each model in full detail; in particular, the algorithms that have been advanced for the numerical solution of some of them will not be given. But the taxonomy should permit us to give the essence of each model, and to show the interrelations between them. The models designed for projects of water resource development are presented first, followed by a model for transportation, and concluding with more general models for economic development planning.

B. U.S. GOVERNMENT PRACTICE IN EVALUATION OF WATER RESOURCE PROJECTS

The federal government evaluates water resource projects by means of benefit-cost analysis. There is no single model which is employed by all agencies;[64] one of the problems in evaluation practice has been the lack of uniform methods among agencies. But there are certain characteristics from which an "ideal" model of federal practice can be derived.[65]

The objective function of this model has two kinds of benefits; "direct" benefits which are largely net additions to individual incomes, and "indirect" benefits which are miscellaneous repercussion effects. At least in principle, the difference between benefits and costs is to be maximized in determining the scale of projects, while project ranking is to be based on the ratio of total benefits to total cost, the crude benefit-cost ratio. Except for the inconsistency between pursuing the scale of individual projects to a point where marginal benefit equals marginal cost while the benefit of marginal projects has to exceed costs at a rate equal to some benefit-cost ratio greater than one, this procedure corresponds to a model in which benefits minus costs are maximized subject to a constraint on cost. A dollar of benefit is given the same weight, no matter "to whom it may accrue," suggesting an objective function of the form (1.3) in Section 1.

The constraint is not applied to the same concept of cost by all agencies. *Proposed Practices* . . . suggests project costs as the proper denominator of the ratio, and hence implicitly as the proper constraint. These are all the costs incurred on the project itself and are contrasted with associated costs, the costs of associated enterprises.

[64] A detailed account of actual practices can be found in my *Water Resource Development.*

[65] The classical statement of the general approach of the government can be found in Federal Interagency River Basin Committee, Subcommittee on Benefits and Costs, *Proposed Practices of Economic Analysis of River Basin Projects,* May 1950.

The Corps of Engineers follows this concept. The Bureau of Reclamation uses federal cost, the cost borne by the federal government. The constraint is applied to costs occurring in all periods, present and future, discounted by the interest rate.

The interest rate needed to discount benefits and costs to derive present values[66] is specified independently and related to government borrowing costs. Since the funds for projects are rarely borrowed but rather raised by taxation, the government borrowing rate is irrelevant. Being rather low, it may be a reflection of social time preference however.

Repercussion effects are measured in the form of "indirect" benefits. These include profits created in processing and in sales to the project, in increased production and wage payments made possible by eliminating floods, and in several other ways. Most of these "indirect" benefits cannot be derived from any reasonable objective function unless a particularly heavy weight is attached to the income—and particularly to the profits—earned in the immediate proximity of the projects, and no weight at all is attached to the offsetting losses elsewhere in the economy.

There is relatively little adjustment for risk and uncertainty. Benefits that are particularly uncertain, usually only expected to begin to accrue in the future, are to be discounted at a higher rate of interest, injecting a slight risk premium. Also, there is general use of engineering contingency allowances in cost estimation. In flood control, some provision is made to stress control of rare "disaster" floods.

Without seeking to subject the federal techniques to systematic critique, five points should be made: (1) The objective function is consistent with the traditional individualist welfare economics and can fairly be interpreted as representing the national interest. (2) In the ranking procedure, there is recognition of the existence of a budget constraint, though the resultant implications for project design are not followed. Some ambiguity remains about the concept of cost to which the constraint is applied. (3) The opportunity cost of budget money is not brought into the analysis; no test is performed to assure that benefits on marginal outlays exceed these opportunity costs. (4) Measurement of repercussion effects largely seeks to measure irrelevant effects. Finally, (5) the model presented is an

[66] In actual practice annual equivalents are employed. These correspond to present value concepts, expressed as an annual average figure.

"ideal," with actual practice rarely utilizing the rankings by the benefit-cost ratio; the analysis is primarily used as a test by which projects with ratios less than 1.0 are rejected, with the scores above 1.0 having only a minor influence in project selection. Also, marginal principles are frequently not followed in project design, particularly in choice of scale.

C. A MODEL FOR BENEFIT-COST ANALYSIS

In my book, *Water Resource Development, the Economics of Project Evaluation*, I present a decision model which was designed to be appropriate to the budgeting problem of the federal water resource programs. This model maximizes the increase in real national income, assuming equal marginal utilities for individuals, subject to a constraint on federal cost. This constraint applies to both capital and operating and maintenance costs; in particular, it applies to the present value of these costs, measured with the interest rate of the analysis. This constraint was chosen over several others. A constraint only on capital was rejected because operating and maintenance costs represent a serious drain on the federal budget in several fields, particularly flood control and navigation; and in the others, e.g., irrigation, these costs are borne by local interests, and hence would automatically fall outside the constraint as far as appropriate. It was also chosen over separate constraints applicable to the funds used in different periods because there was no evidence to suggest drastic changes in the future pattern of availability of funds, and so a perennial constraint equal to present conditions was selected; this assumes that project opportunities are generated at the same rate as funds become available. Finally, while there is some exploration of constraints that include the funds generated by the reimbursable portions of a project, I reject this constraint, because, in actual federal practice, revenues of projects go into general treasury funds, and not into further expenditures for water resource programs.

The interest rate is to be chosen as an expression of social time preference. To bring the opportunity cost of budget money into the analysis, the marginal benefit-cost ratios which correspond to the opportunity costs of budget money raised by taxation are given. In the event that the benefit-cost ratio of marginal projects that can be undertaken within the budget constraint falls below this opportunity cost rate, the latter rate serves as a cutoff, and not all of the available budget money is to be spent.

Repercussion effects are limited in the model, because it is assumed to be applied in full employment conditions and in the mature market economy of the United States, where prices are on the whole, adequate indicators of value. Where there are genuine external economies, largely of a physical nature, these should be measured, of course. Also, in the case of decreasing cost transportation industries, marginal costs rather than actual freight rates measure value.

The treatment of risk and uncertainty is confined to "crude adjustments," particularly risk premiums in the interest rate. In connection with flood control, some recognition is taken of the effect of diminishing marginal utility of income, justifying departure from minimizing the expected total cost of floods in the direction of paying more attention to "disaster-type" floods, but no specific criteria are advanced.

