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Volume Author/Editor: Stanley Diller

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Chapter Author: Stanley Diller

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CHAPTER 2

Methods of Seasonal Adjustment

INTRODUCTION

IT IS OFTEN USEFUL to separate a time series into components that differ in the frequency of their recurrence. Some series, national income, for example, tend to increase over time, to have a positive trend—although any given observation may be smaller than its predecessor because it happens to fall on the down side of a different component, whether cyclical, seasonal, irregular, or some anonymous component. Since each observation is assumed to be the result of the separate influences of each component, to capture the effect on the series of any given component requires that the others be in some way filtered out. Perhaps the main reason for using seasonally adjusted data is to facilitate identification of the cyclical component.¹

It is sometimes profitable to study the behavior of a particular component for what it reveals about the behavior of the composite series. Investment and interest rates, for example, are sometimes observed to vary together over the course of the business cycle because shifts in the demand for investment goods dwarf the movements along the curve. By abstracting from the cyclical component of either

¹ See Julius Shiskin and Harry Eisenpress, *Seasonal Adjustment by Electronic Computer Methods*, Technical Paper No. 12, NBER, 1958. Represented in *Business Cycle Indicators*, Volume 1, Princeton for NBER, 1961.

series it may be possible to observe the expected inverse relation, at a given time, between the two series. Ultimately, since the several components of a series are distinct only because they are determined by different factors (by different variables or by different patterns of variation of the same variables), the identification of the components is a step toward the goal of explaining their behavior.² In practice, because of the difficulty of specifying those models describing the behavior of each component, the two goals are distinguished and, ordinarily, the second one eschewed.

One way to eliminate from the variation of the combined series that part due to one of the components is to smooth the series with a moving average of which the number of terms equals the period of the recurring variation of that component. Consider, for example, a daily series of retail sales: Every Sunday there is a sharp drop. By replacing each daily value of the series with the value of a centered seven-term moving average—in other words, by replacing, say, the original Wednesday value with the average of the preceding three days, that Wednesday, and the following three days, and similarly for the other days—one obtains a series that is free of the recurring variation. When the moving average is subtracted from or divided into the original series, a new series emerges containing samples of all the components for which the variation has a period of seven or less.

Since seasonal variation is defined as variation that recurs in a period not greater than a year, there is no need to distinguish, for the purpose of seasonal adjustment, among components whose variation recurs in periods exceeding one year. It is enough to smooth the seasonal variation with a twelve-term moving average (assuming monthly data) and use the deviations of the composite series from the moving average as estimates of the components, the period of whose recurring variation does not exceed one year. The moving average is then an estimate of the trend-cycle or low frequency component of the series, and the deviations from it of the seasonal and irregular components. These deviations, usually expressed as ratios of the composite series to the trend-cycle component, are the

² Horst Mendershausen, "Annual Survey of Statistical Technique: Methods of Computing and Eliminating Changing Seasonal Fluctuations," *Econometrica*, 1937, pp. 234–62.

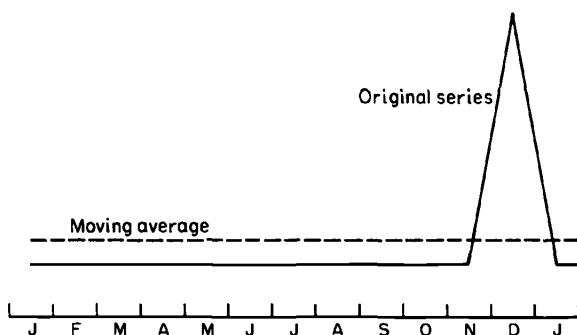


FIGURE 1

raw data for the seasonal adjustment. They are called the seasonal-irregular (SI) ratios.³

SEASONAL-IRREGULAR RATIOS

The SI ratios, expressed as percentages, are computed for each month of each year. The twelve SI ratios for each year are adjusted to force their sum to 1200. This adjustment precludes any discrepancy between the *expected values* of the totals for any consecutive twelve months (as distinct from the observed totals for a given year) ⁴ of the seasonally adjusted series and the totals of the original series. Year to year changes of the series are therefore unaffected, on average,

