lag is greater for "all manufacturing" than for "all industries reporting unfilled orders" (compare the entries for these in column 11), since the exclusion of those nondurables for which N_t and S_t are taken to be equal (i.e., the assumed lag is zero) ought to increase the lag. But there is no inconsistency, only a difference in the periods covered. The lags of S decreased in recent years. For 1953–65, the average lag for all manufacturing was 2.0 when rounded (it is a fraction of a month smaller than the corresponding figure for all industries reporting). For total nondurables, the average lag, analogously computed, was negligible (about 0.6 months).

[11] See J. Durbin and G. S. Watson, "Testing for Serial Correlation in the Least-Squares Regression, I and II," *Biometrika*, December 1950 and June 1951. The second article includes tables of significance points for the d statistics. For n, the number of observations, equal to 100, and for m, the number of independent variables, equal to 4 and 5 (these are the largest values of n and m in the tables), the lower and upper points, d_L and d_u, are as follows:

Level of Significance	$m = 4$		$m = 5$	
	d_L	d_u	d_L	d_u
5.0%	1.59	1.76	1.57	1.78
1.0%	1.46	1.63	1.44	1.65

If the calculated d is less than d_L, the residuals are probably positively autocorrelated; if d is greater than d_u, they are probably not; and if d falls between d_L and d_u, the test is inconclusive. A two-sided test may be performed by applying these rules to both d and $(4 - d)$.

cept electrical, and two estimates are presented for each, one derived by relating S_t to three and the other to seven N_{t-i} terms. The computed series [denoted as $(S_{2m})_{est}$ and $(S_{6m})_{est}$, respectively] clearly resemble new orders too much in that they show considerably larger cyclical movements and earlier timing than estimated shipments should. The fits are particularly poor in times of the greatest discrepancies between new orders and shipments: the Korean period from mid-1950 to mid-1952 and the recessions of 1953–54 and 1957–58. Adding more of the N_{t-i} terms (that is, introducing the longer delivery lags) definitely does improve the fits: The fluctuations of $(S_{6m})_{est}$ have smaller amplitudes and later timing than those of $(S_{2m})_{est}$, hence resemble the actual shipments better. However, the improvements are not large and cannot be considered adequate for either total durables or nonelectrical machinery.

Applications to Quarterly Series

In Table 5-3, shipments are related to new orders received in the same quarter and in each of the two preceding quarters. Seasonally adjusted data are used throughout, as before.

If the true unit period of adjustment were one month but, instead of months, quarters were used in an otherwise identical distributed-lag equation, the results could be seriously distorted by a systematic aggregation error. However, the basic unit period is typically unknown, and there is no exception from this rule in the present case. Moreover, these quarterly equations include fewer terms than the monthly ones, so as to cover the same lag range (six months when measured between the midpoints of the intervals). In this situation, the choice between the monthly and the quarterly units must depend on the verdict of the data. If the true structure of the lagged adjustment process is not substantially obscured by the use of the longer units, the latter should be preferable, since aggregation over time reduces the magnitude of measurement errors relative to the true values of the data.[12]

Comparisons of the corresponding estimates in Tables 5-2 and 5-3 do, in fact, suggest that some gains are made when quarterly rather than monthly series are used. The highest adjusted determination

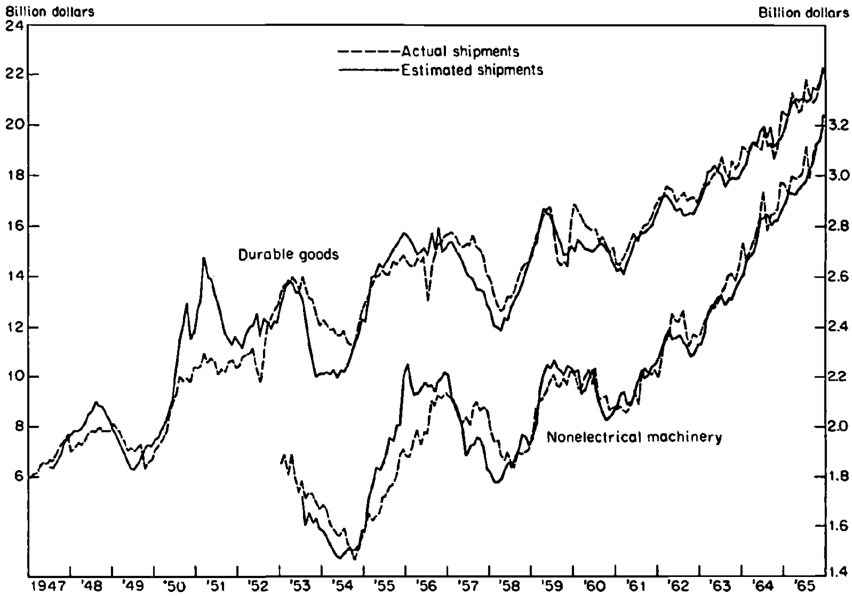
[12] See Yair Mundlak, "Aggregation Over Time in Distributed Lag Models," *International Economic Review*, May 1961, pp. 154–63. Also, see Lester G. Telser, "Discrete Samples and Moving Sums in Stationary Stochastic Processes," *Journal of the American Statistical Association*, June 1967, pp. 484–99.

Chart 5-1

Regressions for Shipments of Durable Goods Industries and Nonelectrical Machinery, Based on Three and Seven Lagged Terms in New Orders, Monthly, 1947–65

PART A

$$(S_t)_{\text{est}} = a + b_1 N_t + b_2 N_{t-1} + b_3 N_{t-2}$$



coefficients in Table 5-3 (columns 6 and 10)[13] exceed the highest $\bar{R}^2$ values in Table 5-2 for each industry. More importantly, the Durbin-Watson statistics give much less evidence of residual autocorrelation for the quarterly than for the monthly regressions. For several industries, these tests (Table 5-3, columns 7 and 11) either suggest that there is no autocorrelation (primary metals, blast furnaces) or are inconclusive on the 0.05 level (fabricated metals, other durables).

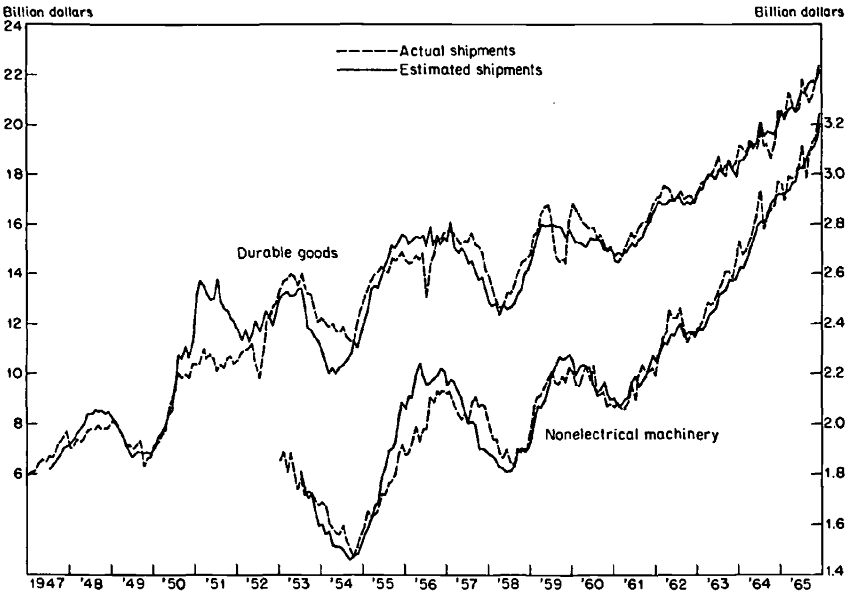
The average lags estimated from the two sets of regressions are closely similar. More figures in Table 5-3, column 8, exceed than fall short of their counterparts in Table 5-2, column 11, but the margins of

[13] The highest observed $\bar{R}^2$ values for primary metals and blast furnaces, etc., are those in column 10, which result from relating S_t to the two terms N_t and N_{t-1}; for all other industries, the highest $\bar{R}^2$ in the table are those in column 6, which result from relating S_t to three terms, N_t, N_{t-1}, and N_{t-2} (the subscripts referring to quarterly intervals).

Chart 5-1 (continued)

PART B

$$(S_t)_{est} = a + b_1N_t + b_2N_{t-1} + b_3N_{t-2} + b_4N_{t-3} + b_5N_{t-4} + b_6N_{t-5} + b_7N_{t-6}$$



difference are very small fractions, sometimes as little as hundredths, of a month.

The influence on S_t of the same-quarter orders N_t is dominant in all but two cases, but particularly large for nondurables and the other durables group, as would be expected of industries that sell a large proportion of their output from stock (Table 5-3, column 2). The effects of N_{t-1} are dominant in primary metals and blast furnaces, significant for total nondurables and other durable goods, and quite weak elsewhere (column 3). The coefficients of N_{t-2}, conversely, are statistically not different from zero in the nondurables and primary metals regressions, but relatively large (mostly three to four times the size of their standard errors; see column 4) for all other industries.

The three regression coefficients add up to approximately 1.0 for the comprehensive aggregates (first three lines, column 5) and the group of nondurable goods industries reporting unfilled orders (fifth line, column 5). The constant terms in these regressions are probably not

Table 5-3
Relations Between Shipments and New Orders Received in the Current and in the Preceding One or Two Quarters, Multiple Correlation and Regression Measures, Major Manufacturing Industries, 1947–65 and 1953–65

	Regression[a] Based on N_t, N_{t-1}, and N_{t-2}								Regression[f] Based on N_t and N_{t-1}		
	Con-stant Term[b]	Regression Coefficients[b]									
Industry		b_{20}	b_{21}	b_{22}	Sum[c] of b's	$\bar{R}^2$	Durbin-Watson Statistic[d]	Av. Lag[e] (mos.)	Sum[g] of b's	$\bar{R}^2$	Durbin-Watson Statistic[d]
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
					1947–65 (74)[h]						
All manufacturing	.242 (.412)	.527 (.080)	.138 (.118)	.330 (.081)	0.995	.984	0.567	2.4	0.986	.980	0.507
Durable goods	.434 (.369)	.480 (.104)	.092 (.157)	.397 (.104)	0.969	.951	0.488	2.7	0.953	.941	0.437
Nondurable goods	.042 (.051)	.738 (.032)	.209 (.040)	.055 (.032)	1.001	.999	1.296	0.95	0.9996	.9989	1.333
					1953–65 (50)[h]						
Reporting unfilled orders											
All industries[i]	2.344 (.098)	.454 (.164)	.132 (.164)	.305 (.102)	0.892	.968	0.935	2.2	0.885	.954	1.126
Nondurables industries[j]	−.056 (.034)	.450 (.054)	.311 (.081)	.265 (.056)	1.027	.997	1.851	2.2	1.043	.992	1.683
Primary metals	.755 (.172)	.266 (.092)	.448 (.129)	.025 (.093)	0.740	.772	1.827	1.50	0.733	.777	1.794
Blast furnaces, steel mills	.593 (.137)	.209 (.103)	.458 (.135)	−.030 (.103)	0.638	.584	1.949	1.20	0.649	.592	1.977
Fabricated metal prod.	.258 (.037)	.419 (.062)	.052 (.086)	.386 (.063)	0.857	.9695	1.412	2.5	0.831	.946	1.197

Machinery exc. elect.	.282	.258	.199	.424	0.882	.9688	0.539	3.1	0.850	.956	0.501
	(.051)	(.092)	(.138)	(.096)							
Elect. machinery	.236	.489	.164	.236	0.888	.9694	0.650	1.9	0.867	.965	0.506
	(.046)	(.079)	(.115)	(.083)							
Transport. equip.	.980	.460	.121	.194	0.775	.881	0.791	1.53	0.740	.872	0.827
	(.174)	(.082)	(.107)	(.089)							
Other durable goods [k]	.078	.627	.163	.191	0.981	.991	1.488	1.64	0.966	.988	1.186
	(.045)	(.058)	(.088)	(.056)							

[a] The estimating equation is

$$S_t = a_2 + b_{20}N_t + b_{21}N_{t-1} + b_{22}N_{t-2} + u_{2t}.$$

[b] Standard errors of the parameter estimates are given underneath in parentheses.

[c] Sum of columns 2–4 ($b_{20} + b_{21} + b_{22}$).

[d] Durbin-Watson Statistic $(d) = \Sigma(u_t - u_{t-1})^2/\Sigma u_t^2$. Shown below are the exact upper and lower bounds for critical values of d for n, the number of observations (75 or 50) and for m, the number of independent variables in the regression (2 or 3). On the interpretation and source of these significance points, see note 11 in this chapter.

n	Level of Significance	$m = 2$ d_L	$m = 2$ d_u	$m = 3$ d_L	$m = 3$ d_u
50	5.0%	1.46	1.63	1.42	1.67
50	1.0	1.28	1.45	1.24	1.49
75	5.0	1.57	1.68	1.54	1.71
75	1.0	1.42	1.53	1.39	1.56

[e] Computed as the sum $b_{21}(-3) + b_{22}(-6)$, analogously to the estimates in Table 5-2, column 11 (cf. note f to that table and text).

[f] The estimating equation is

$$S_t = a_1 + b_{10}N_t + b_{11}N_{t-1} + u_{1t}.$$

[g] Sum of two regression coefficients ($b_{10} + b_{11}$).

[h] The figure in parentheses is the number of quarterly observations. It is the effective sample size for each of the industries covered in the section that follows.

[i] Includes all durable goods industries and the four nondurable goods industries reporting unfilled orders (see note j).

[j] Includes textile, leather, paper, and printing and publishing.

[k] Includes professional and scientific instruments; lumber; furniture; stone, clay, and glass products; and miscellaneous industries.

different from zero, being either smaller or not much larger than their standard errors (column 1). The results for the group of other durable goods (tenth line) are similar in these respects. However, for each of the remaining industries, the sums of the b coefficients fall appreciably short of unity. The constant terms in these equations are all several times larger than their standard errors, and hence presumably significant.