D. MULTIPLE PURPOSE RIVER DEVELOPMENT

Some closely related models were used in a volume of empirical studies.[67] Four investigations were undertaken; (1) the opportunity cost of tax-raised budget money was estimated; (2) an economic analysis of alternative plans of development of the Hell's Canyon project was prepared, using the social cost of capital, as measured by opportunity cost; (3) the extent to which private development is likely to produce the potential nonmarketable outputs of multi-purpose projects was investigated through a case study of the Coosa River, Alabama, and (4), the income distribution effects of a project in the Pacific Northwest were measured under the alternative conditions of private and local, and federal development, with both costs and benefits allocated to regions and income classes. The fourth of these studies seeks to implement an objective function of the form (1.5) of Section 1, identifying distribution of gains and costs by region and of federal costs by income class. No effort is made to rank the alternatives, a task left to the political process; but the necessary data for judgment are presented. It turns out that federal development redistributes income toward the region, compared with other places. The distribution of federal costs by income class depends on the assumed tax changes, but under some likely assumptions falls heavily on the lower income groups. The third study determines the flood

[67] J. V. Krutilla and O. Eckstein, *Multiple Purpose River Development: Studies in Applied Economic Analysis*, Johns Hopkins, 1958. Also see G. L. Reuber and R. J. Wonnacott, "The Cost of Social Capital in Canada" (to be published), where the opportunity cost of funds raised by borrowing is estimated.

control and other nonmarketable benefits that could be produced by the project of the case study, compares them with the incremental costs, and then analyzes the private plan of development. It turns out that the private plan provides virtually none of the nonmarketable benefits.

The remaining two studies are interrelated. The opportunity cost of tax-raised funds is measured from what Musgrave calls the differential incidence of taxation; if the level of expenditures is changed, what tax changes would accompany it, assuming that stabilization policy requires some offset and that fiscal policy is the device chosen? Specific assumptions are made about these tax changes, based on judgment, the tax burden is traced to its ultimate incidence, and insofar as it falls on investment, the foregone rates of return are estimated. Foregone consumption is valued at the time preferences of the affected consumers, as revealed by their saving-borrowing behavior. The resultant average cost of marginal tax funds turns out to be on the order of 6 per cent. This rate is then applied to the Hell's Canyon case, and using it as a test, it turns out that a two-dam plan that costs less than the actual private three-dam plan of development but produces more output is the best choice. The incremental investment required for the one large dam, the public proposal, yields less than 4.5 per cent, assuming fully integrated operation. With opportunity costs at 6 per cent, this increment is rejected.

This model uses rate-of-return comparisons, though they are applied through a benefit-cost terminology. A strict efficiency point of view is taken, in which the interest rate of the analysis is based on individual time preference of the people who are taxed. Had a social time preference been used, perhaps including a lower interest rate, the results would have been the same. The opportunity cost would have had to be revalued into a present-worth concept at the preferred interest rate, and compared to the incremental benefit-cost ratios of the alternative plans. The empirical conclusions would have been identical, since the benefit-cost ratio of the foregone opportunities would have been on the order of 2.0, assuming an interest rate of 3 per cent, while the incremental investment of the large dam plan has a ratio of only 1.5; the two-dam plan would have continued preferable to the private plan, since it has lower costs and greater benefits.

This analysis assumes no budget constraints. This is justified because Hell's Canyon was not a question of choosing the best public projects, but rather to compare competing private and public plans. A victory for the public plan would have meant that the

additional budget money would have been voted; in fact, such a victory would have resulted in a general expansion of public power programs, since it was a symbolic showdown between public and private power advocates. To have used a budget constraint might have condemned the public plan on the grounds that it prevented other good public undertakings, a line of reasoning which was contradictory to the institutional reality of the situation.

There was no concern with risk and uncertainty, and repercussion effects were limited to physical downstream power benefits. Much of the difference in benefits among the plans proved to be in these repercussions, which are nonmarketable for a private developer and which therefore are not considered in private decisions.

E. THE STUDY BY McKEAN

A recent book by McKean devotes a great deal of attention to the theory of expenditure criteria.[68] I cannot summarize the entire discussion, much of which is devoted to practical problems of implementation, to saving the innocent from fallacy, and to setting out the fundamental principles of selecting criteria—a discussion which to some extent parallels this paper, but from a rather different conceptual point of view. I present only the bare outline of McKean's argument with regard to the criteria he considers appropriate. McKean stresses the many objectives of policy, and the limited weight that is to be attached to criteria that reflect only economic efficiency. In the economics, he seeks to maximize the expected gain of real income, though he also stresses the need for consideration of intangibles and of adjusting to uncertainty.

McKean takes the maximization of the difference of the present values of benefits and costs as the ultimate objective (p. 76). The interest rate he would use to compute present values is the marginal internal rate of return. As McKean points out, this is tantamount to a strict rate of return criterion, though there is still an open question about the interest rate to be used in the design of supramarginal projects. McKean makes clear the assumptions that are required for this to be the correct criterion. Either of two sets of assumptions suffices: (1) funds are available without constraint at an interest rate equal to the marginal rate of return—an assumption which makes many criteria, including benefit-cost ratios, come to the same result.

[68] Roland N. McKean, *Efficiency in Government Through Systems Analysis, With Emphasis on Water Resource Development*, A RAND Corporation Research Study, Wiley, 1958, esp. pp. 25–150.

(2) There is a constraint on investment funds, public and private, and the net returns can be reinvested at the marginal rate of return when they accrue (p. 85). McKean makes the necessity of the reinvestment assumption abundantly clear. He is concerned with the sensitivity of the results to the rate of return at which reinvestment occurs, and in view of the necessary arbitrariness on this matter, he proposes that supplementary data be submitted as part of the analysis which give some idea about the time profile of benefits and costs. By giving this profile, the need for any interest rate is eliminated, and the decision-maker, whether Congress or President, is forced to apply his own time preference. Recognizing the need for simple criteria, however, McKean ultimately does propose the internal rate of return as the best simple decision-rule.

