This PDF is a selection from an out-of-print volume from the National Bureau of Economic Research

Volume Title: Demographic and Economic Change in Developed Countries

Volume Author/Editor: Universities-National Bureau Committee for

Economic Research

Volume Publisher: Columbia University Press

Volume ISBN: 0-87014-302-6

Volume URL: http://www.nber.org/books/univ60-2

Publication Date: 1960

Chapter Title: The Structure and Tempo of Current Fertility

Chapter Author: Norman B. Ryder

Chapter URL: http://www.nber.org/chapters/c2384

Chapter pages in book: (p. 117 - 136)

The Structure and Tempo of Current Fertility

NORMAN B. RYDER

UNIVERSITY OF WISCONSIN

1. Introduction

THE births occurring in any one year are contributed by parents who began their lives in many different years, while the births occurring to any group of parents identified by common time of birth (a so-called 'cohort') are experienced over an extended span of years. If we call any index of the fertility occurring in a particular year a period measure, and any index of a particular cohort's fertility a cohort measure, then we have available for analysis two time series of fertility indexes, one for successive periods, and the other for successive cohorts. Generally speaking, these time series will differ, despite the fact that it is the same flow of experience through time that is being summarized. The divergence of the two is a function of changes in the distribution through time of the childbearing of successive cohorts. Demographers prefer, all other things being equal, to analyze the determinants of fertility through use of a cohort time series, essentially because it is assumed that successive events in life histories are interdependent. But if attention is focused on the most recent experience, the only cohorts with complete records for summarization are those past the menopause, whose principal procreation occurred several decades before, whereas the cohorts which are currently the most important bearers of children have fertility records of a still unknown degree of incompleteness.

No such difficulty is present in summaries of experience in a period, but time series of this kind of index are unsatisfactory for direct analytic use because of the tendency of such a series to diverge from the desired sequence of cohort behavior. This paper presents a way of escaping from the dilemma of distorted period measures vs. incomplete cohort measures. The procedure is based on the observation that knowledge about the distribution through time of cohort fertility would permit us to translate cohort rates into period form, whereas knowledge about the distribution by age of period fertility would permit us to translate period rates into cohort form. Convenient formulae for these purposes are developed and used to estimate fertility indexes for currently incomplete cohorts by making assumptions about trends in the age distribution of period fertility. This procedure provides an interpretation of the great depression in American fertility, as well as an assessment of the current situation, which uses less defective data than have heretofore been available.

2. *The Data*

This paper is based entirely on one type of fertility measure, the birth rate by age of mother by order of birth of child, for individual ages from fifteen through forty-six, for first through seventh births separately and for eighth and higher order births, and for individual years from 1920 through 1956. This rate is obtained by dividing the number of births of a given order to women of a given age in the year in question by the number of women in the age in that year.[1] The population is that of native white females in the United States. The data are taken from P. K. Whelpton's monograph, "Cohort Fertility," Tables A and G, from supplements to Table G for 1951–1954, and from supplements for 1955 and 1956 prepared by the writer. These statistics are the best currently available for American fertility analysis, in terms of accuracy and detail.[2]

The tables provide cumulative birth rates by exact age of mother by order of birth, arranged by birth cohorts.[3] The definition of birth cohort T was women between exact ages a and $a + 1$ in the calendar period bounded by exact years t and $t + 1$, where $T = t - a$. The birth rates computed by this definition are assumed to be identical with those for women born in the twelve-month period centered on exact year T, the so-called "fiscal" birth cohort of year T. In the present paper, individual birth rates for each age are used. These are the first differences by age of the cumulative birth rates of the order concerned, for each cohort. The rates for each order for successive cohorts may be visualized in the form of a surface, with time and age as axes of the horizontal plane, and fertility in the vertical dimension. Each cohort fertility-age function is a (diagonal) plane section of this surface for a particular value of $(t - a)$. Three summary measures of such a fertility-age function are used frequently through this paper: (1) The zero moment of the function, or sum of the birth rates over the fertile age span, indicating for a cohort

[1] The numerator of the birth rate for age 15 includes all births to age 16.0, and the numerator of the birth rate for age 46 includes all births beyond age 46.0.

[2] For a discussion of the reliability of these data, and the reasons for confining attention to native white women, see Pascal K. Whelpton, *Cohort Fertility, Native White Women in the United States*, Princeton University Press, 1954. Whelpton and the writer have recently prepared new estimates. See Appendix B, pp. 420–438, of Wilson H. Grabill, et al., *Fertility of American Women*, John Wiley and Sons, 1958.

[3] The cumulative nth order birth rate to exact age x gives the proportion of women who have had at least n births by that age, under the assumption of no mortality.

the proportion of women who eventually have at least n births, and called here the complete nth order birth rate; (2) the arithmetic mean of the distribution of birth rates by age; (3) the variance of the same distribution. The values for these measures, as computed for periods, are also utilized. Each period fertility-age function is derived from a plane section of the same fertility surface, for a particular value of t.

