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# A Postwar Quarterly Model: Description and Applications

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## *Introduction*

THE National Bureau of Economic Research has often made the point that annual data are inadequate in business cycle analysis. This is not to claim that they are worthless but merely to recognize that we ought to try to do better. Without going to the extreme that the NBER reaches in doing most of its analysis with monthly data, we in econometric model-building research ought to go at least as far as the construction of quarterly systems. Eventually, we shall build monthly models, but the first step is naturally a quarterly model. There is serious doubt whether suitable data could be found for our methods on a monthly basis. The quarterly national income accounts are now plentiful, though not necessarily ultimately refined, and we have had a good span of time since the end of World War II in which to build up a sample of respectable size.

Some prewar quarterly data stretch back as far as World War I. These have already been exploited in econometric model building by Harold Barger and myself, but our investigation dealt only with a small model to be used for methodological purposes.[1] It might be possible to prepare an approximate set of quarterly series covering the period before as well as after World War II on the scale needed for the present model, but the expenditure of time would be enormous. We made a pragmatic decision to confine the analysis to postwar quarterly data. That alone posed substantial problems of data processing. A possible advantage of this decision was that we obtained

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[1] "A Quarterly Model for the United States Economy," *Journal of the American Statistical Association*, September 1954, pp. 413–437.

a more homogeneous sample, but we lost in terms of richness of experience.

We made another basic decision at the outset, namely, to use seasonally adjusted data. The alternative would have been to introduce explicit seasonal variables, as was done in the recent British model.[2] Consumers of economic data and of the results of economic analysis appear to be more receptive to seasonally adjusted than to unadjusted data; therefore, we decided to make our findings available immediately in adjusted form. This freed us from a certain amount of routine work by making the number of variables smaller in each equation. Theoretically, there is much to be said in favor of using seasonal variables with unadjusted data, but an adequate treatment may, in several cases, take us beyond the simple additive process used in the British model.

It may be useful, at the outset, to distinguish the present model from its annual predecessors, using the Klein-Goldberger model as a reference point.[3]

1. The present model is less aggregative. There are more equations in the present model. Some represent obvious decomposition of national product elements; others stem from more subtle theorizing about patterns of behavior.

2. Anticipatory data are used in the present model. In applications of the Klein-Goldberger model to problems of forecasting, frequent use was made of expectations about consumer purchases and investment outlays, but these subjective variables were not built into the models directly. Now we have introduced realization functions which express actual behavior as a function of expectations. In *short-run* forecasting these equations can be used; but since we do not provide an endogenous explanation of expectations, only limited use can be made of such relations. Nevertheless, we feel that this is an important first step in macroeconomic model building.

3. Explicit relations among inventories, sales, backlogs, and order flow appear in the new model. The Klein-Goldberger model slurred over the whole question of inventory investment. Later work has extended that model annually, using more explicit inventory behavior; but the essence of inventory-order patterns probably cannot

[2] Lawrence R. Klein *et al.*, *An Econometric Model of the United Kingdom*, Oxford, Eng., 1961.

[3] Lawrence R. Klein and A. S. Goldberger, *An Econometric Model of the United States, 1929–1952*, Amsterdam, 1955.

be discerned with annual data. Our quarterly model is more promising in this respect.

4. The concept of capacity, together with the rate of utilization, is introduced in the new model. It is difficult to define capacity and to measure it. Nonetheless, this concept figures importantly in much economic analysis. We attempt, at the Wharton School, to measure capacity utilization; and, imperfect though our series may be, it appears to be of significance in the structure of our model.

5. The accounting identities are properly expressed in current prices, while the behavioral and technical equations are, save for appropriate exceptions, in real terms, relative prices, or deflated incomes. There was a distortion in the older annual models, caused by requiring the national income identities to hold in real or deflated variables. When prices change by large amounts, these distortions grow in significance.

There are other points of difference between the new and the older models, but those above are the differences that motivated the present research. Others will be brought out in the discussion of the equations of the model.

## *The Model*

The sample data include the quarters from I-1948 to IV-1958. For lagged values we used some earlier quarters. Time has elapsed since the model was estimated, and quarterly data for 1959, 1960, and 1961 are now available. Eventually, the whole system will be re-estimated. The estimates are limited-information maximum-likelihood estimates. In some cases two-stage least-squares estimates have been used because of problems of multicollinearity. It has been found that limited information estimates are more sensitive than two-stage estimates to the presence of multicollinearity. In cases where the limited-information method gave obviously nonsensical results, we used two-stage estimates instead. Multicollinearity problems among the set of predetermined variables also proved troublesome, and we estimated the system in two major groups of equations with a somewhat different set of predetermined variables in each group.

### LIST OF VARIABLES

*$C_d$ Expenditures on consumer durables, billions of 1954 dollars

*$C_n$ Expenditures on consumer nondurables, billions of 1954 dollars

| | |
|---|---|
| $*C_s$ | Expenditures on consumer services, billions of 1954 dollars |
| $*Y - T$ | Disposable personal income, billions of current dollars |
| $*W$ | Wages, salaries, and other labor income, billions of current dollars |
| $*P$ | Nonlabor personal income, billions of current dollars |
| $C_d^e$ | Index of consumer buying plans for durable goods |
| $*L$ | End-of-quarter cash balances, billions of current dollars |
| $*p_d$ | Implicit deflator, consumer durables, 1954 = 1.00 |
| $*p_n$ | Implicit deflator, consumer nondurables, 1954 = 1.00 |
| $*p_s$ | Implicit deflator, consumer services, 1954 = 1.00 |
| $N$ | Population, millions of persons |
| $*I_p$ | Expenditures on private producers' plant and equipment, billions of 1954 dollars |
| $*I_h$ | Expenditures on nonfarm residential construction, billions of 1954 dollars |
| $*I_i$ | Inventory investment, billions of 1954 dollars |
| $*X$ | Private gross national product, billions of 1954 dollars |
| $*X_c$ | Private gross national product at full capacity, billions of 1954 dollars |
| $I_p^e$ | Intended investment outlays, billions of 1954 dollars |
| $*q_h$ | Implicit deflator, nonfarm residential construction, 1954 = 1.00 |
| $*i_L$ | Average yield, corporate bonds, per cent |
| $F_s$ | Number of marriages, thousands |
| $I_h^s$ | Number of housing starts |
| $*h$ | Hours worked per week, index |
| $*i_s$ | Average yield, ninety-day commercial paper |
| $*O$ | Manufacturers' new orders, billions of 1954 dollars |
| $*U$ | Manufacturers' unfilled orders, billions of 1954 dollars |
| $*S_c$ | Corporate retained earnings, billions of current dollars |
| $*P_c$ | Corporate profits, billions of current dollars |
| $T_c$ | Corporate income taxes, billions of current dollars |
| $*q_p$ | Implicit deflator, plant and equipment expenditures, 1954 = 1.00 |
| $*D_r$ | Capital consumption allowances, replacement cost, billions of 1954 dollars |
| $*N_w$ | Number of employees, millions of persons |
| $N_g$ | Number of government employees, millions of persons |

| | |
|---|---|
| $N_e$ | Number of self-employed, millions of persons |
| $W_g$ | Government wages, salaries, and other labor income, billions of current dollars |
| $*p$ | Implicit deflator, gross national product, 1954 = 1.00 |
| $*w$ | Average annual wage, current dollars |
| $*N_L$ | Labor force, millions of persons |
| $*F_e$ | Exports of goods and services, billions of 1954 dollars |
| $X_w$ | Index of world production, 1954 = 1.00 |
| $*F_{im}$ | Imports of crude food and materials, billions of 1954 dollars |
| $p_i$ | Implicit deflator, imports of goods and services, 1954 = 1.00 |
| $*F_{if}$ | Other imports, billions of 1954 dollars |
| $R$ | End-of-quarter percentage of total bank reserves held in excess of required reserves |
| $i_r$ | Federal Reserve average discount rate |
| $U_d$ | Manufacturers' unfilled orders of durable goods, billions of 1954 dollars |
| $U_n$ | Manufacturers' unfilled orders of nondurable goods, billions of 1954 dollars |
| $*C$ | Total consumer expenditures, billions of 1954 dollars |
| $p_w$ | Index of prices of competing exports, 1954 = 1.00 |
| $*p_e$ | Implicit deflator, exports of goods and services, 1954 = 1.00 |
| $G$ | Government expenditures on goods and services, billions of current dollars |
| $D_a$ | Capital consumption allowances, accounting prices, billions of current dollars |
| $T_i$ | Reconciling item between net national product and national income, billions of current dollars |

* Denotes endogenous variable.

Variables taken from the national income accounts in dollar totals are seasonally adjusted at annual rates. Most other variables are also seasonally adjusted.

In the equations written below, the numbers in parentheses under each coefficient are estimated standard errors. The correlation measures, $\overline{R}$, are computed from the formula

$$R = \sqrt{1 - \left(\frac{\Sigma r^2}{T - m}\right)\left(\frac{T-1}{\Sigma x^2}\right)}$$

where $r$ is the residual, $x$ is the dependent variable, and $m$ is the number of parameters in the equation. The equations that have been estimated by the two-stage, least-squares method are marked TSLS below the number.

ESTIMATED EQUATIONS

(1)
$$C_d = \underset{(51.0)}{-67.1} + \underset{(.15)}{.363}\,\frac{Y-T}{p_d} + \underset{(79.0)}{58.4}\,\frac{P}{W} - \underset{(.86)}{1.14}\,\frac{1}{8}\sum_{i=1}^{8}(C_d)_{-i} + \underset{(.093)}{.174}C_d^e \qquad \bar{R} = 0.40$$

(2)
$$C_n = \underset{(8.1)}{27.7} + \underset{(.044)}{.259}\,\frac{Y-T}{p_n} + \underset{(15.0)}{8.88}\,\frac{P}{W} + \underset{(.095)}{.191}\,\frac{1}{8}\sum_{i=1}^{8}(C_n)_{-i} + \underset{(.055)}{.0056}\left(\frac{L}{p_n}\right)_{-1} \qquad \bar{R} = 0.99$$

(3)
$$C_s = \underset{(19.0)}{-152.0} + \underset{(.017)}{.103}\,\frac{Y-T}{p_s} + \underset{(6.9)}{41.1}\,\frac{P}{W} + \underset{(.13)}{.0188}\,\frac{1}{8}\sum_{i=1}^{8}(C_s)_{-i} + \underset{(.024)}{.0596}\left(\frac{L}{p_s}\right)_{-1} + \underset{(.16\ )}{1.13}N$$
$$\bar{R} = 0.99$$

(4)
$$I_p = \underset{(4.16)}{-8.18} + \underset{(4.76)}{32.5}(X/X_c) + \underset{(.0486)}{.557}I_p^e \qquad \bar{R} = 0.91$$

(5)
$$I_h = \underset{(1.2)}{-11.3} + \underset{(.0091)}{.0764}\,\frac{Y-T}{q_h} - \underset{(.47)}{.776}i_L + \underset{(.0015)}{.0011}F + \underset{(.0007)}{.00812}(I_h^s)_{-1} \qquad \bar{R} = 0.96$$

(6)
$$I_i = \underset{(13.5)}{-48.42} + \underset{(.0707)}{.2675}(X - I_i) - \underset{(.06)}{.2997}\sum_{j=1}^{\infty}(I_i)_{-j} + \underset{(75)}{269.3}(p - p_{-1}) + \underset{(.047)}{.2031}U_{-1}$$
$$\bar{R} = 0.99 \text{ (stockform)}$$