McKean rejects benefit-cost ratios (pp. 113–8). He correctly seizes on the critical issue: What are the financial constraints which limit the program? McKean argues that the constraint only applies in the immediate future when the investment costs are incurred, that operating and maintenance costs are financed out of revenues generated by benefits—including the revenues recaptured through taxation. He also feels no need to distinguish between federal costs and other costs. Finally, he prefers to treat the benefits as being reinvested. In my own work, I have preferred other assumptions on these matters. First, I believe budget money will remain scarce for a long, long time, and operating costs a decade from now will prove as much a drain on a scarce financial resource as current investment outlays. Second, since it is the preparation of a federal program which is at stake, I prefer to treat only federal cost as the constrained financial resource. Third, I assume that there is no reinvestment, partly because the benefits of projects and the institutional arrangements in this particular field are such that there is very little direct revenue generated, and what there is does not return to the water resource field; as for benefits recaptured through taxation, in fields such as flood control and irrigation virtually no taxes are created, while in power and navigation it is not clear that the resultant taxes are more than the taxes that would have been paid by the alternative private investments that might have occurred.[69]

[69] This controversy repeats some of the issues of the Lutz-Hildreth exchange of the 1940's. Lutz rejected the internal rate of return in favor of a strict present-value concept, though he did not select the constraint issue as the critical one. Hildreth, in reply, used an illustration which had the reinvestment property which validates the internal rate of return. See F. A. Lutz, "The Criterion of Maximum Profits in the Theory of Investment,"

On analytical grounds, I believe there are no contradictions between the study of McKean and my own. Different assumptions are made, but these are matters on which reasonable men can disagree.

F. THE STEINER PREEMPTION MODEL

Steiner has extended models of this general type in an important way.[70] He employs the same general objective function as the models discussed above, maximizing the difference between present value of benefits and costs. He stresses the need for specifying an interest rate, not only to compute present values of benefits and costs of projects, but also to compute present values of opportunity costs. There is no treatment of risk and uncertainty. The novelty of his approach lies in a combination of constraints and of sectoral analysis which brings out some interesting features of public development in a predominantly private economy.

Steiner defines four sectors of the economy: (1) the public sector the budget of which is being allocated; (2) the private sector which would contain private alternative developments of the particular public projects being considered; (3) the broader public sector in which funds left over from the particular budget would be spent; and (4) the general private sector containing marginal opportunities into which private funds displaced by public projects are pushed.

The total outlay for projects in sector (1) is limited by a budget constraint. This outlay has certain direct benefits in sector (1) of course, but in addition, it leads to repercussions in the other sectors. Sector (3), the general public sector, may receive some funds from the budget of sector (1). This comes about in two ways: first, some funds may be diverted because the marginal returns in sector (1) fall below the opportunities in sector (3). Thus the introduction of public sector (3) assures that marginal projects yield benefits at a rate equal to the opportunities elsewhere in the public sector. Second, funds spill over into (3) because Steiner employs discrete projects and a fixed budget, and so a small amount of money is likely to be left over because the project costs do not exactly equal the constraint.

The other repercussion effect which emerges is the change in

[70] Peter O. Steiner, "Choosing Among Alternative Public Investments," *American Economic Review*, December 1959.

Quarterly Journal of Economics, November 1945, pp. 56–77 and C. G. Hildreth, "Note on Maximization Criteria," *ibid.*, November 1946, pp. 156–64. Also see the later and much extended discussion which resolves some of the issues in F. and V. Lutz, *The Theory of Investment of the Firm*, 1951, pp. 16–48.

benefits earned on private investments because an investment opportunity has been preempted by the government. This forces private funds from the preempted opportunity into a marginal investment, or in the terminology of Steiner, from sector (2) to sector (4). This creates a loss in the private economy.

Steiner also explores the case where there is no budget constraint, the case where funds are drawn from the private economy and where opportunity costs play a key role. He brings the preemption problem into this case as well.

To summarize his model, Steiner writes a general equation

$$y_{ij} = (G_{ij} - a_1 k_{ij}) - (G_j - a_2 1_j) - a_3 m_{ij},$$

where y_{ij} is the net gain from the ij^{th} project, G_{ij} is the present value of benefits minus costs of the project, a_1 is the opportunity cost in the general public sector (2), k_{ij} is the project's drain on the limited public budget, G_j is the present value of the preempted private opportunity, a_2 is the opportunity cost in marginal investments in the private sector (4), 1_j is the capital cost of the pre-empted private project, a_3 is the opportunity cost of funds transferred from the private sector by taxation, borrowing, inflation, or whatever method is actually employed, and m_{ij} is the amount of such funds actually transferred for project ij. This equation can assimilate combinations of budget constraints and transfers of funds from private to public sectors, can assure full recognition of opportunity costs elsewhere in the public and private sectors as far as this proves appropriate, and can reflect the losses caused by preemption of private opportunities.

The empirical magnitudes necessary to implement the model, other than the usual benefit and cost data for each project, include the three constants a_1, a_2, and a_3 (the three opportunity costs) and a rate of interest. Steiner does not advocate any particular interest rate, nor does he propose any specific method of measurement of the opportunity costs. As an empirical matter, in the general public sector, where many outlays do not produce outputs that can be measured with prices, it is extremely difficult to place a value on alternatives which would be comparable to the values attached to the projects being analyzed. The private opportunity cost of marginal investments could presumably be valued; in fact, in a market economy, money costs should be such a measure and no explicit treatment needed. The opportunity cost of funds transferred from the

private sector to augment the public budget is measured by computations of the sort discussed above in connection with the Hell's Canyon study, or by similar computations applied to funds raised by public borrowing, or perhaps even by inflation. Thus Steiner's emphasis on the opportunity costs in the general public sector, sector (3), is likely to remain a counsel of perfection, but the rest of the analysis could probably be implemented empirically.

G. TINBERGEN'S TRANSPORTATION MODEL

Tinbergen has devised a model designed to measure the change in national income due to projects which improve the transportation system of a country.[71] This model consists of a set of geographical points in which production and consumption are carried on. For each product, supply and demand equations are determined, as well as the transportation costs for each product among all points. Each supply function contains the price of the product and of the other products in the geographical point; the demand functions contain the product's delivered price, and hence reflect transportation costs. Given these functions, it is possible to determine what will be produced in each place, and hence what its total production and income will be.

