Decentralization when tax bases are interdependently mobile: Shared or exclusive tax bases ?

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Abstract

Our paper analyzes the issue of tax base assignment of two mobile and interdependent tax bases that generate tax externalities, in a two-tier setting with multiple states and local jurisdictions. We compare three fiscal architectures: i) centralization, ii) partial decentralization with shared tax bases and iii) partial decentralization with exclusive tax bases. The interdependence between the two tax bases generates cross-base tax externalities and reinforces the standard same-base tax externalities. It results in partial decentralization with exclusive tax bases differing from other fiscal architectures in that tax externalities can lead to an inefficiently high tax rate at either tier. While there is always a level of expenditure decentralization such that partial decentralization with shared tax bases is better than full centralization, this is no longer the case with exclusive tax bases, which even leads to the lowest welfare for a sufficiently high degree of substitutability between the tax bases.

Keywords: tax externalities, multiple tax bases, fiscal decentralization, fiscal architecture, fiscal federalism

JEL Classification: H20, H40, H71, H77

1 Introduction

With the widespread decentralization of expenditures that has occurred in most OECD countries over the last thirty years, the issue of tax base assignment among different tiers of government has become even more acute, with the transfer of competencies to subnational tiers often being coupled with a transfer

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of tax power. Which tier should tax which tax base? How does the interdependence between tax bases affect equilibrium tax rates? What is the role of the degree of decentralization of public goods provision in the presence of tax externalities? Which fiscal architecture is welfare-enhancing?

In this paper, we provide an answer to these questions using a two-tier setting with multiple states and multiple local governments, and comparing the following three fiscal architectures: i) *centralization*, where the state authorities alone provide the whole range of public goods and finance them by taxation of the two tax bases, ii) *partial decentralization with shared tax bases*, where the provision of public goods is split between the two tiers and the two tax bases are co-occupied by both tiers, iii) *partial decentralization with exclusive tax bases*, where the provision of public goods is split between the two tiers and each one of the two tax bases is assigned to a different tier, with no co-occupancy of tax bases being allowed. We model the interdependence between the two tax bases by considering two mobile production factors, with the marginal productivity of one factor depending on the quantity of the other. Labor and citizens are immobile but we can consider physical capital and financial assets as examples of interdependent mobile factors.

Pioneering papers in the literature on tax base assignment have provided the following two general guidelines. First, tax bases with the highest mobility should be assigned to the highest tier (Musgrave (1983), Musgrave and Musgrave (1989), Oates (1994)). Second, the co-occupancy of tax bases must be ruled out or very limited (Flowers (1988), Dahlby (2001)) since, in a system of hierarchical governments, taxation of the same tax base by several tiers may induce vertical tax externalities (bottom-up and/or top-down) and then result in inefficient levels of tax rates when each tier neglects these externalities in setting its tax rate.[1] The formal studies, e.g. Keen (1998), supporting the recommendation of exclusive (rather than shared) tax bases rule out the fact that vertical externalities might still occur with exclusive tax bases when tax bases are interdependent.[2] Due to interdependence, tax rates have an impact across tax bases and "cross-base" horizontal and vertical tax externalities can arise, i.e. the size of one tax base in a jurisdiction may be altered by a change in tax rate on another base.[3] While tax externalities have already been analyzed in the presence of two mobile tax bases assigned to same-tier jurisdictions (Burbidge and Myers (1994); Braid (2000); Duran-Vigneron (2012)), no special focus has been given to how the nature and degree of interdependence between tax bases might affect tax externalities. Except for our paper, the only noticeable contribution which challenges the recommandation of exclusive tax bases is the one from Hoyt (2017). He examined the issue of the assignment of a continuum of uniform taxable commodities between a state and several local governments,

[1] See Conseil Des Prélèvements Obligatoires (2010) for a discussion on the practical issues associated with specialization.

[2] Note the exception of Dahlby (2001), who briefly mentioned the issue but did not provide any formal model.

[3] As opposed to "cross-base" externalities, "same-base" externalities arise when a tax base is elastic with respect to its tax rate.

and whether co-occupancy is desirable, with commodities being either gross substitutes or complements. The main conclusion of Hoyt (2017) was that co-occupancy might be optimal. Although the general question raised in our paper appears to be similar to Hoyt (2017), our model departs from his in four main respects. First, Hoyt (2017) assumed that each jurisdiction (state and local governments) raises a uniform tax on all commodities included in its tax base. This assumption is very strong as it considerably limits the emergence of cross-base externalities. In our model, considering two mobile production factors that can be subject to different tax rates gives rise to cross-base externalities whatever the fiscal architecture. Second, in line with most papers about tax externalities[4], Hoyt (2017) only considered one state, which, even with cross-border shopping, rules out horizontal externalities at the top tier and excludes the possibility of positive bottom-up vertical externalities. Third, Hoyt (2017) assumed two types of policy instruments, tax rates and tax bases, while in our model, only the tax rates can be chosen by states and local governments. However, as mentioned by the author, like in the US, the choice of tax base is usually not at the discretion of local governments, but rather of central or state governments. In our model, we thus assume the tax base assignment to be exogenous. The same applies to the level of expenditure decentralization, but unlike Hoyt (2017), we will analyze several levels of decentralization. Here lies the fourth main difference between our paper and Hoyt (2017)'s. While the latter assumed that each tier provides one type of public goods, we consider, like Wilson and Janeba (2005), that a continuum of types can be split among the two tiers. It follows that depending on how many types of public goods a jurisdiction must provide, the number of its tax bases will matter for the choice of tax rates and the interplay between the different tax externalities will be modified. We then extend the issue of tax base assignment to the more general issue of fiscal architecture.

Our analysis provides the following results. Interdependence between the two mobile tax bases not only generates cross-base tax externalities, but also increases same-base tax externalities, whatever the nature and degree of this interdependence. Complementarity of the tax bases always worsens the downward distortion of tax rates, as the cross-base tax externalities go in the same direction as the same-base tax externalities. By contrast, substitutability reduces the downward distortion of tax rates, as the cross-base tax externalities point in the opposite direction to the same-base tax externalities. For partial decentralization with exclusive tax bases and under some degrees of decentralization, it can even lead to inefficiently high –either state or local– tax rates.

While full centralization always performs better than partial decentralization when considering only one state, this is not necessarily the case in a world with several states. We show that, in some cases, partial decentralization can induce higher welfare than full centralization. This result therefore goes against most recommendations of assigning mobile tax bases to the highest tier. While there is always a level of expenditure decentralization such that the use of shared tax bases is better than full centralization, this is no longer the case with exclusive

[4] See for instance Keen (1998), Keen and Kotsogiannis (2002).

tax bases for a sufficiently high degree of substitutability between the tax bases.

This paper is organized as follows. In Section 2, we present the model and the different fiscal architectures that were considered for the analysis. Section 3 describes the tax externalities in the model and derives equilibrium tax rates. Section 4 analyzes the effect of interdependence between the tax bases on tax choices and section 5 compares tax rates, public goods provision and welfare derived from three different fiscal architectures. Section 6 concludes.

2 The model

Our world economy comprises $n \geq 1$ identical states, indexed by $i = 1, ..., n$, and within each state, $m > 1$ identical local jurisdictions, indexed by $j = 1, ..., m$. Citizens are identical and immobile within each local jurisdiction so that we focus on the behavior of a representative citizen.

2.1 The representative citizen

The representative citizen of each local jurisdiction is endowed with two production factors x and y, respectively in quantity $\overline{x}$ and $\overline{y}$. As usual in the capital tax externality literature, the representative citizen owns the single firm located in her local jurisdiction of residence but can supply the two factors to firms located in any local jurisdiction. The firm in local jurisdiction ij is immobile and produces a composite good from the two factors supplied in quantities x_{ij} and y_{ij}. The composite good can be used for private consumption c_{ij} by the citizen or purchased by the public sector to be transformed into public goods.

The representative citizen derives utility from consumption of the private good c_{ij} −which is financed by the profit $\Pi_{ij}(x_{ij}, y_{ij})$ of her firm and the net returns of their endowments in factors $\overline{x}$ and $\overline{y}-$, and from the consumption of a bundle of public goods that differ according to their type δ with $\delta \in [0, 1]$. The public goods are publicly provided private goods[5] and the marginal rate of transformation between these goods and the private good is unity. As in Wilson and Janeba (2005), we assume the preferences of the representative citizen to be given by the following additively separable log-linear utility function:

$$u_{ij} = c_{ij} + \int_0^1 \ln g_{ij}(\delta)\, d\delta$$

where $g_{ij}(\delta)$ denotes consumption of the public good of type δ. Let us note that all types of public goods δ enter the utility function in a symmetric way but are imperfect substitutes.

[5] There are no scale-economy arguments in favour of centralization, so that we can focus exclusively on the issue of fiscal architecture from a tax externality perspective.

2.2 Fiscal architecture

Tax revenues can be raised through taxes on the two mobile production factors x and y. Let t_{ij}^k be the proportional tax rate levied by local authority ij on production factor k_{ij} invested in the local jurisdiction and T_i^k be the proportional tax rate chosen by state authority i on production factor k_i invested in the state, with $k = x, y$. By construction, the state tax base k_i is the sum of the local tax bases located in its territory, i.e. $k_i = \sum_{j=1}^{m} k_{ij}$, with $k = x, y$.

Tax revenues are the only source of financing of public goods provision; no deficit is allowed.[6] The cut-off between public goods provided by the local authorities and those provided by the state authorities is denoted by D with $D \in [0, 1]$. Therefore D captures the level of decentralization in terms of expenditures. A public good of type δ is provided by the local authorities if $\delta \leq D$, while it is provided by the state authorities if $\delta > D$.[7] Due to the symmetry of the utility function with respect to the public goods and its concavity in $g_{ij}(\delta)$, each government splits its tax revenues equally between all the public goods δ it provides. Let $DG_{ij}^l = \int_0^D g_{ij}(\delta)\, d\delta$ denote the aggregate provision of public goods by local government ij and $m(1-D)\, G_i^s = \int_D^1 \left(\sum_{j=1}^{m} g_{ij}(\delta) \right) d\delta$ denote the aggregate provision of public goods by state government i.[8]

We distinguish three fiscal architectures which differ according to both i) the tax base assignment, i.e. which tier taxes which factor(s), and ii) the share of public goods provision between local jurisdictions and states:

i) *Centralization* (hereafter C), where state authorities provide all public goods δ over the interval $[0, 1]$ and finance them through the taxation of both production factors. This fiscal architecture corresponds to the case where $D = 0$. The state budget constraint is given by:

$$mG_i^s = T_i^x \sum_{j=1}^{m} x_{ij} + T_i^y \sum_{j=1}^{m} y_{ij}$$

Local authorities play no role in C: they neither provide public goods nor raise tax revenues, i.e. $G_{ij}^l = t_{ij}^x = t_{ij}^y = 0$.

ii) *Partial decentralization with Shared tax bases* (hereafter PS), where both local and state authorities provide public goods, i.e. $D \in]0, 1[$, and levy taxes on the same two tax bases. Each one of the two tax bases x and y is thus

[6] We rule out vertical transfers between the two tiers of government and horizontal transfers between jurisdictions of the same tier.

[7] We rule out provision of a given public good of type δ by both tiers simultaneously.

[8] $(1-D)\, G_i^s$ thus denotes the aggregate provision of public goods in each local jurisdiction belonging to state i.

	C			PS			PE		
	Taxation		Expenditures	Taxation		Expenditures	Taxation		Expenditures
Local tier				t^x	t^y	DG^l	t^x		DG^l
State tier	T^x	T^y	mG^s	T^x	T^y	$m(1-D)G^s$		T^y	$m(1-D)G^s$

co-occupied by both tiers. The budget constraints are given by:

$$\begin{aligned} DG_{ij}^l &= t_{ij}^x x_{ij} + t_{ij}^y y_{ij} \\ m\left(1-D\right)G_i^s &= T_i^x \sum_{j=1}^m x_{ij} + T_i^y \sum_{j=1}^m y_{ij} \end{aligned}$$

iii) *Partial decentralization with Exclusive tax bases* (hereafter PE), where both local and state authorities provide public goods, i.e. $D \in]0,1[$, and levy taxes on a separate tax base. There is thus no co-occupancy: tax base x is taxed to finance only the local public goods G_{ij}^l and tax base y is taxed to finance only the state public goods G_i^s. The budget constraints are given by:

$$\begin{aligned} DG_{ij}^l &= t_{ij}^x x_{ij} \\ m\left(1-D\right)G_i^s &= T_i^y \sum_{j=1}^m y_{ij} \end{aligned}$$

The three fiscal architectures can be summarized by the following table:

Let us note that for C and PS, there is symmetry between the two tax bases in terms of tax base assignment, i.e. if, at a given tier, a tax is levied on factor x, a tax is also levied on factor y. In contrast, in PE, no tier raises tax revenues from taxation on both x and y.

