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PLAN OF THE STUDY AND PROBLEMS OF MEASUREMENT

THE first section to follow will indicate the general content of the book and its relationship to the other studies of the series. The next three sections—which nontechnical readers may wish to skip over—pertain, in the order mentioned, to the coverage and limitations of the data, to the nature and significance of special measures of investor experience developed in the investigation, and to certain conceptual problems associated with the method of computing bond yields and yield averages. The chapter closes with a brief outline of the structure of the book from Chapter 2 on.

NATURE OF THE NATIONAL BUREAU STUDIES IN CORPORATE BOND FINANCING

The first report of the series—*The Volume of Corporate Bond Financing since 1900*—was published in 1953. It presents aggregate statistics relating to new offerings, extinguishments, and outstandings of corporate bonds; and to new defaults, default settlements, and outstandings in default. In addition, it presents estimates of money flows to and from the corporate sector of the economy arising from transactions in corporate funded debt.[1] The basic series on offerings, extinguishments, and outstandings were compiled from a complete census of all issues appearing in the financial manuals from the beginning of 1900 through the end of 1943. The other series were constructed from data covering all large issues (for purposes of selecting the sample, a large issue was defined as one whose offerings summed to $5 million or more) and a 10 percent sample of the small issues. The report was comprehensive and general, the only breakdowns given covering the broadest industry and size classifications, i.e. all issues, all large issues, all small issues, rails, utilities, and industrials. With this

[1] Cf. W. Braddock Hickman, *The Volume of Corporate Bond Financing since 1900* (Princeton University Press for the National Bureau of Economic Research, 1953). For brevity, the first book will be referred to hereafter as *Volume of Financing*.

limitation, however, the data were given as frequently as the sources permitted, annually in all cases, and monthly for a number of the most important series. These materials were analyzed against the broad background of economic events during the present century, with particular attention given to secular drifts and cyclical ups and downs.

The present, second, report amplifies and extends in new directions the analysis of the earlier monograph. Stated briefly, its purpose is twofold: first, to trace movements in the volume of securities offered and outstanding that had various detailed characteristics, and second, to examine the experience record of the obligations falling into these classes. In an attempt to determine the volume and characteristics of corporate bond investments available as outlets for the funds of such financial intermediaries as insurance companies, mutual savings banks, personal trust accounts, etc., particular attention is given to quality bonds (bonds rated high grade by the investment rating agencies, bonds eligible for savings-bank and personal-trust investment in selected states, and bonds rated high grade by the market). In addition, the report analyzes new materials bearing on such characteristics as minor industry group, size of issue, asset size of obligor, type of security (secured, unsecured, junior lien, etc.), term to maturity, and marketability (stock exchange listing or over-the-counter trading). Wherever possible, cross-classifications of offerings and outstandings are presented, showing, for example, the industry composition of rated bonds, their legal status, marketability, and so on.

In sections paralleling the analysis of the volume of bonds having various characteristics, the report traces the experience record of different types of securities, using data of price, of obligors' payments of interest and principal, and so on. The particular measures of bond experience used for this purpose include the default rate, the promised yield, the realized yield, and the loss rate, each measuring an important aspect of investment experience: the incidence of default, the yield promised by the obligor to maturity, the actual yield realized by investors over given periods of investment, and the extent of their capital loss or gain.

The report is essentially an analytical summary based on a more extensive body of tabulated materials than that presented here. The supporting data—along with notes on methods of derivation and suggested uses—will be published in a companion volume,

Statistical Measures of Corporate Bond Characteristics and Experience.[2]

Coverage of the Data

To come within the scope of our investigation a security must be in "bond" form, be issued by a "domestic corporation," and be held by the "domestic investing public." Because of the difficulty of obtaining data on certain minor types of issues and because of technical problems involved in calculating their yields, the present report (like the earlier one, for the most part) is further restricted to so-called "straight" bonds. These definitional matters are discussed fully in the first volume of the series, and for convenience are summarized briefly here. The section then takes up certain sampling problems that are special to the present report.

Types of Issues Studied

A bond is defined for purposes of this investigation as a long-term debt instrument running between the obligor and the bondholder, secured by a trust indenture, and having a definite coupon rate, maturity date, and principal amount. The principal, or par, amount is the lump-sum payment that the obligor promises to pay at maturity, and the coupon rate, applied to the principal amount, determines the amount of an annuity that the obligor promises to pay throughout the life of the bond. Thus in the case of a typical $1,000, 4 percent coupon bond, the obligor contracts to pay an annuity of $20 semiannually to maturity plus a lump-sum payment of $1,000 on the maturity date.

In practical terms, our definition of a corporate bond excludes all equity investments (common and preferred stock, owners' equity in real property, etc.), all term loans (except those of life insurance companies, which usually provide for a corporate trustee), all real estate mortgage loans, and all short-term loans with maturities of under one year from the date the loan was made.

A domestic corporate bond is one issued by any business concern, whether incorporated in the strict legal sense or not, that was chartered to do business by one of the forty-eight states, by the United States, or by the territories of Alaska or Hawaii. Government obligations are excluded, as are those of eleemosynary and other nonprofit institutions. The major industry categories

[2] For brevity, the third report will be referred to hereafter as *Statistical Measures.*

included are railroads (including services incidental to railroads); public utilities (electric and gas, telephone and telegraph, street railways, water companies, miscellaneous utilities); and industrials (manufacturing, service, trade, and mining). Excluded are bonds of financial concerns (investment trusts, factoring houses, etc.), as well as securities of the residential real estate group.

In so far as possible we have included only bonds held by private domestic investor groups (households, private nonfinancial corporations, and investment intermediaries). In practice we excluded issues or parts of issues held by the federal government or by any of its agencies, as well as issues expressed in some foreign exchange (principally sterling). These restrictions were dictated by the fact that we are interested in analyzing only the experience of domestic investors on their holdings of domestic corporate bonds. Some idea of the importance of the types of issues studied is indicated by the fact that on January 1, 1949 slightly over two-thirds of net domestic corporate long-term debt and slightly under one-third of all corporate debt outstanding was in the form of corporate bonds.

The first report of the series provided annual statistical estimates on outstandings, offerings, and extinguishments for four principal types of corporate bond issues: straight issues, income issues, equipment issues, and other serial issues. In the present report, attention is restricted solely to "straight" issues—those with fixed coupons and single maturities. One of the most useful measures of investor experience is obtained by comparing the difference between the promised yield at offering (or on some chronological date) with the yield realized to extinguishment (or to some other chronological date). Income bonds (like common stocks) are automatically excluded by this approach, since they promise no definite yield to maturity. For computational reasons, serial issues (and equipment obligations, which are usually in serial form) were excluded as well. Offerings of serial issues typically include a large number of maturities with different promised yields for each. Identification of these small amounts and the calculation and recording of their yields would have been so laborious, that it was decided to restrict attention to the straight issues.

Practically speaking, the minor types of issues were relatively unimportant during the period studied. Thus of the $90 billion total par amount of all issues offered in the period January 1, 1900 to January 1, 1944 (including $6.2 billion outstanding on

January 1, 1900), $77 billion or 86 percent was comprised of straight issues. Moreover, the straight issues were on the whole outstanding for longer periods than the minor types, so that they represented at all times between 90 and 95 percent of total outstandings.[3]

The Sample Covering Detailed Bond Characteristics

Our statistics covering detailed bond characteristics (agency rating grade, market rating, legal status, etc.) are of two types: annual par-amount data on bond offerings, and quadrennial par-amount data on bond issues outstanding in the market. It should be noted that we distinguish between a bond and a bond offering and between a bond offering and a bond issue. A bond is an individual piece of a bond issue—usually in par amount of $1,000—running between the corporation and the bondholder. A bond issue is the entire group of such pieces that have common characteristics and rights (all bonds secured under the same trust indenture). A bond offering is any part of a bond issue that is floated at a particular time.

The amount of detailed information provided by the manual sources on offerings and outstandings is roughly proportional to the general public interest in the issue, which in turn is roughly determined by the size of the issue. For the period 1900–1943 the sources were reasonably complete for the large issues but were fragmentary for the small. For that reason, it was decided to record only the most basic information for all issues (issue size, amounts and dates of offering, etc.) and to restrict the search for detailed data (ratings, etc.) to all large issues, and to a 10 percent sample of small. For purposes of drawing the sample, small issues were defined as those whose offerings summed to less than $5 million. A representative 10 percent sample was selected by arraying a card catalog of all small issues by year of maturity within year of first offering, and pulling every tenth card. Except for the primary control, there-

[3] Comparable figures for numbers of straight issues offered during the period 1900–1943 plus those outstanding in 1900 are 21,189 out of 28,005 issues of all types, or 76 percent. Thus it is clear that the average par-amount size of the straight issues was larger than that of the minor types, which was to be expected since in the corporate field serial issues occur most frequently among industrials, where the average size of issue is smaller than in the other two major industry groups. Moreover, serial issues are usually retired more promptly than straight issues.

Annual comparisions of par-amount totals for straight issues and issues of minor types may be made from Tables A-1 to A-5 of *Volume of Financing*.

fore, the sample is essentially a random one, and should reasonably represent the characteristics of small issues.

Since the first report of this series contains annual estimates of the total number and par amount of all large and small issues floated or outstanding during the period studied, it is possible to appraise the coverage of the sample of issues selected for detailed study. Thus we estimate that between January 1, 1900 and January 1, 1944, $71.5 billion of straight bonds were offered, of which $56.6 billion or 79.1 percent were large, and $14.9 billion or 20.9 percent were small. With inclusion of issues that were outstanding in 1900, large issues comprised $60.5 billion, or 78.1 percent of the total for straight bonds, and small issues comprised $17.0 billion, or 21.9 percent. Because of the dominant position of the large issues in the par-amount totals, it was felt that reasonably reliable universe estimates of the detailed characteristics of all bonds offered and outstanding (large issues plus small) could safely be inferred by adjusting the small issues sample to the universe estimates for all small bonds as given in the first report.[4] This was done by calculating "raising" factors for each major industry group (annually for offerings and quadrennially for outstandings) and applying these on an individual issue basis to the par-amount figures for small issues in the 10 percent sample. The quadrennial breakdowns of outstandings thereby obtained should be quite reliable, owing to the large number of issues outstanding at any one time and the importance of large issues in the total.[5] Because of the small number of offerings that occur in any given year, the breakdowns of offerings may be less reliable. Therefore in tracing the characteristics of bond offerings, the data in this report are usually combined into aggregates covering four-year intervals.[6]

[4] The coverage is much less complete for number of issues than for par-amount totals. For example, out of 21,189 straight issues mentioned in the manuals in the period 1900–1943 only 3,314 (15.6 percent) were large issues and 17,875 (84.4 percent) were small. (Excluding those outstanding in 1900 the numbers are 3,004 large issues, or 16.1 percent; 15,619 small issues, or 83.9 percent.) For this reason estimates of the number of issues having detailed characteristics are seldom presented in the report; when given, they should be interpreted with extreme caution.

