1 Introduction

What may happen at the end of a firm’s life influences all its prior decisions, even whether to enter in the first place. Thus cross-country differences in firm resolution policies may have a quantitatively significant impact on the firm size distribution, misallocation, and output across countries. Kornejew et al. (2024) tackle this important issue.

Specifically, they argue empirically, that business credit booms are followed by severe economic downturns in countries with low bankruptcy efficiency, but not in countries with high bankruptcy efficiency. Further they argue the results are symmetric implying more efficient bankruptcy systems are more stable. They then build a simple model to understand these data facts.

Perhaps one of the most important questions, both empirically and theoretically, is “What is bankruptcy efficiency?” I will focus on that in this discussion with particular emphasis on the applied theory part. The other discussant, Carola Frydman, is much more qualified to focus on the data than me.

Hence, the discussion will be in two parts. Section 2 asks “What is bankruptcy efficiency in the data?” The next set of sections asks “What is bankruptcy efficiency in the model?” To answer that question, I summarize the model in Kornejew et al. (2024) in Section 3 and draw out the implications of their benchmark equilibrium parameterization in Section 4. Specifically, under their benchmark parameterization all liquidations are inefficient. Section 5 first documents that there can be parameterizations consistent with U.S. data which admit the possibility of efficient liquidation. I then draw out the equilibrium implications of that parameterization for aggregate output under different bankruptcy policies. In contrast to the authors’ benchmark model where all firms are ex-ante homogeneous, I introduce a screening technology in Section 6 that provides ex-ante signals about a firm’s ex-post heterogeneous productivity which nests the authors’ benchmark. I use the technology to understand the interaction between ex-ante screening and ex-post bankruptcy resolution on the firm size distribution, allocative efficiency, and their implications for aggregate output. This opens up the question of how much of the greater stability across boom-bust cycles in the data is coming from ex-post bankruptcy efficiency and how much comes from ex-ante screening. Section 7 concludes.

2 Data

What is bankruptcy efficiency in the data? Kornejew et al. (2024) use measures of the quality of a country’s bankruptcy system from Djankov et al. (2008) and World Bank (2020). Those papers ask bankruptcy lawyers across country $i$ and time questions such as:

*I wish to thank Charles Smith for excellent research assistance in preparing this discussion.
1. Would a bankrupt hotel with normalized value 100 if it continues in the face of a transient operating loss but worth only 70 units if sold piecemeal, be kept as a going concern \( GC_i = 1 \) or liquidated \( GC_i = 0 \)?

2. What is the likely duration \( \tau_i \) and cost \( K_i \) to complete the bankruptcy?

The first item is the key hypothetical. Then they compute country \( i \)'s bankruptcy efficiency \( \xi_i \) as:

\[
\xi_i = \frac{100GC_i + 70(1 - GC_i) - 100 \cdot K_i}{(1 + r)^{\tau_i}}
\]

In their full sample, the mean \( \xi_i \) is 39.3 and its standard deviation is 24.6. In their sample that contains data on business credit from the Bank of International Settlements which forms the basis of their regression analysis, the mean of \( \xi_i \) is 63.1 and its standard deviation is 23.5.

How should we interpret this bankruptcy efficiency measure? The hypothetical asked to lawyers bears an implicit assumption that reorganization is more efficient than liquidation with transitory losses since there is a 30% discount in liquidation.

A fundamental question, however, is whether liquidation is always inefficient and reorganization efficient? We know from Hopenhayn (1992) that in the presence of fixed operating costs, it may be efficient for persistently low productivity firms to exit.

To answer such a question, we should model the economic source of inefficiency. One such inefficiency arises from a commitment problem as modeled by Kornejew et al. (2024). Here there is a form of a debt overhang problem; it may be efficient to cancel the prior debt of a positive net present value firm experiencing a temporary loss instead of incurring liquidation costs associated with, say, a costly firesale (at e.g. the 30% discount above). Such reorganization can, however, entail costs itself and there can be inefficiencies in the renegotiation (e.g. bargaining) process. Another source of inefficiency, not explored here, is an information problem (more will be said about that in Section 6).

### 3 Model

What is bankruptcy efficiency in the model? Kornejew et al. (2024) build an extremely simple model to help understand their regression results.

