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SHORT- AND LONG-TERM SIMULATIONS WITH THE BROOKINGS MODEL

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PREDICTIONS with econometric models, even thirty years after Tinbergen's initial attempt, still involve art as well as science. The present version of the Brookings Model is an advance over its predecessors. Yet much remains to be done to improve the specification of certain sectors and to reduce the predictive error of the system as a whole. The equations of the present system, the 1969 BUSEM, are similar to those presented previously in the Fromm-Taubman simulations,[1] but there are a few significant differences.

1 STRUCTURE OF THE MODEL

TO begin, the sample period for earlier versions of the model was 1948–60. Thus, it included the waning years of readjustment to World War II and the Korean War experience. Analysis of covariance tests run for the periods 1948–53, 1954–60, and 1948–60 revealed significant shifts in many coefficients between the earlier and later years of these intervals. It was decided to select a sample period germane to the analysis of current economic problems. Therefore, the present version of the model is estimated over the post-Korean War years, 1954–65. The data employed also are taken from different sources: revised na-

[1] Gary Fromm and Paul Taubman, *Policy Simulations with an Econometric Model* (the Brookings Institution, 1968).

tional income and product accounts and later revisions of unpublished statistics.[2]

The theory underlying some of the equations of the model has been modified as well. Consequently, it is not surprising that certain variables in specific functions no longer have statistically significant coefficients, and these variables have been eliminated. In other instances, new variables or better measures of previous ones have been included.

The 1969 version of the model, for which solutions are presented below, contains 230 equations (118 of which are stochastic) and 104 exogenous variables or parameters. Most of the exogenous variables are of minor importance. The model is estimated using ordinary least squares. Two-stage least squares estimates of a somewhat larger and improved version of the model also have been prepared and will be the basis of complete system solutions to be released in 1970.

A description of the difference in specification of the 1969 BUSEM from the 1968 BUSEM follows. Equations for the 1969 and 1968 models are shown in the appendixes of this paper and in the Fromm and Taubman book, respectively.

CONSUMER DEMAND

The present consumption functions depart from the earlier versions primarily in that they are estimated on a real per capita rather than on a real absolute basis. The principal explanatory variables are real disposable income per capita and relative prices. A credit dummy variable has been added to the consumption of durables other than automobiles. The autos equation now includes a capital stock of autos variable and a dummy variable (scaled by per capita disposable income) to reflect auto strikes. The two nondurables consumption equations (foods and beverages, and other nondurables) are unchanged, while the liquid assets term has been deleted from the services equation.

[2] *The National Income and Product Accounts of the United States, 1929–1965, Statistical Tables* and *Survey of Current Business,* July 1966, July 1967, and July 1968.

RESIDENTIAL CONSTRUCTION

Because of compositional shifts in the late 1950's, housing unit starts have now been disaggregated into three categories: single units, double units, and three or more units. All of the original variables appear in the various starts functions, but in different equations. Single unit starts are dependent on cost and the availability of funds in financial markets (short-term interest rates are used as a proxy), real disposable income per household, and the real market price of the average home. (The latter price is also a function of real disposable income per household.) Lagged single unit starts, together with three-quarter moving averages (lagged one-quarter) of the other variables give a modified Koyck lag effect to the impact of the explanatory factors. The same type of lag distribution is used in the other starts and price equations.

Housing starts of dual units are a function of interest rates, disposable income per household, a time trend and lagged starts. Presumably, a market price variable might be significant but data on dual unit prices are not available separately from other multi-unit prices. Multi-unit starts (three or more units) are mainly dependent on supply conditions and are strongly influenced by interest rates and housing unit vacancies. Vacancies also have a significant influence on the market price of multiple units.

Due to the above disaggregation, the price-quantity identity that defines housing unit expenditures is, of course, slightly modified. The equation for other residential expenditures is unchanged, except now the coefficient of the price of such outlays is significant and bears the correct negative sign. Expenditures for new private nonfarm, nonresidential, nonbusiness construction also are included in this sector; here, the lagged dependent variable has been added as an explanatory factor.

INVENTORY INVESTMENT

The inventory investment equations for durable and nondurable manufacturing are nearly identical to those used previously. Real inventory change is a function of real final sales, beginning of period inventory stocks and unfilled orders, and the previous period's inventory investment. (The lagged real change in unfilled orders also appears in the durables equation.) The one exception is that a change in final sales of nondurable goods has been deleted from the nondurable function.

Trade inventories, because of requirements elsewhere in the model, have been separated into investment in car inventories and in trade inventories other than cars. The latter is made a function of the final sale of goods and the beginning of period stock of such inventories.

By definition, car inventories are held only by the trade sector and not by manufacturers. Therefore, the change in the value of dealer stocks is hypothesized to be dependent on the level and first difference of personal consumption expenditures on automobiles and the value of the stock of cars at the beginning of the period.

For the residual sector, real inventory investment is a function of real final sales, the beginning of period stock, and lagged inventory change. Price changes and interest rate terms have been dropped from this equation.

ORDERS

A few modifications have been made in the orders sector. For durable manufacturing, speculative price changes and lagged final sales terms have been dropped from the real new orders equation. A two-quarter lagged moving average of Department of Defense military prime contract awards has been substituted for current government military expenditures (both variables in real terms). The reciprocal of the rate of capacity utilization has been added to reflect the impact of capacity constraints. The remaining key explanatory variables are real final sales of durables, construction expenditures, and the level of unfilled orders at the beginning of the period.

The level of real unfilled orders in durables manufacturing is given by an identity between the beginning of period level, price changes (the identity is only valid in current dollar terms), real new orders, and real sales. A linear function relating real sales to real output originating is substituted for the sales term in the identity.

The approach for nondurables manufacturing, suggested by Gerald Childs, is somewhat different.[3] Here, using inventory decision rules, the real change in unfilled orders is hypothesized to depend on real values of the first difference in new orders, lagged new orders, the lagged level of unfilled orders, inventory stocks, the level and rate of change in the wholesale price index of nondurables, and the reciprocal of capacity utilization in the industry. New orders are then given by an identity similar to that for unfilled orders for durables manufacturing.

INVESTMENT IN NONFARM BUSINESS PLANT AND EQUIPMENT

The real fixed business investment equations include nearly the same variables as previous formulations. Expenditures are functions of real output originating, long-term interest rates, and beginning of period real capital stocks. Rather than including explicit lag terms of various explanatory factors, as was done previously, Almon lag distributions have been used instead. Also, lagged capacity utilization rates (in the form of real actual to potential output) have been included as an indicator of short-term modification of investment plans. Finally, dummy variables and truncated time-trend variables have been added to act as proxies for the effects of the investment tax credit.

FOREIGN TRADE

Due to modifications in the basic data, equations have been estimated for nondurables and services, and for durables imports rather than for finished and unfinished imports. Real nondurables and services expenditures are a function of the price of these imports relative to the price of consumption, as well as real disposable income, lagged imports, and a dummy variable for dock strikes.

The equation for real export expenditures is unchanged except for the additions of a dock strike dummy. Exports are dependent on the volume of world trade and the price of U.S. exports relative to world export prices.

[3] This is based on his *Unfilled Orders and Inventories: A Structural Analysis* (North-Holland, 1967).

GOVERNMENT TAXES AND TRANSFER PAYMENTS

There are four tax functions for total Federal, state, and local receipts of personal, excise, corporate profits and social insurance taxes. In each of the equations, as before, a variable which corresponds to the tax base is included. Now, however, given the changes in rates after 1960, profits before taxes are multiplied by the normal corporate rate plus surtax. (The elasticity of this combined variable is less than unity because of the lower rate that applies to low income corporations and because of carry-forward and carry-back averaging of losses.) Similarly, the social insurance equation now includes terms which reflect the contribution rate, the percentage of employees covered, and the maximum individual tax base. There is also a term to reflect employers' contribution rates for unemployment insurance. Finally, dummy variables are utilized to capture the cut in tax liabilities of the 1964 and 1965 personal and excise tax reductions and the 1962 investment tax credit.

Government transfer payments are predicted by the same equations used previously. Transfers of unemployment benefits are a function of the number of persons unemployed and a *GNP* potential gap valued in current dollars. The remaining categories of government transfers, social security, veterans', and miscellaneous payments, are treated exogenously.

PRODUCTION FUNCTIONS

Rather than utilize linear employment functions, this version of the model contains a restructuring of the production functions in accord with more recent theoretical developments. Starting with a Cobb-Douglas function, and an inertial adjustment process, Michael McCarthy developed logarithmic equations for man-hour requirements for production workers in the durables manufacturing, nondurables manufacturing, and trade sectors. The explanatory variables are real output originating and real capital stocks. Time trends are included to account for technological change; dummy variables are introduced in the 1960's to reflect apparent shifts in production functions and reductions in manufacturing production labor requirements. Further study is needed to validate the justification for use of the latter dummies.

Data are not available on hours worked by nonproduction workers in the manufacturing and trade sectors. Therefore, employment of these workers is made the dependent variable in logarithmic equations with real output originating and lagged employment as the explanatory elements.

Logarithmic functions also are used to explain total man-hour requirements for the contract construction, regulated, and residual sectors. Again, aside from an inertial adjustment process, the principal explanatory variables are real output originating and a time trend for technological change. Because the residuals of these equations were highly serially correlated, Cochrane-Orcutt corrections were applied.

In the previous model, average weekly hours of production workers in the manufacturing and trade sectors and of all workers in the contract construction and regulated sectors were explained by equations of the form:

$$H = \beta_0 + \beta_1 \frac{\Delta X^{58}}{X^{58}_{-1}} + \beta_2 H_{-1}$$

where

H = average weekly hours
X^{58} = gross product originating in 1958 dollars

In specifying the present model an attempt was made to include the real product wage per man-hour (wage rates divided by the price of output) as an explanatory variable measuring labor substitution effects. Unfortunately, statistically significant coefficients were not found. One member of the project staff estimated an hour's equation using the level and change in real output and the level of wage rates as explanatory variables. This produced statistically significant coefficients and the equations were incorporated in the present model. The complete system solutions for the period 1957–65 for hours of all workers are extremely accurate; the root mean square error is only 0.15 hours per week and the mean error is 0.02 hours (on a base of 40 hours per week).

Given the equations for man-hours and average weekly hours. it is then possible to calculate employment by sector from identities of the form $E = MH/H$.

INDUSTRY PRICES

In the earlier model, prices of output originating in durables and nondurables manufacturing were determined as a function of the level of normal unit labor costs (current wage rates divided by a twelve-quarter average of output per man-hour), the difference between actual and normal unit labor costs and, as an indicator of demand pressures, the deviation of real inventory stocks per dollar of real output from its three-year trend. Wholesale price indexes for these industries then simply were made linear functions of the prices of output originating.

The present model defines normal unit labor costs as a four-quarter average of wage rates divided by normal productivity (the above output per man-hour average). Following Eckstein and Fromm, wholesale prices then were made dependent on normal unit labor costs, actual from normal unit labor cost deviations, capacity utilization, and the prices of materials inputs from other sectors.[4]

The price of output originating for durables manufacturing then was related (in a linear equation) to the wholesale price index for this sector and input materials prices (the latter has a negative sign). Output originating prices for nondurables manufacturing were better predicted by using a function similar to that for its wholesale prices (without the raw materials term) than by relating them to wholesale prices. (The marginally significant average weekly hours term—ceteris paribus, an inefficiency indicator—probably should be deleted from the equation.)

With the exception that normal unit labor costs have been slightly redefined, equation specifications for the prices of output originating for the remaining sectors—trade, contract construction, regulated, and other—are identical to those used previously. The primary explanatory variables are normal unit labor costs and actual from normal unit labor cost deviations. For the trade sector, a ratio of an inventory-stock-to-output variable, see above, is included. For the regulated sector, normal (and the deviation of actual from normal) unit capital consumption allowances are important additional determinants of prices.

[4] O. Eckstein and G. Fromm, "The Price Equation," *American Economic Review*, Vol. 58 (December 1968), pp. 1159–83.

WAGE RATES

The four-quarter percentage change in wage rates previously had been explained by four-quarter percentage changes in the consumer price index and profits per dollar of real output, the reciprocal of an average of unemployment rates, and the dependent variable lagged four quarters. Basically, the same form still applies. The equations have been altered slightly by dropping the profits per unit of output term (which is no longer significant), by taking the reciprocal of a four-quarter, unweighted average of unemployment rates (instead of a five-quarter weighted average), and by adding, as a distributed lag adjustment, the dependent variable lagged one quarter.

Also, the sample period for the present model encompasses the guidepost era from 1962–65, when the government attempted to restrain wage and price movements by moral suasion. Inclusion of guidepost dummy variables in the equations yielded significant negative coefficients for the durables and nondurables manufacturing, regulated, and residual sectors.

FINAL DEMAND AND PRICE CONVERSION

The format for relating the final demands and outputs of industry to *GNP* component demands, and *GNP* component prices to industry prices, was originally presented in the first volume on the model.[5] There, as in the Fromm-Taubman solutions, coefficients in equations relating final demands to *GNP* expenditures were constrained to sum to unity. In further analysis, it was hypothesized that changing mixes within the industry and expenditure aggregates would vitiate the homogeneity constraints.[6] The latter approach has been applied in the present model.

Real final demands by industry are related to real *GNP* com-

[5] F. M. Fisher, L. R. Klein, and Y. Shinkai, "Price and Output Aggregation in the Brookings Econometric Model," in *The Brookings Quarterly Econometric Model of the United States*, J. S. Duesenberry, G. Fromm, L. R. Klein, and E. Kuh, eds. (Rand McNally–North Holland, 1965), pp. 652–79.

[6] G. Fromm and L. R. Klein, "Solutions of the Complete System" in *The Brookings Model: Some Further Results*, J. S. Duesenberry, G. Fromm, L. R. Klein, and E. Kuh, eds. (Rand McNally–North-Holland, 1968), pp. 382–408.

ponent expenditures, which correspond most closely to the output of the industry. For example, for durables manufacturing, the components, in real terms, are inventory change of durables and trade, exports, consumption of durables, producers' durables equipment expenditures, and government purchases of durables. Dummy variables are added in selected periods to account for dock strikes and other unusual phenomena that are imperfectly reflected in the explanatory variables. For sectors other than durables manufacturing, auto-regressive transformations are used to eliminate strong serial correlation of residuals.

As in the past, industry gross product originating is predicted using the input-output relationship:

$$X_t^{58} = D_t^{-1}(I - A)^{-1}F_t^{58}$$

where

X^{58} = a vector of industry gross product originating in 1958 dollars
D^{-1} = the inverse of a diagonal matrix of the ratio of real gross output to real gross product originating for each industry
A = a fixed coefficient input-output matrix
F^{58} = a vector of industry final demands in 1958 dollars.

Previously, the 1947 input-output matrix had been used for complete model solutions; the 1958 table is employed for the current runs. The D matrix bears a time subscript because the ratio of gross output to gross product originating in the case of two industries, agriculture and contract construction, has been shifting over the sample period.

In previous solutions, auto-regressive transformations were applied to the output calculations as the next step. These were not done here because final demands are now corrected for serial correlation of residuals prior to the input-output conversion. Also, previously, the sum of real industry gross product originating was constrained to equal real gross national product. A trial calculation indicated that this discrepancy was small (on the order of one to two billion dollars) and the constraint was not imposed. However, it will be applied in future solutions and, if past experience is a guide, should result in improved accuracy.

The conversion of industry prices into GNP component prices has been refined for the present solutions. Previously, GNP prices were related directly to industry prices of gross product originating using the same combined matrix as for the conversion of real GNP expenditures into industry gross product originating. Now, industry prices of gross product originating (PX) are first transformed into prices of industry final demands (PF) using

$$PF = (I - A)^{-1}D^{-1}PX$$

Then the prices of GNP components (such as the implicit price deflator for personal consumption expenditures on durables automobiles, P_{CDA}) are derived from regressions of these prices on the prices of the relevant industry final demands. For example, P_{CDA} is a function of PF in the durables manufacturing and trade sectors. As previously, autoregressive transformations are needed for some GNP component prices to correct for serial correlation of residuals.

An expenditure weighted combination of these prices yields the over-all implicit deflator for total personal consumption expenditures, P_C. The consumer price index, which appears in the wage rate equations, is then predicted as a linear function of P_C.

FINANCIAL SECTOR

The specification of the financial sector retains the essential features and structure of de Leeuw's condensed simultaneous submodel, which was employed in previous solutions.[7] There have been a few changes.

First, equations for both currency and demand deposits and for demand deposits alone appear in the model. This makes currency an endogenous rather than exogenous variable (unborrowed reserves remain the principal Federal Reserve exogenous policy instrument). In each of these equations, the lagged disposable income and business investment variables have been dropped as explanatory variables. Those that remain are time deposit yields, government bill rates, disposable income, and the beginning of quarter level of the dependent variable. The functions are homogeneous in wealth, which now is de-

[7] F. de Leeuw, "A Condensed Model of Financial Behavior," *ibid.*, pp. 270–316.

fined as the sum of a twenty-quarter exponentially distributed lag of real *GNP* multiplied by the current price of *GNP*. (Previously, the distributed lag was taken on current dollar *GNP*.)

The time deposits equation also remains homogeneous in wealth with the primary explanatory variables being time deposit yields, Treasury bill rates, and the prior level of deposits. An alternative first-difference type specification shows a weak influence of disposable income. However, the *t*-statistic of this variable was only slightly greater than unity, so the simpler version of the equation was used for the present model solutions.

In de Leeuw's original formulation, banks' demand for free reserves as a percentage of demand and time deposits was made a function of government bill rates, the Federal Reserve discount rate, and short-run percentage changes in deposits less required reserves. For the present model, the equation was renormalized and the bill rate was made the dependent variable. The discount rate and the level and changes in free reserves relative to deposits became the principal explanatory factors of bill rate changes; government deficits (which have a negative sign) as a percentage of wealth also are found to have an effect and, other things being equal, exert upward pressure on short-term money rates.

The term structure equation relating bill rates and long-term bond yields also has been modified along lines suggested by Modigliani and Sutch.[8] The bond yield now is a function of the level and change in bill rates and the prior level of bond yields.

The yield paid on time deposits equation has been altered, too. It is assumed that quarterly changes in the time deposit rate, RM_{BDT}, cover a fraction of the gap between desired and actual rates subject to the Federal Reserve's Regulation Q ceiling limit (RM_{BDTM}):

$$\Delta RM_{BDT} = \beta_0 (RM^*_{BDT} - RM_{BDT_{-1}}) + \beta_1 RM_{BDTM}$$

The desired rate RM^*_{BDT} is obtained by maximizing banks' profits on deposits (net of reserve requirements with government bills as the mar-

[8] F. Modigliani and R. Sutch, "Innovations in Interest Rate Policy," *American Economic Review*, Vol. 56 (May 1966), pp. 178–97.

ginal investment outlet) less the yield paid on time deposits. Finally, a dummy variable is included to reflect the issuance of certificates of deposit (CD's) in the early 1960's.

NONWAGE INCOME

There has been no change in specification of the nonwage income equations. Government interest payments are a function of levels and changes in government bill rates and the amount of public debt. Dividend payments, following the work of Lintner and Brittain, depend on profits after taxes and capital consumption allowances and lagged dividends. Entrepreneurial income is related to labor compensation and corporate profits in the private sector other than agriculture. Net interest paid by consumers is dependent on a moving average of bond yields multiplied by the sum of personal consumption expenditures on durables and nonfarm residual construction.

CAPITAL CONSUMPTION ALLOWANCES

These equations are identical in form to those in the previous model. Quarterly changes in capital consumption allowances depend on recent investment and on prior allowances and various dummy variables to account for the transitions between the use of different depreciation methods.

LABOR FORCE

The equation for estimating the number of persons in the civilian labor force remains unchanged. The explanatory elements are the number of persons employed, last quarter's level and change in unemployment, and a time trend to approximate the trend toward higher labor force participation rates of working age women.

IDENTITIES, FIXED PROPORTIONS, AND MISCELLANEOUS RELATIONSHIPS

Most of these equations comprise the aggregation of the national income and product accounts and have not been modified. A few changes have been occasioned by redefinition of data or better approximations to splits in an aggregate quantity.

For example, previously, real producers' durables equipment investment, I^{58}_{PDE}, was taken as a fixed proportion of real business investment. Now the split is made a function of time and dummy variables to reflect the impact of the investment tax credit on desired proportions of plant to equipment.

Several additions have been made for the purpose of defining potential *GNP* and both capacity output and capacity utilization in manufacturing. Potential real *GNP* and real capacity output is estimated to have grown by 3.5 per cent per year from the third quarter of 1953 through 1965 and by 4 per cent thereafter. The model uses the Wharton capacity utilization rates for durables and nondurables manufacturing as explanatory variables. These, in turn, are related to previous utilization rates and changes in the real actual- to capacity-output ratios.

2 TURNING POINT ANALYSIS

A DISTINCTIVE feature of the Brookings Model, among the others being considered at this Conference, is its size and detail. The model generates dynamic solutions of 230 variables every quarter. There is a rich analysis waiting to be undertaken in the study of many individual variables or groups of variables. Nevertheless, in the light of the Conference format, the principal analysis is conducted in terms of solutions for seventeen standardized variables. (See Table 1.)

The sample period for parameter estimation begins after the Korean War; therefore, the first cycle analyzed is the 1953–54 recession. The 1957–58, and 1960–61 recessions are then considered. Given the need for lagged values, the first solution is for the trough of the recession in the third quarter of 1954. The first of the successive six-quarter simulations begins in 1954, in the first quarter or two quarters before the trough. The other turning points have simulations beginning one, two, and three quarters before the peak or trough.

A way of evaluating performance at peaks and troughs is to compare root mean square (RMS) errors at the turning point, near the turning point, or over the whole six-quarter span with root mean square errors from the whole sample period (which includes steady growth

TABLE 1

Long-Run Simulation Errors, 1957–65
(billions of dollars or billions of 1958 dollars unless otherwise indicated)

Variable	Root Mean Square Error
Short-term Treasury bill rate (RM_{GBS3}), per cent	0.226
Long-term Treasury bond yield (RM_{GBL}), per cent	0.177
Real nonagricultural gross capital formation ($I^{58}_{BUS_{EAF}}$)	0.946
GNP delator (P_{GNP}), index, 1958 equals 1.0	0.0072
Unfilled orders (O^{58}_{U})	4.155
Real nonfarm residential construction expenditures (I^{58}_{CNFR})	1.006
Personal income (YP)	3.384
Corporate profits before tax (ZB)	3.650
GNP in current dollars (GNP)	5.841
GNP in 1958 dollars (GNP^{58})	4.998
Unemployment rate (RU), proportion	0.00622
Consumer expenditures in 1958 dollars (C^{58})	2.429
Nonfarm inventory investment in 1958 dollars (INV^{58}_{EAF})	2.653
Net foreign balance (B)	1.350
Employment (EHH), millions of persons	0.534
Hours worked per week (H), hours	0.150
Money wage rate ($RWSS$), dollars per hour	0.0397

NOTE: These are quite favorable results. GNP solutions with root mean square errors of approximately \$5.0 billion are good for ex post solutions. Interest rates are estimated with root mean square errors of approximately 20 basis points. Price level errors are approximately 3/4 of a point movement in the index based on 100. Similarly, the error in the unemployment rate, RU, is about 2/3 of a point in the third decimal place when the rate is expressed as a per cent.

phases between the lower and the succeeding upper turning points). The longer dynamic simulation from given initial values begins in the first quarter of 1957 and runs through the fourth quarter of 1965. Given the small size of the variable, the root mean square error of real inventory investment is large. Similarly, corporate profits before taxes and the net foreign balance are estimated subject to a fair size error.

Simulations of eleven variables around the five sample-period turning points are depicted in Charts 1–11. In general, the pattern of predicted values follows actual experience reasonably well. However, solutions tend to run within the actual cycles, understating peak and overstating trough values. Furthermore, predicted turning points often occur either too early or too late. These phenomena have been found in other studies.[9] In the following discussion of the individual turning points, emphasis is placed primarily on the accuracy of *GNP* and real *GNP* predictions. The reader is encouraged, however, to examine all the charts in conjunction with the text discussion.

THE 1954 TROUGH

Most variables show a substantial negative residual (actual minus computed) at the trough, the third quarter of 1954. In some cases the negative residual is one quarter on either side of the trough. In the case of the unemployment rate, the small negative residual at the trough does not indicate underestimation of amplitude; the residual is much smaller than the root mean square value for *RU* in Table 1. For the other variables, the root mean square values are both larger and smaller than the reference values; but the residuals almost all have the same sign, supporting the view that when the economy makes a large movement in either direction, model solutions vary with lower amplitude. The nonstochastic solution is often a smooth series compared to the actual data and cuts off extreme peak and trough fluctuations.

On the whole, performance is good at this turning point. The errors in the solution are frequently lower than those in Table 1. Several

[9] George R. Green in association with Maurice Liebenberg and Albert A. Hirsch, "Short- and Long-Term Simulations with the OBE Econometric Model," this volume, pp. 25–123.