(7)
$$S_c/q_p = \underset{(2.5)}{-.448} + \underset{(.061)}{.938}\frac{P_c - T_c}{q_p} - \underset{(.17)}{.853}\frac{1}{8}\sum_{i=1}^{8}\left(\frac{P_c - T_c - S_c}{q_p}\right)_{-i} \qquad \bar{R} = 0.96$$

(8)
$$P_c = \underset{(5.1)}{5.49} + \underset{(.10)}{.627}\left(P - \frac{1}{3}^{P-1}\right) \qquad \bar{R} = 0.59$$

(9)
$$D_r = \underset{(3.10)}{10.8} + \underset{(.017)}{.0664}X + \underset{(.0034)}{.00599}\sum_{i=1}^{\infty}(I_p + I_h - D_r)_{-i} \qquad \bar{R} = 0.94$$

(10) TSLS
$$X = \underset{(60.83)}{90.9} + \underset{(1.485)}{1.758}[h(N_w - N_g) + N_e] + \underset{(.062)}{.196}(X/X_c)\sum_{i=0}^{\infty}(I_p + I_h - D_r)_{-i} + \underset{(.640)}{.135}t \qquad \bar{R} = 0.99$$

(11) TSLS
$$X_c = \underset{(60.83)}{90.9} + \underset{(1.485)}{1.758}N_L + \underset{(.062)}{.196}\sum_{i=0}^{\infty}(I_p + I_h - D_r)_{-i} + \underset{(.640)}{.135}t \qquad \bar{R} = 0.93$$

(residual variance about mean)

(12)
$$\frac{W - W_g}{p} = \underset{(7.4)}{7.19} + \underset{(.015)}{.254}X + \underset{(.015)}{.254}X_{-1} + \underset{(.083)}{.221}t \qquad \bar{R} = 0.99$$

(13)
$$w - w_{-4} = \underset{(46.0)}{169.0} - \underset{(15.0)}{38.2}\frac{1}{4}\sum_{i=0}^{3}(N_L - N_w - N_e)_{-i} + \underset{(540.0)}{2110}\frac{1}{4}\sum_{i=0}^{3}(p - p_{-4})_{-i} + \underset{(.80)}{1.56}t \qquad \bar{R} = 0.56$$

(14)
$$h = \underset{(.047)}{.721} + \underset{(.052)}{.320}(X/X_c) + \underset{(.0006)}{.00217}(X - X_{-1}) - \underset{(.00017)}{.00026}t \qquad \bar{R} = 0.51$$

(15)
$$N_L = \underset{(.21)}{61.2} - \underset{(.075)}{.308}(N_L - N_w - N_e) + \underset{(.0053)}{.226}t \qquad \bar{R} = 0.99$$

(16) TSLS
$$U = -101 + \underset{(44)}{} 2.12\underset{(.84)}{O} + 111\underset{(55)}{(X/X_c)} \qquad \bar{R} = 0.62$$

Standard errors: (44) (.84) (55)

(17)
$$O = 2.56 + .0589(X - I_i)_{-1} + 387(p - p_{-1})$$
(3.2) (.0098) (72.0)
$$\bar{R} = 0.60$$

(18)
$$F_e = 2.98 + .160(X_w)_{-1} \qquad \bar{R} = 0.90$$
(1.112) (.0115)

(19)
$$F_{im} = 3.82 + .0065X_{-1} - 1.04(p_i/p)_{-1} \qquad \bar{R} = 0.53$$
(.867) (.0015) (.804)

(20)
$$F_{if} = 8.11 + .039\frac{Y - T}{p_i} - 24.1\frac{P}{W} + .286\frac{1}{8}\sum_{i=1}^{8}(F_{if})_{-i} \qquad \bar{R} = 0.97$$
(2.62) (.0082) (5.3) (.15)

(21) TSLS
$$\frac{L}{pX + W_g} = .815 - .0743i_L - 1.38(p - p_{-1}) \qquad \bar{R} = 0.64$$
(.058) (.0131) (.92)

(22)
$$i_L = .0541 + .0497i_s + .959(i_L)_{-1} \qquad \bar{R} = 0.97$$
(.15) (.034) (.060)

(23)
$$i_s = .502 - .146R_{-1} + 1.18i_r \qquad \bar{R} = 0.96$$
(.399) (.060) (.096)

(24)
$$p_d = .548 + .422p + .00067(U_d)_{-1} \qquad \bar{R} = 0.94$$
(.034) (.039) (.00017)

(25)
$$p_n = .346 + .618p + .00946(U_n)_{-1} \qquad \bar{R} = 0.97$$
(.027) (.024) (.0021)

(26)
$$p_s = .716 + .000179w - 1.08(C_s/C) \qquad \bar{R} = 0.99$$
(.090) (.000005) (.29)

(27)
$$q_p = -.508 + 1.52p \qquad \bar{R} = 0.99$$
(.028) (.029)

(28)
$$q_h = .492 + .000144w \qquad \bar{R} = 0.96$$
(.021) (.000006)

(29)
$$p_e = .374 + .0688p_w + .572p \qquad \bar{R} = 0.83$$
(.063) (.088) (.12)

(30)
$$p_dC_d + p_nC_n + p_sC_s + q_pI_p + q_hI_h + pI_i + p_eF_e - p_i(F_{im} + F_{if}) + G = pX + W_g$$

(31) $W + P + S_c - pX = W_g - D_a - T_i$

(32) $hwN_w = W\,10^8$

(33) $W + P = Y$

(34) $C = C_d + C_n + C_s$

## *Discussion of the Equations*

To make the system more comprehensible before we discuss applications of the model and its actual performance, we shall comment briefly on each equation or group of equations, comparing it with related work in econometrics.

### THE CONSUMPTION EQUATIONS

Consumer expenditures have been split into three obvious components—durables, nondurables, and services. Starting from the time-honored proposition that consumption (or consumption type) is dependent on aggregate income, we introduce the following qualifications:

1. Income should be adjusted for taxes and transfers. We use disposable income.
2. Relative prices might be relevant when dealing with subgroups of consumption. We deflate disposable income by the price index of the consumption type considered.
3. Income distribution as well as aggregate income may affect consumption. We use a separate variable to measure the ratio of wage to other personal income.
4. There may be lags in consumer behavior. We introduce average consumption (by type) of the past eight quarters to show the effect of the past.[4]
5. Consumer wealth as well as income may influence behavior. We used total stock of cash as a particular wealth variable of strategic importance in consumer spending.
6. Population growth may affect consumption. We introduced an explicit population variable, although we could have measured

[4] In the Barger-Klein quarterly model, last quarter's consumption was used, in direct analogy to the successful use of last year's consumption in the annual models. There is so much pure autocorrelation that this kind of quarterly relation was not satisfactory. If past consumption is to represent a standard or norm from which adjustments to current conditions take place, it seems better to use average consumption of the recent past. All these schemes using past consumption are transformations of distributed lag processes.

consumption, income, and cash balances on a per capita basis as an alternative.

We made these adjustments uniformly to all three consumption functions, but in the end settled for selective use of certain variables in certain equations. This was an empirical selection that has been used throughout the model. Many experimental calculations were made for each equation. We finally settled upon a set of parameter estimates for each equation that looked reasonable. Reasonableness was based on a priori notions about sign and order of magnitude of coefficients. The degree of experimentation was limited because we committed ourselves to a set of predetermined variables for the whole group of calculations by the method of limited information. We introduced one special variable in the equation for consumer durables. This variable is an index of consumer buying plans for new cars and other household items. We do not explain this variable within the system.

### THE INVESTMENT EQUATIONS

Capital formation is divided into producers' plant and equipment, residential construction, and inventory investment. In the plant and equipment equation, investment intentions are introduced explicitly. These are the data of the Office of Business Economics—Securities and Exchange Commission on first intentions deflated by the price index of capital goods as of the (future) date to which the intentions refer. One may justifiably argue that we should deflate them as of the date at which the intentions are expressed. A similar anticipatory variable appears in the residential construction equation. It is the lagged value of starts.

The system is open with respect to these two anticipatory variables; i.e., we offer no endogenous explanation of investment intentions or housing starts; therefore, the extrapolation period for the model is limited. Our treatment here is parallel to that in the equation for consumer durables, where we introduce the index of consumer buying plans. In the inventory investment equation, we have proceeded somewhat differently. We have the backlog of orders as a kind of anticipatory variable there, but we attempt to give, at a later stage in the model, an endogenous explanation of unfilled orders, bringing new orders into the system as well.

The positive correlation between inventory investment and unfilled orders may seem to be strange, for businessmen ought not to be accumulating stocks while they still have backlogs of unfilled orders on hand. Our disaggregation was not carried far enough in this system to distinguish among inventories of raw materials, goods in process, and finished goods. The first two ought to be positively associated with unfilled orders, while the third ought to be negatively associated. A similar result is found by Duesenberry, Eckstein, and Fromm in their quarterly model.[5]

In the housing demand equation we use a long-term interest rate variable to show the effect of credit terms, and a marriage variable to show the effect of demographic pressures on facilities.

The capacity variable, which we have estimated with considerable expenditure of research effort, appears to be highly significant in the equation for plant and equipment.

The inventory equation, apart from the usual transactions and stock adjustment terms, contains an indication of price speculation. We did not separate farm from nonfarm inventories. This is another direction in which future disaggregation ought to go.

### THE ELEMENTS OF NONWAGE INCOME

There are three equations for nonwage income components. One covers corporate saving; one relates noncorporate (excluding wages) to corporate income; and one deals with depreciation. The fundamental national accounting identity equating national income to national product, with appropriate reconciling items, requires the separate explanation of corporate saving. In the explanation of corporate saving a variable measuring corporate income, as distinct from other nonwage income, must be used. This necessitates an equation. Finally, depreciation in the system must be explained; for the capital formation variables are measured gross, and they must be cumulated for measurement of capital stock.

Corporate savings are made to depend on corporate income (after taxes) and lagged dividend payments of the past eight quarters. The explanation of the particular lag scheme here is the same as in the consumption equations.

When we use depreciation variables in the model for the purposes of measuring capital stock, we reckon depreciation at replacement costs. In other instances, we reckon in accounting prices. Here, we

[5] James S. Duesenberry, Otto Eckstein, and Gary Fromm, "A Simulation of the United States Economy in Recession," *Econometrica*, October 1960, pp. 749–809.

are interested in relating depreciation to the accumulated stock of fixed capital and the rate of economic activity.

The relation between corporate and total nonwage income is purely empirical. It may be wiser to separate dividend and interest income from the nonwage noncorporate amounts, explaining this slow-moving component by a simple trend or autoregression, and to relate corporate income to income from noncorporate self-employment. The particular combination of variables used in the estimation of this empirical relation has been chosen so as to avoid some complications of multicollinearity.

## PRODUCTION FUNCTIONS

The ordinary version of the production function is estimated by equation 10. There we have a relationship between real private output [GNP less government wages and salaries, deflated by the GNP deflator (see equation 30)], the input of labor, the input of capital, and a technological time trend. Labor input is measured as private employment $(N_w - N_g)$ adjusted by an index of hours worked $(h)$ plus the number of self-employed $(N_e)$. Since $h$ is an index value on a unit base, we express adjusted employees and self-employed in conformable units: $h(N_w - N_g) + N_e$. Capital input is measured as the accumulated stock of capital, based on statistics of net investment in fixed capital, times the rate of utilization of capacity. Strictly speaking, we would want to have the rate of utilization of capital as the multiplying factor, but lacking a direct estimate of capital utilization we use an over-all measure of capacity utilization.