A transportation project will change some of the transportation costs in the model. The equations can be solved again assuming the new, lower transportation costs, and the change in total production and income can be seen from the difference between the two solutions.

This model is a technique for estimating benefits of transportation projects. It allows for the repercussions on production caused by broadening the markets in which the output of a place can compete. This increase in production and of income leads to further increases in demand and production. The resultant estimate of the impact on national income is greater than the estimate produced by conventional benefit-cost analysis, where the impact on national income is limited to the savings in transportation cost. The extent of the difference depends particularly on the supply elasticities, high elasticities implying large increases in production.

To apply the model as an expenditure criterion, a symmetrical analysis must also be carried out for the cost side. Presumably,

[71] J. Tinbergen, "The Appraisal of Road Construction: Two Calculation Schemes," *Review of Economics and Statistics*, August 1957. Also see H. C. Bos and L. M. Koyck, "The Appraisal of Investments in Transportation Projects: a Practical Example" (to be published).

repercussion effects on production and income would also result from alternative uses of the resources. Some assumptions would also have to be made about interest rates and budget constraints.

H. CHENERY'S SMP MODEL

H. B. Chenery has advanced an expenditure model designed to aid in the planning of investment budgets for economic development.[72] The objective is to maximize the present value of benefits minus costs, i.e., to maximize the present value of the real national income. In the closed-economy model, a constraint is applied to capital funds, with the resultant criterion, the Social Marginal Product (SMP), consisting of incremental ratios of present values of benefits minus operating costs divided by the requisite increment of capital. This criterion can be applied to the design of projects, and to project selection, with individual projects treated as increments in the determination of a program. Thus the technique is similar to the use of incremental benefit-cost ratios, except that the denominator contains only capital costs.

The criterion requires an interest rate. Chenery avoided this issue by confining his criterion to projects within the same field and with very similar capital intensities, so that the rankings of projects would be unaffected.

Chenery also applied the model to an open economy where foreign exchange has a higher opportunity cost than the nominal exchange rate. The SMP in this case consists of two terms.

$$\text{SMP} = \frac{B - M}{K} + f\frac{E}{K},$$

where B is present value of benefits, M of operating costs, f is the premium on foreign exchange, E the total effect of the project on the balance of payments, and K is the capital cost. Chenery has a very sophisticated repercussion analysis to estimate the balance of payment effect, including direct foreign exchange needs of the project, import savings made possible, as well as the import demands generated by the increase in the national money income caused by the multiplier effects of the project. These models are applied to development planning in several countries.

[72] H. B. Chenery, "The Application of Investment Criteria," *Quarterly Journal of Economics*, February 1953, pp. 76–96. There is an earlier literature by N. S. Buchanan, A. E. Kahn, and J. J. Polak, which is discussed by Chenery. Kahn introduced the SMP criterion.

I. CHENERY'S PROGRAMING MODELS

More recently, Chenery, in collaboration with others, has used programing techniques to solve the same type of problem. The practical advantage of programing is the great potential of empirical implementation. While some simplifying assumptions must be made to make the problem fit the apparatus of linear (and nonlinear) programing, complete solutions of the investment allocation problem of rapidly changing economics are possible. The marginalist approach, based on Lagrangean multipliers, is fundamentally a partial equilibrium approach (though in principle it could of course be applied to centralized planning of an economy as well).[73] When applied to expenditure decisions, it usually requires, at the least, that prices be projected. In advanced economies, particularly where the programs being planned are a small part of the economy, such projections can be made and are likely to be more accurate than prices which emerge from a programing computation. But where an economy is being transformed by rapid development, the supply and demand relations are so strongly modified by the development program itself that prices cannot be assumed. Even prices for planning must emerge from the planning computations; the programing technique produces such prices, in addition to solving for the over-all quantities.

Without seeking to present the results of the programing approach, the key characteristics of these models will be presented, particularly the assumptions made about objective functions, constraints, interest rates and the other matters with which we have dealt above.

In a study of development planning for Southern Italy, Chenery and Kretschmer[74] employed the following model: the economy is divided into 14 sectors, each of which is an industry aggregate. The sectors are divided into subsectors which have the property that they have the same input-output structure except for differences in capital inputs. A set of targets is specified, a list of goods, which is derived from demand projections based on income elasticities. The objective of the program is to meet these targets at a minimum total investment, with the total availability of labor and of foreign exchange acting as constraints. The production relations of the economy consist of two

[73] At a high level of abstraction, linear programing and the quadratic programing problems used by Chenery are logically equivalent to a Lagrangean problem, following the Kuhn-Tucker Theorem. See Chs. 1, 3, 4 and 5 of *Studies on Linear and Nonlinear Programing*, K. J. Arrow, L. Hurwicz and H. Uzawa, eds.

[74] H. B. Chenery and K. S. Kretschmer, "Resource Allocation for Economic Development," *Econometrica*, October 1956, pp. 365–99.

parts: first there is the input-output matrix of the 14 sectors (applied also to the subsectors). This matrix, together with the capital coefficients, defines one method of production for each subsector. Purchase from abroad, at a given import price, is an alternative method. The good of each subsector also has an export demand curve, relating the price the good can command abroad to the amount being sold. This foreign demand curve, assumed to be a declining straight line, introduces a nonlinearity into the model and makes it a case of quadratic programming.[75] When solved, the model reveals what demands should be met by production in domestic subsectors as well as their total outputs, what and how much should be imported, and how much of various goods should be exported. It also reveals the total amount of investment that is required and in what subsectors it has to be placed. In the event more capital is available than is needed, the targets can be raised, of course.[76]

Models of this type clearly have an enormous potential for expenditure analysis in many areas. In water resource planning, for example, the most efficient program of meeting specified needs could be derived. Similarly in planning regional development, the most economic means of raising, say, the average income of substandard regions could be approximated.

J. REINVESTMENT MODELS

Galenson and Leibenstein[77] proposed that several sets of repercussion effects which had not been considered previously in formal analysis ought to be given an important place in decision models. They stress three effects: first, education of the labor force on the job is considered a benefit of some projects; second, if per capita growth of income is in the objective function, differential effects of projects on population growth should be included in the criteria.