³ When the deviations are expressed as ratios, the adjustment is called multiplicative; when expressed as differences, additive. In the Census Bureau's X-11 seasonal adjustment program, as well as in earlier versions of the program, computation of the SI ratios is a much more complicated procedure than the one described above. First a thirteen-term moving average is computed (half weights at the end points) and divided into the original data. The preliminary SI ratios are adjusted for extreme values and smoothed into moving seasonal factors (to be described below). The preliminary factors are divided into the original series to get a preliminarily adjusted series. A Henderson curve, an elastic moving average of varying terms, is fitted to the adjusted series, and a new set of SI ratios computed, adjusted for extremes and smoothed with a moving average into the final seasonal factors. See Shiskin and Eisenpress, *op. cit.*, and the *X-11 Variant of the Census Method II Seasonal Adjustment Program*, Washington, D.C., U.S. Department of Commerce, Bureau of the Census, Technical Paper No. 15, 1965 (hereafter referred to as the X-11 manual).

⁴ Arthur Burns and Wesley C. Mitchell, *Measuring Business Cycles*, New York, NBER, 1946, p. 51.

by seasonal adjustment, while the variation within years is redistributed to eliminate the effect of systematic intrayear variation.

The symmetry thus imposed on the SI ratios reveals an important property of seasonal adjustment; the interrelatedness of the seasonal factors among several months. Consider a series in which the values for the first eleven months of the year are equal (they could even be zero) but for the twelfth month very high.⁵ Graphically, the series is shown in Figure 1. To maintain the symmetry of the SI ratios the high point in December must be balanced by a low point elsewhere. Since the datum for the highs and lows is the trend cycle curve, in this case a twelve-term moving average, the symmetry is achieved by raising the datum above the values for the first eleven months of the series thereby imputing seasonal lows to these values. Through its influence on the moving average, the December high in effect creates the January-to-November lows.

There is clearly a certain arbitrariness in imputing significance to the deviations from the moving average. The arbitrariness resides in the connotation of normality attached to the moving average from which the deviations appear, therefore, to be atypical and to invite behavioral explanations. Using a crude method for ascertaining the extent of seasonality in the capital markets, a method that did not abstract from the effects of trend, Kemmerer drew the following conclusion:

The national bank-note circulation curves do not appear to exhibit any considerable seasonal elasticity, i.e., rise and fall according to the seasonal variations in the demands of trade; it is noteworthy, however, that the increase in the circulation, which takes place normally from year to year, takes place largely in the fall and early winter. Apparently banks intending to increase their circulation postpone doing so until the crop-moving season approaches, so that the year's normal increase takes place principally in the latter part of the year. There is no evidence of contraction when the crop-moving demands are over, the national bank-note elasticity being (to use a rather inelegant expression) of the chewing gum variety. Here, however, . . . it is fortunate that the increase which normally does take place each year takes place in the season when it is needed most.⁶

⁵ This hypothetical series is chosen only for simplicity. The principle involved, however, does not depend on the simplicity of the sample.

⁶ Edwin W. Kemmerer, *Seasonal Variations in the Demands for Currency and Capital*, Washington, D.C., National Monetary Commission, Vol. XXII, 1911, p. 153.

Had the ratio-to-moving average method of seasonal analysis been available at the time of his study, Kemmerer would probably have affirmed the presence of seasonality in the series because of the effect of the high values in the autumn on the trend-cycle values for the rest of the year. Whether a series declines seasonally in a given month depends not on its average value in the month relative to its average value in the previous month—the averages being taken across years—but on the average of its values relative to what they would have been in the absence of seasonality. A series with positive trend is expected to increase from, say, April to May; by remaining constant, on average, it in effect declines relative to the expectation. The arbitrariness resides, therefore, only in the assumed existence of smooth, independent components in fixed relation in the hypothetical population from which the series is drawn.⁷

STABLE SEASONAL FACTORS

Estimated stable seasonal factors are defined as the mean value of the SI ratios for each month. They are called stable or constant because they are computed once for the entire sample period. Any deviation of the SI ratios computed for a given month from the factor computed for that month is attributed to the irregular component. In other words, the SI ratios consist of a systematic and a random component called, respectively, the seasonal and the irregular components. Since the irregular component, expressed as a fraction of the seasonal component, is defined to vary with equal likelihood above and below the seasonal component, its ratio to the seasonal component is on average equal to 1. (In the additive case the mean of the irregular component is zero.) Therefore, the mean SI ratio,

⁷ Using a twelve-month average instead of, say, a six-month or eighteen-month average as the datum for seasonality is also arbitrary unless "the activity represented by a series has a 'natural' business year, with a definite beginning and end, as in movements of products from farms. . . . In the absence of a natural year, there is no basis other than convention for selecting the boundaries of the year; . . . the final seasonal adjustment will then vary with the boundaries selected." Burns and Mitchell, *op. cit.*, p. 49 fn. In commenting thus Burns and Mitchell were concerned with a particular method of adjustment, the Kuznets amplitude-ratio method (described later); although the principle is apropos of any method.

say for January, is equal to the seasonal component, assumed to be a constant, multiplied by 1. In the absence of seasonality the mean SI ratios would not differ significantly from each other or from 1.