There are two periods \( t \in \{1, 2\} \). There is a unit measure of ex-ante identical risk neutral firms with discount rate \( \beta_f \). There are also a large number of competitive risk neutral lenders with discount rate \( \beta > \beta_f \). At \( t = 1 \) firms borrow \( b \) at price \( q \) to fund an investment project requiring an injection of goods of fixed size \( I \). The project pays off \( z \in [z_-, z_+] \) drawn from a uniform distribution with density \( \phi(z) \) at \( t = 2 \) where \( z_--z_+=1 \). All firms are ex-ante \((t=1)\) homogeneous but ex-post \((t=2)\) heterogeneous.

There is limited liability; if \( b > z \) a firm declares bankruptcy. In that case, with probability \( \xi \) the project is reorganized and continues to operate at cash flow \( z_{\text{reorg}} = z \) while with probability \( 1 - \xi \) the project is liquidated yielding \( z_{\text{liq}} = z_+ \). Importantly, the parameter \( \xi \) captures bankruptcy efficiency; with a unit measure of firms this also measures fraction of firms which are reorganized (i.e. Chapter 11 in the U.S. bankruptcy code).

Since the paper is about bankruptcy on debt, the most important equilibrium conditions involve the pricing of that debt which in turn depends on how much debt is chosen. Given there are no future repercussion to default at \( t = 2 \), there is a simple threshold default decision rule at \( t = 2 \), default if \( b > z \) (i.e. default is more likely the higher amount you need to repay). Creditors recognize the borrowers decision rule so that the competitive price of debt under the
above bankruptcy policy is given by

\[ q(b, \beta, \xi) = \beta \mathbb{E}_z \left[ 1_{\{b \leq z\}} + 1_{\{b > z\}} \left( \frac{(1 - \xi) z^{\text{liq}} + \xi z^{\text{reorg}}}{b} \right) \right] \]  

(1)

where \( \mathbb{E}_z [\cdot] \) integrates \( z \) using the density \( \phi(z) \). Importantly, given \( z^{\text{reorg}} = z \geq z^{\text{liq}} = z \) creditors receive more in efficient bankruptcy systems, hence interest rates are decreasing in \( \xi \).

Taking prices as given, firms recognize that their debt choice affects the price they pay yielding the following borrowing decision rule:

\[
b^*(\beta, \beta_f, \xi) = \arg \max_{b \geq 0} q(b, \beta, \xi) \cdot b - I + \beta_f \int_b^\infty (z - b) \phi(z) dz
\]

\[ = \bar{z} - (1 - \xi)/(2 - \xi - \beta_f/\beta) \]  

(2)

Kornejew et al. (2024) establish in Proposition 1 that firms borrow more (i.e. \( \frac{\partial b^*(\beta, \beta_f, \xi)}{\partial \xi} > 0 \)) since (ceteris paribus) creditors receive more (and hence charge lower interest rates) in more efficient bankruptcy systems.

### 4 Benchmark Equilibrium: Inefficient Liquidation

The objective of the paper is to explore the macroeconomic implications, say on aggregate output, of bankruptcy policy. Given a bankruptcy rule with \( z^{\text{liq}} = z \geq 0 \) all liquidation is inefficient since it destroys a non-negative value firm with a debt overhang problem. Aggregate output is given by

\[
Y(b, \xi) = \mathbb{E}_z [z_j] - (1 - \xi) \int_{\bar{z}}^b (z_j - \bar{z}) \phi(z_j) dz_j
\]

(3)

with composition effect \( \xi \) and extensive margin effect \( b = b^* \).

It is clear from (3) that to minimize output loss under the assumed exogenous bankruptcy system, the optimal policy is to set \( \xi = 1 \). But then by (2), in the most efficient bankruptcy system

\[
b^*(\beta, \beta_f, \xi = 1) = \bar{z}
\]

(4)

and hence 100% of firms default since there’s no cost to bankruptcy. This illustrates why modeling bankruptcy efficiency exogenously in a two period model with \( z^{\text{liq}} = z \geq 0 \), as in the paper, may be circumspect.

I think it is helpful to understand what is going on in the benchmark model through a set of figures. Specifically, Figures 1-3 assume: \( z^{\text{liq}} = z = 0, \bar{z} = 1, \beta = 1, \beta_f = 0.9, I = 0.2 \). This parameterization satisfies Assumption 1 of Kornejew et al. (2024).²

Figure 1a graphs the set of equilibrium debt price menus (i.e. equation (1c)) for three different bankruptcy efficiency measures (i.e. the price charged \( q(b, \beta = 1, \xi) \) for each level of debt \( b \) for \( \xi \in \{0, 0.5, 1\} \)). The figure illustrates: (i) for any given \( \xi \), interest rate offerings are increasing in debt \( b \) (standard result in equilibrium models of default), and (ii) for any given debt

\[ 0.2 = I < \frac{\beta}{2} \left( \frac{1 - \xi - \frac{\beta_f}{\beta}}{2 - \xi - \frac{\beta_f}{\beta}} \right) + \beta \bar{z} = \frac{1}{2} \left( \frac{1 - 0.9 \xi}{2 - \xi - 0.9} \right) \]

for all values of \( \xi \in [0, 1] \).