2.3 The factor markets

The market for each factor $k = x, y$ is modeled as in the literature on capital tax externality in a two-tier setting (Wrede (1997); Breuillé and Zanaj (2013)). However, we depart from the previous papers by considering two factor markets rather than one. The quantities x_{ij} and y_{ij} of the two factors invested in local jurisdiction ij are used jointly by the firm located in that same jurisdiction. All firms across the world use the same technology of production that is described by the function $F\left(x_{ij}, y_{ij}\right)$. $F\left(.,.\right)$ is twice-differentiable and concave. We thus have $F_{xx}^{ij} < 0$, $F_{yy}^{ij} < 0$ and $F_{xx}^{ij}F_{yy}^{ij} - F_{xy}^{ij}F_{yx}^{ij} > 0$.[9] The profit of the firm located

[9] We denote F_k^{ij} and F_{kk}^{ij} respectively the first and second derivatives of the production function in region ij w.r.t. input k_{ij}, with $k_{ij} = x_{ij}, y_{ij}$. Similarly, we denote F_{xy}^{ij} and F_{yx}^{ij} the cross derivatives of the production function w.r.t. the two inputs.

in local jurisdiction ij amounts to $\Pi_{ij} = F(x_{ij}, y_{ij}) - r^x_{ij}x_{ij} - r^y_{ij}y_{ij}$, where r^x_{ij} is the gross return for factor x_{ij} and r^y_{ij} is the gross return for factor y_{ij}. Firm profit maximizing behavior implies that both factors are remunerated at their marginal productivity, that is $F^{ij}_x = r^x_{ij}$ and $F^{ij}_y = r^y_{ij}$ for all i,j. The implicit demand functions are thus $x_{ij}(r^x_{ij}, r^y_{ij})$ and $y_{ij}(r^x_{ij}, r^y_{ij})$ with

$$\frac{\partial x_{ij}}{\partial r^x_{ij}} = \frac{F^{ij}_{yy}}{F^{ij}_{xx}F^{ij}_{yy} - F^{ij}_{xy}F^{ij}_{yx}} < 0 \text{ and } \frac{\partial x_{ij}}{\partial r^y_{ij}} = \frac{-F^{ij}_{xy}}{F^{ij}_{xx}F^{ij}_{yy} - F^{ij}_{xy}F^{ij}_{yx}}$$

$$\frac{\partial y_{ij}}{\partial r^y_{ij}} = \frac{F^{ij}_{xx}}{F^{ij}_{xx}F^{ij}_{yy} - F^{ij}_{xy}F^{ij}_{yx}} < 0 \text{ and } \frac{\partial y_{ij}}{\partial r^x_{ij}} = \frac{-F^{ij}_{yx}}{F^{ij}_{xx}F^{ij}_{yy} - F^{ij}_{xy}F^{ij}_{yx}}$$

When $F^{ij}_{k,-k} \neq 0$, the mobility of the two factors is interdependent. The complementarity between the inputs, i.e. $F^{ij}_{k,-k} > 0$ for $k = x, y$ (super-modular function), then results in complementarity between the tax bases and substitutability between the factors, i.e. $F^{ij}_{k,-k} < 0$ for $k = x, y$ (sub-modular function), results in the substitutability of factors. The complementarity between tax bases implies that a higher cost of factor $-k$ in jurisdiction ij reduces both demand for factor $-k$ and demand for factor k, i.e. $\frac{\partial k_{ij}}{\partial r^{-k}_{ij}} < 0$. In contrast, the substitutability between tax bases implies that a higher cost of factor $-k$ in jurisdiction ij reduces demand for factor $-k$ while it increases demand for factor k, i.e. $\frac{\partial k_{ij}}{\partial r^{-k}_{ij}} > 0$. For $F^{ij}_{xy} = F^{ij}_{yx} = 0$, the two factor markets work independently.[10]

The after-tax return of factor k is $\rho^k_{ij} = r^k_{ij} - t^k_{ij} - T^k_i$ for $k = x, y$. As each factor is perfectly mobile in the world, it moves across all local jurisdictions, and thus across states, to locate in the local jurisdiction where the net return is the highest. Perfect mobility implies that at equilibrium, the net returns for each factor are the same everywhere, i.e.,[11]

$$\begin{matrix} \rho^x = \rho^x_{ij} & \forall i,j \\ \rho^y = \rho^y_{ij} & \forall i,j \end{matrix} \iff \begin{matrix} r^x_{ij} = \rho^x + t^x_{ij} + T^x_i & \forall i,j \\ r^y_{ij} = \rho^y + t^y_{ij} + T^y_i & \forall i,j \end{matrix}$$

Supply of each factor in the world is exogenous. Aggregate supply thus amounts to $nm\overline{x}$ for factor x and $nm\overline{y}$ for factor y and the system of market-clearing conditions is:

$$\begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}(r^x_{ij}, r^y_{ij}) = nm\overline{x} \\ \sum_{i=1}^{n}\sum_{j=1}^{m} y_{ij}(r^x_{ij}, r^y_{ij}) = nm\overline{y} \end{cases}$$

[10] In that case: $\frac{\partial x_{ij}}{\partial r^x_{ij}} = \frac{1}{F^{ij}_{xx}} < 0$ and $\frac{\partial x_{ij}}{\partial r^y_{ij}} = 0$; $\frac{\partial y_{ij}}{\partial r^y_{ij}} = \frac{1}{F^{ij}_{yy}} < 0$ and $\frac{\partial y_{ij}}{\partial r^x_{ij}} = 0$. The implicit demand functions are $x_{ij}(r^x_{ij})$ and $y_{ij}(r^y_{ij})$.

[11] Our production function is thus such that we exclude the case where, at equilibrium, there is a local jurisdiction with no factor x and/or factor y.

We differentiate this system to obtain the response of ρ^k, and thus also the response of r_{ij}^k to local and state taxation (see Appendix 1). At symmetric equilibrium, we obtain:

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho^k}{\partial t_{ij}^k} = \frac{\partial r_{ij}^k}{\partial t_{ij}^k} - 1 = \frac{\partial r_{ij}^k}{\partial t_{-(ij)}^k} = -\frac{1}{nm} < 0$$

$$\frac{\partial \rho}{\partial T} = \frac{\partial \rho^k}{\partial T_i^k} = \frac{\partial r_{ij}^k}{\partial T_i^k} - 1 = \frac{\partial r_{ij}^k}{\partial T_{-i}^k} = -\frac{1}{n} < 0$$

$$\frac{\partial \rho^k}{\partial t_{ij}^{-k}} = \frac{\partial r_{ij}^k}{\partial t_{ij}^{-k}} = \frac{\partial r_{ij}^k}{\partial t_{-(ij)}^{-k}} = \frac{\partial \rho^k}{\partial T_i^{-k}} = \frac{\partial r_{ij}^k}{\partial T_i^{-k}} = \frac{\partial r_{ij}^k}{\partial T_{-i}^{-k}} = 0$$

The equilibrium values of the net returns are:

$$\rho^x \left(\mathbf{T^x}, \mathbf{t_1^x}, ..., \mathbf{t_i^x}, ..., \mathbf{t_n^x}\right) \quad \text{and} \quad \rho^y \left(\mathbf{T^y}, \mathbf{t_1^y}, ..., \mathbf{t_i^y}, ..., \mathbf{t_n^y}\right)$$

with $\mathbf{T^k} = (T_1^k, ...T_n^k)$ and $\mathbf{t_i^k} = (t_{i1}^k, ..., t_{ij}^k, ..., t_{im}^k)$ $\forall i$ and for $k = x, y$. A tax rate levied on factor x (resp. y) has no impact on the net return of factor y (resp. x) at symmetric equilibrium.[12] In addition, as shown in the tax externality literature, local taxation is more distortive than state taxation, i.e. $\frac{\partial \rho}{\partial t} > \frac{\partial \rho}{\partial T} = m\frac{\partial \rho}{\partial t}$, since horizontal tax externalities involve fewer jurisdictions at the state tier (n) than at the local tier (nm).[13]

Before deriving equilibrium tax rates chosen by local and state authorities for each fiscal architecture successively, we introduce two additional assumptions. First we assume the same supply in both factors, i.e. $\overline{x} = \overline{y} = \overline{e}$. Second, we assume that the production function is perfectly "symmetric" regarding the two factors, such that $F_{xx}^{ij} = F_{yy}^{ij}$ and $F_{xy}^{ij} = F_{yx}^{ij}$ when $x_{ij} = y_{ij}$. Let us then use the following notations at symmetric equilibrium: $F_{xx} = F_{yy} = -b\left(\overline{e}\right) \equiv -b < 0$ and $F_{yx} = F_{xy} = p\left(\overline{e}\right) \equiv p$. It follows, assuming that $b > p$:

$$\frac{\partial x_{ij}}{\partial r_{ij}^x} = \frac{\partial y_{ij}}{\partial r_{ij}^y} = -\frac{b}{(b^2 - p^2)} < 0 \text{ and } \frac{\partial x_{ij}}{\partial r_{ij}^y} = \frac{\partial y_{ij}}{\partial r_{ij}^x} = -\frac{p}{(b^2 - p^2)}.$$

These assumptions allow us to focus exclusively on the role of tax externalities in the tax decisions in the context of two mobile tax bases.

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[12] When $F_{k,-k}^{ij} = 0$, i.e. the mobility of the two factors is not interdependent, the outflow of one factor from a jurisdiction does not affect the allocation of the other factor, and thus $\frac{\partial k_{ij}}{\partial r_{ij}^{-k}} = 0$ which leads to $\frac{\partial \rho^k}{\partial t_{ij}^{-k}} = \frac{\partial \rho^k}{\partial T_i^{-k}} = \frac{\partial r_{ij}^k}{\partial t_{ij}^{-k}} = \frac{\partial r_{ij}^k}{\partial T_i^{-k}} = 0$.

When $F_{k,-k}^{ij} \neq 0$, a change in t^{-k} modifies the allocation of factor $-k$ and thus the allocation of factor k (through marginal productivity). Finally, the reallocation of each factor affects the gross return of k in opposite directions. The two effects cancel each other out, such that $\frac{\partial \rho^k}{\partial t_{ij}^{-k}} = \frac{\partial \rho^k}{\partial T_i^{-k}} = \frac{\partial r_{ij}^k}{\partial t_{ij}^{-k}} = \frac{\partial r_{ij}^k}{\partial T_i^{-k}} = 0$.

[13] In the extreme case of a single state, horizontal tax externalities disappear at state level. For $n = 1$, $\frac{\partial \rho^k}{\partial T_i^k} = -1$ and thus $\frac{\partial r_{ij}^k}{\partial T_i^k} = 0$. Since aggregate supply of factor is inelastic, a change in the tax rate of the single state does not affect the tax base.

3 Equilibrium

3.1 Description of the game

Local and state governments are both benevolent. They play a Nash game together. Local authorities simultaneously select their tax policy to maximize the welfare of their representative citizen, taking tax policies chosen by the other local jurisdictions and the states as given. State authorities simultaneously select their tax policy to maximize the sum of the welfare of the representative citizens from the local jurisdictions belonging to their territory, taking tax policies chosen by the other states and the local jurisdictions as given. When they choose their tax strategy, local and state authorities take into account the mobility of both factors x and y. Public goods are determined as residuals, after taxes are collected. Given these tax policies, migration of factors and then production take place. Finally, profits are distributed, and citizens enjoy the consumption of both private and public goods. These last two stages are implicitly introduced in our analysis.

3.2 The optimization problem

3.2.1 Centralization

We first analyze the case where the only tier is the state one. The state governments provide all the public goods $\delta \in [0,1]$ and can tax both factors x and y. The optimization problem of the state government i is:

$$\max_{T_i^x, T_i^y} \sum_{j=1}^{m} \left(\Pi_{ij}(x_{ij}, y_{ij}) + \rho^x \overline{x} + \rho^y \overline{y} + \ln \left(\frac{T_i^x}{m} \sum_{j=1}^{m} x_{ij} + \frac{T_i^y}{m} \sum_{j=1}^{m} y_{ij} \right) \right)$$

We obtain the following first-order conditions:

FOC $/T_i^x$:

$$\begin{aligned} & \sum_{j=1}^{m} \left[-\left(\frac{\partial \rho^x}{\partial T_i^x} + 1 \right) x_{ij} + \frac{\partial \rho^x}{\partial T_i^x} \overline{x} \right] \\ + \frac{1}{G_i^s} & \left(\sum_{j=1}^{m} x_{ij} + T_i^x \sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r_{ij}^x} \frac{\partial r_{ij}^x}{\partial T_i^x} + T_i^y \sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r_{ij}^x} \frac{\partial r_{ij}^x}{\partial T_i^x} \right) = 0 \end{aligned} \tag{1}$$

FOC $/T_i^y$:

$$\begin{aligned} & \sum_{j=1}^{m} \left[-\left(\frac{\partial \rho^y}{\partial T_i^y} + 1 \right) y_{ij} + \frac{\partial \rho^y}{\partial T_i^y} \overline{y} \right] \\ + \frac{1}{G_i^s} & \left(\sum_{j=1}^{m} y_{ij} + T_i^x \sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial T_i^y} + T_i^y \sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial T_i^y} \right) = 0 \end{aligned} \tag{2}$$

3.2.2 Partial decentralization with shared tax bases

Local and state tiers coexist and both tiers share the two tax bases x and y. Local governments can levy tax rates t^x and t^y on each tax base, in order to finance the provision of public goods $\delta \leq D$. State governments simultaneously select their additional tax rates T^x and T^y on each tax base, in order to finance the provision of public goods $\delta > D$.

