Artificial Intelligence, Competition, and Welfare

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Abstract

We propose a policy-relevant research agenda examining how market power in upstream artificial intelligence (AI) affects downstream prices, industry structure, factor returns, and welfare—especially whether labor-displacing AI leaves workers worse off. In our open-economy general equilibrium model, AI is a priced, imported input. Our main model features two nontraded sectors and firms making discrete adoption decisions about technology. Adoption reduces unit costs, displaces some types of workers, and depresses wages for those workers via diminishing returns elsewhere, while leaking AI fees abroad. We identify conditions under which market power in AI leads to a "double harm" for displaced workers, who may experience real wages cuts when AI becomes available at low prices, and then experience further harm from increases in AI prices. Strategic AI pricing reduces welfare by raising downstream marginal costs (via usage fees) and limiting entry and variety (via access fees). We derive an adoption frontier linking feasible usage fees to displaced workers' outside options, showing that a monopolist typically makes use of both types of fees and prices on the frontier; capping one fee shifts rents to the other. Regulating both fees, alongside policies that absorb displaced labor, can raise national welfare.

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Introduction

Economists have observed that artificial intelligence (AI) may have a variety of downstream benefits for the economy: it may improve productivity, enable innovation and increase entry of new firms. Alongside potential benefits, AI poses economic risks. Prominent among these are worker displacement and industry restructuring, with consequences for income distribution both within and across countries. For example, unlike a typical productivity shock that may allow a country to increase exports and thus national income (possibly counterbalanced by distributional costs and resource squeezes that raise prices), the global introduction of a labor-displacing technology may decrease the value of labor-intensive exports and thus reduce national income for some countries.

In this paper, we take as a starting point general equilibrium frameworks that can capture effects like these. Our main contribution is to introduce to the models what we view as a critical factor affecting the magnitudes and balance of benefits and risks: *market power* in AI. We propose a research agenda that studies the impact of such market power on downstream industries, including impacts on downstream industry structure, profits, innovation, and consumer prices. We further highlight indirect effects on wages, inequality, productivity in other sectors, and welfare.

We find that market power in AI creates negative effects even for countries that do not rely on exports: when the profits obtained by the AI provider are not recirculated in the economy, for example, if the country imports AI technology, welfare in a country may fall in aggregate as a result of the introduction of AI. Productivity gains are leaked abroad, prices do not reflect the fall in labor costs, and displaced worker groups may be particularly harmed by falling wages without corresponding increases in the variety of goods or decreases in prices.

When considering a future scenario with "transformative" AI, the presence of scale economies and other barriers to entry such as proprietary usage data or distribution imply that oligopoly or even monopoly are outcomes that could emerge in the absence of policy intervention. This paper shows that AI market power can have outsized impacts on a country's welfare, so that policies that reduce the risk of monopoly may play an important role. Competition authorities worldwide have recognized that AI may create new bottlenecks in various layers of the AI stack. Examples include NVIDIA's dominance in advanced chips, concentration among foundation model providers, and the dependence of downstream applications (e.g. search engines, language tutors, writing assistants, customer service platforms) on a few foundation models. Access to data that is crucial for AI performance may be gated by incumbents in both the consumer and enterprise software markets. Distribution channels, such as mobile operating systems, productivity software, and online platforms, may become bottlenecks given that many are controlled by incumbents with substantial market power. When these channels are controlled by firms that also supply AI, or by firms that have control over critical proprietary data, entrants face foreclosure risk, and even efficient AI applications may struggle to gain distribution, data and scale. The United Kingdom's Competition and Markets Authority, the US Federal Trade Commission, and other agencies have issued reports highlighting these issues and are monitoring commercial relationships.[1]

In this paper, we focus on the business-to-business use case, where AI is an input to large parts of the economy. Our starting point is to observe that the arrival of a new technology can be modeled similarly to the availability of a new input factor available through trade. Thus, we consider models of a small economy, considering both open and partially closed economies, where in all cases AI is an imported factor of production. We vary the competitiveness of the AI industry between competition and monopoly. The price of AI is therefore a strategic variable chosen by the upstream supplier who may set the price to maximize revenue, perhaps subject to regulation. We are particularly interested in the impact of AI market power on the restructuring of industry and the real wages of workers, and whether high prices for AI lead to reductions in the real wage despite efficiency benefits in production. We highlight the realistic scenario where AI providers can use nonlinear pricing schemes, influencing entry, innovation and industry structure in downstream industries.

The assumption that AI is imported may be accurate for countries that do not participate in the AI value chain, and it may be a useful approximation for scenarios where domestic firm profits are not broadly recirculated in the domestic economy. We argue that the extent to which AI providers sell at competitive prices has critical implications for the impact of AI on income distribution and welfare. Unlike much of the literature that focuses primarily on tradeoffs between efficiency and distribution, we consider assumptions that are rich enough such that productivity improvements from AI do not necessarily result in improvements in economy-wide welfare. In our model, the firms adopting AI do not internalize the negative externality on worker wages, and they do not internalize that AI payments leave the economy rather than flowing back into country income.

We further identify conditions under which the introduction of AI at a low or moderate factor price hurts a group of displaced workers, while, at the same time, the subsequent exercise of market power inflicts additional loss on the displaced workers. This "double harm" scenario contrasts with traditional results where, if a technology is introduced that disproportionately displaces a group of workers, raising prices for the input helps that group by diminishing the substitution.

[1] see, e.g., Competition and Markets Authority (2023) and Portuguese Competition Authority (2023).

Our model also highlights the key question of whether other sectors, either nontraded sectors or export sectors, can productively absorb displaced workers, an assumption that may be questionable in the face of a major technological shift. This focus underscores the critical role played by government policies that change how productive workers are in sectors *less* affected by AI. For example, the government may increase spending on services like healthcare and education and then help displaced workers transition into those industries, perhaps using AI to be more productive in their new roles.

We show that conclusions about wages and aggregate welfare turn on a small set of modeling choices that are especially salient for AI: whether goods are traded versus nontraded (and in turn, whether there is diminishing marginal utility in the output markets); whether automation reduces demand for some or all groups of labor to (near) zero so that output expansions, if they exist, in newly productive industries do not lead to increases in labor demand; whether displaced workers can be productive in other sectors, and whether they are substitutes or complements for other factors of production in those sectors; and whether the upstream supplier can use nonlinear pricing. We consider both continuous and discrete production functions, where a large fall in the price of an AI input may fundamentally change which factor of production (e.g. skilled or unskilled labor) is dominant, and where with discrete adoption some types of labor demand may be locally insensitive to input prices for technologies that fully automate. We also pay special attention to whether it is optimal for firms to adopt AI when competitors have adopted, since wages will be lower, reducing cost savings from AI adoption.

The popular interest in AI has stimulated a macroeconomic literature modeling possible faster growth from AI. Aghion, B. F. Jones, and C. I. Jones (2019) explore the implications of AI for the labor–capital ratio and the share of expenditure on automated tasks, where outcomes depend on parameters such as substitution elasticities between automated and non-automated goods. Nordhaus (2021) presents a model in which AI induces capital deepening which in turn accelerates growth, emphasizing that returns accrue primarily to capital: *"Capital eventually gets virtually all the cake, but the crumbs left for labor—which are really small pieces of the increasingly huge mountains of cake—are still growing at a phenomenal rate"* (p. 14). In our setting, the question is whether labor actually does benefit from the crumbs, if the things capital produces remain expensive to them.

The literature on growth models AI as a driver of factor usage and total factor productivity (e.g., Aghion, B. F. Jones, and C. I. Jones 2019; Nordhaus 2021); open-economy work on directed technical change treats final goods as traded and incidence of impacts as disciplined by terms-of-trade movements (e.g., Korinek and Stiglitz 2018; Korinek and Stiglitz 2021). We incorporate many of these forces in our model.

Studies of task–based and automation frameworks (for an early example, see Autor, Levy, and Murnane (2003)) study how technology displaces or complements labor when technology costs and output prices are taken as parametric. In our model, we show in a general equilibrium framework how outcomes depend on whether displaced workers are substitutes or complements for other factors of production in other sectors.

Korinek and Stiglitz (2018) and Korinek and Stiglitz (2021) propose a series of models that focus on distributional consequences and policy responses from exogenous technological shocks or changes in the returns to the resources of small countries. A complementary line of work, going back to Stiglitz (1976) and further developed in Delli Gatti et al. (2012b) and Delli Gatti et al. (2012a), studies dual-economy settings in which a constant-returns agricultural sector coexists with an urban sector featuring wage rigidity (e.g., due to efficiency-wage considerations). In their baseline environment, an agricultural productivity improvement is *unambiguously* welfare-reducing: higher rural productivity contracts urban employment when nominal wages cannot adjust, and with flexible wages an induced wage decline can further depress demand and raise unemployment. Our models connect to these results but identify distinct channels that do not rely on nominal rigidities.

Our framework also connects to two classic ideas in economics—Baumol's "cost disease" and the "Dutch disease"—but also shows why the forces they highlight may not apply or may be more nuanced in the case of AI. More precisely, classic Dutch disease logic (Corden and Neary, 1982) raises nontraded prices when resources are pulled into a booming sector, and Baumol and Bowen (1966) shows that uneven productivity growth can raise relative prices in stagnant sectors because wages equalize economy-wide. With AI, we argue that the analogy is incomplete. Adoption of AI often *releases* labor rather than absorbing it, so there is little (labor) resource pull into the "booming" activity; displaced workers crowd into nontraded services, and whether prices rise depends on wages and sectoral productivities rather than on reallocation alone. Further, global availability of AI may limit the ability of AI-adopting industries to expand exports. We formalize benchmark models that capture these ideas, building up to our main model that adds entry/variety and analyzes the welfare impact of market power and nonlinear pricing of AI.

A related perspective comes from growth accounting. Domar (1961) and Hulten (1978) showed that the aggregate effect of a sector's productivity change depends on its "Domar weight"—the ratio of its gross output to GDP. Because an upstream general purpose technology supplies inputs to many final sectors, its Domar weight is disproportionately large and shocks there propagate strongly through the economy. In our setting, AI plays exactly this role: changes in its usage fee resemble productivity shocks in a sector with very high Domar weight. As Baqaee and Farhi (2019) and Baqaee and Farhi (2020) emphasize, however, the Hulten–Domar formula captures only first-order effects. General equilibrium forces can overturn the first-order gain. In our framework, several such general equilibrium effects play a role: (i) **labor displacement**, which lowers wages in the sectors that absorb redundant workers; (ii) **nontraded scarcity**, which raises the cost of living if national income rises; and (iii) **market power**, which allows a monopolist AI supplier to charge high usage and access fees, leak rents abroad, and blunt pass-through of productivity gains. Our contribution is to highlight how market power shapes outcomes in the third channel. The forces we identify imply that even if AI appears beneficial in a Hulten–Domar sense, cheaper AI can still reduce real wages and national welfare once displacement, nontraded scarcity, and monopoly rents are accounted for.

Our model is also related to the literature on offshoring. For example, Grossman and Rossi-Hansberg, 2008 developed a model where firms perform a continuum of tasks, and some can be moved abroad at lower cost. Offshoring acts like a productivity improvement: it reduces production costs, expands output, and has wage effects as tasks intensive in one factor shift abroad. Our model considers the case where the factor price is set by a monopolist, and also where output expansion may be limited.

Benchmarks: Incorporating AI in Standard Models.

We begin by highlighting forces (with details in the Online Appendix) that arise in the standard two–good Heckscher–Ohlin model with AI as an imported factor; in this model we show that, with larger technical shifts that may occur with AI, income inequality can experience what we call "double-harm."

In the first baseline, sector A produces using skilled and unskilled labor (L_S, L_U) alone; sector B combines (L_S, L_U) with AI, purchased at price p_X from the foreign supplier. We write the quantity of labor type $i \in \{S, U\}$ in sector $j \in \{A, B\}$ as L_i^j. Both goods A and B are traded at exogenous world prices $(\bar{p}_A, \bar{p}_B)$.

When $p_X = \infty$, AI is unavailable. When p_X is finite, sector B's costs c_B depend on (w_S, w_U, p_X), where (w_S, w_U) are the equilibrium wages of skilled and unskilled labor.

Zero–profit conditions determine (w_S, w_U) as a function of p_X. For each sector $j \in \{A, B\}$, let $\theta_i^j = w_i a_i^j / c_j$ denote the cost share of factor $i \in \{S, U\}$, and let $R_j = \theta_S^j / \theta_U^j$ denote the sectoral intensity ratio. B is skill–intensive relative to A if $R_B > R_A$, and unskilled–intensive if $R_B < R_A$.

For expositional simplicity, we assume that for sufficiently high p_X, sector B's skill intensity $R_B(p_X) := \theta_S^B / \theta_U^B$ is *decreasing* in p_X (equivalently, *increasing* as AI gets cheaper).

General-equilibrium mapping and reallocation. Totally differentiating zero-profit in A and B at fixed $(\bar{p}_A, \bar{p}_B)$ yields

$$\theta_S^A\, dw_S + \theta_U^A\, dw_U = 0, \qquad \theta_S^B\, dw_S + \theta_U^B\, dw_U + \theta_X^B\, dp_X = 0.$$

Eliminating dw_U,

$$\begin{aligned} &\big(\theta_S^B - \theta_U^B \tfrac{\theta_S^A}{\theta_U^A}\big)\, dw_S = -\, \theta_X^B\, dp_X, \\ \Longrightarrow \quad & \operatorname{sign}\big(\tfrac{dw_U}{dp_X}\big) = \operatorname{sign}(R_B - R_A), \qquad \operatorname{sign}\big(\tfrac{dw_S}{dp_X}\big) = -\operatorname{sign}(R_B - R_A). \end{aligned} \tag{GE}$$

How do the introduction of AI and the exercise of market power affect workers? National income increases with cheaper AI: applying the envelope theorem to national income, we have $\frac{dY}{dp_X} = -M_X \leq 0$. Further, one wage rises while the other falls when p_X changes; which factor gains depends on the relative intensity of A and B. A change in p_X affects R_B through both substitution across sectors as well as across factors. If B is *unskilled-intensive* ($R_B < R_A$) and p_X increases, then w_S rises and w_U falls; activity tilts from B toward A (which is relatively more skill-intensive).

How does market power interact with income distribution? Given R_B decreasing in p_X, let the crossing point with R_A (if it exists) be denoted p_X^*. For $p_X < p_X^*$, B is skill-intensive and low AI prices can depress w_U, but increases in p_X then help. For $p_X > p_X^*$, both of these forces reverse.

This is the standard trade intuition that the losers from ongoing technology adoption in the skill-intensive region may be locally helped by a higher p_X (e.g., a tariff). However, if unskilled workers are hurt by the introduction of AI, its price must be low enough that Sector B is skill-intensive, so that higher p_X helps the unskilled.

Double vs. single reversals.

$R_B(p_X)$ need not be monotone, and for large changes in technology, it may not be. In autarky, sector B may be unskilled-intensive. As the AI price falls, it becomes skill-intensive, but at very low prices, it may again appear unskilled-intensive, as AI substitutes for both labor types. In such a case, the introduction of AI supplied by a firm with market power can deliver a "double-harm": for sufficiently low p_X, the unskilled wage can lie *below* its autarky value ($w_U(\infty) - w_U(p') = \int_{p'}^{\infty} \frac{dw_U}{dp_X}(p_X)dp_X > 0$ *and* be locally *decreasing* in p_X ($\frac{dw_U}{dp_X}(p') < 0$); this holds in Figure 1 for p' just below $p_X^{(2)}$. The local decrease arises because at those very low prices, B is again unskilled–intensive relative to A, so a higher p_X reduces B's output and differentially reduces demand for unskilled labor.

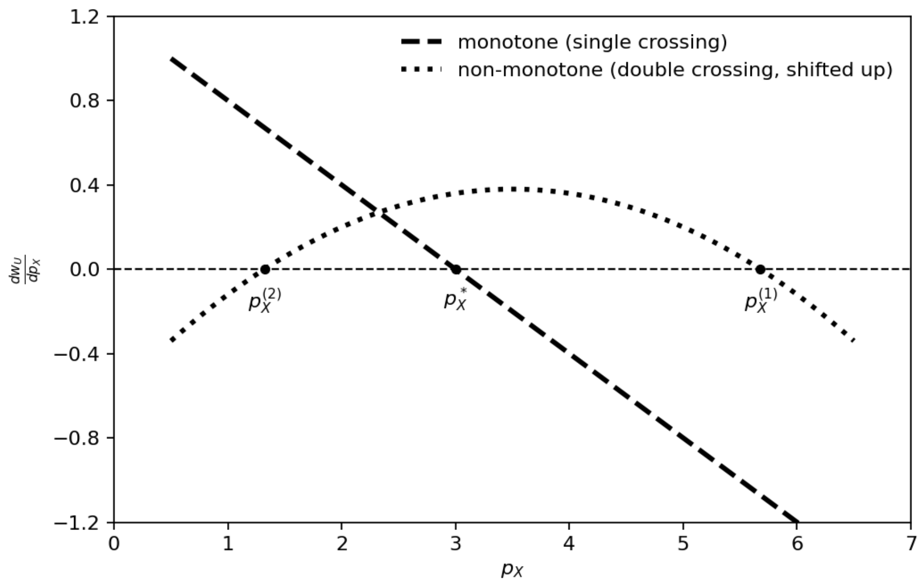

Figure 1: Local incidence of AI input prices on the unskilled wage: $\frac{dw_U}{dp_X}$ vs. p_X. Above the zero line, unskilled wages rise with p_X; below, they fall. A monotone path yields a single crossing of wages as p_X changes (dashed), while a non-monotone path yields two crossings (dotted).

A Nontraded Good and Dutch Disease

Now extend the benchmark model to consider a nontraded good A and a traded good B, with Cobb–Douglas preferences. Aggregate real income (or welfare) satisfies

$$d\log W \;=\; d\log Y \;-\; (1-\alpha)\, d\log p_A, \tag{1}$$

where p_A is the nontraded price. The mechanism is well known: if the nontraded price rises sufficiently relative to income, both groups' real wages can fall, something which cannot occur when all goods are traded.

We assume that AI only lowers costs in B. The Online Appendix presents comparative statics of p_X on income Y and the untraded price p_A, and then on real income via (1). This analysis illustrates the classic logic: as p_X falls, production of the export good B expands, p_A rises, and both groups' real wages can fall simultaneously if the CPI channel dominates. This is the textbook Dutch disease result (Corden and Neary, 1982; Corden, 1984), where countries that discover a valuable natural resource experience a boom in the resource sector, attracting labor and capital away from other industries and raising national income but also making nontraded goods more expensive. This squeezes households through higher cost of living and erodes competitiveness in other tradables. Closely related is the Baumol and

Bowen (1966) cost: when productivity rises in some sectors but not others, wages equalize across the economy at higher levels (supported by rising aggregate income) and stagnant sectors see rising labor costs, leading to higher prices, lower welfare, and distributional harm.