CHART 1

GNP^{58}

Gross National Product in 1958 Dollars

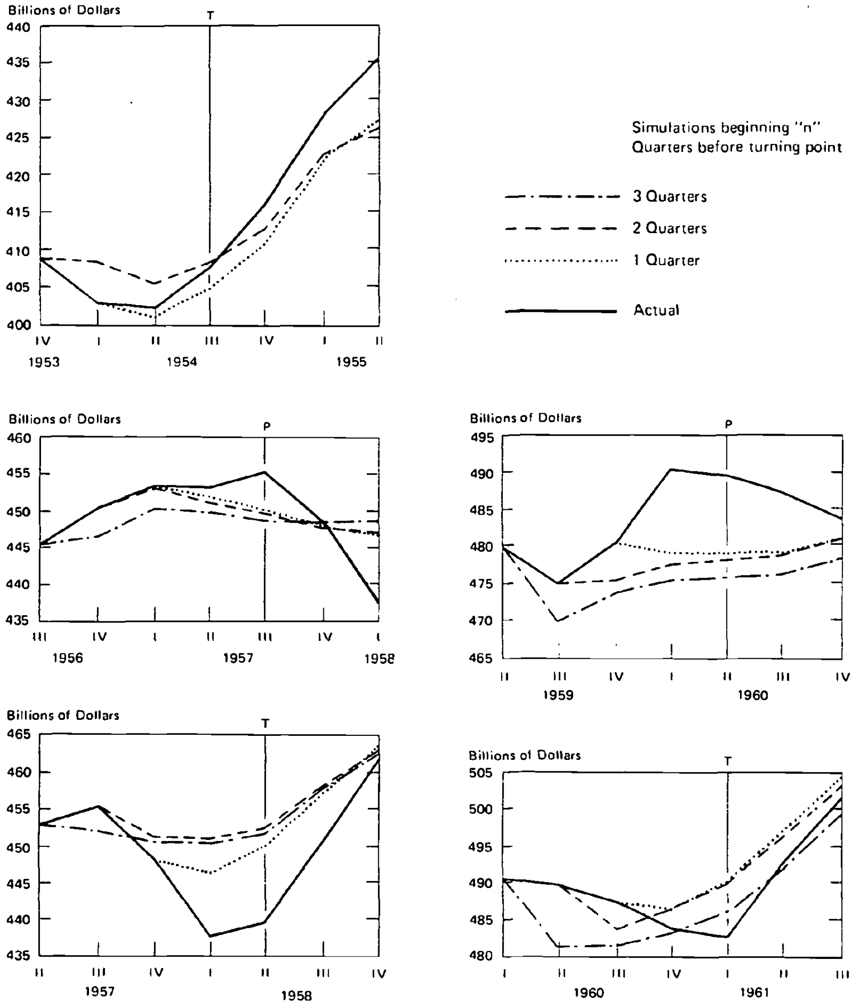


CHART 2

P_{GNP}

Implicit Price Deflator for Gross National Product

(1958 = 100)

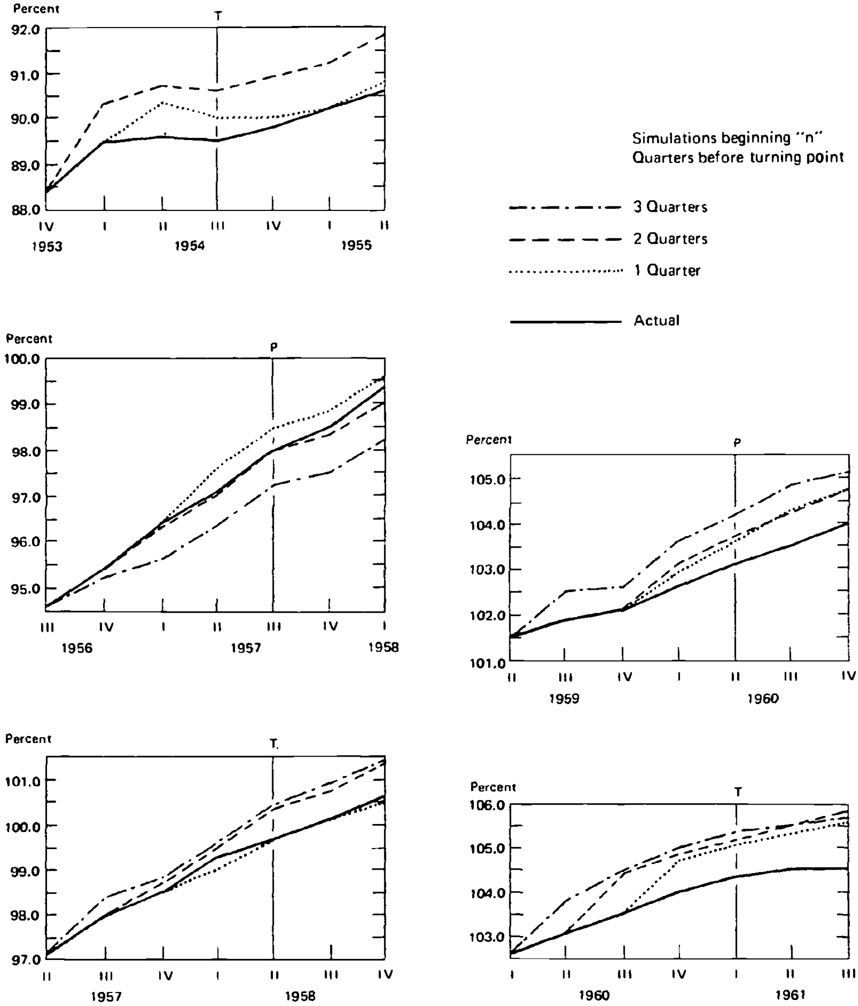


CHART 3

$I^{58}_{BUS_{EAF}}$

Nonagricultural Business Gross Investment

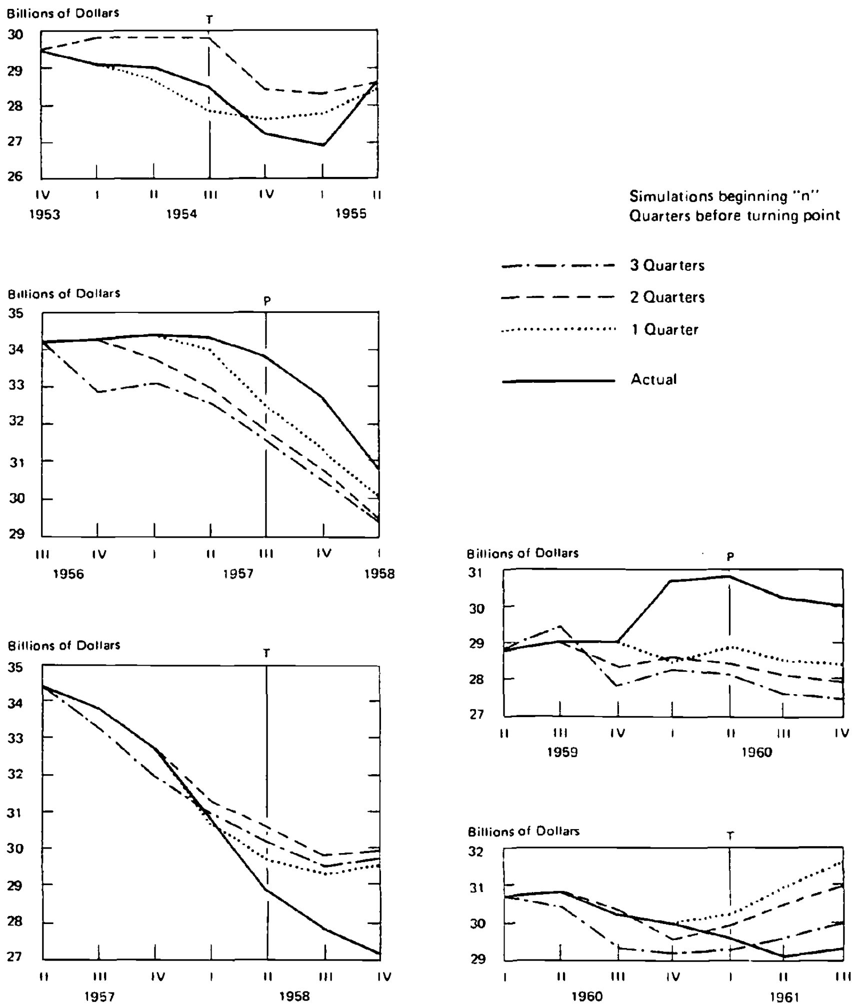


CHART 4

I^{58}_{CNFR}

New Private Nonfarm Residential Construction in 1958 Dollars

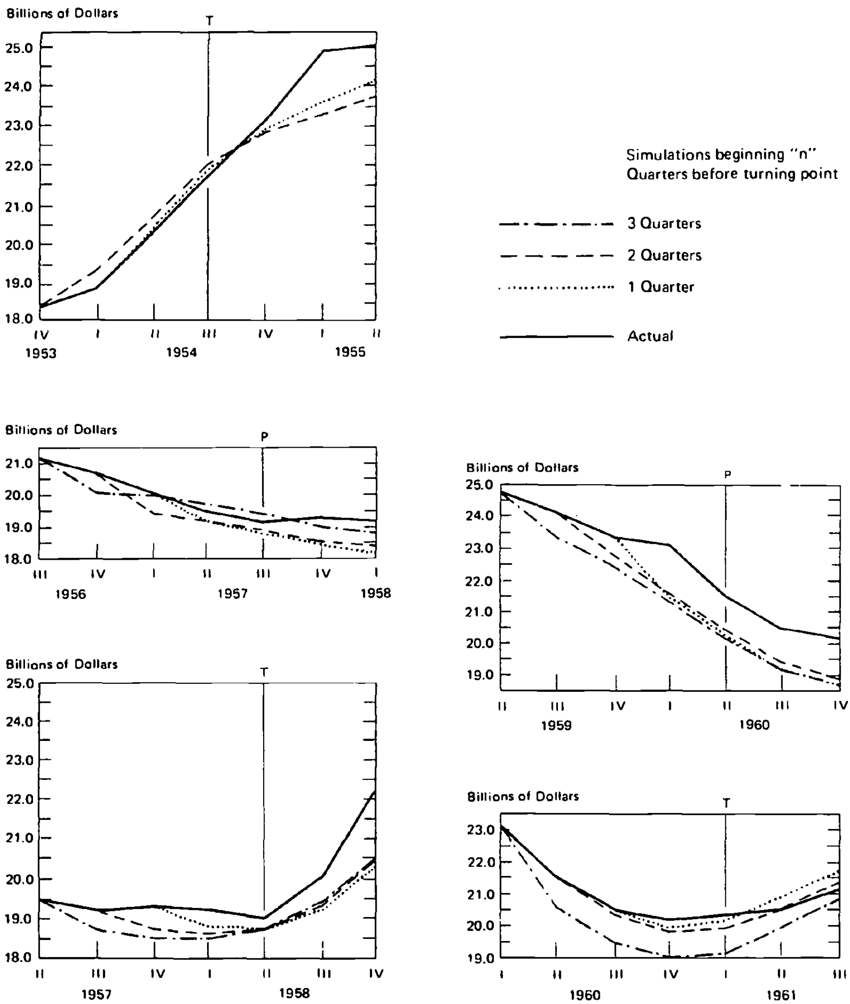


CHART 5

ΔINV^{58}_{EAF}

Nonfarm Inventory Investment in 1958 Dollars

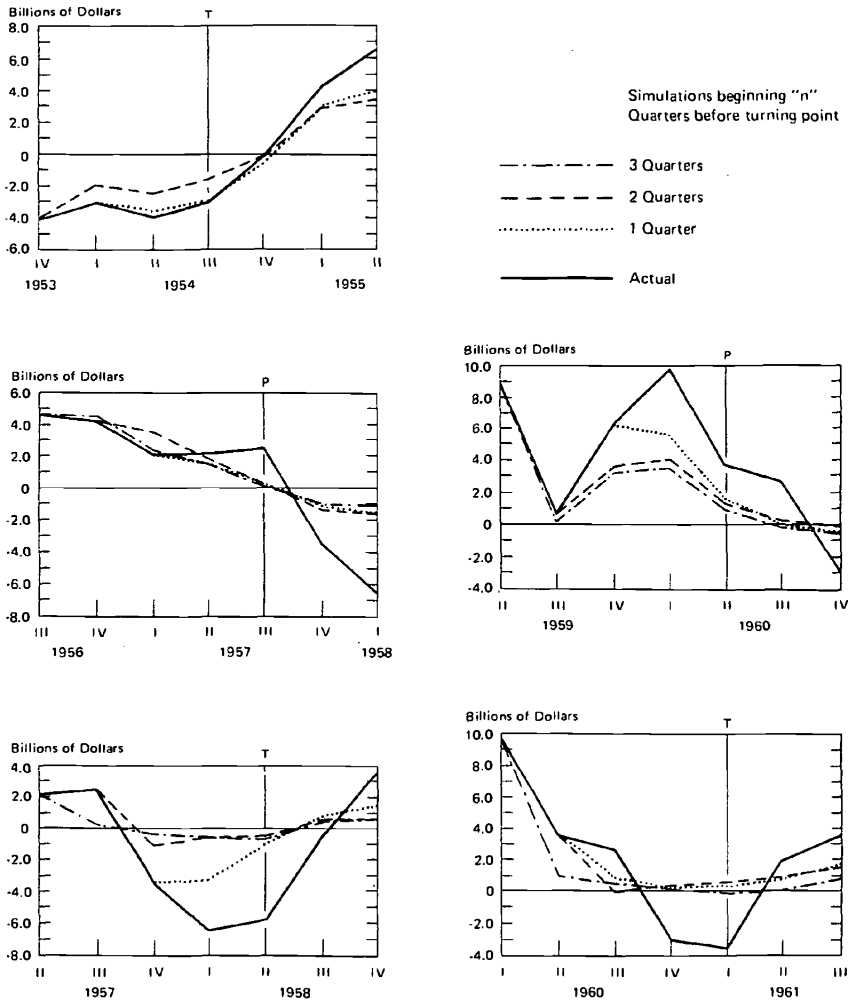


CHART 6

C^{58}

Personal Consumption Expenditures in 1958 Dollars

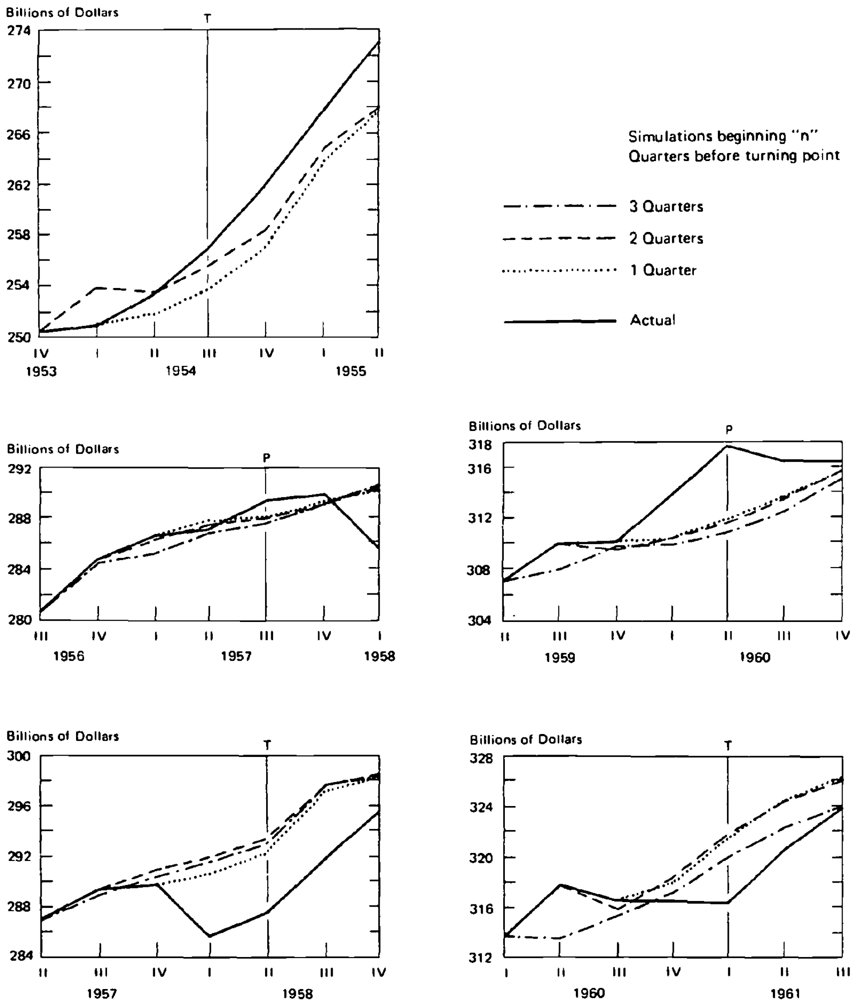


CHART 7

RU
Unemployment Rate

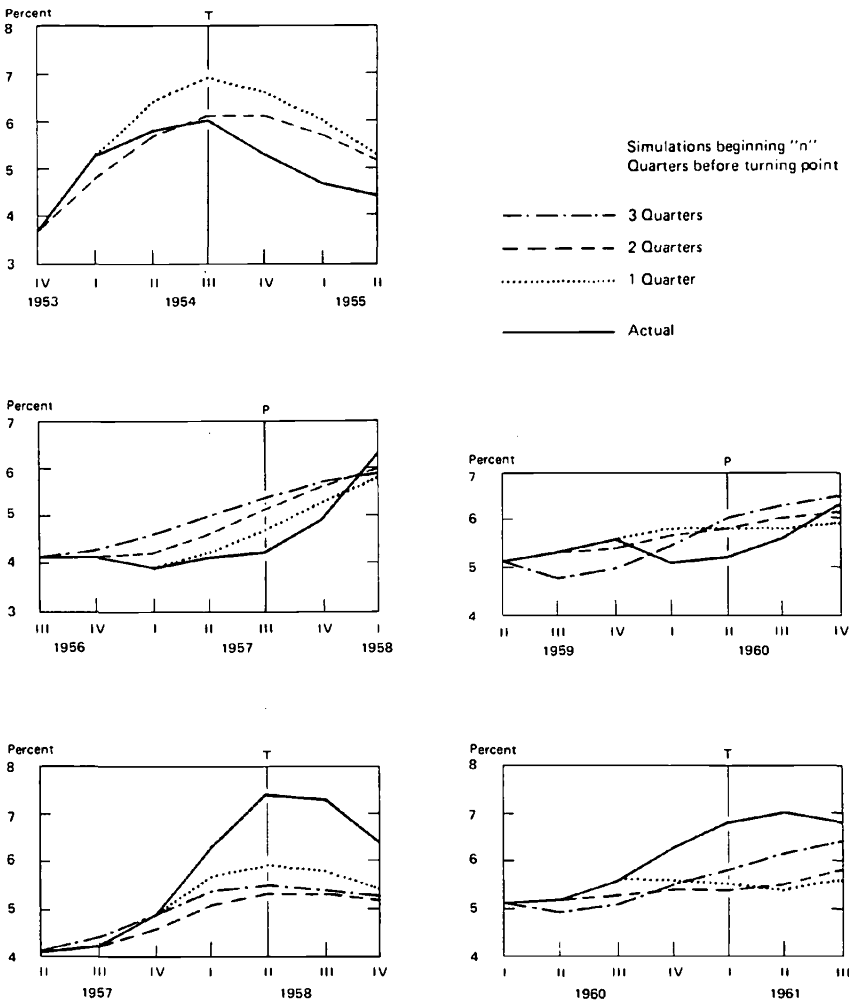


CHART 8

EHH
Civilian Employment

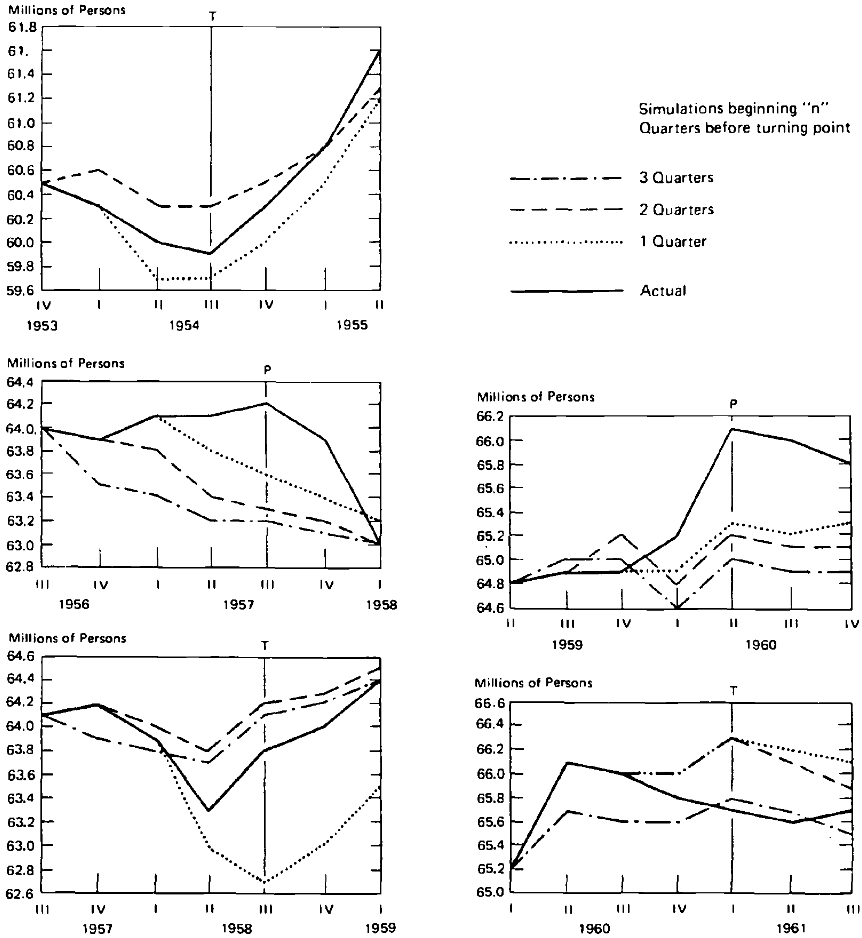


CHART 9

RM_{GBS3}
Short-Term Treasury Bill Rate

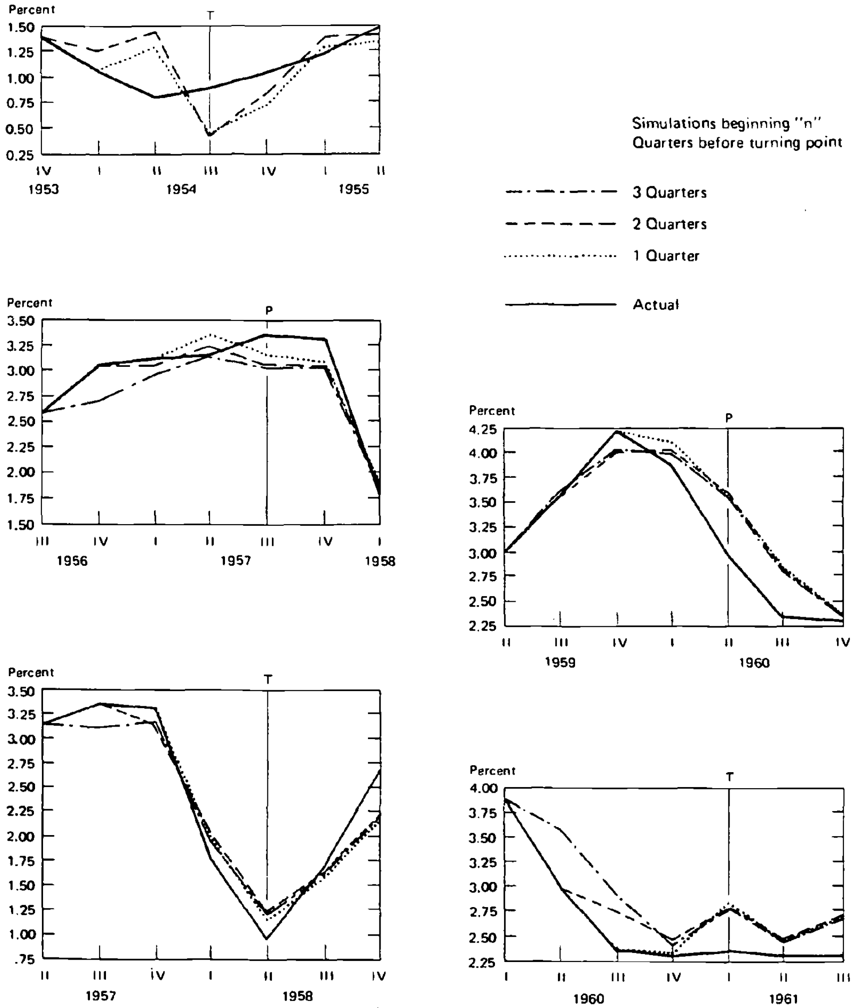


CHART 10

RWSS

Hourly Wage Rate Including Supplements

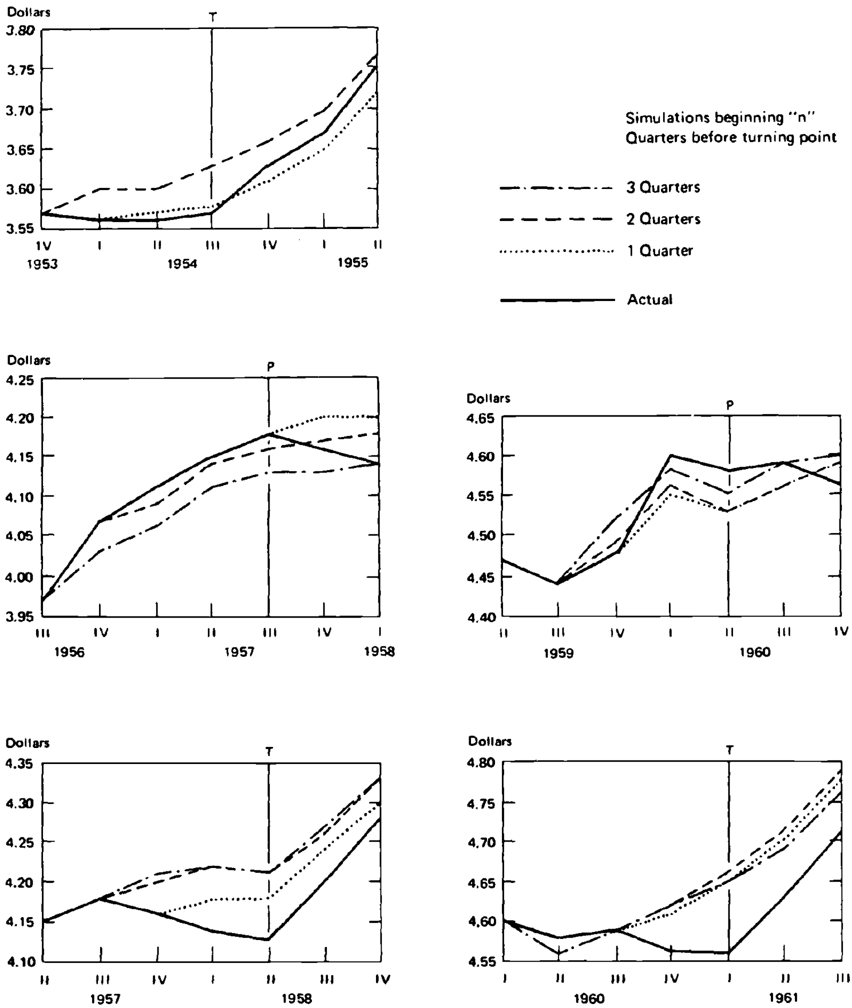


CHART 11

Z_B

Corporate Profits Before Taxes Including Inventory Valuation Adjustment

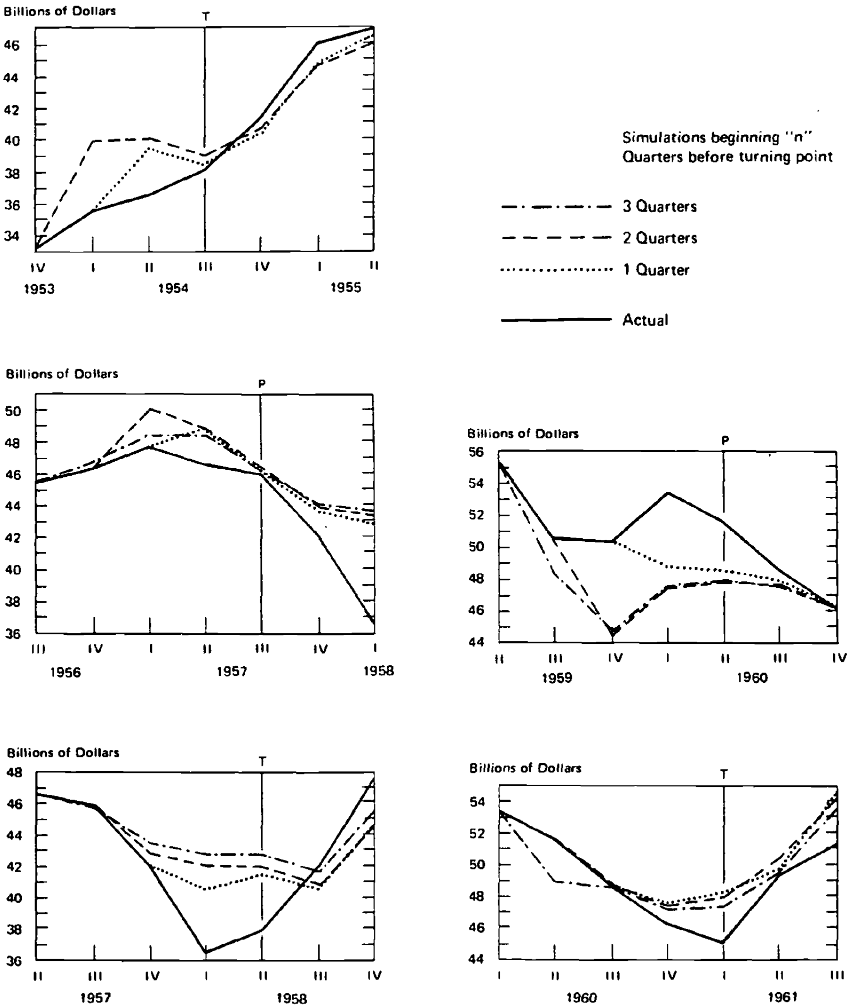


TABLE 2

1954 Recovery: Root Mean Square Errors, Trough (billions of dollars)

	Starting from 1954:1		Starting from 1954:2	
	Six Quarters	1954:3	Six Quarters	1954:3
GNP	4.958	−5.058	4.239	0.615
GNP58	5.184	−0.738	5.079	2.639

turning point errors improve when we start up the solution one instead of two quarters before the trough. (See Table 2.)

