Valuing Government Guarantees:
Fannie and Freddie Revisited

August 2008

Deborah Lucas*
Robert McDonald**

*Northwestern University and NBER
** Northwestern University

We thank Alan Marcus and Dwight Jaffee for helpful suggestions and comments on an earlier draft.

1. Introduction

The federal government explicitly guarantees a portion of deposit obligations of commercial banks and thrifts through deposit insurance, and is thought to provide protection beyond this legal obligation for institutions considered "too big to fail." Although until recently not explicitly guaranteed, Fannie Mae and Freddie Mac also have been longtime beneficiaries of similar federal protection of their debt securities against default.

Despite the perception that these guarantees are valuable and pose significant risk to the government, quantifying the exposure is difficult and there is substantial disagreement in the literature about magnitudes. In general, spread-based estimates of guarantee value for Fannie and Freddie are significantly higher than options-based estimates. Spread-based estimates capitalize the difference between the interest expense of Fannie and Freddie and that of similarly rated financial institutions.[1] Using this approach, Passmore (2005) reports a present value over 25 years in the range of $122 to $182 billion as the subsidy to Fannie and Freddie. At the other extreme, in a study commissioned by Fannie Mae using an options pricing approach, Stiglitz et. al. (2002) concludes that the cost of an implicit guarantee to the government does not exceed $200 million. In a recent paper also using an options pricing approach (Lucas and McDonald, 2006), we estimate a present value cost over 25 years of $28 billion for the two enterprises, still an order of magnitude lower than in Passmore (2005).

There are several possible explanations for the higher subsidy values implied by spread-based analyses. One is that the guarantee may be valued by investors in GSE

[11] In CBO (2001), based on the analysis of Ambrose and Warga (2002), the comparison is made using a "stand alone" rating for Fannie and Freddie, which reflects their risk to the government. As of April 2008, the GSEs had a stand-alone rating of AA- from S&P.

securities not just because of the direct value of protection from default risk, but also because of other benefits such as increased liquidity, or because they satisfy regulatory restrictions. Thus the reduction in the GSEs' borrowing costs may exceed the cost of expected defaults to the government. Whether these other benefits to GSE stakeholders should be included in a calculation of government cost depends on the question at hand. From a broad opportunity cost perspective, since other financial institutions would pay to obtain the same privileges, they are part of the cost. To answer the narrower question of the expected cost of defaults, it is probably appropriate to exclude the value of these sorts of additional benefits.

The theoretical model developed here suggests another reason that spread-based models overestimate guarantee values: they do not correct for the more conservative optimal default policy of an insured firm. To preserve the ability to borrow at a risk-free rate in the future, we show that a guaranteed firm will choose to make debt payments in some states of the world where an otherwise identical uninsured firm would default, lowering the cost to the government relative to what a spread-based estimate would imply. This finding is related to a large body of earlier work on risk taking, charter value, and bank regulation (see, e.g., Demsetz et. al. 1996 and the references therein). As far as we know, however, this analysis is the first to highlight the implications for credit spreads as potentially biased estimators of subsidy value.

A further possibility, however, is that simple options-based models fail to capture important dimensions of risk, and thereby underestimate the cost and risk to the government of providing insurance. In this study we consider several possibilities that have not been taken into account in past options-based estimates for Fannie and Freddie:

(1) the presence of a guarantee may affect the statistical relation between equity and asset value, and hence affect the imputation of asset value and volatility; and (2) a process for the evolution of assets that includes a jump as well as a diffusion component. In light of episodes such as Fannie's accounting restatements and subsequent fall in share price, and the spike in credit losses following the wave of sub-prime defaults, we also explore the sensitivity of options-based estimates to initial conditions for equity value and volatility. In all variations, we report insurance value in terms of an annual premium as well as reporting a present value, making costs easier to interpret and normalizing for the estimation horizon.

The remainder of the paper is organized as follows. Section 2 provides a brief description of Fannie and Freddie, their risk exposure, and the regulatory environment. In Section 3 we present the valuation model, and discuss the effect of the government guarantee on the dynamic relation between the underlying assets and the value of equity. Section 4 describes the calibration used to quantify the value of the guarantee, and reports the results of sensitivity analysis. Section 5 concludes.

2. Background

Fannie Mae and Freddie Mac are government-sponsored enterprises (GSEs) that were created by Congress to provide liquidity and stability in the home mortgage market. They also are required to meet modest goals for low-income lending. The GSEs as currently structured are hybrids of private corporations and federal entities. Although their debt securities explicitly state that they do not bear a government guarantee, their many federal ties and critical role in the housing and financial markets suggest otherwise.

As a consequence, the GSEs raise capital through debt financing at a narrower spread over Treasury rates than similarly rated financial institutions, an advantage that is generally viewed as an un-booked federal subsidy.

Fannie and Freddie participate in the mortgage market in two distinct ways. One is by buying mortgages and financing the purchases with debt issues. Those on-balance-sheet holdings expose the enterprises to default, interest rate, and prepayment risk. The interest rate and prepayment risk is partially hedged with the use of derivatives and dynamic hedging strategies (see Jaffee (2003)). They also securitize mortgages, an off-balance-sheet activity that primarily entails default risk through a credit guarantee.

The rapid growth of on-balance-sheet holdings in the last decade has increasingly raised concerns about the government's risk exposure (Frame and White (2005)). Following Fannie Mae's accounting irregularities,[2] on-balance-sheet growth was temporarily slowed by the consent order from their regulator that limited its mortgage portfolio to $727 billion, down from the $904 billion it held at year-end 2004. Fannie's MBS outstanding continued to grow to a total of $1.77 trillion as of November 2006. Since that time, growth in their on-balance-sheet obligations has resumed. At year-end 2005, Freddie Mac had a comparable exposure to Fannie Mae, with $710 billion of mortgages held on-balance-sheet, and $1.34 trillion in MBS outstanding.

With the sharp downturn in the housing market that began in 2007, concerns about default risk caused the stock price of both companies to plummet. Recent fair value estimates reported by the GSEs indicate that Freddie has negative equity value, and that Fannie is barely solvent. In July 2008, Congress granted Treasury the authority to

[2] Fannie Mae was found by the SEC to have overstated profits by an estimated $9 billion starting in the late 1990s.

infuse funds into the entities as needed over the next 18 months, effectively making the implicit guarantee explicit, and incurring an expected present value cost to taxpayers estimated by the Congressional Budget Office (CBO) to be $25 billion (CBO, 2008). Treasury used this authority two months later to take both GSEs into federal conservatorship.