3. Comparison of Time Series of Period and Cohort Parameters

Some relevant characteristics of the time series of complete birth rates by order, period, and cohort, are presented in Table 1. Rates are provided

TABLE 1

Complete Birth Rates by Order of Birth, Quinquennial Averages, Period, and Cohort

Order of Birth	*1*	*2*	*3*	*4*	*5+*	*All*
	A. Order-specific birth rates (per woman) period 1920–56					
Initial (1920–24)	0.82	0.64	0.45	0.32	0.82	3.05
Minimal (see below)	0.68	0.51	0.32	0.19	0.34	2.24[a]
Terminal (1952–56)	1.04	0.99	0.67	0.35	0.41	3.46
Minimal Years	1931–35	1933–37	1937–41	1938–42	1947–51	1936–40
	B. Order-specific birth rates (per woman) cohorts 1892–1918					
Initial (1892–96)	0.79	0.64	0.45	0.31	0.67	2.86
Minimal (see below)	0.77	0.58	0.36	0.21	0.34[b]	2.30[a]
Terminal	—	—	—	—	—	—
Minimal Cohorts	1901–05	1905–09	1907–11	1909–13	1913–17	1907–11

[a] This is the minimum quinquennium for the complete birth rate for all births and not the sum of the rates for separate orders in the row.

[b] A little projection was required for births occurring since 1957.

for three quinquennia of periods: the initial five years; the intervening quinquennium with the minimum rates for the series; and the terminal five years, covering the time span 1920–1956. This form of comparison was selected because the time series surveyed are all approximately U-shaped. In the lower portion of the table the same kind of information is presented for the cohorts of 1892–1896 and for the subsequent minimum quinquennium of cohorts. The choice of the initial cohort group was based on the fact that its childbearing experience was centered in the five years corresponding to the initial period group.[4] No terminal cohort group is presented because the appropriate cohorts have not yet completed their fertility.

[4] The problem of appropriate temporal juxtaposition of cohort and period parameters is discussed more fully below.

The table reveals: (1) Although the initial period group has similar rates to the initial cohort group, there are considerable differences apparent once the respective minima are considered. This is true despite the fact that the minimum rate for all births, for periods, is not much less than that for cohorts. (2) Although there is no terminal quinquennium of cohorts available for comparison with that for periods, it is apparent that there would have to be large divergences. The period average for first births, 1952–1956, is impossible for a cohort because it exceeds unity, and the average for second births is also impossibly high.

It is clear from these considerations that temporal variations in the fertility of cohorts, measured in various ways, cannot be inferred directly from the movements of the same indexes computed for contemporaneous period aggregations of birth rates, despite the fact that the same surface of fertility by age and time is being summarized in both period and cohort series. This is true of the magnitudes of decrease and increase, and also of their structure by order. If cohort behavior is given analytic priority, then an approach to fertility measurement other than period aggregation must be devised. Yet no direct summary of current cohort behavior is feasible, as the missing line in Table 1 bears witness. The cohorts which are the major contributors to current fertility are by the same token cohorts which are a long way from completing their childbearing. This is the crux of the methodological dilemma to which the next section is devoted.

4. *Types of Interdependency among Period and Cohort Parameters*

Among the various ways in which these two modes of temporal aggregation of birth rates may be distinguished, one approach is to consider the cohort sources of the fertility occurring during a year. Each of 32 cohorts (those in ages 15–46 inclusive) contributes a certain part of its total fertility to the annual output of children. This proportion is identical with the proportion of the cohort's childbearing which occurs in the age it passes through during the year in question. The complete birth rate for a year is from this viewpoint a weighted sum of the complete birth rates of the cohorts represented among the parents that year, the weights being the respective age-distributional components of each cohort's fertility. Now if all cohorts had the same age distribution of fertility, the complete birth rate for the period would simply be a special kind of moving average of cohort birth rates, since the weights would add up to unity. But if, as is generally the case, the age distribution of cohort fertility were changing from cohort to cohort, then these changes would be reflected in modifications of the sum of weights upward or downward, and the consequent period complete birth rate would manifest this condition.

As a way of coming to formal and quantitative grips with this phenomenon, a simple model has been designed which permits the expression of the complete birth rate for a period as a function of parameters of cohort fertility. The formula for distributional distortion derived from this model is $s = S(1 - M' + V'R)$, where S, M and V are the sum, mean, and variance of cohort birth rates by age, M' and V' are the first derivatives of M and V, R is the first derivative of S, divided by S, and s is the complete birth rate for the period which corresponds to the year in which the cohort concerned is at its mean age of fertility, M.[5]

Before proceeding to discuss this formula, a note is appropriate on the logic of cohort dating and period dating with respect to one another. Since thirty-two cohorts are represented in the fertility-age function of each year, and since each cohort is fertile for thirty-two years, there is no one cohort which is necessarily appropriate to compare with any one of these years, and vice versa. There does, however, seem to be an intuitive rationale for comparing the experience in a period with that for the cohort whose childbearing is centered in that year. For two reasons there is also a mathematical rationale for choosing the cohort which is at its mean age of fertility. If there is linear change in the cohort complete birth rate, and no distributional variation, then such a choice yields equality of period and cohort fertility. Thus in this simple model the temporal positioning of the cohort in the manner described permits an assessment of the extent to which distributional variation has caused divergence from equality. In the second place there is an obvious advantage for discussion in the circumstance that such a choice yields a relationship in terms of conventional analytic parameters, namely the mean and the variance.[6]

The formula $s = S[1 - M' + V'R]$ is useful for indicating, in a simple situation, the nature of the distributional distortion present in period fertility measures.[7] If, say, because of a depression, cohorts postpone their childbearing, and also reduce somewhat their eventual output, this

[5] The derivation of the formula is presented in Appendix I-A, below. For convenience, capital letters are used throughout to refer to cohort parameters, and lower-case letters for the same parameters for periods.

