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Appendix B

DERIVATION OF INVESTMENT MODEL FORMULAS

1. THE INVESTMENT DEMAND CURVE

To find the optimal amount of I_{i-1} in the pure investment model, redefine the full wealth constraint as

$$R' = A_0 + \sum \frac{W_i h_i - \pi_i I_i}{(1+r)^i}. \qquad \text{(B-1)}$$

Maximization of R' with respect to I_{i-1} yields

$$\frac{\partial R'}{\partial I_{i-1}} = 0 = \frac{W_i G_i}{(1+r)^i} + \frac{(1-\delta_i)W_{i+1}G_{i+1}}{(1+r)^{i+1}} + \ldots + \frac{(1-\delta_i)\ldots(1-\delta_{n-1})W_n G_n}{(1+r)^n} - \frac{\pi_{i-1}}{(1+r)^{i-1}}. \qquad \text{(B-2)}$$

If (B-2) is combined with the first order condition for I_i by means of the technique outlined in Chapter I, Section 2, then

$$\gamma_i = \frac{W_i G_i}{\pi_{i-1}} = r - \tilde{\pi}_{i-1} + \delta_i. \qquad \text{(B-3)}$$

To satisfy second order optimality conditions, it is necessary that

$$\frac{\partial^2 R'}{\partial I_{i-1}^2} < 0 = \frac{W_i(\partial G_i/\partial H_i)}{(1+r)^i} + \frac{(1-\delta_i)^2 W_{i+1}(\partial G_{i+1}/\partial H_{i+1})}{(1+r)^{i+1}} + \ldots + \frac{[(1-\delta_i)\ldots(1-\delta_{n-1})]^2 W_n(\partial G_n/\partial H_n)}{(1+r)^n},$$

or $\partial G_i/\partial H_i < 0$, all i. Diminishing marginal productivity in all periods is required because (B-3) implies that optimal H must satisfy

$$\frac{\partial \gamma_i}{\partial H_i} = \frac{W_i(\partial G_i/\partial H_i)}{\pi_{i-1}} < 0.$$

In the continuous time model, R' is given by

$$R' = \int e^{-ri}(W_i h_i - \pi_i I_i)\,\mathrm{d}i, \tag{B-4}$$

or

$$R' = \int e^{-ri}(W_i h_i - \pi_i \delta_i H_i - \pi_i \dot{H}_i)\,\mathrm{d}i. \tag{B-5}$$

Hence,

$$R' = \int J(H_i, \dot{H}_i, i)\,\mathrm{d}i, \tag{B-6}$$

where $J = e^{-ri}(W_i h_i - \pi_i \delta_i H_i - \pi_i \dot{H}_i)$. Using the Euler equation outlined in Appendix A, one derives the condition for the optimal path of health capital over the life cycle:

$$\gamma_i = \frac{W_i G_i}{\pi_i} = r - \tilde{\pi}_i + \delta_i. \tag{B-7}$$

In Chapter II, it was indicated that certain production functions of healthy time might exhibit increasing or constant marginal productivity in some regions (see footnote 2). Suppose, for example, that healthy time increased at an increasing rate in the vicinity of the death stock, as in Figure B-1. Then quantities of $H < H^*$ would never be observed because the MEC schedule would be upward sloping in the range $H_{min} < H_i < H^*$.

