

Innovative Growth Accounting

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June 18, 2020

Abstract

Recent work highlights a falling entry rate of new firms and a rising market share of large firms in the United States. To understand how these changing firm demographics have affected growth, we decompose productivity growth into the firms doing the innovating. We trace how much each firm innovates by the rate at which it opens and closes plants, the market share of those plants, and how fast its surviving plants grow. Using data on all nonfarm businesses from 1982–2013, we find that new and young firms (ages 0 to 5 years) account for almost one-half of growth — three times their share of employment. Large established firms contribute only one-tenth of growth despite representing one-fourth of employment. Older firms do explain most of the speedup and slowdown during the middle of our sample. Finally, most growth takes the form of incumbents improving their own products, as opposed to creative destruction or new varieties.

*Klenow: Stanford University and NBER; Li: Federal Reserve Bank of San Francisco. We thank Mark Bills, John Haltiwanger, Hugo Hopenhayn, Chang-Tai Hsieh, Erik Hurst and Chad Jones for useful comments. We also thank Amber Flaharty and Gladys Teng for excellent research assistance. Any opinions and conclusions expressed herein are those of the author(s) and do not necessarily represent the views of the U.S. Census Bureau or the Federal Reserve System. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 1440. All results have been reviewed to ensure that no confidential information is disclosed.

1. Introduction

Solow (1956) famously decomposed output growth into contributions from capital, labor, and productivity. Mankiw, Romer and Weil (1992) further decomposed productivity into human capital versus residual productivity. We take this residual productivity, routinely calculated by the U.S. Bureau of Labor Statistics, as our starting point. We attribute productivity growth to innovation and ask: what form does the innovation take, and which firms do most of the innovation?

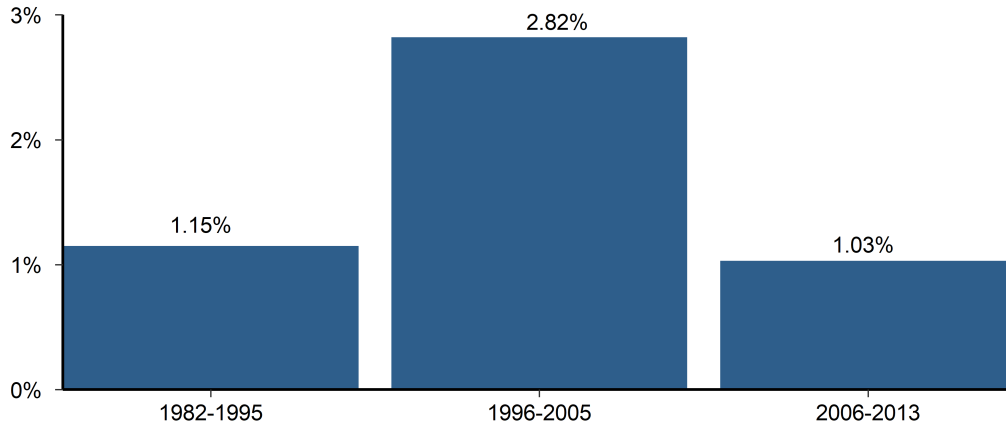
Innovation can take the form of a stream of new varieties that are not close substitutes for any existing varieties (Romer, 1990). Alternatively, growth could be driven by creative destruction of existing products as in Aghion and Howitt (1992). Examples include Walmart stores driving out mom-and-pop stores or Amazon stealing business from physical stores. Yet another possibility is that growth takes the form of existing producers improving their own products (e.g., successive generations of Apple iPhones or new car models). Even conditional on the form of innovation, growth could be led by entrants and young firms (e.g., Uber or Netflix) or by older and larger firms (e.g., Intel or Starbucks).

Figure 1 shows U.S. productivity growth has been lackluster in recent decades, except for a decade-long surge from the mid-1990s through the mid-2000s.¹ At the same time, as shown in Figure 2, plant and firm entry rates have fallen and reallocation of labor across incumbent firms has slowed — coined “declining business dynamism” by papers such as Decker, Haltiwanger, Jarmin and Miranda (2014). Large, established superstar firms have captured a bigger share of markets. Is this declining dynamism and rising concentration responsible for the growth slowdown? Have superstar firms helped growth or hindered it?

We offer a new growth decomposition to shed light on these questions. We decompose growth into three types of innovation: creative destruction, brand new varieties, and innovation by incumbents on their own products. We further decompose each of these sources into contributions by firm age and size.

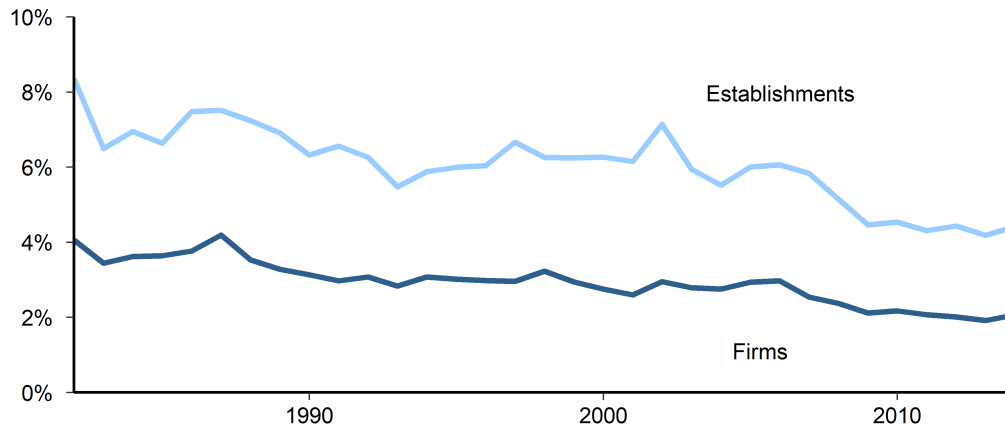
¹See Fernald (2015) for discussion on the timing of the productivity speedup and slowdown.

Figure 1: U.S. productivity growth rate



Source: BLS multifactor productivity series. We calculate yearly productivity growth rate by adding R&D and IP contribution to BLS MFP and then converting the sum to labor augmenting form. The figure plots the average productivity growth within each subperiod. The unit is percentage points.

Figure 2: Employment share of new plants and new firms 1982–2013



Source: Census BDS.

We exploit firm and plant data on employment at all nonfarm businesses from 1983–2013 in the U.S. Census Bureau’s Longitudinal Business Database (LBD). We treat each plant as a unique product (or set of products). This en-

ables us to observe the rate and importance of product entry and exit across firms. We rely on a model in which employment at a plant is isoelastic with respect to the quality of products produced at the plant.² We use the size of entering versus exiting plants to gauge the quality of brand new products and the quality improvements made through creative destruction. We assess quality improvements by firms on their own products using changes in employment over time at surviving incumbent plants. We also allow the lowest quality products (plants) to exit due to obsolescence (i.e, inability to keep up with overall quality and variety growth) as in Hopenhayn (1992).³

We preview our findings as follows. Some 60% of growth on average comes from incumbent innovations on their own products, whereas about 27% comes from new varieties and 13% from creative destruction. Many exiting plants are quite small, suggesting much of exit is due to obsolescence rather than creative destruction. And new plants are smaller than incumbent plants on average, making us infer that most plant entry takes the form of new varieties rather than creative destruction.

We find that own innovation drove both the mid-1990s speedup and the mid-2000s slowdown of growth. TFP growth averaged 1.15% per year from 1982–1995, then sped up to 2.82% per year from 1996–2005 before falling back to 1.03% per year from 2006–2013. Own innovation accounted for 148 of the 167 basis point acceleration, and 144 of the 179 basis point deceleration. Even though entry and job reallocation fell throughout the sample, growth from creative destruction accelerated modestly from 1996–2005 versus 1982–1995.

New firms in a given year account for about 1/3 of growth on average. New and young firms (ages 0–5 years) together contribute roughly 1/2 of all growth, despite employing less than 1/5 of all workers. Older incumbents (ages 6 and

²We model product heterogeneity as coming from quality differences, but these will be isomorphic to differences in process efficiency in our model setting.

³We do not model the arrival rates and step sizes of innovation, but rather back them out from data moments in the LBD. We allow them to vary over time and across firms. In this sense, we are not assuming these arrival rates and step sizes are exogenous. Instead, we are estimating and carrying out a model-based growth decomposition.

above) contribute the other half of growth. Such established firms played a bigger role in the speedup and slowdown of growth, however, accounting for 80% of the pickup and 70% of the dropoff.

The largest firms — those with 5,000 or more workers — accounted for only 11% of growth on average. This is less than half of their 25% share of employment. Small firms, with fewer than 20 employees, explain over 60% of growth despite employing only 21% of all workers. This is very much in the spirit of Haltiwanger, Jarmin and Miranda (2013), who emphasize the outsized contribution of young, small fast-growing firms (“gazelles”) to gross job creation.

The speedup and slowdown of growth occurred uniformly across the size distribution. In particular, firms with (i) less than 20 employees, (ii) between 20 and 249 employees, (iii) between 250 and 4999 employees, and (iv) 5000 or more employees, each produced a speedup and slowdown of around 40 basis points. Thus, in this accounting, superstar firms did not generate the bulk of the boom or bust. Focusing on the role of superstars in creating or hindering growth — at least directly — seems misplaced. It is possible, of course, that the rise of superstar firms discouraged innovation by other firms, as in Aghion, Bergeaud, Boppart, Klenow and Li (2020).

The rest of the paper proceeds as follows. Section 2 relates our paper to the existing literature. Section 3 lays out the model we use to do innovation accounting. Section 4 presents the mapping from model parameters to the moments in the LBD. Section 5 provides our main results. Section 6 discusses further results and directions for future research. Section 7 concludes.