[6] The same is true of higher moments of period fertility, expressed as functions of moments of cohort fertility, in the same type of model, provided the present dating practice is followed. Thus $v = V[1 - Ra_3/\sigma + R^2V]$, where σ is the standard deviation and a_3 is the customary moment type of skew measure.

[7] This is true of all types of period measure of fertility, mortality, nuptiality, and any other processes using the synthetic cohort device for index formation.

means that the mean and the variance of the age distribution rise, (M' and V' are positive) while total fertility declines (R is negative). Thus the distortion factor is less than unity and period fertility is depressed below that for the mean constituent cohort. These conditions obtained during the 1930's in the United States. Conversely, if, in a prosperity period, more births occur, and sooner, then the distortion factor is positive and period fertility is too high.[8] This corresponds with the present situation in the United States. Distributional distortion has also occurred in the long run, because of a secular decline in the mean age of fertility.

At this juncture one issue should be made quite clear. The relationships discussed above are mathematical properties of a surface, expressing the interdependency of vertical plane sections of the surface at one angle to the time axis and vertical plane sections at another angle to the same axis. Cohort complete birth rates may be expressed as a function of the sum, age mean, and age variance of period fertility, rather than, as above, the other way around, and this circumstance is utilized in what follows. The argument for the direction of discussion which considers cohort fertility as intrinsic and period fertility as a distorted reflection of it is founded on considerations entirely outside the realm of the model. For present purposes the writer will simply assert that this seems to be the most fruitful conceptualization for time series analysis, as well as the one which virtually all demographers seem to be employing implicitly, although verbally rather than operationally in most cases.

5. *Techniques for Completing Truncated Cohort Fertility-Age Functions*

In the preceding sections, evidence has been presented for the proposition that time series of period fertility parameters sometimes fail to represent intrinsic cohort trends. The sources of distortion have been identified as distributional variations with time in cohort fertility-age functions. Now if all other things were equal, the message of this paper would simply be: "Accept no substitutes—if you want cohort measures, compute them directly." But all other things are far from equal. Cohort data are just not available in the same convenient sense as period data. The present series of birth records for 1920–1956 contains thirty-seven complete years of information, but there are complete procreative histories for only six cohorts.[9] This number may be increased somewhat with small risk

[8] This is strictly true only in an empirical sense, and that because $V'R$, being of the second order of smalls, tends to be smaller in absolute magnitude than M'.

[9] If it be assumed that the fertile age span is 15–46, then the number of complete cohort histories in 37 years is $(37 - 32 + 1) = 6$.

because of the growing tendency for American women to terminate childbearing well before the menopause. But there is no general procedure yet developed for completing these truncated cohort histories.

Here it seems that the model introduced in the preceding section to demonstrate the relationships among cohort and period fertility-age parameters may be useful. The basis for this impression is the gratifying success the writer has had with attempts to "predict" the time series of period complete birth rates for all births (total fertility rates) using the movements of cohort complete birth rates and cohort mean ages of fertility for Sweden, 1751–1950.[10] There are three important differences between the Swedish experiment and the present situation: (1) The Swedish series used a quinquennial unit for time and age. This would tend to smooth out the most extreme deviations and improve the efficacy of the "predictions." (2) The direction of translation or "prediction" in the Swedish experiment was from cohort parameters to period parameters, and not, as here, vice versa. Now there is no special obstacle to the derivation of a formula for this direction of translation. It is: $S = s(1 + m' + v'r)$, using the symbol system discussed in the previous section. The complete birth rate estimated by this formula is the one for that cohort which is at age m in the period for which the parameters have been computed. But the direction of translation may imply practical difficulties because of the well-known tendency for birth rates to move more erratically from period to period than from cohort to cohort. (3) The third important difference between the Swedish and the American situations is that the former was mostly a continuous development in one direction, whereas the time span for the latter encompasses the most extreme short-run variations in fertility observed in statistical history.

To test the formula $S = s(1 + m' + v'r)$, linear functions were derived for successive fifteen-term moving series of moments of the period fertility-age function for first births, complete first birth rates were derived from the formula on the basis of these values, and the results compared with actual cohort experience. These results are presented in Table 2. First births have been selected because complete rates for relatively recent cohorts can be more confidently estimated for this order than for higher orders (or for all births). The discrepancies between estimated and actual cohort complete birth rates are very small, and particularly in comparison with the period-cohort differences. The three estimates corresponding to

[10] The formula used in that work, $s = S(1 - M')$, was derived from an even more restricted model. See N. B. Ryder, "Problems of trend determination during a transition in fertility," *Milbank Memorial Fund Quarterly*, Vol. 34, No. 1, January 1956, pp. 5–21.