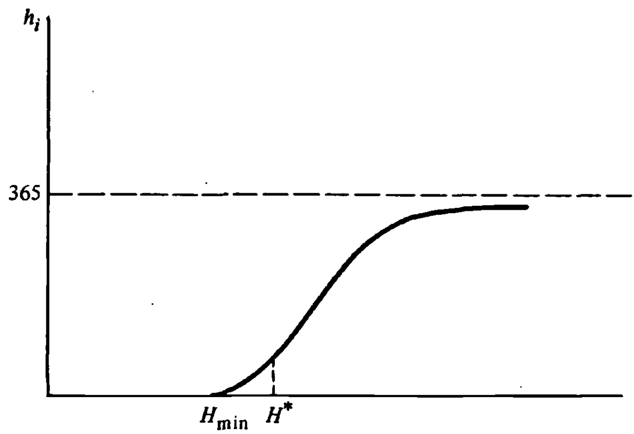

Figure B-1

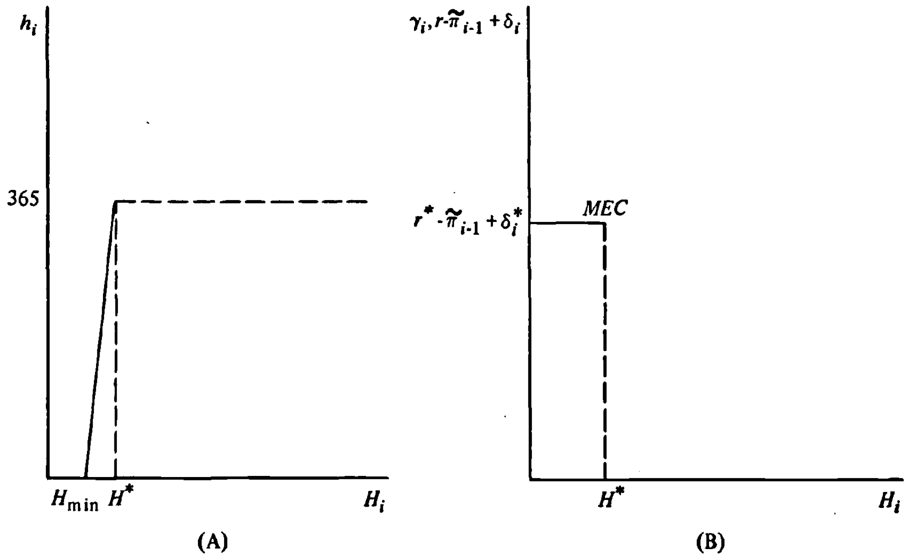

Figure B-2

This implies that individuals would choose an infinite life. Since observed behavior is consistent with finite life spans, segments of increasing marginal productivity can be ruled out around the death age.[1]

Production functions with constant marginal productivity are somewhat more difficult to discard. In panel A of Figure B-2, the number of healthy days is proportional to the stock of health until $H_i = H^*$. Since G_i is constant for $H_i < H^*$ and zero thereafter, the MEC schedule has a discontinuity when $H_i = H^*$. If the cost of health capital were less than $r^* - \tilde{\pi}^*_{i-1} + \delta^*_i$ in panel B, H^* would be the equilibrium stock of health and 365 days would be the equilibrium number of healthy days. At $r - \tilde{\pi}_{i-1} + \delta_i = r^* - \tilde{\pi}^*_{i-1} + \delta^*_i$, any stock between H_{min} and H^* would be optimal, while a higher cost of capital would give rise to an equilibrium stock H_{min}. Although the production function in panel A is not inconsistent with observed behavior, it may be ruled out because "nature does not make jumps." That is, it is reasonable to assume healthy time reaches its upper limit gradually and in a manner that rules out discontinuities in the MEC schedule.

[1] In general, if the production function had alternating segments of increasing and diminishing marginal productivity, the ones with increasing marginal productivity would never be observed.

2. VARIATIONS IN DEPRECIATION RATES

All formulas that were employed to study the effects of life cycle variations in depreciation rates were proved in Chapter II. Therefore, this section develops formulas for the analysis of variations in δ_i among individuals of the same age. If $\tilde{\pi}_{i-1} = 0$, all i, then

$$\ln(r + \delta_i) = \ln W_i + \ln G_i - \ln \pi. \tag{B-8}$$

Differentiation of (B-8) with respect to $\ln \delta_i$ yields

$$\frac{\delta_i}{r + \delta_i} = \frac{\partial \ln G_i}{\partial \ln H_i} \frac{\mathrm{d} \ln H_i}{\mathrm{d} \ln \delta_i},$$

or

$$\frac{\mathrm{d} \ln H_i}{\mathrm{d} \ln \delta_i} = -s_i \varepsilon. \tag{B-9}$$