Program of local government ij

$$\begin{aligned}
&\max_{t_{ij}^x, t_{ij}^y} \quad \Pi_{ij}(x_{ij}, y_{ij}) + \rho^x \overline{x} + \rho^y \overline{y} + D \ln \left(\frac{t_{ij}^x x_{ij} + t_{ij}^y y_{ij}}{D} \right) \\
&+ (1 - D) \ln \left(\frac{T_i^x}{m(1-D)} \sum_{h=1}^m x_{ih} + \frac{T_i^y}{m(1-D)} \sum_{h=1}^m y_{ih} \right)
\end{aligned}$$

We obtain the following first-order conditions:

FOC $/t_{ij}^x$:

$$\begin{aligned}
-\left(\frac{\partial \rho^x}{\partial t_{ij}^x} + 1 \right) x_{ij} + \frac{\partial \rho^x}{\partial t_{ij}^x} \overline{x} + \frac{1}{G_{ij}^l} \left(x_{ij} + t_{ij}^x \frac{\partial x_{ij}}{\partial r_{ij}^x} \frac{\partial r_{ij}^x}{\partial t_{ij}^x} + t_{ij}^y \frac{\partial y_{ij}}{\partial r_{ij}^x} \frac{\partial r_{ij}^x}{\partial t_{ij}^x} \right) & \\
+ \frac{1}{G_i^s} \left(T_i^x \left(\frac{1}{m} \sum_{h=1}^m \frac{\partial x_{ih}}{\partial r_{ih}^x} \frac{\partial r_{ih}^x}{\partial t_{ij}^x} \right) + T_i^y \left(\frac{1}{m} \sum_{h=1}^m \frac{\partial y_{ih}}{\partial r_{ih}^x} \frac{\partial r_{ih}^x}{\partial t_{ij}^x} \right) \right) &= 0
\end{aligned} \tag{3}$$

FOC $/t_{ij}^y$:

$$\begin{aligned}
-\left(\frac{\partial \rho^y}{\partial t_{ij}^y} + 1 \right) y_{ij} + \frac{\partial \rho^y}{\partial t_{ij}^y} \overline{y} + \frac{1}{G_{ij}^l} \left(y_{ij} + t_{ij}^x \frac{\partial x_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial t_{ij}^y} + t_{ij}^y \frac{\partial y_{ij}}{\partial r_{ij}^y} \frac{\partial r_{ij}^y}{\partial t_{ij}^y} \right) & \\
+ \frac{1}{G_i^s} \left(T_i^x \left(\frac{1}{m} \sum_{h=1}^m \frac{\partial x_{ih}}{\partial r_{ih}^y} \frac{\partial r_{ih}^y}{\partial t_{ij}^y} \right) + T_i^y \left(\frac{1}{m} \sum_{h=1}^m \frac{\partial y_{ih}}{\partial r_{ih}^y} \frac{\partial r_{ih}^y}{\partial t_{ij}^y} \right) \right) &= 0
\end{aligned} \tag{4}$$

Program of state government i

$$\max_{T_i^x, T_i^y} \sum_{j=1}^m \left(\begin{array}{c} \Pi_{ij}(x_{ij}, y_{ij}) + \rho^x \overline{x} + \rho^y \overline{y} + \overline{D} \ln \left(\frac{t_{ij}^x x_{ij} + t_{ij}^y y_{ij}}{\overline{D}} \right) \\ + \left(1 - \overline{D}\right) \ln \left(\frac{T_i^x}{m\left(1-\overline{D}\right)} \sum_{j=1}^m x_{ij} + \frac{T_i^y}{m\left(1-\overline{D}\right)} \sum_{j=1}^m y_{ij} \right) \end{array} \right)$$

We obtain the following first-order conditions:

FOC $/T_i^x$:

$$\sum_{j=1}^{m}\left[-\left(\frac{\partial\rho^x}{\partial T_i^x}+1\right)x_{ij}+\frac{\partial\rho^x}{\partial T_i^x}\overline{x}+\frac{1}{G_{ij}^l}\left(t_{ij}^x\frac{\partial x_{ij}}{\partial r_{ij}^x}\frac{\partial r_{ij}^x}{\partial T_i^x}+t_{ij}^y\frac{\partial y_{ij}}{\partial r_{ij}^x}\frac{\partial r_{ij}^x}{\partial T_i^x}\right)\right] \tag{5}$$
$$+\frac{1}{G_i^s}\left(\sum_{j=1}^{m}x_{ij}+T_i^x\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r_{ij}^x}\frac{\partial r_{ij}^x}{\partial T_i^x}+T_i^y\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r_{ij}^x}\frac{\partial r_{ij}^x}{\partial T_i^x}\right) = 0$$

FOC $/T_i^y$:

$$\sum_{j=1}^{m}\left[-\left(\frac{\partial\rho^y}{\partial T_i^y}+1\right)y_{ij}+\frac{\partial\rho^y}{\partial T_i^y}\overline{y}+\frac{1}{G_{ij}^l}\left(t_{ih}^x\frac{\partial x_{ij}}{\partial r_{ij}^y}\frac{\partial r_{ij}^y}{\partial T_i^y}+t_{ij}^y\frac{\partial y_{ij}}{\partial r_{ij}^y}\frac{\partial r_{ij}^y}{\partial T_i^y}\right)\right] \tag{6}$$
$$+\frac{1}{G_i^s}\left(\sum_{j=1}^{m}y_{ij}+T_i^x\sum_{j=1}^{m}\frac{\partial x_{ij}}{\partial r_{ij}^y}\frac{\partial r_{ij}^y}{\partial T_i^y}+T_i^y\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r_{ij}^y}\frac{\partial r_{ij}^y}{\partial T_i^y}\right) = 0$$

3.2.3 Partial decentralization with exclusive tax bases

Two tiers coexist and each tier of jurisdiction taxes a different tax base. Local governments levy a tax rate t^x on tax base x to finance the provision of public goods $\delta \leq D$, while state governments levy a tax rate T^y on the tax base y to finance the provision of public goods $\delta > D$.

Program of local government ij

$$\max_{t_{ij}^x}\ \Pi_{ij}(x_{ij},y_{ij})+\rho^x\overline{x}+\rho^y\overline{y}+D\ln\left(\frac{t_{ij}^x x_{ij}}{D}\right)+(1-D)\ln\left(\frac{T_i^y}{m\,(1-D)}\sum_{h=1}^{m}y_{ih}\right)$$

We obtain the following first-order condition:
FOC $/t_{ij}^x$:

$$-\left(\frac{\partial\rho^x}{\partial t_{ij}^x}+1\right)x_{ij}+\frac{\partial\rho^x}{\partial t_{ij}^x}\overline{x}+\frac{1}{G_{ij}^l}\left(x_{ij}+t_{ij}^x\frac{\partial x_{ij}}{\partial r_{ij}^x}\frac{\partial r_{ij}^x}{\partial t_{ij}^x}\right) \tag{7}$$
$$+\frac{T_i^y}{G_i^s}\left(\frac{1}{m}\sum_{h=1}^{m}\frac{\partial y_{ih}}{\partial r_{ih}^x}\frac{\partial r_{ih}^x}{\partial t_{ij}^x}\right) = 0$$

Program of state government i

$$\max_{T_i^y}\ \sum_{j=1}^{m}\left(\Pi_{ij}(x_{ij},y_{ij})+\rho^x\overline{x}+\rho^y\overline{y}+D\ln\left(\frac{t_{ij}^x x_{ij}}{D}\right)+(1-D)\ln\left(\frac{T_i^y}{m\,(1-D)}\sum_{j=1}^{m}y_{ij}\right)\right)$$

We obtain the following first-order condition:

FOC $/T_i^y$:

$$\sum_{j=1}^{m}\left[-\left(\frac{\partial\rho^y}{\partial T_i^y}+1\right)y_{ij}+\frac{\partial\rho^y}{\partial T_i^y}\overline{y}+\frac{1}{G_{ij}^l}\left(t_{ij}^x\frac{\partial x_{ij}}{\partial r_{ij}^y}\frac{\partial r_{ij}^y}{\partial T_i^y}\right)\right] + \frac{1}{G_i^s}\left(\sum_{j=1}^{m}y_{ij}+T_i^y\sum_{j=1}^{m}\frac{\partial y_{ij}}{\partial r_{ij}^y}\frac{\partial r_{ij}^y}{\partial T_i^y}\right) = 0 \quad (8)$$

3.3 Tax externalities

In our model, tax externalities can be defined along two dimensions: i) horizontal *versus* vertical externalities, i.e. externalities among authorities at the same tier *versus* externalities among authorities at two different tiers, ii) same-base *versus* cross-base externalities, i.e. externalities due to the migration of a tax base k arising from a modification of a tax rate on this base (t^k or T^k), *versus* externalities due to the migration of a tax base k arising from a modification of the tax rate on the other tax base (t^{-k} or T^{-k}). Cross-base externalities only occur when $F_{xy}^{ij} = F_{yx}^{ij} \neq 0$, i.e. when the demand for a factor is affected by the taxation of the other factor.

3.3.1 The four types of externalities

Four different types of externalities are thus at work in our model:

Same-base horizontal tax externalities (SH). Same-base horizontal tax externalities arise in every fiscal architecture: i) at the state level on x and y under centralization, ii) at the local level on x and at the state level on y under PE, iii) at both levels on both x and y under PS. An increase in the tax rate levied on a factor by a jurisdiction induces an outflow of this factor from the jurisdiction and thus an inflow to all other same-tier jurisdictions. We see from the FOCs that jurisdictions do not take into account the horizontal tax externalities, but only the distortive effect of taxation on their own tax base, i.e. respectively $\varepsilon_l^{k,SH}$ for local jurisdictions and $\varepsilon_s^{k,SH}$ for states:[14]

$$\varepsilon_l^{k,SH} = \frac{\partial k_{ij}}{\partial r_{ij}^k}\frac{\partial r_{ij}^k}{\partial t_{ij}^k} < 0, \qquad \varepsilon_s^{k,SH} = \sum_{j=1}^{m}\frac{\partial k_{ij}}{\partial r_{ij}^k}\frac{\partial r_{ij}^k}{\partial T_i^k} < 0 \quad \text{with } k = x, y$$

Same-base vertical tax externalities (SV). Same-base vertical tax externalities arise when a tax base is shared by the two tiers, which is only the case in PS. An increase in the tax rate levied on a factor by a jurisdiction induces an outflow of this factor from the jurisdiction(s) sharing the same tax base at the other tier and thus an inflow to all other jurisdictions of this other

[14] Let us note that due to the fixed supply of factors within the world, same-base horizontal tax externalities on all the other local jurisdictions (resp. states) arising from a change of t_{ij}^k (resp. T_i^k) are positive, equal to $-\varepsilon_l^{k,SH}$ (resp. $-\varepsilon_s^{k,SH}$).

tier. Externalities induced by local taxation are *bottom-up* tax externalities and externalities induced by state taxation are *top-down* tax externalities.

We see from the FOCs that within a state, local jurisdictions internalize only a proportion $\frac{1}{m}$ of the same-base vertical bottom-up externalities imposed on the tax base of their state, which is denoted by $\varepsilon_l^{k,SV}$, since they only care about the welfare of their representative citizen. On the contrary, state authorities fully internalize same-base vertical top-down externalities imposed on the tax base of their local jurisdictions, which are denoted by $\varepsilon_s^{k,SV}$, since they care about the welfare of all the citizens of their local jurisdictions.[15]

$$\varepsilon_l^{k,SV} = \frac{1}{m}\sum_{h=1}^{m}\frac{\partial k_{ih}}{\partial r_{ih}^k}\frac{\partial r_{ih}^k}{\partial t_{ij}^k} < 0, \qquad \varepsilon_s^{k,SV} = \sum_{j=1}^{m}\frac{\partial k_{ij}}{\partial r_{ij}^k}\frac{\partial r_{ij}^k}{\partial T_i^k} < 0 \quad \text{with } k = x, y$$

Cross-base horizontal tax externalities (CH). Cross-base horizontal tax externalities occur when, at a given tier, jurisdictions levy taxes on two factors that are interdependently mobile ($F_{xy}^{ij} = F_{yx}^{ij} \neq 0$), which is the case in C and in PS. An increase in the tax rate raised by a jurisdiction on a factor affects the amount of the other factor available to all other same-tier jurisdictions.

As for the same-base horizontal externalities, we see from the FOCs that a jurisdiction only cares about the cross-base externality on its tax base, respectively $\varepsilon_l^{k,CH}$ for local jurisdictions and $\varepsilon_s^{k,CH}$ for states, and neglects the externalities imposed on the tax base of the other same-tier jurisdictions, respectively $-\varepsilon_l^{k,CH}$ and $-\varepsilon_s^{k,CH}$:

$$\varepsilon_l^{k,CH} = \frac{\partial (-k)_{ij}}{\partial r_{ij}^k}\frac{\partial r_{ij}^k}{\partial t_{ij}^k}, \qquad \varepsilon_s^{k,CH} = \sum_{j=1}^{m}\frac{\partial (-k)_{ij}}{\partial r_{ij}^k}\frac{\partial r_{ij}^k}{\partial T_i^k} \quad \text{with } k = x, y$$

The sign of the cross-base horizontal tax externalities depends on the nature of the interdependence between the two tax bases. The externality on the jurisdiction's own tax base is negative, pointing in the same direction as the same-base horizontal one when tax bases are complementary, i.e. $\varepsilon_l^{k,CH} < 0$ and $\varepsilon_s^{k,CH} < 0$, while it is positive, pointing in opposite direction to the same-base horizontal one when tax bases are substitutable, i.e. $\varepsilon_l^{k,CH} > 0$ and $\varepsilon_s^{k,CH} > 0$.

Cross-base vertical tax externalities (CV). Cross-base vertical tax externalities occur when, in a two-tier setting, a tier levies taxes on one or both factors that are interdependently mobile ($F_{xy}^{ij} = F_{yx}^{ij} \neq 0$), thus in PS and PE. In this case, an increase in the tax raised by a jurisdiction on a factor affects the amount of the other factor available to the other-tier jurisdictions. When $F_{xy}^{ij} = F_{yx}^{ij} > 0$, the externalities correspond to an outflow of factor from the jurisdiction(s) sharing the same tax base to all other jurisdictions.