The Office of Federal Housing Enterprise Oversight (OFHEO), an independent entity within the Department of Housing and Urban Development,[3] has some regulatory authority over Fannie and Freddie, but the GSEs' activities are primarily constrained by statute. By law investments are mostly limited to conforming mortgages, and the enterprises must meet minimum capital requirements.[4] Typically both firms maintain slightly more than the regulatory minimum capital, although capital on occasion has been a binding constraint. As for commercial banks with deposit insurance, economic theory predicts that to maximize the value of the implicit guarantee the enterprises would manage liabilities so that capital remains close to the regulatory minimum.

Historically, the stock of both firms consistently out-performed the overall market. In the last few years, even before the recent turmoil in the housing market, stock price volatility had increased and returns declined. Whether the historically high returns can be attributed to unanticipated growth in the implicit subsidy is a matter of some debate. Many observers contend that their stock holders benefit from their special status, but the enterprises counter that competitive pressure forces any cost advantage to be

---

[3] The 'Housing and Economic Recovery Act of 2008' creates a new and stronger regulator, the Federal Housing Finance Agency (FHFA) with oversight responsibility for Fannie Mae, Freddie Mac and the Federal Home Loan Banks that will supersede OFHEO. At present they have the same powers formerly held by OFHEO.

[4] Current legislative proposals would increase the conforming mortgage limits in high-cost states, allowing GSE holdings of mortgages previously considered jumbo loans.

passed through to borrowers. To the extent that rents are captured by stock holders, returns should be affected by unanticipated changes in the value of the perceived guarantee, and Seiler (2003) presents some evidence of this effect. In any event the returns on the two firms are highly correlated, suggesting that they are affected by common risk factors including common regulatory risk.

3. Modeling Guarantee Value

We take an options pricing approach to modeling the dynamics of guarantee value and risk exposure. The model is based on the fundamental insight of Sharpe (1976) and Merton (1977), that insurance can be valued as a put option on the assets of the firm. To illustrate the basic idea of how the guarantee is valued, and to understand its effect on the relation between observed equity valuations and the unobserved value of operating assets, we begin by analyzing a simple closed-form model where debt is adjusted at fixed intervals as long as the firm remains solvent.

For a firm with guaranteed debt, equity value has two components. The first, analogous to the equity of a levered firm without a guarantee, is a call option on the operating assets of the firm. The second component is the value of the guarantee itself, which is the present value of the (uncertain) stream of savings from being able to borrow at the risk-free rate, rather than at a risk-adjusted rate. The theoretical model is used to explore how the presence of a guarantee affects the dynamics of equity returns and their relation to the dynamics of operating assets. Since the options pricing approach imputes the value and volatility of operating assets from the value and volatility of equity, the answer is critical to correctly imputing guarantee value.

To examine the value of the guarantees quantitatively, in Section 3.3 we numerically implement a more complex version of the theoretical model using an approach similar to that of KMV (as described in Crosbie and Bohn, 2003). It allows for externally financed asset growth, debt adjustment over time, a state-contingent bankruptcy trigger, and state-contingent conditional volatility. Expanding on the related analysis in Lucas and McDonald (2006), we incorporate a jump process, add new internal consistency checks, and investigate of a wider range of parameter values, particularly the sensitivity to initial capital. The value of government insurance is calculated using a Monte Carlo simulation with risk neutral probabilities. We also track the corresponding actual distribution of assets, liabilities, and defaults, in order to report the implied distribution of insurance payouts.

3.1 Single Period Guarantee

We first consider the effect of a guarantee for a firm with a one-period debt contract, where a period has a length T. Consider two firms, one with insured debt and one with uninsured debt. Superscripts *"I"* and *"U"* denote quantities associated with the insured and uninsured firm, respectively; quantities without superscripts are the same for both. Suppose each promises the same debt payment at maturity T, $D_0(T)$, and have the same initial value of operating assets, A_0. The only source of uncertainty is the value of operating assets, which evolve stochastically over time.

At time 0, the equity value of the going concern is the present value under the risk-neutral measure of the expected payoff to equity holders. Because both firms have the same physical assets and the same promised debt repayment, the market value of equity of both firms is: $E_0 = e^{-rT}\mathrm{E}_0[\max(0, A_T - D_0(T))]$, where r is the risk-free rate and $\mathrm{E}_0[.]$ is the expectation conditional on time 0 information under the risk neutral measure. Between times 0 and T, the equity values remain the same: both claims are a call option on the same underlying assets, with identical strike price and maturity.

Unlike for equity, the present value of the debt of the two firms prior to maturity is not equal. At any time $t \leq T$, the value of insured debt is simply the present value of the promised payment: $D_t^I = e^{r(T-t)}D_0(T)$ The realized payment on uninsured debt will be the promised amount, $D_0(T)$, or the asset value at time T, A_T, whichever is less. Hence the value of uninsured debt is the present value of the *expected* payment to debt holders: $D_t^U = e^{-r(T-t)}\mathrm{E}_t[\min(A_T, D_0(T))]$.

The value of the guarantee at time 0, $G_0(T)$ is the difference between the initial value of the insured and uninsured debt:

$$G_0(T) = e^{rT}D_0(T) - e^{-rT}\mathrm{E}_0[\min(A_T, D_0(T))] = e^{-rT}\mathrm{E}_0[\max(D_0(T) - A_T, 0)]. \quad (1)$$

The expression on the right-hand side of (1) is the value of a put option on the operating assets of the firm, where the strike price is the promised payment on debt. When assets are lognormally distributed, the value can be computed using the standard Black Scholes formula for a put option.

We assume that the guarantee value accrues to equity holders.[5] Thus at time 0, after the guarantee is announced but before debt is issued, the market value of equity is G_0+E_0. Since we assume that the scale of operating assets is not affected by the presence

[5] To the extent that Fannie and Freddie are able to act as duopolists rather than as competitors, we expect the guarantee value to accrue to their equity holders rather than to mortgage borrowers or other stakeholders. Some of the benefit may be passed to borrowers in the form of lower rates. As long as the pass-through is a constant proportion of guarantee value, the implications for imputing equity value are similar.

of a guarantee, G_0 can be thought of as being immediately distributed either via a dividend, a share repurchase, or equivalently, as a reduction in the initial investment required from the original equity holders. Following the cash distribution, equity price dynamics, as described above, are identical to that of the uninsured firm.

For the government -- both from a production cost and opportunity cost perspective -- the value of the guarantee is also $G_0(T)$. The guarantee is equivalent to the government writing a put option worth $G_0(T)$, and the firm would be willing to pay up to $G_0(T)$ for the insurance.