1957–58 RECESSION

In this case the peak is in the third quarter of 1957 and the trough is in the second quarter of 1958. At the peak, the performance appears to be good. Many cyclical variables have the correct quarter-to-quarter movements, and the root mean square errors are either smaller or not significantly larger than the sample average values either at the peak point or over an entire six-quarter solution period. There is a tendency, however, for underestimation to occur at the peak.

Usually, models such as the present one do better at business cycle troughs than at peaks. (See Table 3.) This does not seem to be the case for the Brookings Model in the 1957–58 recession. Corresponding to the comparatively good performance at the peak, the root mean square errors are mostly larger for the trough calculations, both at the trough point and for the whole six-quarter simulation period.

With the two *GNP* series, there is consistent underestimation at peaks and overestimation at troughs. The timing and change-of-sign correspondence was good at the troughs in spite of large errors.

THE 1960–61 RECESSION

One of the mildest sets of actual turning points of the economy is the 1960–61 recession. The movement is so slight that it is more difficult to predict this recession than other postwar downturns. At both the peak and trough, there are discrepancies in the movement of principal series. The inability to deal with 1960–61 does not necessarily mean that the model has serious defects; fluctuations in narrow ranges are difficult to project. Some variables, however, perform well in this set of solutions (1960–61). The *GNP* series generally misses the turning point, a defect that needs to be corrected. (See Table 4.)

Generally speaking, there are large errors in comparison with root mean square values for the whole sample period. This is true for both measures of *GNP* at peaks. Behavior near the trough is better. The root mean square errors for GNP^{58} in six quarters covering the trough are low and the error for trough quarters are not excessive. At the peak quarters, observations exceed computed values of *GNP* and GNP^{58}. This again is a failure to reach the complete range of the observed amplitude. At troughs, the reverse result holds, and the model fails to fall enough, leaving negative errors.

TABLE 3

1957–58 Recession—Recovery: Root Mean Square Errors (billions of dollars)

	Peak Starting from 1956:4		Peak Starting from 1957:1		Peak Starting from 1957:2	
	Six Quarters	1957:3	Six Quarters	1957:3	Six Quarters	1957:3
GNP	6.180	8.904	5.582	5.573	6.993	2.781
*GNP*58	5.488	5.339	5.850	5.405	5.808	5.092
	Trough Starting from 1957:3		Trough Starting from 1957:4		Trough Starting from 1958:1	
	Six Quarters	1958:2	Six Quarters	1958:2	Six Quarters	1958:2
GNP	9.741	−15.213	9.635	−15.266	6.423	−10.374
*GNP*58	7.928	−12.100	8.235	−12.750	7.025	−10.544

TABLE 4

1960–61 Recession—Recovery: Root Mean Square Errors (billions of dollars)

	Peak Starting from 1959:3		Peak Starting from 1959:4		Peak Starting from 1960:1	
	Six Quarters	1960:2	Six Quarters	1960:2	Six Quarters	1960:2
GNP	6.517	9.339	6.661	9.144	5.838	8.465
*GNP*58	10.356	14.118	8.334	11.801	7.433	10.939
	Trough Starting from 1960:2		**Trough Starting from 1960:3**		**Trough Starting from 1960:4**	
	Six Quarters	1961:1	Six Quarters	1961:1	Six Quarters	1961:1
GNP	4.929	−8.538	7.472	−11.911	8.364	−11.245
*GNP*58	4.467	−3.557	3.850	−7.289	3.877	−7.407

3 LONGER-RUN SIMULATIONS

THERE are two basic ways of generating long-run solutions of the model. One approach is limited to the sample period, with possibly a few post-sample-period observations added. The other approach extrapolates the series entirely beyond the sample. There is no limit, in principle, to the time duration of simulations outside the sample period. First, sample period simulations for thirty-six quarters are examined, starting in the first quarter of 1957 and ending with the fourth quarter of 1965.

The general impression of these thirty-six-quarter dynamic simulations is that computed values track the course of observed values rather closely, especially at the beginning and end of the period. This point is supported by the relatively small residuals shown in Table 1 and Charts 12–15.

CHART 12

GNP^{58}

Gross National Product in 1958 Dollars

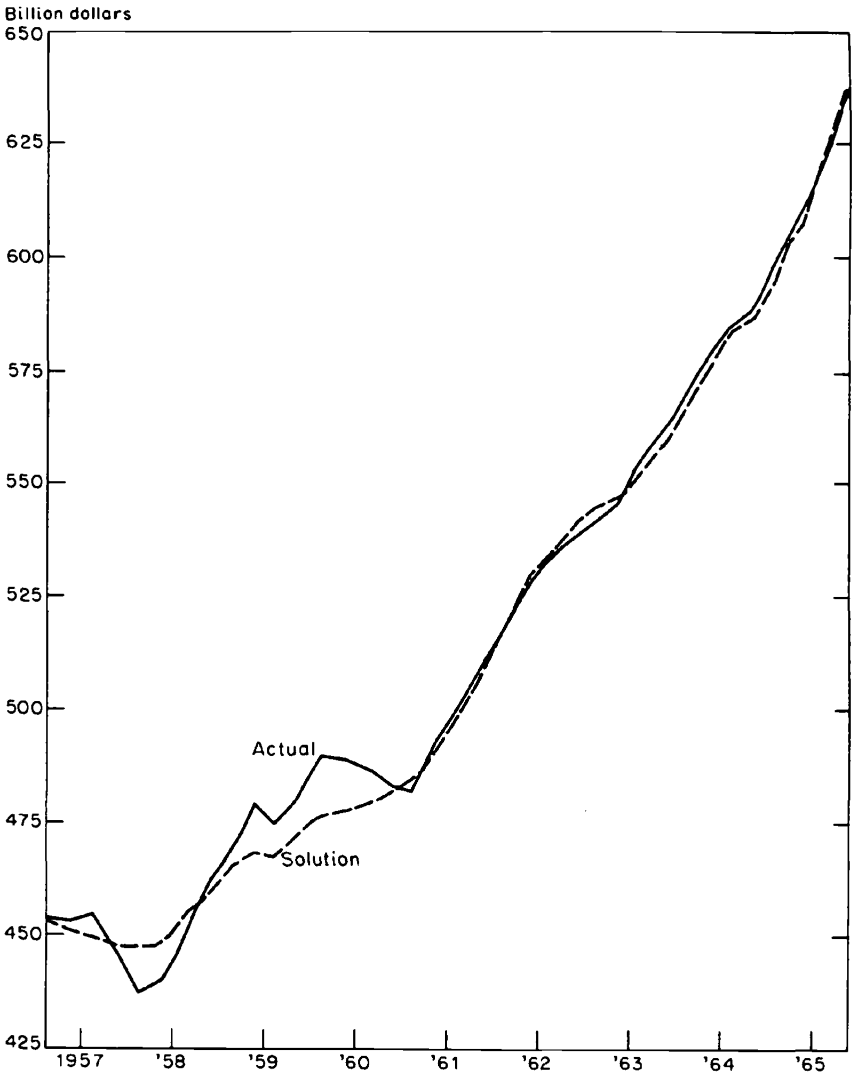


CHART 13

P_{GNP}

Implicit Price Deflator for Gross National Product

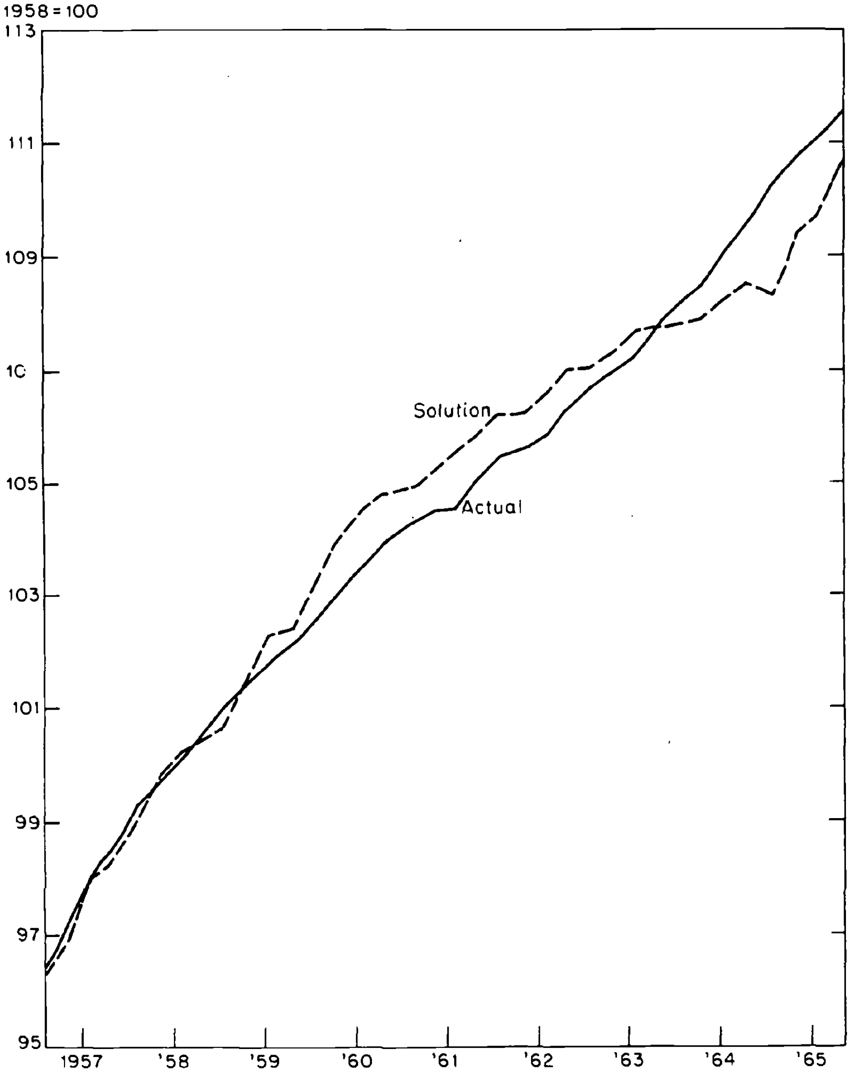


CHART 14

$I^{58}_{BUS_{EAF}}$

Nonagricultural Business Gross Investment

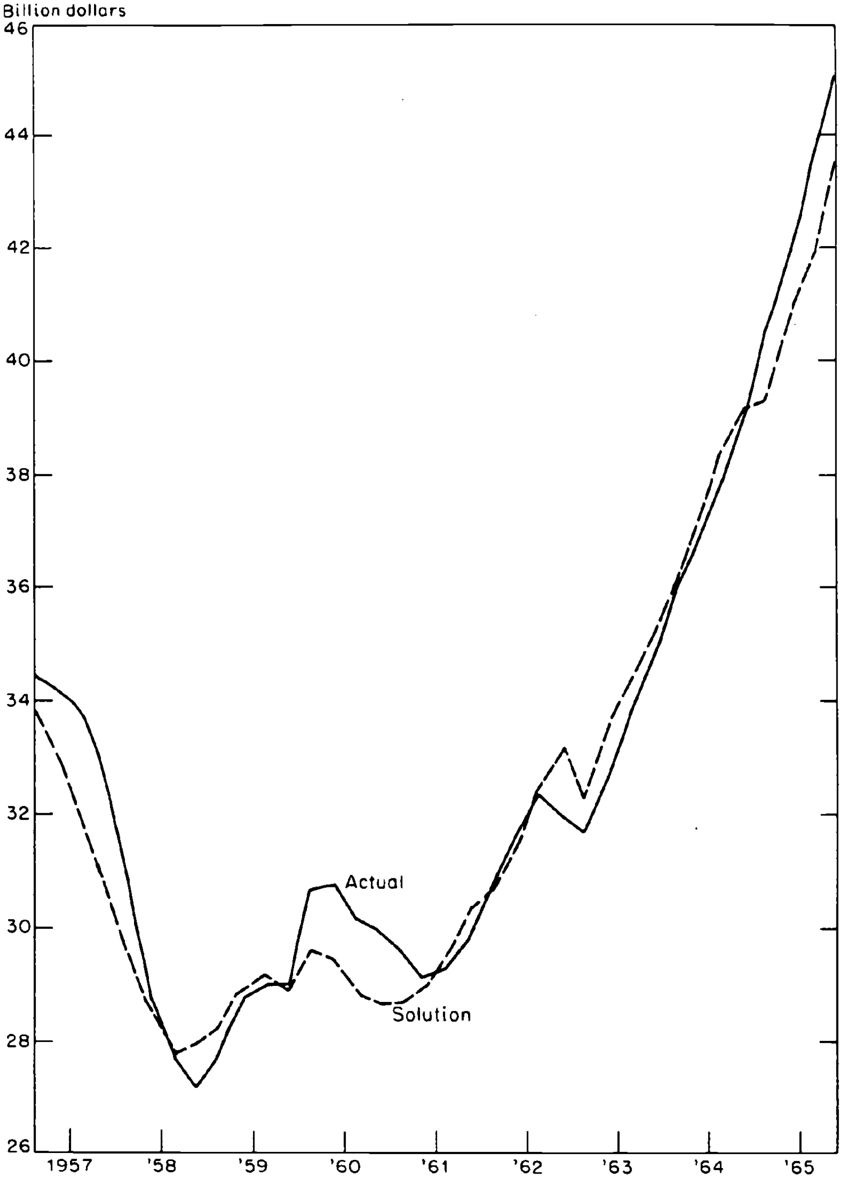
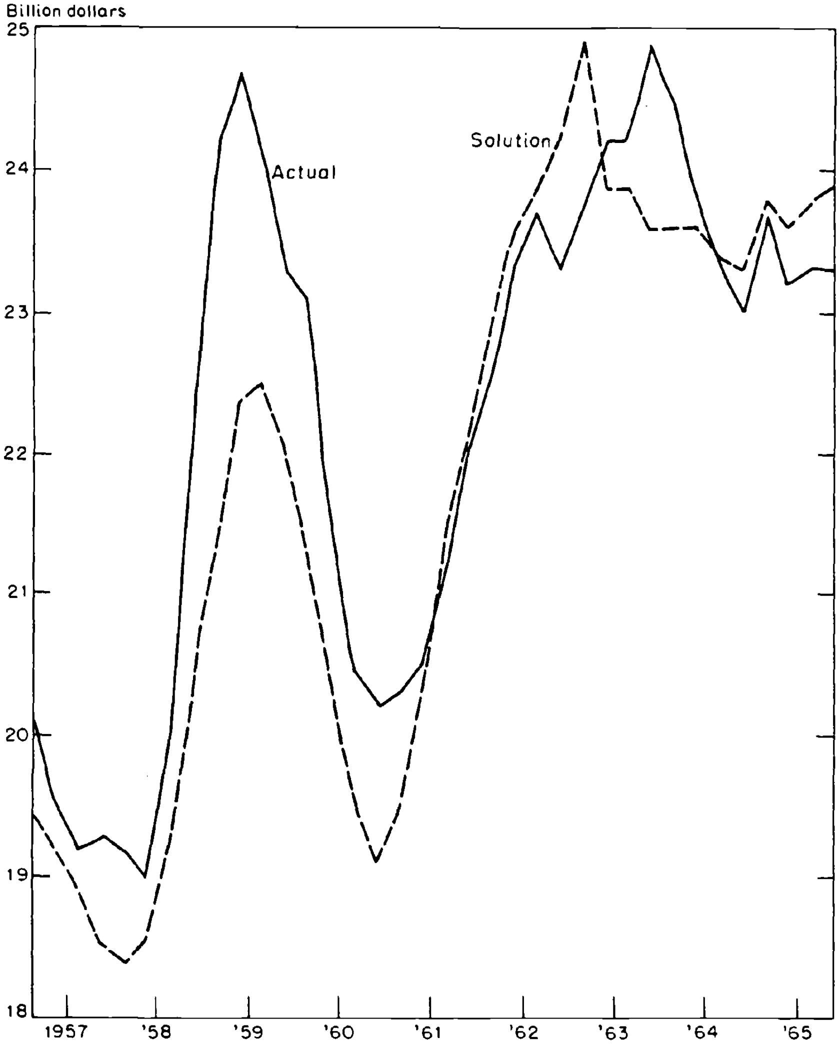


CHART 15

I^{58}_{CNFR}

New Private Nonfarm Residential Construction in 1958 Dollars



Some larger errors occur in the middle ranges of the simulation. The model picks up the trend component very well. Cyclical behavior is mixed. The solution for real *GNP*, however, does follow the 1957–58 period, but it fails to decline in 1960–61. Current dollar *GNP*, reflecting steady price inflation, has only one quarterly decline in the whole nine-year period. It misses the peak and trough points.

The quantitative nature of the solution, however, is good. Among the seventeen simulated variables, the quarterly sign change is correct in most instances. The percentage of correct signs estimated for quarterly changes exceeds 75 per cent in all except four cases.

Some strong trend variables and some highly cyclical variables are displayed in Charts 12–15. Real *GNP* and the *GNP* deflator both have the appropriate trend growth corresponding to the actual data. In addition, real *GNP* has some of the appropriate cyclical content. Price projections are one or two points low at the end of the calculation, but this is not a large error. In the case of the capital formation series—both business investment and residential construction—the main cyclical swings are well delineated by the computed values. Conformity is acceptable not only at the reference cycle peaks and troughs, but also at the specific cycle peaks and troughs. In 1959–60, the amplitudes of the peak and trough in residential construction are underestimated, but the timing is approximately correct.

In comparison with the previous version of the Brookings Model, these results represent improvement. Earlier, there were biases that gave rise to a regular discrepancy between estimated and actual *GNP*. Also, the older model implied more price inflation than actually occurred in the early part of the 1960's. The absolute errors in the present simulation, though for a different time span, are generally one-third to one-half of those reported by Nagar in his study of stochastic and non-stochastic simulations over the sample period.[10]

Two methodological points are highlighted by these results: (1) the estimates of the model at the present time have been computed only by ordinary least squares (*OLS*); (2) some of the estimated structural equations have serially correlated errors.

Although the *OLS* estimates appear to be functioning very well in

[10] A. L. Nagar, "Stochastic Simulation of the Brookings Model" in *The Brookings Model: Some Further Results*, *op. cit.*, pp. 423–56.

these exercises on complete system performance, other studies have found that least squares bias sometimes appears to be more significant in complete system solutions than in separate equations, and that biases tend to accumulate as simulations are conducted over long time periods.[11] Consistent estimates of the new Brookings Model will be forthcoming.

In the paper on the OBE Model, and also on the Wharton Model, it was found that auto-regressive transformations of individual equations (to take account of the presence of serial correlation in calculated residuals) brought modest improvement to sample period simulations. That technique was not used for solution of the Brookings Model, but some of the equations were originally estimated with second-order auto-regressive corrections. For a relationship

$$y_t = \alpha_0 + \alpha_1 x_t + e_t$$

$$e_t = \rho_1 e_{t-1} + \rho_2 e_{t-2} + u_t$$

the estimated form was

$$(y_t - r_1 y_{t-1} - r_2 y_{t-2}) = a_0(1 - r_1 - r_2) + a_1(x_t - r_1 x_{t-1} - r_2 x_{t-2})$$

$$a_i = \text{est } \alpha_i, \qquad r_i = \text{est } \rho_i$$

These estimates were made for investment functions, where it was found that second-order corrections were needed in order to eliminate serial correlation of residuals. Although the estimates of α_0 and α_1 should be more efficient by this correction, the transformed equations gave very poor results in the complete system simulations. The alternative of solving the system using

$$y_t = a_0 + a_1 x_t$$

admitting serial correlations in residuals was not undertaken. Instead,

$$y_t = a_0^* + a_1^* x_t$$

was re-estimated in the presence of serially correlated errors. Although a_i^* should be less efficient than a_i, the equations with a_i^* coefficients gave better system solution results than the transformed set using a_i.

[11] L. R. Klein, "The Estimation of Interdependent Systems in Macroeconometrics," *Econometrica* (April, 1969), pp. 171–192.

The transformed equation uses more lagged values, and probably goes astray through error build-up in the process of dynamic solution. At present, we are experimenting with other estimates of investment equations using a combination of new lag distributions and first-order autoregressive transformations to see if they perform better in simulations.

It should be pointed out that the complete system dynamic simulations all have serially correlated residuals (between computed and actual values). This serial correlation is part of the complete solution of the finite difference equation system and arises through the solution process.

4 TWENTY-FIVE-YEAR SIMULATIONS

THE Brookings Model is essentially a short-run forecasting model and as such is not designed for simulations over a twenty-five year period. Therefore, certain adjustments, discussed below, were necessary in order to produce a reasonable control solution. The control solution is a hypothetical growth path over the period from the first quarter of 1966 through the fourth quarter of 1990. However, actual values of some of the exogenous variables were used for the first three and one-half years.[12] Also, government spending and employment were adjusted to reflect the possible slowdown of the Vietnam War during 1970.

Shocked simulations were produced by introducing random additive disturbances to the stochastic equations of the model. These disturbances are selected so that they have the same asymptotic variance-covariance properties, of all orders, as the original single equation residuals. The procedure used to generate these shocks is discussed in the paper by Michael D. McCarthy.[13] A slight change from McCarthy's procedure was necessary because residuals from the equation estimates were not available. The residuals were computed over the

[12] The variables set to exact values were G, G^{58}, WSS_G, WS_G, E_G, DOD_{MPCA}, WSS_A, and E_A. The components of G^{58} were adjusted to add up to G^{58}.

[13] M. D. McCarthy, "Some Notes on the Generation of Pseudo Structural Errors for Use in Stochastic Simulation Studies," Appendix to "Short-Run Prediction and Long-Run Simulation of the Wharton Model," this volume, pp. 185–191.

sample period using the simulation program, which leads to different results in cases where equations are renormalized. For example, the man-hours equations were estimated in log form but the residuals for the man-hours equations as computed in the simulation program were in man-hours.

The exogenous variables for the twenty-five year simulation were generated mainly by extrapolation along their actual trends over the period from 1954 through 1965. In cases where actual data were used through the second quarter of 1969, the trend was extended from that point. The government sector exogenous variables were lowered in 1970 to simulate the end of the Vietnam War and then raised to their trend levels during 1971. The beginning and ending values along with annual percentage rates of change are shown for the principal exogenous variables in Table 5.

As is mentioned above, certain equations had to be modified in

TABLE 5

Exogenous Variables for the Twenty-five Year Simulation

	Principal Exogenous Assumptions		
	1965:4	1990:4	Annual Rate of Change
G^{58}	118.4	286.6	3.6
G	143.3	694.9	6.5
E_G	10.26	19.86	2.7
WSS_G	78.5	283.9	5.3
V_{OASIGF}	18.6	100.1	7.0
V_{VET}	5.7	13.1	3.4
V_{OG}	11.5	47.0	5.8
$IBUS_{AF}^{58}$	4.6	9.6	3.0
WSS_A	3.6	5.2	1.5
E_A	4.20	1.81	−3.3
PEX_W	1.030	1.065	0.1
EX_W^{58}	164.8	356.5	3.1
PM	1.047	1.537	1.5
INT_{BUS}	6.9	17.70	3.8
INT_{CON}	11.7	26.75	3.4
N^R	194.73	267.67	1.3

order to arrive at a reasonable growth path over the twenty-five year period. The two sets of functions where modifications were necessary were the tax, man-hours and hours equations. The modified tax functions are presented in Table 6. The personal income tax function was adjusted to approximate actual tax yields from 1966 through the second quarter of 1969. The surcharge was cut in half for the first two quarters of 1970 and then terminated. The tax rate was raised gradually through 1976 and then held constant in order to keep disposable income from growing too rapidly and also to limit government deficits.

The corporate profits tax function was adjusted so as to yield an approximately correct value given the actual data over the initial three and one-half years. The suspension of the investment tax credit in the fourth quarter of 1966 and first quarter of 1967 was accounted for by setting the tax credit dummy (DMY_{ITC}) to zero. The expected termination of the credit in the third quarter of 1970 was simulated by both a change in DMY_{ITC} and the corporate tax rate TC_{RT}. The corporate tax surcharge was cut in half in the first quarter of 1970 and terminated in the third quarter.

The indirect business tax function was adjusted, through changes in DMY_{TX}, to approximate actual excise tax collections from the first quarter of 1966 through the second quarter of 1969. The value of DMY_{TX} was held constant from that point onwards. The contributions for social insurance function was similarly adjusted.

Assumptions about old age and survivors insurance (OASI) contribution rates, and maximum individual wage and salary tax bases, and unemployment insurance contribution rates are taken from Pechman, Aaron and Taussig, and from Pechman, respectively.[14] The percentage of employees covered by OASI was raised over the period from 89 to 93 per cent.

The rate of productivity increase implicit in the production man-hours equations for the manufacturing nondurables, trade, and other sectors was too moderate, so the time trends and constant terms in these equations were appropriately adjusted. The long-term rate of productivity increase in the construction sector was also negligible,

[14] Joseph A. Pechman, Henry J. Aaron, and Michael K. Taussig, *Social Security: Perspectives for Reform* (Brookings, 1968), p. 272; and Joseph A. Pechman, *Federal Tax Policy* (Brookings, 1966), p. 251.

TABLE 6

Tax Function Assumptions
(by year and quarter, unless otherwise indicated)

I. Personal Income Tax

$$TP = -5.8DMY_{TP} + 0.14DMY_{TP}Y_P$$

(DMY_{TP})

Year	1	2	3	4
1966	0.923	0.950	0.975	1.000
1967	1.000	1.000	1.010	1.020
1968	1.020	1.030	1.120	1.140
1969	1.150	1.150	1.150	1.150
1970	1.100	1.100	1.050	1.050
1971	1.060	1.070	1.080	1.090
1972	1.100	1.110	1.120	1.130
1973	1.140	1.150	1.160	1.170
1974	1.180	1.190	1.200	1.210
1975	1.220	1.230	1.240	1.250
1976	1.250	1.250	1.250	1.250
1990	1.250	1.250	1.250	1.250

II. Corporate Profits Tax

$$TC = 0.9303 - 0.6567DMY_{ITC} + 0.8360(TC_{RT})(Z_{BU})$$

(DMY_{ITC})

Year	1	2	3	4
1966	1.000	1.000	1.000	0.000
1967	0.000	1.000	1.000	1.000
1968	0.949	0.949	0.950	0.948
1969	0.945	0.945	0.945	0.945
1970	0.967	0.967	0.988	0.988
1971	0.988	0.988	0.988	0.988
1990	0.988	0.988	0.988	0.988
		(TC_{RT})		
1966	0.480	0.480	0.480	0.480
1967	0.480	0.480	0.480	0.480
1968	0.539	0.539	0.539	0.539
1969	0.543	0.543	0.543	0.543
1970	0.518	0.518	0.493	0.493
1971	0.493	0.493	0.493	0.493
1990	0.493	0.493	0.493	0.493

TABLE 6 (*concluded*)

III. Indirect Business Tax

$$TX = -7.5578 + 2.1879DMY_{TX} + 0.1014GNP$$

(DMY_{TX})

Year	1	2	3	4
1966	−1.37	−1.23	−1.10	−1.42
1967	−1.37	−1.23	−1.28	−1.14
1968	−1.46	−1.14	−0.87	−0.69
1969	−0.70	−0.70	−0.70	−0.70
1990	−0.70	−0.70	−0.70	−0.70

IV. Contributions for Social Insurance

$$TW = -5.8424 + 0.1552(t-4) + 0.0286UINS_{RT}EHH$$
$$-0.2765OASI_{PR\text{-}RT}(OASI_{BA}EHH - WSS)$$
$$+ 0.7199OASI_{PR\text{-}RT}OASI_{BA}EHH + DMY_{CR}$$

Years	$OASI_{PR\text{-}RT}$	$OASI_{BA}$	$UINS_{RT}$
1966	0.075	6.6	3.1
1967	0.079	6.6	3.1
1968	0.079	7.8	3.1
1969	0.086	7.8	3.1
1970	0.087	7.8	3.1
1971–72	0.094	7.8	3.1
1973–75	0.103	7.8	3.1
1976–77	0.104	7.8	3.1
1978–79	0.105	7.8	3.1
1980–82	0.107	7.8	3.1
1983–86	0.108	7.8	3.1
1987	0.110	7.8	3.1
1988–90	0.111	7.8	3.1

(DMY_{CR})

Year	1	2	3	4
1966	−1.1	−0.9	0.1	0.2
1967	0.1	0.8	1.0	1.3
1968	−0.6	−0.1	0.4	0.9
1969	1.0	1.0	1.0	1.0
1990	1.0	1.0	1.0	1.0

TABLE 7

Productivity Changes, Control Solution

	1965:4	1990:4	Annual Rate of Change (per cent)
$X_{MD}^{58}/MH_{P_{MD}}$	6.7804	16.3988	3.6
$X_{MD}^{58}/E_{O_{MD}}$	42.7671	92.0779	3.1
$X_{MN}^{58}/MH_{P_{MN}}$	6.4520	18.5101	4.3
$X_{MN}^{58}/E_{O_{MN}}$	39.9040	59.5704	1.6
$X_{T}^{58}/MH_{P_{T}}$	5.7309	15.1120	4.0
$X_{T}^{58}/E_{O_{T}}$	34.4419	45.9639	1.2
X_{R}^{58}/MH_{R}	6.8742	21.0194	4.6
X_{C}^{58}/MH_{C}	3.7652	3.9542	0.2
X_{O}^{58}/MH_{O}	5.9058	10.0364	2.1
GNP^{58}/EHH	8.8653	16.1126	2.4

but was modified only slightly. Productivity was adjusted upward in the overhead employment functions. All the hours functions showed a consistent upward bias and were adjusted to maintain an approximately constant workweek. Productivity figures at the start and end of the twenty-five year control simulation and average annual rates of change are shown in Table 7.