With AI, the dynamics differ: adoption raises productivity in one sector but pushes, rather than pulls, at least some categories of workers into the rest of the economy, where their marginal productivity and thus wages fall. In the context of the benchmark models, if labor-displacing AI is cheap and efficiency gains remain domestic, higher income boosts demand for nontradables, raising their relative prices and worsening the cost of living. If instead a monopolist extracts large fees, national income may not increase and workers face lower wages without the offsetting demand-driven price surge.

Further, unlike resource booms, global adoption of AI may shrink export markets by lowering the world price of previously labor-intensive exports, reducing national income but also alleviating Dutch disease and Baumol cost pressures.

Main Model

This model builds on the benchmarks, tailoring assumptions to capture mechanisms we argue may be important in practice. We incorporate two new features. First, *both* sectors are non–traded, so that output prices and the expansion of the AI-augmented sector are limited by diminishing marginal utility of consumers. We leave for future work the possibility for a discrete fall in previously labor-intensive exports, which would exacerbate the challenges we highlight here, and simply start from a scenario without exports. Second, sector B consists of many differentiated varieties under Constant Elasticity of Substitution (CES) demand (elasticity $\sigma > 1$) with constant markup $\mu = \sigma/(\sigma - 1) > 1$ and free entry subject to a per-firm domestic license fee F (rebated lump-sum and shared equally across households, for example this could be a government license fee or rent for a resource used by firms) and a fixed access fee, ϕ, that is collected by the foreign monopolist. This allows us to consider the impact of AI on industry structure and entry, and further opens the door for more realistic pricing strategies by the AI monopolist. Note that our qualitative results can be extended to the more general case of Hierarchical Structure of Aggregation (HSA) preferences following Matsuyama (2019).

Studying AI in a fixed-cost framework is natural, since digital technologies involve high up-front investments (training, deployment, access) but low marginal costs of use. A differentiated product and free-entry structure makes it possible to analyze how AI pricing reshapes industry structure (the equilibrium number of firms, variety, and the quality-adjusted price index). Moreover, because AI is a general-purpose technology that enables a wide range of applications, entry and variety are themselves first-order welfare channels.

Primitives and price indices. Households spend a constant share $\alpha \in (0,1)$ on B and $1-\alpha$ on A:

$$U = C_A^{1-\alpha} C_B^{\alpha}, \qquad E_B = \alpha Y, \quad E_A = (1-\alpha)Y. \tag{2}$$

Symmetry across the N active firms in B yields that the CPI is

$$P = p_A^{1-\alpha} P_B^{\alpha}, \quad P_B = \mu m_B N^{\frac{1}{1-\sigma}}. \tag{3}$$

Without loss of generality, we normalize $p_A \equiv 1$ so that all prices are relative to the price index in Sector A; although at times we discuss the price effects in Sector A, these should be interpreted as relative prices.

Pricing and free entry in B. Let m_B be the unit marginal cost and $p_B = \mu m_B$ the symmetric price (cf. (25)). With per-firm outlays $F + \phi$ and free entry,

$$(p_B - m_B)q = (F + \phi) \quad \Rightarrow \quad p_B\, q = \tfrac{\mu}{\mu-1}(F+\phi). \tag{4}$$

Because the expenditure in Sector B is $E_B = \alpha Y = N p_B q$, we have

$$N = \tfrac{\mu-1}{\mu}\, \alpha\, \frac{Y}{F+\phi}, \qquad P_B = \mu\, m_B \left(\tfrac{\mu-1}{\mu}\, \alpha\, \frac{Y}{F+\phi} \right)^{\frac{1}{1-\sigma}}. \tag{5}$$

Thus, holding Y and $(F+\phi)$ fixed, changes in m_B move p_B but do *not* move N directly. In general equilibrium, however, p_X and ϕ move Y and hence N through free entry.

AI technology and marginal cost in B. A foreign upstream supplier charges a per-firm access fee ϕ and a per-unit usage fee p_X (we start by setting $\phi = 0$ to facilitate comparisons to the benchmarks, and then generalize the model). Adoption in B is discrete and leads to marginal cost $m_B = s_B w_S + p_X$ with no need for unskilled labor ($u_B = 0$), where we say automation is partial if $s_B > 0$ and (informally) full if s_B is significantly lower than the no-AI baseline, which requires s_0 units of skilled labor and u_0 units of unskilled labor. For simplicity of exposition, we consider the case where under partial automation s_B is at least as large as the skilled labor used per unit in the benchmark without AI. The case where marginal cost is zero ($s_B = 0$) is an edge case given our assumptions about free entry, so we rule it out for simplicity. In both cases, adoption implies $u_B = 0$, so displaced unskilled labor reallocates to A. In contrast to the benchmarks, a marginal expansion of B does not *pull* U from A. The effect of p_X on prices now arises primarily through p_B (via m_B and N), not through p_A (via Q_A).

Sector A and factor markets. Following the benchmarks, sector A is produced competitively with both skilled and unskilled labor, and labor markets clear:

$$L_S = L_S^A + L_S^B, \qquad L_U = L_U^A + L_U^B. \tag{6}$$

With diminishing returns in A, the crowding of unskilled labor into A lowers w_U, while w_S reflects the allocation of L_S across sectors and the demand for L_S in B. For some results below, we assume that production in Sector A is CES with factor shares (where factors are skilled and unskilled labor) parameterized by β and substitution parameter ρ.

Income and external payments. Domestic income includes wages and the rebated F but excludes foreign AI payments, where Q_B is output in B:

$$Y = w_S L_S + w_U L_U + FN, \qquad \text{Foreign outflow} = \phi N + p_X Q_B. \tag{7}$$

Unlike F, ϕ reduces both entry and *domestic absorption.*

Exogenous AI Prices

Adoption frontier defined. AI is adopted by all firms in Sector B only if each firm sees higher profits from adopting AI when all rivals adopt. As we discuss in more detail below, this yields a downward-sloping frontier of the set of incentive-compatible (ϕ, p_X) pairs, where the frontier can be written as $p_X \leq p_{X,\max}(\phi)$. Importantly, adoption constraints are more challenging when other firms adopt, since adoption frees up labor and pushes down wages, and the firm's alternative without AI involves producing B using unskilled labor. In this section, we first consider comparative statics on usage and access fees within the frontier, and then return to consider the frontier in more detail when we consider the problem faced by a monopolist AI provider.

Equilibrium. Let $Z = (\alpha, \sigma, F, s_B;\ L_S, L_U)$ denote the vector of exogenous primitives (preferences, technologies, and endowments) for Model 3. Given (Z, ϕ, p_X), a (competitive) equilibrium consists of prices, quantities, and allocations

$$\Big(p_A, p_B, P_B, m_B, N, q, Q_B, Y, w_S, w_U, L_S^A, L_S^B, L_U^A, L_U^B\Big)^{*r}\Big(Z, \phi, p_X\Big),$$

where the superscript ^{*r} denotes equilibrium values in Model 3 for regime $r \in \{AI, 0\}$ ($r = 0$ being no AI), that jointly satisfy equations (2)-(7). The feasible (ϕ, p_X) must also satisfy an incentive constraint for technology adoption $p_X \leq p_{X,\max}(\phi)$, developed in more detail below. For any equilibrium variable X in the list above, we use the superscript notation $X^{*r}(Z, \phi, p_X)$ to denote its equilibrium value as a function of (Z, ϕ, p_X).

Welfare Ratio In the Online Appendix, we show that applying equilibrium conditions yields the following expression for the ratio of welfare (real income) across regimes:

$$\frac{W^{*AI}}{W^{*0}} = \underbrace{\left(\frac{m_B^{*0}}{m_B^{*AI}}\right)^{\alpha}}_{\text{unit cost / } B \text{ price index}} \underbrace{\left(\frac{F}{F+\phi}\right)^{\frac{\alpha}{\sigma-1}}}_{\text{variety / entry}} \underbrace{\left(\frac{Y^{*AI}}{Y^{*0}}\right)^{1+\frac{\alpha}{\sigma-1}}}_{\text{aggregate income}} \tag{8}$$

The first term reflects unit costs (the sector-B price index), the second variety/entry, and the third aggregate income (with p_A normalized to 1). Low AI fees lower unit costs, raise income, and increase variety.

The same decomposition can also be applied separately for skilled and unskilled workers. Displaced workers may have lower labor income, so that unskilled workers may be harmed even by cheap AI, depending on parameter values, similar to the analysis of the benchmark models.

Linear pricing (usage–only, $\phi = 0$). Under linear pricing the foreign supplier sets the per–unit usage fee p_X with no access fee ($\phi = 0$).

We model the introduction of the technology as a shift from autarky ($p_X = \infty$) to finite p_X. Two new forces arise relative to the benchmarks. First, a *variety channel*: lower p_X reduces unit costs in B, raises output and income, and—through free entry—supports more firms and more varieties. More varieties lower the quality–adjusted price index P_B, a channel absent when B was traded at a fixed world price. Second, a *displacement channel*: once AI is adopted, all L_U is pushed into sector A. With Cobb–Douglas in A, this crowding reduces w_U through diminishing returns, an additional effect beyond the benchmark.

In equilibrium, a lower p_X reduces unit costs in B and raises variety. Under partial automation, the additional production in B pulls L_S out of A, which increases costs and thus prices in B. Under mild conditions, welfare rises relative to autarky, as the unit–cost and variety gains outweigh the rise in p_A. But unskilled workers' real wage typically falls, since the unskilled nominal wage is depressed by crowding in A while the CPI rises. Skilled workers may gain under partial automation (more demand for L_S in B), but under full automation, they too may lose as almost all labor is absorbed into A while gains leak abroad through p_X.

A small increase $dp_X > 0$ when the AI adoption constraint is slack. Suppose the adoption constraint is slack. Then, raising p_X increases unit costs in B and contracts its output. Because B has CES demand and free–entry, this also reduces the number of firms, raising P_B via the variety channel. At the same time, fewer firms in B release some L_S back to A, lowering p_A.

Unlike the benchmarks, these local changes do not induce substitution back to L_U in B: adoption is discrete, so a marginally higher p_X worsens P_B without undoing displacement. When the expenditure share α is moderate, the worsening of P_B and the fall in income dominate any decrease in p_A, so the CPI rises and welfare falls. In this case both w_S/P and w_U/P typically decline. In the Online Appendix, we analyze these comparative statics under CES production in sector A, showing how the incidence depends on the substitution parameter ρ. The outcomes are most stark when ρ is large (labor types highly substitutable): the decrease in p_A vanishes, and both real wages fall unconditionally. Conversely, when ρ is small (close to Leontief), unskilled labor can be partially cushioned or even gain, so the negative incidence is less certain. Thus, as long as α is not too small and A is not extremely unskilled-intensive, the local and global incidence can align negatively for at least one worker group, reproducing the "double harm" result highlighted in Models 1 and 2.

Two–part tariffs (ϕ, p_X). Allowing the foreign AI supplier to set both a per–unit price p_X and an access fee ϕ adds a second lever. A higher ϕ directly reduces entry in B, raising P_B, and, unlike F, ϕ's proceeds leak abroad, lowering domestic income.

Relative to the benchmark models without foreign ownership, the CES environment introduces two additional channels when the AI supplier raises p_X. First, higher unit costs and lower variety both raise P_B, strengthening the CPI channel. Second, because profits include foreign revenues, a new income–leakage effect arises: as p_X rises, domestic license rebates FN shrink, depressing Y. Together these channels imply the price index rises and real wages fall. Moreover, the loss of domestic license rebates increases inequality across workers: all lose in real terms, but the erosion of FN magnifies relative differences between skilled and unskilled labor.

Overall, the combination of discrete displacement, endogenous variety, and two–part tariffs makes it far easier for market power in AI supply to depress welfare, and it is more likely that the introduction of AI harms unskilled workers, and that local exercise of market power exacerbates the harm.

AI Monopolist Choice of Usage and Access Fees

Sector B Firm Profits. We can also examine the impact of access and usage fees on Sector B production and gross profits. Define the gross firm–side aggregate operating surplus before fees as follows (where we substitute in equilibrium conditions):

$$\mathcal{E}^{\text{gross}}(Z,\phi,p_X) \;\equiv\; (\mu-1)\, m_B^{*AI}(Z,\phi,p_X)\, Q_B^{*AI}(Z,\phi,p_X) = \frac{(\mu-1)\alpha}{\mu}\, Y^{*AI}(Z,\phi,p_X). \tag{9}$$

Note that AI access and usage fees impact this *only* through Y^{*AI}. If we think of

$$\mathcal{E}^{\text{gross}}(Z,\phi,p_X) - N^{*AI}F = \frac{(\mu-1)\alpha}{\mu}\, Y^{*AI}(Z,\phi,p_X)\Big(\frac{\phi}{F+\phi}\Big)$$

as the "size of the pie" to be extracted by the monopolist through access and usage fees, we can see that both ϕ and p_X affect the size of the pie through their effect on income. This contrasts with the typical nonlinear pricing problem from industrial organization, where the number of firms is fixed and there are no general equilibrium effects, so that access fees do not distort production while usage fees do. In the latter case, it is optimal for a monopolist to keep usage fees as low as possible and extract surplus using access fees; in contrast, in our model both fees increase the share of profits that go to the AI monopolist.

The slope of income with respect to ϕ is

$$\frac{\partial \mathcal{E}^{\text{gross}}}{\partial \phi}(Z,\phi,p_X) = \frac{(\mu-1)\alpha}{\mu}\,\frac{dY^{*AI}}{d\phi}(Z,\phi,p_X) \;<\; 0, \tag{10}$$

where $dY^{*AI}/d\phi$ incorporates a *direct* income effect, where ϕ affects the share of consumer expenditure retained by firms, and an *indirect* effect via the induced change in Q_B^{*AI}. In contrast, p_X has only an indirect effect on $\mathcal{E}^{\text{gross}}$, through wages and output that affect income.

Adoption frontier in equilibrium. To analyze strategic pricing by the monopolist, we develop the adoption frontier which characterizes the set of fees where AI adoption is an equilibrium. When rivals adopt, wages (w_S^{*AI}, w_U^{*AI}) fall for the displaced factor, so the baseline alternative improves; sustaining adoption therefore requires *lower* p_X or *lower* ϕ.

Consider one firm that *deviates* to the baseline (non-adopting) technology while all rivals keep adopting and charging $p_B^{*AI} = \mu\, m_B^{*AI}$. Let the deviator set the usual markup price $p_{\text{dev}} = \mu\, m_{\text{dev}}$, where its baseline marginal cost is evaluated at the AI equilibrium wages:

$$m_{\text{dev}}(Z,\phi,p_X) \;\equiv\; s_0\, w_S^{*AI}(Z,\phi,p_X) \;+\; u_0\, w_U^{*AI}(Z,\phi,p_X). \tag{11}$$

Under CES demand and substituting in equilibrium conditions, the deviator's quantity at (Z, ϕ, p_X) is

$$q_{\text{dev}}(Z, \phi, p_X) = \alpha\, Y^{*AI}\, (\mu\, m_{\text{dev}})^{-\sigma}\, (P_B^{*AI})^{\sigma-1} = \frac{\alpha}{\mu} \frac{Y^{*AI}}{m_{\text{dev}}} \left(\frac{m_B^{*AI}}{m_{\text{dev}}}\right)^{\sigma-1} \frac{1}{N^{*AI}}. \tag{12}$$

The deviator does not pay the AI access fee (it does not adopt), so its profit is

$$\pi_{\text{dev}}(Z, \phi, p_X) = (\mu - 1)\, m_{\text{dev}}\, q_{\text{dev}} - F = \frac{(\mu-1)\alpha}{\mu}\, Y^{*AI} \left(\frac{m_B^{*AI}}{m_{\text{dev}}}\right)^{\sigma-1} \frac{1}{N^{*AI}} - F. \tag{13}$$

Using (5), the no-deviation condition $\pi_{\text{dev}} \leq 0$ is equivalent to

$$\left(\frac{m_B^{*AI}}{m_{\text{dev}}}\right)^{\sigma-1} \leq \tfrac{F}{F+\phi} \iff p_X \leq m_{\text{dev}} \left(\tfrac{F}{F+\phi}\right)^{\frac{1}{\sigma-1}} - s_B w_S^{*AI}. \tag{14}$$

Usage fee. Consider linear pricing with $\phi = 0$, so the monopolist's revenue is

$$\Pi_{\text{lin}}(p_X) \;=\; p_X\, Q_B \;=\; \frac{\alpha}{\mu} \frac{Y}{m_B}\, p_X,$$

where $m_B = s_B w_S + p_X$ and $Q_B = (\alpha/\mu)(Y/m_B)$ in equilibrium. Differentiating shows that profits rise with p_X so long as the negative impact of p_X on income Y is not too strong relative to the positive cost-share transfer from skilled labor to the monopolist. In this case the profit function is increasing up to the adoption cap, so the optimal usage fee is set at the boundary.

Intuitively, when sector B still requires some skilled labor ($s_B > 0$), a higher p_X both raises m_B and shifts part of the cost burden away from domestic wages, increasing the monopolist's margin. The opposing force is the contraction of income Y, which lowers overall expenditure on B. Profits rise with p_X provided this income contraction is not too severe. This condition is more likely to hold when the expenditure share α on B is moderate, when skilled labor's cost share in m_B is sizable, and when demand for B is relatively elastic (so markups μ are modest). It is also easier to satisfy when technology in sector A allows factors to substitute smoothly: in that case the fall in skilled wages is cushioned, the rise in unskilled wages is limited, and the overall income decline is modest. By contrast, when technology in A is close to fixed-proportions, the rise in p_X depresses Y more strongly, making it harder for the cost-share transfer to dominate. Note, however, that at intermediate levels of ρ (the parameter governing factor substitution in Sector A), outcomes can be nonmonotone in ρ.

Access fee. Now consider the AI monopolist's choice of both usage and access fees,

$$\Pi^{\rm acc}(Z,\phi \mid p_X) \;\equiv\; \underbrace{\phi\, N^{*AI}(Z,\phi,p_X)}_{\text{access revenue}} \;+\; \underbrace{p_X\, Q_B^{*AI}(Z,\phi,p_X)}_{\text{usage revenue}}. \tag{15}$$

Using the CES/free–entry identities, differentiation gives

$$\frac{\partial \Pi^{\rm acc}}{\partial \phi} = N^{*AI}\left[\frac{F}{F+\phi} \;-\; \frac{\phi}{Y^{*AI}}\frac{dY^{*AI}}{d\phi} \;-\; \frac{p_X}{(\mu-1)\, m_B^{*AI}}\cdot\frac{F+\phi}{Q_B^{*AI}}\cdot\frac{dQ_B^{*AI}}{d\phi}\right].$$

The first term is strictly positive. The profit slope in ϕ reflects a direct positive channel and indirect negative channels through Y and Q_B. When adoption is slack, any interior optimum $\phi^\star(Z \mid p_X)$ requires these effects to balance exactly. If the adoption frontier is binding, the optimum lies on the boundary at the highest ϕ consistent with adoption.