2. Related Literature

The classic reference on creative destruction models is Aghion and Howitt (1992). Klette and Kortum (2004) developed potential implications of creative destruction for firm dynamics. Lentz and Mortensen (2008) formally estimated a generalization of the Klette-Kortum model using data on Danish firms. See Aghion,

Akcigit and Howitt (2014) for a survey of the literature on both models and evidence of creative destruction. Aghion, Bergeaud, Boppart, Klenow and Li (2019) analyze how creative destruction can be missed in measures of aggregate productivity growth. Hsieh, Klenow and Nath (2019) develop and calibrate a two-country version of the Klette-Kortum model.

Romer (1990) is the seminal paper on expanding variety models. Rivera-Batiz and Romer (1991) followed up with a two-economy version. Acemoglu (2003) and Jones (2016) are two of many subsequent models built around variety growth. Chapter 12 of Acemoglu (2011) provides a textbook treatment. Feenstra (1994) and Broda and Weinstein (2006) estimate variety gains from imports. Broda and Weinstein (2006) estimate growth in the range of consumer products available.

Krusell (1998) was an early paper modeling innovation by incumbent firms on their own products. Lucas and Moll (2014) can be interpreted to be in this vein. See chapter 12 of Aghion and Howitt (2009) and chapter 14 of Acemoglu (2011) for models combining own innovation with creative destruction. Other papers with multiple types of innovation include Luttmer (2011), Akcigit and Kerr (2018), and Peters (2019). Atkeson and Burstein (2019) analyze optimal policy in the presence multiple innovation channels. They emphasize that the social return to creative destruction is smaller than to own innovation or to the creation of new varieties. From a policy perspective, therefore, we care about the importance of each type of innovation in overall growth.

A very large literature looks at R&D spending and/or patenting to assess innovation inputs or outputs. A classic entry is Kortum (1997), who relates rates of patenting to productivity growth. More recent examples include Kogan, Papanikolaou, Seru and Stoffman (2017) on patent counts and quality, Bloom, Jones, Van Reenen and Webb (2020) on R&D trends, and Acemoglu, Akcigit, Alp, Bloom and Kerr (2018) on both patents and R&D. Such studies are clearly useful, but are somewhat constrained by the propensity to patent (which is much higher in manufacturing) and the propensity to report formal R&D spending.

Argente, Baslandze, Hanley and Moreira (2018) illustrate the limits of patenting data even within consumer products manufacturing, where they find a much higher propensity to patent a new product by large firms compared to small firms. In comparison, an advantage of our indirect approach is that it can be applied to the entire nonfarm business sector, including firms and plants that do not patent or report any R&D spending.

Perhaps the closest paper to ours is Garcia-Macia, Hsieh and Klenow (2019). They too use LBD data to infer the types of innovation behind U.S. growth in recent decades. The key distinction is that they look only at firms. By exploiting data on plants and assuming plants proxy for varieties produced by a firm, we are able to *directly* infer the sources of growth. Garcia-Macia et al. resort to indirect inference, and also make simplifying assumptions such as that the step-size of innovations is the same for own innovation and creative destruction. We are able to relax this assumption, and even allow the step sizes to differ by the size and age of firms.

Our model also features exit of the lowest quality products due to obsolescence. Hopenhayn (1992) is a pioneering model with this feature. Hopenhayn, Neira and Singhania (2018) applies this model to understand the falling entry rate. Akcigit and Ates (2019) and Peters and Walsh (2019) analyze the causes of both falling entry and falling growth. Other efforts to explain lackluster recent growth include Engbom (2017) and Liu, Mian and Sufi (2019).

Our approach builds on Feenstra (1994), which infers quality and variety growth from a product's market share. Feenstra showed that one can back out underlying quality and variety using a CES demand elasticity and the market share of a product. Hottman, Redding and Weinstein (2016) follow the Feenstra approach, and have the advantage of directly seeing products at a detailed level. They also observe prices and quantities, so they can adjust for markups when inferring a product's quality. Due to data constraints, we abstract from markup dispersion across products. The tradeoff is that we look beyond consumer packaged goods to the entire nonfarm business sector.

With data on prices and quantities and products, one could adjust not only for markups but for other factors driving a wedge between a plant's employment and the quality and variety of its products. This includes adjustment costs, financial frictions, and capital intensity of production. One could focus more narrowly on U.S. manufacturing and control for such factors. Following Foster, Haltiwanger and Syverson (2008), one could separate "TFPQ" from "TFPR" by netting out the influence of markups, adjustment costs, factor intensity and so on — and then apply our direct inference to TFPQ data to determine the sources of innovation. See Hsieh and Klenow (2009, 2014) and Eslava and Haltiwanger (2019) for efforts along these lines.

Just as growth accounting does not require that capital or labor be determined exogenously, our innovative growth accounting does not require that innovation rates be exogenous. For this reason, our accounting is not inconsistent with the literature arguing that innovation rates are sustained only because of growth in research efforts. See, for example, Jones (1995) and Bloom, Jones, Van Reenen and Webb (2017).

Less consistent with our approach are decompositions of productivity growth into within-firm, reallocation, and entry-exit terms by Baily, Hulten and Campbell (1992), Griliches and Regev (1995), Olley and Pakes (1996), and Melitz and Polanec (2015). A prominent implementation is Foster, Haltiwanger and Krizan (2001), and more recent versions are Decker, Haltiwanger, Jarmin and Miranda (2017) and Gutiérrez and Philippon (2019). Baqaee and Farhi (2020) show that such statistical decompositions are not derived from explicit general equilibrium models. Thus, although these decompositions provide useful moments that models should match, they do not provide a direct way to infer the sources of innovation and growth.

In particular, the above decompositions use growth in firm labor productivity to evaluate innovation within firms. Figures 3 and 4 compare growth in revenue per worker (labor productivity) to growth in employment for Wal-

Figure 3: Walmart

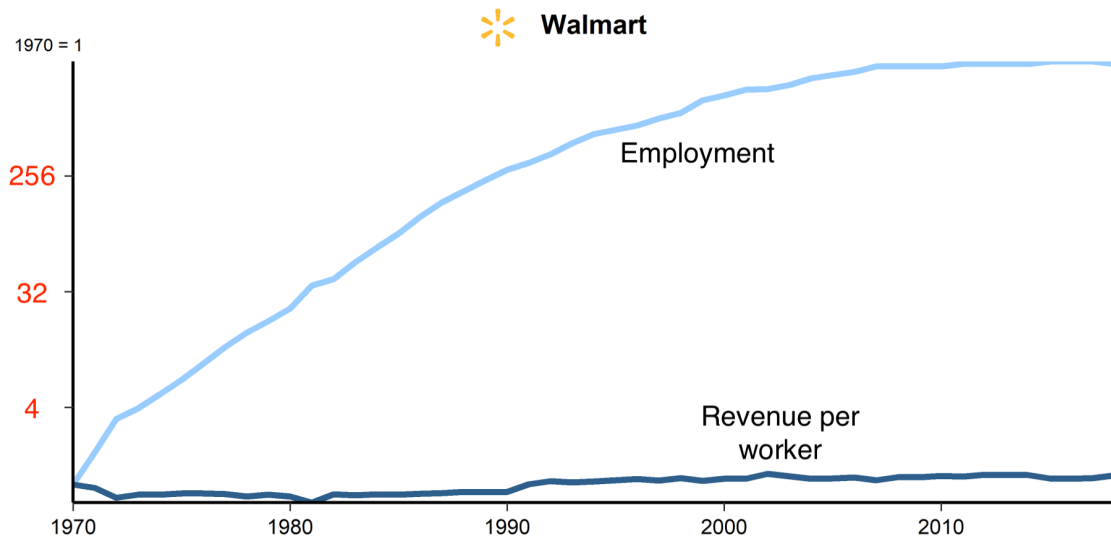
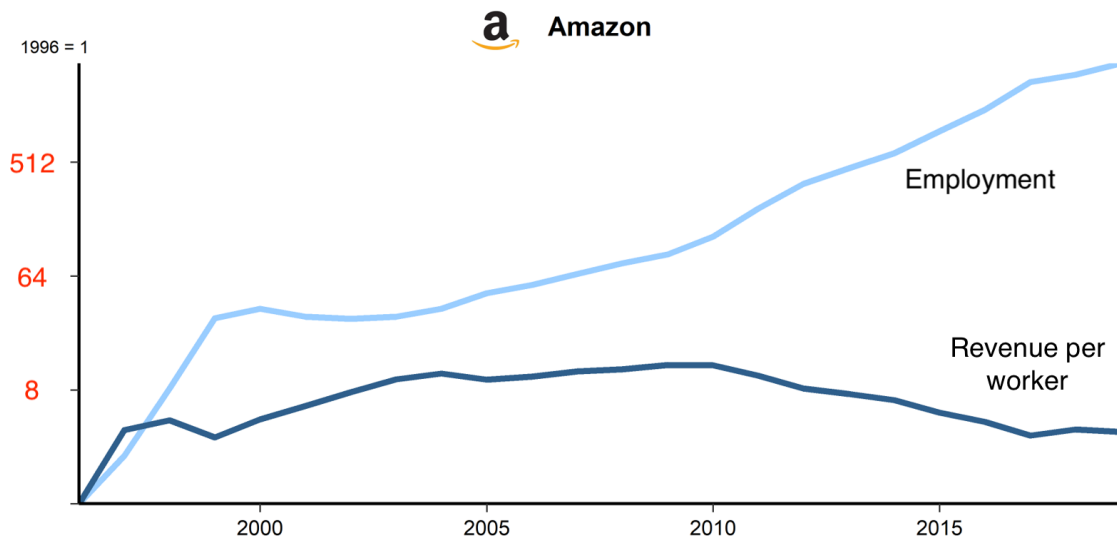


Figure 4: Amazon



Source: WRDS Compustat and BEA Table 1.1.9. Revenue per worker = “REVT / (GDP Deflator x EMP)”, Employment = “EMP” and Year = “FYEAR”. The graph displays revenue per worker and employment relative to the first year when both series are available for the firm.

mart and Amazon based on Compustat data.⁴ For Walmart, employment grew rapidly while labor productivity grew comparatively little. For Amazon, revenue productivity did grow markedly, but employment growth was much faster still. Figures 9 through 13 in the Appendix cover Microsoft, Google, Facebook, Apple and Starbucks. For all of these famously innovative firms, employment growth dwarfed growth in revenue per worker. Perhaps these firms innovated by adding more markets and products with similar price-cost markups, thereby boosting their employment rather than their revenue productivity.⁵

In our simple framework, revenue productivity is actually the same across plants and firms, as each plant equates its marginal revenue product of labor to the common wage. In our setup, a firm's revenue productivity does not contain any information about a firm's contribution to growth. A firm's contribution to growth will instead be manifested in its rising market share (which we proxy with employment, as it is easy to observe and measure). From this perspective, the rising market share of Walmart, Amazon, and other superstar firms is consistent with their outsized innovation and contribution to growth.