The natural logarithm of gross investment may be written

$$\ln I_i = \ln H_i + \ln(\tilde{H}_i + \delta_i). \tag{B-10}$$

Hence,

$$\frac{\mathrm{d} \ln I_i}{\mathrm{d} \ln \delta_i} = \frac{\mathrm{d} \ln H_i}{\mathrm{d} \ln \delta_i} + \frac{\delta_i + (\mathrm{d}\tilde{H}_i/\mathrm{d} \ln \delta_i)}{\tilde{H}_i + \delta_i}.$$

It was shown in Chapter II that $\tilde{H}_i = -s_i \varepsilon \tilde{\delta}$. Thus, $\mathrm{d}\tilde{H}_i/\mathrm{d} \ln \delta_i = -s_i(1 - s_i)\varepsilon\tilde{\delta}$. Utilizing (B-9) and the last two expressions, one has

$$\frac{\mathrm{d} \ln I_i}{\mathrm{d} \ln \delta_i} = \frac{(1 - s_i\varepsilon)(\delta_i - s_i\varepsilon\tilde{\delta}) + s_i^2\varepsilon\tilde{\delta}}{\delta_i - s_i\varepsilon\tilde{\delta}}. \tag{B-11}$$

3. MARKET AND NONMARKET EFFICIENCY

Wage Effects

To obtain the wage elasticities of medical care and the time spent producing health, three equations must be partially differentiated with respect to the wage. These equations are the gross investment production and the two first order conditions for cost minimization:

$$I(M, TH; E) = Mg(t; E) = (\tilde{H} + \delta)H$$

$$W = \pi g'$$

$$P = \pi(g - tg').$$

Since I is linear homogeneous in M and TH,

$$\frac{\partial(g - tg')}{\partial M} = -t\frac{\partial(g - tg')}{\partial TH}$$

$$\frac{\partial g'}{\partial TH} = \frac{1}{t}\frac{\partial(g - tg')}{\partial TH}$$

$$\sigma_p = \frac{(g - tg')g'}{I\partial(g - tg')/\partial TH}.$$

Therefore, the following relationships hold:

$$\frac{\partial(g - tg')}{\partial M} = -\frac{t(g - tg')g'}{I\sigma_p}$$

$$\frac{\partial g'}{\partial TH} = -\frac{1}{t}\frac{(g - tg')g'}{I\sigma_p} \qquad \text{(B-12)}$$

$$\frac{\partial(g - tg')}{\partial TH} = \frac{(g - tg')g'}{I\sigma_p}.$$

Carrying out the differentiation, one gets

$$g'\frac{\mathrm{d}TH}{\mathrm{d}W} + (g - tg')\frac{\mathrm{d}M}{\mathrm{d}W} = -\frac{H(\bar{H} + \delta)\varepsilon}{\pi}\left(\frac{\mathrm{d}\pi}{\mathrm{d}W} - \frac{\pi}{W}\right)$$

$$1 = g'\frac{\mathrm{d}\pi}{\mathrm{d}W} + \pi\left(\frac{\partial g'}{\partial TH}\frac{\mathrm{d}TH}{\mathrm{d}W} + \frac{\partial g'}{\partial M}\frac{\mathrm{d}M}{\mathrm{d}W}\right)$$

$$0 = (g - tg')\frac{\mathrm{d}\pi}{\mathrm{d}W} + \pi\left[\frac{\partial(g - tg')}{\partial TH}\frac{\mathrm{d}TH}{\mathrm{d}W} + \frac{\partial(g - tg')}{\partial M}\frac{\mathrm{d}M}{\mathrm{d}W}\right].$$

Using the cost-minimization conditions and (B-12) and rearranging terms, one has

$$I\varepsilon\frac{\mathrm{d}\pi}{\mathrm{d}W} + W\frac{\mathrm{d}TH}{\mathrm{d}W} + P\frac{\mathrm{d}M}{\mathrm{d}W} = \frac{I\varepsilon\pi}{W}$$

$$I\sigma_p\frac{\mathrm{d}\pi}{\mathrm{d}W} - \frac{1}{t}P\frac{\mathrm{d}TH}{\mathrm{d}W} + P\frac{\mathrm{d}M}{\mathrm{d}W} = I\frac{\pi}{W}\sigma_p \qquad \text{(B-13)}$$

$$I\sigma_p\frac{\mathrm{d}\pi}{\mathrm{d}W} + W\frac{\mathrm{d}TH}{\mathrm{d}W} - tW\frac{\mathrm{d}M}{\mathrm{d}W} = 0.$$