[75] In a subsequent paper, this is generalized to declining demand curves both at home and abroad. See H. B. Chenery and H. Uzawa, "Nonlinear Programing in Economic Development," in *Studies in Linear and Nonlinear Programing*, K. J. Arrow, L. Hurwicz, and H. Uzawa, eds., 1958, Ch. 15.

[76] In a paper to be published in the *Essays in Honor of E. S. Mason*, Chenery applies a similar model to illustrate several problems in development planning. He shows, with realistic empirical magnitudes, how much is gained (1) by using cost figures that reflect real costs rather than money costs, (2) by using a changing price structure suggested by the programing solution rather than constant prices, and (3) by including urbanization costs in the analysis. He also shows (4) how programing can be used to measure the value of generating reinvestible funds in a dynamic (three-period) program.

[77] W. Galenson and H. Leibenstein, "Investment Criteria, Productivity and Economic Development," *Quarterly Journal of Economics*, August 1955, pp. 343–70.

Finally, if a government finds it impossible to achieve an optimal level of investment, the capability of projects to generate further capital out of benefits should be considered, and a marginal reinvestment coefficient is advanced as a measure. All three of these repercussion effects, it is argued, would favor industrial projects in urban locations as opposed to agricultural or handicraft investments in the countryside. Galenson and Leibenstein do not propose a formal criterion;[78] they make their points by illustrative example.

In a subsequent model, I sought to incorporate the reinvestment factor in a formal decision model. The present value of benefits minus costs, or real national income, is maximized subject to a capital constraint. Each alternative has a reinvestment coefficient which states what fraction of its benefits is reinvested, either through private saving or through taxation. The resultant criterion has two components: an efficiency term indicating the present value of benefit minus operating cost per marginal dollar of investment, plus a term which places a premium on that portion of the output which is to be reinvested. This premium has to be derived from the productivity of the reinvestible capital. Because of the long perspective over time, the resultant criterion is very sensitive to the choice of interest rate, and it was in this connection that the analysis of planner's time preference in Section 3 above was worked out.

K. THE MODELS OF A. K. SEN[79]

A. K. Sen has advanced a series of theoretical models designed to illustrate the problem of development planning in an underdeveloped country. These models are not meant to be used in practical planning, but to provide the theoretical underpinning for rules-of-thumb that are empirically feasible.

Sen is particularly interested in exploring the right degree of capital intensity for development, particularly when viewed in relation to the level of reinvestment that might be generated and to balance-of-payments effects. He sets up a simple sectoral model for an underdeveloped country, and by means of it evaluates the alternative strategies of development.

There are two sectors, a backward sector containing lots of

[78] Subsequent criticism interprets the reinvestment coefficient as a decision criterion, and shows it to be wrong or incomplete. But I think this interprets their position too broadly.

[79] A. K. Sen, "Some Notes on the Choice of Capital-Intensity," *Quarterly Journal of Economics*, November 1957, pp. 561–84.

unemployment, which can supply labor in any amount without loss of output, and an advanced sector which contains two departments, one producing capital goods, the other "corn." Two techniques can be employed to add to the output of "corn," one requiring relatively little capital, with labor having relatively low productivity, the other being more capital intensive but having a higher productivity. Following Ricardo, all of wages are consumed, all of profits constitute a surplus and are reinvested. In order to maximize the rate of growth, the rate of reinvestment per dollar of original investment is to be maximized, and this requires that the technique be chosen which produces the greatest surplus. With labor productivity greater under the capital-intensive technique, the rate of surplus per unit of output will also be greater. But there will be less output per unit of investment. The empirical question, which can only be answered by getting magnitudes for the parameters of the model, then becomes this: is the extra surplus per worker made possible by the more capital-intensive technique sufficiently great to offset the loss of total surplus caused by the smaller output which results from sinking the capital into intensive uses?

In a second model, Sen adds foreign trade to this scheme. He assumes that the capital-intensive technique requires imports of foreign machinery, which can be purchased by means of the export of some of the corn being produced. The rate of surplus of corn still needs to be maximized, but in addition to the corn going into wages, the corn absorbed by exports must be subtracted from the total to derive the reinvestible surplus.

Maximization of the rate of growth of output is an odd objective function, and in realistic cases with alternative time profiles, it is ambiguous. However, in Sen's model, if the parameters are assumed to remain unchanged, the growth rate remains constant unless there is a switch in technique. And so, assuming the target date is chosen far enough in the future, the higher growth rate will always dominate short-run losses of output. Maximization of the rate of growth is considered by Sen to be a polar case in which only the economic situation at a remote point in time is considered.[80] Sen views simple turnover criteria, which only takes the first period into account, as the other polar case.

To bring time discount back into the analysis, Sen employs the concept of a "recovery" period. If it is true that the capital-intensive

[80] He identifies Galenson-Leibenstein with this particular case.

technique produces less output in the early years but more later on, the only case in which there is a real problem of choice, then there must be some number of years over which both techniques produce the same amount of output. It is up to the government to decide how many years of output it wishes to consider in its objective function, and by comparing the "recovery" period of the capital-intensive technique with the government's time horizon, a choice of technique can be made.

These models allow the analyst to bring certain important empirical features of underdeveloped countries into the analysis. Particularly where broad strategic choices are concerned, such as the concentration on urban industry or rural cottage industries, empirical evaluation of models of the type proposed by Sen may prove valuable. It is my feeling, however, that whatever can be done by means of these explicit sectoral models, which must simplify reality enormously in order to keep the mathematics from getting out of hand, can be done more easily and more completely by means of programing techniques.

In Sen's particular illustrative models there is a weakness, I think, in the choice of objective function. Maximizing the rate of growth will, among interesting choices, bury more detailed time preferences of the objective function; the decision-maker will not be applying sufficient judgment to the issue, and will essentially leave it to chance. Similarly, the "recovery" period, which is the same as the "pay-out" period of private investment criteria, is arbitrary, placing equal value on output at any time within the period, and a zero value on any output thereafter.