A simple test for the presence of stable seasonality is therefore to determine the statistical significance of the differences among the computed mean SI ratios for each month. The test is a one-way analysis of variance of the monthly SI ratios, twelve columns of them for the twelve months, and the number of rows equalling the number of annual observations.⁸ The *F*-statistic, computed for *N* full years, is equal to

$$\frac{N \text{ times variance of monthly means}}{\text{total variance minus } N \text{ times variance of monthly means}}$$

Except for the graphic method of fitting smooth curves to SI ratios of a given month over the years most of the earlier methods and many of those used currently result in estimates of stable seasonal factors. For example, the widely used dummy variable technique, whereby the monthly series *X* is regressed on twelve dummy variables, eleven of which assume the value 1 when the series is measured in a particular month and zero otherwise, the twelfth dummy variable always assumes the value zero, estimates stable seasonal factors.⁹ The computation of stable seasonals is justified when there are good reasons for believing that the parameters are stable in the hypothetical population from which the observed series is drawn. This belief is *not* analogous to the assumed constancy of the parameters in the structural relation of, say, consumption and income. A change in the seasonal parameters (as distinct from the estimated factors) of interest rates does not require that the structural relation between, say, interest rates and money supply, change but only that the seasonal pattern of money supply change. In fact, by assuming the structural relation fixed one can, in principle, estimate it by relating the changes in the seasonal pattern of one series to those in the other.¹⁰ Therefore, the assumption

⁸ The X-11 performs the analysis of variance test on the SI ratios after they have been modified to reduce the effect of extreme observations. See X-11 manual, *op. cit.*, p. 5.

⁹ Michael C. Lovell, "Seasonal Adjustment of Economic Time Series and Multiple Regression Analysis," *American Statistical Association Journal*, December 1963, p. 993.

¹⁰ The distinction here is between a structural relation between two economic variables, on the one hand, and between an economic variable and time, on the other. Economic models assume constancy in the first case but not the

of fixed seasonal parameters for a given series implies the assumption of fixed structural relations between this series and others, as well as of fixed seasonal patterns in these related series.

MOVING SEASONAL FACTORS

Typically there are changes not so much in the original cause of the seasonal movement but in the economy's adaptation to it. The increased demand for funds in the fall months will result in a seasonal high in interest rates only in the absence of a corresponding increase in the supply of funds. The willingness or ability of the banking system to supply the funds will determine whether the increased demand will result in higher rates. Seasonal increases in the demand, as well as in the supply of funds have varied in the postwar period, and the seasonal increase in interest rates has varied along with it.

Changes in seasonal patterns are difficult to distinguish from irregular movements—the more difficult, the greater is the variance of the irregular component relative to that of the total series. One identifies with greater confidence very small changes in the seasonal variation of money supply, a series with an almost negligible irregular component,¹¹ than changes in the seasonal variation of the highly irregular Treasury bill rate series. The problem of identifying these changes is analogous to one of identifying the components themselves: The method already described for that is predicated on the assumed smoothness of each of the components within its particular frequency; a method for dealing with changes in the components is to assume these changes themselves evolve along a smooth path. This method involves either fitting by eye a smooth curve to the SI ratios for all the Januaries, another for the Februaries, and so on, instead of a straight line at the means of the ratios for each month as in the stable factors, or computing a moving average, one month at a time, of the SI ratios adjusted for extreme values. If a simple three-term moving average were used, for example, the seasonal factor for

second. The fact that a timing relationship has changed does not imply a change in structural relationship since it may reflect merely the timing change of the variables with which it is structurally related.

¹¹ The variation of the rate of change of the money supply has, of course, a much larger irregular component.