The strength of the indirect terms depends on how easily factors can reallocate in A. When technology in A allows smooth substitution between skilled and unskilled labor, the contraction in Y from a higher ϕ is relatively muted. In this case, the direct access-revenue channel dominates, so interior optima are less likely and the monopolist tends to push ϕ to the frontier. By contrast, when A is closer to fixed proportions, the fall in income is sharper, the negative terms dominate sooner, and an interior optimum in ϕ is more plausible. At the extreme, with highly substitutable A, the indirect contraction is small enough that unskilled wages can still fall with ϕ, while with fixed-proportions A, unskilled wages may rise in nominal terms even as welfare declines. Thus, with smooth substitution in A the monopolist relies more on the per–unit fee, while with rigid A technology the fixed access fee is relatively more attractive to the monopolist. At intermediate levels of ρ, outcomes can be non-monotone in ρ.

Summary of results

Market power in AI depresses welfare and can harm both skilled and unskilled workers through several distinctive mechanisms:

(i) *No unskilled pull in B.* Adoption is discrete, so marginal fee changes do not restore unskilled demand in B; displaced labor must be absorbed by A.

(ii) *Two CPI channels.* The per–unit usage fee raises unit costs in B, while the fixed access fee reduces the number of active firms and thus variety. Both channels raise the sector-B price index and contribute to CPI inflation.

(iii) *Income leakage.* Both fees transfer income abroad, leading to declines in national income and a more concentrated domestic industry with less variety.

(iv) *Distributional impacts.* Unskilled nominal wages can rise with higher p_X, but real wages typically fall once CPI effects are considered. Access fees further magnify inequality by reducing entry and thus eroding domestic recirculation of firm fixed costs.

(v) *Labor productivity and substitutability in A.* With high substitution between skilled and unskilled labor in A, both groups lose in real terms; with fixed proportions, unskilled labor may gain nominally from higher access fees.

(vi) *Access and Usage Fees.* In general, the monopolist allocates extraction across fixed and variable fees, adjusting one upward if the other is capped, so effective regulation considers both.

A Policy-Relevant Research Agenda

The models developed above are deliberately stylized, but we argue that they help identify and prioritize open questions for research and considerations for policy-makers. First, in the models, broad welfare gains require that *AI prices fall with real cost savings* (either through competition enforcement or regulation of p_X and ϕ) *and* that displaced labor be *absorbed productively* in sectors less affected by AI. Second, we show that policies that only cap one instrument risk rent-shifting into the other; disciplining both levers is necessary when the upstream supplier prices on the adoption frontier.

What policies increase competition? In some cases, multiple global providers may exist, but not all make investments in customizing and distributing to small countries, reducing local competition and leaving a role for industrial policy. The AI stack spans many layers, including chips, training, models, and applications. A single bottleneck can lead to high downstream prices. Where bottlenecks are likely, how pricing at each layer translates into end-user prices and pricing structure, and which policy tools best counteract market power are all important research topics. The presence of open models, competitive entrants, or fast followers might discipline incumbents and reduce the scope for sustained market power.

From a country's perspective, a key factor is where AI value added accrues. If domestic firms earn the profits and those profits recirculate broadly in the economy, they raise national income and help sustain demand for nontradables. Future research might consider what policies enable a country to participate in the high-value-added parts of the AI stack. Competition policy and industrial policy might be able to improve a country's bargaining leverage with outside suppliers. For example, a country may be able to control access to data or financial systems. It might consider whether open models (such as Meta's Llama) should be regulated for national security reasons (though the arrival of DeepSeek demonstrated that it may be difficult to keep countries from fast-following others' innovations), considering the role that open source may play in constraining AI prices and promoting competition in AI services. Outcomes may depend on where a country's local assets (e.g., data, finance, compute, talent) create favorable outside options.

From a modeling perspective, the simple structure of CES/HSA demand, a monolithic AI industry, and two sectors could be generalized, yielding more nuanced insights. For example, a more general model might include multiple sectors and illustrate the role that government procurement of services plays in stimulating labor demand, in combination with the role of labor-augmenting technologies, as well as in improving consumer welfare. AI may also affect entrepreneurship and idea generation. A long tradition in industrial organization (see, e.g. Lee, Whinston, and Yurukoglu (2021) for a recent survey) and a recent literature in trade (e.g. Grossman, Helpman, and Sabal (2024)) microfound markups in supply chains, and the latter explores general equilibrium implications.

Another risk is capability loss: if adoption displaces domestic production that is costly to rebuild (learning-by-doing, organizational capital, supply-chain agglomeration), dependence on external suppliers increases. A country's outside option, and therefore ability to bargain over and regulate AI import prices, may deteriorate. That raises more industrial-policy questions: how should local capabilities be maintained, and how should any short-run efficiency gains be weighed against longer-run resilience and expertise? Closely related is the risk that a dominant AI provider falls under control of a hostile trading partner, so that objectives other than profit—e.g., degrading local capabilities—become salient.

For workers, the key question is whether displaced labor can be reallocated productively. Policy levers include education and training, procurement of nontraded services (teaching, nursing, care), and targeted subsidies that raise productivity where displaced workers are absorbed. AI assistants may ease both transitions and on-the-job productivity. The political economy around these large fiscal and operational decisions for government will have a big impact on outcomes such as who pays, how efficient redistribution is, and whether AI owners shape policy to avoid bearing social costs. Concentrated ownership, even by domestic firms, may increase their political power and make taxation more difficult.

In standard models, if AI-enabled industries are exportable, higher productivity and increased output mitigate the decreases in per-unit labor demand. However, this result depends on the ability to increase exports at similar prices, when in practice world-wide adoption of AI may compress prices and limit export-led adjustment. Some traded services (e.g., callcenter outsourcing) may shrink materially. The implications of AI for developing countries' competitiveness and the global distribution of production are critical to understand.

The interaction between AI and innovation itself also opens a rich research agenda. On the one hand, AI can erode market power in existing downstream industries by lowering costs and enabling entry. On the other hand, our analysis shows that if fixed costs rise, industry structure may become more concentrated, reducing the variety available to end consumers and slowing innovation. From the perspective of AI innovation, if monopoly rents are the primary reward for investing in frontier AI, restricting those rents too aggressively might dull incentives for innovation. Understanding how different forms of pricing, competition, and regulation shape innovation incentives in AI, and how this interacts with broader patterns of technological progress, remains a pressing challenge.

Conclusions

This paper argues that the payoff from preserving competition in AI has been underappreciated in macroeconomic discussions. Growth models and popular narratives often assume that AI will deliver cheap goods and services in abundance. Our observation is that this outcome is unlikely in the absence of competition, particularly if the profits from the technology accrue outside a country or are not shared throughout the economy. Instead, we show how market power in AI factors of production allows the provider to extract rents and prevent prices from falling to match declines in labor. A profit-maximizing downstream firm may be just indifferent about adopting AI, but the decision redistributes rents outside the country and away from displaced workers within the country. Because AI is a general-purpose upstream technology, monopoly harms include increased concentration and higher prices within downstream sectors but extend well beyond directly impacted sectors, justifying a general equilibrium framework. Our models demonstrate that the welfare impact of AI depends critically on market structure; they further highlight the important role of worker transitions and their unique value and productivity in alternative sectors.

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ONLINE APPENDIX

Artificial Intelligence, Competition, and Welfare

Susan Athey and Fiona Scott Morton

1 Benchmark: Open Economy, Two Traded Goods

This is a baseline model with two traded goods, a small open economy where consumption is not modeled.

1.1 Setup, accounting, and objects

Goods, prices, and factors. The economy consists of two traded final goods A, B sold at exogenous world prices $(\bar{p}_A, \bar{p}_B)$ (small open economy) and produced with two primary factors: skilled labor S and unskilled labor U, with endowments (L_S, L_U) and factor prices (w_S, w_U). A foreign input X is used *only* by sector B at price p_X set abroad. Technologies are CRS; markets are competitive.

Unit costs, unit input coefficients, and zero profit. Let $c_A(w_S, w_U)$ and $c_B(w_S, w_U, p_X)$ be unit-cost functions. Zero-profit (unit-cost) equalities:

$$\bar{p}_A = c_A(w_S, w_U), \qquad \bar{p}_B = c_B(w_S, w_U, p_X). \tag{ZP}$$

Define *unit input coefficients* (Hicksian, i.e., per unit of output):

$$a_S^A := \frac{\partial c_A}{\partial w_S}, \quad a_U^A := \frac{\partial c_A}{\partial w_U}; \qquad a_S^B := \frac{\partial c_B}{\partial w_S}, \quad a_U^B := \frac{\partial c_B}{\partial w_U}, \quad a_X^B := \frac{\partial c_B}{\partial p_X}.$$

Define cost shares: $\theta_S^j = \frac{w_S a_S^j}{\bar{p}_j}$, $\theta_U^j = \frac{w_U a_U^j}{\bar{p}_j}$, and in B also $\theta_X^B = \frac{p_X a_X^B}{\bar{p}_B}$, with $\theta_S^A + \theta_U^A = 1$ and $\theta_S^B + \theta_U^B + \theta_X^B = 1$.

Full employment.

$$a_S^A Q_A + a_S^B Q_B = L_S, \qquad a_U^A Q_A + a_U^B Q_B = L_U. \tag{FE}$$

Imports and national income (GNP at world prices). The quantity of X imported is $M_X = a_X^B Q_B$; the import bill is $p_X M_X$.

$$Y \;=\; \bar{p}_A Q_A + \bar{p}_B Q_B \;-\; p_X M_X \;=\; w_S L_S + w_U L_U. \tag{NI}$$

The first equality is value-added (final output value minus imported intermediates); the second uses zero profits in A, B. *Envelope in p_X:*

$$\frac{dY}{dp_X} \;=\; -\,M_X \;\le 0. \tag{Y–p_X}$$

Hence a fall in p_X raises national income (aggregate welfare at traded-goods prices), while a rise lowers it.

Trade and consumption. We do not model home demand; production and factor prices are determined by (ZP)–(FE). The trade-balance identity is $\bar{p}_A(C_A - Q_A) + \bar{p}_B(C_B - Q_B) + p_X M_X = 0$; the country can export B ($Q_B > C_B$). Welfare statements use Y (equivalently, any homothetic utility at prices $(\bar{p}_A, \bar{p}_B)$).

1.2 Comparative statics

The relative factor intensities and sign patterns for wage changes are stated in the main text. For completeness, define the *relative factor-intensity indices*:

$$R_A := \frac{\theta_S^A}{\theta_U^A}, \qquad R_B(p_X) := \frac{\theta_S^B}{\theta_U^B}.$$

We say B is *skill-intensive (relative to A)* if $R_B(p_X) > R_A$, and *unskilled-intensive* if $R_B(p_X) < R_A$. Totally differentiating the zero-profit conditions at fixed $(\bar{p}_A, \bar{p}_B)$ yields the sign pattern

$$\begin{cases} R_B(p_X) > R_A \text{ (B skill-intensive)} \;\Rightarrow\; \dfrac{dw_S}{dp_X} < 0,\; \dfrac{dw_U}{dp_X} > 0, \\ R_B(p_X) < R_A \text{ (B unskilled-intensive)} \Rightarrow\; \dfrac{dw_S}{dp_X} > 0,\; \dfrac{dw_U}{dp_X} < 0. \end{cases} \tag{Signs}$$

Aggregate income moves monotonically: $dp_X \downarrow$ raises Y, $dp_X \uparrow$ lowers Y.

Hicksian substitution. We say X is a Hicksian substitute for U in B if the Hicksian unit demand satisfies

$$\frac{\partial a_U^B(w_S, w_U, p_X)}{\partial p_X} > 0.$$

This ensures that when p_X rises, demand for unskilled labor a_U^B also rises. We do not assume smoothness at technique switches, if the production function allows firms to select among different techniques with different unit costs; at the point of a switch, a_U^B may jump upward.

Single intensity reversal. A *single intensity reversal* occurs at some $p_X^* \in (0, \infty)$ if

$$R_B(p_X) \begin{cases} < R_A, & p_X > p_X^*, \\ = R_A, & p_X = p_X^*, \\ > R_A, & p_X < p_X^*. \end{cases} \tag{Rev}$$

1.3 The introduction of AI and divergence near a reversal

Fix a low input price p_X^ℓ and compare to autarky in X ($p_X = \infty$). The *level* change in the unskilled wage is

$$w_U(p_X^\ell) - w_U(\infty) \;=\; -\int_{p_X^\ell}^{\infty} \frac{dw_U}{dp_X}\, dp_X,$$

where the integrand's sign at each p_X is given by (Signs). The *local* effect at p_X^ℓ is sign $\left(\frac{dw_U}{dp_X}\Big|_{p_X^\ell}\right)$ and depends only on the current ordering $R_B(p_X^\ell)$ vs. R_A.

Proposition 1 (Single intensity reversal)**.** *Suppose p_X^* satisfies* (Rev). *Then in neighborhoods of p_X^*:*

- *If $p_X^\ell \downarrow p_X^*$ from above (so $R_B(p_X^\ell) < R_A$), then*

  $$w_U(p_X^\ell) - w_U(\infty) \;>\; 0 \quad \textit{and} \quad \frac{dw_U}{dp_X}\Big|_{p_X^\ell} \;<\; 0.$$

  Thus introducing a cheap X is good *for unskilled, and a marginal increase in p_X at that point is* bad *for unskilled.*

- *If $p_X^\ell \uparrow p_X^*$ from below (so $R_B(p_X^\ell) > R_A$), then $w_U(p_X^\ell) - w_U(\infty) > 0$ while $\frac{dw_U}{dp_X}\Big|_{p_X^\ell} > 0$: opening to cheap X is* good *for unskilled, and a marginal increase in p_X is* good *for them. That implies that the opening is* bad *for skilled, and a marginal increase in p_X is also bad for skilled workers.*

In all cases, aggregate income moves with p_X by (Y–p_X).

Proof. By (Signs), $\operatorname{sign}(dw_U/dp_X) = \operatorname{sign}(R_B - R_A)$ in a neighborhood of p_X^*. If $p_X^\ell \downarrow p_X^*$ from above, then $R_B < R_A$ so $dw_U/dp_X < 0$ locally, while the level change satisfies $w_U(p_X^\ell) - w_U(\infty) = \int_\infty^{p_X^\ell} (dw_U/dp_X)\, dp_X > 0$ because the integrand is positive on most of the path before the crossing. The case approaching from below is analogous with $R_B > R_A$. $\square$

Proposition 2 (Double-harm in the all-traded model with non-monotone skill intensities)**.** *Suppose there exist thresholds $p_X^{(1)} > p_X^{(2)} > 0$ such that a double-reversal holds*

$$R_B(p_X) \begin{cases} < \ R_A, & p_X > p_X^{(1)} \qquad \textit{(very high } p_X\textit{: B unskilled-intensive),} \\ > \ R_A, & p_X^{(2)} < p_X < p_X^{(1)} \qquad \textit{(intermediate } p_X\textit{: B skill-intensive),} \\ < \ R_A, & p_X < p_X^{(2)} \qquad \textit{(very low } p_X\textit{: B unskilled-intensive again).} \end{cases} \tag{TwoX}$$

Then it is possible that for $p_X^\ell \in (0, p_X^{(2)})$ sufficiently close to $p_X^{(2)}$,

$$\underbrace{w_U(p_X^\ell) - w_U(\infty)}_{\textit{level vs. autarky}} \ < \ 0 \qquad \textit{and} \qquad \underbrace{\left.\frac{dw_U}{dp_X}\right|_{p_X^\ell}}_{\textit{local at } p_X^\ell} \ < \ 0.$$

Hence introducing (very) cheap X makes unskilled worse off in levels and *a marginal increase in p_X at p_X^ℓ also makes them worse off locally.*

Sketch. By (Signs), $\operatorname{sign}\left(\frac{dw_U}{dp_X}\right)$ equals the sign of $R_B(p_X) - R_A$. Under (TwoX), the derivative is positive on the intermediate band $(p_X^{(2)}, p_X^{(1)})$ and negative on the tails $(p_X^{(1)}, \infty)$ and $(0, p_X^{(2)})$. The level change is the path integral $w_U(p_X^\ell) - w_U(\infty) = \int_\infty^{p_X^\ell} \frac{dw_U}{dp_X}\, dp_X$, whose sign is the net of those regions. For p_X^ℓ close to $p_X^{(2)}$, the (large) positive-derivative region in $(p_X^{(2)}, p_X^{(1)})$ dominates, making the integral negative; at $p_X^\ell < p_X^{(2)}$, we have $R_B < R_A$ so the local derivative is negative. $\square$

Example of Double Reversal We provide an example demonstrating that the conditions of Proposition 2 are feasible.

For production functions, we let Sector A be CRS and let B choose among three techniques T_k, $k \in \{1, 2, 3\}$: (i) T_1 uses no AI and is unskilled-intensive; (ii) T_2 uses moderate AI and is skill-intensive; (iii) T_3 uses heavy AI and reverts to unskilled-intensive. Standard duality results imply the equilibrium wage ratio $\omega(p_X) := w_S/w_U$ lies in a compact interval as p_X varies. The technique-switch prices $p_{12}(\omega)$ and $p_{23}(\omega)$ are determined by cost equalization; by appropriate choice of AI input coefficients, we can ensure $p_{12}(\omega) > p_{23}(\omega)$ throughout. As p_X falls from ∞, firms switch from T_1 to T_2 (B becomes skill-intensive) and then to T_3 (B becomes unskilled-intensive again), generating the double crossing in Figure 1.

Numerical example. With $\Omega = [0.8, 1.5]$, Leontief A with $(a_S^A, a_U^A) = (0.50, 0.45)$, and techniques $T_1 : (0.40, 1.20, 0)$, $T_2 : (1.00, 0.40, 0.60)$, $T_3 : (0.15, 0.45, 3.50)$, the intensity ordering $R_B^{(1)} < R_A < R_B^{(2)}$ and again $R_B^{(3)} < R_A$ holds on Ω, with switch prices $p_{12} \approx 0.333 > p_{23} \approx 0.276$ at unit wages. Proposition 2 then applies.