Over the 1966–90 period the control solution exhibited very little fluctuation, especially after 1970. Smooth extrapolation of the exogenous variables most likely caused this steady growth. Fluctuations over the first four years are due largely to the use of some actual exogenous data and to the initial conditions.

Simulation of the slowdown attributable to the end of the Vietnam War and its aftermath resulted in fluctuations in 1970–71. Only a slowdown in growth, and no actual downturn, resulted with the unemployment rate rising to only 4.4 per cent.

Control solution time paths over the full twenty-five year simulation period for P_{GNP}, GNP^{58}, $I_{BUS_{EAF}}^{58}$, and I_{CNFR}^{58} are shown in Charts 16–19, respectively. Values for the seventeen variables considered in this study are presented in Table 8 for the fourth quarters of 1965 and 1990, together with their annual rates of change (where relevant).

Fifty stochastic simulations were run over the period from the first quarter of 1966 through the fourth quarter of 1990. In computing these simulations, random disturbance terms were introduced having the same variance-covariance and serial correlation properties as the residuals from the sample period equation estimates. While some of the stochastic simulations drifted, the majority fluctuated about the control path. The means of the stochastic simulation values were almost identical to their control solution values. Charts 16–19 show the control solution, a representative stochastic simulation, and the control solution plus and minus two standard errors for real gross national product (GNP^{58}), the implicit deflator for GNP (P_{GNP}), real nonfarm business gross fixed investment ($I^{58}_{BUS_{EAF}}$), and real new private nonfarm residential construction (I^{58}_{CNFR}).

In order to determine whether the stochastic simulations produced

TABLE 8

Summary of Control Solution

	1965:4	1990:4	Annual Rate of Change (per cent)
RM_{GBS3}	4.160	6.942	–
RM_{GBL}	4.350	5.155	–
$IBUS^{58}_{EAF}$	45.2	141.0	4.7
P_{GNP}	1.115	1.709	1.7
O^{58}_{U}	63.4	114.6	2.4
I^{58}_{CNFR}	23.3	55.4	3.5
Y_P	558.4	2088.0	5.4
Z_B	80.3	372.3	6.3
GNP	710.0	2779.8	5.6
GNP^{58}	636.6	1626.9	3.8
RU	0.041	0.039	–
C^{58}	409.2	1075.9	3.9
ΔINV^{58}_{EAF}	8.1	10.1	–
B	5.7	−0.3	–
EHH	71.81	100.97	1.4
H	39.93	38.59	–
$RWSS$	5.691	16.283	4.3

CHART 16

GNP[58]

Gross National Product in 1958 Dollars

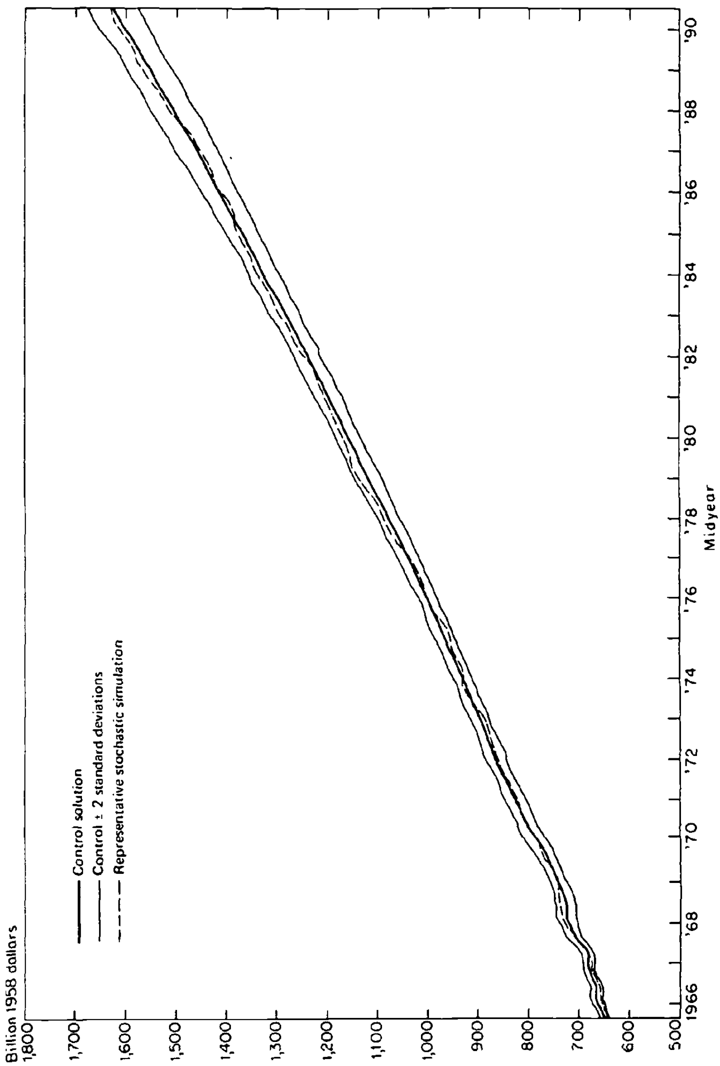


CHART 17

P_{GNP}

Implicit Price Deflator for Gross National Product

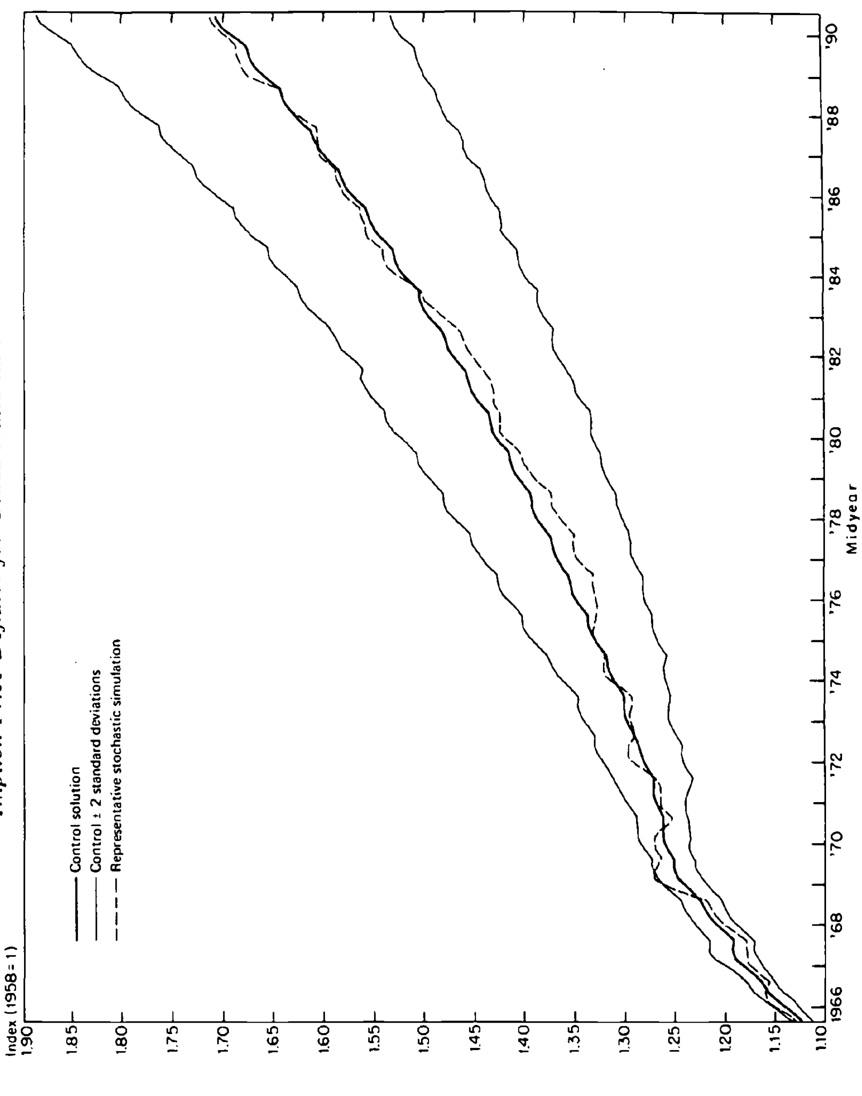


CHART 18

I^{58}_{BUSEAF}

Nonagricultural Business Gross Investment

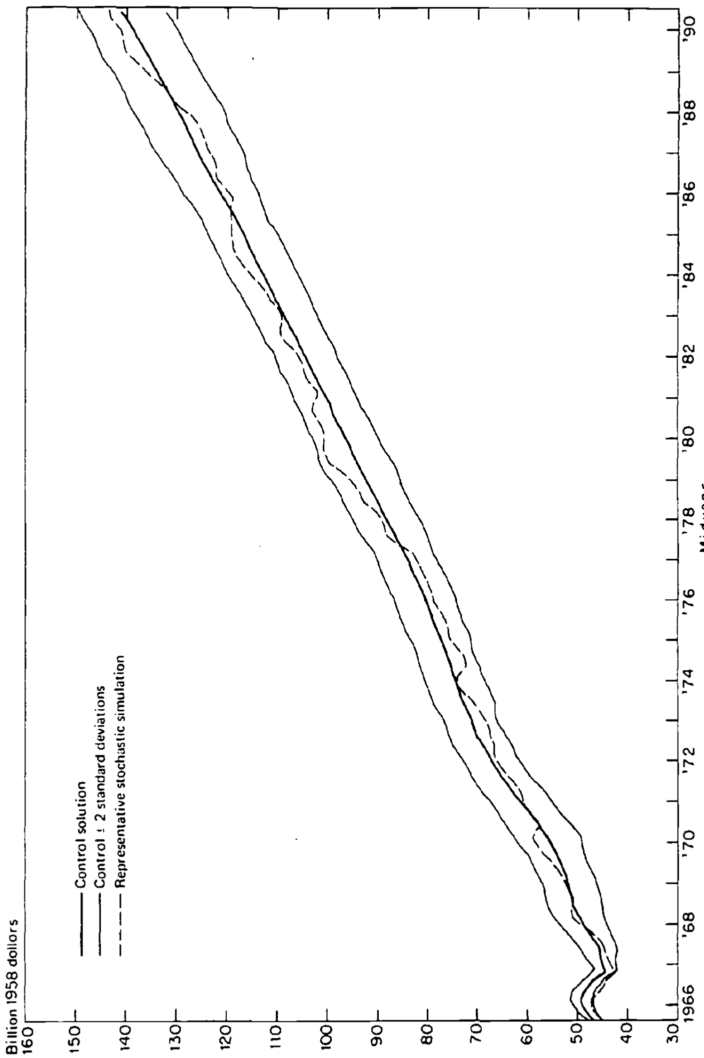
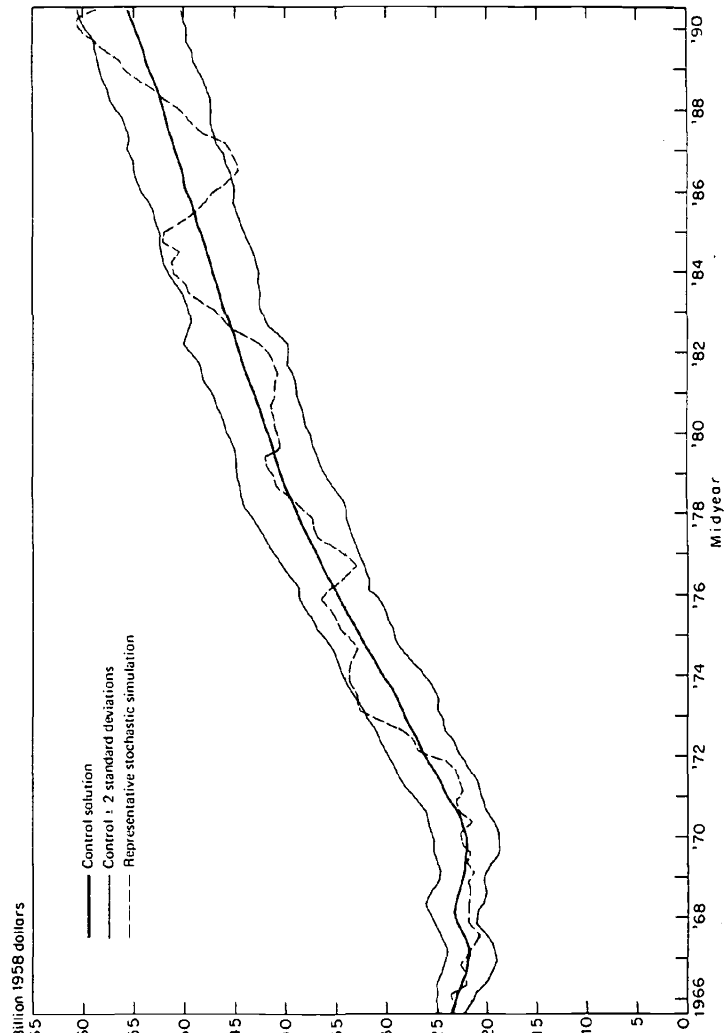


CHART 19

I^{58}_{CNFR}

New Private Nonfarm Residential Construction in 1958 Dollars



cyclical movements similar to those observed in historical data, spectral analysis was applied to the fifty series generated for each of these four variables. This required the removal of trends in the series. Denoting the original stochastically generated series as Y_S, the control solution as Y_C, and t as a time index, the detrended series, X, may be represented by

$$X_t = Y_{S,t} - a - bY_{C,t} - ct, \text{ for } t = 1966{:}1, \ldots, 1990{:}4$$

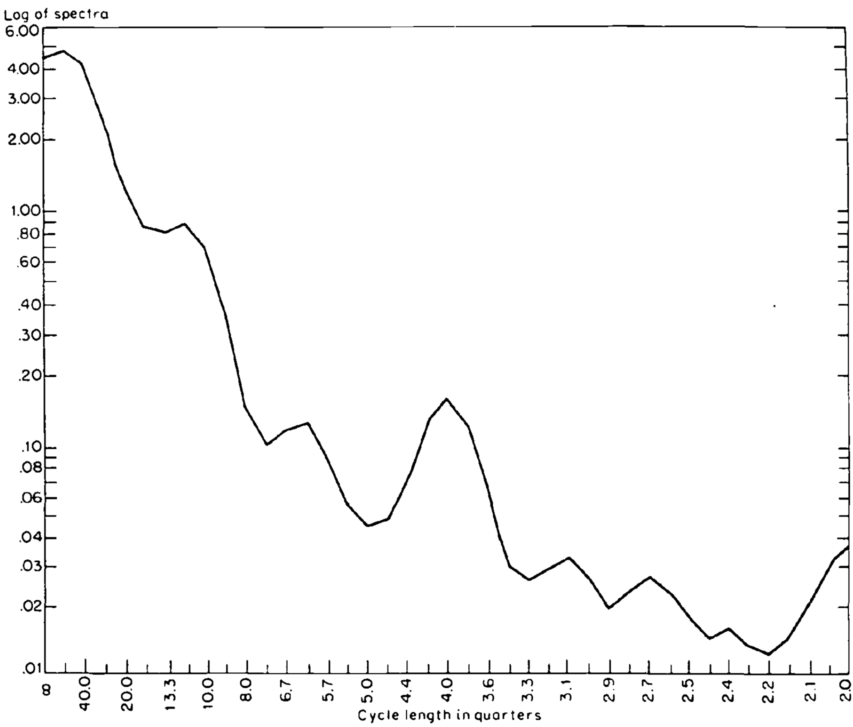
where a, b and c are determined by ordinary least squares regressions of $Y_{S,t}$ on $Y_{C,t}$ and t. The effectiveness of this detrending procedure was tested by comparing the means and variances of X_t computed over the first and second halves of the period first quarter of 1966

CHART 20

Average Spectra for Fifty Real National Product Series

CHART 21

Average Spectra for Fifty GNP Implicit Price Deflator Series



through the fourth quarter of 1990. In almost all cases, the two subsample means and variances were not significantly different.[15]

Average spectral densities are shown for each of the four series in Charts 20–23. Chart 24 shows frequency counts of the most prominent spectral peaks in the fifty series. A Parzen window and a lag length of 40 were used for all spectral calculations.[16] All the average

[15] The *t*- and *F*-tests were used to test equality of the means and variances, respectively. Even though an implicit normality assumption was required, especially for the *F*-test, no test for normality was made.

[16] C. W. J. Granger and M. Hatanaka, *Spectral Analysis of Economic Time Series* (Princeton University Press, 1964), pp. 52–73.

CHART 22

Average Spectra for Fifty Nonfarm Business Gross Fixed Investment Series

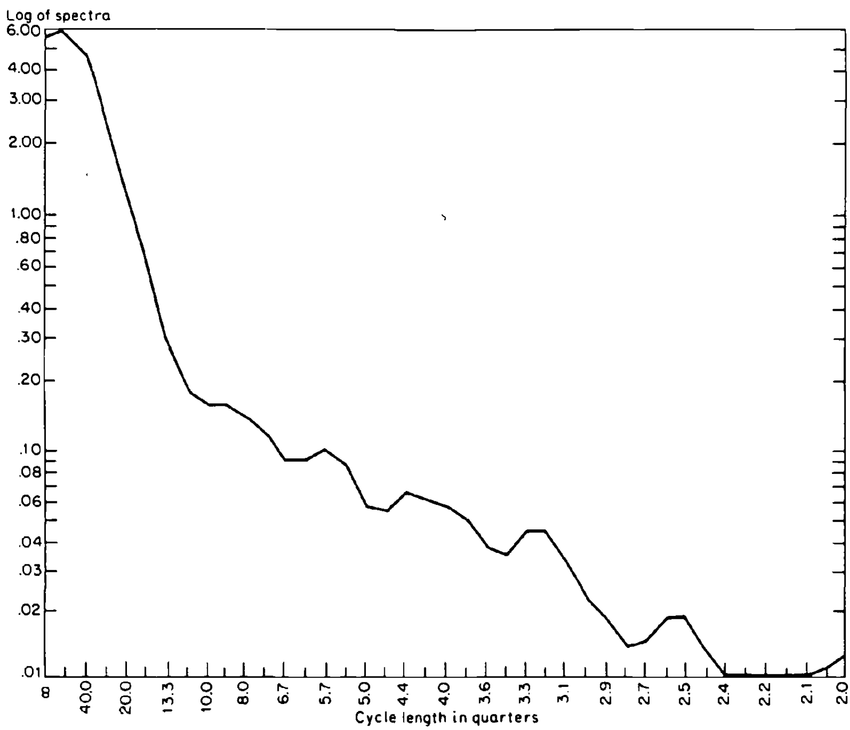


CHART 23

Average Spectra for Fifty Real New Private Nonfarm Residential Construction Series

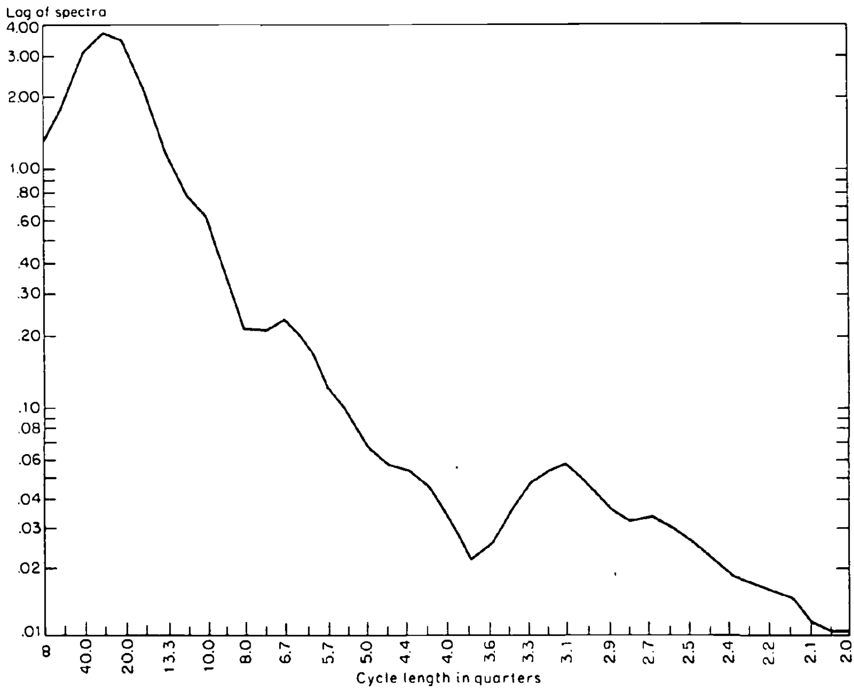
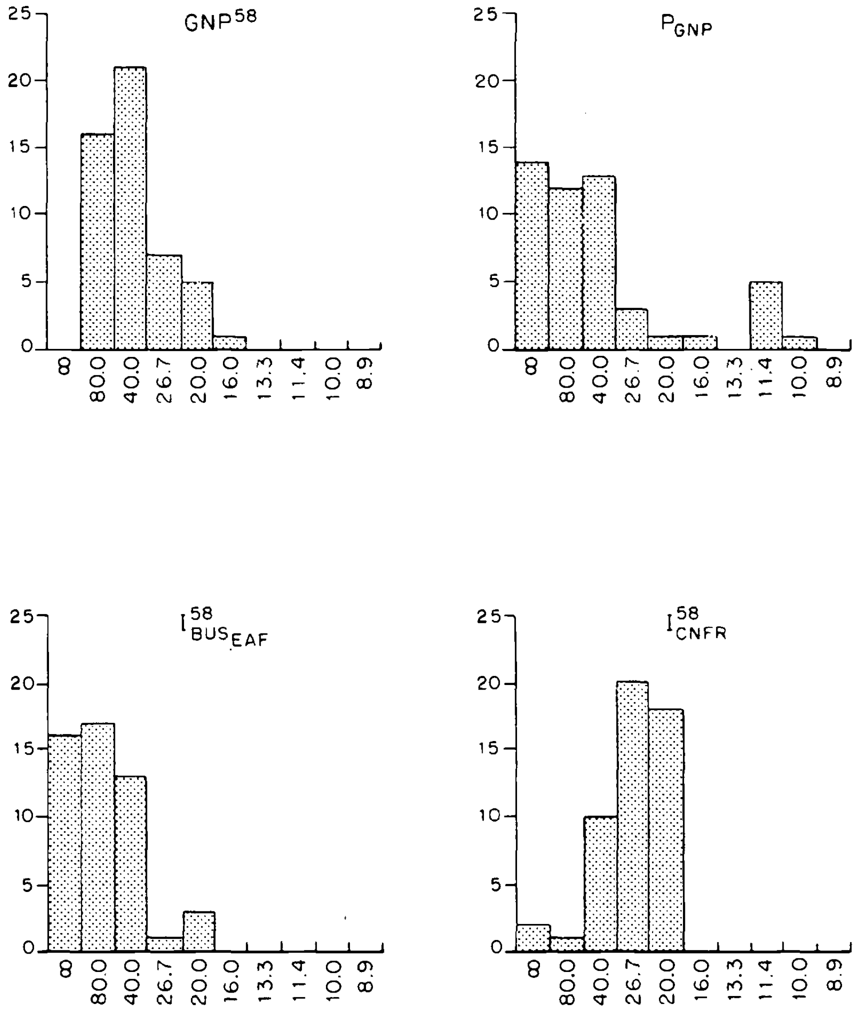


CHART 24

Frequency Bar Charts of the Most Prominent Spectral Peaks for Fifty Series Generated Using Serially Correlated Random Disturbances



spectra have their highest power at low frequencies (long cycle length) and all exhibit minor cyclical movements at the highest frequencies (less than one year cycle length). These latter movements probably reflect seasonal fluctuations. The average spectra for real *GNP* also exhibits peaks at approximately four and six quarters. P_{GNP} tends to exhibit both six quarter and three year cycles. While the three year cycle is not obvious in its average spectra, examination of the individual spectra shows that the cycle occurs in twenty-six of the fifty series—in six series the three year cycle was most prominent (see Chart 24). Real fixed nonfarm business investment ($I^{58}_{BUS_{EAF}}$) does not exhibit any significant cycles of intermediate length. The most prominent spectral peaks for real private nonfarm residential construction (I^{58}_{CNFR}) almost all lie in the five to ten year cycle length range—five to seven year cycle lengths are the most common. For the average spectra of I^{58}_{CNFR}, the most prominent spectral peak also occurs in the five to seven year range.

5 CONCLUSION

THE present, 1969 version of the Brookings Model exhibits sample period properties similar to the earlier Fromm-Taubman version. The model tracks trends quite well. Also, although leads and lags at turning points and cyclical amplitudes are not always predicted accurately, the model portrays the actual cyclical fluctuations.

Judging by root mean square errors, the 1969 version exhibits improved complete system performance over earlier versions. Problem areas basically remain the same, notably inventories, wage rates, and prices. But even here differences between predicted and actual values have been reduced to some extent.

Twenty-five year nonstochastic and stochastic simulations beyond the sample period were run with the model for the first time. It was found that the nonstochastic path of the solution depends primarily on the values chosen for principal exogenous variables but that, given these, endogenous variables take values that accord well with prior historical experience. Although some had trend deviations, stochastic solutions generally fluctuated about the nonstochastic control. A spectral analysis of these results revealed a general pattern of falling spectral densities from low to high frequencies without much evidence of distinct peaks except in isolated cases of highly cyclical variables. It might be said that the average spectral diagram exhibited the typical spectral shape of economic variables suggested by Granger.[17] Analysis of the frequency distribution of spectral peaks did, however, reveal some short-run cyclical content for many of the variables.