Graphs of the quarterly shipment estimates computed from regressions that include from one to three terms N_{t-i} ($i = 0, 1, 2$ quarters) show them to suffer from the same basic deficiency as that observed in Chart 5-1 for the analogously derived monthly estimates. The computed values resemble new orders too much, moving earlier than the actual shipments S and in wider swings—like the monthly estimates. Here too, adding additional lagged terms improves the fit. But even the estimates that incorporate the full range of the lags used, from zero to two quarters, still retain too much similarity to the path of new orders to be really satisfactory. Chart 5-2 illustrates this fact for total durables and machinery except electrical. Of course, the quarterly series are much smoother than the monthly series shown for the same industries in Chart 5-1.

Assumed Forms of Time-Lag Structure

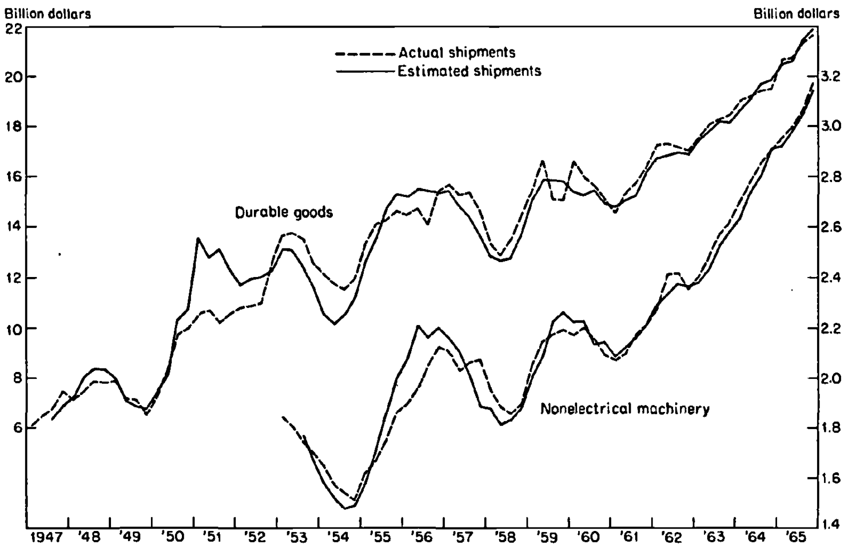
The direct method of adding the lagged terms successively as long as their coefficients do not show "wrong" signs or erratic behavior provided the earliest approach to the estimation of distributed lags.[14] The main difficulty with it is that the lagged terms employed as independent variables are often highly intercorrelated, so that their coefficients cannot be reliably estimated. By imposing upon the data a specific form of lag distribution, this multicollinearity problem can be largely overcome, but not without cost. In general, the proper specification of the time-lag structure is unknown and cannot be readily inferred from either theory or data. Statistical difficulties of estimation are encountered in using the models, and the results based on them may admit of different interpretations.

[14] Irving Fisher and C. F. Roos both used the concept of a distributed lag first in 1925, and G. C. Evans used it in 1930. For a summary and references, see Franz L. Alt, "Distributed Lags," *Econometrica*, April 1942, pp. 113–28.

Chart 5-2

Regressions for Shipments of Durable Goods Industries and Nonelectrical Machinery, Based on Three Lagged Terms in New Orders, Quarterly, 1947–65

$$(S_t)_{\text{est}} = a + b_1N_t + b_2N_{t-1} + b_3N_{t-2}$$



Suppose that the influence on S_t of N_{t-i} steadily declines as i increases; the more remote values of N have less effect than the more recent ones. A very simple form of this hypothesis is that the coefficients α in

$$S_t = \alpha_0N_t + \alpha_1N_{t-1} + \alpha_2N_{t-2} + \cdots + u_t \tag{2}$$

(where u_t is the disturbance term) decline exponentially, so that $\alpha_i = \alpha\beta^i$ ($i = 0, 1, 2, \ldots$), and

$$S_t = \alpha\Sigma_{i=0}^{\infty}\beta^iN_{t-i} + u_t, \tag{3}$$

where both α and β are natural fractions (i.e., $0 < \alpha, \beta < 1$). As shown by Koyck,[15] (3) reduces to

$$S_t = \alpha N_t + \beta S_{t-1} + \epsilon_t, \tag{4}$$

[15] L. M. Koyck, *Distributed Lags and Investment Analysis*, Amsterdam, 1954.

where $\epsilon_t = u_t - \beta u_{t-1}$.[16] Equation (4) includes only two explanatory variables, and thus appears to be quite manageable and probably not subject to a serious multicollinearity problem. But the presence in the regression of lagged values of the dependent variable on the right-hand side may lead to a serious bias of the least-square estimates in small samples (in sufficiently large samples this defect is likely to be substantially reduced).[17] Moreover, if the original disturbance terms u_t in (3) were serially independent, then ϵ_t in (4) would not be; in other words, if the lag structure was properly specified in (3), then estimation by means of (4) will produce autocorrelated residuals. The latter can be particularly troublesome when combined with the complication of lagged variables (autoregressive schemes).[18]

In the present context, there is no strong presumption that the process whereby past new orders are translated into shipments is so well approximated by the geometric lag distribution (3) as to leave the residuals u free of autocorrelation. In some industries, production to stock is of major importance, which would make the coefficient of N_t much greater relative to those of the other terms than is here implied. (In the extreme case of pure production to stock, $\alpha = 1$ and $\beta = 0$.) In other industries, production to order may be so dominant and the delivery lags so long that, for short unit periods, the influence of N_t and perhaps of N_{t-i} would actually be relatively weak—again

[16] Equation (4) is derived from (3) as follows. From

$$S_t = \alpha N_t + \alpha\beta N_{t-1} + \alpha\beta^2 N_{t-2} + \cdots + u_t$$

subtract

$$\beta S_{t-1} = \alpha\beta N_{t-1} + \alpha\beta^2 N_{t-2} + \cdots + \beta u_{t-1}$$

to get

$$S_t - \beta S_{t-1} = \alpha N_t + \cdots + u_t - \beta u_{t-1}$$

or

$$S_t = \alpha N_t + \beta S_{t-1} + \epsilon_t.$$

[17] This is because the assumption that the disturbance term is distributed independently of the explanatory variables does not hold for autoregressive schemes. The least-squares estimates, however, will have the desirable asymptotic (large-sample) properties of consistency and efficiency. See Johnston, *Econometric Methods*, pp. 211–14.

[18] The estimates may be inconsistent in this case. Furthermore, if the autocorrelation of the residuals is positive, the estimated coefficients of the lagged dependent variables (such as that of S_{t-1} in equation (4) would be biased upward, which, as will be shown later, implies overestimation of the average lags involved. See Johnston, *Econometric Methods*, pp. 215–16, and references there; also, Zvi Griliches, "Distributed Lags: A Survey," *Econometrica*, January 1967, pp. 16–49 (particularly pp. 33–42).

unlike (3).[19] Moreover, aggregation over industries with very different types of orders-shipments relations could result in "multimodal" lag distributions.[20] If only for these reasons, one may well doubt the applicability of a fixed lag structure, such as (3), to the diverse industry processes under consideration. On the other hand, if the disturbances u are autocorrelated, then ϵ_t *may* be serially independent with a constant variance, in which case (4) would at least yield consistent estimates of α and β.[21] In any event, model (4) is easy to apply and potentially instructive in suggesting modified and different approaches; hence, considerable use was made of it in the course of this exploration and a report on the results is in order.

Estimates of Geometric Lag Distributions

Table 5-4 presents regressions of the form

$$S_t = k + aN_t + bS_{t-1} + v_t, \tag{5}$$

which follow Koyck's model of equation (4), except that a nonzero intercept k is admitted [a, b, and v_t correspond to α, β, and ϵ_t in (4), respectively]. If a is small and b large, then a slow lagged response (long distributed lag) is indicated; if, on the contrary, a is large and b small, then the response is prompt (i.e., the average and the dispersion of the lag distribution are both small). The sum of the regression coefficients implied in (5) is $a + ab + ab^2 + \cdots = a/(1 - b)$. It should ideally show the complete ultimate response of S to a unit change in N maintained forever, and hence should equal unity (in which case, of course, $a + b = 1$, also).

This estimated "total effect," $\Sigma = a/(1 - b)$, is indeed close to 1.000

[19] In pure production to order, the index i in (3) would start, not from zero, but from some positive value j representing the minimum period needed for production and delivery. The derived form analogous to (4) would then read

$$S_t = \alpha_1 N_{t-j} + \beta_1 S_{t-1} + \epsilon_{1t}; \quad j > 0,$$

thus embodying the assumption that this interval j also represents the "normal" or most frequent delivery lag. On some early experiments in applying this derived form to the 1948–58 OBE figures for N and S, see Appendix G.

[20] For example, if the aggregate consisted of an industry working largely to stock and of another industry, about as large, with a typical delivery lag of four months, then the coefficients of N_t and N_{t-4} would tend to dominate the others in the equation for the combined two-industry totals. Our data are, of course, very comprehensive and they undoubtedly combine much larger numbers of different patterns with varying weights; hence the outcomes are not nearly as simple.

[21] This will be so if u follows the first-order autoregressive (Markov) scheme in which $u_t = \beta u_{t-1} + \epsilon_t$.

Table 5-4
Regressions of Shipments on New Orders, Based on Geometric Lag Distribution, Major Manufacturing Industries, Quarterly, 1947–65 and 1953–65

Industry	Constant Term[a] (1)	Regression Coefficients[a] *a* (2)	Regression Coefficients[a] *b* (3)	Sum of Implicit Coefficients[b] (4)	$\bar{R}^2$ (5)	Lag[c] (mos.) Needed to Account for 50% of Column 4 (6)	70% (7)	90% (8)	Av. Lag[d] (mos.) (9)
				1947–65 (74)[e]					
All manufacturing	–.008	.397	.608	1.014	.994	4.2	7.3	13.9	4.7
	(.245)	(.035)	(.036)						
Durable goods	.047	.311	.692	1.009	.986	5.6	9.8	18.7	6.7
	(.199)	(.037)	(.039)						
Nondurable goods	.027	.736	.265	1.002	.999	1.6	2.7	5.2	1.1
	(.048)	(.028)	(.029)						
				1953–65 (50)[e]					
Reporting unfilled orders									
All industries[f]	.825	.417	.548	0.922	.981	3.5	6.0	11.5	3.6
	(.425)	(.044)	(.055)						
Nondurables industries[g]	–.045	.504	.514	1.036	.996	3.1	5.4	10.4	3.2
	(.036)	(.048)	(.051)						

Primary metals	.403 (.216)	.495 (.064)	.367 (.084)	0.783	.745	2.1	3.6	6.9	1.7
Blast furnaces, steel mills	.380 (.183)	.471 (.079)	.298 (.102)	0.671	.513	1.7	3.0	5.7	1.3
Fabricated metal products	.105 (.042)	.401 (.047)	.542 (.057)	0.876	.972	3.4	5.9	11.3	3.5
Machinery exc. elect.	.031 (.027)	.278 (.025)	.715 (.031)	0.976	.994	6.2	10.8	20.6	7.5
Elect. machinery	.047 (.032)	.330 (.039)	.655 (.048)	0.956	.990	4.9	8.5	16.3	5.7
Transport. equip.	.391 (.183)	.353 (.061)	.562 (.085)	0.806	.922	3.6	6.3	12.0	3.8
Other durable goods [h]	.036 (.042)	.588 (.042)	.405 (.047)	0.988	.992	2.3	4.0	7.6	2.0

[a] The form of the estimates is given by equation (5) in the text. The figures in parentheses are standard errors.

[b] The sum equals $a/(1-b)$.

[c] Computed according to equation (7); see text and note 23.

[d] The average length of the lag equals $3b/(1-b)$.

[e] The figure in parentheses is number of quarterly observations. It is the effective sample size for each of the industries covered in the section that follows.

[f] Includes all durable goods industries and the four nondurable goods industries reporting unfilled orders (see note g).

[g] Includes textiles, leather, paper, and printing and publishing.

[h] Includes professional and scientific instruments; lumber; furniture; stone, clay, and glass products; and miscellaneous industries.

for all of the comprehensive aggregates, the two machinery industries, and the group of other durable goods industries (column 4). The constant terms k are in these cases apparently not significantly different from zero, except for the industries reporting unfilled orders in 1953–65 (column 1). For the metalworking industries and transportation equipment, however, Σ is considerably lower (varying from 0.67 for blast furnaces to 0.88 for fabricated metal products) and k is positive and significant.[22]

The estimated values of a and b are roughly 0.4 and 0.6, respectively, for total manufacturing, and 0.3 and 0.7 for the durable goods sector (columns 2 and 3). Conversely, a is larger than 0.7 and b smaller than 0.3 for the total nondurable goods sector. The part of that sector that reports unfilled orders illustrates the intermediate situation where a and b differ little, each being close to 0.5. Among the major industries, nonelectrical machinery has the lowest a and the highest b, and electrical machinery comes next, with coefficients similar to those for all durables. In contrast, the other durables group shows the highest a (nearly 0.6) and a correspondingly low b (0.4). In terms of broad interindustry comparisons, these results appear sensible and consistent with other relevant information. The equations for total primary metals and the blast furnaces subdivision, however, are much less satisfactory. They yield not only relatively low Σ statistics but also relatively low values of $\bar{R}^2$ (column 5). The a's are high and the b's are lower than for any of the component industries.

It is clear that b is inversely related to the speed with which the orders are filled, that is, translated into shipments. Let q represent the proportion of the "total effect" of new orders on shipments [i.e., of $a/(1-b)$] that is accounted for by an interval of n unit periods. Then

$$q = [a(1-b^n)/(1-b)] \div [a/(1-b)] = 1 - b^n \qquad (6)$$

or

$$n = \log (1-q)/\log b. \qquad (7)$$

Columns 6–8 in Table 5-4 show, according to equation (7), the length of time necessary to account for 50, 70, and 90 per cent of the total

[22] Multiplying a and $a/(1-b)$ by $\bar{N}/\bar{S}$, the ratio of the average values of new orders and shipments, gives elasticity expressions evaluated at the means. Over a sufficiently long period of time, the means $\bar{N}$ and $\bar{S}$ will tend to be equal. The true long-run elasticity of S relative to $N(\epsilon_L = (\bar{N}/\bar{S})\Sigma)$ is 1, as is the true value of Σ. Actually, the estimates for ϵ_L corresponding to those for Σ in column 4 are very close to the latter. Where $\Sigma \simeq 1, \epsilon_L \simeq 1$ also; and for the three industries with estimated Σ of less than 1, ϵ_L is likewise significantly smaller than unity.

long-run reaction, Σ.[23] Most of the impact of orders spends itself within the first few months, while the remainder tapers off very slowly. This, of course, is implicit in the adopted lag structure. But the speed of the process varies greatly among the industries. Thus, according to column 6, the "half-life" of the process is 5.6 months for the durable goods sector but only 1.6 months for the nondurables (1947–65). When the manufacturers not reporting unfilled orders are excluded, however, the estimate for all industries is only slightly larger than that for the nondurable goods industries (the figures for 1953–65 are a little in excess of 3 months). The half-life lags come to 5–6 months in the machinery industries and 2–3 months in the metalworking industries.