7. *Concluding Comments*

Since this paper is a commentary on the problem of expenditure criteria and models, little further remains to be said. I have tried to bring out the major issues on which the choice of economic criteria turn. A deliberately narrow economic point of view has been taken, not because noneconomic factors are unimportant, but rather because we ought to be clear about things about which we can be clear. I would also pass the judgment that there is no excessive preoccupation with the economic aspect in public expenditures decisions, and that improvement of the economics of government activities can be justified by higher criteria.

COMMENTS

JACK HIRSHLEIFER, University of Chicago

Professor Eckstein has given us the benefits of his thinking on a wide variety of topics covering the field of public expenditure decisions. His over-all conceptual organization of the subject I find unexceptionable, and about a great deal of his content all that I could do would be to record agreement. To avoid such a dull proceeding, I shall center my remarks about one general subject on which Eckstein does, I feel, stray somewhat from the true path—the question of what interest or discount rate to use in making decisions on adoption or rejection of public expenditure alternatives. This discussion will, therefore, concentrate on Eckstein's Section 3 ("Interest Rates"), though there are certain spillovers (to use our private jargon) to a number of other Sections—most especially to Section 5 ("The Treatment of Risk and Uncertainty").

The first topic I shall discuss under this heading is "The Interest Rate as a Measure of Value of Outputs at Different Points in Time." Eckstein's argument here may be summarized as follows, I believe. The acceptance of the market interest rate for public decisions involves the acceptance of consumers' saving-spending decisions. But these decisions based on personal time-preference rates have been criticized as representing "myopia" or intertemporal selfishness. Eckstein provides a theoretical foundation for planners' time-preference free of "myopia"—based on expected growth of income together with diminishing marginal utility of income. That is, given the growth rate and the schedule of diminishing marginal utility, the relative utility value of marginal units of income in different periods, and consequently the interest rate, can be inferred. If marginal utility of income declines 2 per cent for each per cent rise in income, and we expect a 4 per cent higher per capita income next year, a planners' interest rate of 8 per cent is implied (Table 1). Next, Eckstein shows (as did Rae and Fisher long before) that on the purely individual level, a certain amount of time-preference is "rational" because of the risk of dying, an eventuality known to impair enjoyment of deferred consumption. Using plausible numbers, Eckstein finds that both of these considerations under present world conditions indicate positive but low interest rates. His conclusion is that the planners'

decision as to what "social" rate of discount to use remains necessarily a value judgment.

This approach to the question of what discount rate to use is defective, I believe, in overemphasizing one element of the problem, time preference. Following Fisher's analysis, we know that the interest rate ruling in the market represents the interaction of the factors of *time preference* and *time productivity* of savings, given the initial distribution of consumable income as between individuals and also over time. Eckstein is of course aware of the influence of these other elements of the problem, but his analysis brings them in only through a side door—for example, postulating a rate of growth of income in Table 1 may be regarded as representing either an initial distribution of income over time or alternatively the product of a quantum of savings and the average productivity thereof. It is clear, however, that this is not the correct form of analysis: the rate of discount adopted for investment decisions will affect the rate of growth of income, which cannot therefore be taken as a datum in determining a "social" rate of discount.

The important practical implication of this theoretical consideration is that the choice of a rate of discount for a particular public project, or for the public sphere in general, is not quite as unconstrained a value judgment as Eckstein indicates. Since the interest rate represents a marginal balance of time preference and productivity, a 4 per cent market rate implies not merely a 4 per cent rate of marginal time preference but also a marginal productivity of investment equal to 4 per cent, setting aside for the moment market imperfections. In such a case the use of a 2 per cent rate in public investment decisions will be inefficient—public investments yielding just over 2 per cent will displace private investments yielding 4 per cent. Now, of course, value judgments can still enter; for example, there may be a political preference for "socialist" versus "capitalist" projects. But it is not true that a planner can correctly reject the market 4 per cent rate and use his own value-judgment 2 per cent rate instead unless he is prepared to accept a loss of economic efficiency.

I cannot forbear from making two additional comments, though they are not centrally relevant here. There is much confusion between time preference as a particular rate and time preference as a *schedule*, akin to the elementary confusion between the demand schedule and a particular quantity demanded. A positive interest rate need not

imply time preference "myopia" in the schedule sense: that is, the time-preference utility isoquant may be perfectly symmetrical when diagrammed on the axes of present income versus future income. On the other hand, the productive opportunity locus for current versus future incomes may be (and usually is) of such a shape as to indicate positive marginal time preference *at the optimal point.*

The second comment relates to Eckstein's "paradoxical" result that the lower the rate of growth, the lower should be the interest rate. (This is his planner's optimal time preference interest rate, based on diminishing marginal utility of income combined with a postulated rate of growth of income.) The paradox is that poor countries having low rates of growth should then use a low public discount rate, but in these countries there is pressure for early consumption (that is, high market interest rates). Eckstein explains the high interest rates there as being based on "pure time-preference" at low levels of income. A more satisfactory analysis of this phenomenon would separate different possible reasons for low growth of income in poor countries. First, the productivity of available investments may be low (Fisher's "hard-tack" illustration), in which case the real rate of interest will necessarily also be low, regardless of time preference. More frequently, perhaps, highly productive investments will be available and the market interest rate consequently high even in the absence of "pure" time preference (in the schedule sense), though this possibility need not be excluded. The low growth nevertheless observed of per capita income may be due largely to population expansion, or else may be low only in an absolute sense while still large relative to the initial level. Perhaps the most important explanation of all is that the observed high interest rates are probably not the real riskless rates we have been talking about, but are nominal monetary rates incorporating adjustment for the very high default and confiscation risks faced by potential investors in such countries as well as a very high inflation risk. I will turn again to this matter of risk and uncertainty later.

The next major topic I will discuss is Eckstein's attempt to reconcile his "social" discount rate with the opportunity cost of capital. His basic error, I believe, is in arbitrarily establishing the social rate independently of the time productivity of savings. If it is intertemporal selfishness that we are worried about, what we want to do is to increase current sacrifice for the benefit of the future. Once we do this, by a system of taxes and subsidies for example, the

interest yield on projects will be driven down to lower real values over the economy in general. No separate "social rate of discount" is necessary, and if one is used in the public but not the private sphere inefficiency will result.