[15] As the supply of factors is fixed within the country, the externalities imposed by a local jurisdiction on other tax revenues of the state are of the opposite sign, equal to $-m\varepsilon_l^{k,SV}$. Similarly, the externalities imposed by a state on the tax revenues of other states' local jurisdictions are of the opposite sign, equal to $-\varepsilon_s^{k,SV}$.

	Same-base	Cross-base
Horizontal externalities	C, PS, PE	C, PS
Vertical externalities	PS	PS,PE

When $F_{xy}^{ij} = F_{yx}^{ij} < 0$, the externalities correspond to an inflow of factor to the jurisdiction(s) sharing the same tax base. These externalities are called cross-base vertical bottom-up externalities when arising from local taxation and cross-base vertical top-down externalities when arising from state taxation.

We see from the FOCs that, as for the same-base vertical externalities, local jurisdictions internalize a proportion $\frac{1}{m}$ of the cross-base vertical bottom-up externalities imposed on their state, denoted by $\varepsilon_l^{k,CV}$, and states internalize all cross-base vertical top-down externalities, denoted by $\varepsilon_s^{k,CV}$, imposed on their local jurisdictions.

$$\varepsilon_l^{k,CV} = \frac{1}{m}\sum_{h=1}^{m}\frac{\partial\left(-k\right)_{ih}}{\partial r_{ih}^k}\frac{\partial r_{ih}^k}{\partial t_{ij}^k}, \qquad \varepsilon_s^{k,CV} = \sum_{j=1}^{m}\frac{\partial\left(-k\right)_{ij}}{\partial r_{ij}^k}\frac{\partial r_{ij}^k}{\partial T_i^k} \text{ with } k = x, y$$

The sign of the cross-base vertical tax externalities depends on the nature of the interdependence between the two tax bases. The cross-base vertical tax externalities internalized by a jurisdiction are negative, pointing in the same direction as the same-base vertical ones when tax bases are complementary, i.e. $\varepsilon_l^{k,CV} < 0$ and $\varepsilon_s^{k,CV} < 0$, while they are positive, pointing in opposite direction to the same-base vertical ones when tax bases are substitutable, i.e. $\varepsilon_l^{k,CV} > 0$ and $\varepsilon_s^{k,CV} > 0$.

To sum up, the nature of externalities depends on the interdependence between the tax bases and on the fiscal architecture which we summarize with the following table:

3.3.2 Externalities at symmetric equilibrium

At symmetric equilibrium, the expressions of the externalities simplify into:

$$\begin{cases} \varepsilon_l^{k,SH} = -\frac{\partial r_{ij}^k}{\partial t_{ij}^k} \frac{b}{(b^2-p^2)} = -\frac{nm-1}{nm} \frac{b}{(b^2-p^2)} \equiv \varepsilon_l^{SH} < 0 \\ \varepsilon_s^{k,SH} = -\left(\sum_{j=1}^{m} \frac{\partial r_{ij}^k}{\partial T_i^k} \right) \frac{b}{(b^2-p^2)} = -\frac{m(n-1)}{n} \frac{b}{(b^2-p^2)} \equiv \varepsilon_s^{SH} < 0 \end{cases}$$

$$\begin{cases} \varepsilon_l^{k,SV} = -\frac{1}{m} \left(\frac{\partial r_{ij}^k}{\partial t_{ij}^k} + (m-1) \frac{\partial \rho^k}{\partial t_{ij}^k} \right) \frac{b}{(b^2-p^2)} = -\frac{n-1}{nm} \frac{b}{(b^2-p^2)} \equiv \varepsilon_l^{SV} < 0 \\ \varepsilon_s^{k,SV} = -\left(\sum_{j=1}^{m} \frac{\partial r_{ij}^k}{\partial T_i^k} \right) \frac{b}{(b^2-p^2)} = -\frac{m(n-1)}{n} \frac{b}{(b^2-p^2)} \equiv \varepsilon_s^{SV} < 0 \end{cases}$$

$$\begin{cases} \varepsilon_l^{k,CH} = -\frac{\partial r_{ij}^k}{\partial t_{ij}^k} \frac{p}{(b^2-p^2)} = -\frac{nm-1}{nm} \frac{p}{(b^2-p^2)} \equiv \varepsilon_l^{CH} \\ \varepsilon_s^{k,CH} = -\left(\sum_{j=1}^{m} \frac{\partial r_{ij}^k}{\partial T_i^k} \right) \frac{p}{(b^2-p^2)} = -\frac{m(n-1)}{n} \frac{p}{(b^2-p^2)} \equiv \varepsilon_s^{CH} \end{cases}$$

$$\begin{cases} \varepsilon_l^{k,CV} = -\frac{1}{m} \left(\frac{\partial r_{ij}^k}{\partial t_{ij}^k} + (m-1) \frac{\partial \rho^k}{\partial t_{ij}^k} \right) \frac{p}{(b^2-p^2)} = -\frac{n-1}{nm} \frac{p}{(b^2-p^2)} \equiv \varepsilon_l^{CV} \\ \varepsilon_s^{k,CV} = -\left(\sum_{j=1}^{m} \frac{\partial r_{ij}^k}{\partial T_i^k} \right) \frac{p}{(b^2-p^2)} = -\frac{m(n-1)}{n} \frac{p}{(b^2-p^2)} \equiv \varepsilon_s^{CV} \end{cases}$$

with $k = x, y$.

As the state tax base is the sum of the tax bases of the m symmetric local jurisdictions that belong to that same state, the horizontal and vertical externalities internalized by a state authority are the same, i.e., $\varepsilon_s^{SH} = \varepsilon_s^{SV}$ and $\varepsilon_s^{CH} = \varepsilon_s^{CV}$. This is not the case for local taxation, as we have $\varepsilon_l^{SV} = \frac{1}{m}\left(\varepsilon_l^{SH} + \sum_{h \neq j} \frac{\partial k_{ih}}{\partial r_{ih}^k} \frac{\partial r_{ih}^k}{\partial t_{ij}^k}\right)$ and $\varepsilon_l^{CV} = \frac{1}{m}\left(\varepsilon_l^{CH} + \sum_{h \neq j} \frac{\partial (-k)_{ih}}{\partial r_{ih}^k} \frac{\partial r_{ih}^k}{\partial t_{ij}^k}\right)$. A local jurisdiction ij not only internalizes the negative externalities ε_l^{SH} and ε_l^{CH} on its own tax bases but also the externalities on the tax bases of each other local jurisdiction of the state, i.e. $\sum_{h \neq j} \frac{\partial k_{ih}}{\partial r_{ih}^k} \frac{\partial r_{ih}^k}{\partial t_{ij}^k}$ and $\sum_{h \neq j} \frac{\partial (-k)_{ih}}{\partial r_{ih}^k} \frac{\partial r_{ih}^k}{\partial t_{ij}^k}$.

In the particular case of a single top-tier jurisdiction in a two-tier setting, i.e. $n = 1$, both horizontal externalities at the state tier ($\varepsilon_s^{SH} = \varepsilon_s^{DH} = 0$) and vertical bottom-up and top-down externalities ($\varepsilon_l^{SV} = \varepsilon_s^{SV} = \varepsilon_l^{CV} = \varepsilon_s^{CV} = 0$) disappear due to the fixed supply of factors, which is similar to the mechanism described by Keen and Kotsogiannis (2002). With more than one top-tier jurisdiction, although the supply of factors is fixed for the world, it is flexible from the state's perspective, and vertical tax externalities as well as horizontal tax externalities occur at each tier.

3.4 Equilibrium tax rates

At symmetric equilibrium, we get the following tax rates[16]:

- for centralization (C):

$$T^C = T^{xC} = T^{yC} = \frac{1}{2\overline{e} - \frac{\varepsilon_s^{SH} + \varepsilon_s^{CH}}{m\overline{e}}} = \frac{1}{2\overline{e} + \frac{n-1}{n(b-p)\overline{e}}} \tag{9}$$

- for partial decentralization with shared tax bases (PS):

$$\begin{aligned} t^{PS} = t^{xPS} = t^{yPS} &= \frac{D}{2\overline{e} - \frac{D\left(\varepsilon_l^{SH} + \varepsilon_l^{CH}\right)}{\overline{e}} - \frac{(1-D)\left(\varepsilon_l^{SV} + \varepsilon_l^{CV}\right)}{\overline{e}}} \\ &= \frac{D}{2\overline{e} + \frac{D(nm-1)}{nm(b-p)\overline{e}} + \frac{(1-D)(n-1)}{nm(b-p)\overline{e}}} \end{aligned} \tag{10}$$

$$\begin{aligned} T^{PS} = T^{xPS} = T^{yPS} &= \frac{(1-D)}{2\overline{e} - \frac{(1-D)\left(\varepsilon_s^{SH} + \varepsilon_s^{CH}\right)}{m\overline{e}} - \frac{D\left(\varepsilon_s^{SV} + \varepsilon_s^{CV}\right)}{m\overline{e}}} \\ &= \frac{(1-D)}{2\overline{e} + \frac{n-1}{n(b-p)\overline{e}}} \end{aligned} \tag{11}$$

- for partial decentralization with exclusive tax bases (PE):

$$\begin{aligned} t^{PE} = t^{xPE} &= \frac{D}{\overline{e} - \frac{D\varepsilon_l^{SH}}{\overline{e}} - \frac{(1-D)\varepsilon_l^{CV}}{\overline{e}}} \\ &= \frac{D}{\overline{e} + \frac{D(mn-1)}{mn(b^2-p^2)\overline{e}} b + \frac{(1-D)(n-1)}{mn(b^2-p^2)\overline{e}} p} \end{aligned} \tag{12}$$

$$\begin{aligned} T^{PE} = T^{yPE} &= \frac{(1-D)}{\overline{e} - \frac{(1-D)\varepsilon_s^{SH}}{m\overline{e}} - \frac{D\varepsilon_s^{CV}}{m\overline{e}}} \\ &= \frac{(1-D)}{\overline{e} + \frac{n-1}{n(b^2-p^2)\overline{e}}((1-D)b + Dp)} \end{aligned} \tag{13}$$

The levels of equilibrium tax rates (9-13) are potentially influenced by:

- the share of public good provided by the tier where the tax is levied, which is captured by the numerator.
- the aggregate tax base per capita available at the tier where the tax is levied, i.e. $2\overline{e}$ in C and PS and $\overline{e}$ in PE.
- the weighted horizontal tax externalities internalized (e.g. $\frac{D\left(\varepsilon_l^{SH} + \varepsilon_l^{CH}\right)}{\overline{e}}$ for t^{PS}). The wider the range of public goods provided by the tier, the stronger the horizontal tax externality effect.

[16] When there is symmetry between the two tax bases in terms of tax base assignment, the tax rates set on both tax bases by a given tier for a given fiscal architecture are identical, i.e. $T^{xC} = T^{yC} = T^C$, $t^{xPS} = t^{yPS} = t^{PS}$ and $T^{xPS} = T^{yPS} = T^{PS}$. This comes from the fact that the two factors enter the production function symmetrically and the endowment in factors is the same, i.e. $\overline{x} = \overline{y} = \overline{e}$.

- the weighted vertical tax externalities internalized (e.g. $\frac{(1-D)\left(\varepsilon_l^{SV}+\varepsilon_l^{CV}\right)}{\overline{e}}$ for t^{PS}). The smaller the range of public goods provided by the other tier, the smaller the vertical tax externality effect.

Under our assumption that $F_{xx}^{ij}F_{yy}^{ij} - F_{xy}^{ij}F_{yx}^{ij} = b^2 - p^2 > 0$, same-base tax externalities are always larger in absolute terms than their cross-base counterparts, i.e. $\varepsilon_z^{SH} + \varepsilon_z^{CH} < 0$, $\varepsilon_z^{SV} + \varepsilon_z^{CV} < 0$ with $z = l, s$, which implies that all equilibrium tax rates in C and PS are positive.

In PE, horizontal tax externalities only arise from same-base externalities (due to the absence of simultaneous taxation by a given tier of both tax bases) and vertical tax externalities only arise from cross-base vertical externalities (due to the absence of tax base co-occupation). The positivity of the two tax rates t^{PE} and T^{PE} depends on the relative magnitude of the horizontal and vertical tax externality effects and is always satisfied for values of the level of decentralization D within the interval $\left]\frac{-\overline{e}^2+\varepsilon_l^{CV}}{-\left(\varepsilon_l^{SH}-\varepsilon_l^{CV}\right)};\frac{m\overline{e}^2-\varepsilon_s^{SH}}{-\left(\varepsilon_s^{DH}-\varepsilon_s^{CV}\right)}\right[$.[17] Hereafter, this condition is assumed to be satisfied to ensure the existence of a Nash equilibrium.[18]

4 How does tax base interdependence affect tax decisions?

In the absence of same-base and cross-base externalities, that is with immobile tax bases, jurisdictions would choose the following optimal tax rates: $T^{*C} = \frac{1}{2\overline{e}}$, $t^{*PS} = \frac{D}{2\overline{e}}$, $T^{*PS} = \frac{(1-D)}{2\overline{e}}$, $t^{*PE} = \frac{D}{\overline{e}}$ and $T^{*PE} = \frac{(1-D)}{\overline{e}}$.