On the assumption that the distributed-lag coefficients are nonnegative weights that sum to unity (so that $0 < b > 1$ and $a = 1 - b$), it is possible to identify the mean lag of the geometric lag distribution here used as $b/(1 - b)$.[24] For the metalworking industries and transportation equipment the sums of the estimated coefficients actually fall short of unity (column 4), but this is disregarded and the average lags are computed uniformly according to the above formula.[25]

The resulting figures (Table 5-4, column 9) exceed in each case the average lags calculated from the regressions of shipments on new orders received in the current and in the preceding two quarters (Table 5-3, column 8). The differences are negligible (0.1 to 0.4 of one month) for the industries with the shortest estimated lags: primary metals, blast furnaces, other durable goods, and total nondurables all have average lags of one to two months according to either set of measures.

[23] These estimates are computed by inserting into (7) the values 0.5, 0.7, and 0.9 for q, and multiplying the resulting values of n by 3 (it is convenient for comparisons with other findings to translate n from quarters into months).

[24] The lag structure is in this case formally identical to the geometric probability distribution. If $a = 1 - b$ and $c = 0$, (5) can be written as

$$S_t = (1 - b)(1 + bL + b^2L^2 + \cdots)N_t = f(L)N_t,$$

where L is a lag operator; then $L(N)_t = N_{t-1}$; $L^2(N_{t-1}) = N_{t-2}$; etc. The mean lag is obtained as the first derivative of the "generating function" $f(L)$ evaluated at $N_t = 1$, which equals $b(1 - b)(1 - b)^{-2} = b/(1 - b)$. See William Feller, *An Introduction to Probability Theory and Its Applications*, New York, 1950, Vol. I, pp. 210 and 252-53.

[25] In those cases where the sums $(a + b)$ are significantly less than 1.0, the ratios b/a have been computed for comparison with $b/(1 - b)$. The values of $3(b/a)$ are as follows: primary metals, 2.2 (months); blast furnaces, 1.9; fabricated metal products, 4.1; and transportation equipment, 4.8. These figures, of course, exceed the corresponding entries for $3b/(1 - b)$ in column 9 of Table 5-4, since $a < (1 - b)$ for these industries.

Under the assumptions specified above, with the mean lag equal to $b/(1 - b)$, the variance of the geometric lag distribution is given by $b/(1 - b)^2$. The variance, then, is an increasing function of b, being $1/(1 - b)$ times as large as the mean (which is itself increasing with b). For example, if $b = 0.5$, the mean lag is 1.0 and the variance is 2.0; if $b = 0.8$, the mean lag is 4.0 and the variance is 20.0.

Elsewhere, however, the differences are large, varying from about one month for fabricated metals and the two aggregates with advance orders, to 2.3 months for transportation equipment and all manufacturing, to about 4 months for the two remaining items (total durables and the machinery industries).

The average lags estimated from regressions of shipments on a few recent values of new orders are generally small, varying in the range of approximately 1 to 3 months (Tables 5-2 and 5-3). They are probably too small in some cases, thus understating the timing differences among the industries (see above, page 190). If this is correctly recognized as a defect, the estimates in Table 5-4 are free of it. Their large range (from 1.0 to 7.5 months) is similar to the interindustry variation of the average leads derived in Chapter 4 from turning-point comparisons. The present estimates may have an advantage in that they are based on distributed-lag models which in a sense include the entire history of the explanatory variable rather than just its most recent past. However, they could err in the opposite direction, that is, toward overstatement of the lags. Positive autocorrelation of the residuals in (5) could be a source of such bias (see note 18, above). The use of a quarterly instead of monthly unit period might also work in this direction, and some evidence bearing on this point will be presented shortly.

Clearly, it is important to know whether the residuals in the model underlying equation (5) are really free of serial correlation. The Durbin-Watson statistic (d) is itself biased in equations with lagged dependent variables and hence provides no reliable test of the residual autocorrelation in these cases.[26] The d figures have been routinely computed and, as descriptive statistics, they show that the *estimated* disturbance terms contain no detectable correlation, but this gives no assurance that the "true" disturbances are likewise not autocorrelated.[27]

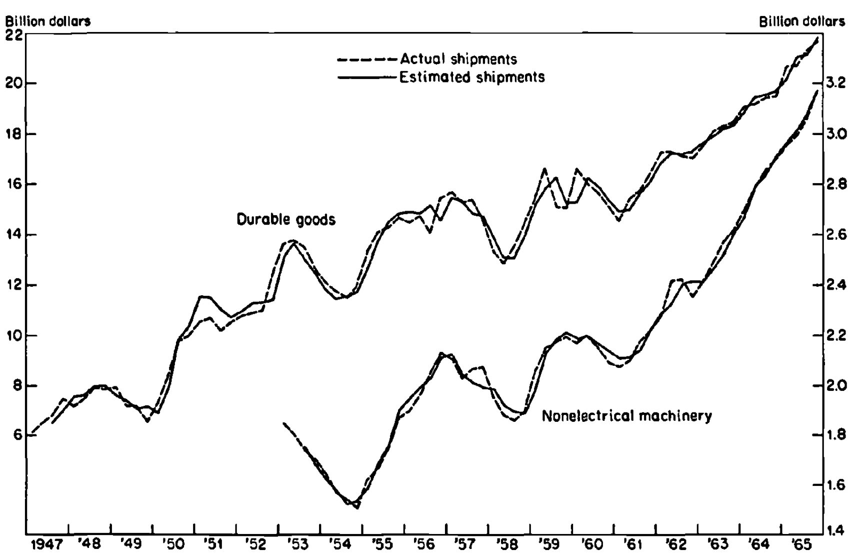
[26] When lagged values of the dependent variable are used as an explanatory factor, a downward bias in the autocorrelation of the estimated residuals is created along with an upward bias in the coefficient of the lagged variable, and for the same reason (see note 18, above). Durbin and Watson warn against the use of their statistic in such cases in their 1951 *Biometrika* article, p. 159 (see note 11, above). See also Zvi Griliches, "A Note on Serial Correlation Bias in Estimates of Distributed Lags," *Econometrica*, January 1961, pp. 65–73; and Marc Nerlove and Kenneth F. Wallis, "Use of the Durbin-Watson Statistic in Inappropriate Situations," *Econometrica*, January 1966, pp. 235–38.

[27] The computed Durbin-Watson statistics vary between 1.720 and 1.830 for the 1947–65 regressions ($n = 74$; see Table 5-4, first three lines), and between 1.573 and 2.386 for the 1953–65 regressions, excluding transportation equipment and other durable goods ($n = 50$). The observed values of d are generally greater than the upper bounds d_u (see Table 5-3, note d), which, taken at its face value, would suggest that the residuals are not positively autocorrelated. In two cases with the highest d values (primary and fabricated metals), the tests against negative serial correlation would be inconclusive at the 5 per cent level (that is, the value of $4 - d$ falls between d_L and d_u or the 5 per cent test, though it exceeds d_u for the 1 per cent test).

As shown in Chart 5-3 for the quarterly data on total durables and nonelectrical machinery, the use of the Koyck equation (5) can produce rather good results when applied to the orders-shipments relations. Certainly, much better fits are obtained here than with the regressions of S_t on the three terms N_{t-i} ($i = 0, 1, 2$ quarters), as will be seen by comparing Charts 5-3 and 5-2. The computed shipments series (S_{est}) fluctuate with amplitudes close to those of actual shipments (S) in Chart 5-3; they do not overstate greatly the cyclical movements of S in 1947–58 as do the S_{est} estimates in Chart 5-2. Furthermore, the timing of S_{est} is much closer to that of S in Chart 5-3 than in Chart 5-2. In the latter, S_{est} shares some of the earliness of N and often leads S at turning points. Here the timing of S and S_{est} is on the average more nearly coincident. Autoregressive forms are known to produce a certain tendency to lag in the estimates, which is favorable for these relations since it helps to offset the leads imparted to the computed series by the use of the early-moving new orders as predictors.

Chart 5-3
Regressions for Shipments of Durable Goods Industries and Nonelectrical Machinery, Based on Geometric Lag Distribution, Quarterly, 1947–65

$$(S_t)_{est} = a + b_1 N_t + b_2 S_{t-1}$$



Further Results on Geometric Lag Models

Some earlier findings of this analysis favored the use of quarterly rather than monthly series in direct estimates of distributed lags of shipments behind new orders, but this does not in any way resolve the issue for the models now under consideration. Here the form of the lag distribution is given, and the choice of an inappropriate basic time period will lead to a misspecified model and overestimation of the average lags.[28] Accordingly, it is desirable to re-estimate model (5) using monthly data, and to compare the results (Table 5-5) with their counterparts for the quarterly regressions (Table 5-4). The regression takes the form $S_t = k' + a'N_t + b'S_{t-1} + v'_t$, again following Koyck.

Not surprisingly, the correlation measures offer little help in discriminating between the two sets of estimates: for most industries, the coefficients $\bar{R}^2$ are very high and very close in both tables (compare columns 5). However, substantial differences in favor of the monthly regressions can be seen for primary metals and blast furnaces (fourth and fifth lines). In these cases, too, the sums of implicit lag coefficients are definitely larger for the monthly data (i.e., $\Sigma' > \Sigma$; compare columns 4 in the two tables). For the other component industries and the sector reporting unfilled orders, $\Sigma > \Sigma'$ by margins varying within a narrow range. In the regressions for all manufacturing, for total durables, and for the two aggregates of nondurables, Σ and Σ' are approximately equal to unity.

The longer the period of reference, the larger will be the proportion of current shipments accounted for by new orders received during the same period; hence it is easy to see why the coefficients of N_t are always greater in the quarterly than in the monthly regressions (that is, $a > a'$; compare columns 2 in Tables 5-4 and 5-5). By the same token, the coefficients of S_{t-1} are in all cases smaller in the quarterly equations ($b < b'$; see columns 3).

More interesting are the differences, expressed in months, between the average lags based on the quarterly and on the monthly regressions [$\bar{n} = 3b/(1-b)$ and $\bar{n}' = b'(1-b')$; see columns 9]. Only in transportation equipment does $\bar{n}'$ exceed $\bar{n}$ by a substantial margin (1.7 months). For both total primary metals and blast furnaces, $\bar{n}' > n$ too, but by minor fractions. Elsewhere $\bar{n} > \bar{n}'$, mostly by 1.0 to 1.5

[28] The latter is due to the positive correlation between the aggregated true disturbances and the lagged values of the aggregate dependent variable. See Mundlak, "Aggregation Over Time."

months (the difference is once more just a small fraction for total nondurables, and it is really large—3.0 months—for nonelectrical machinery). Much larger differences are observed by comparing the lags necessary to account for 50 to 90 per cent of the total reaction of S to additional N (see columns 6–8 in the two tables). On balance, therefore, these results seem consistent with the hypothesis that the use of quarterly data tends to cause some overestimation of the lags, even though the evidence is somewhat mixed.

Inspection of graphs also suggests that in some cases marginal improvements result from the use of monthly rather than quarterly data in the geometric lag models. Because of the large discrepancy between $\bar{n}$ and $\bar{n}'$ observed for nonelectrical machinery, Chart 5-4 shows the estimates for this industry. The fit is indeed extremely close, with some tendency for S_{est} to lag S by one month at turning points.

It may not be correct, however, to assume that the lag coefficients decline from the very beginning of the adjustment process, particularly when the unit periods are very short. As one check on the possibility

Chart 5-4
Regressions for Shipments of Nonelectrical Machinery, Based on Geometric Lag Distribution, Monthly, 1953–65

$$(S_t)_{est} = a + b_1 N_t + b_2 S_{t-1}$$

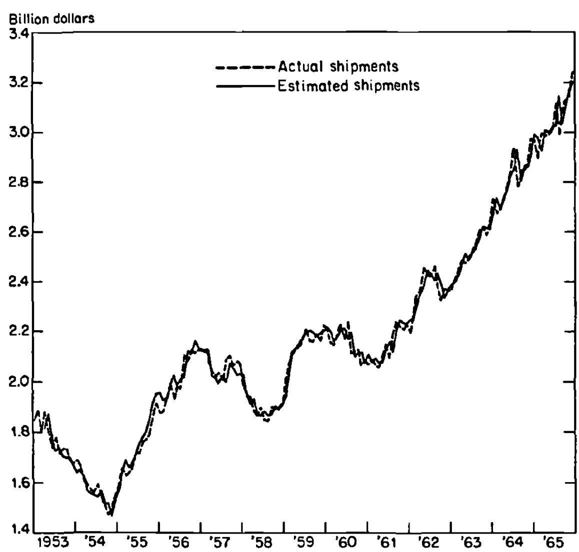


Table 5-5
Regressions of Shipments on New Orders, Based on Geometric Lag Distribution, Major Manufacturing Industries, Monthly, 1947–65 and 1953–65

Industry	Constant Term[a] (1)	Regression Coefficients[a] a' (2)	Regression Coefficients[a] b' (3)	Sum of Implicit Weights[b] (4)	$\bar{R}^2$ (5)	Lag[c] (mos.) Needed to Account for 50% of Column 4 (6)	Lag[c] (mos.) Needed to Account for 70% of Column 4 (7)	Lag[c] (mos.) Needed to Account for 90% of Column 4 (8)	Av. Lag[d] (mos.) (9)
				1947–65 (226)[e]					
All manufacturing	.029	.222	.779	1.004	.995	2.8	4.8	9.2	3.5
	(.125)	(.022)	(.022)						
Durable goods	.046	.152	.847	0.994	.991	4.2	7.2	13.8	5.5
	(.091)	(.019)	(.020)						
Nondurable goods	.030	.598	.401	0.9995	.998	0.8	1.3	2.5	0.7
	(.036)	(.026)	(.026)						
				1953–65 (154)[e]					
Reporting unfilled orders									
All industries[f]	.604	.256	.717	0.904	.983	2.1	3.6	6.9	2.5
	(.222)	(.027)	(.032)						
Nondurables industries[g]	.009	.458	.542	1.000	.991	1.1	2.0	3.8	1.2
	(.031)	(.041)	(.043)						

Primary metals	.200	.291	.642	0.812	.821	1.6	2.7	5.2	1.8
	(.099)	(.034)	(.043)						
Blast furnaces, steel mills	.149	.290	.621	0.767	.680	1.5	2.5	4.8	1.6
	(.080)	(.040)	(.050)						
Fabricated metal products	.097	.266	.679	0.830	.952	1.8	3.1	6.0	2.1
	(.031)	(.032)	(.040)						
Machinery, except electrical	.040	.165	.820	0.913	.990	3.5	6.1	11.6	4.5
	(.020)	(.021)	(.026)						
Electrical machinery	.031	.176	.809	0.923	.986	3.3	5.7	10.9	4.2
	(.020)	(.025)	(.031)						
Transportation equipment	.178	.116	.845	0.750	.936	4.1	7.2	13.7	5.5
	(.090)	(.028)	(.040)						
Other durable goods[h]	.070	.481	.499	0.960	.980	0.8	1.3	2.6	1.0
	(.037)	(.039)	(.043)						

[a] The form of the equation is

$$S_t = k' + a'N_t + b'S_{t-1} + v'_t$$

as discussed in the text. The figures in parentheses are standard errors of the statistics shown.