Capacity as expressed in equation 11 must be explained.[6] It is an important, but elusive, concept in its own right, and it plays an important role in this model. By capacity output, in the aggregate, we mean a *point* on the macro-economic production function corresponding to full utilization of inputs—labor and capital in this case. We might write

$$X_c = \alpha_0 + \alpha_1 N_L + \alpha_2 K + \alpha_3 t + v$$

where $X_c$ = capacity output
$N_L$ = labor force
$K$ = stock of capital [shorthand for $\sum_{i=0}^{\infty} (I_p + I_h - D_r)_{-i}$]
$v$ = random error.

[6] The discussion of capacity and the production function bears heavily on ideas put forward by Professor Morishima of Osaka University.

Perhaps we should write $0.97(N_L - N_g)$, or some other high fraction of the private labor force, for full-utilization labor input in order to allow for frictional unemployment and public employment. In this paper, $1.0N_L$ is used, since the applied work to be described was based on this value.

This is not an independent relationship. Its parameters should be the same as those of the ordinary production function, expressed in terms of actual output, employment, and utilized capital. Two separate linear functions might be used to approximate a single nonlinear production surface—one linear function approximating actual operations and the other approximating full-capacity operations. In the actual process of equation estimation we found difficulties in estimating the full-capacity version of the production function, because $N_L$, $K$, and $t$ are obviously strongly intercorrelated. They are all smooth trends. The problem in estimating the ordinary production function directly is that direct estimates of capital utilization cannot be obtained. We have direct estimates for labor in the form of employment and hours statistics. We approximated the solution of this problem by estimating

$$X = \alpha_0 + \alpha_1[h(N_w - N_g) + N_e] + \alpha_2(X/X_c)K + \alpha_3 t + u.$$

We were able to do this because we had independent estimates of $X/X_c$.

These independent estimates have come to be known as the Wharton School index of capacity utilization. The index is constructed in the following way: Each of thirty major components of the Federal Reserve index of industrial production is plotted on time charts. Seasonally adjusted monthly series, averaged to quarters, are plotted. Trend lines through peaks are established. These are linear segments connecting pairs of successive peaks. Peaks are established by inspection, with minor or temporary peaks eliminated. Some simple rules are established for recognizing peaks. From the last peak in a series, the trend lines are continued linearly with the same slope as the last completed segment. When actual production is rising and goes above the extrapolated trend, we increase the slope of the extrapolated line until a definite peak is established. When the trend lines are revised, we revise capacity calculations back to the last previous peak. The ratios of actual production to trends drawn through peaks give us figures on the percentage of capacity utilized by industry. The industry figures are averaged with weights into a national figure. The weights are those used to combine the Federal Reserve output series in its national index of production.

It would require an extensive argument and documented research study to give full justification to this method of estimating capacity utilization rates. In this paper, we merely want to describe our procedures and definitions of variables in the model. Many criticisms could obviously be raised about our method of measuring capacity. In our use of this measure we have implicitly assumed that industrial capacity, as we measure it from the FRB index components, is indicative (in an index sense) of capacity to produce private national product.

Using our estimates of the production function in (10), we find that the same coefficients inserted into (11) produce calculated values of $X_c$ that are also close to those independently derived by our method of trends through peaks.

The relation between (10) and (11) may be further clarified by multiplying the production function, on both sides, by $X_c/X$. We then transform

$$X = \alpha_0 + \alpha_1[h(N_w - N_g) + N_e] + \alpha_2(X/X_c)K + \alpha_3 t + u$$

into

$$X_c = \alpha_0(X_c/X) + \alpha_1(X_c/X)[h(N_w - N_g) + N_e] + \alpha_2 K + \alpha_3(X_c/X)t + (X_c/X)u.$$

The employment variable, in brackets, is marked up by the factor $X_c/X$. This should bring it close to $N_L$ or $0.97(N_L - N_g)$. The coefficients of $\alpha_0$, $\alpha_3$, and $u$ make this form differ slightly from the full-capacity version

$$X_c = \alpha_0 + \alpha_1 N_L + \alpha_2 K + \alpha_3 t + v$$

with which we started this discussion.

## WAGES, HOURS, AND LABOR FORCE

Associated with the technical conditions of production are the demand for labor and hours of work. Labor demand is converted into wage payments through valuation of employment by the wage rate.

The private wage bill, deflated by the general price index, is made a linear function of current and lagged output, with an upward time trend. This is a straightforward generalization of the constancy of labor's share. To avoid problems of collinearity between $X$ and $X_{-1}$, we make their coefficients equal before estimating the equation.

The wage rate (quarterly earnings at an annual rate) is made to depend on the state of the labor market, the general price level, and a trend. This is a familiar interpretation of the "law of supply and demand" used in the annual models that preceded the present work. Wage changes (over a four-quarter span) are made to depend on unemployment (averaged over the past four quarters), price changes (over a four-quarter span, averaged over the past four quarters), and a trend.

Unemployment is the residual difference between labor force and employment; therefore, we need an equation for labor force. We considered the standard hypothesis that makes labor supply depend upon the real wage rate, but found no satisfactory relationship. Labor force follows a smooth trend that we represent by a purely chronological variable. There is, however, an elastic cyclical element in the labor supply. This is largely accounted for by housewives, students, and semiretired people. They appear to swell the ranks of the labor force when jobs are plentiful and to withdraw when jobs are scarce. In our equation we represent this by a negative association between labor supply and unemployment.

### ORDERS AND BACKLOGS

The inventory equation discussed above contained a variable representing unfilled orders. In the endogenous explanation of unfilled orders we use the rate of capacity operation and the flow of new orders. This requires an additional equation to explain new orders, which we do in terms of recent sales and price changes. Our orders series are limited to the manufacturing sector, and eventually we would want to extend this part of the model on a disaggregated basis to nonmanufacturing sectors.

### FOREIGN TRADE

In a formal sense, both imports and exports are endogenous in this model. The explanation of exports is carried no further than to relate it directly to world production. Relative prices, as we have been able to measure such a magnitude, have not been found to be of significance in this equation. Overseas reserves, trade liberalization, and other variables may eventually prove to be important in a more detailed study of exports. In the applications we have made with the model, exports have been set at predetermined levels, and the export equation used here has been purely formal.

Import demand, however, has been more closely geared to the domestic economy. We divide imports into two classes, imports of unfinished and imports of finished goods. The former are determined directly from statistics of crude food and material imports. Imports recorded in the GNP accounts less these crude food and material imports are called "finished" imports. They are a residual, consisting of goods and services. We treat them like consumer goods. Equation 20, therefore, is simply an import analogue of the consumption equations.

### MONEY AND INTEREST

Demand for cash balances, which appears as a variable in the consumption equations, is made to depend on the long-run interest rate as a standard formulation of the doctrine of liquidity preference. One version of that theory is to assume that velocity, instead of being a constant, is a function of the interest rate. We have made the reciprocal of velocity our dependent variable. We have extended the dependence of cash holdings to price movements as well as the level of the interest rate.

In most versions of the modern theory of employment, the monetary authorities are assumed to control the stock of cash directly. Our assumption here is that they influence or control bank reserves and the discount rate. These influence the short-term rate, which then has a bearing on the long-term rate. These lines of reasoning are brought out in equations 22 and 23. The long-term rate is assumed to be a Koyck-type distributed lag function of past short-term rates. After transformation, this becomes a linear relation with the current short rate and the lagged long rate as explanatory variables.

### PRICES

In various individual equations of the system, specific price levels occur. For example in equations 1, 2, and 3, there are three separate consumer prices. We follow a general rule on all the specific price variables. Each specific price is related to the general price or wage level and possibly to some particular factor affecting that price.

Our system is interrelated; nevertheless, we can pick out certain main lines of causation. For a given output level, including a rate of capital formation as a component, the production function (10) shows labor requirements. Equation 13 is responsible for wage rate determination, and equation 12 can be transformed into a markup of price over unit labor costs. Thus, both the general price level and the wage rate are determined in the system. In equations 24 through 29, specific prices are related to one of these two general variables. The backlog variables used in (24) and (25) are subclasses of total unfilled orders. While the total is explained within the system, the components are not. The coefficient of $C_s/C$, the fraction of total consumption accounted for by services, is statistically significant but negative. This does not appear to be a reasonable result.

### IDENTITIES

The remaining equations in the systems are identities. Components of national product, valued in current prices, add to the total. This is expressed as private GNP ($pX$) plus government wages and salaries ($W_g$). In the next identity, the components of net national income ($W + P + S_c$) are equated with GNP ($pX + W_g$) less depreciation ($D_a$) and a reconciling item ($T_i$), which consists of indirect taxes less subsidies, the statistical discrepancy, and other small items. In this relation, depreciation is valued at accounting prices. We do not give an explicit relation between accounting price and replacement cost depreciation in the model, but we do use some simple proportions between these two for short-period applications.

Equation 32 expresses the wage bill as the product of employment, hours, and the wage rate. The final two equations are self-evident.

### APPLICATIONS—1961 FORECASTS

In the first trial calculations using this model, we extrapolated beyond the terminal sample date, IV-1958, for predictions of the first three quarters of 1961. These calculations were started in March 1961, and were completed in April. Results for the first quarter were not known but could be guessed in broad outline.

To keep the algebra of solution simple we fixed values over the forecast period for some variables in order to make the system linear. This required the assignment of prices. We were not generally satisfied with equations 24 through 29, in any case, and thought that prices could be predicted a priori for the three quarters of 1961 as well as they could be predicted by these equations. We also set the general price level at predetermined values. Interest rates and exports were similarly fixed at predetermined levels.

In order to solve the remaining equations linearly, we needed to fix values for $P/W$ in (1)–(3) and (20). This required the suppression of equation 13. Capacity output was estimated from (11), using last period's labor force and capital stock with the constant item adjusted so as to make the computed value agree with the first quarter's observation, I-1961; and the denominator of $X/X_c$ could thus be computed in advance of the other variables for each forecast solution. This, too, was done to preserve linearity. We added three equations, determined from recent observations, on tax-transfer variables.

$$T = -45.16 + 0.198Y$$
$$T_c = -4.59 + 0.599P_c$$
$$T_i = -39.86 + 0.213pX$$

Using the values of predetermined variables in Table 1, we solved the system for endogenous variables in I-1961. First, however, we made estimates of variables in this model from a starting point in the fourth quarter of 1960.

We reduced the system algebraically to two equations in $I_i$ and $X$. One was directly obtained from the inventory equation (6) with predetermined values substituted for the other variables. The other was obtained by substitution and algebraic reduction of the other variables in (30). This gave a residual equation in $I_i$ and $X$. We adjusted the constant terms of each equation so that they gave us the correct values, simultaneously, for $I_i$ and $X$ corresponding to our best estimates of these in the observation period, IV-1960. Keeping these adjustments in the constant terms of the two equations in $I_i$ and $X$, we solved the system sequentially in I-, II-, III-1961. We used computed values from one quarter as lagged inputs for successive quarters. *We did not adjust individual equations, apart from the two relations between $I_i$ and $X$, which kept a constant adjustment throughout the time sequence of solutions.* Some component series of national product may therefore be biased, but the quarter-to-quarter variation should not be seriously distorted. *Some of our computed components do not add to national totals.* Selected results are given in Table 2.[7] Actual values are in Table 3.