With mobile but independent tax bases, same-base horizontal externalities (in C, PS and PE) and same-base vertical externalities (in PS) both lead to a downward distortion of optimal tax rates. Since jurisdictions neglect the positive same-base horizontal externalities on all the other same-tier jurisdictions arising from an increase in their tax rate, the standard outcome of the competitive horizontal game is a race to the bottom, with inefficiently low equilibrium tax rates (Wilson (1986), Zodrow and Mieszkowski (1986)). This effect is reinforced in PS as jurisdictions also neglect the positive vertical externalities outside the state.

The interdependence between the two mobile tax bases, i.e. for $F_{yx} = F_{xy} = p \neq 0$, affects the tax externalities in two ways. First, it generates cross-base

[17] Equilibrium tax rates in PE are positive whatever the level of decentralization $D^{PE} \in]0,1[$ in three circumstances: i) no cross-base vertical tax externalities occur, i.e. the tax bases are independent or there is only one top-tier jurisdiction ($n = 1$), ii) cross-base vertical tax externalities point in the same direction as same-base horizontal tax externalities, i.e. the tax bases are complementary, iii) cross-base vertical tax externalities point in opposite direction to same-base horizontal tax externalities but are sufficiently small, i.e. the degree of substitutability between the tax bases is sufficiently small.

[18] Given the form of the utility function of the representative citizen, the existence of a Nash equilibrium requires the provision of all public goods to be strictly positive.

(horizontal and/or vertical) tax externalities. Second, it amplifies the same-base (horizontal and/or vertical) tax externalities. Indeed, when a jurisdiction increases its tax rate on a factor k, the modification of the allocation of the other factor reinforces the outflow of factor k via its effect on marginal productivity, whatever the sign of p. The higher the degree of interdependence $|p|$ between the two tax bases (whether complementary or substitutable), the higher the magnitude of both same-base and cross-base externalities, i.e. $\frac{\partial \left|\varepsilon_z^{SH}\right|}{\partial |p|} > 0$, $\frac{\partial \left|\varepsilon_z^{SV}\right|}{\partial |p|} > 0$, $\frac{\partial \left|\varepsilon_z^{CH}\right|}{\partial |p|} > 0$ and $\frac{\partial \left|\varepsilon_z^{CV}\right|}{\partial |p|} > 0$ with $z = l, s$.

Lemma 1 *Interdependence between the two mobile tax bases not only generates cross-base tax externalities but also increases the magnitude of the same-base tax externalities, whatever the nature and the degree of this interdependence.*

When tax bases are complementary ($p > 0$), same-base and cross-base externalities reinforce each other, thereby worsening the race to the bottom of tax rates. This downward distortion increases with the degree of complementarity, i.e. $\frac{\partial t}{\partial p} < 0$ and $\frac{\partial T}{\partial p} < 0$, in all fiscal architectures.

In contrast, when tax bases are substitutable ($p < 0$), same-base and cross-base externalities push tax rates in opposite directions. In C and PS, the same-base tax externalities always exceed the cross-base tax externalities in absolute terms, thus leading to inefficiently low tax rates. Moreover, the higher the degree of substitutability, the less downward distorted the equilibrium tax rates, due to $\frac{\partial \left|\varepsilon_z^{SH}\right|}{\partial |p|} < \frac{\partial \left|\varepsilon_z^{CH}\right|}{\partial |p|}$ and $\frac{\partial \left|\varepsilon_z^{SV}\right|}{\partial |p|} < \frac{\partial \left|\varepsilon_z^{CV}\right|}{\partial |p|}$ with $z = l, s$. In PE, the effect of the interdependence depends on whether the (same-base) horizontal tax externality effect exceeds the (cross-base) vertical tax externality effect, which in turn depends on the level of decentralization D as well as on the degree of substitutability. PE differs from other fiscal architectures in that it may lead to inefficiently high tax rates at either tier. For $D < \frac{-\varepsilon_l^{CV}}{\varepsilon_l^{SH} - \varepsilon_l^{CV}}$ (resp. $D > \frac{\varepsilon_s^{SH}}{\varepsilon_s^{SH} - \varepsilon_s^{CV}}$), the vertical tax externality effect dominates at the local (resp. state) tier only.[19] The local (resp. state) tax rate is thus upward distorted while the state (resp. local) tax rate is downward distorted. The overall impact of the degree of substitutability on tax rates then depends on the level of decentralization D as depicted in figures 1 and 2 for given values of parameters n, m, b and $\overline{e}$. Figure 1 corresponds to the case of $D \in \left[\frac{n-1}{(nm-1)+(n-1)}, \frac{1}{2}\right]$, i.e. tax rates are always downward distorted ($t^{PE} < t^{*PE}$ and $T^{PE} < T^{*PE}$, $\forall p$), while figure 2 corresponds to the case of $D > \frac{1}{2}$, i.e. an upward distortion occurs at the state tier for a sufficiently high degree of substitutability.[20]

[19] $\frac{-\varepsilon_l^{CV}}{\varepsilon_l^{SH} - \varepsilon_l^{CV}} < \frac{\varepsilon_s^{SH}}{\varepsilon_s^{SH} - \varepsilon_s^{CV}}$ is always satisfied.

[20] With $D < \frac{n-1}{(nm-1)+(n-1)}$, we would obtain a symmetric result with an upward

The above results can be summarized in the following proposition:

Proposition 1 • *Case of complementarity ($F_{yx} = F_{xy} = p > 0$). In all fiscal architectures (C, PS and PE), tax externalities lead to inefficiently low tax rates. The higher the degree of complementarity between the two tax bases, the worse the downward distortion of tax rates.*

- *Case of substitutability ($F_{yx} = F_{xy} = p < 0$). In C and PS, tax externalities lead to inefficiently low tax rates. The higher the degree of substitutability between the two tax bases, the smaller the downward distortion of tax rates.*

  In PE, the existence of cross-base tax externalities leads to inefficiently high: i) state tax rates when $D > \frac{\varepsilon_s^{SH}}{\varepsilon_s^{SH} - \varepsilon_s^{CV}}$, ii) local tax rates when $D < \frac{-\varepsilon_l^{CV}}{\varepsilon_l^{SH} - \varepsilon_l^{CV}}$. However, the upward distortion in PE can never occur at both tiers simultaneously. For high values of p, a higher degree of substitutability reduces the downward distortions of both tax rates. For sufficiently low values of p, a higher degree of substitutability reinforces the (downward/upward) distortions arising at both tiers.

Proof. *See Appendix* 2 ■

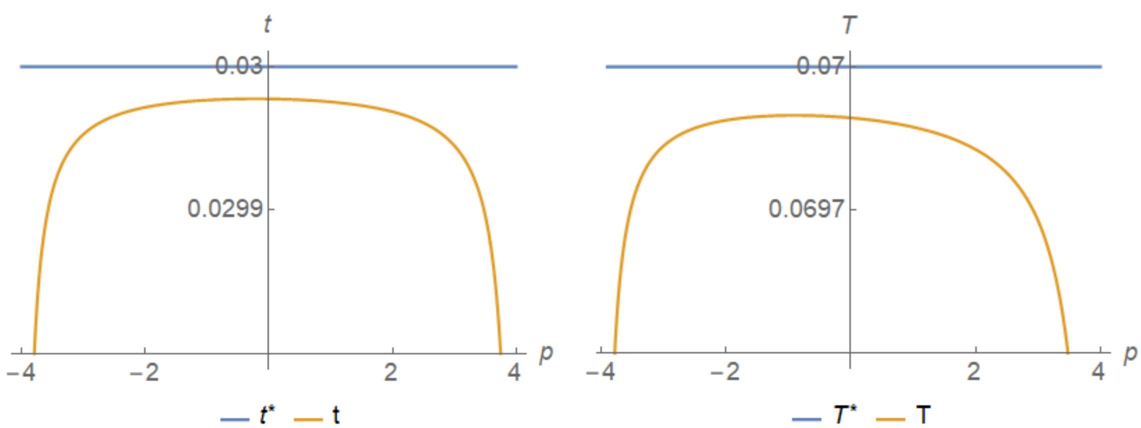

Figure 1: Tax rates in PE for $D = 0.2$, $n = 8$, $m = 20$, $b = 4$ and $\bar{e} = 10$

distortion at the local tier for a sufficiently high degree of substitutability.

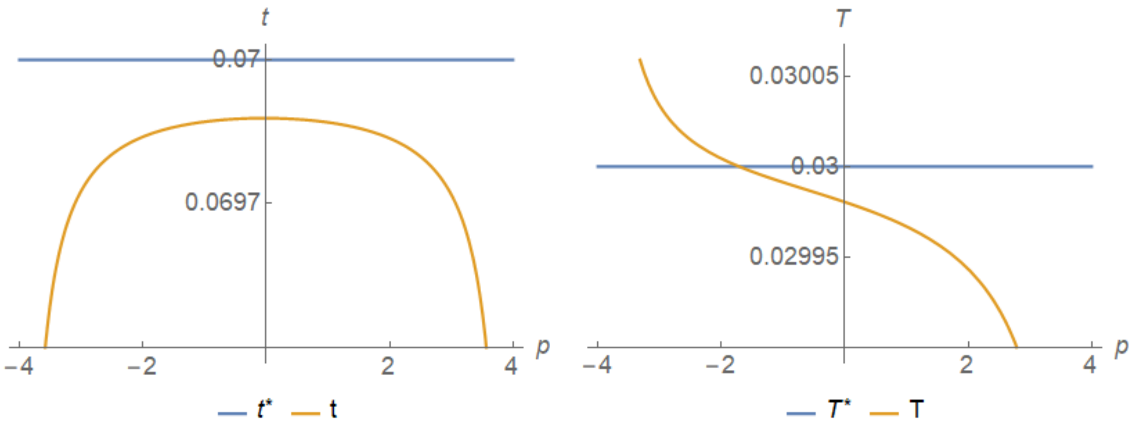

Figure 2: Tax rates in PE for $D = 0.7$, $n = 8$, $m = 20$, $b = 4$ and $\overline{e} = 10$

A high (resp. low) level of decentralization will put more weight on tax externalities affecting the local (resp. state) tax base. For instance, in PE, when setting its tax rate, the state government will take the cross-base vertical externalities into account all the more when the level of decentralization is high. In the case of substitutability, as the cross-base vertical externalities are positive, it can result in an inefficiently high state tax rate.

5 How to decentralize?

In this section, we investigate the issue of tax assignment when partially decentralizing the provision of public goods. We then compare the tax rates selected by jurisdictions and the welfare derived by citizens at symmetric equilibrium. Tables with all comparisons of tax rates are provided in Appendix 3 and the levels of public good consumption in each fiscal architecture are given in Appendix 4. The welfare of the representative citizen is given by the level of its utility function:

- in C

$$u^C = F(\overline{e}, \overline{e}) - G^{s,C} + \ln G^{s,C} \tag{14}$$

- in PS

$$u^{PS} = F(\overline{e}, \overline{e}) + \overline{D}\left(-G^{l,PS} + \ln G^{l,PS}\right) + \left(1 - \overline{D}\right)\left(-G^{s,PS} + \ln G^{s,PS}\right) \tag{15}$$

- in PE

$$u^{PE} = F(\overline{e}, \overline{e}) + \overline{D}\left(-G^{l,PE} + \ln G^{l,PE}\right) + \left(1 - \overline{D}\right)\left(-G^{s,PE} + \ln G^{s,PE}\right) \tag{16}$$

5.1 With only one top-tier jurisdiction ($n = 1$)

In order to interpret our results in the light of the existing literature, we first focus on the particular case of a single top-tier jurisdiction ($n = 1$). In this case, there are no horizontal tax externalities at the state tier and due to the fixed supply of factors, no vertical externalities occur. State tax rates and thus public goods are always set at an efficient level, while local tax rates are inefficiently low leading to an underprovision of public goods. Any fiscal architecture characterized by taxation at the local level leads to a sub-optimal level of welfare. As shown by Keen and Kotsogiannis (2002), it is always better not to decentralize with only one top-tier jurisdiction: C always provides the highest welfare, equal to the socially optimal level $F(\bar{e}, \bar{e}) - 1$. But when decentralizing, is it better to share tax bases or to have exclusive tax bases? Partially decentralizing the provision of public goods generates, at the bottom tier, same-base horizontal externalities in PS and PE and cross-base horizontal externalities in PS only. However, as seen previously, when raising a tax, the magnitude of the cross-base externalities is always lower than the magnitude of same-base externalities, and the aggregate tax base is twice as high in PS than in PE which weakens the impact of tax externalities in PS.[21] For a same level of decentralization, it follows that:

Proposition 2 *For $D^{PS} = D^{PE}$ and $\forall p$, $u^C > u^{PS} > u^{PE}$.*

Proof. See Appendix 5 ■

With only one top-tier jurisdiction, our results do not support the view of the advocates of exclusive tax bases.

5.2 With several top-tier jurisdictions ($n > 1$)

Under centralization (C), the size of the aggregate tax base is $2\bar{e}$ and the whole range of public goods δ has to be provided by a single tier. In addition, no vertical tax externalities occur due to the absence of another tier, and same-base horizontal externalities always exceed cross-base horizontal ones.

With a single mobile tax base, Hoyt (1991) showed that the tax rates, and thus the public good provision and the welfare of residents, increase as the number of jurisdictions at a given tier decreases. The reduction in the number of jurisdictions involved in the tax game reduces the distortive effect of tax externalities and thus lessens the race to the bottom. In our paper, we generalize Hoyt (1991)'s result in a broader framework with two mobile tax bases rather than one. We show that his result holds whether the tax bases are interdependently mobile or not and whatever the nature of the interdependence, i.e. whether the two tax bases are complementary or substitutable. The smaller the number of jurisdictions involved in the tax game in C, the smaller the tax externality effect.