January 1953 would equal the average of the SI ratios for January 1952, 1953, and 1954. In practice, such a simple moving average would be used only for series whose irregular component, being small, is in little danger of distorting the evolution of the seasonal factors. A weighted five-term moving average was used to compute the moving seasonal factors of the interest rate series. When the assumption of gradually evolving seasonal factors is not apposite, the use of this method will impose a spurious similarity on the estimated factors for adjacent years of a given month. However, by observing graphs of the SI ratios themselves one may judge the appropriateness of the method.

There are several alternative methods of calculating moving seasonal factors. The simplest one merely divides the whole sample period into subperiods and computes stable seasonal factors for each of the subperiods. This method is particularly useful for separating the subperiods with clear evidence of seasonality from those without it. There is some evidence, for example, that yields on long-term Treasury securities manifested seasonality during the late fifties but not before or since. A method based on evolving factors would spread the estimated period of seasonality past its true period, at the same time that it dilutes what seasonality there is. The method is also useful when there is an abrupt change in some institutional factor affecting the seasonal pattern, as, for example, when the Treasury-Federal Reserve accord in 1951 removed the peg on U.S. Government bonds. Another method is to compute a set of stable factors for the whole period and, on the assumption that the true factors for any given year remain in fixed relation, compute factors for each year proportional to the stable factors (that is, with an identical pattern but different amplitude). The proportion used for a given year is the regression coefficient obtained by regressing the SI ratios for that year on the stable factors, one regression for each year.¹² The application of this method to the Treasury bill rates would result in an adjustment not very different, and perhaps a little better, than that obtained with the moving average method. This point is considered in the next chapter. Finally, some writers recommend tying the moving seasonal factors of one series to the variation of related variables.

¹² See Simon Kuznets, *Seasonal Variations in Industry and Trade*, New York, NBER, 1933, p. 324.

This method is, of course, limited by the researcher's ability to specify the appropriate relations.¹³

Sometimes the term *moving seasonal* is applied to a different phenomenon than the one described above, where the term referred to changes in the true seasonal component requiring an estimation procedure capable of detecting these changes. If the seasonal component is a function of the trend cycle component of the same series, then an estimation procedure that assumes the components are independent will yield biased estimates of the moving seasonal factors. The seasonal decline in unemployment, for example, is said to be milder when the level of unemployment is low than otherwise because firms are reluctant to temporarily discharge workers at a time when the labor force is fully employed. The appropriate seasonal factor will therefore vary between periods of high and low unemployment. Unlike the other reason for moving seasonals, this one does not involve a change in the relation among the components of the series (sometimes called the structure of the series) but simply a more complicated relation among them.

During the four decades preceding the Federal Reserve Act of 1913, a problem analogous to that alleged for unemployment prevailed on interest rates and the components of the money stock, although with consequences much more severe than the computation of biased estimates of seasonal factors. In his study written for the National Monetary Commission organized in response to the panic of 1907,¹⁴ Kemmerer concluded that the greater incidence of banking panics in the fall than at other times of the year was due not to the normal seasonal tightness in the fall money market but to that tightness coming at the same time as a cyclical crisis. The seasonal movement, in effect, played the role of the proverbial straw.¹⁵

¹³ Mendershausen, *op. cit.*, pp. 254-262.

¹⁴ See also Milton Friedman and Anna Jacobson Schwartz, *A Monetary History of the United States, 1867-1960*, Princeton for NBER, 1963, pp. 171-172.

¹⁵ The evidence on this point is mixed. Kemmerer found that "of the eight panics which have occurred since 1873 [as of 1910], four occurred in the fall or early winter (i.e., those of 1873, 1890, 1899, and 1907); three broke out in May (i.e., those of 1884, 1893, and 1901); and one (i.e., that of 1903) extended from March until well along in November." After discussing the "minor panics or 'panicky periods,'" he concludes that "The evidence accordingly points to a tendency for the panics to occur during the seasons normally characterized by a stringent money market." (*Op. cit.*, p. 232.)

Even if the seasonal component, expressed as a ratio to the moving average, were systematically related to the level of the moving average (or trend-cycle component) the estimated seasonal factors would not reveal this relation. Since the factors are computed from averages of the SI ratios across years, the variation of the SI ratios due to the cyclical variation is canceled out in the averaging process. In the case of an extreme cyclical movement, a related seasonal movement would likely result in an SI ratio that would be regarded as an extreme observation and be eliminated from the computation of the seasonal estimates. However, any relation that exists between the seasonal and cyclical components would present itself in a time series of the SI ratios. Chapter 3 considers this point.