[b] The sum equals $a'/(1 - b')$.

[c] Computed according to equation (7); see text and note 23.

[d] The average length of the lag equals $b'/(1 - b')$.

[e] The number of monthly observations, given in parentheses, represents the effective sample size for each of the industries covered in the section that follows.

[f] Includes all durable goods industries and the four nondurable goods industries reporting unfilled orders (see note g).

[g] Includes textiles, leather, paper, and printing and publishing.

[h] Includes professional and scientific instruments; lumber; furniture; stone, clay, and glass products; and miscellaneous industries.

that the response does not reach its peak immediately or nearly so but rather builds up to it more gradually, N_{t-1} was added as the third independent variable to the regressions of S_t on N_t and S_{t-1} (in quarterly terms). The results suggest that the simple Koyck scheme does not apply to primary metals and blast furnaces. Here the coefficients of N_{t-1} are larger than those of N_t, while the coefficients of S_{t-1} are reduced to insignificance (being smaller than their standard errors and, for blast furnaces, negative).

Primary metals:

$$(S_t)_{\text{est}} = .655 + .291N_t + .389N_{t-1} + .095S_{t-1};\ \bar{R}^2 = .775$$
$$(.220)\quad (.096)\qquad (.143)\qquad (.127)$$

Blast furnaces, steel mills:

$$(S_t)_{\text{est}} = .591 + .210N_t + .445N_{t-1} - .017S_{t-1};\ \bar{R}^2 = .584$$
$$(.181)\quad (.112)\qquad (.148)\qquad (.140)$$

These estimates, then, along with some earlier ones (in Tables 5-2 and 5-3), indicate that in primary metals the lags are concentrated in the current and previous quarters, with the influence of N_{t-1} exceeding that of N_t.

Elsewhere, the coefficients of N_{t-1} are negative and small (for three industries, they are not significantly different from zero). The coefficients of N_t and S_{t-1} are highly significant whether or not N_{t-1} is added, and the values of $\bar{R}^2$ are either slightly lowered or not appreciably raised by the addition. This evidence contradicts the present hypothesis of a somewhat generalized Koyck model (with the added term N_{t-1}), but it is also unfavorable to the alternative hypothesis that S_t is a function of N_t with autocorrelated residuals.[29]

Also examined was the possibility that a trend factor enters the dis-

[29] Suppose that $S_t = \alpha N_t + u_t$ and $u_t = \rho u_{t-1} + \epsilon_t$; then, by substitution,

$$S_t = \alpha N_t + \rho u_{t-1} + \epsilon_t = \alpha N_t + \rho S_{t-1} - \alpha\rho N_{t-1} + \epsilon_t$$

(since $u_{t-1} = S_{t-1} - \alpha N_{t-1}$). In this case, the simple Koyck model, though inappropriate, would produce highly significant results since both N_t and S_{t-1} influence S_t positively. But the addition of N_{t-1} would in this case discredit this model by revealing that N_{t-1} has a negative coefficient approximately equal to the product of the coefficients of N_t and $S_{t-1}(\alpha\rho)$. See Griliches, "Distributed Lags."

In the situations here considered, the inclusion of N_{t-1} has no such implications, since the coefficients of N_{t-1}, whether positive or negative, are insignificant, or in any event much smaller than the products of the corresponding coefficients of N_t and S_{t-1}. Hence the hypothesis, which is implausible in the present context anyway, can be rejected by these tests.

tributed-lag relationship between new orders and shipments. A simple time trend, T (in the form of consecutive numbers for successive periods), was included as another independent variable in the monthly and quarterly regressions based on the Koyck equation (5).[30] The effects of this variable turned out to be either negligible or weak. Even in the most favorable cases—the 1947–65 equations for all manufacturing and all durables—the addition of T had little effect on the regression coefficients of N_t and S_{t-1} and even less effect on the $\bar{R}^2$ coefficients.[31]

Second-Order Distributed Lag Functions

In an attempt to evolve a model which will incorporate a more general and flexible form of lag distribution, another lagged value of the dependent variable was added to the Koyck scheme, that is, S_t was regressed on N_t and S_{t-1} and also on S_{t-2}. The equation

$$S_t = \alpha N_t + \beta S_{t-1} + \gamma S_{t-2} + e_t \tag{8}$$

is capable of producing rather different lag profiles depending on the magnitudes of β and γ. Under certain not too restrictive conditions, the implied lag coefficients or weights will all be nonnegative.[32] They will also often decline throughout, as in the simple geometric case, but at different rates. However, for certain combinations of values of β and γ, the decline of the implied weights will not be monotonic, i.e., occasionally N_{t-i} may have a smaller coefficient than N_{t-i-1}. Frequent

[30] As an alternative, the logarithmic trend expression, log e^t, was also tried, with almost entirely negative results.

[31] The following estimates may be compared with the corresponding entries in the first two lines of Tables 5-4 and 5-5:

	Regression Coefficients of			
	N_t	S_{t-1}	T	$\bar{R}^2$
All manufacturing				
Quarterly	.387	.526	.0029	.995
	(.034)	(.051)	(.0013)	
Monthly	.226	.705	.0075	.996
	(.021)	(.031)	(.0022)	
Durable goods industries				
Quarterly	.299	.613	.0016	.986
	(.038)	(.057)	(.0009)	
Monthly	.151	.788	.0038	.991
	(.019)	(.029)	(.0014)	

[32] The conditions are: (1) $0 < \beta < 2$; (2) $1 - \beta - \gamma > 0$; and (3) $\beta^2 \geqslant -4\gamma$. (They imply that $-1 < \gamma < 1$.) See Griliches, "Distributed Lags," pp. 27–29.

interruptions of this sort would result in an erratic lag form, which may be viewed as unsatisfactory. On the other hand, the *unimodal* pattern produced by certain particular paired values of β and γ has aroused considerable interest. In these distributions, the lag coefficients first increase (as a rule briefly) and then trail off slowly.[33]

While lag structures of the types implied by (8) may be applicable in many situations when the time units are appropriately chosen, the inclusion of the second lagged value of the dependent variable on the right-hand side of the equation again creates the problem of multicollinearity. If that variable is highly autocorrelated, this problem may frustrate estimation and render the model nonoperational.

In quarterly regressions, the addition of S_{t-2} actually fails to improve the results from the simple Koyck model for most of the industries. The coefficients of S_{t-2} are smaller than their standard errors, and in several cases $\bar{R}^2$ is decreased. Better results are obtained for only three industries: fabricated metal products, nonelectrical machinery, and "other durable goods."[34] In all regressions of this type, the coefficients of S_{t-2} are much smaller than those of S_{t-1} and not always positive. The lag distributions implied by these estimates are not unimodal—they would tend to decline from the beginning, though possibly not monotonically, and are in fact rather similar to the distributions produced by the geometric model. The average lag for these functions may be estimated as $\bar{n} = (b + 2c)/(1 - b - c)$. For most of the industries the figures thus obtained are larger than the average lags implied in the quarterly regressions with geometric distributions (Table 5-4, column 9), but the differences are small—typically about one month or less.

Second-order distributed lag regressions yield considerably better statistical results when applied to monthly data for some, but not all, industries. In the five equations shown in the tabulation below, all regression coefficients meet the conventional standards of significance, and the $\bar{R}^2$ slightly exceed their counterparts for the monthly Koyck

[33] Such distributions will be observed when β and γ assume values within the area bounded by $1 < \beta < 2$, $\beta + \gamma = 1$, and $\beta^2 = -4\gamma$. See Griliches, "Distributed Lags." For a discussion of unimodal lag profiles given by the Pascal distributions, see Robert M. Solow, "On a Family of Lag Distributions," *Econometrica*, April 1960, pp. 393–406.

[34] The best is the regression for fabricated metals, as follows:

$$(S_t)_{est} = \underset{(.041)}{.096} + \underset{(.046)}{.418}N_t + \underset{(.112)}{.346}S_{t-1} + \underset{(.092)}{.186}S_{t-2};\ \bar{R}^2 = .974$$

equations (Table 5-5, column 5).[35] The average lags implied by these

Fabricated metal products:

$$(S_t)_{\text{est}} = .076 + .250N_t + .444S_{t-1} + .265S_{t-2};\ \bar{R}^2 = .957;\ \bar{n} = 3.4$$
$$(.030)\ (.031)\qquad(.069)\qquad(.065)$$

Machinery, except electrical:

$$(S_t)_{\text{est}} = .031 + .178N_t + .513S_{t-1} + .299S_{t-2};\ \bar{R}^2 = .991;\ \bar{n} = 5.9$$
$$(.019)\ (.020)\qquad(.072)\qquad(0.66)$$

Electrical machinery:

$$(S_t)_{\text{est}} = .027 + .184N_t + .529S_{t-1} + .278S_{t-2};\ \bar{R}^2 = .988;\ \bar{n} = 5.6$$
$$(.020)\ (.024)\qquad(.076)\qquad(.069)$$

Other durable goods:

$$(S_t)_{\text{est}} = .037 + .463N_t + .252S_{t-1} + .277S_{t-2};\ \bar{R}^2 = .983;\ \bar{n} = 1.7$$
$$(.035)\ (.036)\qquad(.063)\qquad(.055)$$

Nondurable goods industries:

$$(S_t)_{\text{est}} = .024 + .606N_t + .271S_{t-1} + .123S_{t-2};\ \bar{R}^2 = .999;\ \bar{n} = 0.9$$
$$(.035)\ (.025)\qquad(.046)\qquad(.036)$$

equations are larger than their counterparts in Table 5-5 (in fabricated metals and the machinery industries, by relatively large margins of more than one month). These estimates appear sensible by broad tests of consistency with other information and are actually preferable to the other figures obtained earlier for the same industries.

Patterns of lagged response implied by the second-order equations can be estimated by a stepwise method. They indicate declines in the effects on S_t of N_{t-i} as the lag i increases, but there are also some oscillations, e.g., the influence of N_{t-2} is in most cases larger than that of N_{t-1}.[36]

[35] The same is also true of the corresponding equations for primary metals and for blast furnaces and steel mills. Nevertheless, this model seems unsatisfactory for these two industries, since the coefficients of S_{t-2} are negative and the average lags are somewhat lower than those in Table 5-5, column 6. In the light of the earlier results, these lags would be judged too small.

[36] The effects of N_{t-i} on S_t are calculated as follows: for $i = 0$, the effect ("weight") of N_t is $w_0 = a$; for $i = 1$, $w_1 = ab$; for $i = 2$, $w_2 = a(b^2 + c)$; and, in general, $w_i = b(w_{i-1}) + c(w_{i-2})$. To illustrate, the results for fabricated metals are:

Lag (i):	0	1	2	3	4	5	6
Effect of N_{t-i} on S_t:	.250	.111	.116	.081	.066	.051	.040
Cumulative as % of total effect [$= a/(1 - b - c) = 0.859$]	29.1	42.0	55.0	65.0	72.6	78.6	83.2

Finally, N_{t-1} was added to the second-order lag regressions in tests analogous to those performed earlier on the Koyck equations, but the results were on the whole negative. In sum, the gains that can be made by the use of second-order lagged distributions are marginal and limited to monthly data for some industries. No graphs for the results of these calculations are reproduced, since it is generally difficult to establish meaningful differences between them and the graphs for the first-order equations.

Instrumental Variables

It is well known (cf. note 20) that the main difficulty with autoregressive forms such as the Koyck model (equation 5) or the second-order functions (equation 8) is that, say, S_{t-1} is likely to be correlated with the disturbance terms (v_t or e_t) because of the association between these terms and S_t. Following an approach proposed in recent literature,[37] the variable S_{t-1} in (5) was replaced by $(S_{t-1})_{\text{est}}$ as estimated early in this chapter from

$$(S_{6t})_{\text{est}} = a_6 + \Sigma_{i=0}^{6} b_i N_{t-i} = S_t - u_{6t}$$

for the monthly data and

$$(S_t)_{\text{est}} = a_2 + b_{20}N_t + b_{21}N_{t-1} + b_{22}N_{t-2} = S_t - u_{2t}$$

for the quarterly data. In this two-stage procedure, then, $(S_{t-1})_{\text{est}}$ is computed first and then substituted into (5) to give

$$S_t = k + aN_t + b(S_{t-1})_{\text{est}} + v_t + bu_t. \tag{9}$$

This amounts to using a linear combination of past new orders, that is, $(S_{t-1})_{\text{est}}$, as an "instrumental variable." By assumption, the terms N_{t-i} are uncorrelated with the disturbances and, by construction, $(S_{t-1})_{\text{est}}$ consists only of such terms; hence $(S_{t-1})_{\text{est}}$ may be hypothesized to be independent of the disturbances. Thus one may hope that (9) will provide a consistent estimate of b.