This error vitiates, I believe, Eckstein's analysis of the opportunity cost of capital, which is based upon measuring opportunity cost and converting it to a present-value dimension by dividing through by the social rate of discount. Thus, if opportunity cost is 6 per cent, and the social rate is 3 per cent (for investments yielding perpetual streams), present value of $2 is foregone elsewhere for each dollar invested. This leads to Eckstein's prescription, for this case, that project cost and benefits should be discounted at 3 per cent, but that a benefit-cost ratio of 2 to 1 should also be required. The purpose of this peculiar device is to incorporate into the criterion the social time preference rate (3 per cent) while still precluding adoption of projects inferior in yield to alternatives foregone. The first of these aims is misguided, I have maintained. The second will not in general be achieved, since Eckstein's rule is biased in favor of alternatives with higher futurity of yield. For example, his rule would prefer (discounting at 3 per cent) an investment yielding 12 per cent after a lapse of two years (cash-flow sequence: $-1, 0, 1.12$) to one yielding 6 per cent in one year (cash-flow sequence: $-1, 1.06, \epsilon$—the terminal ϵ is an infinitesimal added to give the projects the same life). But if the marginal opportunity rate remains 6 per cent compounding of the quicker-yielding investment will dominate the slower one—whatever the rate of discount used in the comparison.

Eckstein at times implies that the per cent yield of a project cannot in general even be measured except by first postulating a discount rate—e.g., "his social rate of discount." Of course, if the marginal yield on alternative projects cannot be unambiguously determined, we cannot speak of the opportunity cost of capital as an interest rate appropriate for discounting public projects and Eckstein's concentration on time preference as the source of the discount rate becomes understandable.

While Eckstein seems to have been influenced by an article of mine[1] he cites in this connection, that article did not prove or assert inability to measure project yields independently of interest rates. While showing the limitations, as an investment criterion, of project

[1] "On the Theory of Optimal Investment Decision," *Journal of Political Economy*, XVI (August 1958), 329–52.

"rate of return" as familiarly defined—that discount rate which equates the present value of the cost-revenue stream to zero—the article went on to attempt a reconstruction of the important concept of yield of a project. The basic idea of the reconstruction is that project yield for any investment *can* be defined independently of the interest rate, but only as a vector or sequence of two-period yields. This conception has been improved and generalized in a paper by Martin Bailey.[2] In sum, we can continue to speak of opportunity yield foregone as a per cent rate, and the market rate of interest—setting aside market imperfections—as equating marginal time preference *and* marginal productivity.

A possible counterargument to my position on opportunity cost of capital could be based upon imperfection of the capital market—for example, that marginal time preference is not equal to marginal time productivity. In such a case, it might be possible to justify low-yield projects if the funds for them were diverted from low interest-rate sectors of the imperfect market. All that can be here said is that the possibility may be worth empirical study.

Eckstein refers in this paper to his empirical work with Krutilla which estimated the opportunity cost of capital as in the neighborhood of 5 to 6 per cent. His own criticism here of this work, that it fails to incorporate a social rate of discount, I do not accept. It is vulnerable to criticism on another ground, however. While I cannot go into the details here, I believe that the procedure used was faulty in crudely averaging nominal interest rates rather than expected yields. In considering alternative uses of funds secured via personal taxes, for example, a weighted average was made of the earnings of funds in such items as government bonds (3 per cent), stocks (6 per cent?), and reduction of consumer debts (12 per cent). Obviously, these rates incorporate varying risk allowances, and in general the over-all 5 to 6 per cent arrived at is an average of nominal market rates representing some undetermined average riskiness, and thus exceeds the expected yields. My own belief is that the riskless money interest rate for any given term can be estimated by the corresponding rate on United States government obligations, now about 4 per cent for long terms. Yields on such obligations, while (nearly) free of default risk, still represent some allowance for price-level or inflation risk. I think that presently $3\frac{1}{2}$ per cent would be a closer approximation

[2] "Formal Criteria for Investment Decisions," *Journal of Political Economy*, LXVII, October 1959, pp. 476–88.

of the real rate of interest—the rate that a riskless purchasing-power bond would yield. On the other hand, incorporating a risk adjustment comparable to that implicit in the market evaluations of private utilities can be shown, I believe, to lead to the prescription to use rates in the neighborhood of 9 to 10 per cent for government investments—the assumption being that these are of comparable riskiness with those of private utilities.

This discussion has now brought us to the next main topic—risk and uncertainty. While my previous comments on the discount rate have been designed to correct what I believe to be errors in Eckstein's analysis, here my remarks will be mainly of a clarifying nature. First, it is important to distinguish between two logically separate types of "adjustment" for risk. The first or "expected-value adjustment" would correct a nominal or quoted interest rate to allow for the probability of partial or complete default. For example, a nominal 12 per cent rate on consumer debt is not inconsistent with an expected yield to creditors and expected interest cost to debtors of 6 per cent or even 3 per cent. The second or "risk-aversion adjustment" starts from the expected value of a risky (high standard deviation) interest yield and adjusts it downward to allow for risk aversion (or upward for risk preference) as compared with a security with equal expected value but smaller standard deviation of yield. The first adjustment, then, says that a 4 per cent bond of Fly-By-Night, Incorporated really has an expected yield of but 3 per cent, after allowing for default probability. The second says that a particular individual will evaluate Fly-By-Night stock yielding 8 per cent on an expectational basis, but with a high dispersion of outcomes, as no better than the stock of Safe-and-Sure, Incorporated, yielding an expectation of 7 per cent with small dispersion of outcomes. An interest rate correction designed to allow for the notorious cost underestimates of government agencies would be an expected-value adjustment. Incidentally, as Eckstein points out, it is neither necessary nor always convenient to allow for risk through a high discount rate in planning projects; however, this is a familiar way in which the capital market reflects risk.

In the earlier part of Section 5, where Eckstein quotes an example using a specific probability of failure, he is clearly speaking of an expected-value adjustment. Later on, however, and in particular for the discussions of the Roy and Thomas criteria, it is risk aversion which is under discussion, the question being how high an expected

value is required to compensate for higher dispersion of outcomes.