TABLE 2

Actual Cohort Complete First Birth Rates (S), and Estimates of Them (S^*) Based on Period Complete First Birth Rates (s), Multiplied by the Distributional Distortion Factor ($1 + m' + v'r$). (See App. I-B)

(all rates per thousand)

Year	s	S^*	S	*Per Cent of Deviation* S^* *from* S	s *from* S
1927	770	779	775	+1	−1
1928	744	774	772	0	−4
1929	732	766	772	−1	−5
1930	740	764	774	−1	−4
1931	698	764	779	−2	−10
1932	677	760	778	−2	−13
1933	647	753	780	−3	−17
1934	685	758	784	−3	−13
1935	707	769	786	−2	−10
1936	726	782	799	−2	−9
1937	749	785	808	−3	−7
1938	777	791	811	−2	−4
1939	773	818	813	+1	−5
1940	779	852	825	+3	−6
1941	887	875	841	+4	+5
1942	1,028	890	860	+3	+20
1943	954	896	898	0	+6
1944	817	905	891	+2	−8
1945	810	909	894	+2	−9
1946	1,077	905	909	0	+18
1947	1,310	909	916	−1	+43
1948	1,125	911	915	0	+23
1949	1,066	909	918	−1	+16

years 1940–1942 are in fact probably equally as good as the others. The actual complete birth rates for the cohorts concerned are in fact underestimates of true cohort fertility for reasons connected with defects of the estimating procedure used in construction of the original tables. These errors are suggested by the abrupt changes at that point in the actual cohort series. Concerning 1933–1934, the years for which the divergence is a little over 3 per cent, two comments are appropriate: (1) these are the years corresponding to the minimum in period fertility, i.e. that section of the time series for which the assumption of linear change in the period complete birth rate is least justified. This corresponds with the well-known property of simple moving averages at maxima and minima. (2) Even for these years the cohort estimates are much closer to the true cohort level than the basic period values.

The relevance in the present context of the departures of the model from reality is reduced by the consideration that the intention is to project cohort fertility on the basis of assumptions which encompass current types of temporal variation in fertility-age distributions. Provided this is achieved, there is no necessary proscription of a linearity assumption, particularly for the short run. Furthermore, it is both unwise and unnecessary to attempt a prediction of all births for a cohort without using the fertility records which have already been accumulated for it. The formula $S = s(1 + m' + v'r)$ is applicable not only to a distribution covering all ages, but also to the missing tail of a distribution. Thus the procedure developed to complete the truncated fertility-age function for a cohort which is exact age x at the end of 1956 consists of an application of the formula to fertility data for ages x to 46 inclusive, for the years up to and including 1956. An extrapolation of the linear function for cohort complete birth rate thus obtained provides the required estimate of the cohort's fertility beyond age x. Finally, a method has been devised for applying the procedure described separately to fertility within each parity. (See Appendix II.)

6. *The Time Series of Cohort Parity-Specific Fertility*

The procedure outlined in section 5 has been used to complete the records for cohorts up to 1930. The period selected to serve as a basis for determining current trends in period parameters of amount and age distribution was 1948–1956, because it is characterized by modest and approximately monotonic change in these parameters. The results are presented in Table 3, first in terms of parity mean and parity variance,

TABLE 3
Cohort Parity Distributions (Per Cent) and Parameters, 1891–1930

			Parity						
Cohorts	*Mean*	*Variance*	*0*	*1*	*2*	*3*	*4*	*5–6*	*7+*
1891–95	2.90	7.86	21	15	18	14	10	11	11
1896–00	2.70	6.99	22	16	19	14	10	10	9
1901–05	2.43	5.97	23	19	21	14	8	9	7
1906–10	2.31	5.20	22	19	23	14	8	8	6
1911–15	2.35	4.79	20	18	25	16	9	7	5
1916–20	2.59	4.53	14	15	27	19	11	8	5
1921–25	2.89	4.54	10	12	26	22	13	10	6
1926–30	3.21	4.57	8	9	23	24	18	13	6

and second in the form of the more detailed parity distribution. Before making some observations on the results, it is noteworthy that, despite the present youth of the later cohorts, most of the fertility presented has already occurred. Thus the first five quinquennial groups have completed their childbearing, cohorts 1916–1920 are 95 per cent complete, cohorts

1921–1925 are 83 per cent complete, and cohorts 1926–1930 are 64 per cent complete. The completeness varies inversely by parity, so that the most hypothetical entries in the table are those for the higher parities in the youngest cohort group.

The developments displayed in Table 3 fall conveniently into two twenty-year spans. In the earlier half of the experience both the mean and the variance of the parity distribution show large declines The changes in distributional components which yielded these results were decrease in the proportion with more than three births, and increase in the proportion with fewer than three births. In the latter half of the record the mean rises, although the variance remains steady.[11] Examination of the distributional components of this more recent experience reveals a large drop in the proportion with fewer than two births, and a considerable increase in the proportion with more than two births. The modal parity, which is located at zero for the cohorts of 1891–1905, moves to two for 1906–1925, and then to three for the most recent cohort group.[12] Insofar as any influence of the depression can be discerned in the variations of the parity distribution, it appears to be confined to a slight increase in the proportion with fewer than two children, for the cohorts of 1901–1910, whose childbearing was centered in the early thirties. It may be inferred that the principal effects of the depression were to change the time pattern of cohort fertility and perhaps also to retard temporarily a long-run transformation of the parity distribution. The outstanding fact revealed by these data is the reduction to minimal levels of the proportion who fail to bear at least two children. This is the principal reason for the recent rise in mean parity, and not a reversal of the trend away from large families.