As stressed by Hsieh and Klenow (2017) and Garcia-Macia et al, however, not all market share gains are equal when it comes to their contribution to growth. Creative destruction entails more job creation than growth, since only the net improvement beyond existing technology adds to growth. For this reason we are keen to use the entry and exit of plants to disentangle a firm's own innovation and variety creation versus from its creative destruction.

⁴The Census LBD, which we use for almost all of our estimates, does not have data on revenue, and the U.S. Census Bureau forbids us from revealing the identity of firms. Hence we compare measures of revenue labor productivity with employment growth for select firms in Compustat.

⁵The *level* of revenue productivity is high in these firms compared to the average firm. But their rapid growth did not bring down their revenue productivity, as one might expect if their employment growth was a drawn out response to previous shocks due to adjustment costs.

3. Innovation accounting

This section lays out a model wherein aggregate productivity growth comes from innovation by firms. The model connects a firm's growth contribution to the market share of its entering, surviving, and exiting products. The market share dynamics of its surviving products shed light, in particular, on the rate at which the firm improves its own products.

3.1. Final goods production

There is a continuum of intermediate products $i \in [0, N_t]$, with N_t denoting the total measure of varieties in period t . N_t can change over time due to the introduction of new varieties and the obsolescence of low quality products. Omitting time subscripts, aggregate output Y in a given period is a CES aggregate of intermediate varieties:

$$Y = \left[\int_0^N [q(i)y(i)]^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}},$$

where $y(i)$ is the quantity of variety i and $q(i)$ is its quality. Parameter $\sigma > 1$ is the elasticity of substitution between varieties.

Profit maximization by competitive final goods producers generates the demand function for variety i :

$$y(i) = \left(\frac{P(i)}{P} \right)^{\sigma} q(i)^{\sigma-1} Y, \quad (1)$$

where $P(i)$ is the price of variety i and P is the aggregate price index

$$P = \left[\int_0^N [P(i)/q(i)]^{1-\sigma} di \right]^{\frac{1}{1-\sigma}}. \quad (2)$$

Let $p(i) := \frac{P(i)}{P}$ denote the relative price of good i . Demand for a product increases with its quality and decreases with its price.

3.2. Intermediate goods production

Each intermediate good i can be produced by a monopolistically competitive producer at a constant marginal cost equal to the competitive wage W . The profit-maximizing price given demand (1) is the familiar constant markup over marginal cost:

$$P(i) = \mu \cdot W,$$

where $\mu := \frac{\sigma}{\sigma-1} > 1$. The markup is lower when products are closer substitutes. We can also write the relative price as

$$p(i) = \mu \cdot w$$

where $w := W/P$ is the real marginal cost.

Substituting the aggregate price index (2) and the profit-maximizing price into (1) yields the expenditure share (and employment share) of a product:

$$s(i) = \frac{P(i)y(i)}{PY} = \frac{l(i)}{L} = \frac{q(i)^{\sigma-1}}{\int_0^N q(i)^{\sigma-1} di}. \quad (3)$$

We can define an aggregate quality index as

$$Q := \left(\int_0^N q(i)^{\sigma-1} di \right)^{\frac{1}{\sigma-1}}.$$

Substituting this definition into (3), a product's market share can be written as

$$s(i) = \left(\frac{q(i)}{Q} \right)^{\sigma-1}$$

where the market share increases with relative quality with elasticity $\sigma - 1$.

Growth in this model is due to growth in the aggregate quality index Q . Using $y(i) = s(i)L$ in the aggregate production function, aggregate output is

$$Y = \left[\int_0^N (q(i)s(i))^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}} L = Q \cdot L.$$

We are interested in the growth rate of aggregate productivity Y/L :

$$g_t := \ln \frac{Y_t}{L_t} - \ln \frac{Y_{t-1}}{L_{t-1}} = \ln \frac{Q_t}{Q_{t-1}}$$

3.3. Innovation

We next describe the innovation process that drives the growth in Q . We are interested in attributing growth to groups of firms or individual firms. Let \mathcal{M}_t denote the set of all firms in the economy in period t . The set of firms can change over time from the entry of new firms and from the exit of firms that lose all of their products. Let \mathcal{E}_t and \mathcal{X}_{t-1} denote the set of firms entering and exiting between $t-1$ and t . Let $\mathcal{C}_{t,t-1}$ denote the set of continuing firms that operate in both t and $t-1$. By construction, the set of firms in t is the union of new and continuing firms $\mathcal{M}_t = \mathcal{E}_t \cup \mathcal{C}_{t,t-1}$, and the set of firms in $t-1$ is the union of exiting and continuing firms $\mathcal{M}_{t-1} = \mathcal{X}_{t-1} \cup \mathcal{C}_{t,t-1}$.

We denote the set of products produced by firm f in period t as \mathcal{N}_{ft} . The complete set of products produced in t is therefore $\mathcal{N}_t = \{\mathcal{N}_{ft}\}_{f \in \mathcal{M}_t}$. The set of products produced by a firm and the quality of those products can change due to innovation. There are three types of innovation: 1) creative destruction, 2) own-innovation and 3) new varieties. With creative destruction, the producer of an existing product is replaced by a better producer. In own innovation, a producer innovates on its own products. A new variety expands the set of products. We allow the arrival rate and step size of each type of innovation to depend on the innovating firm. In addition, products disappear due to obsolescence when their quality is below a threshold κ_t , which can rise over time.

Let \mathcal{NV}_t , \mathcal{CD}_t , \mathcal{OI}_t and \mathcal{O}_{t-1} denote the set of new varieties, creatively destroyed products, products experiencing own innovation, and those becoming obsolete between t and $t-1$, respectively. Let \mathcal{S}_t denote the set of products that are produced in t and $t-1$ with constant quality. The set of products in t is $\mathcal{N}_t = \mathcal{NV}_t \cup \mathcal{CD}_t \cup \mathcal{OI}_t \cup \mathcal{S}_t$, while the set of products in $t-1$ is $\mathcal{N}_{t-1} = \mathcal{O}_{t-1} \cup \mathcal{CD}_t \cup \mathcal{OI}_t \cup \mathcal{S}_t$.

The timing of events in a period is as follows: firms produce, then obsolete products exit, then new varieties enter and existing products are creatively destroyed, and finally own innovation occurs on surviving incumbent products. Quality levels after innovation apply to production in the next period. Our assumption that obsolete products exit before innovation ensures that firms only carry out innovation on products that are not obsolete.

Let $\tilde{x}_{f,t-1}^O$ denote the share of products of firm $f \in \mathcal{M}_{t-1}$ that exit due to obsolescence:

$$\tilde{x}_{f,t-1}^O = \frac{\int_{i \in N_{f,t-1}} \mathbf{1}\{q_{t-1}(i) < \kappa_t\} di}{N_{f,t-1}} =: G_{f,t-1}(\kappa_t)$$

where $G_{f,t-1}(q)$ is the cumulative distribution of quality for firm f in period $t-1$.

After exit of obsolete products, each surviving firm and new firm $f \in \mathcal{M}_t$ brings in $\nu_{ft}N_{t-1}$ new varieties for production in period t . The quality of a new variety is drawn from the distribution of non-obsolete varieties in period $t-1$ with a step size V_{ft} . These firms may also replace existing producers of $\delta_{ft}N_{t-1}$ non-obsolete products by improving on the quality of those products by step size ω_{ft} . If the firm f was active in period $t-1$, then it loses some of the products it produced in $t-1$ to creative destruction and obsolescence. For the products it keeps, it randomly innovates on share $\tilde{\omega}_{ft}$ with step size O_{ft} .

Let \mathcal{NV}_{ft} , \mathcal{CD}_{ft} and \mathcal{OI}_{ft} denote the set of products of firm f that are new varieties, acquired through creative destruction, or underwent own innovation, respectively. Let $\mathcal{X}_{f,t-1}^{CD}$ and $\mathcal{X}_{f,t-1}^O$ denote the set of products which firm f loses through creative destruction and obsolescence, respectively. Finally, let \mathcal{S}_{ft} denote the set of products that f produces in both t and $t-1$ with constant quality. The set of products f produces in t is therefore $\mathcal{N}_{ft} = \mathcal{NV}_{ft} \cup \mathcal{CD}_{ft} \cup \mathcal{OI}_{ft} \cup \mathcal{S}_{ft}$, while the set it produces in $t-1$ is $\mathcal{N}_{f,t-1} = \mathcal{X}_{f,t-1}^{CD} \cup \mathcal{X}_{f,t-1}^O \cup \mathcal{S}_{ft}$. A firm exits the market when it does not have any products.