Table 5-6 presents the results of applying model (9) to the monthly and quarterly data. The monthly correlations are for the most part somewhat lower than their counterparts for the Koyck equations in Table 5-5; the average lags are larger for nondurable industries reporting unfilled orders and the three metalworking industries, and

[37] For discussion and references, see Griliches, "Distributed Lags," pp. 41–42.

about equal or smaller in the other cases. The quarterly regressions compare more favorably with the corresponding Koyck equations in Table 5-4. The correlations are larger in Table 5-6 for primary metals and blast furnaces, very similar elsewhere in the two sets. The average lags exceed those in Table 5-4 for the three metalworking industries. The quarterly estimates in Table 5-6 appear to be preferable to the others for primary metals and blast furnaces. In particular, the Durbin-Watson statistics d (column 6) show these regressions in a definitely favorable light.[38]

Quarterly estimates of shipments derived from equation (9) for total durables and nonelectrical machinery look very similar to the series for the underlying instrumental variable $(S_t)_{est}$ (see Chart 5-2) and are therefore not reproduced in graphical form. Like that series, these estimates still resemble too much the course of new orders, and they clearly have poorer fits with the observed shipments than do the corresponding estimates from the Koyck regressions in Chart 5-3.

Summary of the Average Distributed-Lag Relations

Different distributed-lag models produce estimates of timing relationships that vary considerably, often even where the models perform equally well according to the usual goodness-of-fit criteria. These timing estimates, which are scattered through several tables and the text, have been brought together so that the varied assortment may be appraised jointly and, if possible, reduced to a few preferred measures.

The result of this undertaking is Table 5-7, which shows first the ranges and then the medians of the average lag estimates for both the monthly and the quarterly regressions (columns 1–3). The lags (all expressed in months) are typically larger when based on quarterly data, though transportation equipment is a clear exception. Also for some industries, such as nondurables and the primary metals groups, the monthly-quarterly differences are small and not uniform in sign (compare the entries on the odd and even lines). The ranges are in most cases large, especially for the quarterly regression estimates (columns 1 and 2).

[38] These statistics are lower for the equations with the instrumental variables than for the corresponding Koyck equations. This would be expected, as the calculated d values for the latter equations are likely to be biased toward 2 (see note 26). The more relevant comparison is between the d statistics in Table 5-6 and those for the regressions in Tables 5-2 and 5-3 from which the S_{est} estimates are computed. Here the comparable d values are usually similar.

Table 5-6

Regressions of Shipments on New Orders and on Lagged Predicted Shipments, Major Manufacturing Industries, Monthly and Quarterly, 1947–65 and 1953–65

Industry	Constant Term[a] (1)	Regression Coefficients[a] *a* (2)	Regression Coefficients[a] *b* (3)	Sum of Implicit Coefficients[b] (4)	$\bar{R}^2$ (5)	Durbin-Watson Statistic[c] (6)	Av. Lag[d] (mos.) (7)
			MONTHLY DATA, 1947–65 (221)[e]				
All manufacturing	.046	.320	.679	0.997	.982	0.344	2.11
	(.256)	(.051)	(.052)				
Durable goods	.075	.246	.749	0.979	.947	0.251	3.0
	(.227)	(.057)	(.061)				
Nondurable goods	.036	.622	.377	0.999	.998	1.156	0.6
	(.041)	(.030)	(.030)				
			MONTHLY DATA, 1953–65 (149)[e]				
Reporting unfilled orders							
All industries[f]	.697	.267	.700	0.890	.961	0.526	2.3
	(.383)	(.054)	(.065)				
Nondurables industries[g]	−.016	.379	.627	1.016	.992	1.706	1.7
	(.030)	(.043)	(.045)				
Primary metals	.164	.216	.728	0.781	.712	0.822	2.6
	(.149)	(.056)	(.083)				
Blast furnaces, steel mills	.126	.197	.726	0.721	.504	0.873	2.66
	(.128)	(.062)	(.105)				
Fabricated metal products	.069	.233	.729	0.858	.945	1.250	2.69
	(.036)	(.039)	(.050)				
Machinery exc. elect.	.038	.157	.828	0.914	.967	0.427	4.8
	(.040)	(.046)	(.058)				
Elect. machinery	.082	.284	.678	0.881	.963	0.504	2.10
	(.036)	(.044)	(.055)				

Transport. equip.	.313	.244	.683	0.772	.847	0.537	2.16
	(.160)	(.046)	(.070)				
Other durable goods[h]	.058	.488	.496	0.968	.979	1.417	1.0
	(.039)	(.043)	(.047)				
QUARTERLY DATA, 1947–65 (73)[e]							
All manufacturing	−.040	.475	.530	1.011	.986	0.519	3.4
	(.397)	(.060)	(.062)				
Durable goods	.095	.402	.592	0.986	.955	0.468	4.4
	(.364)	(.074)	(.078)				
Nondurable goods	.026	.741	.261	1.002	.999	1.258	1.1
	(.053)	(.031)	(.032)				
QUARTERLY DATA, 1953–65 (49)[e]							
Reporting unfilled orders							
All industries[f]	.786	.429	.537	0.927	.973	0.960	3.5
	(.535)	(.064)	(.078)				
Nondurables industries[g]	−.061	.484	.538	1.047	.997	1.865	3.5
	(.033)	(.046)	(.048)				
Primary metals	.282	.426	.477	0.815	.755	2.097	2.7
	(.229)	(.075)	(.108)				
Blast furnaces, steel mills	.275	.396	.435	0.701	.528	2.139	2.3
	(.203)	(.089)	(.147)				
Fabricated metal products	.088	.364	.589	0.886	.970	1.649	4.3
	(.045)	(.053)	(.065)				
Machinery exc. elect.	.043	.291	.697	0.959	.977	0.506	6.9
	(.055)	(.054)	(.068)				
Elect. machinery	.094	.434	.525	0.913	.974	0.452	3.3
	(.053)	(.067)	(.082)				
Transport. equip.	.536	.481	.395	0.795	.888	0.872	1.96
	(.248)	(.086)	(.125)				
Other durable goods[h]	.029	.594	.401	0.991	.992	1.421	2.01
	(.044)	(.048)	(.053)				

Notes to Table 5-6

[a] The form of the estimates is given by equation (9) in the text. The figures in parentheses are standard errors of the statistics shown.

[b] The sum equals $a/(1-b)$.

[c] $d = \Sigma(z_t - z_{t-1})^2/\Sigma z_t^2$, where $z_t = v_t + bu_t$. For the significance points of d, see note 12 and Table 5-3, note d, above.

[d] The average lag equals $b(1-b)$ for the monthly data and $3b(1-b)$ for the quarterly data.

[e] The figure in parentheses is number of observations. It is the effective sample size for each of the industries covered in the section that follows.

[f] Includes all durable goods industries and the four nondurable goods industries reporting unfilled orders (see note g).

[g] Includes textiles, leather, paper, and printing and publishing.

[h] Includes professional and scientific instruments; lumber; furniture; stone, clay, and glass products; and miscellaneous industries.

Table 5-7 incorporates four types of equations: the regressions of S_t on several lagged terms N_{t-i} (labeled "A"); the Koyck model relating S_t to N_t and S_{t-1} ("B"); the second-order equations, which include S_{t-2} on the right-hand side ("C"); and the two-stage model relating S_t to N_t and $(S_{t-1})_{\text{est}}$ ("D"). When these models are ranked according to the average lags they produce, from shortest to longest, A ranks first, followed in order by D, B, and C. These average ranks are the same for both the monthly and the quarterly regression estimates. However, there is considerable variation in the underlying ranks for the different industries.

The "best" estimates of the average lags (columns 4 and 5) for each industry are supplied by those equations which have the three largest values of the sums of calculated or implied coefficients of N_{t-i} (i.e., of the expressions $\Sigma' b$ or Σ in the preceding tables). Typically, but not always, these are also the equations with the highest adjusted determination coefficients $\bar{R}^2$. For the over-all aggregates (first six lines), the sums Σ are all approximately equal to one, so the "best" equations were selected primarily by the highest $\bar{R}^2$. The lowest and highest average lags thus estimated are listed in column 4. The regressions that produce average lags falling into these ranges are identified in column 5. They account for about half of the number of all estimated equations, but include only 4 applications of model A against 15 each of B and C and 14 of D. The average lags for A are generally smaller. Of the selected regressions, fewer are based on monthly than on quarterly data (20 vs. 28).

The "best" average lag estimates, then, point to rather long delivery periods in the machinery industries (6 to 7 months for nonelectrical machinery, 4 to 6 months for electrical machinery) and to short delivery periods (about 2 months or a little more) in primary metals. The midpoints of the selected ranges for fabricated metals and transportation equipment fall in between, at 4 and 3 months. The groups of industries reporting unfilled orders show average lags of about 3 months. For the other durables group, the typical lag is barely 2 months. The 5-month lag for all durable goods industries contrasts sharply with the 1-month lag for the nondurables.

Regression Estimates and Turning-Point Estimates

In Chapter 4, delivery lags were estimated by matching the dates of specific-cycle turns in new orders and shipments and measuring the intervals between them. This method appears to use only a small part of the evidence of monthly time series, but this impression is not correct. It is true that a cycle which may last several years yields only two additional timing comparisons, but the determination of the turning dates requires a thorough examination of the whole sequence of values that the series has assumed.[39] Problems arise because individual comparisons are often influenced by particular configurations of short movements in the vicinity of a turning point. One expects the resulting errors mostly to cancel each other in the average lag measures, but the probability of this is reduced when the available series are so short that the averages cover few observations.

The present approach based on regression and correlation measures avoids these difficulties but presents some problems of its own. It is more objective but also more mechanical. It uses all available information with about equal weights instead of concentrating on turning points, but a discrimination among the observations—e.g., in favor of major turns—may actually be desired for some purposes. The possibly systematic differences between lags at peaks and troughs, or between some other episodes or subperiods, are ignored.

Comparing the median estimates of the average delivery lags as

[39] This becomes clear when one attempts to substitute a mechanical operation for the judgmental process involved. Thus a computer program for selecting turning points that was recently developed by Gerhard Bry and Charlotte Boschan uses all observations for the given series in each of several computational steps; see their *Cyclical Analysis of Time Series: Selected Procedures and Programs*, Technical Paper 20, New York, NBER, 1971.

Table 5-7
Summary of Regression Estimates of Average Lags of Shipments Behind New Orders, Major Manufacturing Industries, Monthly and Quarterly, 1947–65 and 1953–65

	Estimates of Average Lags of Shipments Behind New Orders (months)					Av. Lag of Shipments Estimated from Turning Point Comparisons (months)[d]
				"Best" Estimates[b]		
Industry	Lowest[a] (1)	Highest[a] (2)	Median[a] (3)	Range of Av. Lags (4)	Regression Code[c] (5)	(6)
				1947–65		
All manufacturing	2.1	3.7	2.9	3.4–3.7	B1, C1, D2	3.1
	2.4	5.0	4.0			
Durable goods	2.6	5.6	4.2	4.4–5.6	B1, C1, D2	5.1
	2.7	7.0	5.6			
Nondurable goods	0.6	1.0	0.8	0.9–1.1	A1, A2, B2, C1, D2	2.0
	1.0	1.1	1.1			
				1953–65		
Reporting unfilled orders						
All industries[e]	2.0	2.6	2.4	2.5–3.6	B1, B2, C1, D2	4.1
	2.2	3.6	3.5			
Nondurables industries[f]	1.2	2.3	2.0	2.2–3.5	A1, A2, B2, C1, D2	2.2
	2.2	3.5	3.2			
Primary metals	1.3	2.6	1.6	1.7–2.7	B1, B2, C2, D1, D2	5.0
	1.5	2.7	1.8			
Blast furnaces, steel mills	1.1	2.7	1.4	1.6–2.7	B1, D1, D2	6.1
	1.2	2.3	1.3			

Fabricated metal products	2.0	3.4	2.4	3.4–4.6	B2, C1, C2, D2	3.2
	2.5	4.6	3.9			
Machinery exc. elect.	2.9	5.9	4.6	5.9–7.5	B2, C1, C2, D2	4.9
	3.1	7.5	6.9			
Elect. machinery	1.9	5.6	3.2	4.2–6.0	B1, B2, C1, C2	2.5
	1.9	6.0	4.5			
Transport. equip.	1.6	5.6	3.8	2.0–3.8	B2, C2, D1, D2	3.0
	1.5	3.8	2.8			
Other durable goods[g]	1.0	1.7	1.3	1.7–2.4	B2, C1, C2, D2	2.2
	1.6	2.4	2.0			

[a] For each industry, entry on first line is for estimates from monthly regressions; entry on second line, for estimates from quarterly regressions.

[b] Based on the equations which provide the three highest values of the sums of calculated or implied coefficients of the new-order terms (i.e., the three highest values of Σb or Σ). For the 1947–65 aggregates, where the sums are all about equal to 1, the highest $\bar{R}^2$'s are used instead as criterion. See text.

[c] Identifies the equations referred to in note b and others that produce average lags in the range given in column 4. A refers to the regressions of S_t on N_{t-i}, where $i = 0, 1, \ldots, 6$ months (Table 5-2) or $i = 0, 1, 2$ quarters (Table 5-3); B, to the Koyck equations (Tables 5-4 and 5-5); C, to the second-order functions (see text with tabulations above); D, to the regressions of S_t on N_t and $(S_{t-1})_{est}$ (Table 5-6); 1, to monthly and 2 to quarterly regressions. For example, B1 denotes monthly Koyck equations, C2, quarterly second-order functions, etc.

[d] Based on the timing comparisons in Table 4-5 and note 9 in Chapter 4. For the over-all aggregates, the averages refer to the period 1948–65 and agree with the corresponding entries in Table 4-6, column 9. For the component durables industries, the averages refer to the period since 1953 and hence exclude the observations recorded in Table 4-5, columns 1–4.

[e] Includes all durable goods industries and the four nondurable goods industries reporting unfilled orders (see note f).

[f] Includes textiles, leather, paper, and printing and publishing.

[g] Includes professional and scientific instruments; lumber; furniture; stone, clay, and glass products; and miscellaneous industries.

derived from the monthly regressions with the averages of the turning-point measures for the same periods, one finds that the latter tend to exceed the former (Table 5-7, columns 3 and 6). The differences are very large for the primary metals industries, but elsewhere they are approximately equal to one month and are negligibly small in three cases. Reverse differences are found in only two instances: The median regression lags are somewhat longer than the turning-point lags for electrical machinery and transportation equipment.