1While the authors restrict \( \xi \in (0, 1) \), I consider this limiting case to make the point clear.

2That is,
Taking these menus as given, Figure 1b illustrates the firm borrowing problem $b^*$ which solves (2) off the given menu. Given the uniform distribution of $z$ over a unit support the figure also illustrates the probability of default. The figure illustrates: equilibrium borrowing and default are increasing in bankruptcy efficiency (while the first prediction is consistent with data, it is not clear the default prediction is consistent with data), and (ii) low efficiency bankruptcy systems finance $I$ with mix of debt and “equity” issuance $I - b^*$ which ensures some skin-in-the-game for low bankruptcy efficient systems. This latter result questions how the regression results in Kornejew et al. (2024) should account for other financing options (e.g. internal funds or via an informal sector). Given equilibrium borrowing choice $b^*$, Figure 1c illustrates equilibrium prices $q(b^*, \beta = 1, \xi)$ which are easier to observed data than the price menu. The figure illustrates that the model predicts that low bankruptcy efficient systems, which result in small amounts of borrowing, imply high observed prices $q(b^*, \beta = 1, \xi)$. In other words, the model predicts that observed interest spreads are bankruptcy efficiency, a result which may not be consistent with data.
Dividends $div_t$ accrue to the owners of the firm across the two periods. Specifically, the $t = 1$ dividend is given by $div_1 = q \cdot b - I$ while $div_2 = max\{z - b, 0\}$. Figure 2 illustrates dividend issuance for different bankruptcy systems. It illustrates an important point; equity must be injected (i.e. $div_1 < 0$ in low bankruptcy efficient economies. This provides evidence for an informal sector channel along the lines of D’Erasmo and Boedo (2012). The figure also illustrates that low bankruptcy efficient systems require high backloaded payoffs to induce investment (which yields another potentially testable prediction of the model).

Figure 3 illustrates the macroeconomic implications of bankruptcy systems on aggregate output. As is clear in the figure, with $z^{\text{liq}} = \bar{z} = 0$, it is efficient to reorganize all firms (i.e. the optimal $\xi = 1$ since they are all weakly positive value). But, importantly, the figure illustrates a non-monotonicity arising from a tradeoff:

1. For a given level of debt (and default), as $\xi$ increases, the share of bankrupt firms resolved inefficiently decreases implying an increase in net output $Y - I$. 
2. Since $\frac{\partial b}{\partial \xi} > 0$, default increases implying a decrease in net output $Y - I$.

The composition effect (1) dominates for high $\xi$ while the extensive margin effect (2) dominates for low $\xi$. This non-monotonicity may call into question local (linear) analysis of the data.

5 Alternative Equilibria: Efficient Liquidation

Should all firms be reorganized? As shown above, in the benchmark model version of the model by Kornejew et al. (2024) where $z^\text{liq} = z^\text{opt} = 0$, all firms with a debt overhang problem but with non-negative value should be reorganized (i.e. $\xi = 1$) rather than liquidated. That is, liquidation of positive value firms is inefficient. But what if some firms in the economy are not positive value? In the context of this model, what if $\bar{z} < 0$? Such an assumption is similar to adding fixed operating costs along the lines of Hopenhayn (1992) shifting the entire support $[z, \bar{z}]$ down. In that case, isn’t it efficient to liquidate firms with negative net revenue?

The point that liquidation in the presence of firms with negative operating revenue (i.e. those with $z \in [\bar{z}, 0]$ in the context of the model) is what we see in U.S. Compustat data. Specifically, Figure 4 reproduces Table 1 in Corbae and D’Erasmo (2021). To fix notation, in the U.S., reorganization falls under Chapter 11 bankruptcy rules while liquidation falls under Chapter 7. The figure provides summary statistics on U.S. Compustat firms from 1980-2014 and highlights two particularly important facts relevant for the Kornejew et al. (2024) paper:

1. Bankrupt firms have negative operating income and interest coverage (EBITDA/interest) is declining across non-bankrupt (median 5) / Ch11 (-0.15)/ Ch 7 (-1.07) firms.