Thus, the share of products in $t - 1$ that is subject to creative destruction is

$$\delta_t := \int_{f \in \mathcal{M}_t} \delta_{ft} df.$$

Similarly, the rate of arrival of new varieties between $t - 1$ and t is

$$\nu_t := \int_{f \in \mathcal{M}_t} \nu_{ft} df.$$

We need to specify how firms lose their products to creative destruction. Let $\tilde{x}_{f,t-1}^{CD} N_{f,t-1}$ be the number of products a firm active in $t - 1$ loses between $t - 1$ and t due to creative destruction. In equilibrium, the arrival rate of creative destruction equals the sum of the rate of product exit due to creative destruction. That is,

$$\delta_t = \int_{f \in \mathcal{M}_{t-1}} \tilde{x}_{f,t-1}^{CD} \frac{N_{f,t-1}}{N_{t-1}} df = \int_{f \in \mathcal{M}_{t-1}} x_{f,t-1}^{CD} df,$$

where $x_{f,t-1}^{CD} := \tilde{x}_{f,t-1}^{CD} \frac{N_{f,t-1}}{N_{t-1}}$. For simplicity we assume creative destruction is random so that all firms lose the same fraction of non-obsolete products, denoted by $\hat{\delta}_t$.⁶ This implies that, for all firms in $t - 1$,

$$\tilde{x}_{f,t-1}^{CD} = \hat{\delta}_t [1 - G_{f,t-1}(\kappa_t)].$$

Therefore, the share of products of a firm in $t - 1$ that are not produced by the firm in t is

$$\tilde{x}_{f,t-1} = \tilde{x}_{f,t-1}^O + \tilde{x}_{f,t-1}^{CD} = G_{f,t-1}(\kappa_t) + \hat{\delta}_t [1 - G_{f,t-1}(\kappa_t)],$$

and the aggregate exit rate of product from producers at the end of $t - 1$ is

$$x_{t-1} = \int_{f \in \mathcal{M}_{t-1}} \tilde{x}_{f,t-1} \frac{N_{f,t-1}}{N_{t-1}} df = G_{t-1}(\kappa_t) + \hat{\delta}_t [1 - G_{t-1}(\kappa_t)].$$

The net growth rate in the number of varieties N_t is the difference between

⁶See Appendix [Appendix C](#). for an extension that allows targeted creative destruction.

the rate of new variety arrival and the rate of obsolescence:

$$N_t = [1 + \nu_t - G_{t-1}(\kappa_t)] \cdot N_{t-1}.$$

Recall that \tilde{o}_{ft} is the fraction of the products that survive both creative destruction and obsolescence which experience own innovation by incumbent firm f . The share of products of firm f that experience own innovation is

$$\tilde{o}_{ft} \cdot (1 - \tilde{x}_{f,t-1}^{CD} - \tilde{x}_{f,t-1}^O) = \tilde{o}_{ft} \cdot (1 - \hat{\delta}_t) \cdot [1 - G_{f,t-1}(\kappa_t)]$$

and the share of all products in $t-1$ that experience own innovation is

$$(1 - \hat{\delta}_t) \int_{f \in \mathcal{M}_{t-1}} \tilde{o}_{ft} [1 - G_{f,t-1}(\kappa_t)] \frac{N_{f,t-1}}{N_{t-1}} df.$$

3.4. Aggregate productivity growth

Having laid out the innovation process, how does this process translate into aggregate productivity growth? Using $\mathcal{N}_t = \mathcal{N}\mathcal{V}_t \cup \mathcal{C}\mathcal{D}_t \cup \mathcal{O}\mathcal{I}_t \cup \mathcal{S}_t$, we can decompose growth $g_t = \ln \frac{Q_t}{Q_{t-1}}$ into innovation types:

$$\begin{aligned} e^g &= \left(\frac{\int_{i \in \mathcal{N}_t} q_t(i)^{\sigma-1} di}{\int_{i \in \mathcal{N}_{t-1}} q_{t-1}(i)^{\sigma-1} di} \right)^{\frac{1}{\sigma-1}} \\ &= \left(\frac{\int_{i \in \mathcal{N}\mathcal{V}_t} q_t(i)^{\sigma-1} di + \int_{i \in \mathcal{C}\mathcal{D}_t} q_t(i)^{\sigma-1} di + \int_{i \in \mathcal{O}\mathcal{I}_t} q_t(i)^{\sigma-1} di + \int_{i \in \mathcal{S}_t} q_t(i)^{\sigma-1} di}{\int_{i \in \mathcal{N}_{t-1}} q_{t-1}(i)^{\sigma-1} di} \right)^{\frac{1}{\sigma-1}} \end{aligned} \quad (4)$$

The first term on the right hand side captures new varieties. Since the quality of new varieties is drawn randomly from the distribution of non-obsolete products (plus a firm-specific step size), this term simplifies to

$$\frac{\int_{i \in \mathcal{N}\mathcal{V}_t} q_t(i)^{\sigma-1} di}{\int_{i \in \mathcal{N}_{t-1}} q_{t-1}(i)^{\sigma-1} di} = \frac{\int_{f \in \mathcal{M}_t} \nu_f N_{t-1} \frac{(V_f \hat{Q}_{t-1})^{\sigma-1}}{N_{t-1}} df}{Q_{t-1}^{\sigma-1}} = \left(\int_{f \in \mathcal{M}_t} \nu_f V_f^{\sigma-1} df \right) \left(\frac{\hat{Q}}{Q} \right)^{\sigma-1},$$

where $\widehat{Q}^{\sigma-1} := N_{t-1} E_{G_{t-1}}(q^{\sigma-1} | q \geq \kappa_t)$ is the aggregate quality of non-obsolete products. Since $\widehat{Q} \geq Q$, a given arrival rate of innovation (ν_t) generates more growth when the innovation happens on non-obsolete products. Hence, we will refer to

$$\int_{f \in \mathcal{M}_t} \nu_f V_f^{\sigma-1} df$$

as the growth contribution of new varieties and refer to

$$\left(\int_{f \in \mathcal{M}_t} \nu_f V_f^{\sigma-1} df \right) \left(\left(\frac{\widehat{Q}}{Q} \right)^{\sigma-1} - 1 \right)$$

as the contribution of obsolescence to growth. This Hopenhayn term is the interaction of selection and innovation.

With the assumption that creative destruction is random, we can simplify the second term in (4) to

$$\frac{\int_{f \in \mathcal{M}_t} \int_{i \in \mathcal{C}_{Df,t}} q_{t-1}(i)^{\sigma-1} di df}{Q_t^{\sigma-1}} = \left(\int_{f \in \mathcal{M}_t} \delta_f \int_{i \in \mathcal{C}_{Df,t}} q_{t-1}(i)^{\sigma-1} di df \right) \left(\frac{\widehat{Q}}{Q} \right)^{\sigma-1}$$

The third term in (4) is due to incumbent own innovation and therefore, only applies to continuing firms. It is equal to

$$\begin{aligned} & (1 - \widehat{\delta}_t) \int_{f \in \mathcal{C}_{t,t-1}} \widetilde{o}_{ft} O_f^{\sigma-1} \int_{i \in \mathcal{N}_{f,t-1}, q_{t-1}(i) \geq \kappa_t} \frac{q_{t-1}(i)^{\sigma-1}}{Q_t^{\sigma-1}} di df \\ &= (1 - \widehat{\delta}_t) \left(\frac{\widehat{Q}}{Q} \right)^{\sigma-1} \int_{f \in \mathcal{C}_{t,t-1}} \widetilde{o}_{ft} (1 - G_{f,t-1}(\kappa_t)) O_f^{\sigma-1} \frac{N_{f,t-1} E_{G_{f,t-1}}(q^{\sigma-1} | q \geq \kappa_t)}{\widehat{Q}_t^{\sigma-1}} df \\ &= \left(\frac{\widehat{Q}}{Q} \right)^{\sigma-1} \int_{f \in \mathcal{C}_{t,t-1}} o_{ft} O_f^{\sigma-1} \frac{E_{G_{f,t-1}}(q^{\sigma-1} | q \geq \kappa_t)}{E_{G_{t-1}}(q^{\sigma-1} | q \geq \kappa_t)} df \end{aligned}$$

where $o_{ft} := \widetilde{o}_{ft} (1 - \widehat{\delta}_t) (1 - G_{f,t-1}(\kappa_t)) \frac{N_{f,t-1}}{N_{t-1}}$ denotes the arrival rate of own innovation by f relative to the total number of products in $t-1$.