3.2 Repeated Debt Guarantees

The debt guarantee as just modeled is static: the firm issues debt and then at time T either pays the debt in full or the government makes up the shortfall. This description of the guarantee is overly simplified along several dimensions. First, if the insured firm does not go bankrupt at time T, it will likely have the opportunity to issue additional guaranteed debt. Second, whether or not the insured firm will declare bankruptcy depends on the market value of assets, which is inclusive of current and anticipated future guarantees. Third, the insured firm may readjust its capital structure over time. For example, if assets appreciate the firm may issue more guaranteed debt, whereas if assets fall the firm may buy back some of the guaranteed debt. Such behavior will affect the value of the guarantee and its relation to the value of equity and operating assets. In this section we derive the value of a debt guarantee of an ongoing firm, taking into account these considerations.[6]

[6] For tractability, we take the risk of operating assets as exogenous, but the presence of a guarantee can also affect the characteristics and dynamics of operating assets (see, Keely, 1990).

3.2.1 Operating Asset Dynamics

We distinguish between "operating assets," which denote the financial and physical assets of a firm, and "market assets" which in addition includes the value of credit guarantees. As in Merton (1977) and Merton (1976), we assume that the evolution of firm operating assets over time has three components: an expected return, a random component that is lognormally distributed, and (in the simulations only) a discrete jump in value. Specifically, the percentage change in assets over time is given by the process:

$$dA_t / A_t = (\alpha - \delta)A_t dt + \sigma_A A_t dZ_t + A_t dq \tag{2}$$

where A_t is the asset value, α is the expected return on assets, σ is the volatility parameter, dZ_t is a Brownian motion, and dq is a random variable that over the interval dt is zero with probability $(1\text{-}\lambda)dt$ and $Y\text{-}1$ with probability λdt. The dq term permits the value of assets to jump discretely with probability λdt over an interval dt. The jump takes assets from A_t to YA_t, so the percentage change is $(Y\text{-}1)A_t$. Formulations like equation (2) appear regularly in the literature on debt valuation and bankruptcy.

3.2.2 Valuing a Repeated Guarantee

Here we derive the value of a debt guarantee for a firm with a stationary target debt-to-operating asset ratio. The firm periodically issues debt of fixed, one-period maturity T, setting the amount of new debt to achieve its target debt ratio. For tractability we take the target leverage ratio as given, but a similar policy could arise in response to a regulatory capital requirement, or as an optimal policy in a stationary environment in the presence of fixed adjustment costs. We denote the value of a quantity *X* at time $mT+t$ as $X_m(t)$. We also denote the risk-neutral expectation at time *mT* conditional on information at that time as E_m. We can then express the constant target debt ratio as γe^{rT}, so that for $m = 0, 1, \ldots,$

$$D_m(T) = \gamma e^{rT} A_m(0) \tag{3}$$

The equity value and the default decision for the guaranteed firm will depend on the expected value of current and future credit guarantees. To calculate these quantities, we need to calculate expectations conditional on future solvency. Let $p_m^j(0)$ denote the risk-neutral probability, conditional on information at time *mT*, that firm $j=(I,U\}$ is not bankrupt at time $(m+1)T$. Further, let $\lambda_m^j(0)$ be the expectation of the asset growth rate conditional on no bankruptcy at time $(m+1)T$. Then $\psi_m^j(0)$, which denotes the partial expectation[7] of one plus the expected growth in assets conditional on information at *mT* and not being bankrupt at time $(m+1)T$, is given by $\psi_m^j(0) = \lambda_m^j(0)\ p_m^j(0)$. Finally, let $\phi_m^j(0) = e^{-rT}\psi_m^j(0)$. In the analysis of a stationary equilibrium we drop the time subscripts. These values will depend on the specific condition in any period that determines whether the insured firm declares bankruptcy. To simplify the calculations, we assume that the value of operating assets does not jump, that is, $Y=1$ in equation (2).

As in the one period case, we will compare the value of the guaranteed firm with that of a similar uninsured firm, where both have the same operating assets and target debt ratio, given by equations (2) and (3). For the guaranteed firm, the guarantee remains in place as long as the firm does not experience a default. If the firm does default, we

---

[7] The term "partial expectation of *X*" refers to the probability weighted integral of a random variable *X* over a truncated range of possible realizations.

assume that the value of future debt guarantees is lost forever to current stakeholders. To maintain equivalence of operating assets, we assume that the guarantee value, which is realized through higher proceeds at the time of each debt issue, is paid out immediately as a dividend to the equity holders of the guaranteed firm. We denote the cum dividend equity value at time *mT* as $E_{m-1}(T)$ and the ex dividend equity value as $E_m(0)$.

The one-period guarantee value, and hence the incremental dividend received by the equity holders of the insured firm, is a constant proportion, *g*, of asset value. This follows from the assumption that the amount of newly issued debt is a constant fraction of current asset value, and that the value of a one-period guarantee depends only on the stationary default rule of the *uninsured* firm. Using equations (1) and (3), the proportional guarantee value at a debt reset time *mT* is

$$g = \frac{G_m(0)}{A_m(0)} = \frac{D_m^I(0) - D_m^U(0)}{A_m(0)} = e^{-rT} \mathrm{E}_m[\max(\gamma - \frac{e^{-rT} A_m(T)}{A_m(0)}, 0)]. \tag{4}$$

This can be rewritten, using the Black Scholes formula for a put option and the notation defined above, as

$$g = \left[1 - p_m^U(0)\right]\gamma - \left[1 - \phi_m^U(0)\right] \tag{5}$$

Consider a guaranteed firm, which will continue to operate until it declares bankruptcy, at a debt reset date *mT*. If the firm is solvent, it will issue guaranteed debt maturing at $(m+1)T$. What is the solvency condition at time *mT*? If the firm remains in business, equity holders will receive a call option on the operating assets, and a claim to the present value of current and future dividends generated by the guarantee. Thus, equity holders will pay off the debt coming due, $D_{m-1}(T)$, as long as the value of operating assets plus the guarantee value exceeds the promised debt payment.

Notice that for a comparable uninsured firm, the bankruptcy condition is $A_m(T) > D_{m-1}(T)$. The call option on the operating assets has the same value as for the insured firm, but there is no additional value from the ongoing guarantee. Thus there are states of the world where an insured firm continues to operate to preserve future guarantee value, but an uninsured firm declares bankruptcy. The different solvency conditions imply that the value to the firm of the current one period guarantee, $gA_m(0)$, is no longer equal to the one-period production cost for the government. The former depends on the default policy of the uninsured firm, whereas the latter depends on the more conservative default policy of the insured firm. The additional losses absorbed by the insured firm's equity holders generate a commensurate reduction in cost to the government of the guarantee.