1.4 Output responses in the general comparative statics of usage fees

Stack the full-employment system (FE) as $AY = \bar{F}$ with

$$A := \begin{bmatrix} a_S^A & a_S^B \\ a_U^A & a_U^B \end{bmatrix}, \qquad Y := \begin{bmatrix} Q_A \\ Q_B \end{bmatrix}, \qquad \bar{F} := \begin{bmatrix} L_S \\ L_U \end{bmatrix}.$$

Totally differentiate at fixed endowments:

$$A\,dY \;=\; -\,(dA)\,Y \quad\Longrightarrow\quad dY \;=\; -\,A^{-1}(dA)\,Y. \tag{Mix}$$

With $A^{-1} = \frac{1}{\Delta}\begin{bmatrix} a_U^B & -a_S^B \\ -a_U^A & a_S^A \end{bmatrix}$ where $\Delta := a_S^A a_U^B - a_S^B a_U^A > 0$ (distinct factor intensities), the B-output response is

$$dQ_B \;=\; \frac{1}{\Delta}\Big\{\, Q_A\big(a_U^A\,da_S^A - a_S^A\,da_U^A\big) \;+\; Q_B\big(a_U^A\,da_S^B - a_S^A\,da_U^B\big) \,\Big\}. \tag{$\star$}$$

Here each da_i^j is a Hicksian coefficient response induced by the endogenous wage changes from (Signs) and the direct p_X-effect in B.

What can be signed in general. Let $R_B \lesseqgtr R_A$ denote the local intensity ordering. Under standard Hicksian regularity (negative own-price effects, symmetric substitution) and mild cross-substitution assumptions, we obtain:

Proposition 3 (Output of B as p_X changes)**.** *Fix a point p_X and suppose: (i) B is unskilled-intensive locally, $R_B < R_A$; (ii) X is a (weak) Hicksian substitute for U in B, so $\partial a_U^B/\partial p_X > 0$ (equivalently, lowering p_X reduces a_U^B); (iii) own-price effects are negative for A and B (e.g., $\partial a_U^A/\partial w_U < 0$, $\partial a_S^A/\partial w_S < 0$). Then, for a small change in p_X,*

$$\frac{dQ_B}{dp_X} \;<\; 0.$$

If instead $R_B > R_A$ (skill-intensive B), the sign of $\frac{dQ_B}{dp_X}$ is a priori ambiguous *without further* *structure (e.g., on $\partial a_S^B/\partial p_X$ and cross-substitution magnitudes).*

Proof sketch. Plug the wage responses from (Signs) (for $R_B < R_A$, we have $\frac{dw_U}{dp_X} < 0$, $dw_S/dp_X > 0$) into the Hicksian responses da_i^j and evaluate ($\star$). Under (iii), $da_U^A/dp_X > 0$ and $da_S^A/dp_X < 0$ via the induced wage changes. Under (ii), $da_U^B/dp_X > 0$; da_S^B/dp_X is not needed to be signed if it does not overturn the two summands in ($\star$). Because $a_U^A/a_S^A < R_A^{-1} < R_B^{-1} = a_U^B/a_S^B$ (unskilled intensity of B), the bracketed terms deliver a negative total in ($\star$). □

Interpretation. When B is unskilled-intensive, a rise in p_X makes B less competitive and, via (Signs), lowers w_U and raises w_S. Hicksian coefficients then adjust so that each unit uses *more* U in B and (through wages) more U in A as well; to clear factor markets at fixed endowments, the output mix must tilt *away* from B (hence $dQ_B/dp_X < 0$). Conversely, for $dp_X < 0$, Q_B expands ($dQ_B > 0$). If B is skill-intensive, the induced coefficient changes pull in opposing directions and the sign becomes model-specific.

2 Benchmark Model with One Nontraded Good

We now make sector A nontraded and close the model with Cobb-Douglas preferences over (A, B). This endogenizes the domestic relative price $p_A/\bar{p}_B$ and introduces a demand/price-index channel that interacts with the income and output-mix effects analyzed above.

2.1 Setup and market closure

Preferences and demand. A representative household has utility

$$U(C_A, C_B) = C_A^{1-\alpha} C_B^{\alpha}, \qquad \alpha \in (0, 1),$$

facing the price vector $(p_A, \bar{p}_B)$. Given nominal income Y, Cobb-Douglas demand implies

$$C_A = \frac{(1-\alpha)Y}{p_A}, \qquad C_B = \frac{\alpha Y}{\bar{p}_B}.$$

The exact consumption price index is $P = p_A^{1-\alpha} \bar{p}_B^{\alpha}$.

Nontraded market clearing. Because A is nontraded, $C_A = Q_A$. Hence the equilibrium price of the nontraded good is

$$p_A = \frac{(1-\alpha)\,Y}{Q_A} \qquad \Longleftrightarrow \qquad d\ln p_A = d\ln Y - d\ln Q_A. \tag{NT-P}$$

Intuitively, demand pressure from higher income ($Y\uparrow$) and production scarcity in A ($Q_A\downarrow$) both bid up p_A.

2.2 Equilibrium

Indirect utility and a two-channel welfare map. With CD, indirect utility satisfies

$$\ln U = \text{const} + \ln Y - (1-\alpha)\ln p_A - \alpha \ln \bar{p}_B.$$

Using (NT-P), the welfare differential is

$$d\ln U = \alpha\, d\ln Y + (1-\alpha)\, d\ln Q_A. \tag{W}$$

Let M_X denote domestic spending on input X (Hicksian demand times price). Then the envelope theorem gives $dY/dp_X = -M_X$.

For upstream shocks to p_X,

$$\frac{d\ln U}{dp_X} = -\alpha\frac{M_X}{Y} \; + \; (1-\alpha)\frac{1}{Q_A}\frac{dQ_A}{dp_X}, \tag{W–p_X}$$

combining the effect on income (applying the envelope theorem, $dY/dp_X = -M_X$) with the output response of A.

Real wages. Let $i \in \{S, U\}$. The real consumption wage is w_i/P. Because $d\ln P = (1-\alpha)\,d\ln p_A = (1-\alpha)(d\ln Y - d\ln Q_A)$,

$$d\ln\left(\tfrac{w_i}{P}\right) = d\ln w_i - (1-\alpha)\,d\ln p_A = d\ln w_i - (1-\alpha)\,(d\ln Y - d\ln Q_A). \tag{RW}$$

Thus nominal Stolper–Samuelson–type changes from (Signs) are channeled through a cost-of-living term driven by the nontraded price.

Outputs and factor markets. Production, factor prices, and Hicksian coefficients continue to be determined by (ZP)–(FE). Using the standard full-employment mix mapping,

$$dY = -A^{-1}(dA)Y, \qquad dQ_B \text{ as in } (\star),$$

while dQ_A follows from factor balance and the Hicksian responses. Signs can thus be tracked using the same local intensity and substitution conditions as in Prop. 3.

2.3 Comparative statics: entry, reversals, and incidence

We interpret "entry" as a drop in the foreign upstream price p_X to a low level due to the arrival of an efficient supplier.

Low-price entry. Under the conditions of Proposition 3 with $R_B < R_A$ (sector B locally unskilled-intensive) and X a Hicksian substitute for U in B:

$$dp_X < 0 \;\Rightarrow\; dY > 0, \quad dQ_B > 0, \quad dQ_A < 0, \quad dp_A > 0.$$

Welfare then moves according to (W–p_X): the income gain $\alpha(-M_X/Y) > 0$ is partially offset by the nontraded scarcity channel $(1-\alpha)\, d\ln Q_A < 0$ (a "Dutch disease" effect).

Real-wage dominance of the cost-of-living channel. Nominally, at fixed traded-good prices one wage rises and the other falls (by (Signs)). With a nontraded good, however, *both* real wages can decline locally if the cost-of-living effect dominates:

$$\frac{d}{dp_X}\ln\!\left(\frac{w_S}{P}\right) \;>\; 0, \qquad \frac{d}{dp_X}\ln\!\left(\frac{w_U}{P}\right) \;>\; 0, \tag{BH}$$

that is, as the upstream price p_X falls (AI becomes cheaper), both w_S/P and w_U/P fall. Sufficient conditions include a large α and a sharp contraction of Q_A (hence a strong rise in p_A) relative to the nominal gain of the winning factor.

Reversals at very low entry prices. Two reversals are possible as p_X falls:

1. An *intensity reversal* $R_B(p_X)$ crossing R_A (as in (Rev)), flipping the *nominal* incidence in (Signs).

2. A *welfare reversal* from the nontraded channel: even though $dY/dp_X = -M_X < 0$ everywhere, the sign of $d\ln U/dp_X$ in (W–p_X) can reverse depending on dQ_A/dp_X.

Proposition 4 (Effects of changes in p_X). *1. $dY/dp_X = -M_X < 0$.*

2. $d\ln p_A/dp_X = d\ln Y/dp_X - d\ln Q_A/dp_X$.

3. If $R_B < R_A$ and X (weakly) substitutes for U in B (so $dQ_B/dp_X < 0$ by Prop. 3), then factor reallocation raises Q_A, i.e. $dQ_A/dp_X > 0$. Therefore

$$\frac{d\ln p_A}{dp_X} \;<\; 0, \qquad \frac{d\ln P}{dp_X} = (1-\alpha)\,\frac{d\ln p_A}{dp_X} \;<\; 0.$$

4. Consequently, the welfare effect decomposes as

$$\frac{d\ln U}{dp_X} = -\alpha\frac{M_X}{Y} \;+\; \underbrace{(1-\alpha)\frac{1}{Q_A}\frac{dQ_A}{dp_X}}_{\text{cost-of-living}} \quad\in\; \Big(-\tfrac{M_X}{Y},\;0\Big).$$

That is, the income contraction is partially cushioned *by cheaper nontradables.*

If instead $R_B > R_A$, the sign of dQ_A/dp_X is ambiguous and so is the direction of dp_A; the cushioning may vanish or become amplification.

Corollary 1 (Both workers can be hurt locally). *Under $R_B < R_A$ and weak substitution, if*

$$(1-\alpha)\Big(\Big|\tfrac{d\ln Y}{d\ln p_X}\Big| + \Big|\tfrac{d\ln Q_A}{d\ln p_X}\Big|\Big) \;>\; \max\Big\{\Big|\tfrac{d\ln w_S}{d\ln p_X}\Big|,\;\Big|\tfrac{d\ln w_U}{d\ln p_X}\Big|\Big\},$$

then $\frac{d}{dp_X}\ln\big(\frac{w_S}{P}\big) > 0$ and $\frac{d}{dp_X}\ln\big(\frac{w_U}{P}\big) > 0$, i.e. a local decrease in p_X lowers both real wages.

Remark. The two elasticities $\frac{M_X}{Y}$ and $\frac{1}{Q_A}\frac{dQ_A}{dp_X}$ are sufficient statistics for the sign and size of $d\ln U/dp_X$ in (W–p_X). Likewise, $(1-\alpha)(d\ln Y - d\ln Q_A)$ governs whether both real wages fall in (RW)–(BH).

Equation (16) decomposes *levels* from autarky to p_X^ℓ, while (18) characterizes the *local* derivative at p_X^ℓ. We use these with Propositions 4–5 to isolate when CPI effects dominate.

Proposition 5 (Local–level divergence with a nontraded good: sufficient-statistic conditions). *Suppose p_X^* satisfies an intensity reversal* (Rev), *and let U denote indirect utility under the Cobb-Douglas closure* (W). *Fix p_X^ℓ near p_X^*.*

1. *If $p_X^\ell \downarrow p_X^*$ from above (so $R_B(p_X^\ell) < R_A$), and along the autarky $\to p_X^\ell$ path the* level CPI dominance *condition holds*

$$(1-\alpha)\,\big(\Delta\ln Y - \Delta\ln Q_A\big) \;>\; \Delta\ln w_U,$$

while at p_X^ℓ wage effect dominance *holds*

$$\left|\tfrac{d\ln w_U}{dp_X}\right| \;>\; (1-\alpha)\left|\tfrac{d\ln p_A}{dp_X}\right|,$$

then the unskilled real wage satisfies

$$\frac{w_U(p_X^\ell)}{P(p_X^\ell)} \;<\; \frac{w_U(\infty)}{P(\infty)} \qquad \textit{and} \qquad \frac{d}{dp_X}\left(\frac{w_U}{P}\right)\Big|_{p_X^\ell} \;<\; 0.$$

2. *If* $p_X^\ell \uparrow p_X^*$ *from below (so* $R_B(p_X^\ell) > R_A$*), and the analogous* level CPI dominance *and* wage effect dominance *conditions hold for the skilled group,*

$$(1-\alpha)\big(\Delta\ln Y - \Delta\ln Q_A\big) \;>\; \Delta\ln w_S, \qquad \left|\tfrac{d\ln w_S}{dp_X}\right| \;>\; (1-\alpha)\left|\tfrac{d\ln p_A}{dp_X}\right|,$$

then

$$\frac{w_S(p_X^\ell)}{P(p_X^\ell)} \;<\; \frac{w_S(\infty)}{P(\infty)} \qquad \textit{and} \qquad \frac{d}{dp_X}\left(\frac{w_S}{P}\right)\Big|_{p_X^\ell} \;<\; 0.$$

Moreover, under the primitives in Proposition 4 with $R_B < R_A$ *and weak Hicksian substitution (so* $dQ_A/dp_X > 0$*), the level dominance* (17) *holds whenever the CPI channel is sufficiently strong along the autarky*$\rightarrow p_X^\ell$ *path (large* α *and elastic* p_A*), and the local condition* (19) *follows from the one-group specialization of* (BH)*. Thus the conditions are implied by parameter regions already characterized in this section.*

Proof. From (RW) and (NT-P), we have

$$d\ln\!\left(\tfrac{w_i}{P}\right) = d\ln w_i - (1-\alpha)\big(d\ln Y - d\ln Q_A\big).$$

Integrating from autarky $p_X = \infty$ to p_X^ℓ delivers the level decomposition

$$\Delta\ln\!\left(\tfrac{w_U}{P}\right) = \Delta\ln w_U - (1-\alpha)\big(\Delta\ln Y - \Delta\ln Q_A\big). \tag{16}$$

A sufficient condition for the real wage to fall in levels even when the nominal level rises ($\Delta\ln w_U > 0$) is

$$(1-\alpha)\big(\Delta\ln Y - \Delta\ln Q_A\big) \;>\; \Delta\ln w_U, \tag{17}$$

which ties directly to (W–p_X) and Proposition 4: along the entry path ($dp_X < 0$) we typically have $\Delta\ln Y > 0$ by the envelope theorem and $\Delta\ln Q_A < 0$ when $R_B < R_A$, so the CPI term can dominate.

For the local effect at p_X^ℓ with $R_B(p_X^\ell) < R_A$, (Signs) implies $d\ln w_U/dp_X < 0$, and

Proposition 4 implies $d\ln p_A/dp_X < 0$. Thus

$$\frac{d}{dp_X}\ln\Big(\tfrac{w_U}{P}\Big) = \frac{d\ln w_U}{dp_X} - (1-\alpha)\frac{d\ln p_A}{dp_X}, \tag{18}$$

which is negative whenever

$$\Big|\tfrac{d\ln w_U}{dp_X}\Big| \;>\; (1-\alpha)\,\Big|\tfrac{d\ln p_A}{dp_X}\Big|. \tag{19}$$

Condition (19) is the one-group specialization of the "both types of labor hurt" condition (BH). The case with $R_B > R_A$ follows for skilled workers by symmetry. □

Lemma 1 (Crossing identities: CES in A, Leontief in B)**.** *Assume sector A has CES unit cost $c_A(w_S, w_U) = \big[\beta\, w_S^{1-\rho} + (1-\beta)\, w_U^{1-\rho}\big]^{\frac{1}{1-\rho}}$ with $\beta \in (0,1)$, $\rho > 0$, and sector B (traded) is Leontief with $c_B(w_S, w_U, p_X) = a_S^B w_S + a_U^B w_U + a_X^B p_X$, $(a_S^B, a_U^B, a_X^B) > 0$. Let $\tau := \frac{a_S^B}{a_U^B}$ and $r^* := \frac{1}{\tau}$. At the unique intensity crossing p_X^* where $R_B = R_A$, the wage ratio $\omega^* := w_S/w_U$ solves*

$$\frac{\theta_S(\omega)}{\theta_U(\omega)} = \omega\,\tau, \qquad \theta_S(\omega) = \frac{\beta\,\omega^{\,1-\rho}}{\beta\,\omega^{\,1-\rho} + (1-\beta)}, \quad \theta_U(\omega) = 1 - \theta_S(\omega).$$

Equivalently,

$$\omega^* = \Big[\tfrac{\beta}{\tau(1-\beta)}\Big]^{1/\rho}, \quad k := \frac{\bar\theta_S}{\bar\theta_U} = \tau^{\,1-\frac{1}{\rho}}\left(\frac{\beta}{1-\beta}\right)^{\frac{1}{\rho}}, \quad \bar\theta_S = \frac{k}{1+k}, \quad \bar\theta_U = \frac{1}{1+k}.$$

Lemma 2 (Thresholds at the crossing)**.** *Maintain the definitions and setup from Lemma 1 and let $\alpha \in (0,1)$. The condition*

$$(1-\alpha)\big(\bar\theta_U + \bar\theta_S\, r^*\big) < 1$$

holds if and only if

$$k\Big(\frac{1-\alpha}{\tau} - 1\Big) < \alpha, \qquad k = \tau^{\,1-\frac{1}{\rho}}\left(\frac{\beta}{1-\beta}\right)^{\frac{1}{\rho}}.$$

In particular:

(a) If $\tau \geq 1-\alpha$, the inequality holds automatically.

(b) If $\tau < 1-\alpha$, it suffices that

$$\beta \;<\; \frac{\alpha^\rho\,\tau}{\alpha^\rho\,\tau + (1-\alpha-\tau)^\rho}.$$

Corollary 2 (Both workers hurt locally at the crossing)**.** *Assume CES in A and Leontief in B as in Lemma 1, and* R_B *decreasing in* p_X *with a unique crossing* p_X^**. Fix any* $p_X' \in (0, p_X^*)$ *(approaching from the left, so* $R_B < R_A$*). Let* $(\bar\theta_S, \bar\theta_U)$ *and* r^* *be as in Lemma 1. If*

$$r^* \ > \ \max\left\{ \frac{1-(1-\alpha)\bar\theta_U}{(1-\alpha)\bar\theta_S} \ , \ \frac{(1-\alpha)\bar\theta_U}{1-(1-\alpha)\bar\theta_S} \right\}, \tag{20}$$

then,

$$\frac{d}{dp_X} \ln\!\left(\tfrac{w_U}{P}\right)\Big|_{(p_X')^-} < 0 \qquad \textit{and} \qquad \frac{d}{dp_X} \ln\!\left(\tfrac{w_S}{P}\right)\Big|_{(p_X')^-} < 0.$$

Moreover, (20) *is implied by the primitive thresholds in Lemma 2 together with*

$$r^* > \frac{(1-\alpha)\bar\theta_U}{1-(1-\alpha)\bar\theta_S} \iff 1 + \alpha\,\frac{\bar\theta_S}{\bar\theta_U} \ > \ \frac{1}{r^*}(1-\alpha) \iff k \ > \ \frac{\tau(1-\alpha)-1}{\alpha},$$

which holds automatically if $\tau(1-\alpha) \le 1$*, and otherwise is equivalent to*

$$\beta \ > \ \frac{\big(\tau(1-\alpha)-1\big)^{\rho}\,\tau^{\,1-\rho}}{\alpha^{\rho} + \big(\tau(1-\alpha)-1\big)^{\rho}\,\tau^{\,1-\rho}}.$$

Proposition 6 (Double harm with a single reversal)**.** *Assume the setup of Lemma 1. Let* p_X^* *be the unique crossing and pick any* $p_X' \in (0, p_X^*)$ *with*

$$\Delta \ln\!\left(\tfrac{w_U}{P}\right)\Big|_{\infty \to p_X'} < 0.$$

If the following condition from Lemma 2 holds:

$$(1-\alpha)\big(\bar\theta_U + \bar\theta_S\, r^*\big) \ < \ 1, \tag{21}$$

then

$$\frac{d}{dp_X} \ln\!\left(\tfrac{w_U}{P}\right)\Big|_{(p_X')^-} < 0.$$

At a point where unskilled are already worse off in levels relative to autarky, a marginal increase in p_X *(from the left toward the crossing) makes them even worse off locally. Condition* (21) *is implied by the primitive thresholds in Lemma 2.*

3 Main Model Analysis

This section focuses on the main model outlined in the text, specifically considering the case of CES demand inside sector B. We derive equilibrium identities and comparative statics.