APPENDIX A

CONSUMER DEMAND

(A.1)
$$\frac{C^{58}_{DEA}}{N_R} = \underset{(2.5)}{-0.1378} + \underset{(11.6)}{0.2004}\frac{Y^{58}_D}{N_R} - \underset{(0.4)}{0.0150}\frac{P_{CDEA}}{P_C} - \underset{(1.8)}{0.1499}\left[\frac{K^{58}_{CDEA}}{N_R}\right]_{-1} + \underset{(0.8)}{0.0014}DMY_{55}$$

$$\bar{R}^2 = 0.986 \qquad SE = 0.0028 \qquad DW = 0.94$$

(A.2)
$$\frac{C^{58}_{DA}}{N_R} = \underset{(1.4)}{0.0430} + \underset{(7.5)}{0.0913}\frac{Y^{58}_D}{N_R} - \underset{(2.5)}{0.0723}\frac{P_{CDA}}{P_C} - \underset{(1.3)}{0.0903}\left[\frac{K^{58}_{CDA}}{N_R}\right]_{-1} - \underset{(6.3)}{0.5262}RU + \underset{(6.8)}{0.0214}DMY_{55} + \underset{(1.3)}{0.0039}[DMY_{STR}]\left[\frac{Y^{58}_D}{N_R}\right]$$

$$\bar{R}^2 = 0.893 \qquad SE = 0.0053 \qquad DW = 1.77$$

(A.3)
$$\frac{C^{58}_{NFB}}{N_R} = \underset{(5.5)}{0.4180} + \underset{(8.0)}{0.0655}\frac{Y^{58}_D}{N_R} - \underset{(2.9)}{0.1332}\frac{P_{CNFB}}{P_C}$$

[17] *Ibid.*, pp. 55–59.

$$+ 0.0796 \left(\frac{1}{4}\right) \sum_{i=1}^{4} \left[\frac{C^{58}_{NFB}}{N_R}\right]_{-i}$$

$$\bar{R}^2 = 0.923 \qquad SE = 0.0032 \qquad DW = 0.94$$

(A.4) $$\frac{C^{58}_{NEFB}}{N_R} = \underset{(4.4)}{0.3053} + \underset{(9.8)}{0.1451} \frac{Y^{58}_D}{N_R} - \underset{(4.6)}{0.2636} \frac{P_{CNEFB}}{P_C}$$

$$+ \underset{(1.4)}{0.1710} \left(\frac{1}{4}\right) \sum_{i=1}^{4} \left[\frac{C^{58}_{NEFB}}{N_R}\right]_{-i}$$

$$\bar{R}^2 = 0.989 \qquad SE = 0.0030 \qquad DW = 1.92$$

(A.5) $$\frac{C^{58}_S}{N_R} = \underset{(7.0)}{0.0422} + \underset{(4.9)}{0.0529} \frac{Y^{58}_D}{N_R} + \underset{(33.6)}{0.9272} \left(\frac{1}{4}\right) \sum_{i=1}^{4} \left[\frac{C^{58}_S}{N_R}\right]_{-i}$$

$$\bar{R}^2 = 0.998 \qquad SE = 0.0030 \qquad DW = 0.82$$

RESIDENTIAL CONSTRUCTION

(A.6) $$\Delta HU_{AVL_{EAF}} = -\underset{(0.2)}{0.0035} - \underset{(5.6)}{0.0019}[HU_{AVL_{EAF}}]_{-1}$$

$$+ \underset{(28.9)}{0.2620}[.25(HU_{STS_{EAF}})_{-1} + .50(HU_{STS_{EAF}})_{-2}$$

$$+ .25(HU_{STS_{EAF}})_{-3}]$$

$$\bar{R}^2 = 0.944 \qquad SE = 0.0093 \qquad DW = 0.93$$

(A.7) $$HU_{VAC_{EAF}} = [HU_{VAC_{EAF}}]_{-1} + \Delta HU_{AVL_{EAF}} - \Delta HH_{EAF}$$

(A.8) $$HU^1_{STS_{EAF}} = \underset{(2.4)}{0.7343} + \underset{(9.7)}{0.6413}[HU^1_{STS_{EAF}}]_{-1}$$

$$- \underset{(4.6)}{0.0740} \left(\frac{1}{3}\right) \sum_{i=1}^{3} [RM_{GBS3}]_{-i}$$

$$+ \underset{(2.2)}{0.0957} \left(\frac{1}{3}\right) \sum_{i=1}^{3} \left[\frac{Y_D}{(P_C)(HH_{EAF})}\right]_{-i}$$

$$- \underset{(2.3)}{0.0624} \frac{PM^{1}_{ICRD}}{P_{ICNFR}}$$

$$\bar{R}^2 = 0.924 \qquad SE = 0.0469 \qquad DW = 1.65$$

(A.9) $$HU^{2}_{STS_{EAF}} = \underset{(6.4)}{0.6223}[HU^{2}_{STS_{EAF}}]_{-1} - \underset{(3.4)}{0.0045} \left(\frac{1}{3}\right) \sum_{i=1}^{3} [RM_{GBS3}]_{-i}$$

$$+ \underset{(3.0)}{0.0002}[\text{time-8}] + \underset{(1.5)}{0.0024} \left(\frac{1}{3}\right) \sum_{i=1}^{3} \left[\frac{Y_D}{P_C HH_{EAF}}\right]_{-i}$$

$$\bar{R}^2 = 0.682 \qquad SE = 0.0044 \qquad DW = 1.85$$

(A.10) $$HU^{3+}_{STS_{EAF}} = \underset{(2.6)}{0.1682} + \underset{(13.7)}{0.7796}[HU^{3+}_{STS_{EAF}}]_{-1}$$

$$- \underset{(2.8)}{0.0207} \left(\frac{1}{3}\right) \sum_{i=1}^{3} [RM_{GBS3}]_{-i}$$

$$+ \underset{(3.9)}{0.0031}[\text{time-8}] - \underset{(2.2)}{0.0118} \left[\frac{PM^{2+}_{ICRD}}{P_{ICNFR}}\right]_{-1}$$

$$- \underset{(3.1)}{0.0596}[HU_{VAC_{EAF}}]_{-1}$$

$$\bar{R}^2 = 0.983 \qquad SE = 0.0193 \qquad DW = 1.74$$

(A.11) $$HU^{2+}_{STS_{EAF}} = HU^{2}_{STS_{EAF}} + HU^{3+}_{STS_{EAF}}$$

(A.12) $$HU_{STS_{EAF}} = HU^{1}_{STS_{EAF}} + HU^{2+}_{STS_{EAF}}$$

(A.13) $$\frac{PM^{1}_{ICRD}}{P_{ICNFR}} = \underset{(2.1)}{1.1205} + \underset{(10.3)}{0.7296} \left[\frac{PM^{1}_{ICRD}}{P_{ICNFR}}\right]_{-1}$$

$$+ \underset{(2.6)}{0.3559} \left(\frac{1}{3}\right) \sum_{i=1}^{3} \left[\frac{Y_D}{P_C HH_{EAF}}\right]_{-i}$$

$$\bar{R}^2 = 0.918 \qquad SE = 0.1730 \qquad DW = 2.82$$

(A.14) $$\frac{PM^{2+}_{ICRD}}{P_{ICNFR}} = \underset{(3.2)}{4.3574} + \underset{(5.9)}{0.6390} \left[\frac{PM^{2+}_{ICRD}}{P_{ICNFR}}\right]_{-1}$$

$$- \underset{(2.0)}{0.7727}[HU_{VAC_{EAF}}]_{-1}$$

$$\bar{R}^2 = 0.558 \qquad SE = 0.4480 \qquad DW = 1.80$$

(A.15) $$\frac{PM_{ICRD}}{P_{ICNFR}}$$

$$= \left[\frac{PM^{1}_{ICRD}}{P_{ICNFR}}(HU^{1}_{STS_{EAF}}) + \frac{PM^{2+}_{ICRD}}{P_{ICNFR}}(HU^{2+}_{STS_{EAF}})\right]\frac{1}{HU_{STS_{EAF}}}$$

(A.16) $$I^{58}_{CNFRH} = 0.41\left[\frac{PM_{ICRD}}{P_{ICNFR}}HU_{STS_{EAF}}\right] + 0.49\left[\frac{PM_{ICRD}}{P_{ICNFR}}HU_{STS_{EAF}}\right]_{-1} + 0.10\left[\frac{PM_{ICRD}}{P_{ICNFR}}HU_{STS_{EAF}}\right]_{-2}$$

(A.17) $$I^{58}_{CNFREH} = \underset{(1.5)}{2.6589} + \underset{(16.7)}{.1773}[HU_{AVL_{EAF}}]_{-1} - \underset{(3.3)}{6.9202}\left[\frac{P_{ICNFR}}{P_C}\right]_{-2}$$

$$\bar{R}^2 = 0.891 \qquad SE = 0.2080 \qquad DW = 0.49$$

(A.18) $$I^{58}_{CNFR*} = I^{58}_{CNFRH} + I^{58}_{CNFREH}$$

(A.19) $$I^{58}_{CNFR} = (DMY_{15})I^{58}_{CNFR*}$$

(A.20) $$I^{58}_{CO} = -\underset{(3.7)}{2.2879} + \underset{(3.6)}{0.4240}\left(\frac{1}{8}\right)\sum_{i=1}^{8}\left[\frac{Y_D}{(P_C)(HH_{EAF})}\right]_{-i} + \underset{(12.8)}{0.8148}[I^{58}_{CO}]_{-1}$$

$$\bar{R}^2 = 0.976 \qquad SE = 0.0913 \qquad DW = 1.77$$

INVENTORY INVESTMENT

$$\text{(A.21)}\quad \Delta INV^{58}_{MD} = \underset{(0.3)}{-0.8912} + \underset{(1.9)}{0.0613}[SF^{58}_{D} + GNP^{58}_{IC}]$$
$$- \underset{(1.7)}{0.2922}[INV^{58}_{MD}]_{-1} + \underset{(2.7)}{0.2846}\Delta[INV^{58}_{MD}]_{-1}$$
$$+ \underset{(4.8)}{0.1090}\Delta[O^{58}_{U,MD}]_{-1} + \underset{(1.9)}{0.0427}[O^{58}_{U,MD}]_{-1}$$
$$\bar{R}^2 = 0.725 \qquad SE = 1.2391 \qquad DW = 1.93$$

$$\text{(A.22)}\quad \Delta INV^{58}_{MN} = \underset{(0.1)}{-0.2809} + \underset{(2.7)}{0.0747}SF^{58}_{N} - \underset{(2.2)}{0.5350}[INV^{58}_{MN}]_{-1}$$
$$+ \underset{(2.1)}{0.2480}\Delta[INV^{58}_{MN}]_{-1} + \underset{(3.0)}{.7117}[O^{58}_{U,MN}]_{-1}$$
$$\bar{R}^2 = 0.310 \qquad SE = 0.7103 \qquad DW = 1.94$$

$$\text{(A.23)}\quad \Delta[INV^{58}_{T} - INV^{58}_{CAR}] = \underset{(2.9)}{-5.659} + \underset{(2.7)}{0.0750}\left[SF^{58} - \frac{WS_G}{P_G} - C^{58}_{S}\right]$$
$$- \underset{(2.2)}{0.4559}[INV^{58}_{T} - INV^{58}_{CAR}]_{-1}$$
$$\bar{R}^2 = 0.182 \qquad SE = 1.3300 \qquad DW = 1.69$$

$$\text{(A.24)}\quad \Delta INV^{58}_{CAR} = \underset{(1.0)}{0.9420} + \underset{(4.3)}{0.3642}C^{58}_{DA} - \underset{(1.6)}{0.1649}[C^{58}_{DA}]_{-1}$$
$$- \underset{(2.8)}{1.019}[INV^{58}_{CAR}]_{-1}$$
$$\bar{R}^2 = 0.366 \qquad SE = 0.8516 \qquad DW = 1.93$$

$$\text{(A.25)}\quad \Delta INV^{58}_{O*4} = \underset{(2.0)}{-0.6917} + \underset{(2.5)}{0.0071}\left[SF^{58} - \frac{WS_C}{P_G}\right]$$
$$- \underset{(2.0)}{0.3209}[INV^{58}_{O*4}]_{-1} + \underset{(4.0)}{0.4485}\Delta[INV^{58}_{O*4}]_{-1}$$
$$\bar{R}^2 = 0.435 \qquad SE = 0.2616 \qquad DW = 2.20$$

(A.26) $$INV_j^{58} = \frac{1}{4}\Delta INV_j^{58} + [INV_j^{58}]_{-1}$$

$$j = MD, MN, T\text{-}CAR, CAR, O*4$$

ORDERS

(A.27) $$O_{MD}^{58} = \underset{(6.2)}{264.692} + \underset{(10.1)}{0.9950}[SF_D^{58} + GNP_{IC}^{58}] + \underset{(2.6)}{4.0568}\,\frac{1}{2}\sum_{i=0}^{1}\left[\frac{DOD_{MPCA}}{P_{GF}}\right]_{-i} - \underset{(8.3)}{160.57}\left[\frac{1}{JCAP_{MD}}\right] - \underset{(4.4)}{1.0381}[O_{U_{MD}}^{58}]_{-1}$$

$$\bar{R}^2 = 0.936 \qquad SE = 7.5083 \qquad DW = 0.96$$

(A.28) $$O_{MN}^{58} = 4\left[O_{U_{MN}}^{58} - [O_{U_{MN}}^{58}]_{-1}\left\{\frac{[WPI_{MN}]_{-1}}{WPI_{MN}}\right\}\right] + 2.5650X_{MN}^{58} - 0.6959\Delta INV_{MN}^{58} + 0.3478[\text{time-4}] + 7.1606$$

(A.29) $$O_{U_{MD}}^{58} = [O_{U_{MD}}^{58}]_{-1}\left\{\frac{[WPI_{MD}]_{-1}}{WPI_{MD}}\right\} + 0.25O_{MD}^{58} - 0.25[37.1023 + 1.8387X_{MD}^{58}]$$

(A.30) $$\Delta O_{U_{MN}}^{58} = \underset{(3.6)}{17.4029} + \underset{(1.8)}{0.0255}\Delta O_{MN}^{58} + \underset{(2.2)}{0.0106}[O_{MN}^{58}]_{-1} - \underset{(6.1)}{0.6090}[O_{U_{MN}}^{58}]_{-1} - \underset{(3.9)}{0.1892}[INV_{MN}^{58}]_{-1} + \underset{(1.3)}{3.0671}[WPI_{MN}]_{-1} - \underset{(2.3)}{9.9576}\left\{\frac{WPI_{MN}}{[WPI_{MN}]_{-1}}\right\} - \underset{(3.5)}{5.2507}\left[\frac{1}{J_{CAP_{MN}}}\right]$$

$$\bar{R}^2 = 0.557 \qquad SE = 0.1360 \qquad DW = 2.10$$

INVESTMENT IN NONFARM BUSINESS PLANT AND EQUIPMENT

$$\text{(A.31)}\quad I^{58}_{BUSE_{MD}} = \underset{(5.1)}{-19.8370} + \underset{(2.0)}{7.7521}[DMY_{23} - DMY_{22}]$$

$$+ [\underset{(0.6)}{10.3800} - \underset{(0.7)}{0.1567}t]DMY_{22}$$

$$+ [\underset{(6.5)}{0.2300} - \underset{(2.3)}{0.110}(DMY_{23} - DMY_{22})] \sum_{i=0}^{7} A_i[X^{58}_{MD}]_{-i-2}$$

$$- \underset{(1.1)}{0.3066} \sum_{i=0}^{7} A_i[RM_{GBL}]_{-i-2} + \underset{(5.3)}{10.4768} \left[\frac{X^{58}_{MD}}{X^{58}_{K_{MD}}}\right]_{-1}$$

$$\bar{R}^2 = 0.817 \qquad SE = 0.5051 \qquad DW = 0.56$$

$$\text{(A.32)}\quad I^{58}_{BUSE_{MN}}$$

$$= \underset{(3.3)}{3.9778} - \underset{(5.9)}{11.2909}[DMY_{23} - DMY_{22}]$$

$$+ [\underset{(1.5)}{9.6082} - \underset{(1.5)}{0.1369}t]DMY_{22}$$

$$+ [\underset{(9.1)}{0.3332} + \underset{(5.3)}{0.1641}(DMY_{23} - DMY_{22})] \sum_{i=0}^{7} A_i[X^{58}_{MN}]_{-i-2}$$

$$- \underset{(7.2)}{1.4847} \sum_{i=0}^{7} A_i[RM_{GBL}]_{-2} - \underset{(5.8)}{0.3533}[K^{58}_{E_{MN}}]_{-1}$$

$$\bar{R}^2 = 0.931 \qquad SE = 0.2068 \qquad DW = 1.10$$

$$\text{(A.33)}\quad I^{58}_{BUS_R} = -\underset{(5.4)}{14.0539} - \underset{(6.8)}{2.1526}[DMY_{23} - DMY_{22}]$$

$$+ [\underset{(1.5)}{19.0730} - \underset{(1.5)}{0.2533}t]DMY_{22}$$

$$+ \underset{(7.3)}{0.5494} \sum_{i=0}^{7} A_i[X^{58}_{R}]_{-i-2} - \underset{(3.1)}{1.5925} \sum_{i=0}^{7} A_i[RM_{GBL}]_{-i-2}$$

$$+ \underset{(4.7)}{15.0179}\left[\frac{X_R^{58}}{X_{K_R}^{58}}\right]_{-1} - \underset{(1.5)}{0.0627}[K_R^{58}]_{-1}$$

$$\bar{R}^2 = 0.937 \qquad SE = 0.3862 \qquad DW = 0.87$$

$$\text{(A.34)} \quad I_{BUS_{O*2}}^{58} = \underset{(0.9)}{7.1978} + \underset{(1.5)}{0.0672} \sum_{i=0}^{7} A_i[X_{O*6}^{58}]_{-i-2}$$

$$- \underset{(4.0)}{1.4991} \sum_{i=0}^{7} A_i[RM_{GBL}]_{-i-2} + \underset{(2.1)}{21.4221}\left[\frac{X_{O*6}^{58}}{X_{K_{O*6}}^{58}}\right]_{-1}$$

$$- \underset{(6.0)}{0.7015}[K_{O*2}^{58}]_{-1} + \underset{(3.0)}{0.0971}\ \frac{1}{2}\sum_{i=1}^{2}[X_{O*6}^{58}]_{-i}$$

$$\bar{R}^2 = 0.930 \qquad SE = 0.3189 \qquad DW = 0.85$$

FOREIGN TRADE

$$\text{(A.35)} \quad [M_N^{58} + M_S^{58}] = \underset{(1.7)}{4.9227} - \underset{(2.2)}{5.0961}\frac{PM_{N+S}}{P_C} + \underset{(4.4)}{0.0229}Y_D^{58}$$

$$+ \underset{(4.8)}{0.5508}[M_N^{58} + M_S^{58}]_{-1} + \underset{(5.4)}{0.5789}DMY_{DKSTR1}$$

$$\bar{R}^2 = 0.983 \qquad SE = 0.3521 \qquad DW = 1.91$$

$$\text{(A.36)} \quad M_D^{58} = \underset{(3.1)}{1.9971} + \underset{(9.8)}{0.569}X_{MD}^{58} - \underset{(5.9)}{3.5042}\frac{PM_D}{PX_{MD}} + \underset{(7.7)}{0.4622}[M_D^{58}]_{-1}$$

$$+ \underset{(4.2)}{0.3438}DMY_{DKSTR} + \underset{(7.8)}{0.5395}DMY_{STLWT2}$$

$$\bar{R}^2 = 0.987 \qquad SE = 0.1966 \qquad DW = 1.95$$

$$\text{(A.37)} \quad EX^{58} = \underset{(2.7)}{19.9535} - \underset{(2.6)}{20.5046}\frac{P_{EX}}{P_{EXW}} + \underset{(7.0)}{0.6204}[EX^{58}]_{-1}$$

$$+ \underset{(4.7)}{0.1303}[EX_W^{58} - EX^{58}] + \underset{(4.6)}{1.3051}DMY_{DKSTR1}$$

$$\bar{R}^2 = 0.972 \qquad SE = 0.9543 \qquad DW = 2.13$$

GOVERNMENT TAXES AND TRANSFER PAYMENTS

(A.38)
$$TP = \underset{(12.3)}{-14.5008} + \underset{(1.7)}{13.7186}DMY_{TP} + \underset{(51.7)}{0.1613}Y_P - \underset{(2.5)}{0.0392}[DMY_{TP}Y_P]$$

$$\bar{R}^2 = 0.989 \qquad SE = 1.0715 \qquad DW = 1.08$$

(A.39)
$$TX = \underset{(15.9)}{-7.5578} + \underset{(11.3)}{2.1879}DMY_{TX} + \underset{(100.4)}{0.1014}GNP$$

$$\bar{R}^2 = 0.997 \qquad SE = 0.5355 \qquad DW = 1.10$$

(A.40)
$$TC = \underset{(2.5)}{0.9303} - \underset{(4.1)}{0.6567}DMY_{IC} + \underset{(54.5)}{0.8360}[TC_{RT}Z_{BU}]$$

$$\bar{R}^2 = 0.993 \qquad SE = 0.3176 \qquad DW = 0.45$$

(A.41)
$$TW = \underset{(8.7)}{-5.8424} + \underset{(11.0)}{0.1552}[t - 4] + \underset{(20.1)}{0.7199}[OASI_{RT}OASI_{PR}OASI_{BA}]EHH - \underset{(8.0)}{0.2765}[OASI_{RT}OASI_{PR}][OASI_{BA}EHH - WSS] + \underset{(7.2)}{0.0286}UINS_{RT}[EHH]$$

$$R^2 = 0.999 \qquad SE = 0.2233 \qquad DW = 0.71$$

(A.42)
$$V_{USGF} = \underset{(5.4)}{-1.3555} + \underset{(8.7)}{0.9403}U + \underset{(2.6)}{0.0127}[GNP_K^{58} - GNP^{58}]P_{GNP}$$

$$\bar{R}^2 = 0.930 \qquad SE = 0.2379 \qquad DW = 1.04$$

(A.43) $$V_G = V_{USGF} + V_{OASIGF} + V_{VET} + V_{OG}$$

PRODUCTION FUNCTIONS

(A.44)
$$\begin{aligned}
\Delta \ln MH_{P_{MD}} = {} & \underset{(1.2)}{0.3246} + \underset{(18.5)}{0.7085} \ln X^{58}_{MD} \\
& - \underset{(4.7)}{0.1426} \ln [K^{58}_{E_{MD}}]_{-1} \\
& - \underset{(6.4)}{0.3963} \ln [MH_{P_{MD}}]_{-1} \\
& - \underset{(4.2)}{0.2749} \ln [X^{58}_{MD}]_{-1} \\
& - \underset{(4.2)}{0.270} DMY_1 - \underset{(4.9)}{0.0046} DMY_1 [t - 61] \\
& - \underset{(3.2)}{0.0094} DMY_2 + \underset{(3.1)}{0.0037} DMY_2 [t - 74]
\end{aligned}$$

$\bar{R}^2 = 0.941 \quad SE = 0.0078 \quad DW = 1.52$

(A.45)
$$\begin{aligned}
\Delta \ln MH_{P_{MN}} = {} & \underset{(1.7)}{0.5880} + \underset{(15.4)}{0.7264} \ln X^{58}_{MN} - \underset{(4.1)}{0.3283} \ln [K^{58}_{E_{MN}}]_{-1} \\
& - \underset{(16.9)}{0.0050} [t - 8] - \underset{(15.7)}{0.8675} \ln [MH_{P_{MN}}]_{-1} \\
& + DMY_3 [\underset{(2.4)}{3.8371} - \underset{(2.5)}{0.1102} (t - 8) \\
& + \underset{(2.5)}{0.0007881} (t - 8)^2]
\end{aligned}$$

$\bar{R}^2 = 0.861 \quad SE = 0.0052 \quad DW = 1.61$

(A.46)
$$\ln MH_{P_T} = \underset{(1.1)}{0.2195} + \underset{(3.4)}{0.1780} \ln X^{58}_{T} + \underset{(2.5)}{0.1659} \ln [X^{58}_{T}]_{-1}$$

$$- \underset{(4.8)}{0.0022t} + \underset{(4.5)}{0.4412} \ln [MH_{P_T}]_{-1}$$

$$\bar{R}^2 = 0.984 \qquad SE = 0.0040 \qquad DW = 2.19 \qquad \rho_1 = 0.5940$$

(A.47) $$\ln MH_C = \underset{(3.5)}{-1.0240} + \underset{(5.2)}{0.6899} \ln X_C^{58} - \underset{(1.1)}{0.00044t}$$

$$+ \underset{(3.5)}{0.3848} \ln [MH_C]_{-1}$$

$$\bar{R}^2 = 0.899 \qquad SE = 0.0185 \qquad DW = 1.90 \qquad \rho_1 = 0.2574$$

(A.48) $$\ln MH_R = \underset{(1.5)}{-0.4213} + \underset{(10.5)}{0.4172} \ln X_R^{58} + \underset{(1.9)}{0.1157} \ln [X_R^{58}]_{-1}$$

$$- \underset{(8.7)}{0.00603t} + \underset{(5.1)}{0.4226} \ln [MH_R]_{-1}$$

$$\bar{R}^2 = 0.982 \qquad SE = 0.0092 \qquad DW = 2.46 \qquad \rho_1 = 0.7326$$

(A.49) $$\ln MH_O = \underset{(3.0)}{-0.7940} + \underset{(4.9)}{0.4788} \ln X_O^{58} + \underset{(0.9)}{0.1260} \ln [X_O^{58}]_{-1}$$

$$- \underset{(2.1)}{0.00131t} + \underset{(2.0)}{0.3445} \ln [MH_O]_{-1}$$

$$\bar{R}^2 = 0.999 \qquad SE = 0.0028 \qquad DW = 1.80 \qquad \rho_1 = 0.4653$$

(A.50) $$\ln E_{O_{MD}} = \underset{(3.3)}{-0.1553} + \underset{(3.7)}{0.0461} \ln X_{MD}^{58} + \underset{(41.3)}{0.9495} \ln [E_{O_{MD}}]_{-1}$$

$$\bar{R}^2 = 0.993 \qquad SE = 0.0084 \qquad DW = 0.36$$

(A.51) $$\ln E_{O_{MN}} = \underset{(4.2)}{-0.1363} + \underset{(4.5)}{0.0473} \ln X_{MN}^{58} + \underset{(43.3)}{0.9046} \ln [E_{O_{MN}}]_{-1}$$

$$\bar{R}^2 = 0.998 \qquad SE = 0.0026 \qquad DW = 1.85$$

(A.52) $$\ln E_{O_T} = \underset{(5.2)}{-0.6613} + \underset{(6.6)}{0.2262} \ln X_T^{58} + \underset{(2.5)}{0.0011t}$$

$$+ 0.5801 \ln [E_{O_T}]_{-1}$$
$$(8.1)$$

$$\bar{R}^2 = 0.998 \qquad SE = 0.0053 \qquad DW = 2.12$$

(A.53) $$H_{PMD} = \underset{(125.8)}{37.5601} + \underset{(14.0)}{0.0744} X^{58}_{MD} + \underset{(3.3)}{0.0416} \Delta X^{58}_{MD}$$
$$- \underset{(7.5)}{1.0769} RWSS_{MD}$$

$$\bar{R}^2 = 0.859 \qquad SE = 0.2770 \qquad DW = 0.96$$

(A.54) $$H_{PMN} = \underset{(178.7)}{37.6692} + \underset{(10.3)}{0.1319} X^{58}_{MN} + \underset{(3.8)}{0.1190} \Delta X^{58}_{MN}$$
$$- \underset{(8.8)}{2.4588} RWSS_{MN}$$

$$\bar{R}^2 = 0.803 \qquad SE = 0.185 \qquad DW = 1.13$$

(A.55) $$H_{PT} = \underset{(379.5)}{42.8931} + \underset{(2.5)}{0.0170} X^{58}_{T} + \underset{(0.6)}{0.0108} \Delta X^{58}_{T} - \underset{(9.7)}{2.5898} RWSS_{T}$$

$$\bar{R}^2 = 0.965 \qquad SE = 0.1069 \qquad DW = 0.79$$

(A.56) $$H_{C} = \underset{(30.1)}{36.0767} + \underset{(1.0)}{0.0764} X^{58}_{C} + \underset{(1.5)}{0.2181} \Delta X^{58}_{C} - \underset{(1.3)}{0.2527} RWSS_{C}$$

$$\bar{R}^2 = 0.053 \qquad SE = 0.3891 \qquad DW = 1.50$$

(A.57) $$H_{R} = \underset{(541.7)}{39.7580} + \underset{(13.5)}{0.0816} X^{58}_{R} + \underset{(4.0)}{0.0657} \Delta X^{58}_{R} - \underset{(9.6)}{0.8953} RWSS_{R}$$

$$\bar{R}^2 = 0.915 \qquad SE = 0.0760 \qquad DW = 1.85$$

(A.58) $$MH_{O_j} = (40)(.052)E_{O_j} \qquad j = MD, MN, T$$

(A.59) $$MH_j = MH_{P_j} + MH_{O_j}$$
$$j = MD, MN, T$$

(A.60) $$E_{P_j} = \frac{MH_{P_j}}{(0.052)H_{P_j}}$$

$$j = MD, MN, T$$

(A.61) $$E_j = \frac{MH_j}{(0.052)H_j}$$

$$j = C, R, O$$

PRICES AND WAGE RATES

(A.62) $$ULC_j^N = \frac{\frac{1}{4}\sum_{i=0}^{3}[RWSS_j]_{-i}}{\frac{1}{12}\sum_{i=0}^{11}\left[\frac{X_j^{58}}{MH_j}\right]_{-i}}$$

$$j = MD, MN, T, C, R, O$$

(A.63) $$ULC_j = \frac{WSS_j}{X_j^{58}}$$

$$j = MD, MN, T, C, R, O$$

(A.64) $$WPI_{MD} = \underset{(5.2)}{-0.1632} + \underset{(7.5)}{0.6688}[ULC_{MD} - ULC_{MD}^N]$$
$$+ \underset{(38.7)}{1.1594}ULC_{MD}^N + \underset{(7.0)}{0.2314}J_{CAP_{MD}} + \underset{(3.6)}{0.1393}PR_{MD}$$
$$\bar{R}^2 = 0.982 \qquad SE = 0.0074 \qquad DW = 0.61$$