For many analytic purposes the parity distribution is not as revealing as is a series of proportions which represent the component acts which go into the construction of the parity distribution. These proportions, which have been termed parity progression ratios, indicate the proportion who, having achieved a given parity, advance beyond that parity. Table 4 presents these ratios for the same cohort groups discussed in the last paragraph. The progression ratios for parities zero and one have risen throughout the past twenty cohorts, but biological considerations would

[11] The coefficient of variation (the ratio of the standard deviation to the mean) was approximately constant at about 100 per cent in the first half, and then declined to 67 per cent for the cohorts of 1926–1930.

[12] For the individual cohort of 1930, the mode is firmly placed at three, with the proportions in parities two and four markedly smaller than the proportion in parity three and approximately equal.

TABLE 4

Cohort Parity Progression Ratios (Per Cent), Parities 0–4, 5+, 1891–1930

Cohorts	*0*	*1*	*2*	*3*	*4*	*5+*
1891–95	79	81	72	70	69	68
1896–00	78	79	69	67	67	67
1901–05	77	76	65	64	65	66
1906–10	78	76	61	60	61	65
1911–15	80	78	60	57	58	64
1916–20	86	82	62	56	55	62
1921–25	90	86	66	57	54	61
1926–30	92	90	73	60	52	60

lead us to suspect that they must now be very close to an upper asymptote. The progression ratio for parity three has become relatively stable, while those for parities four and above have fallen throughout the whole series. Only in the parity two progression ratio is there evident a distinct reversal of direction. This would seem to be the decisive stage of procreation for measurement of the net impact of the depression on the amount of fertility, and also the principal problematic feature of the future development of parity patterns.[13]

To complete the analytic picture, reference may be made to another component of the present research which was reported more fully at the Milbank Memorial Fund Conference in October of 1958.[14] By use of translation formulae for higher moments developed in the same way as those outlined in Appendix I of this paper, estimates were prepared of the mean and standard deviation of the fertility-age function for first births, for currently incomplete cohorts. The mean age of first order fertility rose from 23.4 (1900 cohort) to 24.7 (1914 cohort) and then declined to 22.3 (1934 cohort) The standard deviation of the age distribution of first order fertility followed a similar path to that of the mean, rising to a peak during the depression and declining to a new low level in the postwar years. Thus, although the impact of the depression on the parity structure of cohort fertility may have been relatively small, it disturbed greatly the time pattern of that fertility.[15] In several respects, however, the observations on timing dovetail with those on amount: The

[13] It is of interest to note that there has been a distinct decline recently in the rate of increase of the parity two progression ratio.

[14] N. B. Ryder, "An appraisal of fertility trends in the United States," pp. 38–49 in *Thirty Years of Research in Human Fertility: Retrospect and Prospect*, Milbank Memorial Fund, 1959.

[15] Although the observations reported refer only to the timing of first births, it may be asserted rather confidently on the basis of other evidence that most of the variations in fertility timing are concentrated in this order.

postwar fertility pattern is a marked departure from previous experience, in both dimensions; there is a marked increase in the homogeneity of American fertility in both dimensions; and there are signs that temporal variations in both the structure and the tempo of American fertility are approaching an asymptote. Barring developments which have not yet manifested themselves, a decline in period fertility is implied, since the distributional distortion which is the source of the present spurious excess of period over cohort fertility will tend to disappear under conditions of stability of fertility time patterns. Although we may agree on the analytic priority of cohort behavior, problems are posed and policies formulated in terms of the consequences of this behavior period by period. Because of the central influence of variations in the time pattern of cohort child-bearing on period birth rates, as well as the status of the subject as an interesting but neglected area of human behavior, research on the determinants of fertility time patterns deserves high priority.

7. *Conclusion*

The confidence with which the assertions in the present paper have been made can be increased by further methodological improvements. These would include: (1) experimentation with models based on more realistic assumptions than those of linear change; (2) a more elegant solution than presented here of the problem of combining parity-specificity and additivity considerations in the translation formulae; (3) incorporation of marriage data in the parity sequence; (4) extension of the population considered to that of all women. These steps are feasible with currently available data and the writer hopes to implement such a program in the near future. Further methodological refinements will probably require data of types not yet provided by the birth registration process, in particular the dates of birth of previous children and of marriage. The position seems tenable, however, that investments in such purely demographic models are approaching a point of diminishing returns, and that it is now time to establish firm functional links between the vital processes and their socio-economic contexts. At best, the kind of research reported above yields a somewhat more precise statement of the variations to be explained by such analyses.

Appendix I

PERIOD-COHORT TRANSLATION FORMULAE

A. The problem is to determine the value of the complete birth rate for a period, given hypothetical functions of cohort behavior.

Assume that the complete birth rate for cohort T is a linear function of T. $S_T = a_o + b_o T$. Assume that the proportion of S_T in age i of the cohort's experience is also linear. $P_T(i) = c_i + d_i T$. Then the birth rate in age i, cohort T is $(c_i + d_i T)(a_o + b_o T)$. Let $M_T = a_1 + b_1 T$, the mean age, stand for $\Sigma i(c_i + d_i T) = [\Sigma i c_i] + [\Sigma i d_i] \,.\, T$. (Note: all sums $[\Sigma]$ in this presentation are for the complete range of i.)