Unless otherwise noted, all objects are equilibrium functions of (ϕ, p_X) in the AI regime.

Let $Z = (\alpha, F, s_B, L_S, L_U)$ denote primitives. In the AI regime, adoption is discrete and skilled labor is always required in B:

$$m_B \;=\; s_B\, w_S \;+\; p_X, \qquad s_B > 0. \tag{22}$$

Domestic nominal income is

$$Y \;=\; w_S L_S \;+\; w_U L_U \;+\; FN, \tag{23}$$

and foreign outflows equal $\phi N + p_X Q_B$. Sector A is competitive with CRS and $p_A \equiv 1$.

3.1 Primitives, Demand, Markups, and Free Entry

CES in sector B (variety demand and markup). Inside B, preferences are CES with elasticity $\sigma > 1$, so the common markup is constant:

$$\mu \;=\; \frac{\sigma}{\sigma - 1} \;>\; 1. \tag{24}$$

$$p_B \;=\; \mu\, m_B. \tag{25}$$

With symmetry across active adopters, the sector-B price index is

$$P_B \;=\; \mu\, m_B\, N^{\frac{1}{1-\sigma}}. \tag{26}$$

Core pricing and expenditure identities Using the pricing first order condition (25) and the sector-B expenditure identity $E_B = \alpha Y$, symmetry implies that if $Q_B \equiv \sum_{i=1}^{N} q_i = Nq$ denotes the *physical* sum of variety quantities (not the CES aggregator), then

$$m_B\, Q_B \;=\; m_B\, \frac{E_B}{p_B} \;=\; \frac{\alpha}{\mu}\, Y,$$

so

$$Q_B \;=\; \frac{\alpha}{\mu}\, \frac{Y}{m_B}. \tag{27}$$

Notation. Here $Q_B \equiv \sum_{i=1}^{N} q_i = Nq$ is the physical sum of symmetric variety quantities (it is not the CES aggregator). The identity follows directly from $E_B = \alpha Y$ and (25).

Free entry and variety. Zero profit per firm, $(p_B - m_B)q = F + \phi$, together with symmetry and $E_B = \alpha Y$, gives

$$N \;=\; \kappa(\mu)\,\frac{Y}{F+\phi}, \qquad \kappa(\mu) \equiv \frac{\mu-1}{\mu}\,\alpha. \tag{28}$$

Combining (23) and (28) yields

$$\Big(1 - \vartheta(\phi)\Big)\, dY \;=\; L_S\, dw_S + L_U\, dw_U - \Xi(\phi, Y)\, d\phi, \qquad \vartheta(\phi) \equiv \frac{F\,\kappa(\mu)}{F+\phi}, \quad \Xi(\phi, Y) \equiv \frac{F\,\kappa(\mu)}{(F+\phi)^2}\, Y. \tag{29}$$

Remark. Because N satisfies (28), either a higher usage fee p_X (which depresses Y) or a higher access fee ϕ (which raises the denominator) reduces the number of active firms and thus variety.

CES technology in sector A. In order to characterize primitive conditions for comparative statics, we sometimes assume a CES unit-cost function in A. In such cases, we refer to the following:

$$c_A(w_S, w_U) \;=\; \begin{cases} \left[\,\beta\, w_S^{1-\rho} + (1-\beta)\, w_U^{1-\rho}\,\right]^{\frac{1}{1-\rho}}, & \rho \neq 1, \\ w_S^{\beta}\, w_U^{1-\beta}, & \rho = 1 \quad \text{(CD limit)}, \end{cases} \tag{30}$$

with share parameter $\beta \in (0,1)$ and elasticity of substitution $\rho > 0$. Hicksian unit demands are $a_A^S = \partial c_A / \partial w_S$ and $a_A^U = \partial c_A / \partial w_U$, and the corresponding factor-cost shares in A are $\theta_S \equiv w_S a_A^S$ and $\theta_U \equiv w_U a_A^U$, with $\theta_S + \theta_U = 1$. Standard CES properties imply that the Hicksian semi-elasticities are given by

$$\frac{\partial \ln a_A^S}{\partial \ln w_S} = -\,\rho\,\theta_S, \quad \frac{\partial \ln a_A^U}{\partial \ln w_U} = -\,\rho\,\theta_U, \quad \frac{\partial \ln a_A^S}{\partial \ln w_U} = +\rho\,\theta_U, \qquad \frac{\partial \ln a_A^U}{\partial \ln w_S} = +\rho\,\theta_S. \tag{31}$$

With Cobb–Douglas across sectors, $Q_A = (1-\alpha)Y$, so labor usage in A is

$$a_A^U(w)\,(1-\alpha)\,Y \;=\; L_U, \tag{32}$$

$$a_A^S(w)\,(1-\alpha)\,Y \;+\; s_B\,Q_B \;=\; L_S, \tag{33}$$

where Q_B is given by (27).

3.2 Welfare Factorization

Under the assumptions that utility is Cobb–Douglas across sectors and $p_A \equiv 1$, indirect utility satisfies $W \propto Y\, P_B^{-\alpha}$. Using (26),

$$\frac{W^{AI}}{W^0} \;=\; \left(\frac{m_B^0}{m_B^{AI}}\right)^{\alpha} \left(\frac{N^{AI}}{N^0}\right)^{\frac{\alpha}{\sigma-1}} \left(\frac{Y^{AI}}{Y^0}\right). \tag{34}$$

This decomposition isolates the *unit-cost* channel (m_B), the *variety* channel (N), and the *aggregate-income* channel (Y).

3.3 Comparative Statics

We derive how Q_B, w_S, and Y respond to p_X and ϕ by differentiating (27)–(33) together with the income identity (38).

3.3.1 Preliminaries.

Differentiating (27) and using (22):

$$dQ_B \;=\; \frac{\alpha}{\mu}\left(\frac{dY}{m_B} - \frac{Y}{m_B^2}\,(s_B\, dw_S + dp_X)\right). \tag{35}$$

Differentiating (32) gives

$$(1-\alpha)\, a_A^U\, dY + (1-\alpha)\, Y\, da_A^U \;=\; 0 \quad\Longrightarrow\quad dY \;=\; -\,Y\,\frac{da_A^U}{a_A^U}. \tag{36}$$

Differentiating (33) and substituting (35) yields

$$(1-\alpha)\big(a_A^S\, dY + Y\, da_A^S\big) \;+\; s_B\frac{\alpha}{\mu}\left(\frac{dY}{m_B} - \frac{Y}{m_B^2}\,(s_B\, dw_S + dp_X)\right) \;=\; 0. \tag{37}$$

Finally, combining (23) and (28) implies

$$\Big(1-\vartheta(\phi)\Big)\, dY \;=\; L_S\, dw_S \;+\; L_U\, dw_U \;-\; \Xi(\phi, Y)\, d\phi, \tag{38}$$

Equations (36)–(38) form a linear system in (dY, dw_S, dw_U) in terms of $(dp_X, d\phi)$.

3.3.2 Comparative statics of usage fee

Proposition 7 (Usage-fee incidence)**.** *Set $d\phi = 0$. In the CES environment described above (with $s_B > 0$ and CES in A with parameters (β, ρ)), the factor-price responses satisfy*

$$\frac{\partial w_U}{\partial p_X} > 0 \qquad \textit{and} \qquad \frac{\partial w_S}{\partial p_X} < 0. \tag{39}$$

Proof. Let $x := d\ln w_S$ and $y := d\ln w_U$. From (22), $dm_B = s_B\, dw_S + dp_X = s_B w_S\, x + dp_X$. Differentiate the A-sector factor-balance equations (32)–(33) and use the CES Hicksian semi-elasticities in (31):

For notational compactness, write $h_{SS} = -\rho\theta_S,\ h_{UU} = -\rho\theta_U,\ h_{SU} = +\rho\theta_U,\ h_{US} = +\rho\theta_S$ (as in (31)).

$$(1-\alpha)\, a_A^U\, dY + (1-\alpha)\, Y\, da_A^U = 0 \iff \frac{dY}{Y} = -\big(h_{US}x + h_{UU}y\big) = -\rho\big(\theta_S x - \theta_U y\big). \tag{40}$$

Divide (33) after differentiation by Y, substitute (40) and $dm_B = s_B w_S x + dp_X$:

$$\begin{aligned}
0 &= (1-\alpha)\left(\frac{a_A^S}{Y}\, dY + da_A^S\right) + \frac{s_B\alpha}{\mu}\left(\frac{1}{m_B}\frac{dY}{Y} - \frac{1}{m_B^2}(s_B w_S x + dp_X)\right) \\
&= (1-\alpha)\Big[-a_A^S\big(h_{US}x + h_{UU}y\big) + a_A^S\big(h_{SS}x + h_{SU}y\big)\Big] \qquad (41)\\
&+ \frac{s_B\alpha}{\mu m_B}\big(-h_{US}x - h_{UU}y\big) - \frac{s_B\alpha}{\mu m_B^2}(s_B w_S x + dp_X). \qquad (42)
\end{aligned}$$

Using $h_{SS} = -\rho\theta_S$, $h_{UU} = -\rho\theta_U$, $h_{SU} = +\rho\theta_U$, $h_{US} = +\rho\theta_S$, the $(1-\alpha)$ bracket simplifies to $2(1-\alpha)\rho\, a_A^S\big(-\theta_S x + \theta_U y\big)$. .

Collecting coefficients on x and y delivers the linear system

$$\underbrace{\Big(-2(1-\alpha)\rho\, a_A^S\theta_S \ - \ \tfrac{s_B\alpha}{\mu m_B}\rho\theta_S \ - \ \tfrac{s_B^2\alpha w_S}{\mu m_B^2}\Big)}_{:=\Gamma_1} x + \underbrace{\Big(2(1-\alpha)\rho\, a_A^S\theta_U \ + \ \tfrac{s_B\alpha}{\mu m_B}\rho\theta_U\Big)}_{:=\Gamma_2} y \ = \ \underbrace{\tfrac{s_B\alpha}{\mu m_B^2}}_{:=\Gamma_3} dp_X. \tag{43}$$

By construction $\Gamma_1 < 0$, $\Gamma_2 > 0$, and $\Gamma_3 > 0$. Solving (43) together with (40) (Cramer's rule) yields the semi-elasticities

$$\frac{\partial \ln w_S}{\partial p_X} = -\,\frac{s_B}{D}\cdot\frac{s_B\,\alpha}{\mu\, m_B^2} \ < \ 0, \qquad \frac{\partial \ln w_U}{\partial p_X} \ = \ \frac{s_B}{D}\cdot\frac{s_B\,\alpha}{\mu\, m_B^2}\cdot\frac{\theta_S}{\theta_U} \ > \ 0, \tag{44}$$

where the positive denominator is

$$D \ \equiv \ (1-\alpha)\,\rho\,\theta_S \ + \ \frac{s_B\,\alpha}{\mu\, m_B}\cdot\frac{s_B}{w_S} \ > \ 0. \tag{45}$$

The strict positivity of the right-hand side of $\partial \ln w_U / \partial p_X$ in (44) establishes (39). □

Intuition. Sector B uses only skilled labor, so it is skill-intensive relative to A. A higher p_X raises m_B, contracts B, and releases skilled labor. General equilibrium reallocation pushes production toward A, raising relative demand for unskilled labor; hence w_U rises while w_S falls.

Corollary 3. *From* (38) *with* $d\phi = 0$ *and* (44),

$$\frac{\partial Y}{\partial p_X} \;<\; 0. \tag{46}$$

Finally, (27) *and* (22) *give*

$$\frac{\partial m_B}{\partial p_X} \;=\; 1 + s_B \frac{\partial w_S}{\partial p_X} \;\in\; (0,1), \qquad \frac{\partial Q_B}{\partial p_X} \;=\; \frac{\alpha}{\mu}\left(\frac{1}{m_B}\frac{\partial Y}{\partial p_X} - \frac{Y}{m_B^2}\frac{\partial m_B}{\partial p_X}\right) \;<\; 0. \tag{47}$$

Special cases for the A-sector CES parameter ρ

(i) $\rho \to 0$ (Leontief in A) When A has no substitution, the Hicksian responses vanish and the denominator in (44) simplifies to

$$D_{\mathrm{L}} \;=\; \frac{s_B\,\alpha}{\mu\, m_B} \cdot \frac{s_B}{w_S}.$$

Substituting into (44) gives

$$\frac{\partial \ln w_S}{\partial p_X} = -\,\frac{s_B}{D_{\mathrm{L}}}\,\frac{s_B \alpha}{\mu m_B^2} = -\,\frac{w_S}{m_B} \;<\; 0,$$
$$\frac{\partial \ln w_U}{\partial p_X} = \frac{s_B}{D_{\mathrm{L}}}\,\frac{s_B \alpha}{\mu m_B^2}\,\frac{\theta_S}{\theta_U} = \frac{w_S}{m_B}\,\frac{\theta_S}{\theta_U} \;>\; 0.$$

$$\frac{d \ln Y}{d p_X} = \frac{1}{1-\vartheta(\phi)}\,\frac{w_S}{m_B}\left(-\,\theta_S^Y + \theta_U^Y \frac{\theta_S}{\theta_U}\right) = \frac{1}{1-\vartheta(\phi)}\,\frac{w_S}{m_B}\,\theta_S \left(\frac{\theta_U^Y}{\theta_U} - \frac{\theta_S^Y}{\theta_S}\right).$$

Substituting in:

$$\frac{d \ln Y}{d p_X} = \frac{1}{1-\vartheta(\phi)}\,\frac{w_S}{m_B}\,\frac{-\,w_S L_S + \frac{a_A^S}{a_A^U}\frac{w_S}{w_U}\,w_U L_U}{Y} = \frac{1}{1-\vartheta(\phi)}\,\frac{w_S^2}{m_B\, Y}\left(\frac{a_A^S}{a_A^U} L_U - L_S\right).$$

(ii) $\rho = 1$ (Cobb-Douglas in A) At $\rho = 1$, the denominator in (44) becomes

$$D_{\mathrm{CD}} \;=\; (1-\alpha)\,\theta_S \;+\; \frac{s_B\,\alpha}{\mu\, m_B} \cdot \frac{s_B}{w_S} \;>\; 0.$$

Substituting into (44) yields

$$\frac{\partial \ln w_S}{\partial p_X} = -\frac{s_B^2\alpha/(\mu m_B^2)}{(1-\alpha)\theta_S + \frac{s_B\alpha}{\mu m_B}\frac{s_B}{w_S}},$$

$$\frac{\partial \ln w_U}{\partial p_X} = \frac{s_B^2\alpha/(\mu m_B^2)}{(1-\alpha)\theta_S + \frac{s_B\alpha}{\mu m_B}\frac{s_B}{w_S}}\frac{\theta_S}{\theta_U}.$$

$$\frac{d\ln Y}{dp_X} = \frac{1}{1-\vartheta(\phi)}\frac{s_B^2\alpha}{\mu m_B^2}\frac{-w_S L_S + \frac{\beta}{1-\beta}w_U L_U}{Y\left[(1-\alpha)\beta + \frac{s_B\alpha}{\mu m_B}\frac{s_B}{w_S}\right]}.$$

(iii) $\rho \to \infty$ (perfect substitutes in A) As $\rho \to \infty$, the first term in D dominates:

$$D = (1-\alpha)\,\rho\,\theta_S + \frac{s_B\,\alpha}{\mu\,m_B}\cdot\frac{s_B}{w_S} \sim (1-\alpha)\,\rho\,\theta_S \to \infty.$$

Therefore the semi-elasticities shrink to zero:

$$\frac{\partial \ln w_S}{\partial p_X} \approx -\frac{s_B^2\alpha}{\mu m_B^2}\frac{1}{(1-\alpha)\rho\theta_S}, \qquad \frac{\partial \ln w_U}{\partial p_X} \approx \frac{s_B^2\alpha}{\mu m_B^2}\frac{1}{(1-\alpha)\rho\theta_U}.$$

Hence both responses approach zero at rate $O(1/\rho)$, and $\frac{\partial m_B}{\partial p_X} \to 1^-$. Further, substitution gives:

$$\frac{d\ln Y}{dp_X} = \frac{1}{1-\vartheta(\phi)}\frac{s_B^2\alpha}{\mu m_B^2}\frac{-w_S L_S + \frac{\beta}{1-\beta}\left(\frac{w_S}{w_U}\right)^{1-\rho} w_U L_U}{Y\left[(1-\alpha)\rho\frac{\beta\, w_S^{1-\rho}}{\beta\, w_S^{1-\rho}+(1-\beta)\, w_U^{1-\rho}} + \frac{s_B\alpha}{\mu m_B}\frac{s_B}{w_S}\right]}.$$

Local incidence of a usage–price increase with discrete adoption We next characterize the CPI and welfare consequences of a marginal increase in p_X when adoption is discrete and unchanged. In this case, there is no substitution back toward unskilled labor in B.