(A.65) $$WPI_{MN} = \underset{(0.3)}{-0.0228} + \underset{(4.9)}{0.6418}[ULC_{MN} - ULC_{MN}^N]$$
$$+ \underset{(18.3)}{0.6844}ULC_{MN}^N + \underset{(6.6)}{0.3118}J_{CAP_{MN}} + \underset{(9.8)}{0.2995}PR_{MN}$$
$$\bar{R}^2 = 0.917 \qquad SE = 0.0048 \qquad DW = 0.77$$

(A.66) $$PX_{MD} = \underset{(1.2)}{0.0443} + \underset{(40.2)}{1.0987}WPI_{MD} - \underset{(4.9)}{0.1507}PR_{MD}$$

$$\bar{R}^2 = 0.972 \qquad SE = 0.0103 \qquad SW = 1.85$$

(A.67) $$PX_{MN} = \underset{(3.6)}{-0.7470} + \underset{(3.7)}{0.9675}[ULC_{MN} - ULC^N_{MN}] + \underset{(25.1)}{1.7518}ULC^N_{MN} + \underset{(2.1)}{0.2225}J_{CAP_{MN}} + \underset{(1.6)}{0.0108}H_{PMN}$$

$$\bar{R}^2 = 0.947 \qquad SE = 0.0087 \qquad DW = 0.71$$

(A.68) $$PX_T = \underset{(6.5)}{0.1809} + \underset{(3.4)}{0.9021}[ULC_T - ULC^N_T] + \underset{(28.7)}{1.4527}ULC^N_T - \underset{(1.7)}{0.4160}\left[\frac{INV^{58}_T}{X^{58}_T} - \frac{1}{12}\sum_{i=0}^{11}\left(\frac{INV^{58}_T}{X^{58}_T}\right)_{-i}\right]_{-1}$$

$$\bar{R}^2 = 0.953 \qquad SE = 0.0104 \qquad DW = 0.94$$

(A.69) $$PX_C = \underset{(11.2)}{0.1046} + \underset{(24.7)}{1.4501}[ULC_C - ULC^N_C] + \underset{(99.3)}{1.2301}ULC^N_C$$

$$\bar{R}^2 = 0.997 \qquad SE = 0.0076 \qquad DW = 0.39$$

(A.70) $$UCCA^N_R = \frac{1}{12}\sum_{i=0}^{11}\left[\frac{CCA_R}{X^{58}_R}\right]_{-i}$$

(A.71) $$UCCA_R = \frac{CCA_R}{X^{58}_R}$$

(A.72) $$PX_R = \underset{(6.7)}{0.2699} + \underset{(7.2)}{0.8097}[ULC_R - ULC^N_R] + \underset{(10.2)}{0.7517}ULC^N_R + \underset{(5.8)}{1.5635}[UCCA_R - UCCA^N_R] + \underset{(27.4)}{2.3607}UCCA^N_R$$

$$\bar{R}^2 = 0.972 \qquad SE = 0.0064 \qquad DW = 1.03$$

(A.73) $$PX_O = \underset{(20.1)}{0.1483} + \underset{(4.3)}{1.2332}[ULC_O - ULC_O^N] + \underset{(119.9)}{2.3036}ULC_O^N$$

$$\bar{R}^2 = 0.997 \qquad SE = 0.0046 \qquad DW = 0.62$$

(A.74)

$$\left[\frac{RWSS_{MD} - RWSS_{MD-4}}{RWSS_{MD-4}}\right] = \underset{(1.4)}{0.0124} + \underset{(1.7)}{0.00073}\frac{4}{\sum_{i=0}^{3}[RU]_{-i}}$$
$$- \underset{(2.2)}{0.0069}DMY_{GP}$$
$$+ \underset{(1.3)}{0.2010}\frac{1}{4}\sum_{i=0}^{3}\left[\frac{CPI - CPI_{-4}}{CPI_{-4}}\right]_{-i}$$
$$- \underset{(2.4)}{0.2918}\left[\frac{RWSS_{MD-4} - RWSS_{MD-8}}{RWSS_{MD-8}}\right]$$
$$+ \underset{(6.4)}{0.6821}\left[\frac{RWSS_{MD-1} - RWSS_{MD-5}}{RWSS_{MD-5}}\right]$$

$$\bar{R}^2 = 0.649 \qquad SE = 0.0084 \qquad DW = 1.21$$

(A.75)

$$\left[\frac{RWSS_{MN} - RWSS_{MN-4}}{RWSS_{MN-4}}\right] = \underset{(2.1)}{0.0084} + \underset{(5.2)}{0.0012}\frac{4}{\sum_{i=0}^{3}[RU]_{-i}}$$
$$- \underset{(3.9)}{0.0055}DMY_{GP}$$
$$+ \underset{(2.9)}{0.2309}\frac{1}{4}\sum_{i=0}^{3}\left[\frac{CPI - CPI_{-4}}{CPI_{-4}}\right]_{-i}$$
$$- \underset{(7.6)}{0.4551}\left[\frac{RWSS_{MN} - RWSS_{MN-8}}{RWSS_{MN-8}}\right]$$
$$+ \underset{(9.3)}{0.6463}\left[\frac{RWSS_{MN-4} - RWSS_{MN-5}}{RWSS_{MN-5}}\right]$$

$$\bar{R}^2 = 0.898 \qquad SE = 0.0039 \qquad DW = 2.41$$

(A.76)

$$\left[\frac{RWSS_T - RWSS_{T-4}}{RWSS_{T-4}}\right] = \underset{(0.7)}{0.0040} + \underset{(6.1)}{0.0016}\,\frac{1}{RU} - \underset{(0.70)}{0.0011}DMY_{GP}$$

$$+ \underset{(4.2)}{0.5628}\,\frac{1}{4}\sum_{i=0}^{3}\left[\frac{CPI - CPI_{-4}}{CPI_{-4}}\right]_{-i}$$

$$- \underset{(4.7)}{0.4628}\left[\frac{RWSS_{T-4} - RWSS_{T-8}}{RWSS_{T-8}}\right]$$

$$+ \underset{(4.7)}{0.4239}\left[\frac{RWSS_{T-1} - RWSS_{T-5}}{RWSS_{T-5}}\right]$$

$$\bar{R}^2 = 0.794 \qquad SE = 0.0049 \qquad DW = 1.77$$

(A.77)

$$\left[\frac{RWSS_C - RWSS_{C-4}}{RWSS_{C-4}}\right] = \underset{(4.2)}{0.0277} - \underset{(0.4)}{0.0020}DMY_{GP}$$

$$+ \underset{(2.9)}{0.9016}\,\frac{1}{4}\sum_{i=0}^{3}\left[\frac{CPI - CPI_{-4}}{CPI_{-4}}\right]_{-i}$$

$$- \underset{(3.6)}{0.3480}\left[\frac{RWSS_{C-4} - RWSS_{C-8}}{RWSS_{C-8}}\right]$$

$$+ \underset{(3.2)}{0.4060}\left[\frac{RWSS_{C-1} - RWSS_{C-5}}{RWSS_{C-5}}\right]$$

$$\bar{R}^2 = 0.511 \qquad SE = 0.0164 \qquad DW = 2.15$$

(A.78)

$$\left[\frac{RWSS_R - RWSS_{R-4}}{RWSS_{R-4}}\right] = \underset{(3.6)}{0.0283}$$

$$+ \underset{(3.1)}{0.0008}\,\frac{1}{RU} - \underset{(4.2)}{0.0097}DMY_{GP}$$

$$+ \underset{(4.4)}{0.5999}\,\frac{1}{4}\sum_{i=0}^{3}\left[\frac{CPI - CPI_{-4}}{CPI_{-4}}\right]_{-i}$$

$$- \underset{(4.9)}{0.5782} \left[\frac{RWSS_{R-4} - RWSS_{R-8}}{RWSS_{R-8}} \right]$$

$$+ \underset{(4.3)}{0.4718} \left[\frac{RWSS_{R-1} - RWSS_{R-5}}{RWSS_{R-5}} \right]$$

$$\bar{R}^2 = 0.731 \qquad SE = 0.0055 \qquad DW = 2.03$$

(A.79)
$$\left[\frac{RWSS_O - RWSS_{O-4}}{RWSS_{O-4}} \right] = \underset{(1.0)}{0.0071}$$

$$+ \underset{(3.2)}{0.0014} \frac{1}{RU} - \underset{(2.0)}{0.0049} DMY_{GP}$$

$$+ \underset{(1.7)}{0.2674} \frac{1}{4} \sum_{i=0}^{3} \left[\frac{CPI - CPI_{-4}}{CPI_{-4}} \right]_{-i}$$

$$- \underset{(2.6)}{0.3346} \left[\frac{RWSS_{O-4} - RWSS_{O-8}}{RWSS_{O-8}} \right]$$

$$+ \underset{(2.8)}{0.3914} \left[\frac{RWSS_{O-1} - RWSS_{O-5}}{RWSS_{O-5}} \right]$$

$$\bar{R}^2 = 0.587 \qquad SE = 0.0068 \qquad DW = 1.45$$

FINAL DEMAND AND GROSS PRODUCT ORIGINATING

(A.80)
$$F_{MD}^{58} = -\underset{(4.3)}{29.1841} + \underset{(7.2)}{1.2973} \Delta INV_{MD}^{58} + \underset{(2.0)}{0.4711} \Delta INV_{T}^{58}$$

$$+ \underset{(2.3)}{0.6235} EX^{58} + \underset{(9.2)}{0.8437} [C_D^{58} + I_{PDE}^{58}]$$

$$+ \underset{(4.8)}{1.3531} G_{CD}^{58} + \underset{(7.2)}{9.6516} DMY_{24} - \underset{(2.5)}{2.8153} DMY_{25}$$

$$- \underset{(1.5)}{1.3382} DMY_{DKSTR1}$$

$$\bar{R}^2 = 0.979 \qquad SE = 2.1390 \qquad DW = 1.26$$

(A.81) $$F^{58}_{MN} = \underset{(2.2)}{-11.8126} + \underset{(20.8)}{0.6936}[C^{58}_{NFB} + C^{58}_{NEFB} + G^{58}_{CN}]$$
$$+ \underset{(1.7)}{0.2670}[\Delta INV^{58}_{T} - \Delta INV^{58}_{CAR}] + \underset{(2.0)}{0.5848}\Delta INV^{58}_{MN}$$

$\bar{R}^2 = 0.985 \qquad SE = 1.6287 \qquad DW = 1.62 \qquad \rho_1 = 0.6534$

(A.82) $$F^{58}_{T} = \underset{(3.1)}{-17.0536} + \underset{(3.7)}{0.2087}C^{58}_{DA} + \underset{(3.8)}{0.4179}C^{58}_{DEA}$$
$$+ \underset{(4.3)}{0.4726}C^{58}_{NFB} + \underset{(5.7)}{0.5676}C^{58}_{NEFB}$$

$\bar{R}^2 = 0.998 \qquad SE = 0.5325 \qquad DW = 1.93 \qquad \rho_1 = 0.3960$

(A.83) $$F^{58}_{C} = \underset{(0.3)}{-1.9315} + \underset{(10.2)}{1.0547}[I^{58}_{CNFR} + I^{58}_{CER} + I^{58}_{CRAF} + G^{58}_{IC}]$$

$\bar{R}^2 = 0.970 \qquad SE = 0.7993 \qquad DW = 0.8365 \qquad \rho_1 = 0.9603$

(A.84) $$F^{58}_{R} = \underset{(3.1)}{-7.9821} + \underset{(7.3)}{0.2624}C^{58}_{S} + \underset{(1.2)}{0.1316}G^{58}_{CS}$$

$\bar{R}^2 = 0.988 \qquad SE = 0.6182 \qquad DW = 1.92 \qquad \rho_1 = 0.6930$

(A.85) $$F^{58}_{O} = \underset{(3.4)}{14.9166} + \underset{(7.9)}{0.7141}C^{58}_{S} + \underset{(3.6)}{0.0545}G^{58}_{CS}$$

$\bar{R}^2 = 0.997 \qquad SE = 0.9096 \qquad DW = 1.69 \qquad \rho_1 = 0.9900$

OUTPUT CONVERSION

$$
\begin{array}{l}
(A.86)\\(A.87)\\(A.88)\\(A.89)\\(A.90)\\(A.91)\\(A.92)\\(A.93)
\end{array}
\quad
\begin{bmatrix} \hat{X}_A^{58} \\ \hat{X}_C^{58} \\ \hat{X}_T^{58} \\ \hat{X}_R^{58} \\ \hat{X}_O^{58} \\ \hat{X}_{MD}^{58} \\ \hat{X}_{MN}^{58} \\ \hat{X}_{GE}^{58} \end{bmatrix}
=
\begin{bmatrix}
1.46397 & 0.04525 & 0.02645 & 0.03357 \\
0.02976 & 1.01411 & 0.02154 & 0.05480 \\
0.07616 & 0.12910 & 1.03212 & 0.04237 \\
0.06969 & 0.09119 & 0.06838 & 1.15032 \\
0.19137 & 0.16668 & 0.19038 & 0.16624 \\
0.07864 & 0.55824 & 0.06445 & 0.09801 \\
0.25868 & 0.15821 & 0.08993 & 0.10977 \\
0.00811 & 0.01037 & 0.02080 & 0.07036
\end{bmatrix}
$$

(A.94) $$X_A^{58} = \frac{1}{d_{A_t}} \hat{X}_A^{58}$$

(A.95) $$X_C^{58} = \frac{1}{d_{C_t}} \hat{X}_C^{58}$$

(A.96) $$X_T^{58} = \frac{1}{1.281} \hat{X}_T^{58}$$

(A.97) $$X_R^{58} = \frac{1}{1.604} \hat{X}_R^{58}$$

PRICE CONVERSION

$$
\begin{array}{l}
(A.102)\\(A.103)\\(A.104)\\(A.105)\\(A.106)\\(A.107)\\(A.108)\\(A.109)
\end{array}
\quad
\begin{bmatrix} PF_A \\ PF_C \\ PF_T \\ PF_R \\ PF_O \\ PF_{MD} \\ PF_{MN} \\ PF_{GE} \end{bmatrix}
=
\begin{bmatrix}
1.46397 & 0.04525 & 0.02645 & 0.03357 \\
0.02976 & 1.01411 & 0.02154 & 0.05480 \\
0.07616 & 0.12910 & 1.03212 & 0.04237 \\
0.06969 & 0.09119 & 0.06838 & 1.15032 \\
0.19137 & 0.16668 & 0.19038 & 0.16624 \\
0.07864 & 0.55824 & 0.06445 & 0.09801 \\
0.25868 & 0.15821 & 0.08993 & 0.10977 \\
0.00811 & 0.01037 & 0.02080 & 0.07036
\end{bmatrix}
$$

where T superscript stands for the transpose operator.

$$\begin{bmatrix} 0.05342 & 0.05061 & 0.29911 & 0.13344 \\ 0.05363 & 0.01562 & 0.02295 & 0.15350 \\ 0.04555 & 0.07743 & 0.07501 & 0.04890 \\ 0.07312 & 0.09810 & 0.10622 & 0.19775 \\ 1.21264 & 0.15223 & 0.24717 & 0.14496 \\ 0.14863 & 1.61022 & 0.13512 & 0.12618 \\ 0.14432 & 0.16316 & 1.50119 & 0.14877 \\ 0.01795 & 0.01142 & 0.01331 & 1.01571 \end{bmatrix} \begin{bmatrix} F_A^{58} \\ F_C^{58} \\ F_T^{58} \\ F_R^{58} \\ F_O^{58} \\ F_{MD}^{58} \\ F_{MN}^{58} \\ F_{GE}^{58} \end{bmatrix}$$

(A.98) $$X_O^{58} = \frac{1}{1.589} \hat{X}_O^{58}$$

(A.99) $$X_{MD}^{58} = \frac{1}{2.354} \hat{X}_{MD}^{58}$$

(A.100) $$X_{MN}^{58} = \frac{1}{3.255} \hat{X}_{MN}^{58}$$

(A.101) $$X_{GE}^{58} = \frac{1}{1.741} \hat{X}_{GE}^{58}$$

$$\begin{bmatrix} 0.05342 & 0.05061 & 0.29911 & 0.13344 \\ 0.05363 & 0.01562 & 0.02295 & 0.15350 \\ 0.04555 & 0.07743 & 0.07501 & 0.04890 \\ 0.07312 & 0.09810 & 0.10622 & 0.19775 \\ 1.21264 & 0.15223 & 0.24717 & 0.14496 \\ 0.14863 & 1.61022 & 0.13512 & 0.12618 \\ 0.14432 & 0.16316 & 1.50119 & 0.14877 \\ 0.01795 & 0.01142 & 0.01331 & 1.01571 \end{bmatrix}^T \begin{bmatrix} 0.43880 & PX_A \\ 0.41873 & PX_C \\ 0.72587 & PX_T \\ 0.61871 & PX_R \\ 0.62026 & PX_O \\ 0.43317 & PX_{MD} \\ 0.34656 & PX_{MN} \\ 0.51510 & PX_{GE} \end{bmatrix}$$

(A.110) $$P_{CDA} = \underset{(0.9)}{0.1394} + \underset{(4.2)}{0.6925}PF_{MD} + \underset{(0.7)}{0.1480}PF_T$$

$$\bar{R}^2 = 0.954 \qquad SE = 0.0143 \qquad DW = 1.98 \qquad \rho_1 = 0.8316$$

(A.111) $$P_{CDEA} = \left[0.0522 + 0.9472\left(\frac{P_{CDEA}}{P_{CDA}}\right)_{-1}\right] P_{CDA}$$

(A.112) $$P_{CNEFB} = \underset{(8.5)}{0.1779} + \underset{(5.2)}{0.5023}PF_{MN} + \underset{(3.9)}{0.3254}PF_T$$

$$\bar{R}^2 = 0.981 \qquad SE = 0.0058 \qquad DW = 0.69$$

(A.113) $$P_{CNFB} = \underset{(9.8)}{0.8301}PF_{MN} + \underset{(2.0)}{0.1710}PF_T$$

$$\bar{R}^2 = 0.983 \qquad SE = 0.0059 \qquad DW = 1.85 \qquad \rho_1 = 0.9504$$

(A.114) $$P_{CS} = \underset{(20.6)}{0.9667}PF_O + \underset{(1.0)}{0.0456}PF_R$$

$$\bar{R}^2 = 0.999 \qquad SE = 0.0030 \qquad DW = 2.06 \qquad \rho_1 = 0.9504$$

(A.115) $$P_{IBUS} = -\underset{(5.7)}{0.1400} + \underset{(16.8)}{0.9733}PF_{MD} + \underset{(4.2)}{0.1673}PF_C$$

$$\bar{R}^2 = 0.985 \qquad SE = 0.0095 \qquad DW = 1.44$$

(A.116) $$P_{ICNFR} = \underset{(8.9)}{0.2059} + \underset{(35.6)}{0.8074}PF_C$$

$$\bar{R}^2 = 0.963 \qquad SE = 0.0147 \qquad DW = 0.35$$

(A.117) $$P_{EX} = \underset{(5.4)}{0.5719} + \underset{(2.7)}{0.3353}PF_{MD} + \underset{(0.5)}{0.0943}PF_{MN}$$

$$\bar{R}^2 = 0.920 \qquad SE = 0.0085 \qquad DW = 1.55 \qquad \rho_1 = 0.8118$$

(A.118) $$P_G = -\underset{(3.4)}{0.3385} + \underset{(2.8)}{0.2667}PF_{MD} + \underset{(1.9)}{0.3462}PF_{MN}$$

$$+ \underset{(1.4)}{0.1430}PF_C + \underset{(7.0)}{0.0951}\frac{WSS_G}{E_G}$$

$$\bar{R}^2 = 0.997 \qquad SE = 0.0060 \qquad DW = 1.52 \qquad \rho_1 = 0.9009$$

(A.119) $$P_{GF} = \left[0.0587 + 0.940\left(\frac{P_{GF}}{P_G}\right)_{-1}\right] P_G$$

(A.120) $$P_C = \left[\frac{\sum_j P_{Cj} C_j^{58}}{\sum_j C_j^{58}}\right] \qquad j = DA, DEA, NFB, NEFB, S$$

(A.121) $$\underset{(0.6)}{\Delta CPI = -0.0003} + \underset{(8.8)}{1.0626}\Delta P_C$$

$$\bar{R}^2 = 0.620 \qquad SE = 0.0022 \qquad DW = 2.61$$

FINANCIAL SECTOR

(A.122) $$RES_F = RES_{NB} - RES_R$$

(A.123) $$RES_R = \left[\frac{DD_{MB}}{DD_{CB}}\right] RRR_D[DD + DD_{GF}]_{CB} + \left[\frac{DT_{MB}}{DT_{CB}}\right] RRR_T DT_{CB}$$

(A.124) $$WLTH^{58} = .114 \sum_{i=1}^{20} (0.9)^{i-1}[GNP^{58}]_{-i}$$

(A.125) $$WLTH = [P_{GNP}]_{-1} WLTH^{58}$$

(A.126) $$DEF_G = TP + TX + TC + TW - G - V_G - SUB_G - INT_G - V_{FORGF}$$

(A.127) $$\frac{CURR + DD}{WLTH} = \underset{(1.2)}{-0.0232} + \underset{(30.7)}{0.8703}\left[\frac{CURR + DD}{WLTH}\right]_{-1} - \underset{(2.8)}{0.0039} RM_{BDT} - \underset{(6.6)}{0.0026} RM_{GBS3} + \underset{(3.4)}{0.1047}\frac{Y_D}{WLTH}$$

$$\bar{R}^2 = 0.998 \qquad SE = 0.0016 \qquad DW = 1.18$$

(A.128)
$$\frac{DD}{WLTH} = \underset{(0.7)}{-0.0124} + \underset{(26.6)}{0.8522}\left[\frac{DD}{WLTH}\right]_{-1} - \underset{(3.1)}{0.0039}RM_{BDT} - \underset{(6.6)}{0.0023}RM_{GBS3} + \underset{(3.2)}{0.0845}\frac{Y_D}{WLTH}$$

$$\bar{R}^2 = 0.997 \qquad SE = 0.0014 \qquad DW = 1.19$$

(A.129)
$$\frac{DT}{WLTH} = \underset{(1.3)}{-0.0015} + \underset{(72.6)}{1.0023}\left[\frac{DT}{WLTH}\right]_{-1} + \underset{(6.0)}{0.0036}RM_{BDT} - \underset{(7.1)}{0.0020}RM_{GBS3}$$

$$\bar{R}^2 = 0.998 \qquad SE = 0.0013 \qquad DW = 0.75$$

(A.130)
$$RM_{GBS3} = \underset{(2.6)}{-0.4580} + \underset{(0.7)}{0.0860}[RM_{GBS3}]_{-1} + \underset{(6.8)}{1.0050}RM_{FRB} - \underset{(3.3)}{78.1320}\left[\frac{RES_F}{(DD+DT)_{-1}}\right] - \underset{(4.6)}{167.9075}\left[\frac{RES_F - RES_{F-1}}{(DD+DT)_{-1}}\right] - \underset{(1.2)}{3.9087}\frac{DEF_G}{WLTH}$$

$$\bar{R}^2 = 0.942 \qquad SE = 0.2195 \qquad DW = 1.46$$

(A.131)
$$RM_{GBL} = \underset{(1.7)}{0.1933} + \underset{(5.6)}{0.2035}RM_{GBS3} - \underset{(4.9)}{0.1890}[RM_{GBS3}]_{-1} + \underset{(21.3)}{0.9431}[RM_{GBL}]_{-1}$$

$$\bar{R}^2 = 0.966 \qquad SE = 0.0958 \qquad DW = 2.19$$

(A.132)
$$RM_{BDT} = \underset{(1.6)}{0.2261} + \underset{(15.0)}{0.9052}[RM_{BDT}]_{-1} + \underset{(2.9)}{0.0351}[(1 - RRR_{DD})(.65) + (1 - RRR_{DT})]RM_{GBS3} - \underset{(1.5)}{1.1181}\left[\frac{DT}{DD + DT}\right]_{-1} + \underset{(2.5)}{0.1405}DMY_{CD} + \underset{(1.3)}{0.0882}RM_{BDTM}$$

$\bar{R}^2 = 0.990 \qquad SE = 0.0742 \qquad DW = 1.49$

NONWAGE INCOME

(A.133)
$$\Delta INT_G = \underset{(2.6)}{0.0515} + \underset{(3.2)}{0.0152}(RM_{GBS3})\Delta BF_{PUB} + \underset{(2.7)}{0.0008}(BF_{PUB})\Delta RM_{GBS3}$$

$\bar{R}^2 = 0.293 \qquad SE = 0.1240 \qquad DW = 1.62$

(A.134)
$$\Delta DIV = \underset{(1.5)}{-0.0045} + \underset{(6.1)}{0.0671}[Z_A + CCA_{CORP}] - \underset{(5.1)}{0.2511}[DIV]_{-1}$$

$\bar{R}^2 = 0.514 \qquad SE = 0.2000 \qquad DW = 2.70$

(A.135)
$$\Delta Y_{ENT_{EAF}} = \underset{(2.5)}{0.1727} + \underset{(1.2)}{0.0245}\Delta\left[\sum_j WSS_j\right] + \underset{(3.6)}{0.0752}\Delta\left[\sum_j Z_{B_j}\right]$$

$j = MD, MN, T, C, R, O$

$\bar{R}^2 = 0.409 \qquad SE = 0.3000 \qquad DW = 1.48$

(A.136)
$$\Delta INT_{BUS_O} = \underset{(1.8)}{-0.0763} + \underset{(2.8)}{0.00059}\sum_{i=0}^{1}[RM_{GBL}(C_D + I_{CNFR})]_{-i}$$

$\bar{R}^2 = 0.268 \qquad SE = 0.1390 \qquad DW = 1.43$

CAPITAL CONSUMPTION ALLOWANCES

(A.137)

$$\Delta CCA_{MD} = 0.1157 + 0.0318 \left[\frac{1}{2}\sum_{i=0}^{1} (I_{BUSE_{MD}})_{-i} - CCA_{MD_{-1}}\right] + 0.1053DMY_{22} + 0.1059DMY_{23}$$

(A.138)
$$\Delta CCA_{MN} = 0.1146 + 0.0328 \left[\frac{1}{2}\sum_{i=0}^{1} (I_{BUSE_{MN}})_{-i} - CCA_{MN_{-1}}\right] + 0.0722DMY_{22} + 0.0827DMY_{23}$$

(A.139)
$$\Delta CCA_{R} = 0.0807 + 0.0170 \left[\frac{1}{2}\sum_{i=0}^{1} (I_{BUS_{R}})_{-i} - CCA_{R_{-1}}\right] + 0.1231DMY_{22} + 0.0474DMY_{23} + 0.0698DMY_{21}$$

(A.140)
$$\Delta CCA_{O*6} = 0.3031 + 0.0183 \left[\frac{1}{2}\sum_{i=0}^{1} (I_{BUS_{O*2}})_{i} - CCA_{O*6-1}\right] + 0.0681\,\frac{1}{2}\sum_{i=0}^{1} [I_{BUS_{O*2}}]_{-i} + 0.1144DMY_{23}$$

LABOR FORCE

(A.141)

$$\underset{(3.6)}{L = 15.371} + \underset{(14.3)}{0.740EHH} + \underset{(7.4)}{0.520[U]_{-1}} + \underset{(7.0)}{0.594\Delta[U]_{-1}} + \underset{(5.1)}{0.064t}$$

$$\bar{R}^2 = 0.997 \qquad SE = 0.1890 \qquad DW = 1.57$$

(A.142)
$$EHH = E_A + E_{P_{MD}} + E_{O_{MD}} + E_{P_{MN}} + E_{O_{MN}} + E_{P_T} + E_{O_T} + E_C + E_R + E_O + E_G + \epsilon_E$$

(A.143)
$$U = L - EHH$$

(A.144)
$$RU = \frac{U}{L}$$

IDENTITIES AND FIXED PROPORTIONS

Gross national product or expenditures.