These considerations suggest that to find the value of the guarantee to the insured firm, it is convenient to characterize it in terms of two components. The first is the present value of the incremental dividend stream generated by the guarantee, $\Gamma A_m(0)$. On average, operating assets will grow at their expected rate conditional on the insured firm remaining solvent. Thus, the value of the dividend stream associated with the perpetual guarantee, starting with current asset value A, is:

$$\Gamma A = gA\sum_{i=0}^{\infty} e^{-riT}\left[\psi^{I}\right]^{i} = \frac{gA}{\left(1-\phi^{I}\right)} \tag{6}$$

The second component, $\mathrm{H}A_m(0)$, is the cost to equity holders of paying off the debt in states of the world where an uninsured firm would declare bankruptcy. At time mT, the expected difference between $gA_m(0)$ and the one-period guarantee production cost of the government is:

$$\int_{A_{m(0)}(\gamma e^{rT}-\Gamma+\mathrm{H})}^{\gamma e^{rT}A_m(0)} \left[\gamma^{rT}A_m(0)-\alpha\right] f(\alpha \mid A_m(0))\,d\alpha = \eta A_m(0) \tag{7}$$

where $f(\alpha|A_m(0))$ is the probability density of firm asset value at time $(m+1)T$ conditional on asset value the previous period, and η denotes the cost differential as a fraction of asset value. Like guarantee value, the present value of the cost differential depends on the expected future growth rate of assets, conditional on the probability that the firm remains solvent:

$$\mathrm{H}A = \eta A \sum_{i=0}^{\infty} e^{-riT}\left[\psi^{I}\right]^{i} = \frac{\eta A}{\left(1-\phi^{I}\right)} \tag{8}$$

Thus, at mT, if the insured firm is solvent, its equity value exceeds that of the uninsured firm by

$$A_m(0)\left[\Gamma - \mathrm{H}\right]. \tag{9}$$

It follows that one reason previous studies that estimated subsidy cost on the basis of interest rate spreads reported higher costs than derivative-based estimates is that they implicitly set H to 0 in equation (9). The size of the bias, however, is difficult to assess. To the extent that the comparison firms were banks with subsidized federal deposit insurance and access to FHLB advances, it is not clear whether the GSEs or banks have a greater incentive to default conservatively to preserve the value of subsidized insurance.

3.2.3 Asset Value and Volatility

We can observe the value and volatility of market equity, dividend policy, promised debt repayment, debt maturity, and the risk-free rate, but must infer the value and volatility of assets. The problem of finding the value and volatility of market assets is conceptually similar to that considered in Marcus and Shaked (1984), who modeled the value of FDIC insurance in a one-period setting using an options pricing model. As discussed earlier, the value of equity for the guaranteed firm is a call option on market assets, which include both operating assets, with dynamics given by equation (2), and the value of future guarantees. Using equation (9), market assets on a debt reset date *mT* can be written as:

$$A_m(0)^* = A_m(0)[1+\Gamma-\mathrm{H}]. \tag{10}$$

Looking forward to the next reset date, the volatility of market assets is proportional to that of operating assets: $\sigma_{A^*} = \sigma_A[1+\Gamma-H]$. Further, the continuation condition for the insured firm at each debt reset date is:

$$A_m(0)[1+\Gamma-\mathrm{H}] \geq D_{m-1}(T). \tag{11}$$

Then the relation between the distribution of equity returns and asset returns can be found following Merton's approach as the simultaneous solution to two non-linear equations, but with $A_m(0)^*$ in place of $A_m(0)$, and with the dividend yield, δ^*, expressed as a share of $A_m(0)^*$ rather than as a share of operating assets. Let $C(A,D,\sigma_A,\delta,T)$ denote the Black-Scholes value of a European call option with underlying assets *A*, promised debt payment *D*, asset volatility σ_A, dividend yield on market assets, δ^*, and time to maturity *T*. Then the value of equity for an insured firm is:

$$E_m(0) = C(A_m(0)^*, D_m(T), \sigma_{A^*}, \delta^*, T) \tag{12}$$

The value and volatility of market assets is found by solving (12) simultaneously with:

$$\sigma_{A^*} = \sigma_E /(N(d_1)A_m(0)^* e^{-\delta^* T} / E_m(0)) \tag{13}$$

where

$$d_1 = [\ln(A_m(0)^* / D_m(T)) + (r - \delta^* + .5\sigma_{A^*}^{\ 2})T]/(\sigma_{A^*}T^{.5}) \tag{14a}$$

$$d_2 = d_1 - \sigma_{A^*}T^{.5} \tag{14b}$$

Equation (13) comes from the relation, $\sigma_E = \partial E / \partial A \left(\frac{A}{E} \right) \sigma_A$.

3.2.4 Discussion

The preceding analysis is useful for understanding the relation between the value and volatility of operating assets, equity, and a government guarantee on debt. The most straightforward conclusion that emerges is that the market value of debt plus equity exceeds the value of operating assets by the value of the present value of expected guarantee payments. Expected recoveries in the event of default, which depend only on the value of operating assets, must be adjusted discretely downward for this effect. The bankruptcy trigger must also be adjusted to take into account the effect of guarantee value on behavior. However, inferences about the volatility of operating assets made on the basis of stock price volatility, using the framework of (12) and (13) and using observations of equity prices on debt reset dates, are basically the same for a firm with or without a guarantee.

This analysis abstracts from what happens between reset dates. As the value of operating assets evolves so too does the probability of solvency and the expectation of asset value on the next reset date, and hence the expectation of the present value of future guarantees. The fixed proportionality of guarantee value to asset value at the next reset date, however, implies that the dynamics between reset dates are also unaffected by the presence of the guarantee.

In fact government policy may not be stationary, and the value of the guarantee may be perceived by the market as changing over time with economic and political events or as a function of the financial situation of the GSEs. Whether this would make equity value more or less volatile relative to operating assets is unclear, as it would depend on the correlation between the strength of the guarantee and the objective situation of the firm, among other things. Clearly the model can be modified to take other hypotheses into account, but in its stationary form provides a neutral starting point or "guarantee irrelevance theorem" for thinking about these effects.

3.3 Monte Carlo Valuation of the Guarantee

Here we employ a discrete time version of equation (2) that is suitable for simulation. The calibrated model accommodates more complex assumptions about liability management and default behavior, and allows us to explore the effect of a variety of regulatory policies on guarantee cost.