[5] The reliability of the estimates appears to improve towards the end of the period studied. Thus on January 1, 1900, two-thirds of the total par-amount of all issues outstanding consisted of large issues; on July 1, 1920 the comparable figure was 71 percent; on January 1, 1944, 92 percent.

[6] The detailed tables on bond characteristics to be published in *Statistical Measures* provide separate annual estimates for all issues and for large issues. The large bond estimates are exact so far as they go, and, because of the domi-

Although the coverage of our samples on the detailed characteristics of offerings and outstandings appears reasonably good when judged by the par amounts included, the breakdowns by certain characteristics are less reliable than by others. Virtually complete data were obtained, for example, for such variables as size of issue, agency rating grade, legal status, and default status. The coverage is also quite good for the market rating and the asset size of the obligor. It is rather sketchy for the times-charges-earned ratio and the ratio of net income to gross income, and for various minor bond characteristics as well. When data on a particular characteristic could not be obtained, an issue was classified as "information lacking" with respect to that variable. Thus the reader may judge for himself as to the reliability of each breakdown by an examination of the volume of securities for which no information could be obtained.

The Bond Experience Samples

Special problems of sample bias must be faced in interpreting our statistics on corporate bond experience, particularly as they relate to small issues. These measures, it will be recalled, include default rates, and average promised yields, realized yields, and loss rates, constructed for various groups of bonds from records of individual issues. While an attempt was made to calculate yields for each issue included in the primary samples, the necessary data were frequently not available, especially for the small issues. The results are therefore more than usually liable to error in that area.

Generally speaking, the default rates constitute the most reliable data on corporate bond experience. These are based simply on the default status of the issue, which could be determined for virtually all issues in our samples. More serious was the problem of obtaining adequate data for the calculation of realized yields and loss rates. For these it was necessary to know not only the default and extinguishment status of the issue but to have a full record of cash receipts during the various periods analyzed, as well as prices on the terminal dates. Our basic records of investor experience are of three types: (1) the offerings experience record, (2) the default experience record, and (3) the periodic experience rec-

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nant position of large issues in the market, are of primary interest to institutional investors. In the summary tables of the present report we usually present only the all-bond figures.

ord. Evidence as to the availability of data needed for the calculation of yields and loss rates for these records is presented in Tables 3 to 5.

TABLE 3—Coverage of the Offerings Experience Sample
(*dollar figures in millions*)

	OFFERINGS IN		
	Universe[a]	*Experience Sample*[b]	*Percent Included in Sample*
Par Amount			
Large issues	$56,619.2	$55,181.0	97.5%
Small issues	14,924.7	1,006.2	6.7
Number			
Large issues	7,114	6,601	92.8
Small issues	19,804	887	4.5

Offerings in the experience sample are those for which realized yields could be computed from offering to extinguishment (or, in the case of issues extinguished by noncontractual exchange during corporate reorganizations etc., through five years of the life of successor securities), or from offering to the end of 1943.

The search for yield data on small issues was restricted to those in the 10 percent sample.

[a] Straight corporate bond offerings, years 1900–1943 combined, from *Volume of Financing*, Tables A-6 and A-9.

[b] From *Statistical Measures*, Table 182.

The offerings experience sample (Table 3) reflects the behavior of corporate bonds purchased at offering and held to extinguishment. Since we shall wish to compare promised yields at offering with realized yields calculated from offering to extinguishment, an offering is included in this sample only if both types of yield could be computed. As the table shows, the coverage of the offerings experience sample is good for the large issues but only fair for the small. Of the $56.6 billion total par amount of offerings of large issues between 1900 and 1944, $55.2 billion or 98 percent is included in the sample. Of the $14.9 billion total offerings of small issues, however, only 6.7 percent is included, as against approximately 9.8 percent that would have been included if information had been obtainable as readily for small issues as for large. Similar information is provided by comparing the per-

centages of the number of offerings included with the corresponding percentages for the par amounts. The fact that representation for both the large and small issues is less good by number than by amount provides additional evidence of the difficulty of obtaining information on the smaller offerings.

TABLE 4—Coverage of the Default Experience Sample
(*dollar figures in millions*)

	OUTSTANDINGS AT DEFAULT IN		
	Universe[a]	*Experience Sample*[b]	*Percent Included in Sample*
Par Amount			
Large issues	$10,534.1	$9,027.8	85.7%
Small issues	4,381.2	194.0	4.4
Number			
Large issues	710	549	77.3
Small issues	4,380	119	2.7

Issues in the default experience sample are those for which realized yields could be computed from first offering to default and from first offering to extinguishment (or, in the case of issues extinguished by noncontractual exchange during corporate reorganizations etc., through five years of the life of successor securities), or from first offering to the end of 1943.

The search for yield data on small issues was restricted to those in the 10 percent sample.

[a] Par-amount figures from *Volume of Financing*, Table A-18 (new defaults of straight corporate bond issues, years 1900–1943 combined); number figures from Table 135 of *Statistical Measures* after eliminating pledged bonds offered to the public after default.

[b] Number figures from Table 20 herein; par-amount figures from special supplementary tabulations.

The coverage of the default experience sample (Table 4) is roughly similar to that of the offerings experience sample. The table is based on par amounts and numbers of defaulted issues outstanding at date of default. In order to include an issue in the default experience sample we needed the price paid at offering, the market price at default, the amount received at extinguishment, and the full record of cash receipts from offering to extinguishment. A smaller percentage of issues met these more severe requirements than met those of the offerings experience sample,

TABLE 5—Coverage of the Periodic Experience Sample
(*dollar figures in millions*)

BEGINNING OF YEAR	PAR AMOUNT OUTSTANDING			NUMBER OF ISSUES OUTSTANDING		
	Universe[a]	*Experience Sample*[b]	*Percent Covered by Sample*	*Universe*[a]	*Experience Sample*[b]	*Percent Covered by Sample*
			Large Issues			
1900	$ 3,876.9	$ 3,677.6	94.9%	310	282	91.0%
1904	6,483.4	5,939.8	91.6	434	362	83.4
1908	8,944.0	7,825.7	87.5	593	459	77.4
1912	10,928.2	9,824.3	89.9	718	590	82.2
1916	12,646.0	10,669.7	84.4	839	639	76.2
1920	13,668.1	11,654.3	85.3	871	680	78.1
1924	16,176.5	14,889.1	92.0	1,054	918	87.1
1928	21,251.3	18,911.0	89.0	1,338	1,107	82.7
1932	24,233.7	21,850.9	90.2	1,405	1,166	83.0
1936	22,777.5	17,911.1	78.6	1,290	953	73.9
1940	22,683.1	17,078.3	75.3	1,108	776	70.0
1944	20,845.0	15,956.7	76.5	969	703	72.5
			Small Issues			
1900	2,058.6	134.0	6.5	2,256	84	3.7
1904	2,842.3	120.6	4.2	3,309	82	2.5
1908	3,650.1	147.2	4.0	4,424	95	2.1
1912	4,375.1	188.4	4.3	5,606	117	2.1
1916	4,580.6	190.9	4.2	5,870	113	1.9
1920	4,417.0	171.7	3.9	5,492	100	1.8
1924	4,858.8	302.6	6.2	5,383	185	3.4
1928	5,225.2	303.9	5.8	4,912	184	3.7
1932	4,780.3	283.1	5.9	4,099	172	4.2
1936	3,740.3	212.3	5.7	3,264	138	4.2
1940	2,677.4	153.6	5.7	2,368	105	4.4
1944	1,952.8	88.5	4.5	1,710	76	4.4

Issues in the periodic experience sample are those outstanding at the beginning, and having maturity dates later than the end, of the four-year periods starting January 1 of the years given, for which quadrennial realized yields could be computed. For issues extinguished within the quadrennial periods and combinations of them, the period spanned by the realized yield is from the beginning of the period to extinguishment (or, in the case of issues extinguished by noncontractual exchange during corporate reorganizations etc., through five years of the life of successor securities or to the end of the period, whichever is the earlier).

The search for yield data on small issues was restricted to those in the 10 percent sample.

[a] From *Volume of Financing*, Tables A-6 and A-8, for straight corporate bonds.

[b] From *Statistical Measures*, Table 163.

but as the table shows, the coverage is still quite good for large issues. In the case of small issues, however, the coverage is rather poor.

A third basic record relates to the experience on corporate bonds held over selected chronological periods (Table 5). The investment periods analyzed are the four-year intervals beginning and ending on January 1 of the quadrennial years 1900, 1904, etc., and selected combinations of those periods. In order for an issue to be included in this sample we again required prices at the beginning and end of a period and records of intervening cash receipts; and also required, in order to calculate the promised yields, that the issue be in good standing at the beginning of the period. The table shows numbers and amounts of outstanding issues that were included in the sample for the four-year periods, together with universe estimates of the numbers and amounts outstanding at the beginning of each of the quadrennial years. It will be seen that the par-amount coverage ranged from 75 to 95 percent for the large issues and from 3.9 to 6.5 percent for the small. Again the coverage was quite good for the large and only moderately so for the small issues. An interesting feature of the table is the decline in the percent of large issues covered in recent years. One reason is the increasing difficulty of obtaining market quotations for corporate bonds toward the close of the period studied, a reflection of the declining importance of the secondary market in corporate bonds that accompanied the growth of private placements. Another reason is the large volume of issues outstanding in default in the thirties and early forties that were automatically excluded from the sample because the promised yield could not be computed.[7]

The basic questions to be answered in connection with the dis-

[7] An examination of the proportions of the par amounts of outstanding issues not included in the periodic experience sample (comparing the par-amount section of Table 5 with Tables A-6 and A-18 in *Volume of Financing*) indicates that for large issues in 1900, 0.2 percent of the total amount was excluded because in default and 4.9 percent because market quotations or data of cash receipts were not obtainable. In 1920 the corresponding figures are 4.2 percent and 10.5 percent, and in 1944, 12.7 percent and 10.8 percent. The difficulty of obtaining quotations was greater for small issues than for large. This is indicated by the fact that in 1900 only 0.7 percent of the par amount of small issues outstanding in the 10 percent sample was excluded from the periodic experience sample by reason of default and 34.3 percent because of the absence of market quotations or the record of cash receipts. The corresponding figures for small issues in 1920 are 8.4 percent and 52.6 percent, and in 1944, 16.5 percent and 38.5 percent.