Lemma 3 (Signs for m_B, N, P_B)**.** *For $dp_X > 0$ at fixed adoption:*

$$0 < \frac{dm_B}{dp_X} < 1, \qquad \frac{d\ln N}{dp_X} = \frac{d\ln Y}{dp_X} < 0, \qquad \frac{d\ln P_B}{dp_X} > 0.$$

Proof. The first inequality follows from (47). The second uses free entry $N \propto Y$ and $\partial Y/\partial p_X < 0$ (see (46)). For the third, using (26), $\ln P_B = \ln m_B + \frac{1}{1-\sigma}\ln N + \ln\mu$. Since $\sigma > 1$, $\frac{1}{1-\sigma} < 0$ and $d\ln N/dp_X < 0$, so that term is positive; and $d\ln m_B/dp_X > 0$. □

Recall that we have normalized $p_A = 1$ so that Sector A dynamics show up through factor markets and wages. Then, we have

Proposition 8 (CPI increase and welfare loss)**.** *In the CES environment with $p_A \equiv 1$ (sector A is the numéraire), the CPI is $P = P_B^\alpha$ and welfare satisfies $W \propto Y/P$. Then (within the regime that AI is adopted),*

$$\frac{d\ln P}{dp_X} = \alpha\left(\frac{d\ln m_B}{dp_X} - \frac{1}{\sigma-1}\frac{d\ln N}{dp_X}\right) = \alpha\left(\frac{d\ln m_B}{dp_X} - \frac{1}{\sigma-1}\frac{d\ln Y}{dp_X}\right) > 0,$$

and hence

$$\frac{d\ln W}{dp_X} = \frac{d\ln Y}{dp_X} - \frac{d\ln P}{dp_X} < 0.$$

Proof. Within the regime where AI is adopted, $0 < \frac{dm_B}{dp_X} < 1$ and $\frac{d\ln Y}{dp_X} < 0$ (Cor. 3). Since $N \propto Y$ by (28), $d\ln N = d\ln Y$. With $\sigma > 1$, $-\frac{1}{\sigma-1}\frac{d\ln Y}{dp_X} > 0$, so the bracketed term is strictly positive, implying $d\ln P/dp_X > 0$ and therefore $d\ln U/dp_X < 0$. □

Corollary 4 (Real wages)**.** ***(i)*** *Comparative statics for real wages are:*

$$\frac{d\ln(w_S/P)}{dp_X} = \frac{d\ln w_S}{dp_X} - \frac{d\ln P}{dp_X} < 0, \qquad \frac{d\ln(w_U/P)}{dp_X} = \frac{d\ln w_U}{dp_X} - \alpha\left(\frac{d\ln m_B}{dp_X} - \frac{1}{\sigma-1}\frac{d\ln Y}{dp_X}\right).$$

A sufficient *condition for $\frac{d\ln(w_U/P)}{dp_X} < 0$ is*

$$\alpha\left(\frac{d\ln m_B}{dp_X} - \frac{1}{\sigma-1}\frac{d\ln Y}{dp_X}\right) \geq \frac{d\ln w_U}{dp_X}. \tag{48}$$

(ii) *Using* (44) *in* (48) *gives*

$$\alpha\left(\frac{d\ln m_B}{dp_X} - \frac{1}{\sigma-1}\frac{d\ln Y}{dp_X}\right) \geq \frac{s_B^2\alpha}{\mu m_B^2 D}\cdot\frac{\theta_S}{\theta_U}.$$

A sufficient (conservative) condition, since $d\ln m_B/dp_X > 0$, is

$$\frac{\alpha}{\sigma-1}\left|\frac{d\ln Y}{dp_X}\right| \geq \frac{s_B^2\alpha}{\mu m_B^2 D}\cdot\frac{\theta_S}{\theta_U}.$$

(iii) Limiting cases with CES production in sector A.

(a) ***Leontief in** A (ρ → 0)**.***

$$\frac{d\ln(w_U/P)}{dp_X} =$$

$$\underbrace{\frac{w_S}{m_B}\frac{a_A^S}{a_A^U}\frac{w_S}{w_U}}_{\textit{wage}} - \underbrace{\alpha}_{\textit{unit cost}} + \underbrace{\alpha\frac{s_B w_S^2}{m_B}}_{\textit{offset in } dm_B/dp_X} + \underbrace{\frac{\alpha}{\sigma-1}\frac{1}{1-\vartheta(\phi)}\frac{w_S^2}{m_B Y}\left(\frac{a_A^S}{a_A^U}L_U - L_S\right)}_{\textit{variety via } Y}.$$

(b) ***Cobb–Douglas in*** A ***(***$\rho = 1$***).*** *Let*

$$H_{\text{CD}} \;\equiv\; \frac{s_B^2\,\alpha}{\mu\, m_B^2\left[(1-\alpha)\,\beta \;+\; \dfrac{s_B\,\alpha}{\mu\, m_B}\,\dfrac{s_B}{w_S}\right]}.$$

Then:

$$\frac{d\ln(w_U/P)}{dp_X}$$

$$= \underbrace{-\,\alpha}_{\textit{unit cost}} \;+\; \underbrace{H_{\text{CD}}\left[\frac{\beta}{1-\beta} \;+\; \alpha s_B w_S \;+\; \frac{\alpha}{\sigma-1}\,\frac{1}{1-\vartheta(\phi)}\,\frac{-\,w_S L_S + \frac{\beta}{1-\beta} w_U L_U}{Y}\right]}_{\textit{wage + offset + variety}}.$$

(c) ***Perfect substitutes in*** A ***(***$\rho \to \infty$***).***

$$\frac{d\ln(w_U/P)}{dp_X} = \underbrace{\frac{d\ln w_U}{dp_X}}_{=O(1/\rho)} \;-\; \alpha\left[1 - \frac{s_B^3\,\alpha\, w_S}{\mu\, m_B^2 D}\right] \;+\; \underbrace{\frac{\alpha}{\sigma-1}\,\frac{d\ln Y}{dp_X}}_{=O(1/\rho)}.$$

Using the expression for D*,*

$$\lim_{\rho\to\infty}\frac{d\ln(w_U/P)}{dp_X} = \lim_{\rho\to\infty}\frac{d\ln(w_S/P)}{dp_X} = -\,\alpha\,\frac{1}{m_B}.$$

3.3.3 Access fee: Income and Skilled Wage

Throughout this subsection set $dp_X = 0$. Recall

$$\Xi(\phi, Y) = \frac{F\,\kappa(\mu)\,Y}{(F+\phi)^2}, \qquad \kappa(\mu) = \frac{\mu-1}{\mu}\,\alpha, \qquad \vartheta(\phi) = \frac{F\,\kappa(\mu)}{F+\phi} \in (0,1),$$

and let $x := d\ln w_S$, $y := d\ln w_U$.

Step 1: Express (x, y) **in terms of** dY**.** From (36) and the CES Hicksian elasticities (31),

$$\frac{dY}{Y} = -\rho\,\theta_U(x-y) \quad\Longrightarrow\quad y = x + \frac{1}{\rho\,\theta_U}\,\frac{dY}{Y}. \tag{49}$$

With $dm_B = s_B\,dw_S = s_B w_S\,x$, differentiate (33), divide by Y, and substitute (49):

$$\begin{aligned} 0 &= (1-\alpha)\Big(\tfrac{a_A^S}{Y}\,dY + a_A^S(-\rho\theta_S x + \rho\theta_U y)\Big) + \frac{s_B\alpha}{\mu}\Big(\tfrac{1}{m_B}\tfrac{dY}{Y} - \tfrac{s_B w_S}{m_B^2}x\Big) \\ &= (1-\alpha)a_A^S\left[2\,\frac{dY}{Y} - \rho(1-2\theta_U)\,x\right] + \frac{s_B\alpha}{\mu}\left(\frac{1}{m_B}\frac{dY}{Y} - \frac{s_B w_S}{m_B^2}x\right). \end{aligned} \tag{50}$$

Rearranging gives

$$x \;=\; \Lambda_S\,\frac{dY}{Y}, \qquad \Lambda_S := \frac{2(1-\alpha)a_A^S + \frac{s_B\alpha}{\mu}\frac{1}{m_B}}{(1-\alpha)a_A^S\,\rho(1-2\theta_U) + \frac{s_B\alpha}{\mu}\frac{s_B w_S}{m_B^2}}. \tag{51}$$

Step 2: Apply the income identity. Divide (38) by Y and use $dw_S = w_S x$, $dw_U = w_U y$ with (49)–(51). Writing $\theta_i^Y := \frac{w_i L_i}{Y}$,

$$\big(1-\vartheta(\phi)\big)\,\frac{dY}{Y} = \theta_S^Y\,x + \theta_U^Y\,y \;-\; \frac{\Xi(\phi,Y)}{Y}\,d\phi, \tag{52}$$

and substituting $y = x + \frac{1}{\rho\theta_U}\frac{dY}{Y}$ and $x = \Lambda_S\frac{dY}{Y}$ yields

$$\underbrace{\left[1-\vartheta(\phi) \;-\; \theta_S^Y\,\Lambda_S \;-\; \theta_U^Y\Big(\Lambda_S + \frac{1}{\rho\theta_U}\Big)\right]}_{:=\;\Delta_\phi}\frac{dY}{Y} \;=\; -\,\frac{\Xi(\phi,Y)}{Y}\,d\phi. \tag{53}$$

Hence:

$$\frac{dY}{Y} = -\,\frac{\Xi(\phi,Y)/Y}{\Delta_\phi}\,d\phi, \qquad \frac{d\ln w_S}{d\phi} = \Lambda_S\,\frac{1}{Y}\,\frac{dY}{d\phi}. \tag{54}$$

The determinant. Using $\theta_S^Y + \theta_U^Y = (w_S L_S + w_U L_U)/Y = 1-\vartheta(\phi)$ (free entry and rebates), (53) can be written as

$$\Delta_\phi = (1-\vartheta(\phi))\,(1-\Lambda_S) \;-\; \frac{\theta_U^Y}{\rho\,\theta_U}. \tag{55}$$

Proposition 9 (Income and skilled-wage comparative statics)**.** *Suppose* $\Delta_\phi > 0$ *and* $\Lambda_S > 0$. *Then for* $d\phi > 0$*,*

$$\frac{\partial Y}{\partial\phi} < 0 \qquad \textit{and} \qquad \frac{\partial \ln w_S}{\partial\phi} < 0.$$

Proof. From (54), $\Xi(\phi,Y)/Y > 0$ and $\Delta_\phi > 0$ imply $dY/d\phi < 0$. Then $d\ln w_S/d\phi = \Lambda_S(1/Y)(dY/d\phi) < 0$ whenever $\Lambda_S > 0$. □

Primitive sufficient restrictions. A convenient set of primitives ensuring $\Delta_\phi > 0$ and $\Lambda_S > 0$ is:

(i) Either $\theta_U \le \frac{1}{2}$ or $\Big(\theta_U > \frac{1}{2}$ and $\rho < \rho_{\max}\Big)$,

(ii) Determinant bound: $\Lambda_S + \dfrac{1}{\rho\,\theta_U} \;<\; 1-\vartheta(\phi)$, where (for $\theta_U > \frac{1}{2}$)

$$\rho_{\max} = \frac{\alpha\, s_B^2\, w_S}{\mu\, m_B^2\,(1-\alpha)\, a_A^S\,(2\theta_U - 1)}. \tag{56}$$

Condition (i) guarantees the denominator of Λ_S in (51) is positive (hence $\Lambda_S > 0$); (ii) is a conservative way to ensure $\Delta_\phi > 0$ via (55).

Lower bound. Define

$$\epsilon_Y^\phi := -\frac{1}{Y}\frac{\partial Y}{\partial \phi} = \frac{\Xi(\phi, Y)/Y}{\Delta_\phi}.$$

Since the wage–price term in (55) is nonnegative, $\Delta_\phi \leq 1 - \vartheta(\phi)$ and

$$\epsilon_Y^\phi \;\geq\; \underline{\epsilon}_Y^\phi := \frac{\Xi(\phi, Y)/Y}{1 - \vartheta(\phi)} = \frac{F\,\kappa(\mu)}{(F+\phi)\,\big[(F+\phi) - F\,\kappa(\mu)\big]}. \tag{57}$$

Hence $\underline{\epsilon}_Y^\phi$ is strictly decreasing in ϕ, strictly increasing in $\kappa(\mu)$, and single–peaked in F with maximizer $F^* = \phi/\sqrt{1-\kappa(\mu)}$.

Upper bound. If there exists $\overline{\Lambda} < 1$ with $\Lambda_S \leq \overline{\Lambda}$ and $(1-\vartheta(\phi))(1-\overline{\Lambda}) > \frac{1}{\rho\,\theta_U}$ (positivity of the denominator), then (55) implies

$$\epsilon_Y^\phi \;\leq\; \bar{\epsilon}_Y^\phi := \frac{\frac{F\kappa(\mu)}{(F+\phi)^2}}{(1-\vartheta(\phi))(1-\overline{\Lambda}) - \frac{1}{\rho\,\theta_U}}. \tag{58}$$

When $\theta_U \leq \frac{1}{2}$, we can use the bound

$$\overline{\Lambda} = \frac{2(1-\alpha)a_A^S + \frac{s_B\alpha}{\mu}\frac{1}{m_B}}{\frac{s_B\alpha}{\mu}\frac{s_B w_S}{m_B^2}} = \frac{m_B}{s_B w_S} + \frac{2(1-\alpha)\,\mu\, m_B^2}{\alpha\, s_B^2\, w_S}\, a_A^S,$$

and in the Cobb–Douglas case ($\rho = 1$), $a_A^S = \beta/w_S$ gives $\overline{\Lambda}_{\text{CD}} = \frac{m_B}{s_B w_S} + \frac{2(1-\alpha)\,\beta\,\mu\, m_B^2}{\alpha\, s_B^2\, w_S^2}$.

Conditions for $dY/d\phi < 0$ and $d\ln w_S/d\phi < 0$ in special cases of ρ.

- **Leontief in A ($\rho \to 0$).** The denominator of Λ_S is dominated by the Sector B term; $\Lambda_S > 0$ and (55) reduces to $(1-\vartheta)(1-\Lambda_S)$ minus a large positive term $1/(\rho\theta_U)$, so the sign of Δ_ϕ is ambiguous. However, whenever $\Delta_\phi > 0$ holds, $dY/d\phi < 0$ and $d\ln w_S/d\phi < 0$ follow immediately by Proposition 9.

- **Cobb-Douglas in A ($\rho = 1$).** Then $\theta_U = 1-\beta$, $a_A^S = \beta/w_S$, and $\rho_{\max}$ simplifies to

$$\rho_{\max}\Big|_{\rho=1} = \frac{\alpha\, s_B^2\, w_S^2}{\mu\, m_B^2\,(1-\alpha)\,\beta\,(1-2\beta)}.$$

If $\beta \geq \frac{1}{2}$, the Primitive Sufficient Condition (i) holds automatically; if $\beta < \frac{1}{2}$, it suffices that $1 < \rho_{\max}$.

- **Perfect substitutes in A ($\rho \to \infty$).** As $\rho \to \infty$, $\Lambda_S \to 0$ and $\Delta_\phi \to 1 - \vartheta(\phi) > 0$. Thus $dY/d\phi < 0$ and $d \ln w_S/d\phi < 0$ unconditionally for sufficiently large ρ.

3.3.4 Access fees: Incidence on the Unskilled Wage

Throughout this subsection set $dp_X = 0$. From (51)–(53)

$$\Lambda_S = \frac{2(1-\alpha)a_A^S + \frac{s_B\alpha}{\mu}\frac{1}{m_B}}{(1-\alpha)a_A^S\,\rho(1-2\theta_U) + \frac{s_B\alpha}{\mu}\frac{s_B w_S}{m_B^2}}, \qquad \Delta_\phi = 1 - \vartheta(\phi) - \theta_S^Y\,\Lambda_S - \theta_U^Y\Big(\Lambda_S + \frac{1}{\rho\theta_U}\Big).$$

Define the income semi-elasticity and the skilled-technology threshold:

$$\epsilon_Y^\phi := -\frac{1}{Y}\frac{\partial Y}{\partial\phi} = \frac{\Xi(\phi, Y)/Y}{\Delta_\phi}, \qquad T := \frac{\alpha}{\mu\, m_B}\cdot\frac{s_B}{w_S}\cdot\frac{\theta_S}{\theta_U}.$$

Proposition 10 (Unskilled wage response to access fees)**.** *Assume $s_B > 0$, $\sigma > 1$ (so $\mu = \sigma/(\sigma-1) > 1$), CES in A with (β, ρ) and $\rho > 0$, and interior factor shares. Then*

$$\frac{\partial w_U}{\partial\phi} > 0 \quad\Longleftrightarrow\quad \epsilon_Y^\phi < T. \tag{59}$$

Proof. Differentiate the A-sector factor-balance equations and the income identity (cf. (36)–(38)) with $dm_B = s_B\, dw_S$ (since $dp_X = 0$). Solving the linear system yields the semi-elasticity (as in the derivation recorded previously):

$$\frac{\partial \ln w_U}{\partial\phi} = \underbrace{\frac{s_B}{D}\cdot\frac{s_B\,\alpha}{\mu\, m_B^2}\cdot\frac{\theta_S}{\theta_U}}_{\text{(i) substitution in } B} - \underbrace{\frac{\rho}{D}\cdot\theta_U\cdot\epsilon_Y^\phi}_{\text{(ii) income}}, \qquad D \;=\; (1-\alpha)\,\rho\,\theta_S \;+\; \frac{s_B\,\alpha}{\mu\, m_B}\cdot\frac{s_B}{w_S} \;>\; 0. \tag{60}$$

Since $w_U > 0$, $\operatorname{sign}(\partial w_U/\partial\phi) = \operatorname{sign}(\partial \ln w_U/\partial\phi)$. Setting (60) to zero and solving for ϵ_Y^ϕ gives $\epsilon_Y^\phi = \frac{\alpha}{\mu\, m_B}\frac{s_B}{w_S}\frac{\theta_S}{\theta_U} = T$, establishing (59). □

Interpretation. Term (i) in (60) captures the reallocation toward A when higher ϕ tightens entry and contracts B; this pushes up w_U. Term (ii) reflects the reduction in domestic rebates FN; a larger ϵ_Y^ϕ pulls w_U down. The sign turns on which effect dominates.

Bounds and one-sided conclusions. Since the GE wage–price effect in (53) is nonnegative, $\Delta_\phi \le 1-\vartheta(\phi)$, yielding the lower bound

$$\epsilon_Y^\phi \;\ge\; \underline{\epsilon}_Y^\phi := \frac{\Xi(\phi,Y)/Y}{1-\vartheta(\phi)} = \frac{F\,\kappa(\mu)}{(F+\phi)\,\big[(F+\phi)-F\,\kappa(\mu)\big]}\,. \tag{61}$$

Hence a conservative *exclusion* condition is $\underline{\epsilon}_Y^\phi \ge T \;\Rightarrow\; \partial w_U/\partial\phi \le 0$. Moreover, $\underline{\epsilon}_Y^\phi$ is strictly decreasing in ϕ, strictly increasing in $\kappa(\mu)$, and single-peaked in F (max at $F^* = \phi/\sqrt{1-\kappa(\mu)}\,$).