On the surface, this appears to have been a good forecast. The prediction of an upturn in the economy after the low point in the first quarter of 1961 was not surprising. Opinion was much divided,

[7] This table was circulated privately to more than 100 technicians in April 1961. It was a genuine forecast.

TABLE 1

PREDETERMINED VARIABLES USED IN 1961 FORECASTS

| Variables | Value Assumed I | II | III | Actual Value* I | II | III |
|---|---|---|---|---|---|---|
| $P/W$ | 0.36 | 0.36 | 0.36 | 0.36 | 0.35 | 0.35 |
| $\frac{1}{8}\Sigma(C_d)_{-i}$ | 41.0 | computed | | 41.3 | | |
| $C_d^e$ | 110.0 | 110.0 | 110.0 | 117.0 | 110.0 | 113.0 |
| $\frac{1}{8}\Sigma(C_n)_{-i}$ | 140.6 | computed | | 140.3 | | |
| $(L/p_n)_{-1}$ | 227.7 | 228.5 | 229.0 | 227.7 | 231.1 | 238.9 |
| $\frac{1}{8}\Sigma(C_s)_{-i}$ | 111.5 | computed | | 112.1 | | |
| $(L/p_s)_{-1}$ | 210.8 | 211.5 | 212.0 | 212.4 | 214.8 | 219.9 |
| $N$ | 182.5 | 183.3 | 184.1 | 182.5 | 183.2 | (183.95)p |
| $I_p^e$ | 28.5 | 28.0 | 28.0 | 28.9 | 27.9 | (28.6)p |
| $i_L$ | 4.64 | 4.60 | 4.50 | 4.59 | 4.59 | 4.72 |
| $F$ | 296.0 | 450.0 | 461.0 | 291.0 | 430.0 | (430.0)p |
| $(I_h^s)_{-1}$ | 1,003.0 | 1,050.0 | 1,100.0 | 1,003.0 | 1,016.0 | 1,100.0 |
| $X_{-1}$ | 394.0 | computed | | 395.0 | | |
| $t$ | 61.0 | 62.0 | 63.0 | 61.0 | 62.0 | 63.0 |
| $q_p$ | 1.22 | 1.22 | 1.22 | 1.23 | 1.23 | (1.23)p |
| $\frac{1}{8}\Sigma\frac{1}{q_p}(P_c - T_c - S_c)$ | 11.2 | computed | | 11.3 | | |
| $P_{-1}$ | 102.9 | computed | | 101.7 | | |
| $W_g$ | 51.0 | 52.0 | 53.0 | 50.4 | 51.3 | 52.1 |
| $p$† | 1.153 | 1.153 | 1.153 | 1.156 | 1.158 | 1.164 |
| $\Sigma(I_i)_{-j}$ | 239.3 | computed | | 246.4 | | |
| $(X - I_i)_{-1}$ | 396.4 | computed | | 396.2 | | |
| $(p_i/p)_{-1}$ | 0.841 | 0.840 | 0.840 | 0.841 | 0.835 | 0.830 |
| $\frac{1}{8}\Sigma(F_{if})_{-i}$ | 19.72 | computed | | 19.0 | | |
| $G$ | 103.0 | 104.0 | 105.0 | 105.0 | 107.3 | 108.5 |
| $p_e$ | 1.085 | 1.085 | 1.085 | 1.105 | 1.204 | (111.0)p |
| $F_e$ | 25.0 | 25.0 | 25.0 | 25.0 | 21.9 | (24.0)p |
| $D_a$ | 44.5 | 45.0 | 45.5 | 44.2 | 45.0 | 45.5 |
| $p_d$ | 1.045 | 1.045 | 1.045 | 1.048 | 1.055 | 1.055 |
| $p_n$ | 1.085 | 1.085 | 1.085 | 1.085 | 1.081 | 1.081 |
| $p_s$ | 1.175 | 1.180 | 1.185 | 1.167 | 1.174 | 1.174 |
| $q_h$ | 1.170 | 1.170 | 1.170 | 1.170 | 1.170 | (1.175)p |
| $p_i$ | 0.97 | 0.97 | 0.97 | 0.97 | 0.96 | (0.96)p |
| $N_e$ | 9,200.0 | 9,200.0 | 9,200.0 | 9,410.0 | 9,100.0 | 8,820.0 |
| $N_g$ | 8,600.0 | 8,700.0 | 8,800.0 | 8,670.0 | 8,700.0 | 8,450.0 |
| $\Sigma(I_p)_{-i}$ | 2,224.86 | computed | | 2,224.86 | | |
| $\Sigma(I_h)_{-i}$ | 1,006.27 | computed | | 1,006.27 | | |
| $U_{-1}$ | 44.6 | computed | | 44.3 | | |

p = preliminary.
* Available at later date—after the forecast.
† The value for IV-1960 was estimated to be 1.152.

however, on the magnitude of the recovery. There is no doubt that many persons were surprised (in government and business) by the magnitude of our increments from first to second quarter and from second to third quarter. This is not to say that we were alone in pre-

TABLE 2

SELECTED FORECAST VALUES, 1960–61
(billions of 1954 dollars unless otherwise stated)

| Variable | Starting Value 1960 (IV) | Estimate 1961 (I) | Forecast 1961 (II) | Forecast 1961 (III) |
|---|---|---|---|---|
| $C_d$—durable consumption | 43.5 | 41.4 | 43.4 | 47.9 |
| $C_n$—nondurable consumption | 144.7 | 143.9 | 145.4 | 148.2 |
| $C_s$—services consumption | 115.0 | 115.5 | 116.9 | 118.7 |
| $I_p$—plant and equipment | 36.4 | 35.4 | 35.4 | 36.3 |
| $I_h$—residential construction | 18.2 | 17.7 | 18.2 | 19.9 |
| $I_i$—inventory investment | −2.4 | −4.4 | −2.7 | 0.7 |
| $X$—private *GNP* | 394.0 | 388.4 | 396.2 | 412.1 |
| *GNP* (current prices) | 503.8 | 498.8 | 508.8 | 528.2 |
| $X/X_c$ (capacity rate) | 0.89 | 0.87 | 0.88 | 0.90 |

TABLE 3

ACTUAL VALUES OF SELECTED FORECAST VARIABLES, 1960–61
(billions of 1954 dollars)

| Variable | 1960 (IV) | 1961 (I) | 1961 (II) | 1961 (III) |
|---|---|---|---|---|
| $C_d$—durable consumption | 41.6 | 37.6 | 39.8 | 40.3 |
| $C_n$—nondurable consumption | 141.3 | 141.6 | 142.6 | 145.2 |
| $C_s$—services consumption | 116.6 | 117.8 | 119.2 | 121.4 |
| $I_p$—plant and equipment | 38.5 | 36.3 | 36.9 | 36.6 |
| $I_h$—residential construction | 17.5 | 16.5 | 17.6 | 19.9 |
| $I_i$—inventory investment | −1.1 | −3.2 | 2.9 | 3.9 |
| $X$—private *GNP* | 395.1 | 389.6 | 401.4 | 407.0 |
| *GNP* (current prices) | 504.5 | 500.8 | 516.1 | 525.8 |
| $X/X_c$ (capacity rate) | 0.88 | 0.86 | 0.90 | 0.92 |

dicting a substantial improvement in real output, but the model came out in the correct neighborhood when there were great doubts in the minds of many persons that the recovery would be this strong. It is also important to note that the prediction was for a surprisingly large increase in output associated with quite modest increments in our estimate of capacity utilization.

While our estimate of GNP for the third quarter is close to the outcome, the model underestimated the growth from the first to the second quarter and overestimated it from the second to the third. We had too little inventory investment and too much durable consumption. Our other errors were less remarkable.

These are only surface observations. A more detailed appraisal requires two considerations: (1) data revision and (2) accuracy of assumptions. We made our forecast for the second and third quarters on the basis of preliminary estimates of the fourth quarter of 1960 and informed guesses about the first quarter of 1961, which had just passed. Data were not fully collected for the first quarter of 1961, and many of the fourth quarter estimates for 1960 were highly tentative. Our base period (IV-1960) estimates of output were too high by approximately $1 billion, and our inventory estimates were too low by the same magnitude. These two variables were forced by our adjustment process to give the "correct" values as we estimated them at the time for the base period. *We did not adjust the other component equations of the model;* therefore, in the "back" solution, which gives the distribution of values of individual variables, all the identities do not necessarily hold; and we may start off from biased values in the base period. This bias is not serious, though, since we can see its magnitude in IV-1960. $C_d$ is, for example, overestimated by about $2.0 billion in the base period. This bias value in $C_d$ is not adequate to account for the large value of durable consumption in the third quarter. We definitely overestimated the rise in $C_d$. Apart from the underestimate of inventory change, no other GNP component is seriously enough distorted in the forecast to merit special consideration. Our index of capacity utilization was revised in the summer of 1961.[8] Although it is not apparent in the comparison of the values for IV-1960, the new index tends to run about one or two points above the old one that was used in the forecast.

One of the drawbacks of the model is that it contains so many predetermined variables that a large amount of nonmodel forecasting is necessary before the model can be used in forecasting. A month's work at data processing and extrapolation of exogenous variables is required in preparation for a forecast. The large number of predetermined variables in Table 1 indicates the magnitude of initial input. There are many variables, covering many aspects of the economy here. It is easy to be right on some values, too high on some, and too low on others. We underestimated the growth in money supply. Government spending was set too low in the initial period and grew slightly less than was actually the case. The interest rate should have risen slightly instead of declining by a small amount.

[8] The index was computed from the FRB indexes on a 1957 base in the revision. The older indexes on a base of 1947–49 had been previously used.

Housing starts were actually fixed in advance. Price increases were too low. Population growth was closely estimated, and so on.

Major sources of error in the forecast are not to be sought in the assumptions made for predetermined variables or in data revisions. The model is only a statistical estimate of reality and is subject to error. Imperfect knowledge of the true relationships in the economy and some large disturbances probably account for the great part of the forecast error. Strikes and hurricane damage in the third quarter probably had substantial effects on changes in variables between the second and third quarters.

### APPLICATIONS—THE RECESSIONS OF 1953–54 AND 1957–58

Models can be tested by *ex post* as well as by *ex ante* forecasts. In the previous section, we described *ex ante* forecasting. In this section we shall summarize the results of a simulation study prepared for the Joint Economic Committee of the Congress of the United States.[9] This is an example of *ex post* forecasting and has the advantage of controlling error in the assumptions for predetermined variables. Since it is an application after the event, good estimates of the predetermined variables are available.

*Ex post* extrapolations of a model outside the sample data to which the model is fitted provide better tests than do *ex post* calculations using internal sample data. The present example uses internal data and is, therefore, not as stringent a test as we hope, eventually, to apply. At the moment this example is cited as an interesting application.