2. There is substantial heterogeneity involved in net debt to assets (i.e. Ch 11 > Ch.7 > Non-Bankrupts) and size matters for bankruptcy choice: big bankrupt firms choose reorganization while smaller firms choose liquidation.

Fact 1 documents that a relevant case for the model is $z \in [\bar{z}, 0]$. Fact 2 documents that there is substantial heterogeneity in debt and bankruptcy choices which is inconsistent with ex-ante homogeneous borrowing choices and an exogenous probability $\xi$ of reorganization. These two facts, negative value firms and heterogeneity, motivate my discussion in this section and the next Section 6.
Importantly, the presence of negative value firms in Fact 1 has implications for the optimal bankruptcy resolution policy. In Figure 5 we calculate net output comparing an economy where there are negative value firms ($\tilde{z} = -0.1 < 0 = z^{\text{lv}}$) versus the benchmark parameterization where ($z = 0 = z^{\text{lv}}$). Unsurprisingly, the figure illustrates that the economy with lower mean productivity implies lower output. Surprisingly, however, maximum output is no longer achieved at $\xi = 1$ as in the benchmark, but with $\xi = 0$. The maximum at $\xi = 0$ is because inefficient firms with $z < 0$ are liquidated while at $\xi = 1$ they are not (but should be). This example simply illustrates that there are cases where liquidation is not inefficient (as in the benchmark case) but potentially efficient with limited liability.\footnote{In Appendix IA2.6, Kornejew et al. (2024) show that under a different assumption on investment costs, their main result in Proposition 1 on output dynamics following an exogenous increase in credit continues to hold.}

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Figure 5: Efficient Liquidation ($z < 0 = z^{liq}$) vs. Benchmark ($z = 0 = z^{liq}$): Net Output

Note: Left Vertical axis corresponds to Net Output in Benchmark (i.e. $[z, \bar{z}] = [0, 1]$) while right vertical axis correspond to Net Output in efficient liquidation case (i.e. $[z, \bar{z}] = [-0.1, 0.9]$).

6 Ex-Ante Firm Heterogeneity, Information, and Misallocation

As described above in Fact 2 of the Compustat data there is substantial heterogeneity in debt holdings and bankruptcy choice; bigger bankrupt firms have more net-debt and choose reorganization rather than liquidation. As in the Hopenhayn (1992) model, exit depends on persistence of revenue shocks (consistent with the “transient operating loss” question in Djankov et al. (2008)). The more persistent is a low revenue shock, the lower the expected net present value of the firm and the more likely it should efficiently exit. In Corbae and D’Erasmo (2021) we estimate quarterly autocorrelation of operating income in Compustat is 0.9 which is similar to estimates in Cooper and Haltiwanger (2006). Importantly, besides its impact on the value of the firm, ex-ante heterogeneity and persistence provides lenders with information to efficiently separate/screen unproductive firms, to which I now turn.

Suppose that instead of ex-ante homogeneous firms as in the model of Kornejew et al. (2024), we introduce signals $s(z)$ at the start of $t = 1$, before borrowing choices are made, about productivity $z \in [s - \Delta, s + \Delta]$ at $t = 2$. One can interpret these signals as an approximation to a Markov process over productivity as in Corbae and D’Erasmo (2021) or as firm credit ratings along the lines of credit scores in Chatterjee et al. (2023). This nests the previous Benchmark ex-ante homogeneous firm case when $\Delta = 0$ and a perfect signal case where $\Delta = 0$. Provided signals are observable to lenders, prices can be also be conditioned on the signal (i.e. $E_z|s$):

$$q(s, b, \beta, \xi) = \beta E_{z|s} \left[ 1_{\{b \leq z\}} + 1_{\{b > z\}} \left( \frac{(1 - \xi) z^{liq} + \xi z}{b} \right) \right] \quad (5)$$

Figure 6 illustrates an important point that screening technologies (here modeled simply as informative signals) can aid in the “ex-ante” efficient allocation of lending which can obviate the ex-post need for more efficient bankruptcy systems. Specifically, in Figure 6 we set $\xi = 0.7$ which is similar to the fraction of reorganization in the US keeping all other parameters as in the benchmark model (in particular $z^{liq} = \bar{z} = 0$). Panel 6a plots price menus $q(s, b, \beta = 1, \xi = 0.7)$
in equation (5) for a particular signal $s = 0.5$. If the signal is completely uninformative (i.e. $\Delta = 0.5$), prices strictly decrease in the amount borrowed. On the other hand, if the signal is perfectly informative (i.e. $s = z$), given firms only default when $b > z$, lenders supply funds at the risk free rate for $b \leq z$ while further borrowing leads to default, creditors still receive $((1 - \xi)z^{t_q} + \xi z)/b = 0.35b^{-1}$ generating the remaining decreasing price menu. For partially informative signals with $\Delta = 0.2$, any $b < s - \Delta = 0.3$ leads to no default, so that a risk free price is offered, while any borrowing beyond that level is risky and priced accordingly.