Let $N_T = a_2 + b_2 T$ stand for $\Sigma i^2(c_i + d_i T) = [\Sigma i^2 c_i] + [\Sigma i^2 d_i] \,.\, T$. $V_T = M_T - N_T{}^2 = (a_2 + b_2 T) - (a_1 + b_1 T)^2$ is the age variance. The first derivative of $S = S' = b_o$; and of $M = M' = b_1$. The first derivative of V,

$$V' = b_2 - 2a_1 b_1 - 2b_1{}^2 T = b_2 - 2b_1(a_1 + b_1 T) = b_2 - 2b_1 M.$$

Now the birth rate in age i, for the period in which cohort T is in age x is $[a_o + b_o(T + x - i)]\,[c_i + d_i(T + x - i)]$ and the complete birth rate for the period is the sum of products like this over all i's.

$$\begin{aligned} s &= \Sigma[a_o + b_o(T + x) - b_o i]\,[c_i + d_i(T + x) - i d_i] \\ &= [a_o + b_o(T + x)] \,.\, \Sigma[c_i + d_i(T + x)] - [a_o + b_o(T + x)] \,.\, \Sigma i d_i \\ &\qquad - b_o \Sigma i[c_i + d_i(T + x)] + b_o \Sigma i^2 d_i \\ &= S_{T+x}(1 - b_1) - b_o(M_{T+x} - b_2). \end{aligned}$$

Now $S_{T+x} = S_T + b_o x$ and $M_{T+x} = M_T + b_1 x$.

Let $x = M_T$. Then the complete birth rate for the period in which cohort T is at its mean age of fertility M_T equals

$$\begin{aligned} (S + b_o M)(1 - b_1) - b_o(M + b_1 M - b_2) &= S(1 - b_1) + b_o(b_2 - 2b_1 M) \\ &= S(1 - M') + S'V' \\ &= S(1 - M' + RV') \end{aligned}$$

where $R = (S'/S)$. All parameter values in this statement are for cohort T.

Appendix I-B

By the same procedure, i.e., making assumptions about period fertility of the same type as those in Appendix I-A for cohort fertility, and deriving the complete birth rate for the cohort which is at the mean age of period fertility in period t, it is found that $S = s(1 + m' + rv')$ where the lower-case letters have the same meaning for periods as their capitalized counterparts had for cohorts, and the parameter values in the equation are for period t.

Appendix I-C

General formulae have been developed, for both directions of translation, using the assumption that the birth rate for age i is an n-order polynomial function of time. The solution has been generalized to encompass the rth moment of the period or cohort fertility-age function. The result, which involves at each step derivatives of various degree and moments of various order, is logically satisfying, but rapidly approaches the realm of impracticality because of the instability of higher derivatives and higher moments, particularly when the data betray such patterned irregularities as those produced by age misstatement. The general presentation and discussion will be included in a forthcoming monograph.

Appendix II

AN ADDITIVE FORM FOR PARITY-SPECIFIC FERTILITY

A. Zero Parity. If the central first birth rate in age x is $b(1, x)$ and the exposure to risk of a first birth in age x is defined operationally as the women who had not had a first birth by exact age x, i.e. $1 - \sum_{i=1}^{x-1} b(1, i)$, then the fertility rate for age x, parity 0,

$$f(0, x) = [b(1, x)]/[1 - \sum_{i=1}^{x-1} b(1, i)].$$

By using $b(1, x) = [f(0, x)]\,[1 - \sum_{i=1}^{x-1} b(1, i)]$ as a recursion formula, it may be shown that $b(1, x) = f(0, x)$.

$$\prod_{i=1}^{x-1} [1 - f(0, i)] = \prod_{i=1}^{x-1} [1 - f(0, i)] - \prod_{i=1}^{x} [1 - f(0, i)].$$

Then

$$\sum_{x=1}^{a-1} b(1, x) = \sum_{x=1}^{a-1} \left\{ \prod_{i=1}^{x-1} [1 - f(0, i)] - \prod_{i=1}^{x} [1 - f(0, i)] \right\}$$

$$= 1 - \prod_{x=1}^{a-1} [1 - f(0, x)].$$

Thus the complete first birth rate

$$\sum_{x=1}^{w} b(1, x) = 1 - \text{antilog} \sum_{x=1}^{w} \log [1 - f(0, x)]$$

Accordingly, in anticipation of application of the cohort-period translation formulae, the surface of first birth rates by age and time is transformed into a surface in which the "vertical" variable is $\log[1 - f(0, x)]$.[16] The complement of the antilog of S, as yielded by application of the formula below, is the desired complete first birth rate.

B. Parity One. If $f(1, x)$ is defined as $[b(2, x)]/[1 - \sum_{i=1}^{x-1} b(2, i)]$ then, as above,

$$[1 - \sum_{x=1}^{w} b(2, x)] = \text{antilog} \sum_{x=1}^{w} \log[1 - f(1, x)].$$

The dependency of $\sum_{i=1}^{x} b(2, i)$ on $\sum_{i=1}^{x-1} b(1, i)$ is operationalized by computing what may be called the relative change in the first parity nonprogression ratio. Symbolically:

$$\frac{1 - \sum_{x=1}^{w} b(2, x)}{1 - \sum_{x=1}^{w} b(1, x)} = \text{Antilog} \sum_{x=1}^{w} \log\left[\frac{1 - f(1, x)}{1 - f(0, x)}\right]$$

Thus once $\sum_{x=1}^{w} b(1, x)$ is achieved by following Appendix II-A, this formula permits estimation of $\sum_{x=1}^{w} b(2, x)$. Similarly the higher parity computations may be made sequentially.