The regression measures here considered largely reflect the association between fairly short movements in new orders and shipments, measured in months or quarters. For such movements, the maximum-correlation lags are evidently quite small: The highest correlations in Table 5-1 are typically those for the simultaneous timing of N and S. As noted earlier, it is plausible that many random events, e.g., strikes, affect both variables at about the same time. The correlations between the longer, cyclical movements, on the other hand, are likely to involve longer lags. Thus a transition from an expansion to a contraction in new orders, especially if it is gradual rather than abrupt, would require some time to be recognized as such and translated into a similar reversal in shipments. The intervening process is a cumulation of many short-lag effects, in which backlogs of unfilled orders act as a factor that cushions and delays the reaction on the supply side.

An important type of lag distribution in production to order contains a clustering of lags around some typical (modal) delivery period and also smaller frequencies of progressively longer lags. Such a distribution is skewed "to the left," i.e., in the direction of longer lags reaching further into the past. The skewness implies that the mean and median diverge from the mode in the direction of longer lags. The maximum-correlation timing probably often corresponds to the mode,[40] and it is indeed generally coincident or a shorter lag than the average derived from the estimated lag distributions (compare Table 5-1 with the subsequent tables). The average lags at the turning points correspond more nearly to the means than to the modes of the lag distributions, but they

[40] Let a_k be the highest of the coefficients a_i in $S_t = \Sigma a_i N_{t-i} + u_t$, where the summation is over $i = 0, \ldots, \infty$. Under certain conditions, the lag k will coincide with that lag of S relative to N which yields the highest simple correlation between the two variables. The assumptions here are that the autocorrelations of N are lower the longer the intervals over which they are taken (i.e., $r_{12} > r_{13} > r_{14}$, etc., for successive periods $t = 1, 2, 3, \ldots$) and that they are independent of t for a given interval ($r_{12} = r_{23} = r_{34}$, etc.). The variances of N_{t-i} are assumed equal for all values of i. I am indebted to Jacob Mincer for informing me of a proof of these propositions that he has recently developed.

are frequently still longer than the mean regression lags estimated from the monthly regressions.

In several cases, including all manufacturing and total durables (compare columns 4 and 6 in Table 5-7), the turning-point estimates fall within the ranges of our preferred regression estimates. The ranges, being based on quarterly as well as on monthly regressions, include longer lags, which actually exceed the turning-point lags in some instances (notably for the machinery industries). However, turning-point comparisons yield much longer lags than any of the regressions for the primary metals industries. This is due in large measure to particular developments, such as the major steel strike in 1956, which had effects of different intensities upon new orders and shipments (see Chart 3-2 and Table 4-5 with the accompanying text).[41]

Predictive Equations with Variable and Constant Lags

Variable Lag Coefficients

All preceding estimates are based on distributed-lag models with fixed coefficients. However, the lead time required to fill orders lengthens when the rates at which new orders are received are high relative to the desired or optimal levels of capacity utilization. At such times, as noted on earlier occasions, pressures of demand upon capacity are met in large part by backlog accumulation. The build-up of unfilled orders indicates that the average time-span between the receipt of an order and the start of production on it tends to increase. Conversely, when the demand pressures subside and the backlogs decline, this "waiting-period" part of the over-all delivery lag gets shorter. After the decline in the rates of new orders received and, possibly, an increase in the rates of past orders canceled (Chapter 2) had depleted the backlogs sufficiently, little if any waiting would be imposed on currently received orders, that is, work on them would tend to begin promptly after receipt and the delivery lag would be largely limited to the actual worktime required to fill the orders. This argument implies that the lags of shipments behind new orders vary systematically in the

[41] At most turning points, primary metals shipments followed orders by fairly short intervals, but their lags in 1953–54, 1956, and 1960–61 were long. Excluding these episodes leaves nine observations for total primary metals and five for blast furnaces; the average lags for these subsets are 2.4 and 3.0 months, respectively, which are already much closer to the regression estimates in Table 5-7.

course of the business cycle; hence it suggests that the coefficients in the distributed-lag relations between S and N should be variable rather than fixed and such as to make the average lag a positive function of some measure of the relative demand pressures.

A model with such variable lag coefficients was presented recently by Popkin.[42] In his original and very interesting article, Popkin starts from a predictive equation in which shipments are related only to the preceding, and not also to the current, values of new orders. The equation, designed to be applied to quarterly data, reads in our notation:

$$S_t = \alpha_{1,t}N_{t-1} + \alpha_{2,t}N_{t-2} + u_t. \tag{10}$$

The coefficients of the new-order terms in (10) are made to depend on the ratio of unfilled orders to shipments, U/S, and since this ratio varies over time so will these coefficients (it is because of this that time subscripts must be added to α_1 and α_2). In accordance with the preceding discussion, U/S can be viewed as an index of relative demand pressures, which provides the rationale for this approach. When U/S rises, the influence on S_t of orders received in the more distant past should increase, while the influence of the more recent orders should decrease, that is, α_2 is then to become more important relative to α_1. This is expected because when backlogs accumulate faster than output and shipments can be increased, newly received orders pile up and must presumably yield to the older orders which have priority in the production schedule. Thus current shipments would then consist in larger part of the older orders. By the reverse of the same argument, α_1 should gain relative to α_2 when the ratio U/S declines. Popkin assumes that these relations involve a one-period lag and are linear but not necessarily proportional; so

$$\alpha_{1,t} = \beta_0 + \beta_1(U/S)_{t-1}. \tag{11}$$

A further assumption is that all of each quarter's orders will result in shipments over the following two quarters, which implies the constraint $\alpha_{1,t} + \alpha_{2,t+1} = 1$. It follows that

$$\alpha_{2,t} = 1 - [\beta_0 + \beta_1(U/S)_{t-2}]. \tag{12}$$

[42] Joel Popkin, "The Relationship Between New Orders and Shipments: An Analysis of the Machinery and Equipment Industries," *Survey of Current Business*, March 1965, pp. 24–32.

Substituting (11) and (12) into (10) results in

$$S_t = [\beta_0 + \beta_1(U/S)_{t-1}]N_{t-1} + \{1 - [\beta_0 + \beta_1(U/S)_{t-2}]\}N_{t-2} + u_t. \quad (13)$$

Finally, by rewriting a little and dropping two constraints implicit in (13) so as to allow for departures from the hypothesis, one gets

$$S_t = \alpha_0 + \beta_0\Delta N_{t-1} + \beta_1\Delta[(U/S)N]_{t-1} + \beta_2 N_{t-2} + u_t, \quad (14)$$

which is the form to be estimated.[43]

Popkin applied equation (14) to shipments and new and unfilled orders for the market category of machinery and equipment. Since the data were deflated by the BLS wholesale price index for machinery and equipment, the variables are expressed in billions of 1957–59 dollars. For the period from III-1953 through III-1964 (45 quarters), Popkin reports the following result:

$$S_t = \underset{(6.29)}{2.409} + \underset{(5.16)}{1.205}\Delta N_{t-1} - \underset{(3.70)}{0.390}\Delta[(U/S)N]_{t-1} + \underset{(16.09)}{0.717}N_{t-2}; \; \bar{R}^2 = .868 \quad (15)$$

The numbers in parentheses are the t statistics (ratios of the coefficients to their standard errors); they indicate that all the estimates (including the constant) are significant at the 1 per cent level. The equation provides a good fit to the sample data, but the residuals show a significant degree of autocorrelation.[44]

Equation (15) implies that $\alpha_{1t} = 1.035 - 0.390(U/S)_{t-1}$, which corresponds to (11), and that $\alpha_{2t} = 0.318 + 0.390(U/S)_{t-2}$, which corresponds to (12) when allowance is made for the fact that β_2 in (15) is not 1 but 0.717.[45] It is clear that α_1 varies inversely and α_2 varies directly with U/S. As expected, then, a rise in the ratio U/S is associated

[43]Note that (13) can be written as $\beta_0[N_{t-1} - N_{t-2}] + \beta[(U/S)_{t-1}N_{t-1} - (U/S)_{t-2}N_{t-2}] + N_{t-2} + u_t$. The two terms in brackets are changes over time for which the shorter notation ΔN_{t-1} and $\Delta[(U/S)N]_{t-1}$ is adopted. Allowing for the possibility of a nonzero constant term (α_0) and of a coefficient of N_{t-2} which differs from 1 (β_2), this equation is then translated directly into (14).

[44]The adjusted standard error of estimate is $0.271 billion, while the mean value of shipments during the period is $8.46 billion. The residual autocorrelation statistic is 1.292, significant at the 1 per cent level. When charted, the calculated shipments are seen to miss turning points by changing direction one quarter after actual shipments. See Popkin, "Relationship Between New Orders and Shipments," pp. 28–29 and Chart 16.

[45]Accordingly, the values of α_{1t} and α_{2t} always add up to 0.717 (not to 1). The ratio of the constant term in (15) to the average value of new orders is 0.289, which, when added to 0.717 totals approximately 1.

with an increase in the proportion of orders received during $t-2$ and a decrease in the proportion of orders received during $t-1$, within the aggregate of shipments for period t. Thus the higher (lower) the ratio, the longer (shorter) is the average lag of S relative to N.[46]

When production runs at virtually full capacity, increases in shipments may be constrained to smaller amounts than those predicted only by the recent levels and changes of new orders and backlog-shipment ratios. Severe shortages of materials due to strikes, etc., could likewise interfere with the performance of the model. Popkin notes these and some other possible shortcomings of his estimating equations. The other models of the N-S relationship that are considered in this chapter are subject to similar difficulties. However, the limitations to quarterly data and to orders of only two quarters $(t-1)$ and $(t-2)$ may well cause a misspecification of the lag structure, and other models not so restricted are possibly better in this respect. This is the cost paid for the presumably realistic and important feature of variable coefficients combined with the ease of estimation and the predictive nature of the model.[47]

In an effort to examine further the variable lag hypothesis in the context of orders-shipments relations, I have applied equation (14) to the OBE-Census major-industry data. The results are presented in Table 5-8.

For most of the industries, the directly estimated regression coefficients β are definitely significant statistically and have the expected signs. Typically, β_0 is large and positive, while β_1 is small and negative (columns 2 and 3). This means that $\alpha_{1,t}$, the implied coefficients of N_{t-1}, are likely to be large, except at high values of the backlog-shipment ratios [according to equation (11)]. It is not surprising that the effects

[46] The simple average of one-quarter and two-quarter lags is a lag of 4.5 months, which would apply if N_{t-1} and N_{t-2} had equal weights ($\alpha_{1,t} = \alpha_{2,t}$). The value of U/S associated with such a lag is about 1.735, as implied by equation (15). When U/S is as low as 0.815, α_1 and α_2 equal 0.717 and zero, respectively, giving an average lag of 3 months; when U/S is as high as 2.743, $\alpha_1 = 0$ and $\alpha_2 = 0.717$, and the average lag is 6 months. Actually, the ratio of unfilled orders to shipments (quarterly, seasonally adjusted, and deflated) fluctuated in 1953–64 within a narrower range, approximately between 1.0 and 2.0. This implies a low value of the average lag of about 3.3 months (with $\alpha_1 \simeq 0.645$ and $\alpha_2 \simeq 0.072$) and a high value of the average lag of about 4.9 months (with $\alpha_1 \simeq 0.255$ and $\alpha_2 \simeq 0.462$).

[47] Only a few small modifications of the assumed lag distribution were tried, with some success, in the machinery-and-equipment analysis under review. The contents of the quarterly terms N_{t-1} and N_{t-2} were redefined by one-month shifts forward or backward relative to the current quarter t to which S_t refers. The shift forward in time would cause a one-month "overlap" and a shortening of the imposed lag structure; the shift backward would cause a one-month "gap" and a lengthening of the lag. Popkin reports that the shortening worked better than the lengthening.

on S_t of N_{t-1} should be large, especially since they probably absorb much of the influence of the omitted N_t terms.

The β_2 coefficients are approximately equal to 1 for all manufacturing, the nondurables, and the other durables group, and are not much smaller than 1 for total durables and the machinery industries (column 4). In these cases, then, the total effect on S_{t-1} of N_{t-2} does come close to unity (the sums of the variable coefficients $\alpha_{1,t}$ and $\alpha_{2,t}$ equal β_2, up to rounding errors). For the other industries, the values of β_2 fall short of 1.0 by varying but at least appreciable margins. In these cases, the constant terms α_0 are also disturbingly large.

Once more the least satisfactory results are for total primary metals and for blast furnaces. Here the α_0 are very large and highly significant, and the β_2 are only 0.66 and 0.55. Moreover, the β_1 coefficients are either positive, which contradicts the hypothesis that α_1 varies inversely to U/S, or, more likely, are not different from zero, which contradicts the notion of variable lag coefficients.[48]

The determination coefficients, $\bar{R}^2$, like their counterparts for the other models, are very high (exceeding .9) for industries other than primary metals and transportation equipment (column 5). Not surprisingly, these correlations tend to be lower than those for the equations that include N_t (see Tables 5-3 and 5-4), but the differences involved are generally small.

The best feature of the regressions in Table 5-8 is the high Durbin-Watson statistics, d, which give no indications of significant autocorrelation in the residuals. This contrasts with the generally low d values for the quarterly regressions of S_t on N_t, N_{t-1}, and N_{t-2} (see Table 5-3, column 7 and note c).

The values of the variable coefficients $\alpha_{1,t}$ and $\alpha_{2,t}$ can be computed given the estimates of β_0, β_1, and β_2, and the reported figures for the backlog-shipment ratios U/S. The estimates for the mean values of U/S over the regression periods, $\bar{\alpha}_{1,t}$ and $\bar{\alpha}_{2,t}$, are shown in columns 7 and 8 of the table.

These measures are not apt to be satisfactory for interindustry comparisons of the lag structures, since each equation includes only two new-order terms with the same time subscripts $(t-1, t-2)$. Estimates

[48] These results are not inconsistent with the earlier ones which suggest that shipments of primary metal products consist of orders received in the same and (perhaps to a larger extent) in the preceding quarter, while the longer lag terms are unimportant. See the regressions for these industries in Table 5-3.