(A.145) $$GNP^{58} = SF^{58} + \Delta INV^{58}$$

(A.146) $$SF^{58} = C^{58} + I^{58}_{CNFR} + I^{58}_{CO} + I^{58}_{CRAF} + I^{58}_{BUS} + EX^{58} - M^{58} + G^{58} + \epsilon_{IBUS58}$$

(A.147) $$C^{58} = C^{58}_{DA} + C^{58}_{DEA} + C^{58}_{NFB} + C^{58}_{NEFB} + C^{58}_{S}$$

(A.148) $$C^{58}_{D} = C^{58}_{DA} + C^{58}_{DEA}$$

(A.149) $$\Delta INV^{58}_{EAF} = \Delta INV^{58}_{MD} + \Delta INV^{58}_{MN} + \Delta INV^{58}_{T} + \Delta INV^{58}_{O*4}$$

(A.150) $$\Delta INV^{58} = \Delta INV^{58}_{EAF} + \Delta INV^{58}_{AF}$$

(A.151) $$I^{58}_{BUS} = I^{58}_{BUS\,AF} + I^{58}_{BUSE\,MD} + I^{58}_{BUSE\,MN} + I^{58}_{BUS\,O*2}$$

(A.152) $$M^{58} = [M^{58}_{N} + M^{58}_{S}] + M^{58}_{D}$$

(A.153) $$I^{58}_{PDE} = [0.6047 + 0.0007t - 0.6943 DMY_{IC} + 0.0067 DMY_{IC} \cdot t](I^{58}_{BUS} + \epsilon_{IBUS58}) + 11.0393 DMY_{IC} + 0.7895$$

(A.154) $$I^{58}_{CER} = [I^{58}_{BUS} + \epsilon_{IBUS58}] - I^{58}_{PDE} + I^{58}_{CO}$$

(A.155) $$[I^{58}_{CPLEAF} + I^{58}_{CAF}] = I^{58}_{CER} + I^{58}_{CRAF} - I^{58}_{CO}$$

(A.156) $$\epsilon_{IBUS58} = I^{58}_{CER} + I^{58}_{PDE} - I^{58}_{BUS} - I^{58}_{CO}$$

(A.157) $$I^{58}_{C} = I^{58}_{CNFR} + I^{58}_{CO} + [I^{58}_{CPLEAF} + I^{58}_{CAF}]$$

(A.158) $$GNP^{58}_{IC} = I^{58}_{C} + G^{58}_{IC}$$

(A.159) $$C_{D} = [P^{58}_{CDA}][C_{DA}] + [P_{CDEA}][C^{58}_{DEA}]$$

(A.160) $$EX^{58}_{D} = (0.4189 - 0.0011t)EX^{58}$$

(A.161) $$SF^{58}_{D} = C^{58}_{D} + I^{58}_{PDE} + G^{58}_{CD} + EX^{58}_{D} - M^{58}_{D}$$

(A.162) $$EX^{58}_{N} = (0.3897 - 0.0008t)EX^{58}$$

(A.163) $$M^{58}_{N} = (0.5422 - 0.0002t)(M^{58}_{N} + M^{58}_{D})$$

(A.164) $$SF^{58}_{N} = C^{58}_{NFB} + C^{58}_{NEFB} + G^{58}_{CN} + EX^{58}_{N} - M^{58}_{N}$$

(A.165) $$I_{BUS_j} = [P_{IBUS}][I^{58}_{BUS_j}]$$

$$j = MD, MN, R, O*2, AF$$

(A.166) $$\Delta INV = [PM_{AF}][\Delta INV^{58}_{AF}] + [WPI_{MD}][\Delta INV^{58}_{MD}] + [WPI_{MN}][\Delta INV^{58}_{MN}] + [.5462WPI_{MD} + .3722WPI_{MN} + .0816P_{MAF}][\Delta INV^{58}_{O*4}] + \epsilon_{\Delta INV}$$

(A.167) $$I_{CO} = [PICNFR][I^{58}_{CO}] + \epsilon_{ICO}$$

(A.168) $$I_{CNFR} = [PICNFR][I^{58}_{CNFR}]$$

(A.169) $$I_{FIXER} = \sum_j I_{BUS_j} + [P_{IBUS}][\epsilon_{IBUS58}] + \epsilon_{IBUS} + I_{CO}$$

$$j = MD, MN, R, O*2, AF$$

(A.170) $$I^{58}_{BUS_{EAF}} = I^{58}_{BUS} - I^{58}_{BUS_{AF}}$$

(A.171) $$GNP = [P_C][C^{58}] + I_{CNFR} + \Delta INV + I_{FIXER} + [P_{EX}][EX^{58}] - [P_M][M^{58}] + G + I_{CRAF}$$

(A.172) $$P_{GNP} = \frac{GNP}{GNP^{58}}$$

(A.173) $$M^{58} = [M^{58}_N + M^{58}_S] + M^{58}_D$$

Relations among gross national product, national income, personal income and disposable personal income.

(A.174) $$Y_N = GNP - CCA - TX - V_{BUS} - STAT + SUB_G$$

(A.175) $$CCA = \sum_j CCA_j + \epsilon_{CCA}$$

$$j = A, MD, MN, R, O*6$$

(A.176) $$CCA_{CORP} = 0.6086CCA$$

(A.177) $$Z_B = Y_N - WSS - Y_{ENT} - Y_{RENT} - INT_{BUS} + WALD$$

(A.178) $$WSS = \sum_j WSS_j + \epsilon_{WSS}$$

$$j = A, G, W, MD, MN, T, C, R, O$$

(A.179) $$WSS_j = [RWSS_j][MH_j]$$

$$j = MD, MN, T, C, R, O$$

(A.180) $$INT_{BUS} = \sum_j INT_{BUS_j}$$

$$j = A, C, T, R, O, MD, MN, W$$

(A.181) $$Z_{BU} = Z_B - IVA_{CORP}$$

(A.182) $$Z_{AU} = Z_{BU} - TC$$

(A.183) $$Z_A = Z_{AU} + IVA_{CORP}$$

(A.184) $$RE = Z_{AU} - DIV$$

(A.185) $$Y_P = Y_N - RE - TC - IVA_{CORP} - TW - WALD + V_G + INT_G + V_{BUS} + INT_{CON}$$

(A.186) $$Y_D = Y_P - TP$$

(A.187) $$Y_D^{58} = Y_D/P_C$$

(A.188) $$T = TP + TC + TX + TW$$

Miscellaneous relationships.

(A.189) $$X_{Kj}^{58} = [X_j^{58}]_{1953.2}\left[1.0 + \frac{r}{4}\right]^{t-34}$$

$$j = MD, MN, R, O*6$$

$$r = 0.035 \text{ in } 1953{:}3\text{–}1965{:}4$$

$$r = 0.040 \text{ in } 1966{:}1\text{–}1990{:}4$$

(A.190) $$K_{MD}^{58} = K_{MD_{-1}}^{58} + .25[I_{BUS_{MD}}^{58} - .1638K_{MD_{-1}}^{58}]$$

(A.191) $$K_{MN}^{58} = K_{MN_{-1}}^{58} + .25[I_{BUS_{MN}}^{58} - .1118K_{MN_{-1}}^{58}]$$

(A.192) $$K_R^{58} = K_{R_{-1}}^{58} + .25[I_{BUS_R}^{58} - 0.778K_{R_{-1}}^{58}]$$

(A.193) $$K_{O*2}^{58} = K_{O*2_{-1}}^{58} + .25[I_{BUS_R}^{58} - .1575K_{O*2_{-1}}]$$

(A.194) $$GNP_K^{58} = [GNP_K^{58}]_{1953.2}\left[1.0 + \frac{r}{4}\right]^{t-34}$$

$$r = 0.035 \text{ in } 1953{:}3\text{–}1965{:}4$$

$$r = 0.040 \text{ in } 1966{:}1\text{–}1990{:}4$$

(A.195) $$J_{CAP_{MD}} = 0.000673 + 0.99[J_{CAP_{MD}}]_{-1} + 1.0461\left[\frac{X^{58}_{MD}}{X^{58}_{K_{MD}}} - 0.99\left(\frac{X^{58}_{MD}}{X^{58}_{K_{MD}}}\right)_{-1}\right]$$

(A.196) $$J_{CAP_{MN}} = 0.00176 + .9682[J_{CAP_{MN}}]_{-1} + .9012\left[\frac{X^{58}_{MN}}{X^{58}_{K_{MN}}} - 0.9682\left(\frac{X^{58}_{MN}}{X^{58}_{K_{MN}}}\right)_{-1}\right]$$

(A.197) $$K^{58}_{CDEA} = .25C^{58}_{DEA} + .92784[K^{58}_{CDEA}]_{-1}$$

(A.198) $$K^{58}_{CDA} = .25C^{58}_{DA} + .925[K^{58}_{CDA}]_{-1}$$

(A.199) $$X^{58}_{M} = X^{58}_{MD} + X^{58}_{MN}$$

(A.200) $$O^{58}_{U_M} = O^{58}_{U_{MD}} + O^{58}_{U_{MN}}$$

(A.201) $$B^{58} = EX^{58} - M^{58}$$

(A.202) $$H = \frac{\left[\begin{array}{r} H_{P_{MD}}E_{P_{MD}} + H_{P_{MN}}E_{P_{MN}} + H_{P_T}E_{P_T} \\ + H_RE_R + H_CE_C + H_OE_O \end{array}\right]}{[E_{P_{MD}} + E_{P_{MN}} + E_{P_T} + E_R + E_C + E_O]}$$

(A.203) $$RWSS = \frac{[WSS - WSS_A - WSS_G]}{[EHH - E_A - E_G]}$$

LIST OF VARIABLES AND DEFINITIONS

MONETARY variables are in billions of dollars, seasonally adjusted. Monetary stock variables are, unless otherwise indicated, end-of-period; and monetary flow variables, including changes in stocks between ends of periods, are at annual rates. In the definitions, the variables are generally defined as if they are in current dollars. In the equations, the distinction is made between current and constant 1958 dollars. Variables in the latter units are superscripted 58. Other modifiers of the variables are:

1. Sector subscripts. These refer only to producing sectors and government; those that appear in the system of equations presented here are as follows:

A	Agriculture, forestry, and fisheries
AF	Farming
C	Contract construction
EAF	Nonfarm business
F	Federal government (used only as a subscript for government expenditure variables)
G	Government and government enterprises
GE	Government enterprises
GF	Federal government
GSL	State and local government
M	Manufacturing
MD	Durables manufacturing
MN	Nondurables manufacturing
O	Residual industries: mining; finance, insurance, and real estate; and services
$O*2$	Mining, wholesale and retail, services, finance, and contract construction
$O*4$	All industries except manufacturing, wholesale and retail trade, and farming
$O*6$	Wholesale and retail trade and contract construction plus residual industries (mining; finance, insurance, and real estate; and services)
R	Regulated industries: railroad and nonrail transportation, communications, and public utilities
T	Wholesale and retail trade

2. *Other subscripts are defined with the variables to which they apply.* The variables in alphabetical order are:

A_i Almon weights for investment equations

$A_0 = .074 \quad A_{-1} = .132 \quad A_{-2} = .170 \quad A_{-3} = .183$

$A_{-4} = .171 \quad A_{-5} = .138 \quad A_{-6} = .091 \quad A_{-7} = .041$

B	Net exports of goods and services
BF_{PUB}	Marketable Federal debt held outside the Federal Reserve and U.S. government agencies and trust funds, average during quarter

C	Personal consumption expenditures on goods and services
CCA	Capital consumption allowances
C_D	Personal consumption expenditures on durable goods
C_{DA}	Personal consumption expenditures on new and net used automobiles
C_{DEA}	Personal consumption expenditures on durable goods other than automobiles
C_{NEFB}	Personal consumption expenditures on nondurable goods other than food and beverages
C_{NFB}	Personal consumption expenditures on food and beverages
CPI	Consumer price index, 1958 = 1.00
C_S	Personal consumption expenditures on services including imputations
$CURR$	Currency in the hands of the nonbank public, average during quarter
d	Ratio of gross output to output originating
DD_{CB}	Private demand deposit liabilities of commercial banks less interbank deposits, cash items in process of collection, and Federal Reserve float, average during quarter
$DD_{GF_{CB}}$	Federal government demand deposits at commercial banks, average during quarter
DD_{MB}	Demand deposits subject to reserve requirements at Federal Reserve System member banks, average during quarter
DEF_G	Government surplus or deficit on income and product accounts
DIV	Dividends
DMY_1	Dummy variable representing a productivity shift, 0.0 in 1954.1 through 1960.1, 1.0 thereafter
DMY_2	Dummy variable representing a productivity shift, 0.0 in 1954.1 through 1963.1, 1.0 thereafter
DMY_3	Dummy variable representing a productivity shift, 0.0 in 1954.1 through 1963.3, 1.0 thereafter
DMY_{15}	Dummy variable to convert from Bureau of the Census value of new private nonfarm residential

buildings put in place to *GNP* expenditures on private residential nonfarm new construction, both in 1958 dollars

DMY_{21} Dummy variable representing the investment boom in 1955, 1.0 in 1955.1 through 1955.4, 0.0 elsewhere

DMY_{22} Dummy variable representing a change in the investment tax credit, 1.0 in 1962.1 through 1962.4, 0.0 elsewhere.

DMY_{23} Dummy variable representing the investment tax credit, 0.0 in 1954.1 through 1961.4, 1.0 elsewhere

DMY_{24} Dummy variable, 0.0 in 1954.1 through 1960.1, 1.0 thereafter

DMY_{55} Dummy variable representing the 1955 easing of consumer credit, 1.0 in 1955.1 through 1955.4, 0.0 elsewhere

DMY_{CD} Dummy variable representing the establishment of the market for certificates of deposit, 0.0 in 1954.1 through 1960.4, .82 in 1961, 1.0 in 1962, .96 in 1963, .74 in 1964, and 1.0 thereafter

DMY_{DKSTR} Dummy variable representing longshoremen's strikes, −1.0 in 1954.1, 1956.4, 1957.1, 1959.4, 1962.4, 1963.1, 1965.1, 0.0 elsewhere

DMY_{DKSTR1} Dummy variable representing longshoremen's strikes and incorporating anticipatory and make-up effects, −1.0 in 1954.1, 1.0 in 1954.2, 1.0 in 1956.3, −1.0 in 1956.4, 0.5 in 1957.1, 1.0 in 1959.3, −1.5 in 1959.4, 0.5 in 1960.1, 0.5 in 1962.3, −0.5 in 1962.4, −1.0 in 1963.1, 0.5 in 1963.2, −1.0 in 1965.1, 1.0 in 1965.2, 0.0 elsewhere

DMY_{GP} Dummy variable representing the wage guide posts, 0.0 in 1954.1 through 1961.4, 1.0 in 1962.1 through 1965.4

DMY_{ITC} Dummy variable representing the investment tax credit, 0.0 from 1954.1 through 1961.4, 1.0 in 1962.1 through 1965.4

DMY_{STLWT2} Dummy variable representing anticipation of steel strikes occurring after foreign producers became competitive in the U.S. market, 1.0 in 1959.2, 2.0

	in 1959.3, 2.0 in 1959.4, 1.0 in 1965.1, and 0.0 elsewhere
DMY_{STR}	Dummy variable representing strikes in the automobile industry and incorporating make-up effects, −1.0 in 1958.4, 1.0 in 1959.1, −1.0 in 1961.3, −1.0 in 1961.4, −1.0 in 1967.3, −1.0 in 1967.4, 1.0 in 1968.1, 0.0 elsewhere
DMY_{TP}	Dummy variable representing the 1964 tax cut, 0.0 in 1954.1 through 1963.4, 1.0 in 1964.1 through 1965.4
DMY_{TX}	Dummy variable representing a change in the excise tax rate, 0.0 in 1954.1 through 1960.1, 1.0 in 1960.2 through 1965.4
$DODMPCA$	Department of Defense military prime contract awards for work performed in the U.S.
DT_{CB}	Time deposits at all commercial banks other than those due to domestic commercial banks and the U.S. government, average during quarter
DT_{MB}	Time deposits at Federal Reserve System member banks other than those due to domestic commercial banks and the U.S. government, average during quarter
EHH	Employment, as reported in the household survey, millions of persons, average during quarter
E_O	Employment of nonproduction workers, as reported in the payroll survey, millions of persons, average during quarter
E_P	Employment of production workers, as reported in the payroll survey, millions of persons, average during quarter
EX	U.S. exports of goods and services
EX_D	U.S. exports of durable goods
EX_N	U.S. exports of nondurable goods
EX_W	World exports excluding U.S. exports
ϵ_{CCA}	Capital consumption allowances epsilon: the difference between capital consumption allowances in the national income accounts and the sum of the same

concept by industry from quarterly interpolations of annual data (on an establishment basis)

$\epsilon_{\Delta INV}$ Inventory investment epsilon: the difference between current dollar inventory investment in the gross national product accounts and the sum of real inventory investment inflated

ϵ_E Employment epsilon: the difference between employment estimates based on the Bureau of Labor Statistics' household survey, from which unemployment estimates are derived, and the sum of employment by industry from BLS's establishment survey

ϵ_{IBUS} Business investment epsilon: the difference between the sum of expenditures on producers' durable equipment and business construction expenditures as reported in the *GNP* accounts and the sum of such investment by industry

ϵ_{ICO} The difference between the current dollar balance of new private nonfarm, nonresidential, nonbusiness construction put in place and the real value of such construction, inflated by the implicit price deflator for nonfarm residential construction

F Estimated final demand

G Government purchases of goods and services

G_{CD} Government purchases of durable goods

G_{CN} Government purchases of nondurable goods

G_{CS} Government purchases of services

G_{IC} Government expenditures on new construction

GNP Gross national product

GNP_{IC} Construction component of gross national product

GNP_K Potential gross national product

H Average weekly hours of all workers, hours

HH Number of households, millions

H_P Average weekly hours of production or nonsupervisory workers, hours

HU_{AVL} Number of housing units available, millions

HU^1_{STS} Number of single-family housing units started, millions

HU^2_{STS}	Number of two-family housing units started, millions
HU^3_{STS}	Number of multiple-family housing units started, millions
HU_{VAC}	Vacant available housing units, millions
I_{BUS}	Business gross investment in plant and equipment
I_C	New construction component of gross private domestic investment
I_{CER}	Gross private domestic investment in nonresidential structures
I_{CNFR}	New private nonfarm residential construction, *GNP* basis
I_{CNFREH}	Value of new private nonfarm residential construction excluding housing units put in place (additions and alterations plus nonhousekeeping buildings)
I_{CNFRH}	Value of new private nonfarm housing units put in place
I_{CNFR^*}	Value of new private nonfarm residential buildings put in place, Bureau of the Census basis
I_{CO}	Value of new private nonfarm, nonresidential, nonbusiness construction put in place, billions of dollars
I_{CPL}	Business construction
I_{CRAF}	New farm residential construction
I_{FIXER}	Gross private domestic investment in nonresidential structures and producers' durable equipment
INT_{BUS}	Personal interest income paid by business
INT_{CON}	Personal interest income paid by consumers
INT_G	Personal interest income paid by government
INV	The stock of business inventories
INV_{CAR}	Dealers' automobile inventories
I_{PDE}	Investment in producers' durable equipment
IVA	Corporate and unincorporated enterprises' inventory valuation adjustment
K	Stock of business fixed capital
K_{CDA}	Stock of consumers' automobiles
K_{CDEA}	Stock of consumers' durable goods other than automobiles
L	Civilian labor force, millions of persons

M	Imports of goods and services
M_D	Imports of durable goods
M_N	Imports of nondurable goods
M_S	Imports of services
MH	Total man-hours, billions per year
MH_O	Man-hours of nonproduction workers, billions per year
MH_P	Man-hours of production or nonsupervisory workers, billions per year
NR	Total resident population, millions of persons, average during quarter
O	Manufacturers' net new orders
$OASI_{BA}$	Salary base for determining payments to the Old-Age, Survivors, and Disability Insurance program (OASDI), thousands of dollars
$OASI_{PR}$	Percentage of employees covered by the OASDI program
$OASI_{RT}$	Percentage of base salary paid into OASDI, sum of employees' and employers' contributions
O_U	Manufacturers' unfilled orders
P_C	Implicit price deflator for personal consumption expenditures, 1958 = 1.0
P_{CDA}	Implicit price deflator for personal consumption expenditures on new and used automobiles, 1958 = 1.0
P_{CDEA}	Implicit price deflator for personal consumption expenditures on durable goods other than new and used automobiles, 1958 = 1.0
P_{CNEFB}	Implicit price deflator for personal consumption expenditures on nondurable goods other than food and beverages, 1958 = 1.0
P_{CNFB}	Implicit price deflator for personal consumption expenditures on foods and beverages, 1958 = 1.0
P_{EX}	Implicit price deflator for exports of goods and services, 1958 = 1.0
P_{EXW}	Unit value index of world exports excluding U.S. components, 1958 = 1.0

P_F	Implicit price deflators for the final demand sectors, 1958 = 1.0
P_G	Implicit price deflator for total government purchases of goods and services, 1958 = 1.0
P_{GF}	Implicit price deflator for Federal government purchases of goods and services, 1958 = 1.0
P_{GNP}	Implicit price deflator for gross national product, 1958 = 1.0
P_{IBUS}	Implicit price deflator for business gross investment in plant and equipment, 1958 = 1.0
P_{ICNFR}	Implicit price deflator for new private nonfarm residential construction, 1958 = 1.0
P_M	Implicit price deflator for imports
PM_{AF}	Implicit price deflator for value of cash receipts from farm marketing and CCC loans plus value of farm products consumed directly in farm households
PM_{ICRD}	Average cost per unit of private housing starts, thousands of dollars
$PM_{(N+S)}$	Implicit price deflator for imports of nondurable goods and services, 1958 = 1.0
PR	Index of prices of raw materials in manufacturing, 1958 = 1.0
PX	Implicit price deflator for gross product originating, 1958 = 1.0
RE	Undistributed corporate profits
RES_F	Free reserves of Federal Reserve member banks, average during quarter
RES_{NB}	Nonborrowed reserves of Federal Reserve member banks, average during quarter
RES_R	Required reserves of Federal Reserve member banks, average during quarter
RM_{BDT}	Yield on commercial bank time deposits, per cent
RM_{BDTM}	Maximum rate payable on time deposits under Regulation Q
RM_{FRB}	Federal Reserve Bank of New York discount rate, average during quarter, per cent
RM_{GBL}	Yield on U.S. government securities maturing or

	callable in ten years or more, average during quarter, per cent
RM_{GBS3}	Market yield on three-month U.S. Treasury bills, average during quarter, per cent
RRR_D	Effective required reserves ratio for demand deposits at Federal Reserve member banks, average during quarter
RRR_T	Effective required reserves ratio for time deposits at Federal Reserve member banks, average during quarter
RU	Rate of unemployment
$RWSS$	Compensation of employees per man-hour including supplements, dollars
SF	Final sales, gross national product less change in inventories
SF_D	Final sales of durable goods
SF_N	Final sales of nondurable goods
$STAT$	Statistical discrepancy in the reconciliation of gross national product with national income
SUB	Subsidies less current surplus of government enterprises
t	Time trend where 1946:1 = 1 and 1954:1 = 37
T	Government receipts
TC	Corporate profits tax liability
TC_{RT}	Corporate profits tax rate
TP	Personal tax and nontax receipts (or payments)
TW	Contributions for social insurance
TX	Indirect business tax and nontax accruals
U	Unemployed in the civilian labor force
$UCCA$	Unit capital consumption allowances (capital consumption allowances per unit of real gross product originating), dollars per dollar of real product
$UCCA^N$	Normal unit capital consumption allowances
$UINS_{RT}$	The unemployment insurance tax rate
ULC	Unit labor cost (compensation of employees per unit of gross product originating), dollars per dollar of real product

ULC^N	Normal unit labor costs
V_{FOR_G}	Net government transfer payments to foreigners
V_G	Government transfer payments to persons
$V_{OASI_{GF}}$	Old age and survivors insurance benefits
V_{OG}	Government transfer payments to persons other than old age and survivors insurance benefits, state unemployment insurance benefits and veterans' benefits
$V_{US_{GF}}$	State unemployment insurance benefits
V_{VET}	Veterans' benefits
$WLTH$	Wealth, a weighted moving average of GNP
WPI	Wholesale price index, 1958 = 1.0
WS_G	Wages and salaries in government
WSS	Total compensation of employees (wages, salaries, and supplements)
X	Gross product originating, by sector
X_K	Potential gross product in the producing sector
Y_D	Disposable personal income
Y_{ENT}	Proprietors' income
Y_N	National income
Y_P	Personal income
Z_A	Corporate profits after taxes, including inventory valuation adjustment
Z_{AU}	Corporate profits after taxes, excluding inventory valuation adjustment
Z_B	Corporate profits before taxes, including inventory valuation adjustment
Z_{BU}	Corporate profits before taxes, excluding inventory valuation adjustment

DISCUSSION

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The Brookings Model, née the SSRC Model, has now been in existence for over ten years, which is a long enough period of time to establish some trends. For one thing, the model, formerly the biggest one of all, seems to get a little smaller at each appearance. The government sector originally contained some thirty-odd equations, with at least a dozen for state and local receipts and expenditures. Now the model is down to one transfer and four tax equations, and state and local government receipts have vanished entirely. Furthermore, the tax functions themselves look pretty scruffy, which is particularly surprising in light of all the tax research that has been conducted at Brookings. One of the boasts of the original model was that it used tax rates and tax bases, but the present model uses neither. The tax rate, of course, has little meaning if we combine federal receipts with state and local receipts. The tax base or income variable used for estimating personal taxes is the national income accounts variable, personal income. This variable includes transfer payments and excludes personal contributions for social insurance, just the opposite of the Internal Revenue Service definition of taxable income. Indirect taxes are a straight percentage of *GNP*, minus a constant, with a dummy variable apparently intended to reflect changes in federal excise tax rates. Corporate tax liabilities are regressed on the product of corporate profits and "the tax rate" (federal plus state and local, presumably). An advantage claimed for the original model was the endogeneity of many government expenditures. Now we have just one equation: state unemployment insurance payments.