An *upper* bound arises if we impose the positivity restriction

$$(1-\vartheta(\phi))(1-\overline{\Lambda}) > \frac{1}{\rho\,\theta_U} \quad \text{for some} \quad \Lambda_S \le \overline{\Lambda} < 1. \tag{62}$$

Using $\Delta_\phi = (1-\vartheta)(1-\Lambda_S) - \frac{\theta_U^Y}{\rho\theta_U}$ and $\theta_U^Y \le 1$,

$$\epsilon_Y^\phi \;\le\; \overline{\epsilon}_Y^\phi := \frac{\frac{F\kappa(\mu)}{(F+\phi)^2}}{(1-\vartheta(\phi))(1-\overline{\Lambda}) - \frac{1}{\rho\theta_U}}\,. \tag{63}$$

Then $\overline{\epsilon}_Y^\phi \le T$ is *sufficient* for $\partial w_U/\partial\phi > 0$.

Remark 1 (Primitive bound for $\overline{\Lambda}$)**.** *When $\theta_U \le \frac{1}{2}$ the denominator of Λ_S is greater than the B-sector term, so*

$$\overline{\Lambda} = \frac{2(1-\alpha)a_A^S + \frac{s_B\alpha}{\mu}\frac{1}{m_B}}{\frac{s_B\alpha}{\mu}\frac{s_B w_S}{m_B^2}} = \frac{m_B}{s_B w_S} + \frac{2(1-\alpha)\,\mu\, m_B^2}{\alpha\, s_B^2\, w_S}\, a_A^S$$

is a convenient primitive upper bound. In the Cobb–Douglas case ($\rho=1$), using $a_A^S = \beta/w_S$ yields $\overline{\Lambda}_{\rm CD} = \frac{m_B}{s_B w_S} + \frac{2(1-\alpha)\,\beta\,\mu\, m_B^2}{\alpha\, s_B^2\, w_S^2}$.

Corollary 5 (Access-fee incidence on w_U: special cases by ρ)**.** *Let $T = \frac{\alpha}{\mu m_B}\frac{s_B}{w_S}\frac{\theta_S}{\theta_U}$ and ϵ_Y^ϕ as above.*

(a) ***Leontief in*** *A ($\rho\to 0$)*. *$D \to D_{\rm L} = \frac{s_B\alpha}{\mu m_B}\frac{s_B}{w_S}$ and the income term in (60) vanishes, so*

$$\frac{\partial \ln w_U}{\partial\phi} = \frac{s_B}{D_{\rm L}}\cdot\frac{s_B\alpha}{\mu m_B^2}\cdot\frac{\theta_S}{\theta_U} > 0 \quad\Rightarrow\quad \frac{\partial w_U}{\partial\phi} > 0 \textit{ for any } s_B > 0.$$

(b) ***Cobb-Douglas in*** *A ($\rho=1$)*. *With $\theta_S = \beta$, $\theta_U = 1-\beta$,*

$$T_{\rm CD} = \frac{\alpha}{\mu m_B}\cdot\frac{s_B}{w_S}\cdot\frac{\beta}{1-\beta}.$$

A conservative sufficient condition for $\partial w_U/\partial\phi \le 0$ is

$$\underline{\epsilon}_Y^{\phi} \;\ge\; T_{\mathrm{CD}} \quad\Longleftrightarrow\quad \frac{F\kappa(\mu)}{(F+\phi)\big[(F+\phi)-F\kappa(\mu)\big]} \;\ge\; \frac{\alpha}{\mu m_B}\cdot\frac{s_B}{w_S}\cdot\frac{\beta}{1-\beta}.$$

Under (62) *with $\overline{\Lambda} = \overline{\Lambda}_{\mathrm{CD}}$ above, a sufficient condition for $\partial w_U/\partial\phi > 0$ is*

$$\overline{\epsilon}_Y^{\phi} = \frac{\frac{F\kappa(\mu)}{(F+\phi)^2}}{(1-\vartheta(\phi))(1-\overline{\Lambda}_{\mathrm{CD}}) - \frac{1}{1-\beta}} \;\le\; T_{\mathrm{CD}}.$$

(c) ***Perfect substitutes in** A **($\rho\to\infty$).*** *$D = (1-\alpha)\rho\,\theta_S + O(1)$, hence*

$$\frac{\partial \ln w_U}{\partial\phi} = -\,\frac{\theta_U}{(1-\alpha)\theta_S}\,\epsilon_Y^{\phi} + O\Big(\frac{1}{\rho}\Big).$$

From (53)*, $\Delta_\phi \to 1-\vartheta(\phi) > 0$, so*

$$\epsilon_Y^{\phi} \;\to\; \frac{\dfrac{F\kappa(\mu)}{(F+\phi)^2}}{1-\dfrac{F\kappa(\mu)}{F+\phi}} = \frac{F\,\kappa(\mu)}{(F+\phi)\big[(F+\phi)-F\kappa(\mu)\big]} \;> 0.$$

Therefore, for large ρ, $\partial \ln w_U/\partial\phi < 0$ and $\partial w_U/\partial\phi < 0$ unconditionally.

Proof of Corollary 5. (a) Take the limit $\rho \to 0$ in (60); since $D \to D_{\mathrm{L}}$ and $\rho/D \to 0$, the income term vanishes and the substitution term is strictly positive.

(b) Substitute $\theta_S = \beta$, $\theta_U = 1-\beta$ in (59) to obtain the CD threshold T_{CD}. The lower-bound exclusion follows by comparing $\underline{\epsilon}_Y^{\phi}$ in (61) to T_{CD}. Under (62) with the primitive $\overline{\Lambda}_{\mathrm{CD}}$, (63) gives $\overline{\epsilon}_Y^{\phi}$, and the sufficiency claim is immediate.

(c) As stated, $D = (1-\alpha)\rho\theta_S + O(1)$ makes the substitution term $O(1/\rho)$ in (60). Since $\Delta_\phi \to 1-\vartheta(\phi) > 0$, the limit of ϵ_Y^{ϕ} is strictly positive, and the leading term $-\frac{\theta_U}{(1-\alpha)\theta_S}\epsilon_Y^{\phi}$ determines the negative sign for large ρ. □

3.3.5 Sector B Output and Number of Firms

Decomposition. Using $Q_B = \frac{\alpha}{\mu}\frac{Y}{m_B}$ and $m_B = s_B w_S + p_X$,

$$\frac{\partial Q_B}{\partial\phi} = \frac{\alpha}{\mu}\,\frac{\partial Y}{\partial\phi}\,\left[\frac{1}{m_B} - \frac{s_B w_S}{m_B^2}\,\Lambda_S\right].$$

Assuming $\Delta_\phi > 0$ so that $\frac{\partial Y}{\partial \phi} < 0$, we have

$$\frac{\partial Q_B}{\partial \phi} \begin{cases} < 0, & \text{if } \Lambda_S < \dfrac{m_B}{s_B w_S} \quad \text{(income contraction dominates)}, \\ > 0, & \text{if } \Lambda_S > \dfrac{m_B}{s_B w_S} \quad \text{(unit-cost decrease dominates)}. \end{cases}$$

Note. The unit-cost decrease referred to above arises indirectly via the skilled wage: $\frac{dm_B}{d\phi} = s_B \frac{dw_S}{d\phi} = -\dfrac{s_B w_S \Lambda_S}{\Delta_\phi} \dfrac{F\kappa(\mu)}{(F+\phi)^2} < 0$ whenever $\Delta_\phi > 0$ and $\Lambda_S > 0$.

Number of firms. From $N = \kappa(\mu) \dfrac{Y}{F+\phi}$,

$$\frac{\partial N}{\partial \phi} = \kappa(\mu) \left(\frac{1}{F+\phi} \frac{\partial Y}{\partial \phi} - \frac{Y}{(F+\phi)^2} \right) \; < \; 0 \qquad (\text{since } \; \Delta_\phi > 0 \Rightarrow \partial Y/\partial \phi < 0).$$

Per-firm quantity (always rises under $\Delta_\phi > 0$). With symmetry,

$$q := \frac{Q_B}{N} = \frac{\frac{\alpha}{\mu} \frac{Y}{m_B}}{\kappa(\mu) \frac{Y}{F+\phi}} = \frac{F+\phi}{(\mu-1)\, m_B},$$

so

$$\frac{\partial q}{\partial \phi} = \frac{1}{(\mu-1)\, m_B} - \frac{F+\phi}{(\mu-1)\, m_B^2} \frac{\partial m_B}{\partial \phi} \quad \text{with} \quad \frac{\partial m_B}{\partial \phi} = s_B \frac{\partial w_S}{\partial \phi} < 0.$$

Hence $\partial q/\partial \phi > 0$: the direct $(F+\phi)$ effect is positive, and $\partial m_B/\partial \phi < 0$ (via $\partial w_S/\partial \phi < 0$ under $\Delta_\phi > 0, \Lambda_S > 0$) amplifies it. Thus $Q_B = Nq$ can rise even though N falls, if q rises enough— exactly the case $\Lambda_S > \frac{m_B}{s_B w_S}$.

Special cases.

$$\begin{cases} \text{Leontief in } A \ (\rho \to 0): & \Lambda_S > \dfrac{m_B}{s_B w_S} \ \Rightarrow \ \dfrac{\partial Q_B}{\partial \phi} > 0, \\ \text{Cobb-Douglas in } A \ (\rho = 1): & \dfrac{\partial Q_B}{\partial \phi} < 0 \iff 2 < (2\beta - 1) \dfrac{m_B}{s_B w_S} \quad (\text{requires } \beta > \tfrac{1}{2}), \\ \text{Perfect substitutes in } A \ (\rho \to \infty): & \Lambda_S \to 0 \ \Rightarrow \ \dfrac{\partial Q_B}{\partial \phi} < 0 \ \ (\text{unconditionally}). \end{cases}$$

3.4 Adoption Frontier

Consider an AI equilibrium (Z, ϕ, p_X) (using (25) and (22)). A single deviator (no adoption, hence no access fee) uses the baseline technology that employs both factors. Let (s_0, u_0) denote the baseline (non-AI) unit inputs; the deviator's marginal cost is

$$m_{\text{dev}} \;=\; s_0\, w_S \;+\; u_0\, w_U, \tag{64}$$

with (w_S, w_U) evaluated at the AI equilibrium. Under CES, the "no-deviation" condition $\pi_{\text{dev}} \leq 0$ reduces to the adoption cap

$$p_X \;\leq\; m_{\text{dev}} \left(\frac{F}{F + \phi} \right)^{\frac{1}{\sigma - 1}} \;-\; s_B\, w_S, \tag{65}$$

which defines a downward-sloping frontier $p_{X,\max}(\phi)$.

Monotonicity of the cap in ϕ Differentiating (65) *holding the AI-equilibrium wage vector fixed for the local comparison* gives

$$\frac{dp_{X,\max}}{d\phi} \;=\; -\frac{1}{\sigma - 1}\, m_{\text{dev}} \left(\frac{F}{F + \phi} \right)^{\frac{1}{\sigma - 1}} \frac{1}{F + \phi} \;<\; 0, \tag{66}$$

so the adoption cap is strictly decreasing in ϕ.

3.4.1 Joint optimality on the adoption frontier (CES): theorem and primitives

We study the constrained profit maximization problem

$$\max_{\phi \geq 0,\; p_X \leq p_{X,\max}(\phi)} \Pi(\phi, p_X) \qquad \text{with} \qquad \Pi(\phi, p_X) \;=\; \phi\, N(\phi, p_X) \;+\; p_X\, Q_B(\phi, p_X),$$

where $p_{X,\max}(\phi)$ is the adoption cap from (65) and, under CES, $\mu = \sigma/(\sigma - 1)$ is constant, $\kappa(\mu) = \frac{\mu - 1}{\mu} \alpha$.

Standing assumptions.

(A1) *(CES, interior)* Inside B: CES with elasticity $\sigma > 1$; in A: CES unit cost (30) with $\rho > 0$ and $0 < \beta < 1$; adoption requires $s_B > 0$ (cf. (22)). Factor endowments (L_S, L_U) and primitives (α, F) are strictly positive.

(A2) *(Regularity)* The equilibrium correspondences $(Y, N, Q_B, m_B, w_S, w_U)$ are single-valued and C^1 in (ϕ, p_X) on the feasible set $\mathcal{F} = \{(\phi, p_X) : \phi \geq 0,\ 0 \leq p_X \leq p_{X,\max}(\phi)\}$.

(A3) *(Cap monotonicity)* The adoption cap is C^1 and strictly decreasing: $p'_{X,\max}(\phi) \equiv \frac{dp_{X,\max}}{d\phi} < 0$ (cf. (66)).

(A4) *(Compactness)* For any $\bar{\phi} < \infty$, the truncated feasible set $\{(\phi, p_X) : 0 \leq \phi \leq \bar{\phi},\ 0 \leq p_X \leq p_{X,\max}(\phi)\}$ is compact.

Lemma 4 (Profit derivatives under CES)**.** *Fix $\phi \geq 0$. With $\epsilon_Y^X \equiv -\frac{1}{Y}\frac{dY}{dp_X} \geq 0$ and $0 < \frac{dm_B}{dp_X} < 1$, the derivative of Π in p_X is*

$$\frac{d\Pi}{dp_X} = Y\Big[-\epsilon_Y^X \underbrace{\Big(\frac{\phi\,\kappa(\mu)}{F+\phi} + \frac{\alpha}{\mu}\frac{p_X}{m_B}\Big)}_{=:\ \mathcal{C}_Y(\phi,p_X)} + \underbrace{\frac{\alpha}{\mu}\Big(\frac{1}{m_B} - \frac{p_X}{m_B^2}\frac{dm_B}{dp_X}\Big)}_{=:\ \mathcal{C}_M(p_X)}\Big], \tag{67}$$

and

$$\frac{\alpha}{\mu}\frac{s_B w_S}{m_B^2} \ \leq\ \mathcal{C}_M(p_X)\ \leq\ \frac{\alpha}{\mu}\frac{1}{m_B}. \tag{68}$$

Proof. Differentiate $\Pi(\phi, p_X) = \phi N(\phi, p_X) + p_X Q_B(\phi, p_X)$: $\frac{d\Pi}{dp_X} = \phi\frac{dN}{dp_X} + Q_B + p_X\frac{dQ_B}{dp_X}$. Using (28) and (27), we have $\frac{dN}{dp_X} = \frac{\kappa(\mu)}{F+\phi}\frac{dY}{dp_X}$ and

$$\frac{dQ_B}{dp_X} = \frac{\alpha}{\mu}\left(\frac{1}{m_B}\frac{dY}{dp_X} - \frac{Y}{m_B^2}\frac{dm_B}{dp_X}\right).$$

Collecting terms yields (67). Since (22) and $0 < \frac{dm_B}{dp_X} < 1$ (cf. (47)), we have

$$\frac{1}{m_B} - \frac{p_X}{m_B^2}\frac{dm_B}{dp_X} \in \Big[\frac{1}{m_B} - \frac{p_X}{m_B^2},\ \frac{1}{m_B}\Big] = \Big[\frac{s_B w_S}{m_B^2},\ \frac{1}{m_B}\Big],$$

which implies (68). □

Theorem 1 (Boundary optimality at the joint optimum (CES))**.** *Under (A1)–(A4), the constrained problem admits a maximizer $(\phi^\star, p_X^\star)$. Define the* boundary objective $\widehat{\Pi}(\phi) \equiv \Pi\big(\phi,\, p_{X,\max}(\phi)\big)$ *and the* reduced objective $\widetilde{\Pi}(\phi) \equiv \max_{0 \leq p_X \leq p_{X,\max}(\phi)} \Pi(\phi, p_X)$*. Then:*

(i) (KKT at a maximizer) *Any maximizer $(\phi^\star, p_X^\star)$ admits a multiplier $\lambda^\star \geq 0$ s.t.*

$$\frac{\partial \Pi}{\partial p_X}(\phi^\star, p_X^\star) - \lambda^\star = 0, \tag{69}$$

$$\frac{\partial \Pi}{\partial \phi}(\phi^\star, p_X^\star) + \lambda^\star\, p'_{X,\max}(\phi^\star) = 0, \tag{70}$$

$$p_X^\star \leq p_{X,\max}(\phi^\star), \quad \lambda^\star \geq 0, \quad \lambda^\star\big(p_{X,\max}(\phi^\star) - p_X^\star\big) = 0. \tag{71}$$

(ii) (Boundary iff condition) *$(\phi^\star, p_X^\star)$ is* binding *$(p_X^\star = p_{X,\max}(\phi^\star))$ if and only if*

$$\left.\frac{\partial \Pi}{\partial p_X}\right|_{(\phi^\star,\, p_{X,\max}(\phi^\star))} \ \geq\ 0 \quad\Longleftrightarrow\quad \left.\frac{\partial \Pi}{\partial \phi}\right|_{(\phi^\star,\, p_{X,\max}(\phi^\star))} \ \geq\ 0, \tag{72}$$

where the equivalence uses $p'_{X,\max}(\phi^\star) < 0$ from (A3).

(iii) (Boundary selection via the inner problem) *Fix* ϕ. *If* $p \mapsto \Pi(\phi, p)$ *is nondecreasing on* $[0, p_{X,\max}(\phi)]$, *then the inner maximizer is* $p_X^{\text{in}}(\phi) = p_{X,\max}(\phi)$ *and* $\widetilde{\Pi}(\phi) = \widehat{\Pi}(\phi)$.

(iv) (Boundary optimality at the joint optimum) *If* $\phi^\star \in \arg\max_{\phi \geq 0} \widehat{\Pi}(\phi)$ *and, in addition,* $p \mapsto \Pi(\phi^\star, p)$ *is nondecreasing on* $[0, p_{X,\max}(\phi^\star)]$, *then* $(\phi^\star,\, p_{X,\max}(\phi^\star))$ *solves the joint problem.*

Outline. Existence follows from (A2)–(A4) and Weierstrass on truncated compact sets. By standard KKT for a smooth inequality constraint, there is $\lambda^\star \geq 0$ satisfying (69)–(71). If the constraint binds ($p_X^\star = p_{X,\max}(\phi^\star)$), then (69) implies $\lambda^\star = \partial\Pi/\partial p_X\big|_{\text{frontier}} \geq 0$; if interior, $\lambda^\star = 0$ and $\partial\Pi/\partial p_X(\phi^\star, p_X^\star) = 0$. Combining (70) with $p'_{X,\max}(\phi^\star) < 0$ yields the boundary condition (72). If, for fixed ϕ, the inner map $p \mapsto \Pi(\phi, p)$ is nondecreasing on $[0, p_{X,\max}(\phi)]$, the inner maximizer is the boundary $p_{X,\max}(\phi)$; maximizing $\widehat{\Pi}(\phi)$ over ϕ then delivers the joint optimum at $(\phi^\star,\, p_{X,\max}(\phi^\star))$. □

Remark 2 (On part (ii)). *At a boundary optimum,* $\lambda^\star = \partial\Pi/\partial p_X\big|_{\textit{frontier}}$ *and* (72) *is the boundary FOC using* $p'_{X,\max}(\phi^\star) < 0$.