The fourth term in (4) contains products that survive creative destruction

and obsolescence but do not experience own innovation. This term is equal to

$$\begin{aligned} & (1 - \widehat{\delta}_t) \int_{f \in \mathcal{C}_{t,t-1}} (1 - \widetilde{o}_{ft}) \int_{i \in \mathcal{N}_{f,t-1}, q_{t-1}(i) \geq \kappa_t} \frac{q_{t-1}^\sigma(i)}{Q_{t-1}^\sigma} di df \\ &= \left(\frac{\widehat{Q}}{Q} \right)^{\sigma-1} (1 - \widehat{\delta}_t) \int_{f \in \mathcal{C}_{t,t-1}} (1 - \widetilde{o}_{ft}) \frac{(1 - G_{f,t-1}(\kappa_t)) N_{f,t-1} \mathbb{E}_{G_{f,t-1}(\kappa_t)}(q^{\sigma-1} | q \geq \kappa_t)}{N_{t-1} \mathbb{E}_{G_{t-1}}(q^{\sigma-1} | q \geq \kappa_t)} df. \end{aligned}$$

The third and fourth terms in (4) combined equals the net change in quality of products that are produced by the same producer in t and $t-1$:

$$\begin{aligned} & \left(\frac{\widehat{Q}}{Q} \right)^{\sigma-1} \int_{f \in \mathcal{C}_{t,t-1}} o_{ft} (O_f^\sigma - 1) \frac{\mathbb{E}_{G_{f,t-1}}(q^{\sigma-1} | q \geq \kappa_t)}{\mathbb{E}_{G_{t-1}}(q^{\sigma-1} | q \geq \kappa_t)} df \\ &+ \left(\frac{\widehat{Q}}{Q} \right)^{\sigma-1} (1 - G_{t-1}(\kappa_t) - \delta_t) \end{aligned}$$

where the second line comes from

$$\delta_t = \int_{f \in \mathcal{M}_t} \delta_{ft} df = \widehat{\delta}_t \int_{f \in \mathcal{M}_{t-1}} [1 - G_{f,t-1}(\kappa_t)] \frac{N_{f,t-1}}{N_{t-1}} df.$$

Combining our derivations for the right hand side terms of (4), we can express aggregate growth in terms of arrival rates and step sizes.

$$\begin{aligned} e^{g(\sigma-1)} &= 1 + \int_{f \in \mathcal{M}_t} \nu_f V_f^{\sigma-1} + \delta_f (V_f^{\sigma-1} - 1) df + \int_{f \in \mathcal{C}_{t,t-1}} o_{ft} (O_f^\sigma - 1) \frac{\mathbb{E}_{G_{f,t-1}}(q^{\sigma-1})}{\mathbb{E}_{G_{t-1}}(q^{\sigma-1})} df \\ &+ \left(\int_{f \in \mathcal{M}_t} \nu_f V_f^{\sigma-1} + \delta_f (V_f^{\sigma-1} - 1) df \right) \left(\left(\frac{\widehat{Q}}{Q} \right)^{\sigma-1} - 1 \right) \\ &+ \int_{f \in \mathcal{C}_{t,t-1}} o_{ft} (O_f^\sigma - 1) \frac{\mathbb{E}_{G_{f,t-1}}(q^{\sigma-1} | q \geq \kappa_t) - \mathbb{E}_{G_{f,t-1}}(q^{\sigma-1})}{\mathbb{E}_{G_{t-1}}(q^{\sigma-1})} df \\ &- \left(1 - \left(\frac{\widehat{Q}}{Q} \right)^{\sigma-1} [1 - G_{t-1}(\kappa_t)] \right). \end{aligned} \tag{5}$$

The first line of the right hand side of equation (17) is the sum of the growth contributions from new varieties, creative destruction, and own innovation.

The second and third lines on the right hand side of equation (17) are positive contributions from obsolescence through selection, while the last line is the negative contribution based on the market share in $t - 1$ of varieties lost to obsolescence.

3.5. Firm contribution to growth

In addition to decomposing growth into types of innovation and obsolescence, we are also interested accounting for the contribution of firms or groups of firms. Define the contribution of firm $f \in \mathcal{M}_t \cup \mathcal{X}_{t-1}$ to growth

$$\begin{aligned}
 g_f &:= \nu_f V_f^{\sigma-1} + \delta_f (\sigma_f^{-1} - 1) \\
 &+ o_f N_{t-1} (O_f^{\sigma-1} - 1) E_{G_{f,t-1}}[s_{t-1}] \\
 &+ o_f N_{t-1} (O_f^{\sigma-1} - 1) (E_{G_{f,t-1}}[s_{t-1} | q \geq \kappa] - E_{G_{f,t-1}}[s_{t-1}]) \\
 &+ \left(\int_{j \in \mathcal{M}_t} \nu_j V_j^{\sigma-1} + \delta_j (\sigma_j^{-1} - 1) dj \right) \left(\left(\frac{\widehat{Q}}{Q} \right)^{\sigma-1} - 1 \right) \frac{G_{f,t-1}(\kappa) N_{f,t-1}}{G_{t-1}(\kappa) N_{t-1}} \\
 &- E_{G_{f,t-1}}[s_{t-1} | q < \kappa] G_{f,t-1}(\kappa) N_{f,t-1}, \tag{6}
 \end{aligned}$$

where s_{t-1} is the market share of a product in $t - 1$ and we abbreviated the subscript t . The first two lines on the right hand side of (6) are firm f 's contribution through new varieties, creative destruction and own-innovation. The third line is the positive contribution from own-innovation focusing on non-obsolete products. The fourth line captures the contribution of obsolescence through improving the quality distribution that new varieties and creative destruction build upon. To attribute the aggregate contribution of obsolescence to individual firms, we multiply the fourth line by firm f 's share of obsolete products. The final line is firm's f 's negative contribution to growth from losing its obsolete varieties.

For new firms, g_f simplifies to their contributions from new varieties and creative destruction, respectively:

$$g_f = \nu_f V_f^{\sigma-1} + \delta_f (\sigma_f^{-1} - 1).$$

Firms that exit, meanwhile, contribute only through obsolescence:

$$g_f = \left(\int_{j \in 2\mathcal{M}_t} \nu_j V_j^{\sigma-1} + \delta_j (V_j^{\sigma-1} - 1) dj \right) \left(\left(\frac{\widehat{Q}}{Q} \right)^{\sigma-1} - 1 \right) \frac{G_{f,t-1}(\kappa) N_{f,t-1}}{G_{t-1}(\kappa) N_{t-1}} - \mathbb{E}_{G_{f,t-1}}[s_{t-1} | q < \kappa] G_{f,t-1}(\kappa) N_{f,t-1}$$

Using $\ln(1+x) \approx x$, we can approximate the growth rate by

$$g = \frac{1}{\sigma-1} \ln \left(1 + \int_{f \in 2\mathcal{M}_t \cup \mathcal{X}_{t-1}} g_f df \right) \approx \frac{1}{\sigma-1} \int_{f \in 2\mathcal{M}_t \cup \mathcal{X}_{t-1}} g_f df.$$

This allows us to define the share of growth coming from firm f as

$$c_f := \frac{g_f}{\int_{f \in 2\mathcal{M}_t \cup \mathcal{X}_{t-1}} g_f df}.$$

Similarly, we define the contribution of a group of firms $\mathcal{F} \subset (\mathcal{M}_t \cup \mathcal{X}_{t-1})$ as

$$c_{\mathcal{F}} := \frac{\int_{f \in \mathcal{F}} g_f df}{\int_{f \in 2\mathcal{M}_t \cup \mathcal{X}_{t-1}} g_f df}.$$

4. Calibration

In the previous section, we laid out our model of how product innovation contributes to growth. In this section we will describe how to relate the arrival rates and step sizes of innovation to moments we observe in the data.

Let k be any non-negative number. For a set of products \mathcal{P} , denote the k th non-centered moment of market share is

$$S_{\mathcal{P}t}^{fkg} := \int_{i \in 2\mathcal{P}} s_t(i)^k di = \int_{i \in 2\mathcal{P}} \left(\frac{q_t(i)}{Q_t} \right)^{k(\sigma-1)} di.$$

If \mathcal{CD}_{ft} is the group of products that firm f creatively destroyed between $t-1$

and t with step size δ_{ft} ,

$$\begin{aligned}
 S_{CD_{ft}}^{fk_g} &= \int_{i \in 2CD_{ft}} \left(\frac{q_{t-1}(i)}{Q_{t-1}} \frac{Q_{t-1}}{Q_t} \right)^{(\sigma-1)k} di \\
 &= \left(\frac{\nu_{ft}}{e^g} \right)^{(\sigma-1)k} \int_{i \in 2CD_{ft}} \left(\frac{q_{t-1}(i)}{Q_{t-1}} \right)^{(\sigma-1)k} di \\
 &= \left(\frac{\nu_{ft}}{e^g} \right)^{(\sigma-1)k} \int_{i \in 2CD_{ft}} s_{t-1}(i)^k di \\
 &= \delta_{ft} \left(\frac{\nu_{ft}}{e^g} \right)^{(\sigma-1)k} \mathbb{E}_{G_{t-1}}(s_{t-1}^k | q \geq \kappa_t) N_{t-1}
 \end{aligned}$$

The second-to-last equality holds because, conditional on non-obsolence, the arrival of creative destruction innovation is independent of $q_{t-1}(i)$.

For \mathcal{NV}_{ft} the set of new varieties introduced by firm f ,

$$S_{\mathcal{NV}_{ft}}^{fk_g} = \nu_{ft} \left(\frac{V_{ft}}{e^g} \right)^{(\sigma-1)k} \mathbb{E}_{G_{t-1}}(s_{t-1}^k | q \geq \kappa_t) N_{t-1}.$$

Let \mathcal{E}_{F_t} be the set of new products introduced by a group of firms \mathcal{F} with the same arrival rate and step size for creative destruction and new varieties, Applying the above derivations, we map the k th moments of the employment share of these products to

$$\begin{aligned}
 S_{\mathcal{E}_{F_t}}^{fk_g} &= S_{\mathcal{NV}_{F_t}}^{fk_g} + S_{CD_{F_t}}^{fk_g} \\
 \frac{S_{\mathcal{E}_{F_t}}^{fk_g}/E_{F_t}}{\mathbb{E}_{G_{t-1}}(s_{t-1}^k | q \geq \kappa_t)} &= \left(\frac{N_{t-1}\nu_F}{E_{F_t}} \left(\frac{V_F}{e^g} \right)^{(\sigma-1)k} + \frac{N_{t-1}\delta_F}{E_{F_t}} \left(\frac{\nu_F}{e^g} \right)^{(\sigma-1)k} \right), \quad (7)
 \end{aligned}$$

where E_{F_t} is the *number* of new products introduced by \mathcal{F} .