COMMENT

EDWARD P. HUTCHINSON, University of Pennsylvania

As the author points out, this paper relates to two fundamental aspects of fertility study: first, the description or analysis of fertility patterns, and second, the formation of inferences about the course of fertility. It is to the latter aspect of the paper that the following remarks are addressed for the most part, but necessarily include some comment on the former.

The work described in this paper promises to be a significant further step in the development of techniques of fertility analysis and population projection. During the past several decades there has been large advance in the techniques of projection. In the 1920's we were generally satisfied to use a simple arithmetic extrapolation, at least for short run projections, or more elegantly, to use a geometric extrapolation that assumed a constant growth rate. Soon after there was a vogue of curve fitting, especially the logistic curve. On into the 1930's, however, demographers

[16] In practice, to avoid a negative surface, since the parity-specific rates have values between 0 and 1, cologs are used in place of logs.

came to prefer composite estimates derived from projection of the separate components of population change. Here it was the fertility projections that gave by far the most trouble, for fertility was the most variable and least predictable of the components of population change. It was the projection of the fertility trend of the 1920's and early 1930's that led to forecasts of an eventually stationary or declining population, which forecasts were refuted by the rebound of fertility that accompanied returning prosperity and World War II.

In 1936 Himes made the quotable remark that "The whole history of population thought shows that populations adjust to conditions more promptly than do writers on population." And by 1940 this remark had taken on a sharper edge than when it was written. It is worth re-emphasizing in this connection that populations do adjust, and that a large part of the adjustment is in fertility.

This paper well represents the direction recently taken by demographers faced with the limitations of their previous analysis of fertility. Whelpton's work has shown, and this paper demonstrates very neatly, that period fertility may diverge quite deceptively from cohort fertility. Or as Ryder puts it, distributional distortion may cause the two to diverge. The answer to the problem, we can believe, lies in more detailed analysis of the pattern of fertility as demonstrated here.

The paper also gets at several fundamental technical problems. If relative security in analysis and projection of fertility trends lies in the use of generation or cohort measures, then we face an impasse, for we have to wait some 30 years before a given cohort has passed through its childbearing period. Attacking this problem, the paper presents techniques for approximating the relation between period fertility and cohort fertility and for estimating completed fertility from incomplete cohort data. It thus goes to the heart of the methodological problem of getting at generation processes with data for briefer periods.

My reaction to this work, insofar as it related to the problem of projection, is one of admiration mixed with a small feeling of caution. The latter perhaps calls for a little explanation.

My feeling of caution arises from a strong conviction that fertility is a variable quantity, capable of adjusting itself, and rather quickly, to changes in the socio-economic environment, rather than being ruled wholly by its own internal dynamics. Here we can visualize several different types of adjustment bringing changes in the pattern of fertility. One is a temporary change, such as from war or depression, that may have little effect in the long run on the size of completed family. Second is secular change, such as toward earlier marriage or greater concentration of childbearing in the first years after marriage. Third are the periods of transition from one long time trend to another. The formulae have shown remarkable ability to adjust for distributional distortion when tested on past data; but the necessary data are not available for many cohorts of completed fertility in the United States, and we shall need to continue testing out the formulae in order to find out how well they operate under changing conditions from year to year.

A lesser question concerns the formula linking period and cohort fertility, which relates the fertility of a given year and the cohort whose mean age of childbearing falls on that year (if I follow the procedure correctly). At a time of changing fertility pattern, is it possible that no cohort or that more than one cohort centers its fertility on that year? And in any case the basis for the determination of distributional distortion seems a rather narrow one, cohort-wise, especially at a time of change in the distribution of fertility.

Finally, to omit other particular comments, there is a general point that bears not on this paper alone, that needs to be underlined strongly. That is that population and fertility are far from being independent variables. As other papers in this conference point out correctly enough, demographic change may affect labor supply, the volume of demand, the composition of demand, and so on. But at the same time population, especially in its reproductive behavior, is responsive to economic and other changes. So regarded, therefore, the projection of population trends cannot proceed from a purely demographic base, for there are unknowns that are more in the field of the economist than of the demographer.

I should not close, however, without pointing out that my comments on the problem of projecting fertility patterns and population trends do not acknowledge adequately the contributions of the work reported in this paper. It is based on the most detailed dissection of fertility that we have yet achieved with the aid of official data, and presents what seem to me important new tools for the analysis of fertility.

ANSLEY J. COALE, Office of Population Research, Princeton University

Since I feel that Ryder has made an important contribution to the understanding of fertility and its measurement, I have little in the way of comment or criticism to offer. Therefore I shall limit my remarks to a short non-technical exposition of what I conceive as the main point of his paper.

There are two major categories of fertility measurement that convey quite different sorts of information. One category of measures concerns the birth performance of persons at all ages during a specific interval of time. The other category concerns the childbearing performance through the whole reproductive span of persons starting life during a given interval. Measures of the first kind are *period* or *cross-sectional* measures of fertility, while measures of the second kind are *cohort* or *longitudinal* measures. Demographers use the term cohort (or birth cohort) to signify members of a population born in the same year (or quinquennium).