We next provide *primitive sufficient conditions* that guarantee the monotonicity required in Theorem 1(iii)–(iv), expressed solely in terms of CES parameters and cost-share objects.

Corollary 6 (Primitive sufficient conditions for boundary optimality). *Maintain (A1)–(A4). For a given* $\phi \geq 0$, *define the worst-case income semi-elasticity bound*

$$\bar{\epsilon}_Y^X(\phi) \;:=\; \sup_{p \in [0,\, p_{X,\max}(\phi)]} \epsilon_Y^X(\phi, p). \tag{73}$$

$$\mathcal{R}(\phi) \;:=\; \left. \frac{\dfrac{s_B\, w_S}{m_B}}{p_X \;+\; (\mu - 1)\, m_B\, \dfrac{\phi}{F + \phi}} \right|_{p_X = p_{X,\max}(\phi)}. \tag{74}$$

If

$$\bar{\epsilon}_Y^X(\phi) \;\leq\; \mathcal{R}(\phi), \tag{75}$$

then $p \mapsto \Pi(\phi, p)$ *is nondecreasing on* $[0, p_{X,\max}(\phi)]$, *hence the inner maximizer is* $p_{X,\max}(\phi)$ *and* $\widetilde{\Pi}(\phi) = \widehat{\Pi}(\phi)$. *In particular, if* $\phi^\star \in \arg\max_\phi \widehat{\Pi}(\phi)$ *and* (75) *holds at* $\phi^\star$, *then* $(\phi^\star,\, p_{X,\max}(\phi^\star))$ *solves the joint problem.*

Proof. By Lemma 4,

$$\frac{1}{Y}\frac{d\Pi}{dp_X} = -\,\epsilon_Y^X(\phi, p_X)\,\mathcal{C}_Y(\phi, p_X) + \mathcal{C}_M(p_X) \;\geq\; -\,\bar{\epsilon}_Y^X(\phi)\,\mathcal{C}_Y(\phi, p_X) \;+\; \frac{\alpha}{\mu}\frac{s_B w_S}{m_B^2}.$$

For the usage–price shock we have $0 < \frac{dm_B}{dp_X} < 1$ and $\frac{dw_S}{dp_X} < 0$, hence $\frac{d}{dp_X}(p_X/m_B) > 0$ and $\frac{d}{dp_X}(s_B w_S/m_B^2) \le 0$. Therefore the right–hand side is minimized at $p_X = p_{X,\max}(\phi)$, and we obtain

$$\frac{1}{Y}\frac{d\Pi}{dp_X} \;\ge\; -\,\bar{\epsilon}_Y^X(\phi)\,\mathcal{C}_Y\big(\phi, p_X\big) + \frac{\alpha}{\mu}\frac{s_B w_S}{m_B^2} \;\ge\; -\,\bar{\epsilon}_Y^X(\phi)\,\mathcal{C}_Y\big(\phi, p_{X,\max}(\phi)\big) + \frac{\alpha}{\mu}\frac{s_B w_S}{m_B^2}\bigg|_{p_X=p_{X,\max}(\phi)} \;\ge\; 0,$$

where the last inequality is exactly (75). Hence $\frac{d\Pi}{dp_X} \ge 0$ on $[0, p_{X,\max}(\phi)]$, so the inner maximizer is $p_{X,\max}(\phi)$ and $\widetilde{\Pi}(\phi) = \widehat{\Pi}(\phi)$. □

Special cases for substitutability in A (ρ). The boundary condition (75) compares a worst–case income feedback $\bar{\epsilon}_Y^X(\phi)$ to the technological/market ratio $\mathcal{R}(\phi)$ in (74).

- **Leontief in A ($\rho \to 0$).** With limited factor substitution in A, the usage–price income feedback ϵ_Y^X is typically larger; thus (75) is *harder* to satisfy. Boundary optimality can still hold if $\mathcal{R}(\phi)$ is large (e.g., high s_B/w_S, low m_B, high σ).
- **Cobb–Douglas in A ($\rho = 1$).** The criterion (75) applies directly by computing $\bar{\epsilon}_Y^X(\phi)$ over $[0, p_{X,\max}(\phi)]$ and comparing it to $\mathcal{R}(\phi)$.
- **Perfect substitutes in A ($\rho \to \infty$).** In the CES A-sector, wage semi-elasticities and the usage–price income feedback scale as $O(1/\rho)$, hence $\epsilon_Y^X(\phi, p) \to 0$ uniformly on compact feasible sets. Therefore, for sufficiently large ρ, (75) is automatically satisfied and the inner problem selects the boundary $p_{X,\max}(\phi)$.

Proposition 11 (High–ρ boundary optimality)**.** *Fix $\sigma > 1$ and $\phi \ge 0$. There exists $\bar{\rho} < \infty$ (depending on primitives and ϕ) such that for all $\rho \ge \bar{\rho}$ the inner objective $p \mapsto \Pi(\phi, p)$ is nondecreasing on $[0, p_{X,\max}(\phi)]$. Consequently, any joint maximizer satisfies $p_X^\star = p_{X,\max}(\phi^\star)$.*

Proof sketch. For the usage–price shock, the CES A-sector linear system yields wage semi-elasticities and dY/dp_X of order $O(1/\rho)$ (the Sector A determinant is proportional to $(1-\alpha)\rho\theta_S$). Hence $\epsilon_Y^X(\phi, p) = -(1/Y)\,dY/dp_X = O(1/\rho)$ uniformly in $p \in [0, p_{X,\max}(\phi)]$. Since the frontier ratio $\mathcal{R}(\phi)$ in (74) does not scale with ρ through the A-sector elasticities, while $\epsilon_Y^X(\phi, p) = O(1/\rho)$ uniformly in p, choose $\bar{\rho}$ large enough that $\sup_p \epsilon_Y^X(\phi, p) \le \frac{1}{2}\mathcal{R}(\phi)$. Then (75) holds and Corollary 6 implies monotonicity of $p \mapsto \Pi(\phi, p)$ and boundary selection at the joint optimum. □

Interpretation. Boundary optimality is guaranteed for sufficiently high ρ; at moderate ρ (e.g., Cobb-Douglas), it holds whenever the inequality (75) (condition on primitives) is satisfied;

at very low ρ (Leontief), income feedback can overturn monotonicity unless primitives make $\mathcal{R}(\phi)$ large.

Comparative statics intuition (all else equal). Boundary optimality is more likely with large s_B/w_S (strong cost-share transfer), low m_B, high σ (low μ), and *smaller* ϕ relative to F (since $\phi/(F+\phi)$ increases in ϕ and raises the denominator of $\mathcal{R}(\phi)$). It is less likely with large α *and* high markups (both amplify $\mathcal{C}_Y$), or with especially strong income feedback (large $\bar{\epsilon}_Y^X$).

3.4.2 Mixed partial inside the frontier (CES): sign and primitives

We show that raising ϕ reduces the marginal profitability of p_X under primitive conditions, so the instruments are local substitutes.

In this subsection, we work in the feasible interior.

Notation. Let $\epsilon_Y^X \equiv -\frac{1}{Y}\frac{dY}{dp_X} \geq 0$, and write (cf. Lemma 4)

$$\frac{d\Pi}{dp_X} = Y\Big[-\epsilon_Y^X\,\mathcal{C}_Y(\phi,p_X) + \mathcal{C}_M(p_X)\Big], \tag{76}$$

$$\mathcal{C}_Y(\phi,p_X) := \frac{\phi\,\kappa(\mu)}{F+\phi} + \frac{\alpha}{\mu}\frac{p_X}{m_B}, \quad \mathcal{C}_M(p_X) := \frac{\alpha}{\mu}\left(\frac{1}{m_B} - \frac{p_X}{m_B^2}\frac{dm_B}{dp_X}\right). \tag{77}$$

Lemma 5 (Markup derivative)**.** *With* $\mathcal{C}_M(p_X) = \frac{\alpha}{\mu}\left(\frac{1}{m_B} - \frac{p_X}{m_B^2}\frac{dm_B}{dp_X}\right)$*, define*

$$\tau := \frac{dm_B}{dp_X} \in (0,1), \qquad s := \frac{\partial m_B}{\partial \phi} < 0 \quad (\textit{under } \Delta_\phi > 0,\ \Lambda_S > 0).$$

Then

$$\frac{\partial \mathcal{C}_M}{\partial \phi} = \frac{\alpha}{\mu}\left\{-\frac{s}{m_B^2} - \frac{p_X}{m_B^2}\frac{\partial \tau}{\partial \phi} + \frac{2p_X}{m_B^3}\,s\,\tau\right\},$$

and

$$\frac{\partial \mathcal{C}_M}{\partial \phi} \leq 0 \iff \frac{\partial \tau}{\partial \phi} \geq \frac{-s}{p_X\,m_B}\left(1 - \frac{2p_X}{m_B}\tau\right). \tag{78}$$

Corollary 7 (Primitive sufficient condition for $\partial_\phi \mathcal{C}_M \leq 0$)**.** *Let* $\tau = \frac{dm_B}{dp_X} \in (0,1)$*,* $s = \frac{\partial m_B}{\partial \phi} < 0$*, and* $m_B = s_B w_S + p_X$*. A primitive sufficient condition for* $\frac{\partial \mathcal{C}_M}{\partial \phi} \leq 0$ *is*

$$\boxed{\frac{\partial \tau}{\partial \phi} \geq 0 \quad \textit{and} \quad 2p_X\,\tau \geq m_B}.$$

Two facts we use repeatedly:

$$\frac{\partial \mathcal{C}_Y}{\partial \phi} = \frac{\kappa(\mu)F}{(F+\phi)^2} + \frac{\alpha}{\mu}\frac{p_X}{m_B^2}\left(-\frac{\partial m_B}{\partial \phi}\right) \;>\; 0, \tag{79}$$

since $\kappa(\mu), F > 0$ and, by (80), $\frac{\partial m_B}{\partial \phi} < 0$.

Finally, with $dp_X = 0$, $m_B = s_B w_S + p_X$ and Prop. 9 yield:

$$\frac{\partial m_B}{\partial \phi} = s_B \frac{\partial w_S}{\partial \phi} < 0 \qquad (\text{whenever } \Delta_\phi > 0,\ \Lambda_S > 0), \tag{80}$$

i.e. the unit cost in B falls as ϕ rises via the GE effect on w_S.

Monotonicity of the income feedback. We now give primitive sufficient conditions under which ϕ makes the usage–price income contraction weakly larger:

$$\frac{\partial \epsilon_Y^X}{\partial \phi}(\phi, p_X) \;\geq\; 0.$$

Lemma 6 (Monotonicity checks)**.** *In the CES environment with $s_B > 0$, $\sigma > 1$:*

(a) ***Leontief in** A **(**$\rho \to 0$**).*** *With fixed proportions in A, higher ϕ raises $\vartheta(\phi)$ and lowers m_B; the usage–price income feedback is nondecreasing in ϕ: $\frac{\partial \epsilon_Y^X}{\partial \phi} \geq 0$ without additional restrictions.*

(b) ***Cobb–Douglas in** A **(**$\rho = 1$**).*** *A primitive sufficient condition is either $\beta \geq \frac{1}{2}$ or*

$$\frac{s_B^2\,\alpha}{\mu} \cdot \frac{w_S^2}{m_B^2} \;\geq\; (1-\alpha)\,(1-2\beta)\,\beta. \tag{81}$$

Under either inequality, $\dfrac{\partial \epsilon_Y^X}{\partial \phi} \geq 0.$

(c) ***Perfect substitutes in** A **(**$\rho \to \infty$**).*** *As $\rho \to \infty$, the usage–price income feedback is $O(1/\rho)$ and nondecreasing in ϕ due to the entry term $\vartheta(\phi)$; for sufficiently large ρ, $\dfrac{\partial \epsilon_Y^X}{\partial \phi} \geq 0$ holds without further restrictions.*

Proposition 12 (Mixed partial at an interior usage–price optimizer)**.** *Fix $\phi \geq 0$ and suppose $\hat{p}_X \in (0, \infty)$ is an unconstrained interior maximizer of $p \mapsto \Pi(\phi, p)$. Using* (76) *and Lemma 4,*

$$\frac{d\Pi}{dp_X} = Y\big[-\,\epsilon_Y^X\,\mathcal{C}_Y + \mathcal{C}_M\big],$$

and the mixed partial satisfies

$$\frac{\partial^2\Pi}{\partial p_X\,\partial\phi}(\phi,\hat{p}_X) = Y\Big\{-\ \frac{\partial\epsilon_Y^X}{\partial\phi}(\phi,\hat{p}_X)\,\mathcal{C}_Y(\phi,\hat{p}_X)\ -\ \epsilon_Y^X(\phi,\hat{p}_X)\,\frac{\partial\mathcal{C}_Y}{\partial\phi}(\phi,\hat{p}_X)\ +\ \frac{\partial\mathcal{C}_M}{\partial\phi}(\phi,\hat{p}_X)\Big\}. \tag{82}$$

In particular, if

$$\frac{\partial\epsilon_Y^X}{\partial\phi}(\phi,\hat{p}_X)\geq 0 \qquad \textit{and} \qquad \Big(\frac{\partial\tau}{\partial\phi}(\phi,\hat{p}_X)\ \geq\ 0\ \ \textit{and}\ \ 2p_X\,\tau(\phi,\hat{p}_X)\ \geq\ m_B\Big), \tag{83}$$

then

$$\frac{\partial^2\Pi}{\partial p_X\,\partial\phi}(\phi,\hat{p}_X)\ \leq\ 0, \quad \textit{with strict } <0 \textit{ if at least one inequality in } (83) \textit{ is strict.} \tag{84}$$

Moreover, $\frac{\partial\mathcal{C}_Y}{\partial\phi}>0$ *always holds by* (79)*.*

Corollary 8 (Primitive special cases by ρ)**.** *Let* $\mu=\sigma/(\sigma-1)>1$, $m_B=s_Bw_S+p_X$*.*

(a) ***Leontief in*** A ***(***$\rho\to 0$***).*** *If the condition in Corollary 7 holds (i.e.* $\partial_\phi\tau\geq 0$ *and* $2p_X\tau\geq m_B$*), then* $\dfrac{\partial^2\Pi}{\partial p_X\,\partial\phi}<0$*.*

(b) ***Cobb–Douglas in*** A ***(***$\rho=1$***).*** *If either* $\beta\geq\frac{1}{2}$ *or the inequality* (81) *holds, and, in addition, the primitive condition in Corollary 7 holds (i.e.* $\partial_\phi\tau\geq 0$ *and* $2p_X\,\tau\geq m_B$ *with* $\tau:=dm_B/dp_X\in(0,1)$*), then* $\dfrac{\partial^2\Pi}{\partial p_X\,\partial\phi}<0$*.*

(c) ***Perfect substitutes in*** A ***(***$\rho\to\infty$***).*** *For sufficiently large* ρ*, the usage–price income feedback is* $O(1/\rho)$ *and* $\partial_\phi\epsilon_Y^X\geq 0$ *due to the entry term. If, in addition, the primitive condition in Corollary 7 holds (i.e.* $\partial_\phi\tau\geq 0$ *and* $2p_X\,\tau\geq m_B$*), then* $\dfrac{\partial^2\Pi}{\partial p_X\,\partial\phi}<0$*. Moreover, even if that condition fails, there exists* $\bar{\rho}<\infty$ *(depending on* $(\alpha,\mu,s_B,w_S,m_B,p_X,F,\phi)$*) such that for all* $\rho\geq\bar{\rho}$ *the mixed partial remains negative.*

Interpretation. At an interior p_X optimum, the cross-partial in (82) is the sum of: (i) a *negative* income-feedback term (Lemma 6), (ii) a *negative* demand/entry term since $\partial_\phi\mathcal{C}_Y>0$ (79), and (iii) a markup term. The primitive condition in Corollary 7 (i.e. $\partial_\phi\tau\geq 0$ and $2p_X\,\tau\geq m_B$) makes the markup term nonpositive, so raising ϕ reduces the marginal profitability of p_X: the instruments behave as local substitutes.

3.5 Summary: Case of High Sector A Factor Substitutability

For ease of reference, here we summarize the results about the case with high Sector A substitutability between skilled and unskilled labor. When $\rho \to \infty$, the Sector A determinant is proportional $(1-\alpha)\rho\theta_S$, so wage semi-elasticities with respect to usage price fees are, from (44)–(45), $\partial \ln w_S/\partial p_X, \partial \ln w_U/\partial p_X = O(1/\rho)$ and $\partial m_B/\partial p_X \to 1$. With $N \propto Y$ we have $\partial \ln Y/\partial p_X < 0$ (Corollary 3), and within the regime where AI is adopted, $P = P_B^{\alpha}$ implies $\partial \ln P/\partial p_X > 0$ and hence $\partial \ln U/\partial p_X < 0$ (Proposition 8); moreover the real-wage derivatives converge to the same constant:

$$\lim_{\rho\to\infty} \frac{\partial \ln(w_S/P)}{\partial p_X} = \lim_{\rho\to\infty} \frac{\partial \ln(w_U/P)}{\partial p_X} = -\frac{\alpha}{m_B}.$$

For changes in access fees, $\Lambda_S \to 0$ and $\Delta_\phi \to 1-\vartheta(\phi) > 0$, so $\partial Y/\partial\phi < 0$ and $\partial \ln w_S/\partial\phi < 0$ unconditionally (Proposition 9); the unskilled-wage incidence reduces to $\partial \ln w_U/\partial\phi = -\frac{\theta_U}{(1-\alpha)\theta_S}\epsilon_Y^{\phi} + O(1/\rho)$, and since $\epsilon_Y^{\phi} > 0$ for large ρ, also $\partial \ln w_U/\partial\phi < 0$ (Corollary 5). On quantities, the bracket in $\partial Q_B/\partial\phi = \frac{\alpha}{\mu}\,\partial Y/\partial\phi\,[\,\frac{1}{m_B} - \frac{s_B w_S}{m_B^2}\Lambda_S\,]$ converges to $1/m_B > 0$, so with $\partial Y/\partial\phi < 0$ we get $\partial Q_B/\partial\phi < 0$ unconditionally. $N = \kappa(\mu)Y/(F+\phi)$ falls and $q = Q_B/N = (F+\phi)/[(\mu-1)m_B]$ rises. On the adoption frontier, $p'_{X,\max}(\phi) < 0$ holds for any $\sigma > 1$ ((66)), and for sufficiently large ρ the inner usage-price problem is monotone so the joint optimum lies on the frontier, $p_X^{\star} = p_{X,\max}(\phi^{\star})$ (Proposition 11). Finally, inside the frontier the mixed partial is negative for large ρ—the first two terms in (82) dominate and, together with the primitive markup condition of Corollary 7, imply $\partial^2\Pi/(\partial p_X\,\partial\phi) < 0$ (Corollary 8).