Equation (7) says the non-centered k th moment of entering products of firm group \mathcal{F} relative to the non-centered k th moment of all non-obsolete products in the previous period is equal to the average step size of these products' innovation relative to the aggregate growth rate. A group of products that have

higher innovation step size has higher relative k th moment. This relationship holds for all k . We will use data on g (the aggregate TFP growth rate) and on S^k moments for $k = 0, 1, 2, 3$, plus the restrictions $V_F > 0$ and $\nu_F > 1$, to infer ν_F , V_F , δ_F , and ν_F .

To estimate the contribution of an incumbent's own innovation, let \mathcal{P}_f be the set of products that f produces in both $t - 1$ and t . The market share growth for these products is

$$\begin{aligned} \frac{S_{\mathcal{P}_f t}}{S_{\mathcal{P}_f, t-1}} &= \frac{\int_{i \in \mathcal{P}_f} q_t(i)^{\sigma-1} di}{\int_{i \in \mathcal{P}_f} q_{t-1}(i)^{\sigma-1} di} \left(\frac{Q_t}{Q_{t-1}} \right)^{\sigma-1} \\ &= \frac{(1 + \tilde{\delta}_f (O_f^{\sigma-1} - 1))(1 - \hat{\delta}_t) \int_{i \in \mathcal{N}_{f,t-1, q_{t-1}(i) < \kappa_t}} q_{t-1}(i)^{\sigma-1} di}{(1 - \hat{\delta}_t) \int_{i \in \mathcal{N}_{f,t-1, q_{t-1}(i) < \kappa_t}} q_{t-1}(i)^{\sigma-1} di} \left(\frac{Q_t}{Q_{t-1}} \right)^{\sigma-1} \\ &= \frac{1 + \tilde{\delta}_f (O_f^{\sigma-1} - 1)}{e^{g(\sigma-1)}}. \end{aligned} \quad (8)$$

The market share of the firm's continuing products grows if their realized average quality improvement exceeds the aggregate growth rate.

To estimate the negative contribution of obsolescence, we need to distinguish product exits due to obsolescence from product exits due to creative destruction. The share of firm f 's products that it loses to obsolescence or creative destruction is

$$\frac{x_{f,t-1} N_{t-1}}{N_{f,t-1}} = \hat{\delta}_t [1 - G_{f,t-1}(\kappa_t)] + G_{f,t-1}(\kappa_t). \quad (9)$$

The share of firm f 's sales that belong to products it loses to obsolescence or creative destruction is

$$\begin{aligned} \frac{S_{X_{f,t-1}}}{N_{f,t-1} \mathbb{E}_{G_{f,t-1}}(s_{t-1})} &= \hat{\delta}_t [1 - G_{f,t-1}(\kappa_t)] \frac{\mathbb{E}_{G_{f,t-1}}(s_{t-1} | q \geq \kappa_t)}{\mathbb{E}_{G_{f,t-1}}(s_{t-1})} \\ &\quad + G_{f,t-1}(\kappa_t) \frac{\mathbb{E}_{G_{f,t-1}}(s_{t-1} | q < \kappa_t)}{\mathbb{E}_{G_{f,t-1}}(s_{t-1})} \end{aligned} \quad (10)$$

Since creative destruction is random while obsolescence is selected, the share

of firm f 's sales belonging to products it loses is smaller than the share of products it loses when it loses some its products to obsolescence. The gap increases with the share of exit that is obsolescence. This relationship helps us to distinguish between creative destruction and obsolescence.

Summing the exit rates across firms and summing the exiting product market shares across firms yield the aggregate exit rate and aggregate market share of exiting products:

$$x_t = \int_{f \in \mathcal{M}_{t-1}} \left[\widehat{\delta}_t + (1 - \widehat{\delta}_t) G_{f,t-1}(\kappa_t) \right] \frac{N_{ft-1}}{N_{t-1}} df = \widehat{\delta}_t + (1 - \widehat{\delta}_t) G_{t-1}(\kappa_t) \quad (11)$$

$$S_{X,t-1} = \widehat{\delta}_t \int_{i \in \mathcal{N}_{t-1}, q_{t-1} > \kappa_t} s_{t-1}(i) di + \int_{i \in \mathcal{N}_{t-1}}^{\kappa_t} s_{t-1}(i) di. \quad (12)$$

Using equation (11), the exit rate can be rewritten as

$$\widehat{\delta}_t = \frac{x_t - G_{t-1}(\kappa_t)}{1 - G_{t-1}(\kappa_t)}.$$

Substituting this expression into (12) yields

$$S_{X,t-1} = \frac{x_t - G_{t-1}(\kappa_t)}{1 - G_{t-1}(\kappa_t)} \int_{i \in \mathcal{N}_{t-1}, q_{t-1} > \kappa_t} s_{t-1}(i) di + \int_{i \in \mathcal{N}_{t-1}}^{\kappa_t} s_{t-1}(i) di. \quad (13)$$

Holding fixed the exit rate, increasing κ_t lowers the market share of products exiting due to obsolescence or creative destruction. Instead of randomly replacing a fraction of the producers, increasing κ_t removes products with the smallest market share. Conditional on $\widehat{\delta}_t$, we can use (9) to back out $G_{f,t-1}(\kappa_t)$.

Since the $t-1$ market share of a firm's continuing products is

$$(1 - \widehat{\delta}_t) [1 - G_{f,t-1}(\kappa_t)] N_{f,t-1} \mathbb{E}_{G_{f,t-1}}(s_{t-1} | q \geq \kappa_t),$$

we can use data on the $t-1$ market share of a firm's continuing products and (10) to infer the market share of obsolete products of firm f and hence calculate the negative contribution to growth from the firm.

We also need \widehat{Q}/Q to calculate the positive contribution of obsolescence. This term can be rewritten as

$$\frac{N_{t-1}[1 - G_{t-1}(\kappa_t)] E_{G_{t-1}}(s_{t-1} | q \geq \kappa_t)}{1 - G_{t-1}(\kappa_t)} \quad (14)$$

This is none other than the market share of non-obsolete products divided by the share of non-obsolete products. This can be calculated using $\widehat{\delta}_t$, $G_{t-1}(\kappa_t)$ and data on the $t - 1$ market share of incumbent firms continuing products.

4.1. Measuring products

We calibrate the model to the Longitudinal Business Dynamics (LBD) plant-level data from the Census Bureau. To make progress, we assume that each plant produces one product and that all new varieties and creative destruction occur through new plants. This enables us to use plant entry and exit to measure the arrival rate of new varieties and creative destruction within firms.

We partition firms in each period into groups. For example, a group can be new firms (age 0) with less the 20 employees. Then the set of new and exiting products by the firm group, $\mathcal{E}_{F,t}$ and $\mathcal{X}_{F,t-1}$ respectively, maps into the set of new and exiting plants of the firm group in the data. The set of all products \mathcal{N}_t maps to the total set of plants in the data. The arrival rates in the model map to the arrival rate of plants in the data. The market share of products map into to the employment shares of plants in the data. Table 1 summarizes the parameters for calibration and the data targets.

Table 1: Parameters and data targets

Calibrated parameters	
ν_F	Arrival rate of NV by firm group \mathcal{F}
V_F	Step size of NV by firm group \mathcal{F}
δ_F	Arrival rate of CD by firm group \mathcal{F}
F	Step size of CD by firm group \mathcal{F}
$o_F(O_F^\sigma - 1)$	Contribution of OI by firm group \mathcal{F}
κ	Cutoff quality of obsolescence
δ	Total rate of creative destruction
Assigned parameter	
σ	Elasticity of substitution
Data targets	
$\frac{E_{F,t}}{N_{t-1}}$	# of new plants by firm group \mathcal{F}
$\frac{X_{F,t-1}}{N_{t-1}}$	# of exiting plants by firm group \mathcal{F} in $t-1$
$S_{E_{F,t}}$	employment share of new plants by firm group \mathcal{F} in t
$S_{X_{F,t-1}}$	employment share of exiting plants by firm groups \mathcal{F} in $t-1$
$\frac{S_{C_{F,t}}}{S_{C_{F,t-1}}}$	growth in employment share of continuing plants by firm group \mathcal{F}
$S_{E_{F,t}}^{f2g}$	2nd moment of employment share of new plants by firm group \mathcal{F} in t
$S_{E_{F,t}}^{f3g}$	3rd moment of employment share of new plants by firm group \mathcal{F} in t
$S_{N_{t-1}}^{fk g}$	k th moment of employment share of all plants in $t-1$ for $k = 0, 1, 2, 3$
g	TFP growth rate

Notes: $S_{\mathcal{P}_t}^{\{k\}} := \int_{\mathcal{P}} s_t^k(i) di$

We carry out calibration in four steps. First, we use (8) plus the change in the employment share of surviving plants to calibrate the own innovation parameters relative to the aggregate growth rate. Given g and σ , these moments map directly into the composite arrival rate and step size of own innovation by a group of firms: $(1 + \tilde{\delta}_F [O_F^\sigma - 1])/e^{g(\sigma - 1)}$. Second, we calibrate the arrival rate and step size of new varieties and creative destruction $\{v_F, \delta_F, \frac{V_F}{e^g}, \frac{\Delta_F}{e^g}\}$ to fit the employment share moments of entering plants in (7) for $k = 0, 1, 2, 3$.⁷ Third, we sum the calibrated δ_F across \mathcal{F} to arrive at the aggregate rate of creative destruction δ . We substitute δ into the aggregate exit rate and exiting plant employment share equations (11) and (13) to back out the rate of obsolescence and the employment share associated with obsolescence. Fourth and finally, we recover the level of step sizes by multiplying the relative step sizes by the measured aggregate TFP growth rate in the data. When a resulting step size violates a minimum boundary condition, for example when it is negative or when $v_F < 1$, we set that particular step size to the minimum. We check the validity of our estimates by comparing the fit of the model to the growth rate.⁸