TABLE 6—Distribution of Offerings by Default Status and Method of Extinguishment: Universe versus Offerings Experience Sample

	All Offerings	REGULAR OFFERINGS					Irregular Offerings[b]	Total Par Amount (millions)
		Total	Paid in Full at Maturity	Called[a]	Outstanding Jan. 1, 1944 with No Prior Default	Defaulted		
Large Issues								
Universe	100.0%	94.3%	10.8%	37.2%	30.7%	15.6%	5.7%	$56,619.2
Sample	100.0	95.0	12.7	36.8	29.1	16.4	5.0	55,181.0
Small Issues								
Universe	100.0	89.8	16.0	37.2	8.0	28.6	10.2	14,924.7
Sample	100.0	92.6	21.6	42.4	5.6	23.0	7.4	1,006.2

Covers par amount of straight corporate bond offerings, years 1900–1943 combined. Sample data from *Statistical Measures*, Tables 182 and 183; universe data from special supplementary tabulations.

[a] Includes issues extinguished before maturity by other contractual methods as well as by call.
[b] Issues offered by contract modification during corporate reorganizations, etc.

parate coverage of large and small issues are whether there is in fact any evidence as to significant differences in the behavior of the two groups, and, if that is the case, whether there is further evidence as to a possible bias, arising from the loss of information on excluded issues. Some evidence on these questions is presented in Table 6, which compares the distributions by method of extinguishment of the universe of bonds offered during the period 1900-1943 with the corresponding distributions included in the offerings experience record. Because of possible differences in yield and loss experience on bonds offered during corporate reorganizations, such offerings are classified separately as "irregular offerings." Only the regular offerings are classified by method of extinguishment: paid in full at maturity, called, defaulted, or still in good standing at the end of the period of study (January 1, 1944). Since bonds usually are called at a premium, a high percentage called for any group implies a high realized rate of return from offering to extinguishment; the same is true of issues still in good standing in 1944, since most of them were selling at a premium at that time. A high default rate, on the other hand, implies a poor yield record.

Comparison of the percentages for the universe of large and of small issues shows that a larger proportion of the par-amount total of small issues was extinguished after default and a smaller proportion remained outstanding on January 1, 1944, thus indicating heavier losses on small issues. On the other hand, comparison of the percentages for the sample and universe of small issues indicates that the sample contains a somewhat higher percentage of bonds extinguished by payment in full at maturity or by call, and a smaller percentage extinguished after default, a reflection of the fact that it was more difficult to obtain data on the treatment of small issues after default than on those extinguished by call or payment at maturity. In consequence, the small bond sample appears to be slightly biased in favor of the better bonds, i.e., toward better performance than that shown by the universe of all small issues. On the other hand, the coverage for large bonds was virtually complete and the sample shows little evidence of bias in that area.

Since the large issues dominate the totals it is possible to adjust the experience samples to obtain reasonably reliable global estimates for realized yields, promised yields, etc., and we shall

occasionally venture to take this step. In general, however, investor interest centers on the experience with large issues, and we shall not therefore attempt to combine the data for the two size samples when examining the experience of bonds having various detailed characteristics. It will be found that the experience measures for large and small issues frequently differ significantly as to level, but that they are usually mutually substantiating when issues are ranked according to the more important variables affecting bond performance.

Nature and Significance of the Measures of Bond Experience

Of the four measures of investor experience used in this study—the default rate, the promised yield, the realized yield, and the loss rate—only the last two are sufficiently unusual to require special comment. Definitions of each measure follow, and then an illustration of how estimated loss rates may be used to set up reserves to take care of default losses on corporate bonds.

Default Rates

The default rate, as the name implies, is simply the ratio of the par amount of corporate bonds going into default to the par amount exposed to the risk of default. Two types of default rates are used in this report, one based on offerings, the other on outstandings. In analyzing the experience on bonds purchased at offering we shall be interested in the proportions of the amounts offered in given periods that subsequently went into default. While such rates take no account of the amount of the issue outstanding at default or the number of years between offering and default, they are nevertheless useful in interpreting differences among yields on various types of securities.

The second type of default rate employed is a ratio of the par amount of bonds going into default within a given interval of time to the total amount of issues outstanding at the beginning of the period. Such default rates measure the volume of bonds going into default per interval of time. Annual default rates for major industry groupings were presented in the first report of this series. Here quadrennial default rates are used to measure the differential default experience on bonds having various detailed characteristics.

Both types of default rates are useful measures of the ability of bonds of various types to avoid default. Along with similar statistics on the percent of bonds called and of bonds paid off at maturity, they enable us to trace differences in realized yields and loss rates over the periods studied.

Promised Yields

The promised yield is simply the yield calculated on the assumption that the bond will be paid in full at maturity. It is calculated at compound interest, and, since most bond indentures specify semiannual coupon payments, the period of compounding is one-half year. Promised yields are readily obtained from ordinary tables of bond values. Reference to such a table indicates, for example, that a $1,000, ten-year, 4 percent coupon bond purchased at a price of 108.35 will yield 3 percent to maturity (a $20 semiannual coupon return less the appropriate amortization of principal).

The promised yield is sometimes referred to as the expected yield, sometimes as the market yield, and sometimes as the contractual yield; and each of these terms has its advantages and disadvantages. The term "expected yield" emphasizes the fact that on very high-grade investments, particularly those purchased at offering (i.e. in the primary market), the promised yield is usually the yield expected by the purchaser. This is less apt to be true, however, of low-grade issues. For example, few purchasers of a highly speculative bond, selling in the secondary market to yield, say, 15 percent to maturity, would be so naive as to expect to obtain such a high rate of return on the investment.[8] The expression "market yield" is less objectionable but may be confused with the current yield, or ratio of the coupon rate to bond price, and it is inapplicable to bonds purchased at offering. Similarly, the contractual yield may be confused with the coupon rate. The term "promised yield" is perhaps preferable to the others for our purposes, provided it is recognized that the yield referred to is that which the obligor promises to pay to maturity. The difficulty is that other yields may also be "promised" in the bond contract,

[8] In the National Bureau's studies in urban mortgage financing, the term expected yield is the one used. Those studies deal entirely with loans made at par and in the primary, not the secondary, market. Since such loans, when acquired, are not expected to go into default, the term expected yield seems reasonably appropriate for them.

i.e., the yields to any one of a number of call dates specified in the trust indenture. In calculating the promised yields of this study, such options were disregarded and attention was focused solely on the maturity date.

Realized Yields

From the actuarial or mathematical point of view the realized yield is precisely the same as the promised yield except that it takes account of call premiums or default losses actually realized, and that it covers the period the bond was actually outstanding rather than from offering to maturity. Thus in the case of a ten-year 4 percent bond selling to yield 3 percent to maturity, the promised yield is 3 percent, and the same would be the realized yield if the bond were paid in full at maturity. The realized yield would exceed the promised yield if the bond were, say, called five years before maturity at a price of 110 (specifically, a realized yield of 3.9 percent per annum); conversely, the realized yield would fall short of the promised yield if the bond should go into default at maturity and be liquidated promptly at that time by cash payment of 90 cents on the dollar (specifically, a realized yield of 2.1 percent per annum).

Corporate bonds (other than direct placements) are marketable securities and may be purchased or sold by the investor at any time between offering and extinguishment. Given prices paid or received on the terminal dates and the full record of cash receipts, realized yields could be computed for any assumed period of investment. Since it was clearly impossible to compute and analyze all of such yields, we compromised by selecting periods likely to be of interest to most types of investors. The yields may be divided conveniently into two groups, the first covering investor experience over various natural periods in the life of each issue, and the second covering selected uniform chronological periods.

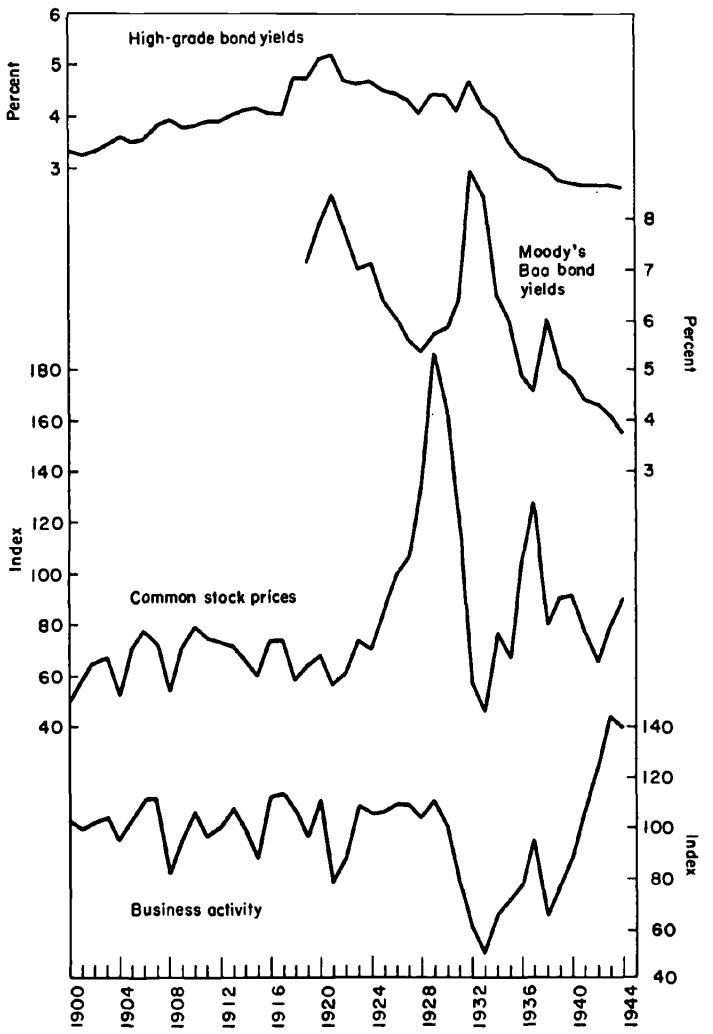
Because many institutional and other investors purchase bonds at offering and hold to extinguishment, the basic natural period for the calculation of realized yields is the period from offering to extinguishment. Such yields were calculated for each offering in the sample for which the requisite data could be obtained. A supplementary set of yields, also covering natural periods, was calculated for each defaulted issue in the samples. These yields, which are designed to show the result of following the requirements of many supervisory agencies that bonds be sold at de-