Table 5-8
Regressions of Shipments on New Orders of Two Preceding Quarters, Variable Lag Coefficients, Major Manufacturing Industries, 1947–65 and 1953–65

Industry	Constant Term[a] (1)	Regression Coefficients[a] β_0 (2)	β_1 (3)	β_2 (4)	$\bar{R}^2$ (5)	Durbin-Watson Statistic[b] (6)	Estimates at Mean Values[c] $\bar{\alpha}_{1,t}$ (7)	$\bar{\alpha}_{2,t}$ (8)
				1947–65 (74)[d]				
All manufacturing	.164	1.202	–.225	1.001	.985	2.026	.762	.246
	(.404)	(0.104)	(.032)	(0.015)				
Durable goods	.358	1.283	–.181	0.976	.964	1.820	.626	.356
	(.314)	(0.123)	(.024)	(0.022)				
Nondurable goods	–.003	0.856	–.095	1.014	.991	1.872	.878	.187
	(.152)	(0.114)	(.109)	(0.011)				
				1953–65 (50)[d]				
Reporting unfilled orders								
All industries[e]	2.020	1.043	–.085	0.908	.958	1.631	.802	.110
	(.588)	(0.158)	(.038)	(0.030)				
Nondurables industries[f]	–.099	0.975	–.090	1.038	.992	1.864	.899	.140
	(.052)	(0.139)	(.064)	(0.014)				

Primary metals	.980	0.662	.021	0.660	.733	1.827	.710	−.051
	(.180)	(0.129)	(.031)	(0.064)				
Blast furnaces, steel mills	.732	0.608	.013	0.548	.550	1.981	.643	−.096
	(.130)	(0.119)	(.022)	(0.082)				
Fabricated metal products	.216	0.760	−.093	0.883	.945	1.955	.519	.367
	(.061)	(0.169)	(.043)	(0.036)				
Machinery, exc. elect.	.113	1.379	−.226	0.954	.982	1.849	.638	.326
	(.045)	(0.145)	(.033)	(0.020)				
Elect. machinery	.145	1.283	−.158	0.934	.955	1.520	.664	.279
	(.063)	(0.198)	(.046)	(0.031)				
Transport. equip.	.929	0.953	−.067	0.789	.836	1.720	.488	.307
	(.209)	(0.147)	(.021)	(0.051)				
Other durable goods[g]	.051	1.103	−.105	0.993	.968	2.018	.933	.062
	(.085)	(0.159)	(.077)	(0.027)				

[a] The estimates are based on equation (14) in the text. The figures in parentheses are standard errors of the statistics shown.

[b] The Durbin-Watson statistic (d) equals $\Sigma[(u_t) - (u_{t-1})]^2/\Sigma(u_t^2)$. For the significance points of d, see text note 11 and Table 5-3, note d.

[c] Computed from the equations for the mean values of the backlog-shipments ratios: $\bar{\alpha}_{1,t} = \beta_0 + \beta_1(\bar{U}/\bar{S})_{t-1}$ and $\bar{\alpha}_{2,t} = \beta_2 - [\beta_0 + \beta_1(\bar{U}/\bar{S})_{t-2}]$.

[d] The figure in parentheses is number of quarterly observations. It is the effective sample size for each of the industries covered in the section that follows.

[e] Includes all durable goods industries and the four nondurable goods industries reporting unfilled orders (see note f).

[f] Includes textiles, leather, paper, and printing and publishing.

[g] Includes professional and scientific instruments; lumber; furniture; stone, clay, and glass products; and miscellaneous industries.

of average lags may be obtained by multiplying $\bar{\alpha}_{1,t}$ by 3 and $\bar{\alpha}_{2,t}$ by 6 and adding the products. When this is done, the results vary only from 3.2 to 4.0 months among industries other than total primary metals and blast furnaces. The actual differences between the average delivery lags in the industries compared are, in all likelihood, considerably larger, as indicated by the measures assembled in Table 5-7.

Variable vs. Fixed Lag Coefficients

Despite the weakness just noted, equation (14) turns out to be a useful model in dealing with the specific question of the variability of shipments lags over time. When compared with estimates which use the same quarterly data to link S_t to N_{t-1} and N_{t-2} by means of fixed lag coefficients (Table 5-9), the results obtained with variable coefficients are for the most part superior.

The implied average coefficients $\bar{\alpha}_{1,t}$ and $\bar{\alpha}_{2,t}$ in Table 5-8 are strikingly (and somewhat reassuringly) similar to the corresponding,

Chart 5-5
Regressions for Shipments of Durable Goods Industries and Nonelectrical Machinery, Based on Variable Lag Coefficients, Quarterly, 1947–65

PART A

$$(S_t)_{est} = a + b_1(N_{t-1} - N_{t-2}) + b_3[(U/S)_{t-1} - (U/S)_{t-2}] + b_3N_{t-2}$$

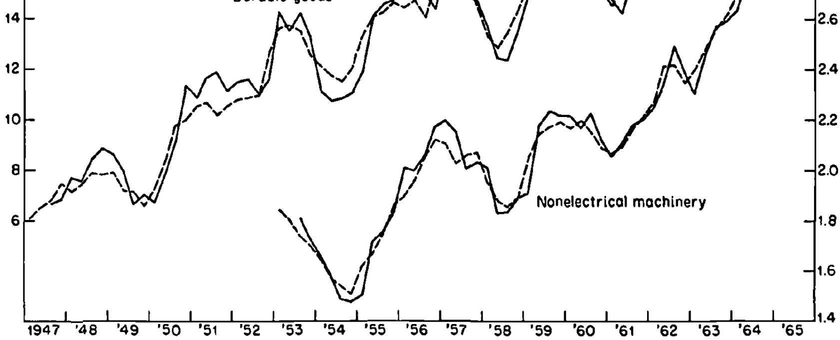
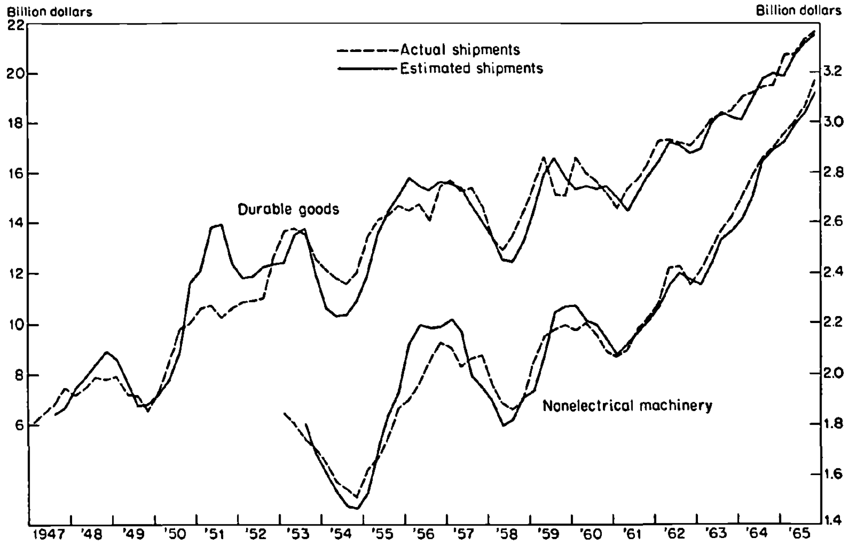


Chart 5-5 (continued)

PART B

$$(S_t)_{est} = a + b_1 N_{t-1} + b_2 N_{t-2}$$



directly estimated coefficients in Table 5-9, that is, to b_{31} and b_{32}, respectively. But the sums of the variable coefficients are in several cases appreciably larger than the sums of the fixed coefficients (compare columns 4 in the two tables). The constant terms are often considerably smaller in the variable lag than in the fixed lag equations (columns 1). The differences between the values of $\bar{R}^2$, while small, also favor the variable lag regressions (column 5). And the Durbin-Watson statistics are low for most of the fixed lag equations, suggesting positive autocorrelations of the residuals, whereas the evidence from the d tests for the variable lag estimates is generally favorable (columns 6).

Only for total primary metals and for blast furnaces are the regressions with fixed coefficients somewhat better than those with variable coefficients, but neither type of equation works really well for these industries; in particular N_{t-2} does not emerge as a significant factor in either model (see sixth and seventh lines in the two tables).

Chart 5-5 compares the estimates of shipments of durable goods manufacturers and the nonelectrical machinery industry based on the

Table 5-9
Regressions of Shipments on New Orders of Two Preceding Quarters, Fixed Lag Coefficients, Major Manufacturing Industries, 1947–65 and 1953–65

Industry	Constant Term[a] (1)	Regression Coefficients[a] b_{31} (2)	b_{32} (3)	Sum of b's (4)	$\bar{R}^2$ (5)	Durbin-Watson Statistic[b] (6)
			1947–65 (74)[c]			
All manufacturing	0.521	.717	.275	0.992	.974	0.945
	(0.519)	(.101)	(.102)			
Durable goods	0.719	.641	.313	0.955	.937	0.664
	(0.412)	(.116)	(.117)			
Nondurable goods	0.011	.796	.218	1.014	.991	1.876
	(0.151)	(.091)	(.093)			
			1953–65 (50)[c]			
Reporting unfilled orders						
All industries[d]	2.535	.776	.109	0.885	.954	1.126
	(0.563)	(.107)	(.112)			
Nondurables industries[e]	−0.110	.817	.226	1.043	.992	1.683
	(0.052)	(.084)	(.088)			

Primary metals	0.942	.722	−.050	0.672	.736	1.989
	(0.172)	(.095)	(.096)			
Blast furnaces, steel mills	0.718	.643	−.087	0.556	.556	2.118
	(0.127)	(.102)	(.102)			
Fabricated metal products	0.294	.448	.391	0.840	.940	1.692
	(0.051)	(.087)	(.088)			
Machinery exc. elect.	0.281	.494	.393	0.886	.964	0.665
	(0.054)	(.097)	(.102)			
Elect. machinery	0.242	.690	.203	0.892	.945	0.900
	(0.062)	(.105)	(.111)			
Transport. equip.	1.076	.506	.253	0.819	.805	1.176
	(0.222)	(.105)	(.113)			
Other durable goods [f]	0.073	.928	.060	0.988	.968	1.761
	(0.084)	(.095)	(.100)			

[a] The estimating equation is

$$S_t = a_3 + b_{31}N_{t-1} + b_{32}N_{t-2} + u_{32}.$$

The figures in parentheses are the standard errors of the statistics shown.

[b] The Durbin-Watson statistic (d) equals $\Sigma(u_{3t} - u_{3t-1})^2/\Sigma(u_{3t})^2$. For the significance points of d, see note 11 and Table 5-3, note d, above.

[c] The figure in parentheses is the number of quarterly observations. It represents the effective sample size for each of the industries covered in the section that follows.

[d] Includes all durable goods industries and the four nondurable goods industries reporting unfilled orders (see note e).

[e] Includes textiles, leather, paper, and printing and publishing.

[f] Includes professional and scientific instruments; lumber; furniture; stone, clay, and glass products; and miscellaneous industries.

variable lag model (Part A) and the fixed lag model (Part B), and refer to the regressions shown in Tables 5-8 and 5-9. Here, as elsewhere, shipments estimated from new-order data (S_{est}) show larger variations than do actual shipments (S), but the use of variable instead of fixed coefficients reduces the amplitudes of S_{est} markedly, thus bringing S_{est} and S closer together. This is because in expansion, when the levels of "older" orders are low relative to those of the more recent orders, the former gain greater influence on the variable lag estimate of shipments, while in contraction, when the reverse is true, the relative influence of the older orders declines.[49]

Generalized Least-Squares Estimators

The disturbance terms are probably positively autocorrelated for several of the relationships in Table 5-9, and it is interesting to observe that this cannot be attributed to the omission of N_t from these regressions. (Actually, there is *more* evidence of autocorrelation in Table 5-3, where S_t is related to N_t as well as to N_{t-1} and N_{t-2}.)[50]

A very simple form of the dependence of disturbances over time is the first-order autoregressive scheme

$$u_t = \rho u_{t-1} + \epsilon_t. \tag{16}$$

This relation is often assumed, although it is actually rather special and may not apply, because knowledge about the true structure of the disturbances is usually lacking and ρ is easy to obtain from the least-squares regression of calculated residuals. The computed autocorrelation coefficient can then be used to transform the variables and re-estimate their relationship so as to get new residuals e_t, which are presumably not significantly autocorrelated.[51]

Applying this approach to the residuals u_{3t} from the regressions of Table 5-9, let us first estimate ρ_1 from

$$u_{3t} = \rho_1 u_{3t-1} + e_{it}, \tag{16a}$$

by least squares. In cases where the coefficients ρ_1 are definitely

[49] However, the systematic error of overestimating the fluctuations of shipments, while substantially reduced, is still quite evident in the variable coefficients case. It should be noted that no provision is made in any of the regressions for a capacity constraint that may limit the expansion of output and shipments at certain times. Limitations of this sort could be responsible for some of the discrepancies between S_{est} and S, particularly in 1951.

[50] Compare the Durbin-Watson statistics in Table 5-9, column 6, with those in Table 5-3, column 7.

[51] Inefficient predictions and underestimation of the sampling variances of the regression coefficients are among the costs of a direct application of the usual least-squares estimation formulas to situations involving autocorrelated disturbances. On the consequences and treatment of this problem, and in particular on the above approach as an implication of "generalized least squares," see Johnston, *Econometric Methods*, pp. 179–88, 193–94.

significant, the next step is to compute least-squares regressions for the transformed variables according to

$$S_t - \rho_1 S_{t-1} = a_3^*(1 - \rho_1) + b_{31}^*(N_{t-1} - \rho_1 N_{t-2}) + b_{32}^*(N_{t-2} - \rho_1 N_{t-3}) + e_{1t}. \quad (17)$$

The regression coefficients b_{31}^* and b_{32}^* in (17) are new estimates of the effects on S_t of N_{t-1} and N_{t-2}, to be compared to b_{31} and b_{32} in Table 5-10.[52] The residuals e_{1t} are subject to tests for the presence of autocorrelation; if they are significantly autocorrelated, an iteration of the procedure would be indicated.