Facing price menus like Panel 6a for each signal $s$, ex-ante heterogeneous firms with signal $s \in [0, 1]$ choose how much they want to borrow (i.e. solve the $s$ dependent analogue of equation (2) in Panel 6b. With uninformative signals $\Delta = 0.5$, we are back in the benchmark case with a homogeneous borrowing level (i.e. independent of $s$). With perfectly informative signals $\Delta = 0$, sufficiently low productivity signals $s = z < I$ are rationed, while firms with higher productivity signals borrow increasing amounts. Again, with partially uninformative signals, creditors face more risk and require higher interest rates which generates lower equilibrium borrowing in Panel 6b.

Given borrowing levels $b^*(s, \beta = 1, \beta_f = 0.9, \xi = 0.7)$ from Panel 6b, Panel 6c plots equilibrium prices $q(s, b^*, \beta = 1, \xi = 0.7)$ across signals $s$ for each level of informativeness $\Delta$. Uninformative signals $\Delta = 0.5$ correspond to the benchmark case which pools all firms with a spread of 16%. In that case, low risk (high $z$) firms at $t = 2$ are cross-subsidizing high risk (low $z$) firms in the credit market. At the other extreme where signals are perfect (i.e. $\Delta = 0$), lenders can efficiently separate and ration firms which are negative net present value (i.e. $z < I$) from those with positive net present value who receive funding at the risk free rate equal to zero. With partially informative signals (i.e. $\Delta \in (0, 0.5)$), firms with $s > I + \Delta$ have guaranteed positive net present value projects and hence receive the risk free rate. Those firms with $s < I - \Delta$ have negative net present value projects and face rationing. Those with $s \in [I - \Delta, I + \Delta]$ are risky so face a spread that is decreasing in $s$. 

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The implications of screening technologies for borrowing in Figure 6 feeds into the determination of net output. Figure 7 illustrates the macro implications of bankruptcy resolution policies in the presence of screening technologies. Better screening technologies minimize the frequency of costly default. Output is lowest in the benchmark economy ($Y = 0.28$), increases by 11% with imperfect signals (some ex-ante screening), and increases a further 3% with perfect signals (i.e. in a separating equilibrium).
In summary, the firm size distribution and misallocation depends critically on ex-ante heterogeneity not modelled in Kornejew et al. (2024). For instance, with perfect signals, ex-ante screening means that no firms ever default so bankruptcy efficiency is not even relevant. Those negative net present value firms are rationed with implications for the firm size distribution and can lead to an efficient allocation of resources.

The impact of bankruptcy policies on the firm size distribution and misallocation in a quantitative model of firm dynamics is taken up in Corbae and D’Erasmo (2021). We parameterize our model to match current bankruptcy code and conduct a counterfactual change to bankruptcy policy like that in Aghion et al. (1992). We find that: (i) better bankruptcy policy generates lower borrowing interest menu offerings, and despite higher equilibrium borrowing, lower (observed) equilibrium interest spreads, (ii) an increase in measured productivity arising from selection and a change in the firm size distribution, and (iii) measures of allocative efficiency rise closer to that of a frictionless economy.

7 Conclusion

Kornejew et al. (2024) have written a very thought provoking and interesting cross-country paper on a fundamental question: How do policies aimed at financially distressed firms affect the boom-bust cycle? The answer to this question must address the sources of inefficiency: bankruptcy policy in light of commitment frictions and incomplete information. In this discussion I have tried to extend the authors’ analysis to understand how ex-post liquidation of unprofitable firms may be efficient and how better ex-ante screening technologies can achieve efficient rationing of unprofitable firms. This raises the question of whether the authors’ empirical findings of greater aggregate stability following a credit boom is coming from better ex-post bankruptcy institutions or better ex-ante credit screening. These extensions can have important implications for the firm size distribution, allocative efficiency, and financial stability. This seems like a fruitful avenue for future research.
References