5. Results

5.1. Data

We use the Longitudinal Business Dynamics (LBD) data from the Census Bureau to calculate moments related to firms. We use the BLS multifactor produc-

⁷We minimize the $L2$ distance between the k th moments in the data and that implied by (7) for $k = 0, 1, 2, 3$. We fit the $k = 0$ and $k = 1$ moments exactly while giving equal weight to $k = 2, 3$ moments. Note that (7) by itself is symmetric in the new variety and creative destruction parameters and hence only identifies a pair of values. That is, $\{v_{\mathcal{F}}, \delta_{\mathcal{F}}, \frac{V_{\mathcal{F}}}{e^g}, \frac{\Delta_{\mathcal{F}}}{e^g}\} = \{\bar{v}_{\mathcal{F}}, \bar{\delta}_{\mathcal{F}}, \frac{\bar{V}_{\mathcal{F}}}{e^g}, \frac{\bar{\Delta}_{\mathcal{F}}}{e^g}\}$ and $\{v_{\mathcal{F}}, \delta_{\mathcal{F}}, \frac{V_{\mathcal{F}}}{e^g}, \frac{\Delta_{\mathcal{F}}}{e^g}\} = \{\bar{\delta}_{\mathcal{F}}, \bar{v}_{\mathcal{F}}, \frac{\bar{\Delta}_{\mathcal{F}}}{e^g}, \frac{\bar{V}_{\mathcal{F}}}{e^g}\}$ generate the same distance. Given the growth rate in the data, restrictions $\Delta_{\mathcal{F}} > 1$ and $V_{\mathcal{F}} > 0$ break the symmetry and help us choose between the two specifications. This identification does not work if both $\Delta_{\mathcal{F}}$ and $V_{\mathcal{F}}$ exceed 1. Then we use the aggregate exit rate for identification.

⁸We fit the growth rate by construction when the boundary conditions are not violated. Any gaps between the model-implied growth rate and the growth rate in the data reflect the extent to which we run into corners.

tivity series to calculate productivity growth. We have access to the LBD data from 1977 to 2013. We partition the firms into age and size bins, where age is the number of years since the first plant of the firm enters the dataset and size is the total employment across all of plants of the firm in the previous year.⁹ The age bins are 0, 1–5, 6–10 and 11+ while the size bins are 0, 1–19, 20–249, 250–999, 1000–4999, 5000–9999 and 10000+. New firms are in the age 0 and size 0 bin.

To observe firms aged 6+, the earliest year we can start is 1982. Productivity growth in the U.S. surged from 1996 to 2005. We divide the years into the prior period (1982–1995, average 1.15% growth per year), the burst period (1996–2005, 2.82% per year) and afterward (2006–2013, 1.03% per year). We target averages of entry, exit, productivity growth and so on over each subperiod.

Table 2 presents some of the key data moments from the LBD that we use to estimate arrival rates and step sizes. For conciseness, we average them across all years in the Table. The moments shown include the plant entry rate, the employment share of new plants, and the employment share of continuing plants in the current year relative to their share in the previous year. Size 0 plants are those of new firms.

Figure 2 in the introduction showed the generally declining employment shares of new plants and new firms over our 1982–2013 sample. Figure 5 illustrates that new plants at new firms are generally smaller than the average new plant; their share of new plant employment is less than half their share of all new plants. Figure 6 similarly shows that the new plants of smaller firms are typically smaller than the new plants of larger firms. Figure 7 indicates that exiting plants are smaller than surviving plants, as their employment share is about one-half as big as their share of plants. Figure 8 demonstrates that the employment share of continuing plants grew modestly in all three subperiods.

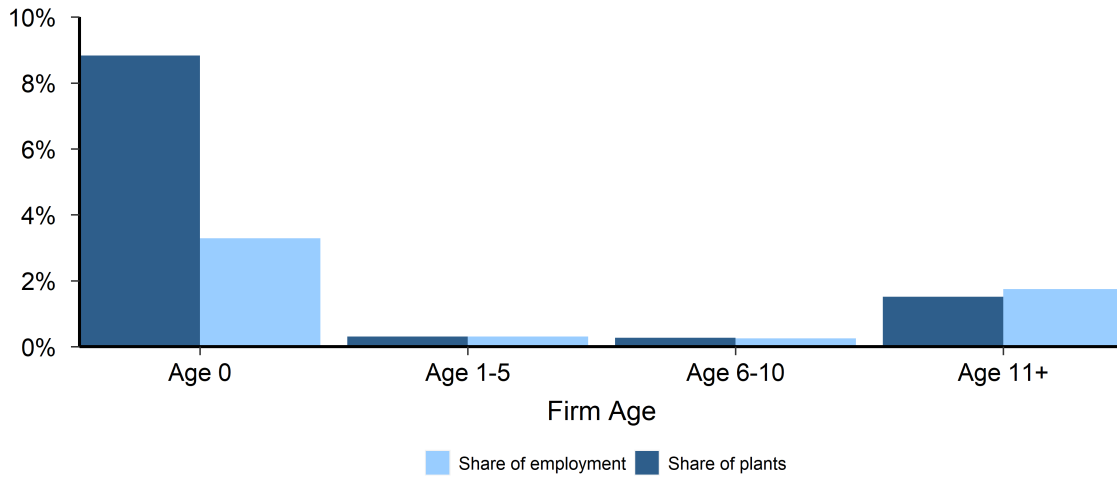
⁹We keep ownership of a plant to the firm that first owns the plant. We use employment in the previous year because the current year employment is affected by innovation.

Table 2: Data moments by age-size groups, 1982–2013

Age	Size	Entry rate	Entry emp. share	$\frac{S_E^2}{S_N^2}$	$\frac{S_E^3}{S_N^3}$	Survivor growth
0	0	8.99	3.10	2.54	3.52	NA
1–5	1–19	0.12	0.06	0.04	0.05	1.15
1–5	20–249	0.11	0.10	0.02	0.01	1.00
1–5	250–999	0.04	0.06	0.02	0.00	0.97
1–5	1k–4999	0.03	0.04	0.04	0.02	0.95
1–5	5k–9999	0.01	0.02	0.05	0.08	0.86
6–10	1–19	0.11	0.05	0.09	0.17	0.99
6–10	20–249	0.07	0.06	0.02	0.02	0.91
6–10	250–999	0.04	0.04	0.01	0.00	0.90
6–10	1k–4999	0.03	0.04	0.04	0.04	0.91
6–10	5k–9999	0.01	0.02	0.01	0.01	0.89
6–10	10k+	0.01	0.02	0.04	0.05	0.89
11+	1–19	0.26	0.18	0.39	0.74	0.98
11+	20–249	0.18	0.14	0.07	0.10	0.91
11+	250–999	0.15	0.15	0.06	0.09	0.91
11+	1k–4999	0.23	0.26	0.19	0.21	0.91
11+	5k–9999	0.13	0.14	0.15	0.22	0.91
11+	10k+	0.58	0.76	0.72	0.87	0.90

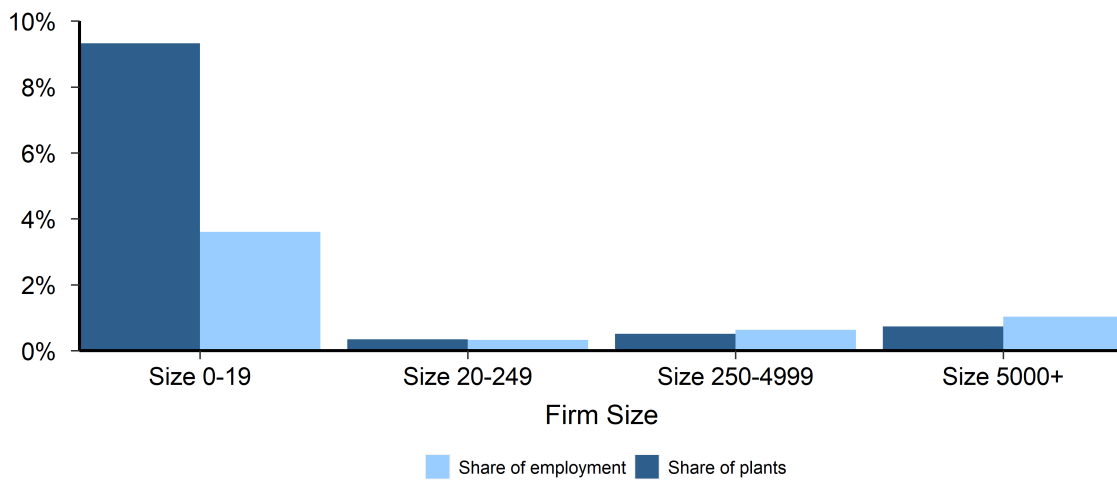
Source: Census LBD. Firm size is the sum of employment across all of its plants. Firm age is the difference between the year of observation minus the birth year of the firm’s first plant. “Entry rate” = Number of new plant divided by lagged total number of plants, “Entry emp. share” = Employment share of new plants, “ $\frac{S_E^k}{S_N^k}$ ” = new plant k th moment relative to all plants. Units are %. “Survivor growth” = continuing plants employment share in t over employment share in $t - 1$

Figure 5: Employment and plant share of new plants by firm age (%)



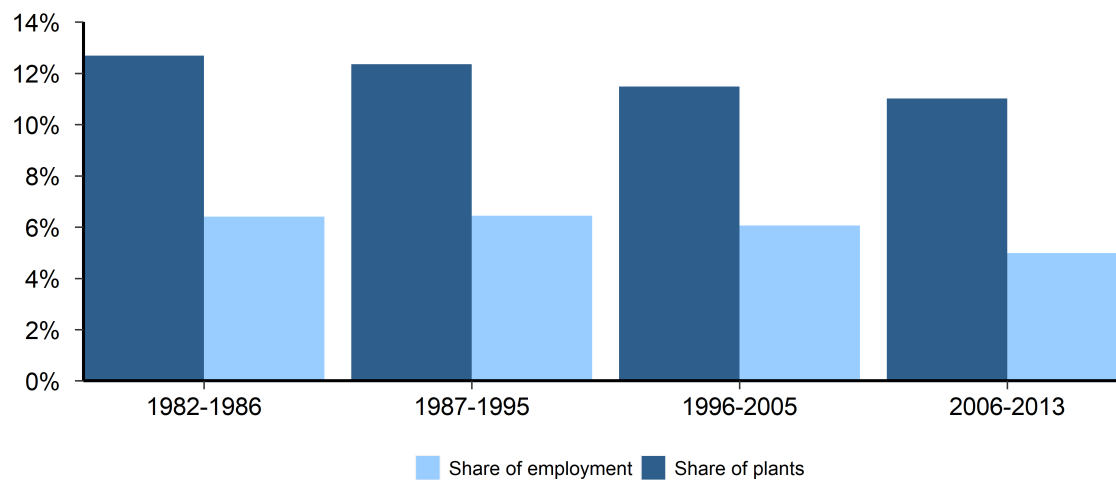
Source: Census LBD.