fault, cover the period from first offering to extinguishment, from first offering to default, and from default to extinguishment. To avoid unsettled market conditions that may have prevailed around the dates of reorganization and settlement (the extinguishment date of the original issue), successor securities, if any, were assumed to have been carried for a period of five years following settlement, or up to the date of their liquidation by cash repayment, if liquidated earlier. The reorganization plan accepted by the majority of the bondholders was followed throughout, primarily because it was of greatest public interest, but also because the treatment afforded the minority usually would not have been feasible for all holders. For the defaulted issues, an additional set of realized yields was calculated to measure the returns obtained by those who purchased bonds at default. In addition to the yield from default to extinguishment already mentioned, these cover the periods from default to two and to five years later (for defaulted issues still outstanding at the end of those periods). For a smaller number of defaulted issues extinguished by exchange for successor securities or by contract modification, yields were calculated to measure the returns obtained by those who purchased the successors at time of exchange and held them until five years later (or until such earlier time as the successors were liquidated by cash principal repayment).

In addition to the yields covering the various natural periods, realized yields for selected uniform chronological periods were calculated for the issues in the samples. These were constructed on the assumption that the investor purchased the security at the beginning of the period and held it until the end (or until the date of extinguishment, for securities liquidated during the period). In the case of securities extinguished by exchange for successor securities, the latter were assumed to be carried for a period of five years or up to the end of the period, whichever was the earlier.

The principal advantage of the periodic yields is that they reflect the experience of various types of investments under homogeneous conditions. For example, by assuming that securities were purchased in 1928 and held until 1940, it was possible to test the ability of securities of different grades, etc., to withstand the heavy default risks of the thirties. A disadvantage of the periodic yields, as compared with realized yields from offering to extinguishment, is that they are quite sensitive to the particular

CHART 1—Bond Yields, Stock Prices, and Index of Business Activity, 1900-1944



High-grade bond yields, index of common stock prices, and index of business activity are from sources cited in Table 7. Moody's Baa bond yields are from "Survey of Current Business," November 1937, p. 19, and later issues.

CHART 2—Par Amount of Outstandings, Offerings, Extinguishments, and New Defaults, 1900-1944

Data are for straight bonds only, from "Volume of Financing," Tables A-2 and A-17.

prices that happened to rule in the market on the terminal dates of the period. To some extent the effects of market prices can be reduced by lengthening the assumed period of investment, but they cannot be entirely eliminated. For that reason, the periodic yields are always analyzed in this report in conjunction with the yields for the natural periods. The periodic data are, of course, of paramount interest to investors who are in a position to trade in and out of the market prior to maturity.

In selecting the uniform periods, an attempt was made to pick those that would be representative of the variety of capital market and general business conditions that have prevailed since the turn of the century, and at the same time were sufficiently long to damp out the effects of minor variations in bond market conditions on the terminal dates. In short, the aim was to select observation dates that would represent as well as possible the major types of economic conditions that occurred in the past and might possibly recur in the future.

In choosing the observation dates particular attention was paid to levels and trends in the following variables:

High-grade bond yields
Prices of speculative securities (low-grade bond yields and stock prices)
The volume of bond financing
General business conditions.

Annual data for a few of the major financial and business series examined are presented in Charts 1 and 2. After some experimentation it was decided that the quadrennial dates 1900, 1904, . . . , 1944 could be used to mark off intervals that were fairly representative of the major changes that have occurred since 1900. Some evidence as to the relevant economic conditions that prevailed on those dates is presented in Table 7. As the table and charts indicate, the observation dates selected reflect the general upward drift in high-grade bond yields and stock prices to 1920, the decline in bond yields and the bull market in corporate stock in the twenties, the near-panic conditions in the early thirties, and the following pronounced decline in bond yields and modest recovery of stock prices up to 1944. The quartiles for the yield spreads in Table 7 mark off differences between yields of the highest grade of bonds and the yields of medium and low-grade issues (for a full explanation of the yield spreads, see Chapter 5 on

TABLE 7—Selected Indicators of Capital Market Conditions and General Business Activity, 1900–1944

FIRST QUARTER OF YEAR	*High-grade Bond Yields*[a]	YIELD SPREADS[b]				*Index of Common Stock Prices*[c]	*Index of Business Activity*[d]
		First Quartile	*Second Quartile*	*Third Quartile*	*Semi-inter-quartile Range*		
1900	3.30%	0.45%	0.82%	1.27%	0.41%	49.8	103.2
1904	3.60	0.33	0.70	1.17	0.42	52.8	95.3
1908	3.95	0.41	0.85	1.56	0.57	54.5	82.3
1912	3.90	0.27	0.61	1.17	0.45	73.6	101.3
1916	4.05	0.34	0.74	1.35	0.51	74.4	114.0
1920	5.10	0.55	1.20	2.28	0.87	68.6	112.1
1924	4.66	0.40	0.94	1.86	0.73	70.8	105.6
1928	4.05	0.36	0.76	1.38	0.51	134.9	104.5
1932	4.70	0.89	2.21	5.82	2.47	57.3	60.6
1936	3.20	0.80	1.55	2.57	0.89	105.0	78.7
1940	2.70	0.66	1.83	4.93	2.13	91.8	88.8
1944	2.60	0.31	0.96	2.42	1.05	90.2	139.0

[a] Thirty-year corporate bond yields from *Basic Yields of Corporate Bonds, 1900-1942*, by David Durand, and *Basic Yields of Bonds 1926-47: Their Measurement and Pattern*, by David Durand and Willis J. Winn (National Bureau of Economic Research, Financial Research Program, 1942 and 1947, respectively).

[b] Based on the par-amount section of Table 13 of *Statistical Measures*, covering all large (straight) issues outstanding and 10 percent of small issues adjusted to universe totals.

[c] Average of monthly prices for the first quarter based on Alfred Cowles III and Associates' *Common-Stock Indexes* (1939), Series P-1, pp. 66f., and on *Survey of Current Business*, 1942 Supplement, p. 86, and 1947 Statistical Supplement, p. 95 (Standard and Poor's index). The latter was chained to the Cowles index on the basis of their relationship in 1935–37.

[d] Average (for the first quarter) of three indexes of business activity—American Telephone and Telegraph Company, Cleveland Trust Company (during the period studied, Ayres index), and Barron's (formerly, Persons)—adjusted for trend by the compilers. Because of changes in base, the Persons indexes were chained before averaging.

TABLE 8—Changes in Selected Indicators of Capital Market Conditions and General Business Activity, 1900–1943

PERIOD	*High-grade Bond Yields*	YIELD SPREADS				*Index of Common Stock Prices*	*Business Activity*[a]
		First Quartile	*Second Quartile*	*Third Quartile*	*Semi-inter-quartile Range*		
1900–1903	+0.30%	−0.12%	−0.12%	−0.10%	+0.01%	+3.0	92.6
1904–1907	+0.35	+0.08	+0.15	+0.39	+0.15	+1.7	86.4
1908–1911	−0.05	−0.14	−0.24	−0.39	−0.12	+19.1	123.1
1912–1915	+0.15	+0.07	+0.13	+0.18	+0.06	+0.8	112.5
1916–1919	+1.05	+0.21	+0.46	+0.93	+0.36	−5.8	98.4
1920–1923	−0.44	−0.15	−0.26	−0.42	−0.14	+2.2	94.2
1924–1927	−0.61	−0.04	−0.18	−0.48	−0.22	+64.1	99.0
1928–1931	+0.65	+0.53	+1.45	+4.44	+1.96	−77.6	58.0
1932–1935	−1.50	−0.09	−0.66	−3.25	−1.58	+47.7	129.9
1936–1939	−0.50	−0.14	+0.28	+2.36	+1.24	−13.2	112.9
1940–1943	−0.10	−0.35	−0.87	−2.51	−1.08	−1.6	156.9
1900–1907	+0.65	−0.04	+0.03	+0.29	+0.16	+4.7	79.9
1900–1919	+1.80	+0.10	+0.38	+1.01	+0.46	+18.8	108.9
1908–1915	+0.10	−0.07	−0.11	−0.21	−0.06	+19.9	138.5
1920–1927	−1.05	−0.19	−0.44	−0.90	−0.36	+66.3	93.3
1920–1931	−0.40	+0.34	+1.01	+3.54	+1.60	−11.3	54.0
1920–1939	−2.40	+0.11	+0.63	+2.65	+1.26	+23.2	79.2
1924–1939	−1.96	+0.26	+0.89	+3.07	+1.40	+21.0	84.2
1928–1939	−1.35	+0.30	+1.07	+3.55	+1.62	−43.1	85.0
1932–1939	−2.00	−0.23	−0.38	−0.89	−0.34	+34.5	146.7

Based on Table 7. The change over 1900–1903 is the difference between first-quarter data for 1904 and for 1900; and so on. Changes are arithmetic differences, except in the case of the business index.

[a] The change over each four-year period is expressed as the relative of the index of business activity at the close to the index at the beginning of the period. Each of the series making up the composite index in Table 7 was treated separately, and the results were then averaged.

the market rating). Thus at the beginning of 1900 roughly one-quarter of the issues outstanding had promised yields to maturity lying between 3.30 percent and 3.75 percent (3.30 percent plus the first-quartile yield spread of 0.45 percent), one-half between 3.30 percent and 4.12 percent, and so on. The quartiles, and the semi-interquartile ranges based on them, measure the tendency for yields on medium and low-grade issues to cluster up around the yields of the highest grade of bonds.[9] The index of industrial activity shown in the table and Chart 1 is a composite based on the American Telephone and Telegraph, Persons, and Ayres indexes, each adjusted to remove the effects of long-term trend.