The problem posed in this application was how to determine, from the model, whether and how much specific dampening of inventory fluctuation in past recessions would have contributed to total output stabilization. This is a hypothetical problem, exemplifying how models can be used in policy formulation, and is not a test of the model. However, the first step in attacking the inventory stabilization issue was to let the model run through the course of each of the two recessions considered to see whether it duplicated actual output fluctuations. Predetermined variables were inserted into the equations for the first quarter of 1953 (and the first quarter of 1957). The

[9] Lawrence R. Klein and Joel Popkin, "An Econometric Analysis of the Post-War Relationship Between Inventory Fluctuations and Changes in Aggregate Economic Activity," *Inventory Fluctuations and Economic Stabilization*, 87th Cong., 1st sess., December, 1961, III, 69–89.

system was then successively solved as a dynamic model through the fourth quarter of 1954 (and the fourth quarter of 1958). Exogenous variables were assigned their actual values for each quarter's solution, but lagged endogenous variables were generated within the model after starting from given initial conditions. As in the case of the 1961 forecasts, the two equations were adjusted in $I_i$ and $X$, so that correct values were obtained for the starting quarter of each simulation. New tax equations were determined for the simulation periods, and the changes in revenue laws during 1954 required the use of different tax equations for the quarters of 1953 and of 1954. The results are given in Table 4.

TABLE 4

ACTUAL AND SIMULATED VALUES OF $X$, 1953–54 AND 1957–58
(billions of 1954 dollars)

| | *1953–54* | | *1957–58* | |
|---|---|---|---|---|
| *Quarter* | Actual | Computed | Actual | Computed |
| I | 334.72 | 334.72 | 371.90 | 371.90 |
| II | 338.87 | 337.64 | 373.03 | 369.82 |
| III | 335.69 | 332.65 | 373.24 | 367.90 |
| IV | 329.64 | 331.29 | 366.76 | 368.94 |
| I | 326.43 | 328.18 | 353.72 | 365.98 |
| II | 325.35 | 341.27 | 355.02 | 378.10 |
| III | 327.40 | 343.62 | 360.25 | 388.62 |
| IV | 335.33 | 349.23 | 370.89 | 397.91 |

Computed output turns up one quarter earlier than output in 1954, and the recovery is stronger. In 1958, the timing is coincident, but the downswing started earlier and was interrupted by a temporary advance in the fourth quarter of 1957. The sharpness of the 1957–58 recession is not duplicated in the computed data. The fall is not as great as the actual output decline, and the revival is stronger. The revival is also stronger in the computed than in the actual output for 1954.

The time paths of other variables can be seen in the tables and charts of the JEC study paper referred to earlier. The policy application of the model made in the study paper can be summarized by noting that if inventory fluctuations are autonomously reduced in amplitude, fluctuations in output, employment, and other variables are also reduced. The model results show that if inventory fluctuation (deviations above and below zero inventory investment) can be reduced by a factor of one-quarter, output fluctuations are moderately reduced. At the cycle troughs, we estimate multiplier values of four to five, i.e., the trough of the production cycle is raised by four or five dollars (1954 prices) for every dollar reduction in the absolute value of inventory investment at the trough. If inventory stabilization is much greater, say, a dampening of fluctuations by a factor of three-quarters, the ordinary business cycle in computed output vanishes.

## SELF-CRITICISM

This is only another one in a series of American models. There will be more to come. The ancestors of this model have been used to make a number of helpful forecasts, provide a setting for computational experiments, and provide tailor-made subjects for critical doctoral dissertations. They have all had a measure of intellectual attack. In anticipation of some points of attack on the present system, the system might be appraised here and now. This will set the stage for work on the models to come.

By the time data are collected, parameters are estimated, and models are tested for performance, ideas about the detailed structure of the economy can change drastically. At the end of this time-consuming process (about three years in the present case) we usually decide that we would have built the system differently if we were starting the project freshly. The price and interest rate equations are the poorest of the lot in the model, and these need revision.[10] It would be possible to use the present price and interest rate equations in a more essential way in forecasting from the present model, but a good and simple computing routine for coping with the nonlinearities caused by these is not fully prepared.

As in past models, we have looked for a balanced estimate of equations as a whole and systems as a whole. Goodness of fit, randomness of residuals, signs of coefficients, approximate magnitude of coefficients, and standard errors have all been used together in deciding whether to accept or reject estimated equations. In these decisions many candidates are accepted for which individual coefficients do not meet some standard test—say a $t$-test for significance at the 5 per cent level. Some of our standard errors are large. If the

[10] In a joint project supported by the work of many scholars, and sponsored by the Social Science Research Council, a new model is being built which appears to be much stronger on the side of price estimation. The price formation equations are quite different.

model were brought up to date and re-estimated with twelve more observations, some of these insignificant results might be changed.

In specific equations there are definite possibilities for improvements. Population might be directly introduced in the consumption equations by expressing all variables in per capita terms. The empirical relation between $P_c$ and $P$ can be improved by extracting dividend and interest income from $P$, estimating that component separately by some simple autoregressive scheme, and relating only entrepreneurial elements of $P$ to $P_c$. This relation can be refined even more if farm entrepreneurial income is taken out of $P$ as well.

Inventories should be subdivided by farm and nonfarm category. In addition the nonfarm category should be disaggregated by stage of process and type of holder (seller versus manufacturer). These disaggregations all call for a substantially larger model. Many of these things are already being done in the Social Science Research Council model referred to in note 10. In the equation for residential construction, housing starts are an important variable. Starts are not really independent in their relation and certainly not for as many time periods ahead as we have tried to use them in applications. In fact, construction expenditure series are prepared by the phasing-in of starts data, using an average construction lag. We should have an equation explaining starts, another showing how construction data are built from starts data, and another on unit structure value.

Similarly, investment intentions and consumer buying plans are not really independent data in our system, although we use them in that way. We need separate equations explaining these expectations, in addition to equations showing how expectations are transformed into realizations.

The government sector is purely exogenous except for the simple tax-transfer equations used in applications of the model. There is much useful work that can be done in distinguishing between induced government expenditures like those for highways and education and purely autonomous categories like defense. Some equations can be developed for the induced parts, and some realization functions associating expenditures with budget appropriations can be constructed. Many more things can be done on the side of government receipts. Tax equations using income distribution and internal revenue reports can be greatly improved. Major transfer items could be usefully separated from taxes and estimated in new equations.

All these improvements require substantial research work, but they are all feasible and can easily be added to the basic framework presented here.

## COMMENT

EDWARD F. DENISON, Brookings Institution

This meeting on models of income determination stemmed in part from a compelling need to strengthen communication between members of this conference who are concerned with national accounting and related economic statistics, on the one hand, and econometricians, on the other. The Program Committee, in accordance with the objective, gave the econometricians who are presenting papers a strong injunction to translate their mathematics as best they could into the verbal language of the econometrically illiterate. The main sanction available to the Program Committee was assignment of a member of the latter group as a discussant. For Lawrence Klein's paper, the Program Committee took no chances at all, but assigned from its own ranks the only member clearly possessing the desired qualification of innocence of mathematical or econometric knowledge.

Let me first, therefore, fill my assigned role as appointed spokesman for the ignorant by stating that Klein has done an exceptionally good job of translation and explanation of his model. Anyone who merely looked at his list of thirty-four equations would not agree. But there is very little difficulty in understanding the individual equations if they are examined in conjunction with the text description. It becomes immediately apparent that the terms that appear most formidable are simple numbers we are all accustomed to using, such as the net stock of plant and equipment, or the average level of consumption during the past two years. I hope these revelations do not cost Klein his license to practice.

The groupings of equations Klein provides are also very helpful in understanding the approach and general structure of the equations. However, while I think I can follow the individual equations, I am sure I do not understand their interrelationships and the structure as a whole in more than a rudimentary sense. This is no criticism of Klein; I feel I am well ahead of where I was before reading the paper.

From the standpoint of those of us who are accustomed to worry about the relative merits of particular economic series, it would be useful to specify in the definitions of the variables the exact series used—for example, for employment and hours worked. The particular notation used is not always as easy to remember as it could be. But these are small defects, easily remedied. Viewed as a description of his model, Klein's paper is an unqualified success.

Now for the model itself. Without questioning his choice of economic variables in his individual equations, let me raise first a few questions about the particular series used to represent the economic phenomena he wishes to represent. These are quarterly series, seasonally adjusted in most cases.

I believe the labor force and employment series used are from the *Monthly Report on the Labor Force*. Quarterly fluctuations around the trend in these series, and in the private employment subtotal, are dominated by irregular fluctuations in the agricultural component. These fluctuations clearly are irrelevant to the quarterly behavior of either total or agricultural GNP. Even nonagricultural employment in the MRLF series is rather erratic, and I would expect the series derived from establishment reports to be much more closely related to short-term GNP behavior. It is also far more relevant to the derivation of the average wage series which enters Klein's system of equations because of its greater statistical consistency between employment and earnings. Now it is true that erratic fluctuations may roughly cancel in deriving the equation, and may cause little trouble in forecasting when the employment and labor force figures are derived rather than observed. But this is not the case when the model is used, as Klein has used it, to study patterns in past periods. I think most persons who follow employment and payroll data closely will be troubled by the use of MRLF rather than establishment series. It may be heretical, but I suspect that little of relevance to income determination would be lost, and something would be gained, if the incompleteness of the establishment data were surmounted by so crude a device as defining the labor force as nonagricultural employees from the establishment series plus unemployment from the MRLF.

A current minor controversy among economic statisticians concerns seasonal adjustment of labor force data. The Labor Department seasonally adjusts the labor force, employment, and unemployment independently; thus, employment plus unemployment does not equal the labor force. A widely used alternative obtains seasonally adjusted unemployment as a residual from the seasonally adjusted labor force and employment data. Inadvertently, rather than by deliberate choice, Klein uses a third alternative that has no apparent advantage. He obtains seasonally adjusted employment as the residual. Klein needs to obtain employment or unemployment as a residual to avoid introducing an additional variable, but unemployment is the better choice and especially so since it follows the logic of his equations.

Indexes of net capital stock and gross stock differ when the average age of capital changes. Klein uses net stock to measure changes in fixed capital input. My own, and I think the more common, view is that gross stock provides the more logical index of capital input; and I wonder whether Klein disagrees.

Use of housing starts rather than building permits in the estimation of residential construction seems questionable, if the model is to be used more than one quarter in advance. Permits are less erratic and have a longer lead time than the new Census series for starts.

Equation 4 for the estimation of plant and equipment expenditures is interesting. The SEC-OBE projection based on the quarterly anticipations survey is used, but as only one term in the equation rather than directly in percentage-change form.

Is the equation primarily an attempt to adjust plant and equipment as defined in the SEC-OBE survey to include farm and other components excluded from the survey but included in private GNP expenditures for producers' durables and nonresidential construction? This seems unlikely. The other variable in the equation, the level of the capacity utilization index, hardly seems appropriate to represent the missing components. Moreover, it appears to be about as important as the anticipations figure in determining Klein's estimate. This greatly exceeds the relative weight of the missing components.

Or, as seems more likely from Klein's preliminary comments on the introduction of anticipations data and on the significance, in equation 4, of the capacity variable, is this equation an attempt to improve on the government plant and equipment anticipation estimates? Does it do so? The government agencies themselves correct for systematic past biases in reported anticipations. However, these adjustments do not take account of the unemployment position of the economy. Klein's does, but with no distinction between the upward and downward phase of the cycle. If the equation can be shown conclusively to furnish better forecasts than the government adjustments, as presently applied, this is an interesting and useful discovery. It warrants exhaustive analysis. I would like to know exactly what anticipations data were used in deriving the equation.

Also, I wonder at what level of unemployment the implied adjustment of the anticipations series switches from plus to minus.