This distinction clarifies the familiar argument about the advantages of "pooling" of risks—that government has a natural advantage in undertaking risky projects because it can pool the risks of a great many such together. Eckstein has some remarks on this subject, whose point I did not grasp. The contention about the advantage of pooling is correct insofar as the argument relates to private risk aversion, assuming that risk aversion rather than risk preference dominates in the private sphere (since the law of large numbers reduces the dispersion of the average outcome). It is not correct insofar as it relates to expected-value adjustment: projects will fail for governments as well as for private investors, and this must be allowed for. The 9 to 10 per cent figure mentioned above incorporated only an expected-value adjustment, and is just as applicable for pooled as for unpooled investment alternatives.

As a minor comment, I think that the main lesson to draw from Eckstein's flood-control example in this Section is that risk aversion follows from an assumption of diminishing marginal utility of income. Of course, the same point was made by Marshall in "proving" the irrationality of gambling.

In conclusion, let me say that Eckstein's abstract formulation of the problem of social decision in this field is admirable, despite my criticism of particular points, in its logical organization, its many original touches, and its attempt to boldly measure parameters that others have merely hypothesized about. I must admit to an uneasy feeling, however, that all our logical improvement of the theory of decision may be getting us no closer to stopping, for example, the federal reclamation scandal. Perhaps the fruitful topic for research in this field is "Imperfections of the Political Decision Process"; if we knew more about this, I would feel more confident of the usefulness of theories of optimal public expenditure criteria.

BENJAMIN TOREN, Bureau of the Budget, Ministry of Finance, Jerusalem, Israel.

Both Mr. Eckstein and Mr. Hirshleifer[1] present a very strong case in favor of the present value approach (or cost benefit analysis), rather than the internal rate of return approach (or profitability

[1] Hirshleifer presents his case in a paper cited by Eckstein, "On the Theory of Optimal Investment Decision," *Journal of Political Economy*, August 1958.

analysis). This preference is based on a rather theoretical argument—as more limiting assumptions are needed in the case of the internal rate of return to lead to ideal allocation of resources than in the case of the present value approach.

We do not know how much more actual limitation is introduced into the analysis by the rate of return approach, and to what extent resources may be misallocated by following this efficiency criterion. The only thing we do know is that a highly elaborate and complicated procedure is recommended.

The rate of return analysis uses as its basic tool a detailed profit and loss estimate, with adjustments on income and expenditure sides. The data is on an annual basis, as in any financial report of any business. And it goes without saying that to the public whatever in government looks like business, looks sound.

The present value argument embraces 20–50–100 years in its analysis and can be understood only by the quite complex notion of present value for future money flows, computed by using a compound rate of discount.

There is no difference whatsoever in the two approaches, neither in the process of price adjustments, i.e., replacing market prices by "real" accounting prices, nor in the process of introducing indirect benefits or costs into the analysis.

There are further the enormous statistical problems inherent in all of these studies, which make one worry much less about the perfection of the model, than about the availability of reliable data. The internal rate of return approach makes it possible to do the analysis without specifying the rate of interest. It is precisely this problem of specifying the applicable price of capital which is the least clear and most disputed element in the whole benefit cost analysis, as the discussion of Eckstein, Hirshleifer, Vickerey, and Buchanan clearly indicates.

In arranging a list of investment projects according to priority, the rate of interest does not enter the picture. It does enter the picture only if instead of a list of priorities, a classification between "good" and "bad" projects, is necessary, because then a borderline of minimum efficiency has to be fixed. The rate of interest may also be a necessary consideration, when we compare projects with different pay-off intervals.

Public opinion in Congress, government, business or elsewhere is very much inclined to reject rational efficiency evaluations as the

sole basis for decision making, as it narrows the sphere where "mutual partisan adjustment" rules.

Congress recently has had a very good opportunity to prove its fullest interest in the regional allocation of water programs, and its fullest disinterest in the application of a rational way to evaluate these programs. In hearings before the Committees of Interior and of Public Works, the Bureau of the Budget and its circular A-47 were sharply criticized. The benefit cost approach in this circular was denounced as a political device to fight against federal water resource development.[2]

Nothing would be easier to rationalize an opposition to efficiency evaluation of government programs, than the mere fact that nobody understands the tool used, or that experts cannot agree on the right prices involved.

We should be cautious not to trade too much clarity, public acceptance, and statistical needs for an elaborate model with conceptual perfection.

Reply by Mr. Eckstein

The fundamental issue between Professor Hirshleifer and myself is quite simple: the interest rate in the model underlying his discussion serves its classical function, of equating the marginal rates of substitution in production and consumption. The profusion of interest rates in the economy is considered to be due to differing risk premiums. My model assumes the capital market to be imperfect, to be rife with rationing, ignorance, differential tax treatments, reluctance to finance investment from external funds, slow adjustment processes, etc., which destroy the normative significance of actual rates found in the market. The enormous gaps between the investing and savings rates in the system, with savers typically receiving 3 per cent and investment decisions being made at 20 per cent and more (and yielding similar *ex post* returns), is strong evidence against the pure, classical view.

Once the interest rates in the markets are denied their normative role, the rate for public decision-making must be derived from other considerations. It can be derived from individual revealed preference, from a planner's preference model, or from a vision in a dream; it is a value judgment, pure and simple.

[2] Report No. 2686, Senate, 84th Cong., 2nd sess., on S. Res. 281.

To assure a correct allocation of funds, capital or otherwise, marginal yields must be equated—the yield being present value, measured at the social rate of discount. The productivity of capital enters into the analysis as the rate at which present value can be created in alternative employment.

I agree that the determination of the social discount rate cannot be divorced from the over-all investment plan, and have tried to show the interrelation elsewhere.[1] The notion of schedules of rates, related to growth rates of output per capita or elasticities of the marginal utility functions is part of the model, of course.

Finally, I want to echo Professor Hirshleifer's call for getting on with the job of applying the criteria to public expenditures. The potential of this type of economic analysis is large and it is important that our theoretical quarrels not interfere with our empirical performance in this area.

[1] "Investment Criteria . . . ," *op. cit.* pp. 78–82.