The periods selected for special study—each of the eleven four-year periods defined by the quadrennial observation dates, and nine longer periods—are indicated in Table 8, along with the changes in the major indicators from the beginning to the end of each period. The changes in the indicators will be used later to interpret differences among yields for the various periods. Thus, from the standpoint of a fully invested fund, a rise in the basic rate on high grades over a period is equivalent to a fall in the value of the fund, which, other things equal, implies low realized return on the investment. Similarly a rise in the interquartile range, and of the quartiles generally, reflects a spreading out of yields and a general deterioration in the quality of medium and low-grade issues. The changes in these and other indicators shown in Table 8, in conjunction with default rates, call rates, and other statistics to be presented later, thus serve to explain the behavior of various grades of investments over different periods. It is sufficient to note at this point the wide variety of economic changes covered by the periods selected for special study.

Loss Rates

The loss rate is simply the difference between the yield promised at offering (or at date of assumed purchase in the market) and the yield actually realized from then until the date of extinguishment (or sale in the open market). The loss rate, if positive, shows that less was actually earned on the investment during the time that it was on the books of investors than was promised at date of

[9] The semi-interquartile range is one-half the difference between the third and first quartiles. It serves as a rough measure of dispersion.

acquisition, which, under conventional accounting practices, usually implies that a capital loss occurred.[10] Similarly, a negative loss rate indicates the reverse situation: more was earned than was promised, so that a capital gain occurred. Properly interpreted the loss rate measures the extent of the defect of contract (i.e., the total capital loss expressed as a percent of the amount invested and amortized over the life span of the investment).[11] Loss rates thus supplement the default rates of this study, which measure only the incidence of default.

Although the realized yield on an investment portfolio can never be known precisely at the time of investment, past experience may help us to estimate the extent of the expected defect of contract. With such an estimate at hand, it would then be possible to "amortize" the "expected" loss, much as the difference between the promised yield on a bond and its coupon rate is now used to amortize bond premiums.

To fix our ideas as to how this might be done, two hypothetical amortization schedules are presented in Table 9, one built according to the customary rules for amortizing a bond premium, and the other constructed to take account of a default loss. For illustrative purposes, it is assumed that on July 1, 1952 the investor purchased a $1,000, 4¼ percent coupon bond due July 1, 1962, at a price to yield 4 percent to maturity. Under the conventional rules assumed in the first schedule of the table, one-half of the

[10] This assumes that the full promised yield on the investment is applied to amortized book value to determine investment income (i.e. coupon receipts adjusted for amortization of premium or accrual of discount). For a small proportion of the bond issues a positive loss rate may be treated as a temporary reduction of income rather than a loss of capital; but such cases, where an interest default was settled by payment of back interest, were very infrequent. See *Volume of Financing*, p. 185.

[11] To avoid any misunderstanding it should be noted that the difference between a negative loss rate (capital gain) or a positive loss rate (capital loss) is essentially a matter of bookkeeping. The sign of the loss rate does not indicate whether or not the investment income was satisfactory to the investor. Many investors would doubtless prefer a zero loss rate on payment at maturity to a capital gain resulting from a call prior to maturity, particularly if bond yields were low at the time of call (and high at the time of maturity). Other investors might prefer the call if they had a pressing need for funds on the call date or if returns in other sectors of the capital market (e.g. the mortgage market) were more attractive at that time. Or again, it is conceivable that losses resulting from default might actually be welcome to investors if they reflected an extension of the issue at maturity at a time when interest rates and bond yields were very low. Such complex factors are not reflected in our loss rates, which simply measure the extent of the capital loss or gain when investment accounts are kept on the conventional accounting basis.

annual coupon, or $21.25, is debited to the cash account, and one-half of the annual interest on the initial book value, or $20.41, is credited to the income account. The remainder—$0.84—is credited to a bond premium account, which would, if all goes well, accumulate to an amount sufficient to wipe out the premium at the end of ten years.

TABLE 9—Conventional Schedule for Amortization of Bond Premium, and a Suggested Schedule for Amortizing Bond Premium and "Expected" Loss

($1,000, ten-year, 4¼ percent coupon bond purchased July 1, 1952 to yield 4 percent to maturity)

	CONVENTIONAL SCHEDULE			
	Coupon Received	*Interest at 4% on Book Value*	*For Amortization of Premium*	*Conventional Book Value*
July 1, 1952	–	–	–	$1,020.44
Jan. 1, 1953	$21.25	$20.41	$0.84	1,019.60
July 1, 1953	21.25	20.39	0.86	1,018.74
Jan. 1, 1954	21.25	20.37	0.88	1,017.86
July 1, 1954	21.25	20.36	0.89	1,016.97
Jan. 1, 1955	21.25	20.34	0.91	1,016.06
July 1, 1955	21.25	20.32	0.93	1,015.13
TOTAL			5.31	

	SCHEDULE PROVIDING FOR CAPITAL LOSS (LOSS RATE = 1 PERCENT)					
			FOR ALLOCATION TO RESERVES			
	Coupon Received	*Interest at 3% on Book Value*[a]	*Total*	*Amortization of Premium*[b]	*Loss Reserve*	*Adjusted Book Value*
July 1, 1952	–	–	–	–	–	$1,020.44
Jan. 1, 1953	$21.25	$15.31	$5.94	$0.84	$5.10	1,014.50
July 1, 1953	21.25	15.22	6.03	0.86	5.17	1,008.47
Jan. 1, 1954	21.25	15.13	6.12	0.88	5.24	1,002.35
July 1, 1954	21.25	15.04	6.21	0.89	5.32	996.14
Jan. 1, 1955	21.25	14.94	6.31	0.91	5.40	989.83
July 1, 1955	21.25	14.85	6.40	0.93	5.47	983.43
TOTAL			37.01	5.31	31.70	

[a] Promised yield at 4 percent less loss rate at 1 percent.
[b] From conventional schedule above.

We shall further assume, however, that all does not go well; that the bond goes into default at the end of three years; and that it is then immediately liquidated by cash payment (or sold in the open market) at a price of $983.43. Under those conditions, the realized yield over the three years would be exactly 3 percent on the book value; the loss rate 1 percent; and the lump-sum capital loss, $31.70, the difference between the book value of $1,015.13 and the liquidating price of $983.43. Multiplied manyfold, such amounts (which may be positive or negative) not only cause undesirable gyrations in the surplus account but also introduce an inequity as between the treatment of previous income recipients and residual legatees.

These consequences could, of course, be obviated if it were known in advance that loss rates on bonds of this type would average around 1 percent; and it is assumed, in constructing the second schedule, that the investor possessed such information, and acted accordingly. He would then estimate the realized yield at 3 percent (4 percent promised less the loss rate of 1 percent) and carry only that amount semiannually into the income account. Cash would be debited as before by the coupon payment of $21.25, and the remainder set aside in reserves to amortize the bond premium and the expected capital loss.

Since the accountant hesitates to confuse in one account that which is known with certainty and that which is only estimated, one method of distributing the difference between earned income and coupon receipts would be to proceed as in the first schedule with the bond premium account; i.e., to credit that account with the difference between the coupon and the promised return, so that the premium would be wiped out at maturity. The remainder, $5.10 for the first investment period, would be credited to a loss-reserve account. The accumulated amounts of the two reserves, provided that the loss rate was correctly estimated, would then wipe out the capital loss ($31.70) and the premium ($5.31).[12]

In practice the loss rate is never known in advance, but may be estimated from historical data, and one of the principal purposes

[12] The same result may be achieved by keeping the books in the conventional way so far as income is concerned and setting aside a portion of surplus in a loss reserve account. This is akin to the procedure now followed by U.S. life insurance companies under a resolution adopted December 5, 1951 by the National Association of Insurance Commissioners, which provides for annual appropriations from surplus to a so-called "mandatory security valuation reserve."

of this report is to obtain actual loss rates for various types of bonds and to indicate how these rates have varied under changing economic conditions. Purely apart from this practical purpose, however, the loss rate is an important tool of analysis, since it measures the annual rate of gain or loss on the investment.

Problems in the Computation of Yields and Yield Averages

Before proceeding to the substantive analysis of bond characteristics and behavior, several computational problems affecting the interpretation of the data call for discussion. These relate to the method of pricing bonds, the method of computing interest, and the problem of interpreting yield averages.

The Pricing Problem and Investment Costs

Initial and terminal prices for the uniform chronological periods were obtained by averaging high and low sale prices for bonds traded on organized securities exchanges and by averaging the highest bid and lowest asking prices for bonds traded over the counter. To avoid unsettled market conditions at year ends, monthly quotations were averaged for the first quarter of the quadrennial years. Market prices exclude commissions paid by investors to brokers and dealers. Adjustments to include them were not made. They would have been troublesome and rather trivial in view of the lengths of the periods spanned by most of the realized yields.

For the offering price, the price given in the advertisement was generally used, i.e. the price at which the bonds were available to the general public. Offering prices therefore include fees and commissions paid by the issuing corporation to the underwriters. In the case of noncash offerings (principally bonds offered in exchange or by contract modifications) market quotations for the three-month period following the date of offering were used.

For the extinguishment price (i.e., price paid investors on date of final extinguishment), par was used for bonds extinguished by payment in full at maturity, and the call price for issues extinguished by call. In the case of bonds extinguished after default we used the lump-sum principal payment received by majority interests, or, when defaults were settled by exchange, the market prices of successor securities five years after exchange. Issues or their successors still outstanding at the end of the period studied

(January 1, 1944) were assumed to have been liquidated at prices ruling in the first quarter of 1944.

For prices on natural dates other than offering and extinguishment, market quotations for the three months following the observation dates were used. For example, to avoid unsettled market conditions on the exact date of default, and at the same time to reflect the market's reappraisal of the altered status of the issue, prices (or bid and asked quotations) were averaged over the three months following default and centered at one and one-half months after default. To simplify the calculation of realized yields from default to two and five years later, the same method was employed: prices were averaged for three-month periods centered at two years and one and one-half months, and at five years and one and one-half months, after default.

Just as the market prices used in computing promised and realized yields exclude commissions and other expenses incidental to the acquisition and disposal of investments, so the records of cash receipts used to determine the realized yields exclude any expenses that may have been incurred by bondholders in collecting those receipts. Such expenses are ordinarily negligible except in default situations. For lack of adequate information, no allowance was made for such incidental costs as lawyers' fees, expenses of bondholders' committees, and any other costs borne by investors, which in some instances represented a major offset to the gross yields obtained on defaulted obligations.