The preceding comments refer mainly to the choice of statistical series to represent the economic variables, like employment, used in the equations. A thorough critique would consider the choice of economic variables themselves, but I cannot attempt this. However, I am sufficiently surprised at the extensive use of the division of personal income between labor and other income to ask why. Is it perhaps serving as a proxy for something else?

Estimation of corporate profits from nonlabor personal income, which is the sum of farm and nonfarm proprietors' income and of private and government interest, dividends, and rental income of persons, is surprising. In the text, Klein suggests a shift to proprietors' income, presumably confined to the nonfarm sector. Since he now has no equation for proprietors' income, this would seem to lead nowhere. There is, to be sure, a relationship between profits and proprietors' income. The Office of Business Economics, insofar as it can, bases the movement of noncorporate business income on corporate profits. But what Klein would seem to need, if he changes his model in the way indicated, is a method of forecasting both corporate and noncorporate profits, which he now lacks.

In applying his model to forecast 1961, Klein did not actually use all of his equations. He used assumed or independently estimated values for prices, interest rates, and exports. Although Klein indicates he was not too unhappy about this, because he lacks confidence in the price equations anyway, the main reason was inability to solve the system of equations quickly when they lost linearity. This would seem to raise questions concerning the practical value of completeness in a short-term forecasting model. Incidentally, my instincts suggest that the definitional relationships among changes in productivity, unit costs, and prices could cause real trouble if prices are predetermined. Other things equal, the larger the productivity gain, the smaller the price increase.

Although Klein's paper is, in general, commendably lucid, I would appreciate more extended discussion of the procedure to tie in the model calculations with the most recent period for which actual figures are available. In the 1961 projection, Klein adjusted the constants in the key equations to force the most recent observation to fall on the regression line. Would it be quicker and about equally good to solve the equations for the most recent "actual" quarter and future quarters, and base the forecast on changes? Might not this also minimize the problem of getting the GNP components to add up to the total? I do not know the answers to these questions.

How good is the model? Klein compares forecasts from the model with actual data for three time periods. Two are past periods, each extending over eight quarters, encompassing the 1953–54 and 1957–58 recessions. The other is a forecast for the quarters of 1962. The past comparisons show rather poor agreement between actual and calculated GNP values, even though they fall within the period upon which the model is based, and use actual data for exogenous variables, and actual tax rates, which could not have been known in advance. It is true, however, that in both cases a recession and recovery is predicted, two of the four turning points are correctly calculated, one is off by only one quarter, and one by two quarters. I am not clear whether this degree of success depends upon the use of anticipatory data not available at the beginning of the periods.

The 1961 forecast shows rather good agreement, even though the exogenous variables were estimated and the values of many of the endogenous variables were simply assumed rather than obtained from the model.

My greatest doubt about Klein's whole system is an obvious one, and I am a little surprised he did not discuss it more. It is generally recognized that structural changes that accompany a given change in GNP or employment while unemployment is excessive are quite different from the structural changes that occur when the economy is growing under high-employment conditions. This raises the question whether one system of equations can describe both situations accurately.

If it can, I should suppose it to be only by systematic inclusion in the model of some such variable as the unemployment rate in order to distinguish the two situations. Even then I doubt it could be done with functions like Klein's, in which the difference between 3 per cent and 4 per cent unemployment has the same effect as that between 6 and 7, because the change in structural behavior patterns occurs rather abruptly instead of continuously. I suppose it would be possible to work into each equation a variable that would take effect only under depressed conditions, or only under prosperous conditions. Perhaps this might be the arithmetic excess of the unemployment rate above, or below, the postwar low or average, or some carefully selected number. Brown's Canadian model has such a term in the establishment of wage rates. But if done systematically, this is really similar to having two models.

Actually, only a few of Klein's equations explicitly or, I think, even implicitly include any indication of how far from capacity the economy is operating. The current or recent percentage unemployed or the capacity utilization index, the two relevant variables in his system, enter explicitly into only six equations. One of these, that for plant and equipment, I have already mentioned. Four are the equations determining the year-to-year increase in the average wage, average hours of work, the size of the labor force, and unfilled orders. At least equally obvious candidates, such as corporate profits, have no such determinant. Finally, the capacity utilization index enters the private GNP production function, equation 10, but not in the way I have in mind. It enters only as a multiplier for the net capital stock to arrive at capital input.

I presume Klein did not use unemployment or capacity utilization more often because he found they did not improve the estimates. But might this not indicate only that two models or the equivalent are needed, rather than only one?

In short, I wonder whether recession-recovery models such as we have begun to obtain, and companion short-term growth models for periods when the economy is operating at high employment, are not more promising than a single model, both for forecasting and for describing structural relationships.

This reaction to the Klein *model* is that of an outsider and should not be weighed heavily. I am really doing no more than asking questions. My judgment of the Klein paper as an educational document to describe the model for the ignorant, on the other hand, is altogether favorable, and my qualifications for this judgment are solid.

FRANCO MODIGLIANI, Massachusetts Institute of Technology

## *Over-All Considerations*

Before I attempt a critical evaluation of Klein's quarterly model, one in a long string of past and future similar undertakings, I want to make it quite clear that I am a great admirer of his indefatigable labor in this area and that I share with him the conviction that this type of endeavor is very much worthwhile and will contribute with increasing effectiveness to economic forecasting and policy-making, as well as to a basic understanding of the working of our economy. I am, therefore, quite pleased to find that the United States economy decided to encourage his activities by producing in the third quarter of 1961 a GNP within a couple of billion dollars of the forecast made in April 1961, and largely on the basis of information relating only to the last quarter of 1960. The accuracy of the model is even more impressive if stated in terms of change over the last quarter of 1960. On this basis, Klein's forecast is only about 10 per cent over the actual change of some $21 billion. To be sure, this accuracy is somewhat misleading, for it results, in part, from considerably larger offsetting errors. Thus, the change in the price level was understated by 100 per cent, while the change in real private gross national product was overstated by 50 per cent. But even so, I at least feel that the model performed a rather creditable job on this occasion.

I am also quite aware of two difficulties that beset my role as commentator of this paper. The first is that any criticism I might advance is open to the obvious objection: if I claim that Klein's hypotheses should be replaced by better ones, why don't I build my own model? Unfortunately, only in a few instances can I claim that my suggestions have been explicitly tested by myself or others. The second difficulty is that Klein, in presenting the outcome of his labors, could not take the time to tell us how many other things he tried before settling on his final choice. Hence, at least some of my suggestions may have been tried out and rejected as empirically inadequate. Given my assigned role I have no choice but to ignore these difficulties. In setting forth my criticism of Klein's model I do, however, wish to make it quite clear that all of my criticism, right or wrong, is offered in the constructive spirit of advancing a common cause.

Let me, finally, indicate that in trying to assess the strength and shortcomings of the model and in suggesting some promising directions for further improvement I will assume that Klein's construction has also purposes other than that of arriving at accurate short-run forecasts of the course of economic activity. I assume his goal is also to provide an increased empirically supported understanding of the *modus operandi* of our present-day economic system and to develop a tool for testing the effects of alternative economic policies.

From this point of view goodness of fit to historical data and even initial forecasting success, though not unimportant, are clearly not the only relevant criteria for assessing the model and its parts. Other criteria are equally important and, in particular, that the hypotheses on which the model rests make "economic sense" or can be derived from other hypotheses having this property.

In the light of these considerations Klein's latest model strikes me, on the whole, as a very significant improvement over earlier ones. Some of these improvements consist in the incorporation of features that I would have liked to have seen in earlier models; in other words, it comes closer to my own notations, which, of course, I must regard as improvements, although others may not share this view. I am referring here, for instance, to the systematic incorporation of anticipatory variables, to the treatment of the purchase of consumers durables more nearly as a type of investment, and to the increasing attention to monetary phenomena and their interaction with real phenomena. Other improvements reflect, instead, advances in the general state of knowledge, which Klein has been quick to incorporate in his model, e.g., with respect to the inventory equation and the interrelationship of wages and prices. In some cases, as indicated below, I feel that Klein may not have gone far enough; but this is a debatable point; and, in any event, the movement is certainly in the right direction.

My only general complaint is a minor and readily remediable one; I wish he had provided us with a measure of goodness of fit for each of his equations. While such measures must of course be taken with a grain of salt, they are useful in providing an idea of how close we are coming to an explanation of the behavior of the dependent variable. This information is especially valuable where the reader may have serious qualms about the adequacy of the hypothesis.

Let me now abandon generalities and take a closer look at certain major groups of equations.

## *Comments on Some Specific Components of the Model*

### THE CONSUMER SECTOR

One feature of the present model is the disaggregation of consumption expenditure into three sectors: durables, nondurables, and services. While I see little point in disaggregation for its own sake, I believe that in the present instance the separation of the three sectors is worthwhile, both because it should help provide a more reliable explanation of total consumption expenditures, and because the behavior of the three components of consumption may be expected to affect differently the rest of the economy.

This separation is especially important in the case of durable goods purchases, since, in my view, this type of expenditure is really in the nature of an investment. It should, therefore, be controlled by somewhat different forces than those controlling other types of consumer outlays and should, in fact, be explainable along the general lines of the "acceleration principle." That is, one might visualize an "optimum" stock of durable goods, the size of which should be controlled by the demand for durable goods *services* and hence, finally, by the level of income and relative prices (although the relevant measure of income might well be something akin to the "permanent income" of the Friedman model or the "total resources" of the Modigliani-Brumberg model, and these might not be too well approximated by measured income in the current quarter). Let us denote this optimum stock by $D = D[Y_t, (p_d)_t]$. The current purchases of durable goods might then be expected to be proportional to the gap between optimum stock and initial stock, $D_{t-1}$, adjusted for depreciation. In other words, the basic hypothesis I would favor, and with which I have done some encouraging experiments, at least for yearly data, would be of the form

$$C_d = g\{D[Y_t, (P_d)_t] - hD_{t-1}\}$$

where $g$ is the speed of adjustment, presumably smaller than 1.0, and $1 - h$ is the rate of depreciation. The coefficient $g$ might well be a function of certain other variables, while $h$ could probably be approximated by a constant, at least in the short run.

The model actually used by Klein can be regarded as a linear approximation to this hypothesis, except that instead of using initial stock, he uses purchases of the last two years. Since the typical life of durable goods is appreciably longer than two years, this approximation strikes me as inadequate. It might be noted that in Klein's equation 1, the coefficient of past purchases is negative, as expected. It is appreciably larger than 1.0 because the stock is several times annual purchases (in recent years, around four times, according to Goldsmith's estimates).[1] Since estimates of the stock of durables and the depreciation thereof are available at least for part of the period (and can be readily approximated for later years), it would seem desirable to try out the formula suggested here. Eventually, one may

[1] Cf. Raymond W. Goldsmith, *A Study of Saving in the United States*, Princeton, N.J., 1955, Vol. III, and *The National Wealth of the United States in the Postwar Period*, Princeton for NBER, 1962, Statistical Appendix.

also want to break out new automobile purchases from the other durables, in view of the importance of this commodity and the availability of promising hypotheses relating specifically to it.

