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APPENDIX C

ANALYTICS OF THE TWO-STAGE PROCEDURE FOR ESTIMATING GROSS PRICE EFFECTS

THE ANALYTICS of the two-stage procedure used in estimating the gross price effects and gross price coefficients in Detroit are easily described. The "correct" estimating technique using separate equations for each household class would have

$$\Pi(J, H, K) = A(H, K) * X(H) + \sum_{K'=2}^{KT} B(K, K', H) * P(K', J, H) + e(H, K, J); \qquad \text{(C.1)}$$

where:

$\Pi(J, H, K)$ = the proportion of households of class H employed at workplace J choosing house-type K;
$A(H, K)$ = the probability that household class H chooses house-type K independent of workplace effects;
$B(K, K', H)$ = coefficient of the relative gross price of house-type K' for household H choosing house-type K;
$P(K', J, H)$ = the relative gross price of house-type K' for the household H at workplace J;
$e(K, H, J)$ = the random error term (mean = zero) for H choosing K;
$X(H)$ = a series of binary variables for household class H.

In fact, because suitable price information was unavailable, the effect of household characteristics was first estimated as

$$PR(H, K) = AA(H, K) * X(H) + V(H, K, J); \tag{C.2}$$

where:

$PR(H, K)$ = the proportion of household class H choosing K, ignoring workplace-specific prices;
$AA(H, K)$ = the probability that household class H chooses house-type K, estimated without controlling for workplace;
$V(H, K, J)$ = a workplace-specific error term.

Assuming the X's and P's are uncorrelated, $AA(H, K) = A(H, K)$, i.e., the estimates of the coefficients of household class are unbiased. Setting equation C.2 equal to equation C.1, it is clear that

$$V(H, K, J) = \sum_{K'=2}^{KT} B(K, K', H) * P(K', J, H) + e(H, K, J). \tag{C.3}$$

Over any significant subsample, such as all households at workplace J who choose house-type K, the mean of the random error, $e(H, K, J)$, is expected to be zero; or

$$\sum_{H} e(H, K, J) = 0.$$

Now define $U(H, K, J)$ such that

$$V(H, K, J) = U(H, K, J) + e(H, K, J). \tag{C.4}$$

This definition enables us to set

$$\begin{aligned} R(K, J) &= \sum_{H} [V(H, K, J) / N(H, K, J)]; \\ &= \sum_{H} \{[U(H, K, J) + e(H, K, J)] / N(H, K, J)\}; \\ &= \sum_{H} [U(H, K, J) / N(H, K, J)]; \\ &= \sum_{H} \{[\sum_{K'=2}^{KT} B(K, K', H) * P(K', J, H)] / N(H, K, J)\}; \end{aligned} \tag{C.5}$$

where:

$R(K, J)$ = the average residual variation in the proportions of households at workplace J choosing house-type K;

$N(H, K, J) =$ the number of sample observations of household H choosing house-type K at workplace J.

Equation C.5 shows that the mean within-workplace residual is simply the weighted sum of relative prices, the weights being the relative price coefficients of the original equation, C.1.

The Gross Price Equations

From a set of KT gross prices it is possible to form $KT - 1$ relative housing prices for each workplace. For this analysis the set of $KT - 1$ relative gross prices, $P(K', H, J)$ is formed from KT gross prices, $PP(K, H, J)$ by using $PP(1, H, J)$ as the numeraire price. The operation is thus

$$P(K', H, J) = \frac{PP(K', H, J)}{PP(1, H, J)} \tag{C.6}$$

where K' varies from 2 to KT.

For several reasons it is undesirable to employ the relative prices directly (untransformed) in the mean workplace residual–relative price equations. The residual (dependent variable) is constrained to lie in the interval -1 to $+1$. Therefore, a desirable property of the residual–relative price equations would be a declining derivative of the residual with respect to the relative price (declining in absolute value). A linear equation utilizing untransformed relative prices would imply a constant derivative of the residual with respect to any relative price.

The form of the relative prices ultimately chosen is the natural logarithm of $[1 + P(K', H, J)]$. Thus the residual equations are of the form

$$R(K, J) = \sum_{K'} B(K, K', H) * [1 + P(K', H, J)]. \tag{C.7}$$

Then the derivative of the residual with respect to any relative price $P(K', H, J)$ is given by

$$\frac{dR}{dP} = \frac{B(K, K', H)}{1 + P(K', J, H)}$$

which declines in absolute value as $P(K', J, H)$ increases. [Note that $P(K', J, H)$ will be greater than zero if all absolute housing prices are constrained to be nonnegative.]

It is important to consider the restrictions which can be placed on the relative price coefficients a priori. First, the coefficient of the "own relative price" must be zero or negative. An increase in the relative price of housing type K^*, holding the relative prices of the remaining $K - 2$ housing types constant, should decrease, or at least not increase, the proportion of households in a workplace consuming K^*.

Second, an increase in the relative price of any other housing type $K(\neq K^*)$, holding all other relative prices constant, should lead to a nonnegative change in the consumption of housing type K^*. That is, households should not respond to an increase in the price of any one type of housing by reducing their consumption of some other type of housing, the relative price of which has remained constant. For the discrete housing types employed here, inferior and complementary goods are precluded by definition.

The degree to which estimated coefficients conform to the a priori restrictions that own-price coefficients be nonpositive and cross-price coefficients be nonnegative is an important test of the consistency of the estimated price coefficient equations.