The same method was also applied to the relations that link S_t to N_t as well as to N_{t-1} and N_{t-2}, so as to learn more about the uses of this approach in the present context and in particular to check on the effects of the omission of N_t from (17). That is, using the residuals u_{2t} from the regressions of Table 5-3, the values of ρ_2 were estimated from

$$u_{2t} = \rho_2 u_{2t-1} + e_{2t} \quad (16b)$$

and used to compute the transformed variables for equations

$$S_t - \rho_2 S_{t-1} = a_2^*(1 - \rho_2) + b_{20}^*(N_t - \rho_2 N_{t-1}) + b_{21}^*(N_{t-1} - \rho_2 N_{t-2}) + b_{22}^*(N_{t-2} - \rho_2 N_{t-3}) + e_{2t}. \quad (18)$$

The estimated autocorrelation coefficients ρ_1 range from 0.019 to 0.735; the values of ρ_2 vary from 0.064 to 0.830. Both ρ_1 and ρ_2 are very small for the metalworking industries, and at least one of them is also small for the nondurables and the other durables group. The sectors or industries for which both ρ_1 and ρ_2 are large enough to appear significant include all manufacturing, total durables, the group of industries reporting unfilled orders, the two machinery industries, and transportation equipment.

[52] From

$$S_t = a_3 + b_{31}N_{t-1} + b_{32}N_{t-2} + u_{3t}$$

subtract

$$\rho_1 S_{t-1} = \rho_1 \alpha_3 + \rho_1 b_{31} N_{t-2} + \rho_1 b_{32} N_{t-3} + \rho_1 u_{3t-1}.$$

The result is

$$S_t - \rho_1 S_{t-1} = a_3(1 - \rho_1) + b_{31}(N_{t-1} - \rho N_{t-2}) + b_{32}(N_{t-2} - \rho_1 N_{t-3}) + e_{1t}.$$

This indicates the general relationship between the model used in Table 5-9 and equation (17). Of course, the estimated coefficients differ, since direct least-squares procedures are applied without any constraints in both cases: to the basic data on S and N in Table 5-9 and to the transformed variables $(S_t - \rho_1 S_{t-1})$, $(N_t - \rho_1 N_{t-1})$, etc., in (17).

Table 5-10

Lagged Relations Between Shipments and New Orders Estimated from Data Transformed to Account for Autocorrelated Disturbances, Selected Industries, Quarterly, 1947–65 and 1953–65

Industry	Autocorrelation Coefficient[a] (1)	Constant Term[b] (2)	Regression Coefficients[c] (3)	(4)	(5)	$\bar{R}^2$ (6)	Durbin-Watson Statistic[d] (7)
			I. EQUATION (17)[e]				
1. All manufacturing	.526	0.561		.568	.401	.924	1.693
		(0.437)		(.079)	(.080)		
2. Durable goods	.668	0.823		.492	.337	.725	1.684
		(0.295)		(.083)	(.083)		
3. All industries reporting unfilled orders[f]	.420	1.758		.699	.164	.904	1.804
		(0.479)		(.105)	(.110)		
4. Machinery except elect.	.658	0.147		.462	.359	.872	1.711
		(0.035)		(.067)	(.069)		
5. Elect. machinery	.526	0.172		.524	.314	.873	1.585
		(0.044)		(.076)	(.080)		
6. Transport. equip.	.508	0.766		.345	.300	.550	1.732
		(0.172)		(.086)	(.091)		

				II. EQUATION (18)[g]			
7. All manufacturing	.715	0.304	.541	.256	.164	.929	1.697
		(0.262)	(.051)	(.052)	(.052)		
8. Durable goods	.725	0.422	.476	.230	.179	.803	1.657
		(0.217)	(.063)	(.068)	(.064)		
9. All industries reporting unfilled orders[f]	.53	1.132	.532	.201	.151	.923	1.779
		(0.358)	(.090)	(.113)	(.083)		
10. Machinery exc. elect.	.735	0.106	.325	.308	.191	.896	1.059
		(0.025)	(.055)	(.055)	(.057)		
11. Elect. machinery	.647	0.116	.439	.286	.117	.910	1.413
		(0.028)	(.059)	(.057)	(.058)		
12. Transport. equip.	.601	0.391	.473	.143	.154	.723	1.850
		(0.120)	(.068)	(.064)	(.065)		

[a] On estimation, see equations 16a and 16b and text.

[b] Standard errors of the constant terms are given in parentheses.

[c] For equation 17, b_{31} is shown in column 4, and b_{32} in column 5. For equation 18, b_{20}, b_{21}, and b_{22} are shown in columns 3–5 in that order. The figures in parentheses are standard errors of the statistics shown.

[d] Durbin-Watson statistic (d) equals $\Sigma(e_t - e_{t-1})^2/e_t^2$, where e stands for e_1 in lines 1–5 and for e_2 in lines 6–10. For the significance points of d, see note 11 and Table 5-3, note d, above. There are 74 quarterly observations for the series on all manufacturing and durable goods industries and 50 for the others.

[e] See text and note 52.

[f] Includes all durable goods industries and four nondurable goods industries: textiles, leather, paper, and printing and publishing.

[g] See text.

Table 5-10 presents the estimates of equations (17) and (18) for these six industries. The Durbin-Watson statistics are considerably higher here than before the transformation of the variables (compare the figures in Table 5-10, column 7, lines 1–6, with the corresponding entries in Table 5-9, column 6). The d tests gave strong indications of positive residual autocorrelations in Table 5-9 (all manufacturing, durables, electrical and nonelectrical machinery, transportation equipment); they suggest that there is little if any residual autocorrelation in the equations of Table 5-10, lines 1–5.

Consistently with other findings (see note 52 and text above), $\rho_1 < \rho_2$ for each of the industries covered (Table 5-10, column 1). A few differences of the same kind still exist between the estimates from the transformed variables, that is, between equations 17, which do not, and equations 18, which do, include N_t. Unlike the former regressions in the first six lines [from equation (17)] the ones from (18) do, in some cases, give considerable evidence of autocorrelated residuals. This is shown by the values of d for the machinery industries, particularly nonelectrical.

The regression coefficients in Table 5-10 are lower for the current and immediate past values of new orders and higher for the more distant past values, when compared with the corresponding coefficients in Tables 5-3 and 5-9. For illustrations of the statements in this paragraph, see the accompanying table. (Similar results can be obtained

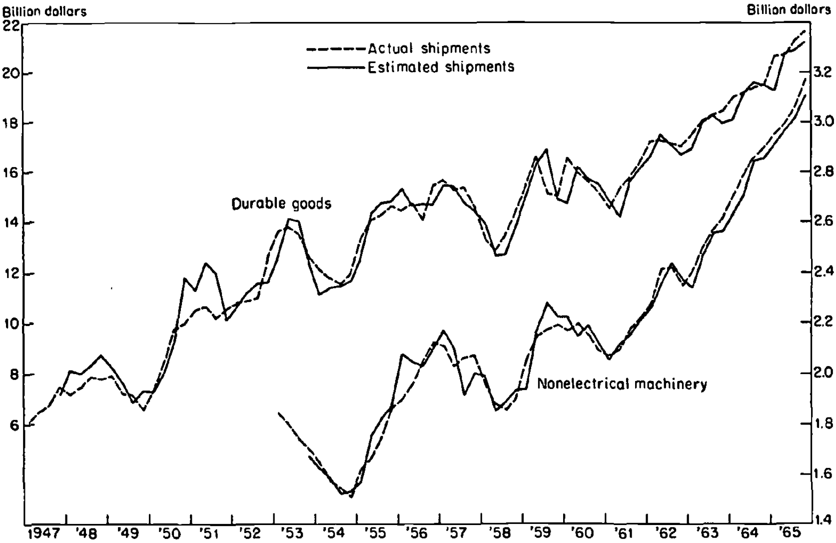
	All Manufacturing, from Table				*Durable Goods Industries, from Table*			
	5-9	*5-10 (Part I)*	*5-3*	*5-10 (Part II)*	*5-9*	*5-10 (Part I)*	*5-3*	*5-10 (Part II)*
Coefficient of N_t			.527	.541			.480	.476
Coefficient of N_{t-1}	.717	.568	.138	.256	.641	.492	.092	.230
Coefficient of N_{t-2}	.275	.401	.330	.164	.313	.337	.397	.179
Sum of coefficients	.992	.969	.995	.961	.954	.829	.969	.885
$\bar{R}^2$	.974	.924	.984	.929	.955	.725	.951	.803

Chart 5-6

Regressions for Shipments of Durable Goods Industries and Nonelectrical Machinery, Based on Transformed Variables, Quarterly, 1947–65

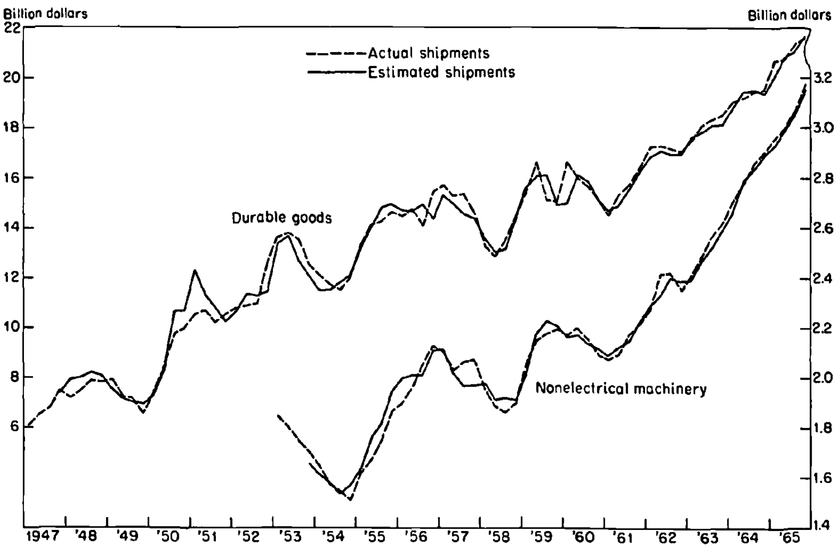
PART A

$$S_t - \rho_1 S_{t-1} = a + b_1(N_{t-1} - \rho_1 N_{t-2}) + b_2(N_{t-2} - \rho_1 N_{t-3})$$



PART B

$$S_t - \rho_2 S_{t-1} = a + b_1(N_t - \rho_2 N_{t-1}) + b_2(N_{t-1} - \rho_2 N_{t-2}) + b_3(N_{t-2} - \rho_2 N_{t-3})$$



for the other industries from the same tables.) Moreover, the transformation of the variable results in "well-behaved" patterns of a monotonic decrease of the lag coefficients in the direction of the past: $b^*_{20} > b^*_{21} > b^*_{22}$ in each case (Table 5-10, lines 7–12). In contrast, b_{21} is often smaller than b_{22} in the regressions of Table 5-3, and b_{20}, while typically the largest of the coefficients there, is not always so. The sums of the coefficients are somewhat smaller in Table 5-10 than in the other tables. Since the present regressions essentially use changes, they also produce lower $\bar{R}^2$ than the other regressions which use levels, but the values of $\bar{R}^2$ in Table 5-10, column 6, are still comfortably high.

Finally, Chart 5-6 shows that the applications of equations (17) and (18) to the quarterly data for total durables and nonelectrical machinery produce estimates that fit the recorded shipments quite well. The bias of too large amplitudes is reduced to small proportions in the values calculated by relating S_t to N_{t-1} and N_{t-2} only [equation (17)], and it is almost eliminated in the improved estimates that also incorporate N_t [equation (18)]. In the former series, the peak values are somewhat overestimated and the trough values underestimated on several occasions, but few of the deviations are disturbingly large (the largest ones refer to the durables in the early Korean phase in 1950–51). In the latter series, the only sizable overestimates can be seen at peaks, in the first quarter of 1951 (for total durables) and in the third quarter of 1959 (for nonelectrical machinery). The timing of the computed and actual shipments tends to be coincident, on the average, with relatively small deviations in either direction (one significant exception here being the early downturn of the computed machinery series in 1959).

Summary

Shipments may be viewed as a weighted sum of past (and perhaps current) values of new orders, for the dependence of S_t on N_{t-i} ($i = 0, 1, \ldots, m$) is basically a distributed-lag relation. In a regression of S_t on the terms N_{t-i}, the sum of the coefficients of the latter is expected to tend toward unity, given that new orders are taken net of cancellations and for all the relevant past periods m. Various distributed-lag regressions, including those that include up to seven monthly or three quarterly terms for N_{t-i} and those with assumed forms of time-lag structure (geometric, second-order), generally confirm this expectation and the implied view of the lagged production (order-filling) process.

Regressions with several lagged terms in new orders suffer from multicollinearity and positive autocorrelation of residuals. Typically, the $\bar{R}^2$ coefficients are already high for equations that contain only the shortest lags, but they do increase by small steps as successively earlier values of new orders are included to account for the longer delivery lags.

Geometrical lag distributions produce definitely better results. The constant terms in most of these regressions are not significant, the correlations are very high, and the fits, as shown by the graphs, are good. On the other hand, the second-order distributed-lag functions give no improvements except a few small ones for some of the monthly data.

Autoregressive equations using linear combinations of past new orders as "instrumental variables" compare favorably with the geometric lag equations in a few cases. In particular, good results were obtained for primary metal industries—just where the other regressions had been found least satisfactory.

Median estimates of the typical delivery lags as derived from the preferred regression models suggest rather long lags in the machinery industries and short ones in primary metals and the "other durables" group, with fabricated metals and total transportation equipment in intermediate positions. The five-month lag for the durable goods sector contrasts with the one-month lag for all nondurables. These estimates tend to be smaller than the average lags of shipments at the turning points in new orders, but, except for primary metals, the differences are minor. The two types of timing measures have different meanings and it is neither possible nor necessary to reconcile them precisely, but some plausible reasons for the observed differences are suggested. One of these is that the correlations between the short movements in N and S may involve smaller lags than those between the longer cyclical movements.

There is reason to believe that the lags of shipments behind new orders are subject to systematic variation during the business cycle; so the coefficients in these relations should be variable rather than fixed. A model embodying variable lag coefficients has been applied to the quarterly major-industry data and has been found in most cases to perform well—definitely better than the estimates applying fixed lag coefficients to the same data. The variable lag approach consists in making the lag coefficients depend on the value of the backlog-shipment (U/S) ratios, which serve as an index of the pressure of demand upon capacity.