Concerning the remaining consumption equations, I continue to have the most serious doubts about the relevance of cash balances as a determinant of consumption expenditure, except possibly for durable goods, where, however, this variable does not in fact appear (cf. equation 1). I have of course stressed in past and forthcoming writings the importance of consumers' initial net worth as a determinant of consumption. However, in the first place, I believe this variable to be more relevant to the long-run behavior of consumption than to an explanation of quarterly movements, where the effect of this variable can probably be conveniently proxied by other lagged variables, such as Klein's previous consumption. Second, and more important, I do not believe that "cash balances" are an adequate proxy for wealth, especially Klein's total balances, which include a sizable portion of business cash holdings. I am, therefore, not surprised to find that the contribution of this variable to the explanation of consumption is quantitatively negligible in all cases. It is also statistically insignificant, except possibly in the demand-for-services equation, where, however, its relevance is, a priori, most doubtful. I hope, therefore, that Klein will see fit to drop this variable at the next opportunity.

As for the remaining variable common to all consumption equations, $P/W$, which measures the distribution of income as between labor and property income, I am somewhat bothered by its appearance as a ratio, which creates dimensionality problems. More seriously, I am puzzled because its sign is positive, and, except possibly for durables, this would seem to be contrary to expectation and to some previous evidence. However, this variable, too, contributes very little, and is not significant except in the case of services.

Finally I have some reservations about the way in which the index of buying plans is used in the demand-for-durables equation, but I propose to take up this problem below in connection with the investment equations.

## INVESTMENT IN FIXED CAPITAL

The two equations relating to investment in fixed capital, namely, (4) and (5), are notable for the inclusion of anticipatory data, a procedure which, of course, I heartily endorse. However I have some qualms about the specific way in which these data are incorporated in the equations. In my view, Klein's formula does not fully exploit the information such data convey, and also gives rise to hypotheses which, whatever their empirical accuracy, are rather difficult to rationalize.

Take first the case of investment in plant and equipment, which is expressed as a function of anticipations and the rate of utilization of capacity. As I have argued extensively elsewhere,[2] if investment plans are meaningful—a proposition which, of course, cannot be assumed a priori, but is by now supported by a number of empirical investigations—then they embody all the information pertaining to the appropriate level of investment in the current period, as seen at the time the plan is made. This information includes, in particular, all relevant initial conditions and anticipations of future variables, such as sales, profits, availability of funds, etc. If so, actual investment should be expected to deviate from plans only in so far as the actual course of the anticipated variables differs from the anticipations. In other words, the discrepancy between $I_p$ and $I_p^e$ should depend on the error of anticipation. Symbolically, $I = F(I_p^e, A - E)$, where $A$ denotes actual variables and $E$ their anticipation. I have labeled the function $F$ the "realization function." Thus, the realization function should include, in addition to plans, variables measuring the error of anticipations $(A - E)$ rather than the actual course $(A)$, and should not include initial conditions which are already absorbed in plans, unless the initial conditions themselves can be expected to control the extent to which plans are revised in the light of later information.

Now Klein's equation does not include initial conditions (except possibly for the variable $X_c$), which is in line with my suggestion. However, it includes actual output, $X$, instead of the error of expectations, $X - X^e$. It is true that he could not very well have used $X^e$, since this information has not been available on a quarterly basis, at least until quite recently. Nevertheless, I submit that $X$ is a poor approximation to $X - X^e$. A more adequate approximation in terms of readily available observables might be something like the change in sales, on the assumption that, on the average, sales expectations are close to current sales; or one might try to infer quarterly expec-

[2] Franco Modigliani and Kalman J. Cohen, *The Role of Anticipations and Plans in Economic Behavior and Their Use in Economic Analysis and Forecasting*, Urbana, Ill., 1961.

tations from yearly data. Eisner, in a recent paper,[3] has actually tried both approaches, apparently with good results (although I cannot say how they compare with the results obtained from Klein's hypothesis).

An essentially initial condition such as existing capacity may, of course, also play a role, since it is conceivable that the revision of plans might be more responsive to errors of anticipation at high rates of utilization. However, my purpose here is not so much to suggest a specific alternative to Klein's, but rather to indicate the assumptions on which a hypothesis embodying anticipatory data should be based.

Similar considerations apply to the housing equation. This equation again does not contain initial conditions of the type one would include in the ordinary formulation, such as the initial stock of housing. However, the remaining variables that appear here, in addition to the *ex ante* variable "starts," are essentially those that would seem relevant to the explanation of starts rather than to their rate of completion. However, in this instance, my objections are weaker, since starts are in physical units and are, therefore, not the same as planned expenditure. It is certainly conceivable that the actual amount of expenditure per unit, or even the speed of completion and, hence, the rate of expenditure, might be influenced by the variables Klein has used. However, here too I would strongly urge that some attempt be made at reformulating the hypothesis along lines more consistent with the nature of the anticipatory data included in the equation.

## THE PRODUCTION FUNCTION

Klein's handling of the aggregate production function, embodied in equations 10 and 11, is quite interesting and ingenious. It has, however, two drawbacks. One of these is of an essentially logical nature, while the other is, I believe, also of considerable practical relevance. Let me first remark that in his model the production function can be looked at as providing a short-run relation between output, $X$, and equivalent full-time nongovernment employment, $h(N_w - N_g) + N_e$, which I shall, for brevity, denote by $E$. In time, this short-run relation shifts with the accumulation of capital (the sum of all previous net investment), which I shall denote by $K$, and with technological progress, proxied in his equation by the time trend. The function of this

[3] Robert Eisner, "Investment Plans and Realizations," *American Economic Review*, May 1962.

relation in the model is essentially that of establishing the employment implication of a given output (since the output itself is largely determined by other mechanisms). Now suppose equation 10 is explicitly solved for $X$, which in the form stated appears on both sides of the equation. We then obtain

$$X = \frac{1}{1 - .196(K/X_c)}(91 + 1.76E + .135t) \tag{M-1}$$

Now, as can be seen from equation 11, $X_c$ is a function of the labor force, $N$. Therefore, an increase in the labor force will tend to reduce the ratio $K/X_c$ and, hence, the quantity $1/[1 - .196(K/X_c)]$. It follows directly that an increase in the labor force would reduce output, even though employment is kept constant. This is an awkward implication, to say the least, and constitutes my logical objection to the production function.

My second and more practical objection is that, as far as I can see, Klein's production function implies an elasticity of full-time employment with respect to output appreciably below unity. From equation M-1 above it is in fact apparent that the derivative of output with respect to employment—i.e., the marginal productivity of labor—is $dX/dE = 1.7[X_c/(X_c - .196K)]$. From the data in Klein's paper, it can be inferred that, at least in recent years, the expression $X_c/(X_c - .196K)$ is of the order of two (which, incidentally, implies that the proportion of total output imputed to the productivity of capital is surprisingly high, namely, of the order of one-half). Thus $dX/dE$ is of the order of 3.5 (reckoning in thousands of dollars per man-year). On the other hand, the average productivity of labor is much higher, of the order of 6 to 7 (in thousands of dollars per man-year). Thus, the elasticity of output with respect to employment, which is the ratio of marginal to average productivity, is only of the order of 0.6. In other words, according to Klein's equations, a 1 per cent increase in employment would increase output by 0.6 per cent; and conversely a 1 per cent increase in output would increase employment by about 1/0.6, or 1.7 per cent. This very high elasticity of employment with respect to output is in sharp contrast with the results of several recent studies. I might call attention in particular to the results reported by Robert Solow in his paper "Technical Progress, Capital Formation, and Economic Growth," presented in December 1961 at a joint session of the American Economic Association and the Econometric Society. His estimate of the elasticity of output with respect to employment is not far from 1.5, instead of Klein's 0.6; and, consequently, his elasticity of employment with respect to output is about two-thirds, in contrast to Klein's figure which is well above unity. Solow further reports similar results as having been obtained by Arthur Okun.[4]

I am not in a position to say with confidence which of these two widely different estimates is closer to the truth. Perhaps I am inclined to attach somewhat greater credence to Solow's than to Klein's estimates because Solow's analysis was primarily focused on this particular issue, whereas Klein's estimate is unavoidably part of a mass production process. The essential point, however, is that the difference between the two estimates needs to be closely scrutinized and resolved, since an accurate estimate of the responsiveness of employment to short-run fluctuations in output would seem to me one of the essential ingredients of a satisfactory short-run model, whether for purposes of forecasting or of economic policy.

### MONEY, INTEREST RATES, AND THEIR EFFECT ON MONETARY AND REAL VARIABLES

A number of equations are devoted to the description of the money market and to the role of interest rates—notably equations 21, 22, 23, and 5. This is, of course, a desirable development, and is in line with the revival of interest in and understanding of the role of monetary policy. Unfortunately, closer examination reveals that, even in this latest model, money plays in fact a very minor role in the short run, at least in the extent of effective interaction between the money markets and the real markets.

It may be noted, first, that interest rates appear in only one of the real markets, namely, in the equation describing investment in housing, which contains $i_L$, the yield on corporate bonds. But from equations 22 and 23, it can be seen that even this variable is completely determined by exogeneous or lagged variables and is completely unrelated to the quantity of money. To establish this point, observe that according to (22), $i_L$ depends on a lagged variable $(i_L)_{-1}$ and on $i_s$, the yield on ninety-day commercial paper. But from (23), we see that $i_s$ in turn is unrelated to the quantity of money either nominal or real, depending instead on a lagged variable, excess

[4] These results are reported in A. M. Okun, "Potential GNP: Its Measurement and Significance," *American Statistical Association, 1962 Proceedings of the Business and Economic Statistics Section*, pp. 98–104.

reserves, $R_{-1}$, and on an exogenous variable, the Federal Reserve discount rate, $i_r$. Since (22) and (23) are sufficient to determine both $i_L$ and $i_s$, it follows that neither rate depends on the quantity of money ($L$) and, hence, that this variable has no direct short-run effect on investments.

The only other place where $L$ appears is in consumers' demand for nondurable goods and services—a rather unexpected and unconventional vehicle for monetary policy! In fact, however, as pointed out earlier, *even if* Klein's results are taken at face value, the role of $L$ in the consumption equations is quantitatively quite negligible—e.g., a 10 per cent expansion of the money supply in one quarter, implying a rate of 40 per cent per year, would increase consumption in the quarter by well below a billion. Hence, this variable could clearly be dropped out of equations 2 and 3 without appreciably affecting the solution of the system. But if we do so, then, since money appears nowhere else, we must conclude that Klein's system omitting equation 21 is sufficient to determine the value of all the remaining variables, including both real variables and prices, without reference to the quantity of money. Furthermore, the only way in which this solution could be affected by the monetary authority in the short run would be through manipulation of the rediscount rate (which affects the short rate, which affects the long rate, which affects housing expenditure), a conclusion that must certainly come as somewhat of a shock to many Federal Reserve officials.

The above considerations illustrate and support my contention that the description of the monetary mechanism embodied in equations 21, 22, and 23 is exceedingly weak and is in urgent need of mending. Though I cannot enter into details here, let me indicate that this requires at least the following steps: (1) a more careful distinction and specification of the *demand* and the *supply* side of the money market; (2) a more adequate explanation of the short rate and its relation to the quantity of money; (3) a more refined approach to the relation between short and long rates, exploiting recent contributions in this area and, in particular, the very promising line of inquiry opened up by David Meiselman[5] and followed up by, among others, Reuben Kessel.[6]

[5] Cf. his *The Term Structure of Interest Rates*, Englewood Cliffs, N.J., 1962.

[6] "The Cyclical Behavior of the Term Structure of Interest Rates," National Bureau of Economic Research manuscript.