Even in the absence of a true relation between the seasonal and cyclical components the inappropriate use of either an additive or multiplicative adjustment, that is, the use of one when the other is required, will result in an apparent relation between the seasonal and cyclical components.

When the level of rates is low the basis point equivalent of a multiplicative adjustment factor is smaller than when the level of rates is high. If the true seasonality were additive, and a multiplicative adjustment method were used, there would result an inverse relation between the SI ratios and the level of rates. Assume, for example, the true seasonal difference for a particular month to be 50 basis points. If in estimating the seasonal variation a multiplicative method were used, the SI ratio computed for this month would be high when the level of rates were, say, 100 basis points (i.e., $150/100$) and low when the level were, say, 400 basis points (i.e., $450/400$). Application of this test to the computed SI ratios for the Treasury bill rates does not reveal a systematic inverse relation between these ratios and the level of rates. However, the opposite procedure designed to test the efficacy of a multiplicative adjustment, assuming the true seasonal were multiplicative and the estimates additive, fails to confirm the appropriateness of a multiplicative adjustment. The question is therefore open and invites deference to convention—which is to use a multiplicative adjustment unless an additive one is clearly indicated.¹⁶

There is a method that combines elements of both the additive

¹⁶ See Julius Shiskin and Harry Eisenpress, *Seasonal Adjustment by Electronic Computer Methods*, New York, NBER Technical Paper No. 12, 1958, p. 434.

and multiplicative adjustment. Seasonal irregular ratios are computed for each month and the set for each month regressed on the trend cycle component for that month; twelve regressions in all. The constant term of each regression is an estimate of the additive component of that month's seasonal factor and the regression coefficient of the multiplicative component. This method, however, assumes stable seasonality in the sense used earlier in this report.¹⁷

SEASONAL ADJUSTMENT ON COMPUTERS

The X-11 program, used in this study, embodies a series of refinements in the original ratio-to-moving average technique that Macaulay developed in the 1930's. By reducing the cost and virtually eliminating the tedium of the vast number of elementary calculations this method of adjustment requires, the program makes feasible the use of complex weighting schemes in computing moving averages that are both elastic (i.e., remain faithful to the original series) and smooth (i.e., avoid the irregular wiggles). It allows, moreover, the extensive use of iteration to mitigate the obscuring influence of the irregular component on the separation of the seasonal from the trend-cycle components. Its most important advantage is the reduction in the time cost and skills required in manual adjustments.

The program's advantages are particularly obvious through the stage in which the modified seasonal irregular ratios or differences are computed as, of course, are their mean values, or the stable seasonal factors, when relevant. An experienced draftsman, however, can graphically fit moving seasonals to the SI ratios as well as the program does, and probably better than the program does when the

¹⁷ There is clearly room here for variations on a theme. One can adapt the regression method to allow for a moving seasonal by applying the regression method as stated and applying the X-11 method to the residuals of the regressions. In that case one could obtain: an additive component, a component related to the trend cycle, and a moving seasonal component. Since this study uncovered no evidence of a relation between the seasonal and trend-cycle components of the interest rate series there was no reason to experiment with this method. In his exhaustive article Mendershausen (*op. cit.*) describes many exotic techniques for circumventing this or that problem of conventional methods; virtually all of them are in desuetude either because they introduced other problems or they were too unwieldy.

series has a prominent irregular component. Moreover, judgment is often required in determining whether an adjustment for any sub-period should be undertaken at all. Since the analysis of variance is a test of the means of the SI ratios for the whole sample period, there is some danger that the presence of a relatively strong seasonal component in one subperiod will affect the means for the whole period sufficiently to lend an apparent significance to the computed differences among them. (This effect would have to be large enough to overcome the increased within-group variance as a result of the greater heterogeneity of the SI ratios of a given month that a moving seasonal implies.) The program adjusts the whole series regardless of the results of the analysis of variance. The user cannot rely on the F -test alone to decide whether to accept the adjustment in its entirety.

The absence of objective criteria for selecting the period of adjustment, the extent of the adjustment, or the quality of the results¹⁸ precludes an elaborate tabulation of this study's findings replete with standard errors. Nevertheless, from the descriptive statistics, the diagrams, and the verbal entourage, patterns emerge that are worth noting. Chapter 3 presents this material for seventeen interest rate series.

¹⁸ "A statistician who has struggled with seasonal adjustments of numerous time series is not likely to underestimate the part played by 'hunch' and 'judgment' in his operations." Burns and Mitchell, *op. cit.*, p. 44.