Under a risk-neutral representation in discrete time, operating assets evolve according to:

$$A_{t+h} = (1 - I_{J,t}\omega)A_t Exp\left[(r_f + p_j\omega + \theta_t - \delta\frac{E_o}{A_o} - .5\sigma_A^2)h + \sigma_{A,t}\varepsilon\sqrt{h}\right] \qquad (15)$$

where h is the time step, t subscripts represent time, E is equity, r_f is the risk-free rate, θ_t is externally financed firm asset growth, δ is the dividend yield on equity (hence $\delta\frac{E_0}{A_0}$ is taken to be the dividend yield on assets), $\sigma_{A,t}$ is the possibly time-dependent volatility of operating assets, ε is a draw from a standard normal distribution, $\omega = Y\text{-}1$ is the non-stochastic jump size, $I_{j,t}$ is an indicator that a jump has occurred, $p_j h$ is the probability of a jump over an interval of length h. The actual evolution of operating assets is identical except that r_f is replaced by the expected return on assets r_A.

Here A_t represents the value of all of the firm's operating and investment activities, both on and off balance sheet: It includes the mortgage portfolio, the MBS business, derivative market activities, etc. Asset value is affected by a variety of factors, including interest rate, credit, and other risks. Unhedged interest rate risk on the retained portfolio, and the associated prepayment and extension risk that arise due to the prepayment option on residential mortgages, until recently has been considered the greatest source of risk. Credit risk arises both from mortgages held on balance sheet, and from the MBS they guarantee. This risk is mitigated by the collateral value of the underlying real estate. The remaining risks -- political, accounting, fraud, liquidity, model, counterparty, etc. -- are potentially important but difficult to quantify. Political risks include the possibility of legislation that restricts growth or increases competition, reducing franchise value. Accounting misrepresentations or fraud may cause downward jumps in perceived asset value, and can prolong the time between when a problem arises and is recognized, increasing the severity of losses.

Importantly, this measure of operating assets represents the true financial condition of the company, and we take it to be the recovery value in bankruptcy. The market value of assets, however, also includes the value of current and future expected guarantees, G_t,. As suggested by the analysis of section 3.2 we assume that the guarantee value is a constant proportion of the market value of assets:

$$A_t{}^* = (1 + \Gamma - \mathrm{H})A_t \tag{16}$$

We do not, however, attempt to identify the two components of guarantee value separately.

To summarize the different roles of operating assets and market value assets in the calibrations: operating assets are identified with the recovery value of the firm in bankruptcy and determine the dividend stream to equity holders, whereas market value assets determine the continuation condition for the firm. Since the procedure for setting the initial conditions only identifies the market value of assets, finding the proportionality factor (1 + Γ – H) is a fixed point problem. We can guess what the factor is, compute the implied value of the guarantee, and check for consistency.

3.3.1 Liabilities

Representing debt as having a single fixed maturity, as we did in sections 3.1 and 3.2, abstracts from the possibility of more complex debt rebalancing strategies and future growth opportunities. Closed form solutions for the value of debt under optimal or stationary debt policies have been derived for a few special cases (e.g., Leland (1994), Collin-Dufresne and Goldstein (2001)), but those do not allow for state dependent changes in debt policy or continuation rules. To allow for more complicated patterns of behavior we choose instead to specify a liability process that allows for gradual adjustment of debt towards a target ratio, with asymmetry in the upward and downward speed of adjustment reflecting the relative difficulty of reducing debt when asset value falls. Book liabilities, L, evolve according to:

$$L_{t+h} = L_t e^{(r_d + \gamma\theta_t)h} + I_t \alpha_t h \left[\lambda * - L_t e^{r_d h} / A_t\right] A_t \tag{17}$$

where α_t is the annual rate of adjustment, which may be state dependent, $\lambda*$ is the target liability to operating asset ratio, and I_t is an indicator variable that equals one in a period where liabilities are adjusted, and 0 otherwise. Liabilities grow at a rate r_d to cover promised interest.[8] In addition, a fraction γ of externally financed growth is supported by debt. This representation applies to both the actual and risk-neutral calculations, but the realized paths differ because the return on debt and externally financed growth take on different values in each instance, and the ratio of assets to liabilities displays different dynamics. Although computationally it would be straightforward to add volatility to liabilities, we assume instead that the estimated volatility of assets implicitly captures volatility arising from all sources including liabilities.

The promised interest rate, r_d, depends on what one assumes about the strength of the government guarantee. If it were completely firm, and abstracting from other differences between Treasuries and other securities, then setting r_d equal to r_f would be appropriate. In the calibrations we assume a positive rate spread that is somewhat smaller than the average observed in the data. This is consistent with our view that the guarantee is not risky, but that there are some other features that make Treasury debt more valuable than agency securities.

3.3.2 Insolvency Trigger

Consistent with the analysis in section 3.2, we assume that the solvency condition depends on the market value of assets relative to book liabilities. We do not, however, restrict the default policy to be optimal, but instead consider the effect on guarantee cost of alternative continuation rules.

As in Merton (1977), we assume that bankruptcy only occurs during periodic audits. If the shut-down condition is met, the auditor closes the firm and makes a

[8] An alternative would be to reduce assets by the amount of a periodic interest payment, which would reduce the scale of the enterprises over time relative to what is assumed here.

guarantee payment to debt holders. Several insolvency triggers have been proposed in the literature. One that is roughly consistent with observed bankruptcy experience is to liquidate the firm when the market value of assets falls below the level of current liabilities plus half of the book value of long-term liabilities. Another is that the market value of assets falls below a fraction of the total book value of liabilities. We use the latter type of rule, since distinguishing between the long and short-term liabilities of Fannie and Freddie is complicated by the frequent maturity conversions taking place through derivatives market transactions.

The cost of a drawn-out reorganization or closure process, or regulatory forbearance, is assessed by varying the ratio of assets to liabilities that triggers bankruptcy, and be varying the time between audits. A drawn-out closure procedure can also be more costly if asset volatility increases with financial distress. This could occur, for instance, because of a correlation between conditions that cause distress and overall market volatility, because distress raises the cost of hedging, or through a purposeful increase in risk-taking by management to try to make up for past losses. This increase in volatility may not be easily discernable in historical data, both because its occurrence is a low probability event, and because it is likely to persist for relatively short periods of time when it does occur. In Lucas and McDonald (2006) we found this to be a significant potential driver of guarantee cost, and we also incorporate it into these estimates.