[21] Recall that in PS, each tier can raise taxes on two tax bases ($2\bar{e}$) while only one tax base ($\bar{e}$) is available for each tier in PE.

Lemma 2 $\forall p,\ \frac{\partial u^C}{\partial n} < 0$

Proof. We know that $\varepsilon_s^{SH} < 0$, $\left|\varepsilon_s^{SH}\right| > \left|\varepsilon_s^{CH}\right|$ and $G^{s,C} = \frac{1}{1 - \frac{\varepsilon_s^{SH} + \varepsilon_s^{CH}}{2m\bar{e}^2}}$. With $\frac{\partial\left|\varepsilon_s^{SH}\right|}{\partial n} > \frac{\partial\left|\varepsilon_s^{CH}\right|}{\partial n} > 0$, the result follows. ■

We then compare C with partial decentralization with shared tax bases (PS). Our results can be summarized as follows:

Proposition 3 *When $n \neq 1$ and whatever the nature and degree of interdependence between the tax bases (i.e. $\forall p$), there is always a level of decentralization $D < 1 - \frac{1}{n}$, such that partial decentralization with shared tax bases (PS) prevails over centralization (C), i.e. $T^C < t^{PS} + T^{PS}$, $G^{s,C} < DG^{l,PS} + (1-D)\, G^{s,PS}$ and $u^C < u^{PS}$ $\forall p$. For $D > 1 - \frac{1}{n}$, C prevails over PS and for $D = 1 - \frac{1}{n}$, they are equivalent.*

Proof. See Appendix 6 ■

Proposition 3 not only generalizes the result of Wilson and Janeba (2005) to the case of more than two states, but also extends it by assuming more than one mobile tax base and potential interdependence in the mobility of the two tax bases.

In PS, same-base and cross-base vertical tax externality effects add to same-base and cross-base horizontal ones already at play at the state tier in C. However, the range of public goods provided by the state tier in C is by definition wider than the range of public goods provided by the state tier in PS. This affects the tax rates in two ways. First, fewer resources are required to produce the optimal level of public goods at the state tier in PS compared to C, which pushes down the tax rates. Second, a weaker horizontal tax externality effect, i.e. horizontal tax externalities weighted by $D < 1$, occurs at the state tier in PS compared to the horizontal tax externality effect in C, which pushes up tax rates.

At the state tier, horizontal externalities being equal to vertical externalities ($\varepsilon_s^{SH} = \varepsilon_s^{SV}$ and $\varepsilon_s^{CH} = \varepsilon_s^{CV}$), the decrease in the horizontal tax externality effect due to partial decentralization is perfectly compensated for by the emergence of the vertical tax externality effect, such that the overall tax externality effect is the same in PS as in C. Therefore, the difference in state tax rates between PS and C is solely determined by the difference in the range of public goods provided at the state tier. Although it results that $T^{PS} = (1-D)\, T^C < T^C$, a public good δ is provided in the same quantity at the state tier in PS as in C, i.e. $G^{s,PS} = G^{s,C}$ $\forall p, \forall D$.

As vertical externalities arising from local taxation in PS are smaller than the state horizontal externalities in C ($\left|\varepsilon_l^{SV}\right| < \left|\varepsilon_s^{SH}\right|$ and $\left|\varepsilon_l^{CV}\right| < \left|\varepsilon_s^{CH}\right|$), there is a critical level of decentralization $D = 1 - \frac{1}{n}$ such that the overall tax externality effect is the same at the local tier in PS and in C. The smaller D, the smaller the horizontal tax externality effect, but the stronger the vertical tax externality effect. Therefore a low enough level of decentralization, i.e. $D < 1 - \frac{1}{n}$, leads to an overall smaller tax externality effect at the local tier in PS than in C. It then translates into a public good provided in a larger quantity at the local tier in PS than in C, i.e. $G^{l,PS} > G^{s,C}$, $\forall D < 1 - \frac{1}{n}$. It follows that $\forall D < 1 - \frac{1}{n}$, $DG^{s,PS} + (1 - D) G^{l,PS} > G^{s,C}$ and thus $u^{PS} > u^{C}$. However, the level of welfare in PS is always socially sub-optimal, characterized by an underprovision of public goods at both tiers.

We proved that partial decentralization is preferable to centralization for $D < 1 - \frac{1}{n}$ but would decentralization with exclusive tax bases (PE) lead to a higher level of welfare? The comparison between PE and C/PS comes down to weighing differences in two effects: i) the differences in the overall tax externality effect and ii) the differences in the aggregate tax base.

5.2.1 No tax base interdependence ($p = 0$)

As stated in proposition 3, the comparison between C and PS holds whatever the nature and the degree of interdependence between the tax bases, and thus in particular for $p = 0$. Let us then focus on the comparison between C and PE. With $p = 0$, all externalities are same-base horizontal externalities and at a given tier, the tax externality effect depends positively on the range of public good provided, that is on D^{PE}, and on the number of same-tier jurisdictions (n at the top tier and nm at the bottom tier). Moreover, as each tier in PE is able to tax only one factor, the aggregate tax base available at each tier is twice as small in PE as in C. It follows that $G^{s,C} > G^{s,PE}$ and $G^{s,C} < G^{l,PE}$ for a low level of decentralization D^{PE}, and vice-versa for a high level of decentralization D^{PE}. We then obtain $G^{s,C} > D^{PE} G^{l,PE} + \left(1 - D^{PE}\right) G^{s,PE}$, $\forall D^{PE}$.

Proposition 4 *When $n > 1$ and $p = 0$, $u^{C} > u^{PE} \quad \forall D^{PE}$.*

Proof. See Appendix 7 ■

Corollary 5 *When $n > 1$ and $p = 0$, $u^{PS} \geq u^{C} > u^{PE} \quad \forall D^{PS} \leq 1 - \frac{1}{n}$ and $\forall D^{PE}$*

Proposition 6 *$u^{C} > u^{PS} > u^{PE}$ for $\frac{(n-1)(2mn-1)}{n(mn-1+m(n-1))} \geq D^{PS} > 1 - \frac{1}{n}$ and $\forall D^{PE}$. For particular values of $D^{PS} > \frac{(n-1)(2mn-1)}{n(mn-1+m(n-1))}$ and D^{PE}, it is possible that $u^{PE} > u^{PS}$.*

Proof. See Appendix 8 ■

While there is always a level of decentralization such that decentralizing with shared tax bases leads to higher welfare than with centralization, it is never the case when using exclusive tax bases. Moreover, when decentralizing, using exclusive tax bases is better than using shared tax bases only for a sufficiently high level of decentralization, i.e. when the smaller aggregate tax base in PE is offset by the smaller tax externality effect. Again, with more than one top-tier jurisdiction, our results do not support the view of the advocates of exclusive tax bases.

5.2.2 Tax base interdependence ($p \neq 0$)

We now analyze the case of tax base interdependence. Since the main drawback of the use of shared tax bases rather than exclusive tax bases comes from vertical tax externalities, we first look at the difference in terms of tax externality effect between PE and PS. When the tax bases are independent, no vertical tax externalities occur in PE (since there are no cross-base tax externalities) and the overall tax externality effect is always smaller in PE. This result holds when tax bases are complementary, since more tax externalities are at work in PS than in PE, all pointing in the same direction. When the tax bases are substitutable, the overall tax externality effect can be either stronger or smaller at a given tier in PS than in PE, depending on the level of decentralization. Furthermore, as previously mentioned, the aggregate tax base is twice smaller in PE than in any other fiscal architecture and even in the case where the overall tax externality effect is weaker in PE, jurisdictions might not be able to provide a higher level of public goods in PE than in PS.

The comparison with PS thus comes down to weighing differences in two effects: i) the differences in the overall tax externality effect and ii) the differences in the aggregate tax base. Although comparisons in tax rates between PE and PS provided in Appendix 3 were made for an identical level of decentralization D, we must allow for different D for the welfare comparisons between PE and PS in order to be able to draw some conclusions about the optimal fiscal architecture. Since the direct comparison of welfare functions does not give general results for any level of decentralization and degree of interdependence between the tax bases, we turned to simulations. As production $F(\overline{e}, \overline{e})$ is the same in every fiscal architecture at symmetric equilibrium, comparing welfare amounts to comparing $V^Z = u^Z - F(\overline{e}, \overline{e})$, for $Z = C, PS, PE$, the social optimal level of which is -1. We then determine the levels of decentralization, denoted D^{PE*} and D^{PS*} for PE and PS respectively, that maximize welfare for a given degree of interdependence p between the tax bases in PE and PS. We then obtain $D^{Z*}(p) = \underset{D^Z}{\arg\max} V^Z$, for $Z = PS, PE,$. For a given set of parameters n,m,b and $\overline{e}$, we can now compare the three fiscal architectures for any value of p. All our simulations provide the same qualitative results as the ones in figure 3.[22]

[22] The results are provided for the same values of parameters as in figures 1 and 2.

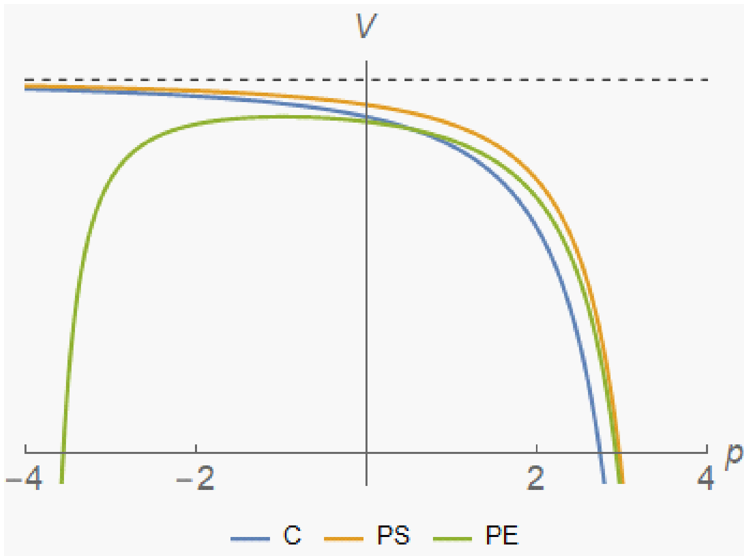

Figure 3: Welfare comparisons

In figure 3, for the optimal levels of decentralization D^{PE*} and D^{PS*}, we see that PS is better than any other fiscal architecture, whatever the nature and the degree of interdependence between the two tax bases.

Moreover, figure 3 illustrates the proposition 1. As a decrease in p reduces the downward distortion arising from tax externalities in C and PS, it results in an increase of welfare. Should the local authorities have the choice between different mobile tax bases, it is always better to levy taxes on tax bases that are substitutable.

The same analysis prevails in PE as long as the degree of substitutability is not too high. Otherwise, too high a degree of substitutability between the tax bases induces large distortions of tax rates in PE and thus lower welfare.

When the level of decentralization is exogenous, the question then remains whether PE can ever be better than PS in terms of welfare. We then used a 3D plot of the difference $u^{PS} - u^{PE}$ to compare welfare in PS and in PE for every combination of p and D (with D being the same in PS and PE), with given values of parameters n,m,b and $\overline{e}$. It turned out that we could not find any set of parameters such that $u^{PE} - u^{PS} > 0$. Therefore, even when the level of decentralization is given, we can make the conjecture that PS always provides a higher level of welfare than PE, whatever the nature and degree of interdependence between the tax bases.

Proposition 7

- *Whatever the level of decentralization D, partial decentralization with exclusive tax bases is always outperformed by all other fiscal architectures for a sufficiently high degree of substitutability between the tax bases.*

- *There are sets of parameters n,m, b and $\overline{e}$ for which the use of shared tax bases is better than partial decentralization with exclusive tax bases for all levels of decentralization D and whatever the nature and degree of interdependence between the tax bases.*

Proof. See Appendix 9 ■

With more than one top-tier jurisdiction, depending on the degree of interdependence and the combination between the level of decentralization and the tax base assignment, we show that partial decentralization can induce higher welfare than full centralization. However, while there is always a level of expenditure decentralization such that a fiscal architecture characterized by shared tax bases is better than C $(\forall p)$, this is no longer the case with exclusive tax bases for a sufficiently high degree of substitutability between the tax bases (see figure 3), due to the very large distortions of tax rates.

6 Conclusion

The issue of tax base assignment in a multi-tier setting, i.e. which tier should tax which tax base(s), cannot be dealt with in isolation from the issue of expenditure decentralization. Our paper demonstrates that interdependence between two mobile tax bases and the level of decentralization (measured by the share of public goods provision assigned to lower-tier jurisdictions) are crucial parameters that affect the impact of tax externalities on tax choices and thus the welfare of citizens. On the one hand, interdependence between tax bases complicates the tax interactions arising in a two-tier setting by: i) introducing cross-base horizontal and vertical tax externalities and ii) reinforcing the standard same-base horizontal and vertical tax externalities. On the other hand, the level of decentralization affects the weights of these tax externalities and thus their effect on tax policy.

We show that, depending on the interdependence between the tax bases *and* the share of public goods provision assigned to lower-tier jurisdictions, partial decentralization with exclusive tax bases may lead to inefficiently high tax rates (although not simultaneously at both tiers), while tax rates are always inefficiently low in all other fiscal architectures, i.e. centralization and partial decentralization with shared tax bases. A higher degree of complementarity between the two tax bases pushes tax rates down and thus deteriorates welfare in all fiscal architectures. Conversely, a higher degree of substitutability reduces the downward distortion of tax rates and thus improves the welfare for all fiscal architectures, but only for a low degree of interdependence in partial decentralization with exclusive tax bases. In the latter case, with a high degree of interdependence between the tax bases, substitutability reinforces the (downward and/or upward) distortions of tax rates and thus reduces welfare. It follows that authorities should always favor taxation on tax bases which are substitutable, although not with a high degree of interdependence in case of partial decentralization with exclusive tax bases.