The common period measures are the *crude birth* rate (births in an interval divided by person-years lived), and *general fertility rate* (births divided by person-years lived by women in childbearing ages), and the *total fertility rate* (the sum over all ages of birth rates for each age of mother). The last measure is independent of the age-distribution of the population; it represents the total number of children at current child-bearing rates that would be born this year if there were one woman at each age in the childbearing span.

There are more refined period measures of fertility, giving fertility rates for each order of birth, i.e., first births per person-year lived by women in the childbearing ages, second births per person-year, etc. Incidentally, the first order fertility rate in the United States reached a sharp peak in 1947, fell until 1950 and has been stable since. Second-birth fertility reached a new high plateau in 1950, while third and higher order fertility rates were still rising in 1956, the most recent year for which rates for the various orders are available.

Period measures of fertility are indispensable tools in analyzing (or estimating) the flow of births through time. It is by definition the fertility of a *period* and not of a *cohort* that determines the number of births in 1958 or 1965. Moreover, temporal variations in fertility in response to economic fluctuations can scarcely be fully analyzed without period measures. Thus to trace the relation in either direction between economic variations and variations in fertility or to construct population projections calls for the use of period measures of fertility.

The advantage of cohort measures over period measures arises from the greater stability of fertility from cohort to cohort than from period to period. A rise or fall in the total fertility of a period may connote no change in the total fertility of *any* of the cohorts currently in the fertile ages, but simply a temporary concentration or avoidance of birth experience during the period in question.

As an example of the effect of changes in timing among cohorts on period fertility, consider a change in age at marriage. With no change in fertility at each duration of marriage (and consequently no change in completed size of family), a sudden reduction by one year in the average age at marriage would, in effect, move forward by one year the whole childbearing cycle of all cohorts subsequently coming into marriageable ages. During the year of the change the number of marriages would be approximately doubled. During the following 20 years there would be extra births equal to the total offspring of one cohort. The "transient" surplus of births due to younger marriage would gradually disappear, but no offsetting birth deficit would ever appear, unless age at marriage rose at some later time. The extra crop of babies (and their offspring) would be a permanent addition to the population.

The so-called baby boom that began in the late thirties and reached a sustained high level after the war is complicated by just such a shift in timing. There has been an important decline in age at marriage as well as a rise in the fraction getting married, and marked reductions in the proportion of couples with no children, or with only one or two children. At the same time it appears that the tendency to move above four children (once having achieved four) has continued to decline.

It is obvious that the significance of fertility changes, and particularly the likelihood that high fertility rates will persist, can be better appreciated by an understanding of the changing fertility of *cohorts*. However, the basic weakness of measures of cohort fertility is that we have the full record only of cohorts that have passed age 40 or 45 and made their major contribution to total births 10 to 20 years ago. When one attempts to appraise fertility in the near future, it is disturbing to note that recent cohorts have been setting twentieth century records at each stage they pass. We have no apparent assurance that they won't continue to set records through the higher birth orders. In other words, cohort analysis indicates that the baby boom *may* connote only relatively moderate increases in completed size of family, but such an inference is permissive, not obligatory.

Ryder has constructed a method (*inter alia*) for inferring the likely complete fertility records of cohorts still in the childbearing span from period fertility data. The technique is basically simple and elegant, and somewhat fragmentary tests indicate that it is successful under quite difficult circumstances.

The problem can be visualized as follows:

Fertility is a function of time and age. We may visualize age on one axis of a horizontal plane, and time on the other, while the third dimension, fertility, forms a surface above this plane. The age-schedule of fertility (births per woman at each age) at any period is represented by a vertical section (perpendicular to the time axis) through this surface. The fertility schedule of a cohort is represented by a vertical section that bears off at 45° to the time axis, starting at the cohort's date of birth. We wish to estimate the remaining fertility of a cohort (the remaining surface along a 45° angle to the time axis), knowing only its fertility to date and the changing fertility distribution of slices perpendicular to the time axis.

Ryder shows that the completed fertility of cohorts only partly through childbearing may be estimated by a simple formula on the straightforward assumption that period completed fertility, period mean age of childbearing, and the second moment of the period fertility schedule by age all change linearly with time. The suitability of these assumptions is confirmed by the close estimates of first-order cohort fertility that Ryder derives from period data for 1927–1949.

His preliminary results indicate (among other interesting findings) that the cohort now at the middle of childbearing will have larger average completed fertility than any cohort within the preceding 40 years. However, current cohorts will have very few childless or one-child women, a sharp concentration of women with 2, 3, and 4 children, and sharply diminished proportions with very large numbers. In contrast, cohorts who bore children early in the century had large proportions childless and with only one or two children, but the distribution by birth order was flat, with high proportions of very large families also.

Limitations

Ryder's techniques permit us to make systematic allowance for the changing age pattern of period fertility as well as for changing total fertility in estimating the complete experience of cohorts not yet through with childbearing. However, it is a technique based on mechanical extrapolation (albeit of higher *moments* rather than a simple total) and can be accepted as reliable only in estimating the remaining fertility of cohorts well along in childbearing. His techniques can give us little guide to the fertility of cohorts not yet married, or not yet beyond the initial stages of family formation. But it is precisely these cohorts who will determine the size of future birth crops, even in the next few years. It is of course a tribute to the soundness of Ryder's techniques that it should not yield such prophecies, since purely internal analysis of demographic factors, no matter how ingenious, could not foretell future behavior of groups that have as yet compiled no fertility record.