3.3.3 Equity

Equations (15) and (17), which govern the evolution of firm operating assets and book liabilities respectively, implicitly define the cash flows to equity. Those consist of the dividend payment each period, and cash raised from subsequent debt issues not used to finance exogenous asset growth. Exogenous asset growth not assumed to be debt financed further implies a negative cash flow to initial equity holders, or equivalently an equity issue.

The time 0 value of equity is the present value of all future cash flows to equity. That value is computed in the Monte Carlo simulations under the risk-neutral measure, by discounting cash flows at the risk-free rate. As a proxy for cash flows beyond the simulation horizon T, the terminal value of equity at time T is approximated by $A_T - L_T$. This neglects the value of the guarantee after time T, but that effect becomes small as T increases. Calculating the implied equity value using this approach provides a valuable check on the internal consistency of the model, since it can be compared to the observed equity value used to determine the initial value of assets and liabilities.

3.3.4 Deriving Initial Conditions and Accounting for Guarantee Value

The initial market value and volatility of firm assets must be estimated since these quantities are not directly observable. The analysis of section 3.2 suggests that we can do this using Merton's framework, where equity can be valued as a call option on the firm's market assets. Specifically, we use equations (12) and (13), calibrated with market and balance sheet data from Fannie and Freddie, to estimate the initial market value of assets and their volatility.[9] What is tricky conceptually is to choose a horizon for debt, since liabilities follow (17) and there is no specific maturity date. We use the reported average

[9] We also used this approach for deriving initial conditions for asset value and volatility in Lucas and McDonald (2005). Marcus and Shaked (1984) show that the same equations can be used to estimate the value of the government guarantee, and use that insight to estimate the value of deposit insurance for U.S. banks
.

effective maturity of debt as a proxy, and consider the sensitivity of the results to varying the assumed debt maturity.

4. Calibration and Results

The model in the base case is calibrated to year-end 2005, a time when the reported financial condition of both firms was strong. We will then look at how the estimated cost of the guarantee to the government changes as their financial condition deteriorates, and the sensitivity to other parametric assumptions and policy variables.

Three critical inputs for guarantee valuation are market value equity, equity volatility, and liabilities. Table 1 reports these statistics, along with the other parameters used for the base case. Data acquisition for 2005 was complicated because Fannie Mae delayed in filing financial reports since it had to restate its financial statements through 2004. As of December 2006 it had not filed any further financial reports. Fannie did, however, provide monthly information on the size of their mortgage portfolio and MBS outstanding. We have imputed some of the missing information for Fannie by relying on Freddie's disclosures. Specifically, we estimate book liabilities for Fannie, assume that the ratio of liabilities to retained mortgages is the same for both firms.

We infer base case equity volatility using historical implied annualized 30-day volatility from option prices. The series are shown for Fannie Mae and Freddie Mac in Figures 1 and 2. Both series are graphed against 30-day implied volatility for the S&P 500 index (the VIX index) in order to highlight changes in volatility, which are firm specific rather than market-wide. Implied volatility for both firms ranges from 20% to

60%, with an average of about 30%, our base estimate for both firms. Implied volatility in 2006 and 2007 remained at similar levels.

Estimates of guarantee value are based on 20,000 Monte Carlo runs, for 10 and 20 year horizons. As in Lucas and McDonald (2006), asset volatility is assumed to increase to four times its normal level when assets fall to 101 percent of liabilities, representing increased volatility in periods of financial distress. Management and regulatory decisions (debt adjustment and solvency determination) are evaluated at a quarterly frequency, while assets returns are calculated at a monthly frequency. Several variables are parameterized differently than in our previous study. The ability to adjust down liabilities is more constrained, a change that achieves greater consistency between observed and computed equity values. We set exogenous asset growth to zero (in contrast to the 6% previously assumed), because it seemed in 2005 unlikely that future growth would match historical rates. Liabilities still grow on average at about 9% annually, however, because of the assumption that interest accumulates as increased debt and because the expected return on assets exceeds the dividend rate, creating growth from retained earnings that on average causes the target debt level to grow.

As discussed in the previous section, using (12) and (13) to estimate initial asset value and asset volatility is problematic because it requires a fixed debt maturity as an input. Nevertheless, it is a useful starting point for estimation. In 2004, the last year for which we have obtained average maturity data, Fannie's effective debt maturity was 2.65 years and Freddie's was 3.05 years. Since the agencies normally match the duration of assets and liabilities, it seems likely that their effective maturity of debt has increased since then with the lengthening effective maturity of mortgages. Table 2 illustrates the effect of the debt maturity assumption on implied asset value and volatility, using the parameter assumptions in Table 1, for maturities of 2.5, 5 and 7.5 years. Both implied asset value and volatility increases with assumed debt maturity. Although the increases appear small in percentage terms, model estimates are very sensitive to assumed asset volatility, and hence to the initial maturity assumption.

Table 1: Base Case Parameter Values, Year-end 2005

Short Name	Value	Description
Fannie Mae		
FLinit	$ 744	initial imputed book value of liabilities ($ billions)
MVEquity	$48,750	initial market value of equity ($ millions)
dividend yield	0.028	
Freddie Mac		
FLinit	$727	initial book value of liabilities ($ billions)
MVEquity	$47,056	initial market value of equity ($ millions)
dividend yield	0.03	
Common Values		
FAvol_h	FAvol*4	firm asset volatility in high volatility state
rf	0.045	risk free rate
rd	.0475	promised return on debt
FAer_a	0.05307	firm assets expected return (actual)
FAer	0.045	firm assets expected return (risk-neutral)
FLrate_d	0.03 / 4	quarterly adjustment of liabilities to lower target
FLrate_u	0.8 / 4	quarterly adjustment of liabilities to higher target
growth	0.0	externally financed growth if enough capital
growth_trig	.93	if target liability to asset ratio met, then growth
growth_debt	1	proportion of external financing that is debt
trigger	0.98	bankruptcy trigger assets/liabilities
trig_volh	1.01	trigger of assets/liabilities for higher volatility
look	4	frequency of checking bankruptcy trigger per year
look_l	4	frequency of updating debt
FLFAtarget	.93	target liability to asset ratio
newFLFA	1	proportion of debt financed exogenous asset growth
nmonte	20,000	number of Monte Carlo simulations
nyear	10	number of years in each simulation run
nfreq	12	time steps per year

Table 2: Implied Asset Value and Volatility as Function of Assumed Debt Maturity						
	Fannie	Freddie	Fannie	Freddie	Fannie	Freddie
Horizon (yrs)	2.5	2.5	5	5	7.5	7.5
Asset Value	797.0	778.4	800.4	781.2	803.1	780.9
Asset Vol	.0208	.0185	.0225	.0204	.0238	.0230

Table 3 reports the guarantee and equity values in the base case with no jumps in asset value, and using initial conditions assuming Fannie's (Freddie's) effective debt maturity is 2.65 (3.05) years. The combined guarantee value over 20 years is $40 billion. The guarantee value expressed as a premium rate on liabilities is 17 to 20 bps. For both firms, the implied equity values are somewhat lower than the observed values used to estimate asset volatility and value, but small changes in parameters (e.g., volatility) can easily reconcile the equity values.