More specifically, with the interdependence between the tax bases, partial decentralization with exclusive tax bases does not prevent vertical tax externalities and the tax externality effect can even be stronger than in other fiscal architectures, leading to lower welfare. Moreover, a weaker tax externality effect does not guarantee higher welfare as the exclusive use of tax bases reduces the sources of tax revenues at each tier, thereby pushing up the tax rates.

With a single top-tier jurisdiction and a fixed supply of factors, no vertical tax externality occurs while horizontal tax externalities only takes place at the bottom tier. It is then optimal to follow the recommendations of assigning mobile tax bases to the highest tier and thus opt for full centralization. Considering more than one top-tier jurisdiction competing for mobile tax bases modifies this conclusion. Partial decentralization may induce a higher welfare than full centralization. However, this result crucially depends on the degree of interdependence and the combination between the level of decentralization and the tax base assignment. Compared to full centralization, partial decentralization with exclusive tax bases always deteriorates the welfare for a sufficiently high degree of tax base substitutability, while there is always a level of decentralization such that the use of shared tax bases is welfare enhancing, whatever the nature and degree of interdependence between the tax bases.

The results suggest that partial decentralization combined with appropriate tax base assignment is always preferable to centralization when the share of public good provision between the two tiers can be adjusted freely.

In practice, there might be some constraints on the level of decentralization. For instance, heterogeneous preferences for public goods as well as scale economies can influence the level of decentralization. While some public goods should be provided at a local level to better match preferences following Tiebout (1956)'s argument, scale economies may be achieved by a provision of public goods at a higher level. These elements are absent from our framework as we focused exclusively on the issue of tax externalities, but could be introduced in the model at the expense of some complexity.

Finally, it ought to be remarked that assuming a world economy with perfectly identical states amounts to assuming homogeneity in fiscal architecture across states. However, the issue of the coexistence of different fiscal architectures may arise when introducing some form of asymmetry between states. Moreover, although recommendations can be made about the optimal fiscal architecture(s), our analysis assumes the fiscal architecture to be exogenous and is thus silent about which one would actually be adopted if the fiscal architecture could be used as a strategic device by states. With two mobile tax bases, states would then have to decide on both the tax bases and the share of public good provision assigned to each tier.[23]

[23] Although Wilson and Janeba (2005) assumed the level of decentralization to be endogeneous, they only considered two top-tier jurisdictions and one mobile tax base, which excludes the case of partial decentralization with exclusive tax bases.

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References

Braid, R. M. (2000). A spatial model of tax competition with multiple tax instruments. *Journal of Urban Economics 47*(1), 88–114.

Breuillé, M.-L. and S. Zanaj (2013). Mergers in fiscal federalism. *Journal of Public Economics 105*, 11–22.

Burbidge, J. B. and G. M. Myers (1994). Population mobility and capital tax competition. *Regional Science and Urban Economics 24*(4), 441–459.

Conseil Des Prélèvements Obligatoires (2010). La fiscalité locale. Technical report.

Dahlby, B. (2001). Taxing choices: issues in the assignment of taxes in federations. *International Social Science Journal 53*(167), 93–101.

Duran-Vigneron, P. (2012). When transparency goes against efficiency: Fiscal equalization in a context of firm and household mobility. *Journal of Public Economic Theory 14*(1), 99–130.

Flowers, M. R. (1988). Shared tax sources in a leviathan model of federalism. *Public Finance Quarterly 16*(1), 67–77.

Hoyt, W. H. (1991). Property taxation, nash equilibrium, and market power. *Journal of Urban Economics 30*(1), 123–131.

Hoyt, W. H. (2017). The assignment and division of the tax base in a system of hierarchical governments. *International Tax and Public Finance 24*(4), 678–704.

Keen, M. (1998). Vertical tax externalities in the theory of fiscal federalism. *Staff Papers 45*(3), 454–485.

Keen, M. J. and C. Kotsogiannis (2002). Does federalism lead to excessively high taxes? *American Economic Review*, 363–370.

Musgrave, R. (1983). *Who should tax, where and what?*, pp. 2–19. Centre for Research on Federal Financial Relations, Australian National University Canberra.

Musgrave, R. A. and P. B. Musgrave (1989). *Public Finance in Theory and Practice*. Mcgraw-Hill College; 5 Sub edition (February 1989).

Oates, W. E. (1994). *Federalism and government finance*, Book section 5, pp. 368. Harvard University Press.

Tiebout, C. M. (1956). A pure theory of local expenditures. *The journal of political economy*, 416–424.

Wilson, J. D. (1986). A theory of interregional tax competition. *Journal of urban Economics 19*(3), 296–315.

Wilson, J. D. and E. Janeba (2005). Decentralization and international tax competition. *Journal of Public Economics 89*(7), 1211–1229.

Wrede, M. (1997). Tax competition and federalism the underprovision of local public goods. *FinanzArchiv/Public Finance Analysis*, 494–515.

Zodrow, G. R. and P. Mieszkowski (1986). Pigou, tiebout, property taxation, and the underprovision of local public goods. *Journal of urban Economics 19*(3), 356–370.

7 Appendix

7.1 Response of net returns to taxation

The system of market-clearing conditions is:

$$\begin{cases} \sum_{i=1}^{n}\sum_{j=1}^{m} x_{ij}(r_{ij}^x, r_{ij}^y) = nm\overline{x} \\ \sum_{i=1}^{n}\sum_{j=1}^{m} y_{ij}(r_{ij}^x, r_{ij}^y) = nm\overline{y} \end{cases}$$

With $r_{ij}^x = \rho^x + t_{ij}^x + T_i^x$ and $r_{ij}^y = \rho^y + t_{ij}^y + T_i^y$, $\forall i, j$

From the differentiation of market-clearing we derive the response of ρ^x to local and state taxation:

$$\frac{\partial \rho^x}{\partial t^x_{ij}} = \frac{-\frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^y_{ij}} + \frac{\partial y_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^y_{ij}}}{\sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^y_{ij}} - \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^x_{ij}}} \leq 0$$

$$\frac{\partial \rho^x}{\partial t^y_{ij}} = \frac{-\frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^y_{ij}} + \frac{\partial y_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^y_{ij}}}{\sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^y_{ij}} - \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^x_{ij}}} \leq 0$$

$$\frac{\partial \rho^x}{\partial T^x_{i}} = \frac{-\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^y_{ij}} + \sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^y_{ij}}}{\sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^y_{ij}} - \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^x_{ij}}} \leq 0$$

$$\frac{\partial \rho^x}{\partial T^y_{i}} = \frac{-\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^y_{ij}} + \sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^y_{ij}}}{\sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^x_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^y_{ij}} - \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial x_{ij}}{\partial r^y_{ij}} * \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{\partial y_{ij}}{\partial r^x_{ij}}} \leq 0$$

7.2 Proof of proposition 1

7.2.1 In C and PS:

All tax rates depend on the sum of same-base and cross-base externalities $\varepsilon_z^{SH} + \varepsilon_z^{CH}$ and $\varepsilon_z^{SV} + \varepsilon_z^{CV}$ with $z = l, s$. Therefore, as same-base externalities always exceed cross-base ones in absolute terms, we obtain $\varepsilon_z^{SH} + \varepsilon_z^{CH} < 0$ and $\varepsilon_z^{SV} + \varepsilon_z^{CV} < 0 \; \forall p$ with $z = l, s$. All tax rates are thus inefficiently low, i.e. $T^{*C} > T^C$, $t^{*PS} > t^{PS}$ and $T^{*PS} > T^{PS} \; \forall p$.

Differentiating the expressions of equilibrium tax rates (9-11) with respect to p, provides the results directly, i.e. $\frac{\partial t}{\partial p} < 0$ and $\frac{\partial T}{\partial p} < 0$.

7.2.2 In PE:

Whether the local (resp. state) tax rate is inefficiently low, i.e. $t^{*PE} > t^{PE}$ (resp. $T^{*PE} > T^{PE}$), or inefficiently high, i.e. $t^{*PE} < t^{PE}$ (resp. $T^{*PE} < T^{PE}$), depends on the sign of $\frac{D\varepsilon_l^{SH}}{\bar{e}} + \frac{(1-D)\varepsilon_l^{CV}}{\bar{e}}$ (resp. $\frac{(1-D)\varepsilon_s^{SH}}{m\bar{e}} + \frac{D\varepsilon_s^{CV}}{m\bar{e}}$) . It follows that when tax bases are independent or complementary, both tax rates are always inefficiently low due to $\varepsilon_z^{SH} < 0$ and $\varepsilon_z^{CV} \leq 0 \; \forall p \geq 0$ with $z = l, s$. When tax bases are substitutable, same-base and cross-base externalities are of opposite signs and we observe an upward distortion of: i) the state tax rate T^{PE} for $D > \frac{\varepsilon_s^{SH}}{\varepsilon_s^{SH} - \varepsilon_s^{CV}}$, ii) the local tax rate t^{PE} for $D < \frac{-\varepsilon_l^{CV}}{\varepsilon_l^{SH} - \varepsilon_l^{CV}}$.

Moreover, since $\frac{-\varepsilon_l^{CV}}{\varepsilon_l^{SH}-\varepsilon_l^{CV}} < \frac{\varepsilon_s^{SH}}{\varepsilon_s^{SH}-\varepsilon_s^{CV}}$, an upward distortion of tax rates can never occur simultaneously at both tiers.

Complementarity of tax bases, i.e. $p > 0$:
As the magnitude of both same-base and cross-base externalities increases with p, a higher degree of complementarity increases the downward distortion, i.e. $\frac{\partial t^{PE}}{\partial p} < 0$ and $\frac{\partial T^{PE}}{\partial p} < 0$, $\forall p > 0$.
Substitutability of tax bases, i.e. $p < 0$:
The relative effect of cross-base vertical tax externalities to same-base horizontal tax externalities depends on the level of decentralization D as well as on the degree of substitutability p. By assumption $|p| < b$, an upward distortion at either the state or the local tier can never occur for a level of decentralization $D \in [\frac{(n-1)}{(mn-1)+(n-1)}; \frac{1}{2}]$.
For t^{PE}, we find that :

1. $\forall D > \frac{(n-1)}{(mn-1)+(n-1)}$, $\forall p < 0$, t^{PE} is strictly concave in p and with a maximum for value lower than the optimal tax rate.
2. $\forall D < \frac{(n-1)}{(mn-1)+(n-1)}$, $\forall p < 0$, $\frac{\partial t^{PE}}{\partial p} < 0$, with $\lim_{p \to 0} t^{PE} = \frac{D}{\bar{e} + \frac{D(mn-1)}{mnb\bar{e}}} < t^{*PE} = \frac{D}{\bar{e}}$. and $\lim_{p \to \widehat{p}} t^{PE} = +\infty$, where $\widehat{p}$ is the lowest value of p such that $t^{PE} > 0$.
3. for $D = \frac{(n-1)}{(mn-1)+(n-1)}$, $\forall p < 0$, $\frac{\partial t^{PE}}{\partial p} < 0$, and $t^{PE} = \frac{1}{\frac{(mn-1)+(n-1)}{(n-1)}\bar{e} + \frac{(mn-1)}{mn(b-p)\bar{e}}} < t^{*PE} = \frac{1}{\frac{(mn-1)+(n-1)}{(n-1)}\bar{e}}$

For T^{PE}, we find that :

1. $\forall D < \frac{1}{2}$, $\forall p < 0$, t^{PE} is strictly concave in p with a maximum for value lower than the optimal tax rate.
2. $\forall D > \frac{1}{2}$, $\forall p < 0$, $\frac{\partial T^{PE}}{\partial p} < 0$, with $\lim_{p \to 0} T^{PE} = \frac{(1-D)}{\bar{e} + \frac{(n-1)(1-D)}{nb\bar{e}}} < T^{*PE} = \frac{(1-D)}{\bar{e}}$ and $\lim_{p \to \widetilde{p}} T^{PE} = +\infty$, where $\widetilde{p}$ is the lowest value of p such that $T^{PE} > 0$.
3. for $D = \frac{1}{2}$, $\forall p < 0$, $\frac{\partial T^{PE}}{\partial p} < 0$, and $T^{PE} = \frac{1}{2\bar{e} + \frac{(n-1)}{n(b-p)\bar{e}}} < T^{*PE} = \frac{1}{2\bar{e}}$

The last part of the propositions follows directly from this analysis.

7.3 Comparisons of tax rates

7.3.1 Symmetry between tax bases in terms of tax base assignment

Comparisons with PS give:

	t^{PS}	T^{PS}	$t^{PS}+T^{PS}$
T^C	$T^C > t^{PS}$	$T^C > T^{PS}$	$\begin{cases} T^C \geq t^{PS}+T^{PS} \text{ if } D \geq 1-\frac{1}{n} \\ T^C < t^{PS}+T^{PS} \text{ if } D < 1-\frac{1}{n} \end{cases}$

Where $\widetilde{D} = \frac{-\frac{\varepsilon_l^{SH}+\varepsilon_l^{CH}}{\overline{e}}+\frac{\varepsilon_s^{SH}+\varepsilon_s^{CH}}{m\overline{e}}}{\left(2\overline{e}-\frac{\varepsilon_l^{SH}+\varepsilon_l^{CH}}{\overline{e}}\right)} = \frac{(m-1)}{(2mn(b-p)\overline{e}^2+mn-1)}$

7.3.2 PE versus all other fiscal architectures

Comparisons between PE and PS are made for an identical level of decentralization D.