Compound versus Simple Interest

Bond yields are customarily calculated on the assumption of semiannual compounding (that is, on the assumption of a fund growing at a constant semiannual rate). To conform with accepted practice and to preserve internal consistency of the data, both the promised and the realized yields of this study were ordinarily computed by the method of semiannual compounding. The only exceptions were a few of the realized yields that were calculated at simple interest.

The decision to calculate simple interest returns in selected cases was based largely on computational convenience. Because of the approximate nature of the price data the yields are of doubtful significance beyond the first decimal place and were therefore tabulated only to tenths of one percent, e.g., 4.0 percent, 4.1 percent, etc. Within that limit of accuracy the method of sim-

ple interest gives a close approximation to yields under compound interest, provided the investment period is short or the yields are close to zero. [The simple interest return is exactly equal to the compound return in a number of special situations, including those for which the length of the investment period is one period of compounding or the return on the investment is zero; see the technical note at the end of the chapter, especially sections (b) and (e).]

Two types of yields were calculated at simple interest in this investigation: negative realized yields for all periods, and positive realized yields on bonds purchased at default and held over selected periods, i.e., bonds held from default to extinguishment, from default to two and five years later, and, for successor securities, from extinguishment of the old issue to five years later. All other yields were calculated at compound interest. Negative realized yields occur in about 10 percent of the cases from offering to extinguishment, but are usually fairly small; they are substantial only for securities liquidated shortly after investment. For these reasons the majority of such negative realized yields are quite close to those that would be obtained under the more laborious method of semiannual compounding. Negative yields are more important when only defaulted issues are considered, especially for the period from offering to default. About half of the yields on defaulted issues were negative from offering to default, and some of these were appreciable. Since the absolute values of yields at simple interest are larger than at compound interest, the effect of averaging negative yields at simple interest with positive yields at compound interest is to give the yield averages a downward bias as compared with the result that would be obtained by computing both positive and negative yields at compound interest. This bias is believed to be negligible for the yield averages of this study covering the various chronological periods and the life spans of the issues, but may be significant (i.e., large enough to affect the last place of the yield averages carried to tenths of one percent) for the yields of defaulted issues from offering to default.

A further factor militating against the use of the compound interest principle in cases of negative returns is a doubt as to its conceptual applicability. The principle of compound interest is applicable to funds that can be conceived of as growing upon themselves at some constant rate of return. Thus a promised yield

of, say, 4 percent is the rate at which a fund would have to grow to provide a return equivalent to that on the investment. Since it is hardly conceivable that a fund would voluntarily be invested to decline at a compound rate of return, the principle of compounding is of doubtful applicability to negative yields on investments.

As it works out in practice, the decision originally taken to calculate simple interest returns for selected investment periods beginning with the date of default raises somewhat knottier questions of interpretation. Although the periods from default to extinguishment and from default to two and five years later, and, for successors, from extinguishment of the old issue to five years later, are fairly short, the rates of return obtained by investors who held over these periods were found to be substantial. Sample tests give the impression that simple interest returns are quite reliable for purposes of comparing the relative merits, for investors purchasing at default, of liquidating over shorter or longer periods in the future. However, the simple interest returns on defaulted issues, when compared with their counterparts figured at compound interest, show a significant upward bias. Fortunately, the general reasonableness of these returns can be checked against the realized yields from offering to default and from offering to extinguishment, as well as against data on discounted values of receipts following default, all of which (except for the negative yields) were computed at compound interest. Likewise, returns on successor securities held for five years after extinguishment of the old issue can be checked against realized yields on irregular offerings.

Yield Averages

When comparing the yield experience on corporate bonds of various groups or classes, it is convenient to obtain summary measures, or averages, that will reasonably represent the typical behavior of bonds in the different groups. The principal types of summary statistics used for this purpose in the present investigation are simple, or unweighted, averages of promised and realized yields, and weighted averages of the yields, with par amounts of offerings or of outstandings as weights.

The following schedules illustrate the implications of the two types of averages for yield analysis. Under Section A, the investor is assumed to have purchased two bonds at par, one bearing a coupon rate of 3 percent and the other a rate of 4 percent. Both

are further assumed to have been repaid in full at maturity at the end of four years, with no prior delay in the payment of interest. Thus the promised and realized yields on the two securities considered individually are equivalent to their coupon rates, or

Hypothetical Portfolio Yields, Weighted and Unweighted (investment period, 4 years)

AMOUNT INVESTED		PAYMENT DURING			
		First Year	*Second Year*	*Third Year*	*Fourth Year*
		A. Equal Investment in Two Issues at Par			
First issue	$1,000	$30	$30	$30	$1,030
Second issue	1,000	40	40	40	1,040
Total	2,000	70	70	70	2,070
Average	1,000	35	35	35	1,035
		B. Unequal Investment in Two Issues at Par			
First issue	$2,000	$60	$60	$60	$2,060
Second issue	1,000	40	40	40	1,040
Total	3,000	100	100	100	3,100
Weighted average	1,000	33.33	33.33	33.33	1,033.33

3 and 4 percent, respectively. As the schedule indicates, aggregate cash receipts on the total investment of $2,000 are the $70 annual coupons, plus the return of the principal of $2,000 at the end of four years, a stream of income equivalent to the cash receipts obtained by the purchase of two bonds at par, both carrying coupon rates of 3½ percent. Thus the return on the aggregate, or pooled, investment is 3.5 percent—precisely the unweighted average of the promised yields on the two bonds. The illustration may clearly be generalized to cover any number of bonds. Provided all bonds are purchased at par and equal amounts are invested in each, an unweighted average of yields is an exact measure of the rate of return on the aggregate investment.

Schedule B illustrates the case of unequal investments in the two bonds. Both are purchased at par as before, but twice as much is invested in the 3 percent as in the 4 percent bond. Hence the rate of return on the aggregate, or pooled, investment is now 3⅓ percent, which is equivalent to a weighted average of yields, with the par amounts used as weights.

In this monograph the promised yields at offering and the

realized yields from offering to extinguishment are usually presented in the form of weighted averages, with par amounts of offerings as weights. The same is true of the yields for the uniform chronological periods, except that outstandings are used as weights instead of offerings. Such yields, as we have seen, are relevant to an investor who diversifies his portfolio so that the holdings are in rough proportion to the total amounts offered or outstanding. Weighted averages are also appropriate for the analysis of interest returns when the problem is approached from the aggregative, or social, point of view. For the investing public necessarily takes up all issues floated and holds all issues outstanding on any particular date. When analyzing returns on defaulted bonds apart from nondefaulted bonds, unweighted yields were used rather than weighted averages; the former, in effect, give equal weight to issues of various sizes.

The use of weighted or unweighted averages of bond yields as the true yield on the aggregate investment is strictly applicable only in certain limiting situations, such as those discussed above in which the issues are purchased at par. (For further details, see footnote 13; also the technical note at the end of the chapter.) The point may be illustrated by assuming that equal amounts are invested in two twenty-year bonds, one carrying a coupon rate of 3 percent and selling at par to yield 3 percent to maturity, and the other carrying a coupon rate of 5 percent and selling at 113.7 to yield 4 percent. We shall assume as before that the bonds are repaid in full at maturity with no prior delay in interest, so that realized yields equal promised yields. In this case the rate of return on the aggregate investment, with interest compounded semiannually, works out at 3.49 percent, as compared with an average of the individual yields of 3.50 percent. Since our yield averages are tabulated only to tenths of one percent, the difference is negligible in this case, despite the substantial premium on the 5 percent coupon bond. Since most bonds sell close to par at offering, and since the differences between the aggregate returns and the averages are slight even when a substantial number of bonds are selling at premiums or discounts, it did not seem worth while to attempt to adjust the yield averages for such differences. In fact, up to the degree of accuracy recorded in our tables, the averages may generally be interpreted as though such adjustments had actually been made. Thus the unweighted averages of yields provide close approximations to rates of return

on aggregate investments in which equal cash amounts are invested in each issue. Similarly the weighted averages provide close approximations to the aggregate return when cash commitments are proportional to the par amounts of offerings or of outstandings.[13]

The Problem of Reinvestment Rates

In addition to minor discrepancies between the aggregate return on a group of investments and an estimate of that return obtained from a yield average, a further difficulty arises from the fact that the securities in a given portfolio may be held for different lengths of time. When securities are liquidated for any reason during an investment period the proceeds are usually reinvested, but the rate of return on the new bonds will depend upon the condition of the market at the time of reinvestment, the type of security purchased, etc.

Strictly speaking, yield averages of the type presented in this report are applicable only to a fixed portfolio of securities held uniformly throughout a given interval of time. If certain investments are in fact liquidated during the period, the computed averages are not entirely appropriate unless the proceeds are reinvested to yield the same return as that obtained on the original investments. Consider, for example, an investor who purchases equal amounts of two bond issues at the beginning of one of our four-year periods. Bond A, which is held throughout the period, yields a return of, say, 3 percent, while bond B, which is called at the end of two years, yields 4 percent. The average yield of 3.5 percent will be the true rate earned on the aggregate investment only if the proceeds of bond B are reinvested to yield 4 percent, the return obtained on the original investment.

[13] In a technical note at the end of the chapter it is shown that the discrepancy between the true aggregate yield on a group of investments and an average of the individual yields vanishes under certain conditions but not under others. Mathematically, the rate of return on an aggregate is equal to an average of individual yields under the following conditions: (1) when all rates are calculated at simple interest; (2) when the rates are calculated at compound interest and (a) all of the yields are equal, (b) all of the bonds are purchased at par, (c) the bonds are held over a single period of compounding, or (d) the bonds are held over a period of infinitely long duration. Under condition 2(a) any system of weights gives the same average, and under conditions 2(b)–(d) the yield at simple interest is the same as the compound rate. In all other cases discrepancies occur, but they are usually quite small. In view of the approximate nature of our price data, they seemed practically unimportant for the present study.