<PLEASE NOTE, ESTIMATES IN TABLES 3&4 MAY CHANGE IN THE FINAL DRAFT. WE'RE IN THE PROCESS OF RECALCULATING, BUT SHOULDN'T CHANGE QUALITATIVE RESULTS.>

Table 3: Base Case 2005 Guarantee Value Estimates				
	Fannie	Freddie	Fannie	Freddie
Horizon	10	10	20	20
Guarantee Cost ($ billions)	11.11	9.16	21.14	18.6
Premium Rate	.0017	.00140	.00203	.00175
Implied Equity Value ($ billions)	47.9	44.7	47.7	42.6
Default Prob. (actual)	.1	.06	.21	.15

Next, we consider the effect of discrete jumps down in asset value, where trend growth is adjusted up so that average asset growth is the same as the Table 3 calculations.

The probability of a jump is 3% per year, and the jump size is 5%. The results are reported in Table 4. The effect is to increase the probability of default and the value of the guarantee, but not significantly. Increasing the size of the jump to 10% increases the 20-year cost for Fannie to $26.2 billion, but also increase the equity value to $56.5 billion, significantly higher than its observed value. It appears that plausible jump processes increase estimated cost, but not nearly enough to reconcile options-based and spread-based cost estimates.

Table 4: 2005 Guarantee Value Estimates 3% annual probability of 5% reduction in asset size				
	Fannie	Freddie	Fannie	Freddie
Horizon	10	10	20	20
Guarantee Cost ($ billions)	11.98	10.20	21.9	19.7
Premium Rate	.00188	.00160	.00217	.00193
Implied Equity Value ($ billions)	49.3	46.3	50.8	45.3
Default Prob. (actual)	.14	.11	.28	.23

Options-based estimates of guarantee value are quite sensitive to equity prices, which of course can be quite volatile. An interesting question is what the market signaled about the guarantee value of Fannie and Freddie in the months prior to their being put into receivership? To illustrate the sensitivity to changes in leverage ratios, Table 5 reports on guarantee values as a function of the initial ratio of market liabilities to market assets, holding other parameters the same as in the base case.

Table 5: 2005 Guarantee Value Estimates, Varying Initial Equity for Fannie			
	Fannie	Fannie -20% equity	Fannie -50% equity
Horizon	20	20	
Guarantee Cost ($ billions)	21.14	21.97	23.43
Premium Rate	.00203	.0023	.0028
Implied Equity Value ($ billions)	47.7	40.1	30.9
Default Prob. (actual)	.21	.24	.31

Conclusion

<TBA>

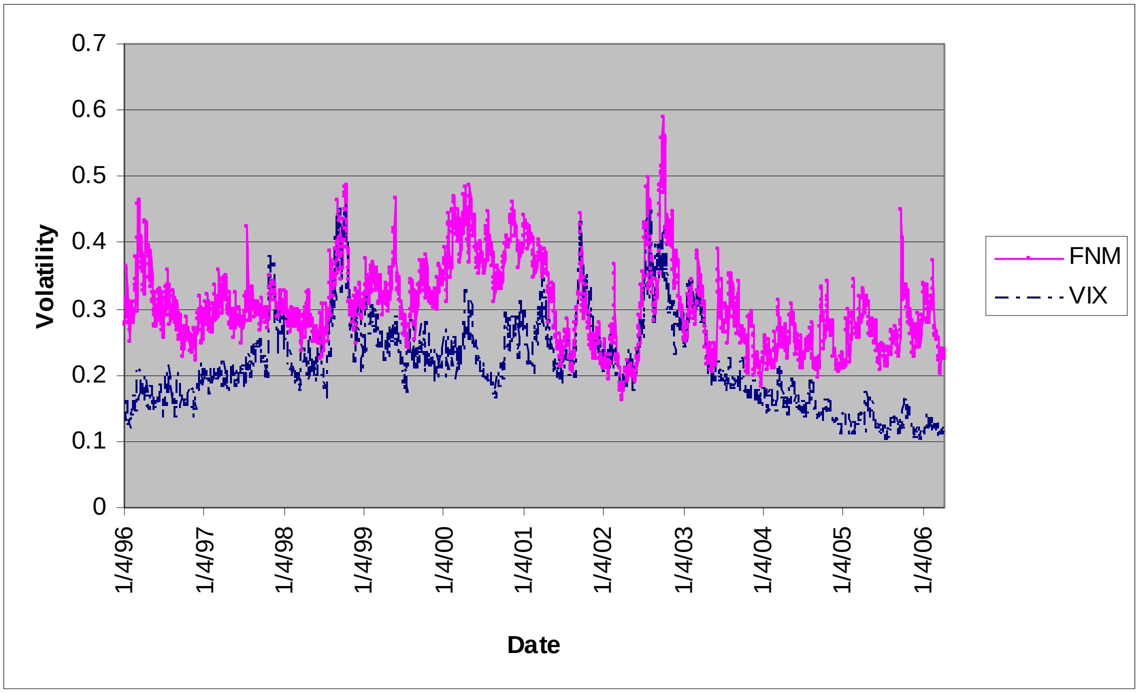

Figure 1: Implied volatility for Fannie Mae and for the S&P 500 (VIX), 1996-2006. Source: Optionmetrics and Yahoo.