Another area of shrinkage in the model involves the estimation of the labor force. Originally there were thirty or forty equations explaining participation rates and even marriage rates. Now we have one equation making the labor force a function of employment, lagged unemployment, and a time trend. While it may be unreasonable to expect an econometric model to predict the marriage rate, I do think that demographic factors should have some influence on the labor force.

Another trend that I think I observe in the model is an increased willingness to use dummy variables. This is the way the investment credit is handled, for example. I am rather surprised that the credit is estimated to reduce corporate taxes by only $0.7 billion. That is a lot lower than Treasury estimates. The impact of the investment credit on business expenditures shows up as three additive and multiplicative dummy terms. Other variables determining business investment are the long-term interest rate on government bonds and capacity utilization or capital stock or both, depending on the sector being examined. Some of Dale Jorgenson's pioneering work on investment functions was done in connection with this model, and I question whether the present formulations are an advance over his work.

Dummy variables used for personal tax changes yield curious results. The 1964 tax cut is represented by a dummy that serves both as a constant term and as a multiplier of personal income. I have two observations to make on this procedure. First, only two-thirds of the tax cut was effective in 1964; the rest came in 1965. Second, even at 1965 income levels, the implied estimate of the tax cut is only $7 billion, about three-fourths of the Treasury estimate.

Perhaps the ultimate in dummy variables is shown in the equation for consumer expenditure on durables excluding automobiles, where a dummy variable is included even though its coefficient is less than its standard error!

Another difference between this and earlier versions of the model is the sample period. Formerly, the equations were fit to 1948–60, but the authors state that analysis of covariance tests indicated significant shifts in many coefficients between 1948–53 and 1954–60, so the present model was fit to 1954–65 "to select a sample period germane to the analysis of current economic problems" (page 201). This seems to me a mistaken procedure. Granted that the Korean War period was one of great instability, with horrifying effects on correlation coefficients and standard errors, I think we should hesitate before restricting ourselves to more homogeneous observations. Limiting the sample period to 1954–65, for example, means that we have *no* observations in which unemployment was below 4 per cent! Is this sample really "germane to the analysis of current economic problems"? I am not surprised that statistically significant shifts occurred in the period after 1953. The economy entered a period of stagnation from which it did not emerge until it reached its potential again in 1966. Is this the experience most germane to an analysis of the current inflationary situation? In light of the relative homogeneity of the sample period, it seems a shame the authors did not make use of observations after 1965. In addition to different unemployment levels and price movements, we have experienced large swings in residential construction since 1965 and reached the end of the strong upward trend in nonresidential construction in the fourth quarter of 1965. I would be willing to bet that the model failed to pick up either of these last two phenomena.

Turning to applications of the model, there are three analyses in the present paper, the simulation of five National Bureau "turning points"; a sample period simulation covering 1957–65; and a post-sample, twenty-five-year simulation extending from 1966 to 1990.

The format of this conference provided no criteria for the performance of models at turning points, so it is difficult, if not impossible, to evaluate them. For example, is it better to forecast the precise timing of a turning point but badly miss the numerical magnitude, or to be close to the correct magnitude even if the direction of movement is wrong? Past disagreements between econometricians and adherents of the indicators approach have been based on just this distinction. The econometric model builder has concerned himself with minimizing squared residuals, be they dollars, unemployment percentages, or interest rates. The sign of the derivative of a variable with respect to time does not really matter to him, and a change in this sign is important only if it affects the error of the equation or model. Given this approach, it is hardly surprising that most econometric models move much more smoothly than the economy, lagging behind turning points, underestimating amplitudes (both high and low), but "on the average" being not far off.

A "business cycles indicator" researcher, on the contrary, is concerned with dating and forecasting turning points. Less interested in the magnitude of a series than in the sign of its first difference, he deals with indexes of economic performance that are aggregated differently from those of econometric models (e.g., diffusion indexes). It is, therefore, hardly surprising that forecasts based on this approach tend to be qualitative, focusing on the probability that a turning point will or will not occur. The quantitative aspects of such a forecast tend to be adjectival ("vigorous," "weak") rather than numerical.

We cannot label either of these approaches the "correct" one. If we are at, or think we are at, a turning point, a leading indicator's analysis may be of more interest to us; otherwise, we may prefer to focus on the output of an econometric model. As long as the two approaches remain as different from one another as they are at present, the one to which we turn depends on what information we have at hand and what questions we are asking.

In line with this, the Brookings Model simulations turn out to have root mean square errors that are larger for turning points than for non-turning point periods, but not very much larger. While the magnitude of the variables is forecast rather well, the turning points are not, and the peaks and troughs are underestimated.

The "longer-run simulations" cover the last three-fourths of the sample period. Charts 12–15 indicate that the model tracked real *GNP* and business investment rather well for the post-1960 expansion, but did not perform very well during the earlier, less stable years. The price estimates appear subject to severe serial correlation errors, and the residential construction simulation does not seem particularly good. Attempts to simulate the 1966 "credit crunch," which was outside the period of fit, might have been instructive.

For the purposes of the twenty-five-year simulation (1966–90), certain adjustments were necessary in tax rates and productivity equations. Specifically, after expiration of the surcharge, the personal tax rate was *increased* each quarter until 1976, then held constant. The authors state that this was done "in order to keep disposable income from growing too rapidly and also to limit government deficits." This result is certainly different from most long-run projections, which typically show the necessity of periodic tax *cuts* to reduce what used to be referred to as "fiscal drag." The model's low personal tax elasticity, combined with the assumed rising government share of current dollar *GNP*, changes projections of "fiscal dividends" to "fiscal deficits," and certainly warrants further discussion by the authors.

The authors also felt it necessary to alter some of the time trends in the production man-hour equations in order to raise productivity increases to what they considered more "reasonable" levels. Since the productivity equations seem to be influenced mainly by various "shift" dummy variables during the sample period, I wonder if productivity should really be considered an endogenous variable in this model.

The control solution produced very smooth paths to 1990, presumably the result of smooth extrapolation of the exogenous variables. None of the variables in Charts 16–19 seems to display any cyclical behavior after 1970 in the control solution, although there seems to be considerably more variation in the "representative stochastic simulation." In fact, the residential construction series (Chart 19) exhibits strong cycles that look as if they would become explosive in the late 1980's.

In conclusion, let me say that I approach econometric models as both a producer and a consumer. As a producer who has spent some years attempting to develop improved forecasts, I am filled with humility for my own efforts and admiration for the success of others in building models. As a consumer of models, who is supposed to provide technical assistance to policymakers in the government, I am often appalled at how inappropriate models can be. Consider the economic policy issues that agitated the government during the last ten years: the investment credit and its suspension and revocation, the 1964 tax cut, the 1965 excise cuts, and the 1968 surcharge with its extensions. To be honest, I think the only time econometric models had a major impact on policy decisions came in 1968. At that time, everyone's model showed that the Federal Reserve should ease up on monetary policy to avoid a recession in 1969. I am afraid we still have quite a way to go. If I have seemed critical of the Brookings Model, it is because I was speaking as a consumer of econometric models. Speaking as a producer, I will confess that I agree with the opening sentence of the paper: "Predictions with econometric models, even thirty years after Tinbergen's initial attempt, still involve art as well as science."

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The paper by Fromm, Klein, and Schink (hereafter FKS) is the fourth version of the Brookings Model to appear in print. Version I, the set of equations presented in the individual chapters of the 1965 Brookings volume [7], has been extensively criticized [16] but never solved nor simulated. Version II was the abbreviated version presented at the end of the 1965 volume, which was solved but never simulated. The equations of version III are presented by Fromm and Taubman in an appendix of [13] and have been used to derive dynamic policy multipliers, but the transition from version II to version III has never been rigorously justified and the equations of the latter are presented in [13] denuded of all measures of goodness of fit or other statistical information.

Version IV replaces version III, as version III replaced version II, with scarcely a word of explanation. Old variables are dropped and new ones appear, with goodness of fit or a "structural change" during the sample period as virtually the only criteria for replacement offered in the cursory explanation by FKS. During the span of roughly six years since the articles in the original Brookings volume were written, almost no published or unpublished articles have been written to justify either theoretically or econometrically the changes made from version to version. In contrast with the MIT-FRB and Wharton Models, which are both supported by a considerable body of theoretical and econometric literature, the Brookings Model has been transformed so many times since its inception that it is now a model almost devoid of theory, with equations altered and dummy variables added wherever necessary to maximize the model's ability to produce a control solution that accurately tracks *GNP* during the sample period.[1] In version IV many vari-

[1] Despite the recent publication of a second set of econometric papers by the Brookings project [8], none of the equations in version IV has incorporated any of the results of the new papers. In contrast, the equations of the Wharton-EFU Model have been copiously justified and defended ([9] [11] and other references cited there). The MIT-FRB Model is based on several well-known theoretical models [1] [3] [19], and the specific assumptions underlying the financial sectors and monetary channels are described in [5] and [6].

ables are included in the equations and thus influence the simulation results even if their coefficients are not significant (there are thirty-six coefficients with "t" ratios below 2.0 in the first seventy-nine equations). As the specification of the model becomes more arbitrary in successive versions, the less one is likely to trust its policy multipliers or secular simulations, yet richness of simulation detail has always been an important justification for the continuation of research on a model as large as Brookings. And even the details are gradually being sacrificed as the model shrinks in over-all size between successive versions. In light of these developments, which continually reduce the model's margin of disaggregation over competing models with no offsetting theoretical or statistical innovations, one is left with the impression that the model project has lost its sense of direction.

A natural point of departure for these comments is the "menu of revisions" suggested in my recent critical review [15] of version III. To what extent have the major weaknesses of that version been corrected in the new version IV used for the present simulations? Are any important weaknesses introduced in version IV that were absent in the previous version? Do the simulation results appear to be accurate representations of the cyclical and secular features of the real world, or are some of the results of questionable validity due to the particular assumptions made in specifying the model?

1. THE MODEL

Final Expenditure Equations. As in previous versions, consumption is disaggregated into five components. In version III the absence of a flexible accelerator in the auto equation contributed to the sluggishness of the model in simulations. This defect has now been cured, since the lagged stock of automobiles appears with a negative coefficient in the auto equation as well as that for nonauto durables. Previous critics [9] [16] noted that in earlier versions the long-run marginal propensity to consume implied by the five equations taken together was much lower than the average postwar propensity to consume of about .92. Version IV appears to err in the opposite direction, with a long-run marginal propensity to consume of 1.175. This high propensity will cause long-run policy multipliers to be misleadingly high when these

TABLE 1

Propensity to Consume

	Brookings Marginal Propensity to Consume		Actual Average Propensity to Consume, 1969
	Impact (1)	Long-Run (2)	(3)
Autos	.2204	.1322	.0786
Nonauto durables	.0913	.0700	.0640
Food and beverages	.0655	.0710	.1910
Other nondurables	.1451	.1755	.1964
Services	.0529	.7260	.3890
	.5752	1.1747	.9190

Source by column: Columns 1 and 2—Calculated from equations (A.1)–(A.5), (A.197), and (A.198) in appendix to FKS paper; *Column 3*—*Survey of Current Business* (April 1970), Table 11, p. 9.

are eventually calculated for version IV, and it is responsible for the increase in the ratio of real consumption to *GNP* in the 1965–90 simulations presented in the FKS paper. Table 1 suggests that the equation for services is the primary culprit responsible for the excessively high long-run marginal propensity to consume.

The present set of consumption equations, as in previous versions, fails to allow for any direct influence of monetary policy on consumption. Thus monetary policy multipliers calculated for the Brookings Model are likely to be smaller than those for the MIT-FRB Model, where total consumption is a function of real wealth (which is influenced by monetary policy via stock prices) and where durables consumption depends on interest rates.[2] And we might expect overpredic-

[2] Although econometric evidence is preferable to anecdotes, direct monetary influence on consumption is supported by frequent reports in the financial press in 1969–70 of reduced consumption of luxury goods, attributed to the drop in stock prices. One also notes the marked decline in the average propensity to consume between the first and last halves of 1966, and the first and last halves of 1969, both of which were years characterized by much slower rates of growth of monetary aggregates in the last half than in the first half.

tions of *GNP* in simulations following periods of monetary tightness.

The residential construction equations suffer from a failure to distinguish between the separate influence of monetary factors on the demand for housing and the supply of credit for housing. One would expect demand to be a function of the mortgage rate, which is much less volatile than the Treasury bill used by Brookings. (Housing demand would also be expected to depend on household formations, tax rates and the expected rate of capital gains.) The supply of housing credit, on the other hand, depends on the gap between short-term market interest rates and deposit rates at banks and savings institutions. The Brookings Model would probably underpredict housing expenditures for periods like 1967 and the last half of 1968, when the Treasury bill rate was relatively high compared to the 1954–65 sample period but the supply of credit to the housing market was ample because deposit rates were high relative to the Treasury bill rate. Another weakness, which the housing equations have in common with many others, is that lags on interest rates and other variables are fixed arbitrarily rather than estimated statistically by the numerous methods now available.

The change in inventories causes difficulties in all models, but the Brookings equations do an unusually poor job of fitting the sample period in all sectors but manufacturing durables. This is unfortunate, since inventory change has been the main contributor to the timing pattern of postwar recessions, and models which explain inventories badly are likely to track badly in simulations of recession. Because of the difficulty of explaining inventory change, it is suggested below that the ability to track final sales rather than *GNP* should be the criterion for judging dynamic simulations of alternative models.

The investment equations were extremely weak in version III, but version IV is even worse. The new equations repeat the earlier error of representing the cost of capital with a nominal rather than a real interest rate. The previous arbitrary "spiked" lag distributions (in which virtually all of the influence of a change in output and interest rates occurs in the fifth quarter after the change, rather than being spread out over several quarters) have been replaced by a smooth distributed lag pattern. But these new lag weights should have been estimated by the Almon technique separately for the output and interest rate variables in each of the four sectors. Instead, however, the authors have used a *single* set of weights for output and interest rates in each sector, and these weights were not estimated for the Brookings sectors, variables, or sample period but were simply copied from weights estimated by Shirley Almon for a different variable (the lag of expenditures behind appropriations), a different sector (all of manufacturing) and a different sample period (1954–61). Why create a disaggregated model if a single inappropriate lag pattern is going to be imposed on all sectors?[3]

The extension of the end of the sample period of version IV from 1960 to 1965 forces the authors to deal with the investment tax credit and liberalized depreciation allowances introduced in 1962. The approach is a completely ad hoc use of dummy variables and stands in contrast to numerous recent articles [3] [4] [17] [18], one of which was written by an author of the FKS paper, that attempt to base the treatment of investment incentives on theoretical considerations. And the effect of the dummy variables is very peculiar. They raise the constant and reduce the output elasticity of investment in durable manufacturing, but lower the constant and raise the elasticity in nondurables and lower the constant and leave the elasticity unaffected in the regulated sector. The most dubious feature of the equations is the result that, ceteris paribus, investment incentives *reduced* real investment spending between 1961 and 1963 by about $7 billion![4] Very little confidence should be placed in the long-run simulations calculated with these equations.

Other equations. In general, the equations outside of the final expenditures sector are not as weak as other models and require less extended comment that the expenditure equations. The production functions warrant attention, since they determine how rapidly productivity will grow in the twenty-five-year simulations. The durables manufac-

[3] Exactly the same use of nonestimated weights is employed in the Wharton-EFU Model (see [9]).

[4] To perform the ceteris paribus experiment, fix the interest rate at 4.0; durables output originating at the approximate 1963 figure of $90 billion; nondurables output at $60 billion; and the durables utilization rate at .85. The equations then predict durables investment of $13.3 billion with 1961 values of the dummy variables and $8.9 billion with 1963 values; $10.1 billion in 1961 and $9.3 billion in 1963 for nondurables; and a straight $2.1 billion reduction in the regulated sector.

turing equation calls for reexamination, since its steady-state version has an unreasonably high degree of increasing returns (1.26) and erratic fluctuations in the rate of disembodied technical change (a zero annual rate before 1960:1, a 4.6 per cent annual rate between 1960:2 and 1963:1, and 0.9 per cent annual rate thereafter). The degree of increasing returns is even stronger in nondurables (1.65).

The hours equations introduce a novel theory of money illusion in the long-run labor supply curve. An increase in the nominal wage rate reduces hours per week, no matter how rapidly prices are rising. An increase in nominal wages of $1.00 due entirely to inflation would reduce hours by 2.5 hours per man per week. The coefficients in these equations are influenced by the slow rate of inflation during the sample period and will overestimate the secular decline in hours during periods of faster inflation.

In the twenty-five-year simulation the price-wage sector generates a remarkably low 1.7 per cent steady-state annual rate of inflation at a 3.9 per cent unemployment rate, a much lower rate of inflation than is implied in other econometric work (for my own simulation results see [14]). In the price equation the elasticity of prices to changes in unit labor cost is between 1.27 and 1.75, implying an increasing secular ratio of profits to wages. If corporations are so aggressive in raising the profit share, why is the rate of inflation so slow in the long-run simulations? The coefficients in the wage equation suggest implausibly docile behavior by workers. The wage equations of the old version III imply a plausible annual rate of wage increase of 6.7 per cent at a steady 4 per cent unemployment rate [15, Table 2], but in version IV workers have become more timid. The steady-state rate of increase of wages can be calculated for a 4 per cent unemployment rate on the assumption of a unitary elasticity of product prices to changes in unit labor cost, a rate of productivity growth of 3.0 per cent per annum in each sector, and an increase in consumer prices at the same rate as in product prices.[5] The resulting figures are extremely low: 6.0 per cent in durables, 3.8 per cent in nondurables, 3.4 per cent in trade, 4.8 per

[5] These assumptions are the same as those used in [16] and [15, Table 2], except for the arbitrary assumption of a unitary elasticity of prices to changes in unit labor cost, which is introduced here to judge the coefficients of the wage equations in combination with a "more reasonable" set of price equations than those in version IV.

cent in regulated, 3.6 per cent in the residual sector, and approximately zero in contract construction. These low estimates in version IV compared to version III may be due to the exclusion from the sample period of any years with an unemployment rate below 4.0 per cent, to the ending of the sample period in 1965:4 before the falling unemployment rate of 1964–65 had much time to influence wage rates, and perhaps to the arbitrary lag distributions. These weak equations also ignore the recent emphasis in the literature on price expectations and disguised unemployment as determinants of wage rates (e.g., [12] [20]).

An improvement in the financial sector of version IV compared to version III is the elimination of the investment variable in the demand for money equation, the coefficient of which in earlier versions was negative and caused the model to generate misleading policy multipliers (see [15]). But the model will still predict that a cut in personal tax revenues will increase interest rates more than an equal increase in government expenditures, due to the use in the demand for money equations of disposable income rather than some broader income concept. Finally, an extremely important flaw in the financial sector is the failure to incorporate any influence on interest rates of changes in the expected price level. In 1968 and 1969 an increase in the rate of expected inflation was a major factor causing a rapid rise in nominal interest rates, and by ignoring inflationary expectations the Brookings Model in a prediction experiment would presumably have underpredicted nominal interest rates. As noted above, the model makes no distinction between the nominal interest rates that enter the demand for money function and the real interest rates that should influence the demand for commodities.

2. THE SIMULATIONS

Turning points. On what criteria should we judge the turning point simulations? A one- or two-quarter error in predicting the exact timing of peaks and troughs is not serious if the order of magnitude of the boom or recession is tracked accurately. And the FKS criterion of comparing six-quarter simulation errors around turning points with errors in a nine-year simulation is uninformative, since a good performance by this criterion might be due to the shorter time span of the turning point simulations rather than a relatively accurate performance of the model at turning points. Instead, I would ask of these simulations whether a policy maker having confidence in the model and using it for forecasting postwar recessions would have been misled into making an incorrect policy decision. The simulations of the 1958 and 1961 troughs shown in Charts 1–11 are pessimistic on this score. For instance, the trough unemployment rate in 1958:2 (Chart 7) is estimated to be 5 per cent instead of 7 per cent, and the rate of inflation (Chart 2) is overestimated by almost 1 per cent per annum between 1957:2 and 1958:2. These forecasts would have thus supported the arguments of those like Secretary Humphrey who stood against a stimulative monetary and fiscal policy during the Eisenhower recessions.

Once simulation errors have been judged to be serious, the equations that make the largest contributions to the errors should be sought out, a task not attempted by FKS. One first notes that the unemployment error is only about half due to the error in tracking real *GNP*.[6] At the 1958 trough, for instance, the overestimate of real *GNP* is about 3 per cent of real *GNP*, which by application of Okun's law should cause an underestimate of the unemployment rate by one percentage point.[7] Since the unemployment rate is actually underestimated by about two percentage points, about half of the unemployment error is contributed not by the expenditure equations but on the supply side by the productivity-hours-participation equations. The small employment errors (Chart 8) relative to the unemployment errors suggest that the participation equation is an important source of the trouble. Until the supply equations in this and other large-scale econometric models are improved, model builders would be well advised to rely on a simple Okun's law equation to minimize errors in estimating unemployment for a given estimate of real *GNP*.

Considering the large underestimates of unemployment in the 1958 and 1961 troughs, the overestimate of wage and price changes is

[6] To simplify the following discussion, we consider only the simulations starting four quarters before the 1958 and 1961 troughs.

[7] For recent statistical evidence that Okun's law is valid for the 1951–69 period, see [14, Appendix B, equation (2)].

surprisingly small. The slight response of wage changes to unemployment errors is consistent with comments on the wage equations made above, and with the impossibly low rates of inflation generated by the model in the twenty-five-year secular simulation.

Turning now to the real *GNP* errors at troughs, these appear to be about half due to incorrect predictions of inventory change. The model seems to generate a flat and smooth, rather than cyclical, pattern of inventory change in all postwar recessions (Chart 5), and it would be interesting to know whether the same is true of the residuals in the underlying inventory equations. The model does a much better job of tracking real final sales than real *GNP* and large-scale models will make a much better impression on readers of simulation reports if increased emphasis is placed on the ability to track final sales. Of the components of final sales, residential construction is tracked very closely, and the nonresidential investment predictions behave quite well except at the 1960 peak. Consumption contributes most (in absolute, not relative terms) to final sales errors, due to the model's inability to predict a drop in the propensity to consume in 1958 and a marked increase in 1960. These results cannot fail to give support to the monetarist argument that velocity is relatively more stable than the Keynesian multiplier.

1957–65 simulation. The $5.0 billion root mean square error in tracking real *GNP* during a nine-year simulation between 1957:1 and 1965:4 appears quite impressive. By contrast the MIT-FRB Model in a similar simulation for 1958:1 through 1967:2 generates a $7.0 billion error [2]. But the lustre of the Brookings achievement dims somewhat when we consider the heavy dependence of the results on dummy variables. Excluding all strike and tax rate dummies, the remaining dummy variables change values in eight of the thirty-six quarters included in the simulation. The dummy variables in the investment equations, which change in 1962:1 and 1963:1 are particularly important in keeping the model on target in the 1961–65 period.

But even without dummy variables most models fitted to the post-Korea era do well in simulations of 1961–65, simply because most of the variance of expenditure components between 1954 and 1965 occurs during 1961–1965, so that these years play a dominant role in determining the coefficients in the underlying expenditure equations. A much more challenging test would have been an extension to 1966–69, an experiment in which the Wharton Model goes completely off the rails [10].

The long-term 1966–90 simulations. The authors introduce this section with the puzzling statement: "The Brookings Model is essentially a short-run forecasting model and as such is not designed for simulation over a twenty-five-year period." Whatever its original intent, however, the model has never been used as a forecasting device (to my knowledge no *ex ante* forecasts have ever been released), partly because the large size of the model inhibits the maintenance of an up-to-date data file. Thus the only possible justification for the Brookings Model is that its large scale yields superior representation of the true structure of the economy than smaller models. If so, a secular simulation is one of the few experiments in which the Brookings Model should have a comparative advantage.

One is tempted to apply a microscope to the graphs of the 1966–69 portion of the 1966–90 simulation values depicted in Charts 16–19 to test the model's ability to track outside of its sample period. Lacking a microscope, I shall eschew comment on this aspect of the simulations, except to remark that the Brookings Model exhibits the universal failing—common to all large-scale econometric models which misspecify the channels by which monetary policy influences real spending—of predicting an economic slowdown in late 1968 and speed-up in the last half of 1969.

Given the extrapolations of steady growth in government spending, it is not surprising that the economy is relatively stable after 1972. Tax rates are manipulated to maintain the economy at full employment, so we would expect a stabler economy than occurred with the highly unstable full-employment surplus of 1953–69. As one looks down the list of exogenous variables, however, it is apparent that Hamlet is missing. Nowhere do FKS mention the assumption made about the secular behavior of unborrowed reserves, the major exogenous monetary variable, although I am told privately that a constant growth rate was assumed. The assumed stability of monetary growth compared to the instability of 1953–69 makes a contribution to the steadiness of the economic advance of 1970–90, even in a model like Brookings where money plays a marginal role.

In their discussion of exogenous assumptions, the authors state that tax rates are raised steadily through 1976 "to keep disposable income from growing too rapidly." At first glance this appears to conflict with the widespread assumption that steady economic growth yields a "fiscal dividend" that allows a reduction in tax rates. But in this secular simulation the fiscal dividend is negative, since an assumption of a roughly constant ratio of real government spending to real *GNP* combined with a rising relative price of government requires an increase in the *G/GNP* ratio in current dollars from 20.2 per cent in 1965:4 to 25.0 per cent in 1990:4, and revenues must also increase the same relative amount to maintain a balanced full-employment budget. This increase in tax revenues could be attained over a twenty-five-year period with income-elasticity of tax revenues of only 1.16, but the Brookings tax equations understate the income elasticity of the U.S. tax system, forcing an increase in the "dummy" coefficients in the tax equations.[8]

Do the secular rates of growth in Table 8 tell us anything about what is likely to occur in the real world? Virtually all of the results can be traced to some of the peculiar features of the model, described in Part I above. For instance, the ratio of profits to *GNP* (both in current prices) increases from the already high level of .113 in 1965:4 to .134 in 1990:4 (as compared to .101 in the prosperous year of 1968). This is caused by the unreasonably high elasticity of changes in prices to changes in unit labor cost in the price equations. The low rate of wage increase and low rate of inflation originate in a very weak set of wage equations. The increase in the ratio of consumption to *GNP* is due to the long-run marginal propensity to consume of greater than 1.0 in the consumption equations. A surprising result is the rapid increase in the ratio of business nonresidential investment to *GNP* in light of the predicted increase in interest rates. An increase in this ratio would imply a reversal of the secular decline in the capital-output ratio which has continued in the United States since 1919. In fact the behavior of this ratio tells us more about the effect of dummy variables in the investment equations than it tells us about the real world.

[8] The ad hoc adjustment that FKS apply to the hours and productivity equations confirm the critical comments made above about the supply sector of the model.

All in all, version IV of the Brookings Model does not appear to be a net improvement over previous versions. Its improved performance in sample-period simulations rests on a shortening of the sample period, the widespread adoption of dummy variables, and ad hoc techniques for specifying equations to maximize simulation performance and goodness of fit. The long-run implications of the resulting equations are questionable in many cases, but the creation of a plausible set of long-run implications is a basic test which must be passed by any model, and this is particularly true of a model like Brookings, which is so unwieldy that it has never been used to fulfill its primary purpose of short-term forecasting. My unhappy conclusion is that, rather than attempt to move on to another version, the Brookings Model builders should merge the best features of their equations into the MIT-FRB Model, which in my judgment is the only large-scale model robust enough to withstand the onslaught of the St. Louis monetarists on either the simulation or forecasting front.

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