Since the reinvestment rate actually obtained will depend upon investor preferences and programs peculiar to the investor, as well as upon the general condition of the market, etc., the assumption of any given reinvestment rate common to all investors is essentially arbitrary. Our solution was to make no explicit assumption as to the reinvestment rate: yields of the liquidated securities (principally bonds called or bonds extinguished following default) were simply averaged with the yields of bonds outstanding throughout the entire period. As has been noted, the assumption implicit in the averaging process is that the funds were reinvested at rates obtained on the old securities. This assumption is as arbitrary as any other but it has the virtue of simplicity.

To some extent the problem of assuming a reinvestment rate may be obviated, or its dimensions may be reduced, by appropriate alternative assumptions as to the types of securities studied. Thus in the analysis of investor experience over uniform chronological periods, we assume that only long-term bonds are held at the beginning of the period, that is, only bonds maturing after the close of the period. Although in every period studied some bonds were called or were extinguished after defaulting, the majority were outstanding at the end of each period. Moreover, experiments conducted for several periods on the basis of assumed levels of reinvestment rates indicated that reasonable assumptions in this respect have relatively little effect on the over-all rate of return. The reason appears to be that major drifts in general economic and capital market conditions, of the type that occurred within several of the periods studied, usually carried through to the terminal dates. Thus we shall find in the next chapter that in periods of falling interest rates the volume of bonds called at a premium is usually high, but as interest rates continue to fall throughout the period, new issues eligible for reinvestment also go to a premium. Under such conditions realized returns on the new investments, on the assumption of general liquidation at the end of the period, are roughly comparable with those obtained on the old bonds. Similar relationships usually prevail—although in an opposite direction—in periods of heavy defaults and deteriorating market conditions.

The problem of the reinvestment rate becomes acute in the case of yields calculated over natural periods, such as from offering to extinguishment. Very few bonds offered in any given short period

will be outstanding for the same length of time; e.g., few bonds offered in the single year 1900 were extinguished in the year 1920; yet it is only for issues whose periods of existence correspond, that closely comparable reinvestment rates may be assumed. At the same time, reasonably firm statistical conclusions depend upon combining a sufficiently large number of offerings. We have attempted to meet each of these two problems half way by pooling bonds offered and extinguished over fairly long, but roughly homogeneous periods. The detailed experience records for these periods are presented in *Statistical Measures.* Because of space limitations the records of bonds offered and extinguished at different times are usually grouped in the present report over still broader investment periods. Global statistics of the latter type provide convenient summary measures of experience. It is perhaps unnecessary to add that they typify bond behavior in the subperiods only to the extent that they are confirmed by the detailed data in *Statistical Measures.*

Plan of the Remainder of the Book

The subject matter of the book falls naturally into three parts: the first (Chapter 2) deals with the aggregate default and yield experience on corporate bond investments; the second (Chapters 3 to 6), with various groups of bonds classified according to quality, where quality is defined by the rating agencies, the legal lists, and the market; and the third (Chapters 7 and 8), with other significant characteristics of bonds and their effects on bond performance.

In Chapter 2, attention will be devoted to aggregate experience on corporate bonds over the various natural and chronological periods selected for study, as well as to the behavior of the major industry and size components. Selected materials are also presented on the performance of bonds in minor industry divisions.

Chapter 3 treats of the volume, the characteristics, and the default and yield experience of bonds with different agency ratings, using a composite rating derived from the individual ratings assigned by Fitch Investors Service, Moody's Investors Service, and Standard and Poor's Corporation. Chapter 4 is similar to Chapter 3, but relates to bonds eligible for savings bank and trust fund investment in the states of Maine, Massachusetts, and New York. The analysis of bonds classified by quality is continued in Chapter 5, with an examination of the ratings "assigned" to the bonds

by the market. The market rating used is a yield spread or risk premium, defined as the difference between the promised yield of a given bond (at offering or on any of the quadrennial observation dates) and the yield of the highest grade of bond outstanding at that time and having the same term to maturity.

The rating agencies, the legal lists, and the market provide three systems of rating corporate bonds. One of the problems confronting those interested in broadening the lists of securities eligible for purchase by financial intermediaries and personal trust accounts is that of the inclusiveness, consistency, and stability of alternative rating systems. Another is the comparative performance of bonds rated differently under different rating systems. Problems of this type are examined in Chapter 6.

The next chapter (Chapter 7), on earnings coverage and lien position, pertains to the contributions made by these two important factors to bond safety and yield.

Chapter 8, on size of issue and asset size of obligor, investigates the structure and behavior of the bond market as it is influenced by these two variables—the dominant position of large issues in total outstandings, the degree of concentration shown by the size distributions, and the performance of bonds in different issue- and asset-size classes.

As an aid in locating areas of special interest, each chapter is preceded by a brief summary of findings. Cross-references are given throughout the book to supporting data in other reports of the series.

TECHNICAL NOTE ON BOND YIELDS AND YIELD AVERAGES

The purpose of this note is to develop somewhat more fully than in the text the concepts of promised and realized yields as applied to corporate bonds, and to indicate the type of problem that arises when such yields are combined by averaging. The yield of a bond, whether promised or realized, is a special application of the concept of "rate of return over cost" encountered in the economic theory of capital and interest. The rate of return over cost is the rate that equates the sum of discounted values of future receipts flowing from an investment to its present value or cost. Let the future receipts be represented by $\pi_1, \pi_2, \ldots, \pi_n$ (where n is the number of periods of compounding that elapse between the dates of receipt and investment). Also let the present value or cost of the investment be represented by P; the rate of return over cost by i; and the discount factor $(1 + i)^{-\alpha}$ by v^α. Then the formula for the present worth of an investment may be written as follows:

$$P = \sum_{\alpha=1}^{n} \pi_\alpha v^\alpha \tag{1}$$

If P and the π's are given in advance, then i, the rate of return over cost, is the resulting solution of equation (1).

Promised Yields

The promised yield of a bond is a particular solution of equation (1) obtained by interpreting P as the price of the bond (offering price if purchased at offering; market price if purchased on some subsequent date), and by inserting for the π's the amounts promised the bondholder in the bond contract, on the assumption that the issue will be extinguished by payment in full at maturity and that all interest payments will be made when due.

A straight bond contract may be looked upon as a promise to pay a certain lump-sum amount (P_n) on the maturity date (the principal or par amount of the bond), plus an annuity of C dollars (the coupons) at the end of each of n periods of compounding. Since the coupons are usually paid semiannually, bond yields are customarily calculated on the basis of semiannual compounding, despite the fact that the yields themselves, and the coupon rates too, are normally quoted on an annual basis. Thus if i_p is the promised yield for one interest period, and c is the coupon rate for the same period, then the quoted yield is $2i_p$ and the quoted coupon rate is $2c$. Bond prices, yields, and coupon rates are quoted as percents of par in the market place, and have been presented in the text in that form, but it will be observed that the common unit in which these variables are measured is immaterial so far as the solution of equation (1) is concerned. To simplify the notation, the variables in this note are expressed in the decimal notation (i.e. as cents per dollar rather than as percents of par). In decimal form, the principal amount is equal to unity ($P_n = 1$), and the amounts of the semiannual coupons are equal to the coupon rates ($C = cP_n = c$). The price (or

yield) of a bond may therefore be determined directly from equation (1) by substituting c for each $\pi_\alpha(\alpha = 1, \ldots, n-1)$, and $(c+1)$ for π_n, viz.,

$$P = c \sum_{\alpha=1}^{n} v^\alpha + v^n \tag{2}$$

The expression $\sum_{\alpha=1}^{n} v^\alpha$, which reduces to $(1 - v^n)/i$, is usually represented by the symbol $a_{\overline{n}|}$ (read a angle n). In that notation formula (2) becomes:

$$P = ca_{\overline{n}|} + v^n \tag{3}$$

The functions $a_{\overline{n}|}$ and v^n, which are available in most standard collections of mathematical tables, are tabulated as functions of the arguments n and i. From the definitions of $a_{\overline{n}|}$ and v^n and from equation (3), it is evident that the promised yield (i_p) is an implicit function of P, c, and n. To remind ourselves of this fact it is sometimes convenient to write equation (3) as follows:

$$P = c\,(a_{\overline{n}|} \text{ at } i_p) + v^n \text{ at } i_p \tag{4}$$

In general, equations (3) and (4) cannot be inverted to express i_p as an explicit function of P, n, and c. A solution must therefore be obtained by the method of successive approximations or from standard bond yield tables, whenever the tables cover the appropriate range of values of P, n, and c. In the bond yield tables, the variables are expressed in percents of par, and i and c are converted to annual rates.

Realized Yields

The realized yield (i_r) is determined from equation (1) by inserting the bond price for (P) and the amounts actually received by the bondholder for the π's. As a general rule, the promised yield and the realized yield are equal only for nondefaulted bonds held by the investor from purchase date to maturity. In other cases (calls, defaults, sales prior to maturity, etc.), the two yields usually differ considerably, and the realized yield must then be computed independently, either by means of equation (1), or by some short-cut formula derived from it. For bonds extinguished by call, for example, the realized yield is the solution of the equation

$$P_0 = c\,(a_{\overline{n}|} \text{ at } i_r) + P_n(v^n \text{ at } i_r)$$

where P_0 is the purchase price, P_n the call price, and the other variables are as defined above. For nondefaulted bonds held over a quadrennial period, the equation to be solved is similar, viz.,

$$P_0 = c\,(a_{\overline{8}|} \text{ at } i_r) + P_8(v^8 \text{ at } i_r)$$

where P_0 is the purchase price (market price at the beginning of the period), and P_8 is the sale price (market price at the end of eight semi-

annual periods of compounding). For bonds extinguished after default, the equation to be solved is usually more involved, particularly when successor securities received by the bondholder on the settlement date provide an irregular stream of cash payments. But no matter how involved the computations, a solution for the realized yield can always be obtained in principle upon substitution of the appropriate values in equation (1).

The Problem of Interpreting Yield Averages

The section on yield averages, in Chapter 1, touched upon the question of interpretation. It may be stated more formally as follows: If an amount X_1 is invested in a bond issue B_1 having a coupon rate c_1 and selling at a price P_1 to yield i_1, and another amount X_2 is invested in a second issue B_2 having a coupon rate c_2, and selling at a price P_2 to yield i_2, under what conditions will the yield on the aggregate or "pooled" investment of an amount $(X_1 + X_2)$ be equal to the weighted average $(X_1 i_1 + X_2 i_2) \div (X_1 + X_2)$? It will be shown that in some circumstances the yields may be combined without error by averaging, but that in others—not practically important for the problems of this report—some error is involved. To simplify the discussion, it will be assumed that the two obligations selected by the investor are held over the same period of time, and are paid in full at maturity, with no prior delay or impairment in the payment of interest. Under such conditions, the realized yield equals the promised yield in each case, and the amounts paid the investor at maturity are unity (i.e., $i_r = i_p$ and $P_n = 1$ for B_1 and B_2).