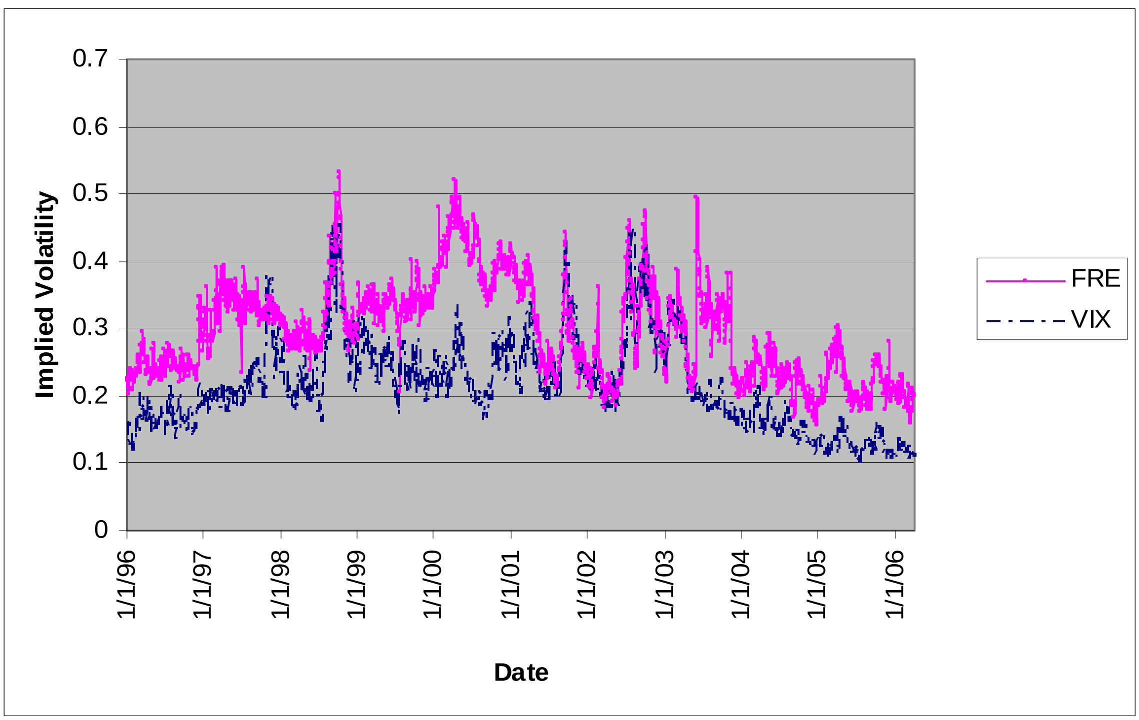

Figure 2: Implied volatility for Freddie Mac and for the S&P 500 (VIX), 1996-2006. Source: Optionmetrics and Yahoo..

References

Ambrose, Brent, and Arthur Warga. 2002. Measuring potential GSE funding advantages. *Journal of Real Estate Finance and Economics* 25(2/3): 129-150.

Carnell, Richard S. 2005. "Handling the Failure of Fannie Mae and Freddie Mac," Conference on Receivership Powers, American Enterprise Institute, Discussion Draft

Collin-Dufresne, P. and R.S. Goldstein, 2001, "Do credit spreads reflect stationary leverage ratios?", *Journal of Finance,* 1929-1957.

Congressional Budget Office 2001, "Federal Subsidies and the Housing GSEs"

Congressional Budget Office 2004a, "Updated Estimates of the Subsidies to the Housing GSEs"

Congressional Budget Office 2004b, "Estimating the Value of Subsidies for Federal Loans and Loan Guarantees"

Congressional Budget Office 2008, Cost Estimate for H.R. 3221, Housing and Economic Recovery Act of 2008, July 2008

Crosbie, Peter and Jeff Bohn 2003, "Modeling Default Risk," Moody's KMV White Paper; available at http://www.moodyskmv.com/research/whitepaper/ModelingDefaultRisk.pdf

Dempsetz, R., M. Saidenberg, and P. Strahan 1996, "Banks with Something to Lose: The Disciplinary Role of Franchise Value," *Economic Policy Review*, Vol. 2, No. 2,.

Feldman, Ron. 1999. "Estimating and managing the federal subsidy of Fannie Mae and Freddie Mac: Is either task possible?" *Journal of Public Budgeting, Accounting, and Financial Management* 11 (Spring): 81–116.

Frame, W. Scott and Lawrence J. White. 2005. "Fussing and Fuming over Fannie and Freddie: How Much Smoke, How Much Fire?" *Journal of Economic Perspectives*

Hubbard, R. Glenn, 2004. "The Relative Risk of Freddie and Fannie," Fannie Mae papers, Vol. III, Issue 3.

Jaffee, Dwight. 2003. "The interest rate risk of Fannie Mae and Freddie Mac," *Journal of Financial Services Research*, 24(1): 5-29.

Kane, Edward J. and Chester Foster. 1986. "Valuing conjectural government guarantees of FNMA liabilities," In *Proceedings: Conference on Bank Structure and Competition*. Chicago: Federal Reserve Bank of Chicago.

Keeley, Michael. 1990. "Deposit Insurance, Risk, and Market Power in Banking." *American Economic Review* 80, no. 5: 1183-200.

Leland, Hayne E., "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure", *Journal of Finance,* 49 (4), 1994, 1213-52.

Lucas, Deborah and Robert McDonald, 2006, "An Options-Based Approach to Evaluating the Risk of Fannie Mae and Freddie Mac," *Journal of Monetary Economics*

Marcus, Alan and Israel Shaked, 1984, "The Valuation of FDIC Deposit Insurance Using Option-Pricing Estimates," *Journal of Money, Credit and Banking*, Vol. 16, No. 4, 446-460.

Merton, Robert C., "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates", *Journal of Finance*, 29 (2), 1974, 449—470.

Merton, Robert C., "An Analytic Derivation of the Cost of Loan Guarantees and Deposit Insurance: An Application of Modern Option Pricing Theory, *Journal of Banking and Finance,* 1(1), 1977 (June), 3—11.

Naranjo, Andy and Alden Toevs. 2002. "The effects of purchases of mortgages and securitization by government sponsored enterprises on mortgage yield spreads and volatility," *Journal of Real Estate Finance and Economics*. 25 (September-December): 173-195

Nothaft, Frank E., James E. Pearce, and Stevan Stevanovic. 2002. "Debt spreads between GSEs and other corporations," *Journal of Real Estate Finance and Economics* 25 (September-December): 151:172.

Panageas, Stavros, 2006. "Bailouts, the Incentive to Manage Risk, and Financial Crises," manuscript, The Wharton School.

Passmore, Wayne. 2005. "The GSE implicit subsidy and the value of government ambiguity," *Real Estate Economics*

Seiler, Robert S. 2003. "Market discipline of Fannie Mae and Freddie Mac: How do share prices and debt yield spreads respond to new information?" Office of Federal Housing Enterprise Oversight Working Paper 03-4.

Sharpe, William F. (1976), "Corporate Pension Funding Policy", *Journal of Financial Economics,* 3(3), (June), 183—193.

Stiglitz, Joeseph E., Jonathan M. Orsag and Peter Orsag, 2002, "Implications of the New Fannie Mae and Freddie Mac Risk-Based Capital Standards," Fannie Mae papers, Vol 1, Issue 2.