### PRICE FORMATION EQUATIONS

The model includes price formation equations for all the major components of GNP. As indicated earlier I regard this as a very desirable development. To be sure, many of these equations are open to criticism on a number of counts. However, I shall not take the time for a detailed criticism, since Klein himself is clearly well aware of the limitations of his current hypothesis, and we also know that the task force now at work under his and Duesenberry's direction for the purpose of constructing an improved model is developing an ambitious and promising fresh approach to this problem.

### INVESTMENT IN INVENTORIES

I hold the conviction, which I am sure is shared by most scholars interested in this area and is also strongly supported by the paper of Friend and Jones at this conference, that a reliable explanation of investment and disinvestment in inventories largely holds the key to successful short-term forecasting. Thus, while I have no basic quarrel with Klein's inventory equation 6, I believe it would be particularly worthwhile to pay closer attention to, and provide further scope for, this sector in his model. Once more, it is not possible in this comment to enter into details, but I should like to indicate certain directions for further development which I hold to be very promising, partly on the basis of my own work in this area.

1. We know that, in the short run, inventory changes partly reflect intentional adjustments and partly errors of sales forecasts (and of delivery schedules). The relative importance of the error component will be greater the shorter the period of time over which the change is measured. One should, therefore, try to recognize explicitly this double mechanism—although admittedly the case for doing so is not as strong for a quarterly model as it might be for, say, a monthly model. In order to achieve this goal one has somehow to introduce sales expectations in the model. A good deal of, hopefully, reliable information on short-run sales expectations is now becoming available and should be tried out. For the past, one may have to rely on various kinds of proxy variables, of the type reviewed in Michael Lovell's contribution to this conference.

2. Inventory behavior strikes me as an area where we may stand to gain significantly from disaggregation. There are indications, for

instance, that much of the fluctuation in the postwar inventory cycles has been concentrated in the manufacturing sectors, in the face of relatively negligible fluctuations in sales to final users. One should, therefore, investigate the possibility of separating the retail-wholesale complex from the manufacturing sector. Also, one should explore the possibility of exploiting disaggregation of the consumer sector so as to disaggregate inventory investments in the durable and nondurable goods sectors. While I am fully aware of the "costs" of disaggregation, notably in terms of a considerable enlargement in the number of equations and unknowns, I feel that this is an area very much worthy of further careful exploration.

### *The Workings of the Model and Its Solution for Short-Term Forecasting*

I propose to wind up this already lengthy comment with a few remarks on the working of the model and its utilization for short-run forecasts. A close examination of Klein's model reveals that its workings are fairly intricate, as there is a great deal of genuine interaction between its various parts. Furthermore, recognition of monetary as well as real variables results in a system which is definitely not linear —although it is hard, for a superficial critic like myself, to judge just how essential these nonlinearities are. The nonlinearity shows up most clearly in the equilibrium condition (30), which is, in essence, a glorified version of the standard Keynesian condition that consumption demand, investment, and government expenditure must equal gross national product. However, this condition is stated in terms of current values, which are *products* of prices and real variables, which, in turn, depend on both prices and real variables.

In his reported application of the model to the first three quarters of 1961 (and apparently, also, in his later application through the second half of 1962),[7] Klein has, however, disposed of the nonlinearity essentially by treating prices as exogenous variables, assumed constant or forecasted through *ad hoc* devices. One gathers that this procedure was followed to facilitate the task of solution. While this is an understandable consideration, it should be recognized that it amounts to throwing overboard some of the very features that make this latest model a potential improvement over

[7] Cf. the release of the Econometric Research Unit, Wharton School, University of Pennsylvania, December 21, 1961.

its predecessors. I should like to suggest that it may be possible and worthwhile to retain, at least in part, both the advantage of simplicity and the richness of the model by having recourse to iterative procedures. That is, having assumed a set of prices and solved the resulting linear system, one goes back and tests how closely the assumed prices and the derived real variables satisfy the various price equations. If the discrepancies are judged unreasonable—which, of course, depends partly on the confidence one is willing to place in the price formation equation—one could change the price assumptions accordingly and iterate. This method should converge to a solution, nor is this an unreasonable hope, if the system makes sense. Although I have not tried out this suggestion, I venture the guess that had Klein followed it, he would have been led to modify his assumption—unwarranted at least *ex post*—that prices would remain constant over the first three quarters of 1961.

Obviously, this comment can in no way do full justice to Klein's paper. For one thing, I have concentrated on the shortcomings of his model. Let me therefore repeat that the only reason for doing so is that there is, on the whole, very broad agreement between Klein and myself on the role of econometric models and on the strategy of model construction and testing, and that, furthermore, the significance and quality of Klein's contributions are so obvious that there is hardly any need for a discussant to point them out.

REPLY by L. R. Klein

I am very fortunate to have such stimulating and constructive comment by my two discussants. I greatly appreciate their remarks. Denison knows his way among Washington figures far better than I do, and I respect his judgment as to the relative accuracy of alternative series where choice is possible. I hope in future revisions of this model to look into his data suggestions.

I have long been bothered about the relationship between corporate and total nonwage income. A rough empirical relation that serves to close the system has been suggested, but in a current revision and re-estimation of the model, we are taking rentier (dividend and interest) income out of nonwage income and forming the relationship between corporate income and nonrentier, nonwage income. We would do better if we were to exclude farm income as well. Rentier income will be treated as a smooth trend corrected possibly for autoregression and possibly for interest rate changes. I disagree, however, with Denison that the relationship being constructed leads nowhere. Profits are treated as a residual in this system, and they are determined in the over-all set of equations. This seems to be reasonable. A separation into corporate and noncorporate profits, however, cannot be made by a rationally constructed equation of behavior. This is a purely institutional relationship that results from some legal distinctions in the functioning of enterprise.

In a completely linear model, the forecasting of *change* (from last period's *estimated* values) in each equation is equivalent to adding last period's error to each equation so that each equation is exactly satisfied if error is unchanged. In our solution process, we did this only after we had reduced the system to two relationships in $I_i$ and $X$. Our system, after simplification—as explained in the paper—is solved by linear steps. It is not, however, a linear system. Time lags enable us to solve it in linear steps even though it is nonlinear in the variables.

Denison raises some very fundamental questions about reversibility and the use of a single model for different cyclical phases. This system has performed fairly well at both peaks and troughs. There is some plausibility in irreversibility, but much more work needs to be done to establish an empirically sound irreversible model. Eventually, I would like to work on that aspect and revise the model accordingly. For the moment, though, there are a number of other problems to be tackled, and I feel that these have higher priority. I am pleased by the uses we have been able to make so far of capacity and unemployment variables. I am not as disappointed with the results achieved to date as is Denison.

I appreciate Franco Modigliani's suggestions about treating the stock of consumer durable capital in the equation for durables demand. Subsequent to his remarks, we changed the moving average term in this equation to cover twelve past years (forty-eight past quarters) of gross durables expenditure. This gave a better estimate of the stock in consumer hands, but did not improve the equation. We have found only one promising lead for the improvement of this equation, namely, the use of the Survey Research Center's index of consumer attitudes in place of our previous index of buying plans. The attitudinal index shows much higher correlation than buying plans, or almost any other available variable, with durables expenditures. We are now investigating this lead and the interpretation of the attitudinal index more closely.

The coefficient of income in our equation for durables demand seems to be high. Our model in extrapolation has shown some tendency to overestimate durable consumption. In subsequent work on this equation in connection with further testing of the Survey Research Center index, we have, in fact, selected a new equation that has a considerably lower coefficient of income.

All our consumption equations were first formulated on a uniform basis, with liquid assets, factor share ratios, and population as potential variables. Both statistical significance and reasonableness of the whole equation (sign and general size of some coefficients) were used to weed out some alternatives. To my way of thinking there is no a priori basis for choosing between liquid assets and total real wealth as a possible variable. On many occasions, I have considered both of these variables in experimental consumption function calculations. There is no clear-cut empirical case for preferring one or the other. Liquid asset wealth gives us a more direct tie with the monetary sector, and this has been one of our motivations in using this variable. As for the effect of $P/W$, the factor share ratio, I fail to see how Modigliani arrives at his a priori notions about the effect of this variable. Surely services contain many luxury items (entertainment, travel, personal services, medical services, dental services), so we could not object to the finding of a significant positive effect for $P/W$. The argument about dimensionality, as far as I am concerned, is pointless. On grounds of elegance, we might want all variables of a linear relation in the same dimension; but elegance, of course, is well known to be a concern of tailors.

Modigliani is extremely rigid and supremely confident of the correctness of his views on the structure of realization equations. We really do not know much about the parametric structure of such subjective relationships. In our experiments, we tried some formulations like those suggested. The change in output or the change in nonwage income was used as a separate variable, together with investment expectations. No formulation looked as good, however, as the version finally selected, with capacity utilization and anticipated outlays as the explanatory variables. There is a good deal of independent evidence that our capacity series is close to what producers call their "preferred" operating level. The ratio of actual output to a "preferred" level of output is a measure of the deviation of actual from desired (not expected) and, in this sense, is a reasonable variable for explaining investment apart from its relation to planned investment.

I have long been an admirer of the time-honored work of Paul Douglas in measuring production functions. I cannot, therefore, feel unhappy about the estimation of production elasticities (w.r.t., employment) in the neighborhood of 0.6. I would regard any alternative figure of 1 5 as utterly ridiculous. There is much evidence against the plausibility of this figure. In earlier models based heavily on observations from the period of the Great Depression, elasticities larger than unity are acceptable. If we think of production functions (in two dimensions) as being of the standard sigmoid shape, we should not be surprised that approximations to sections of the function in the neighborhood of low output values give high elasticities But in the postwar period, our approximations are for a different section of the function and ought to give elasticities less than unity.

My theoretical predilections are very much in favor of a theory of the *real* economy. The monetary economy, if in good housekeeping order, will not have a dominant influence on real affairs. Nevertheless, I have tried hard over the years, in several models, to give the benefit of every doubt to money and interest rates when making statistical estimates. My empirical verdict, thus far, is that little evidence can be found for the actual influence of money or interest on real activity. It is this weak influence that Modigliani finds here, and about which he is concerned.

The links that we have finally tried to establish, and the statistical significance measures, leave much to be desired; they finally boil down to a dependence on the discount rate and excess reserves. Were it not for the weak measures of significance, I would have concluded that these were almost ideal monetary variables to have at the end of a series of relationships tying the real to the monetary sector. These are the variables that the monetary authorities directly control or use as guidelines. In pedagogical models it is assumed for simplicity that the authorities control the money supply; but this is very indirect—through the use of open-market operations, discount policy, and the variation of reserve requirements. What could be a better indication of money supply than excess reserves? What are open-market operations other than means of influencing reserves?

I am not pleased with the sharpness of my coefficients, but I would not want to change the sequence of steps in relating the monetary to the real sector. There are other instruments of monetary action having to do with the regulation of mortgage, share, foreign exchange, and consumer credit markets that ought to be built into a more detailed system, but the broad patterns of the existing scheme ought to be retained. It is hard to see why Federal Reserve officials would be shocked to learn about this pattern, since customary expositions of their lines of influence run just in the terms implied by the model. It would be helpful and constructive if Modigliani would be able to show more explicitly how the real and monetary sectors of the economy are related.

The comments on inventory relations and their importance, on the use of sales expectations, and on the treatment of prices are all well taken. Work is being pursued on all these fronts now, but there are no definite results to present yet. Some iterative and approximation methods have been tried to bring in price forecasts in an endogenous way in the nonlinear model, but the results obtained so far are unsatisfactory.