(a) *Simple interest.* Few of the yields utilized in the present investigation were computed at simple interest; but statement of that case will illuminate the rest.

At simple interest the yield (i_1) on X_1 dollars invested in B_1 is

$$i_1 = \left(\frac{X_1}{nP_1}\right)\left(\frac{1 - P_1 + nc_1}{X_1}\right) = \frac{1 - P_1 + nc_1}{nP_1}$$

Similarly, at simple interest, the yield (i_2) on X_2 invested in B_2 is

$$i_2 = \left(\frac{X_2}{nP_2}\right)\left(\frac{1 - P_2 + nc_2}{X_2}\right) = \frac{1 - P_2 + nc_2}{nP_2}$$

Finally, the yield (i_3) on $X_1 + X_2$ dollars invested at simple interest in B_1 and B_2 is

$$i_3 = \left(\frac{X_1}{nP_1}\right)\left(\frac{1 - P_1 + nc_1}{X_1 + X_2}\right) + \left(\frac{X_2}{nP_2}\right)\left(\frac{1 - P_2 + nc_2}{X_1 + X_2}\right)$$

$$= \frac{X_1 i_1 + X_2 i_2}{X_1 + X_2}$$

In other words, at simple interest the over-all yield on the pooled investment is equal to the weighted average of the yields on the individual investments.

(b) *Compound interest for bonds held over a single period of compounding.* When calculated for a single period, the yield is the same at compound interest as at simple interest. To prove that this is so, each side of equation (2) may be multiplied successively by X_1 and X_2 to obtain the following pair of equations:

$$X_1P_1 = c_1X_1 \sum_{\alpha=1}^{n} v^\alpha + X_1v^n,$$

and

$$X_2P_2 = c_2X_2 \sum_{\alpha=1}^{n} v^\alpha + X_2v^n$$

If $n = 1$, these equations reduce to

$$i_1 = \frac{1 - P_1 + c_1}{P_1}$$

$$i_2 = \frac{1 - P_2 + c_2}{P_2}$$

But these are precisely the expressions for the simple interest returns discussed in section (a) above when $n = 1$. It follows that at compound interest the over-all yield on the pooled investment held over a single period of compounding is equal to the weighted average of the yields on the individual investments.

(c) *Compound interest for bonds held over a period of infinitely long duration.* Since n is the duration of the period of investment, a period of infinitely long duration may be defined as one having length $n > k$, for any preassigned k.

Reference back to the definitions of $a_{\overline{n}|}$ and v^n shows the following to be true:

$$\lim_{n\to\infty} v^n = 0$$

$$\lim_{n\to\infty} a_{\overline{n}|} = i^{-1}$$

Upon substitution of these limiting values in equation (3) we obtain

$$i_1 = c_1/P_1,$$

and

$$i_2 = c_2/P_2$$

The formulas of section (a) on simple interest reduce to the same expressions for obligations held over a period of infinitely long duration. It follows as an immediate consequence of the results of that section that the over-all yield on the aggregate investment is equal to the weighted average of the yields on the individual investments, for bonds held over a period of infinitely long duration.

(d) *Compound interest for bonds selling at par.* For bonds selling at par, $P = 1$. Since the bond yield at simple interest is

$$i = \frac{1 - P + nc}{nP}$$

it follows immediately that $i = c$ when $P = 1$. The same is true when the yields are calculated at compound interest; for, since $P = 1$, we have from equation (3),

$$1 - v^n = ca_{\overline{n}|}$$

By definition

$$a_{\overline{n}|} = \frac{1 - v^n}{i}$$

It follows that

$$ia_{\overline{n}|} = ca_{\overline{n}|}$$

or

$$i = c$$

Since the yield at compound interest is equal to the yield at simple interest for bonds selling at par, it follows from results previously established that the true yield on the aggregate investment is the weighted average of yields on the individual investments.

(e) *Compound interest for equal yields.* Consider two issues B_1 and B_2 with the respective coupon rates c_1 and c_2, and selling at the respective prices P_1 and P_2 such that $i_1 = i_2$. From equation (4) it is evident that the following equations must hold:

$$X_1 = \frac{X_1 c_1}{P_1}(a_{\overline{n}|} \text{ at } i_1) + \frac{X_1}{P_1}(v^n \text{ at } i_1)$$

$$X_2 = \frac{X_2 c_2}{P_2}(a_{\overline{n}|} \text{ at } i_2) + \frac{X_2}{P_2}(v^n \text{ at } i_2)$$

For the aggregate investment of $X_1 + X_2$ in B_1 and B_2 the following equation must hold:

$$X_1 + X_2 = \left(\frac{X_1 c_1}{P_1} + \frac{X_2 c_2}{P_2}\right)(a_{\overline{n}|} \text{ at } i_3) + \left(\frac{X_1}{P_1} + \frac{X_2}{P_2}\right)(v^n \text{ at } i_3)$$

Since $i_1 = i_2$ is the solution of the first two equations, it is necessarily a solution of the third equation, i.e., $i_1 = i_2 = i_3$. Thus the weighted average of yields, which in this case is the same as the individual yields, again equals the true yield on the aggregate investment.

The situation in which the yields are very small (to all intents and purposes, zero) is an interesting special case of the more general class of cases in which the yields are equal. It is noteworthy that in this particular situation the yields at compound interest and at simple interest are equal, so that the result follows immediately from the conclusions of section (a) on simple interest. To prove that this is so it is sufficient to show that the yield at simple interest vanishes with the yield at compound interest. From the definitions of $a_{\overline{n}|}$ and v^n, it follows that

$$\lim_{i \to 0} v^n = 1,$$

and

$$\lim_{i \to 0} a_{\overline{n}|} = n$$

from which it follows by substitution in equation (3) that

$$P = cn + 1$$

Substitution of this value for P in the simple interest formula of section (a) shows that the yield at simple interest vanishes with the yield at compound interest.

(f) *Compound interest in the general case.* Thus far it has been demonstrated that under several limiting conditions the weighted average of yields on individual investments is equal to the true yield on the aggregate, or "pooled," investment. The matter is conceptually significant for the present report since it ensures that a definite interpretation may be placed upon the yield averages studied, whenever it may reasonably be assumed that the limiting conditions are satisfied approximately. As a matter of brute fact, one or the other of the limiting conditions is nearly fulfilled in many practical situations; and in most other situations, the "error" committed by interpreting the yield averages as the true yield appears to be very small. Most bonds, for example, are priced close to par at offering, so that an average of promised yields at offering is necessarily quite close to the true over-all promised yield. Moreover, in many cases yields realized from offering to extinguishment are close to the yields promised at offering, so that the average life-span yields should usually also be close to the true yield for the aggregate. In the present report, yields are frequently averaged at offering and on quadrennial dates for issues in selected quality and industry classes, and such issues not only sell on about the same promised yield basis, but often provide roughly similar realized returns to investors. Section (e) indicates that in all such cases the weighted average of yields is close to the true return. Moreover, the errors associated

with the combining of yields on broader groups of issues may be roughly offsetting (as, for example, when yields of issues selling above and below par are averaged), so that the errors tend to cancel out in large aggregates such as those presented in this report. Most important, actual calculations covering a number of hypothetical situations show that the weighted averages usually match the true yields when carried to tenths of one percent, even when there are marked departures from the limiting conditions under which the error vanishes.[14]

From the foregoing considerations it is believed that most (possibly all) of the yield averages presented in this report can be interpreted without error as true over-all yields, up to the degree of accuracy presented. On the other hand, since the true yields for the large number of aggregates studied are not known and cannot be determined short of intolerably complex calculations, it cannot be guaranteed that the averages may always be interpreted in this fashion.

The essential difficulty arises from the fact that a weighted average of bond yields is not in general precisely equal to the true yield. To illustrate that the difficulty is not an imaginary one, it will be assumed that an investor places equal amounts in two obligations, B_1 and B_2, and holds them over two periods of compounding (i.e., one year under semiannual compounding), at the end of which time both issues are paid off at par. To further simplify the algebra, it will be assumed that the two issues have different coupon rates, but sell at the same price other than par (i.e., $c_1 \neq c_2$, but $P_1 = P_2 = P \neq 1$). Substitution of $(1 + i)^{-\alpha}$ for v^α and rearrangement of terms in equation (2) shows that under such conditions the following equations must hold:

$$(P)i_1^2 + (2P - c_1)i_1 + (P - 2c_1 - 1) = 0$$

$$(P)i_2^2 + (2P - c_2)i_2 + (P - 2c_2 - 1) = 0$$

It will be observed that the coefficients of the i's in these equations are unequal, from which it follows that $i_1 \neq i_2$. Since equal amounts are assumed to have been invested in the two issues, the true aggregate yield (i_3) must satisfy the following equation:

$$(P)i_3^2 + \left(2P - \frac{c_1 + c_2}{2}\right)i_3 + (P - c_1 - c_2 - 1) = 0$$

If we substitute $(i_1 + i_2)/2$ (the weighted average of yields under conditions of equal investment) for i_3 in this equation, expand, and elimi-

[14] Actuaries consulted on the matter have expressed the opinion that the error involved in the use of weighted averages of bond yields is practically unimportant. In day-to-day work, weighted averages are usually calculated to hundredths of one percent (one place beyond those presented here) and are freely interpreted as true yields on the bond portfolio. A search of the actuarial literature failed to locate any discussion of the errors that may arise from this practice.

nate expressions satisfying the first pair of equations, there remains the following equation:

$$2P(i_1i_2 + i_1 + i_2 + 1) - 2(c_1 + c_2 + 1) - i_1c_2 - i_2c_1 = 0$$

Since the left-hand member of this equation does not vanish identically, $i_3 = (i_1 + i_2)/2$ is not a general solution. Particular solutions are obtained if $c_1 = c_2$ (which implies that $i_1 = i_2$) or if $P = 1$ (which implies that $i_1 = c_1$ and $i_2 = c_2$); but these solutions must be rejected, since they violate the initial conditions.