- At the local tier:
  - For $n=1$:

	$p < p^*$	$p^* = p$	$p^* < p$
$/t^{PS}$	$\begin{cases} t^{PS} \geq t^{PE} \text{ if } D \geq D^{**} \\ t^{PS} < t^{PE} \text{ if } D < D^{**} \end{cases}$	$t^{PS} < t^{PE}$	$t^{PS} < t^{PE}$

  - For $n>1$:

	$p < p^{**}$	$p = p^{**}$	$p^{**} < p < p^*$	$p = p^*$	$p^* < p$
$/t^{PS}$	$\begin{cases} t^{PS} \geq t^{PE} \text{ if } D \geq D^{**} \\ t^{PS} < t^{PE} \text{ if } D < D^{**} \end{cases}$	$t^{PS} < t^{PE}$	$\begin{cases} t^{PS} \geq t^{PE} \text{ if } D \geq D^{**} \\ t^{PS} < t^{PE} \text{ if } D < D^{**} \end{cases}$	$t^{PS} < t^{PE}$	$t^{PS} < t^{PE}$

Where $p^* < 0$ satisfies $\overline{e} - \frac{\varepsilon_l^{CH}}{\overline{e}} = 0$; $p^{**} < 0$ satisfies $\overline{e} - \frac{\varepsilon_l^{CV}}{m\overline{e}} = 0$

And $D^* = \frac{\overline{e}-\frac{\varepsilon_l^{CV}}{\overline{e}}}{\left(2\overline{e}-\frac{\varepsilon_l^{CH}}{\overline{e}}-\frac{\varepsilon_l^{CV}}{\overline{e}}\right)}$, $D^{**} = \frac{\overline{e}-\frac{\varepsilon_l^{SV}}{\overline{e}}}{-\left(\frac{\varepsilon_l^{SV}}{\overline{e}}-\frac{\varepsilon_l^{CH}}{\overline{e}}\right)}$

- At the state tier:
  - For $n=1$: $\begin{cases} T^C \geq T^{PE} \text{ if } D \geq \frac{1}{2} \\ T^C < T^{PE} \text{ if } D < \frac{1}{2} \end{cases}$, $\forall p$ and $T^{PS} < T^{PE}$, $\forall p, D$
  - For $n>1$:

	$p < p^{***}$	$p = p^{***}$	$p^{***} < p$
$/T^C$	$\begin{cases} T^C \leq T^{PE} \text{ if } D \geq \frac{1}{2} \\ T^C > T^{PE} \text{ if } D < \frac{1}{2} \end{cases}$	$T^C = T^{PE}$	$\begin{cases} T^C \geq T^{PE} \text{ if } D \geq \frac{1}{2} \\ T^C < T^{PE} \text{ if } D < \frac{1}{2} \end{cases}$
$/T^{PS}$	$\begin{cases} T^{PS} \leq T^{PE} \text{ if } D \geq D^{***} \\ T^{PS} > T^{PE} \text{ if } D < D^{***} \end{cases}$	$T^{PS} < T^{PE}$	$T^{PS} < T^{PE}$

Where $p^{***} < 0$ satisfies $\overline{e} - \frac{\varepsilon_s^{CH}}{m\overline{e}} = 0$

And $D^{***} = -\frac{\overline{e} - \frac{\varepsilon_s^{CH}}{m\overline{e}}}{\left(-\frac{\varepsilon_s^{SV}}{m\overline{e}} + \frac{\varepsilon_s^{CH}}{m\overline{e}}\right)}$

7.4 Levels of public good consumption in each fiscal architecture

$$\text{In } C \quad : \qquad G^{s,C} = 2T^C\overline{e} = \frac{1}{1 - \frac{\varepsilon_s^{SH} + \varepsilon_s^{CH}}{2m\overline{e}^2}} = \frac{1}{1 + \frac{n-1}{2n(b-p)\overline{e}^2}}$$

$$\text{In } PS \quad : \quad \begin{cases} G^{l,PS} = \frac{2t^{PS}\overline{e}}{D} = \frac{1}{1 - D\frac{\varepsilon_l^{SH} + \varepsilon_l^{CH}}{2\overline{e}^2} - (1-D)\frac{\varepsilon_l^{SV} + \varepsilon_l^{CV}}{2m\overline{e}^2}} = \frac{1}{1 + \frac{D(nm-1) + (1-D)(n-1)}{2nm(b-p)\overline{e}^2}} \\ G^{s,PS} = \frac{2T^{PS}\overline{e}}{(1-D)} = \frac{1}{1 - (1-D)\frac{\varepsilon_s^{SH} + \varepsilon_s^{CH}}{2m\overline{e}^2} - D\frac{\varepsilon_s^{SV} + \varepsilon_s^{CV}}{2m\overline{e}^2}} = \frac{1}{1 + \frac{(n-1)}{2n(b-p)\overline{e}^2}} \end{cases}$$

$$\text{In } PE \quad : \quad \begin{cases} G^{l,PE} = \frac{t^{PE}\overline{e}}{D} = \frac{1}{1 - D\frac{\varepsilon_l^{SH}}{\overline{e}^2} - (1-D)\frac{\varepsilon_l^{CV}}{m\overline{e}^2}} = \frac{1}{1 + \frac{D(mn-1)b + (1-D)(n-1)p}{mn(b^2-p^2)\overline{e}^2}} \\ G^{s,PE} = \frac{T^{PE}\overline{e}}{(1-D)} = \frac{1}{1 - (1-D)\frac{\varepsilon_s^{SH}}{m\overline{e}^2} - D\frac{\varepsilon_s^{CV}}{m\overline{e}^2}} = \frac{1}{1 + \frac{n-1}{n(b^2-p^2)\overline{e}^2}((1-D)b + Dp)} \end{cases}$$

7.5 Proof of proposition 2

For $n = 1$, we have $G^{s,C} = G^{s,PS} = G^{s,PE} = 1 > G^{l,PS} = \frac{1}{1 + \frac{D(m-1)}{2m(b-p)\overline{e}^2}} > G^{l,PE} = \frac{1}{1 + \frac{D(m-1)}{m(b^2-p^2)\overline{e}^2}b}$

The result follows.

7.6 Proof of proposition 3

For $n > 1$, we have $G^{s,C} = G^{s,PS}$ and $G^{s,C} \leq G^{l,PS}$ if $D \leq 1 - \frac{1}{n}$

The result follows.

7.7 Proof of proposition 4

Let us denote $h(x) = -x + \ln x$, a strictly concave function with a maximum for $x = 1$.

We can easily show that $\forall D^{PE}$ and $p = 0$, $1 > G^C > D^{PE}G^{l,PE} + \left(1 - D^{PE}\right)G^{s,PE}$. We thus have $h(G^C) > h\left(D^{PE}G^{l,PE} + \left(1 - D^{PE}\right)G^{s,PE}\right) > D^{PE}h(G^{l,PE}) + \left(1 - D^{PE}\right)h(G^{s,PE})$.

It follows that $\forall D^{PE}$, $u^C > u^{PE}$.

7.8 Proof of proposition 6

$u^{PE} = F(\bar{e},\bar{e}) + D^{PE}\left(-G^{l,PE} + \ln G^{l,PE}\right) + \left(1 - D^{PE}\right)\left(-G^{s,PE} + \ln G^{s,PE}\right)$

$\frac{\partial u^{PE}}{\partial D} = \left(-G^{l,PE} + \ln G^{l,PE}\right) - \left(-G^{s,PE} + \ln G^{s,PE}\right) - \left(\frac{1}{\frac{1}{D^{PE}}\frac{(mn-1)}{mnb\bar{e}^2}+1}\right)^2 + \left(\frac{1}{\frac{1}{(1-D^{PE})}\frac{(n-1)}{nb\bar{e}^2}+1}\right)^2 \leq (>) 0$ for $D^{PE} \geq (<) \frac{m(n-1)}{(mn-1+m(n-1))}$

We thus have maximum welfare in PE for $D^{PE} = D^{PE*} = \frac{m(n-1)}{(mn-1+m(n-1))}$

For $D^{PE} = D^{PE*}$, $G^{PE} = G^{l,PE} = G^{s,PE} = \frac{1}{1+\frac{m(n-1)(mn-1)}{(mn-1+m(n-1))mnb\bar{e}^2}}$

Since $G^{s,PS} = G^{s,C}$, then $G^{PE} < G^{s,PS}\ \forall D^{PS}$

We can easily show that $G^{PE} \leq G^{l,PS}$ for $D^{PS} \leq \frac{(n-1)(2mn-1)}{n(mn-1+m(n-1))}$

It follows that $u^{PE} < u^{PS}$ for $D^{PS} \leq \frac{(n-1)(2mn-1)}{n(mn-1+m(n-1))}$.

Given that $\frac{(n-1)(2mn-1)}{n(mn-1+m(n-1))} > 1 - \frac{1}{n}$, we obtain that .$u^{PE} < u^{PS} < u^C$ for $1 - \frac{1}{n} < D^{PS} \leq \frac{(n-1)(2mn-1)}{n(mn-1+m(n-1))}$ and $\forall D^{PE}$

$u^{PS} = F(\bar{e},\bar{e}) + D^{PS}\left(-G^{l,PS} + \ln G^{l,PS}\right) + \left(1 - D^{PS}\right)\left(-G^{s,PS} + \ln G^{s,PS}\right)$

$\frac{\partial u^{PS}}{\partial D} = \left(-G^{l,PS} + \ln G^{l,PS}\right) - \left(-G^{s,PS} + \ln G^{s,PS}\right) - \frac{D^{PS}n(m-1)\left(D^{PS}(nm-1)+\left(1-D^{PS}\right)(n-1)\right)}{\left(2nmb\bar{e}^2+D^{PS}(nm-1)+(1-D^{PS})(n-1)\right)^2}$

Since $G^{s,PS} > G^{l,PS}$ when $D^{PS} > 1 - \frac{1}{n}$ and $\frac{D^{PS}n(m-1)\left(D^{PS}(nm-1)+\left(1-D^{PS}\right)(n-1)\right)}{\left(2nmb\bar{e}^2+D^{PS}(nm-1)+(1-D^{PS})(n-1)\right)^2} > 0\ \forall D^{PS}$, $\frac{\partial u}{\partial D} < 0$ for $D^{PS} \geq \frac{(n-1)(2mn-1)}{n(mn-1+m(n-1))}$

$\lim_{D^{PS}\to 1} u^{PS} = F(\bar{e},\bar{e}) + \left(-G^{l,PS} + \ln G^{l,PS}\right)$

$\left(\lim_{D^{PS}\to 1} u^{PS}\right) - u^{PE*} = \left(-G^{l,PS} + \ln G^{l,PS}\right) - \left(-G^{PE} + \ln G^{PE}\right) < 0$

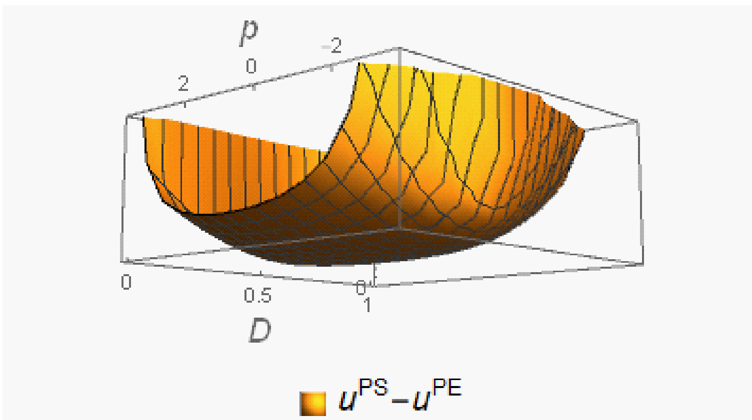
This gives the last result.

7.9 Proof of proposition 7

When p approaches $-b$, tax rates in PE are highly distorted (proposition 1), such that the limits of $G^{l,PE}$ and $G^{s,PE}$ are infinity or zero for any level of decentralization D^{PE} (except for $D^{PE} = \frac{1}{2}$ for which $G^{s,PE}$ has a finite limit and $D^{PE} = \frac{n-1}{(nm-1)+(n-1)}$ for which $G^{l,PE}$ has a finite limit). It follows that the limit of welfare is minus infinity when p approaches $-b$.

In all other fiscal architectures, the limits of G^l and G^s belong to the finite interval $\left[\frac{1}{1+\frac{nm-1}{4nmb\bar{e}^2}}, \frac{1}{1+\frac{(n-1)}{4nmb\bar{e}^2}}\right]$ when p approaches $-b$. It follows from the form of the utility function that the welfare also belong to a finite interval such that it is always higher than in PE.

For the set of parameters $n = 8$, $m = 50$, $b = 4$ and $\bar{e} = 0$ (same parameters as in the other figures), we plot the difference $u^{PS} - u^{PE}$ to compare welfare in PS and in PE for every combination of p and $D = D^{PS} = D^{PE}$. As shown in the figure below, PS always provides a higher level of welfare than PE, whatever the nature and degree of interdependence between the tax bases and the level of decentralization D.



We did find many other sets of parameters such that $u^{PS} - u^{PE} > 0, \forall p, \forall D$. However, we could not find any set of parameters such that PE provides a higher welfare than PS for some combinations of p and D.