Since (B-13) is a system of three equations in three unknowns, $\mathrm{d}TH/\mathrm{d}W$, $\mathrm{d}M/\mathrm{d}W$, and $\mathrm{d}\pi/\mathrm{d}W$, Cramer's rule can be applied to solve for, say, $\mathrm{d}M/\mathrm{d}W$:

$$\frac{\mathrm{d}M}{\mathrm{d}W} = \frac{\begin{vmatrix} I\varepsilon + W & +I\varepsilon\pi/W \\ I\sigma_p - (1/t)P & +I(\pi/W)\sigma_p \\ I\sigma_p + W & -0 \end{vmatrix}}{\begin{vmatrix} I\varepsilon + W & +P \\ I\sigma_p - (1/t)P & +P \\ I\sigma_p + W & -tW \end{vmatrix}}.$$

The determinant in the denominator reduces to $I\sigma_p\pi^2 I^2/THM$. The determinant in the numerator is

$$\frac{I\sigma_p}{THM}\left(I\pi\sigma_p THM + I\pi\varepsilon\frac{P}{W}M^2\right).$$

Therefore,

$$\frac{\mathrm{d}M}{\mathrm{d}W} = \frac{THM}{I\pi}\left(\sigma_p + \frac{\varepsilon P}{WTH}\right)M.$$

In elasticity notation, this becomes

$$e_{M,W} = (1 - K)\varepsilon + K\sigma_p. \tag{B-14}$$

Along similar lines, one easily shows

$$e_{TH,W} = (1 - K)(\varepsilon - \sigma_p). \tag{B-15}$$

The Role of Human Capital

To convert the change in productivity due to a shift in human capital into a change in average or marginal cost, let the percentage changes in the marginal products of medical care and own time for a one unit change in human capital be given by

$$\frac{\partial(g - tg')}{\partial E}\frac{1}{g - tg'} = \frac{g\hat{g} - tg'\hat{g}'}{g - tg'}$$

$$\frac{\partial g'}{\partial E}\frac{1}{g'} = \hat{g}'.$$

If a shift in human capital were factor-neutral, the percentage changes in these two marginal products would be equal:

$$\hat{g}' = \frac{g\hat{g} - tg'\hat{g}'}{g - tg'},$$

or

$$\hat{g}' = \hat{g} = r_H. \tag{B-16}$$

The average cost of gross investment in health is defined as

$$\bar{\pi} = (PM + WTH)I^{-1} = (P + Wt)g^{-1}.$$

Given "factor-neutrality,"

$$\frac{d\bar{\pi}}{dE}\frac{1}{\bar{\pi}} = -\hat{g} = -r_H. \tag{B-17}$$

This coincides with the percentage change in marginal cost since

$$\pi = P(g - tg_t)^{-1},$$

and

$$\frac{d\pi}{dE}\frac{1}{\pi} = -\left(\frac{g\hat{g} - tg'\hat{g}'}{g - tg'}\right) = -\hat{g}' = -\hat{g} = -r_H. \tag{B-18}$$

Chapter III outlined a derivation of the human capital parameter in the demand curve for medical care but did not give a rigorous proof. Taking the *total* derivative of E in the gross investment function, one computes this parameter:

$$\frac{dI}{dE}\frac{1}{I} = \frac{M(g - tg')}{I}\hat{M} + \frac{THg'}{I}\widehat{TH} + r_H.$$

Since $\hat{M} = \widehat{TH}$ and $\hat{H} = \hat{I}$, the last equation can be rewritten as

$$\hat{H} = \hat{M} + r_H\varepsilon.$$

Solving for $\hat{M}$ and noting that $\hat{H} = r_H\varepsilon$, one gets

$$\hat{M} = r_H(\varepsilon - 1). \tag{B-19}$$