Figure 6: Employment and plant share of new plants by firm size (%)



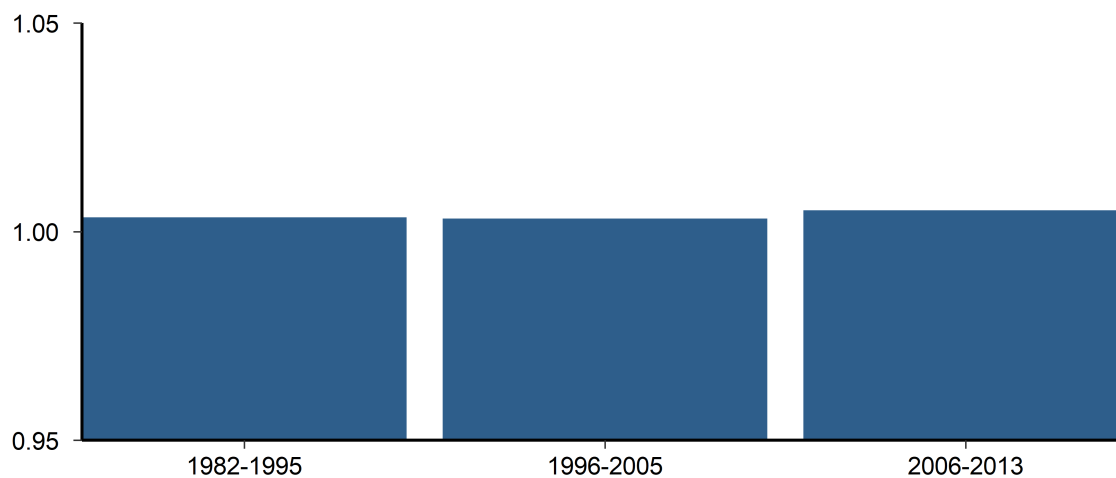
Source: Census LBD.

Figure 7: Employment and plant share of exiting plants (%)



Source: Census LBD.

Figure 8: Growth rate of employment share of continuing plants



Source: Census LBD.

5.2. Results: parameter estimates and model fit

Table 3 presents the parameters we estimate based on the moments in Table 2. As above, the lower cases are arrival rates and upper cases are step sizes. The Table provides these statistics by age-size group.

In Table 4 we compare the growth rate in the data to that in our model. When we do not hit any corners, our calibration strategy fits the growth rate by construction. The Table shows that we deviate slightly from the actual growth rates. This is because some parameters hit corner values (such as step sizes being no lower than 1 for creative destruction). Overall, the deviation is small and the model fits the data well. The largest deviation is in the 1982–1986 period.

5.3. Results: types of innovation

We first decompose growth by innovation type over the entire 1982–2013 period in Table 5. We estimate that, on average, 60% of growth comes from own innovation while new varieties contribute about 27% and creative destruction the remaining 13%.

Looking at subperiods in Table 6, we see that own innovation generated 50% of growth in the slow-growth periods (1982–1995 and 2006–2013), and 70% of growth in the high-growth middle period (1996–2005). Table 7 shows that all three sources of growth rose in the 1996–2005 period and fell back again afterward. But the bulk of the acceleration (91%) and the subsequent slowdown (82%) were due to innovation by incumbent firms on their own products. This finding may be surprising in light of Figure 8, which showed that the growth of continuing plants did not accelerate from 1996–2005 or decelerate afterwards. Growth of all surviving plants is not a sufficient statistic, however, for the contribution of own innovation to growth. Consider a hypothetical wherein the arrival rates of all types of innovation increase by the same proportion. The market share of *innovating* plants who survive stays the same, while the market share of *non-innovating* plants who survive shrinks more rapidly. The market

Table 3: Calibrated innovation parameter by age-size groups, 1982–2013

Age	Size	v_f	V_f	δ_f	f	$\theta_f(O_f - 1)$
0	0	0.081	0.602	0.009	1.184	0.000
1–5	1–19	0.001	0.657	0.000	1.032	0.213
1–5	20–249	0.001	0.985	0.001	1.009	0.056
1–5	250–999	0.000	1.113	0.000	1.116	0.027
1–5	1k–4999	0.000	1.210	0.000	1.211	0.013
1–5	5k–9999	0.000	0.916	0.000	1.351	0.000
6–10	1–19	0.001	0.301	0.000	1.193	0.123
6–10	20–249	0.001	0.929	0.000	1.046	0.039
6–10	250–999	0.000	1.071	0.000	1.070	0.027
6–10	1k–4999	0.000	1.202	0.000	1.102	0.033
6–10	5k–9999	0.000	1.297	0.000	1.184	0.014
6–10	10k+	0.000	0.956	0.000	1.259	0.017
11+	1–19	0.002	0.028	0.001	1.232	0.104
11+	20–249	0.001	0.943	0.001	1.222	0.036
11+	250–999	0.000	0.994	0.001	1.020	0.037
11+	1k–4999	0.000	1.183	0.002	1.059	0.037
11+	5k–9999	0.000	1.280	0.001	1.039	0.035
11+	10k+	0.000	1.164	0.006	1.119	0.029

Source: Census LBD. Firm size is the sum of employment across all of its plants. Firm age is the difference between the year of observation minus the birth year of the firm's first plant.

Table 4: Fit of model

Period	Data g	Model g
1982–1986	1.25	1.33
1987–1995	1.10	1.13
1996–2005	2.82	2.83
2006–2013	1.03	1.03

Source: BLS Multifactor Productivity series. Percentage points of average yearly productivity growth within the specified period. Yearly growth is the sum of R&D and IP contributions to BLS MFP growth, converted into labor augmenting form.

share of all continuing plants is the combined market share of those who innovated and those who did not innovate. This combined market share *declines* even though the share of growth from own innovation stays the same in this hypothetical. By comparison, the fact that the market share of continuing plants in the data stayed constant during the speedup and slowdown suggests own innovation contributed disproportionately to the speedup and slowdown.¹⁰

5.4. Results: firm age groups

Table 8 displays the contribution of each firm age group to growth. Age 0 firms, who are in their first year of operation, contribute almost 1/3 of growth on average despite employing only 3% of workers. Firms ages 1–5 and 6–10 generate similar amounts of growth as their share of employment. Firms age 11+, in contrast, contribute much less growth (41%) than their share of employment (72%).

The high share of growth from new firms in Table 8 can be better understood with three additional points. First, as the final row in the Table shows, new firms

¹⁰This explanation has the testable prediction that the dispersion of employment growth should have increased (decreased) among surviving plants during the speedup (slowdown), due to the divergence between innovating and non-innovating survivors.

Table 5: Growth by innovation type, 1982–2013

g	CD	NV	OI
1.64	0.21	0.38	1.06
	13.0%	27.2%	59.8%

Source: BLS Multifactor Productivity series. Percentage points of average yearly productivity growth within the specified period. Yearly growth is the sum of R&D and IP contributions to BLS MFP growth, converted into labor augmenting form. All other variables are calculated using the Census LBD. CD = Creative Destruction, NV = New Varieties, and OI = Own Innovation (incumbents improving on their own varieties).

Table 6: Growth by innovation type, subperiods

Period	g	CD	NV	OI
1982–1995	1.15	0.16	0.42	0.57
1996–2005	2.82	0.33	0.41	2.09
2006–2013	1.03	0.13	0.28	0.62

Source: BLS Multifactor Productivity series. Percentage points of average yearly productivity growth within the specified period. Yearly growth is the sum of R&D and IP contributions to BLS MFP growth, converted into labor augmenting form. All other variables are calculated using the Census LBD. CD = Creative Destruction, NV = New Varieties, and OI = Own Innovation (incumbents improving on their own varieties).

Table 7: Share of the *change* in TFP growth from innovation types

Period	g	CD	NV	OI
1982–1995 vs 1996–2005	1.67	9.8%	-0.5%	90.8%
1996–2005 vs 2006–2013	-1.79	11.0%	7.1%	81.9%

Source: BLS Multifactor Productivity series. Percentage points of average yearly productivity growth within the specified period. Yearly growth is the sum of R&D and IP contributions to BLS MFP growth, converted into labor augmenting form. All other variables are calculated using the Census LBD. CD = Creative Destruction, NV = New Varieties, and OI = Own Innovation (incumbents improving on their own varieties).

are responsible for almost 60% of all employment at new plants. Second, as reported by Hsieh and Klenow (2017), new and young firms account for 50% of all gross job creation. Third, the employment gains associated with new varieties contribute more to growth than identical employment gains associated with creative destruction. This is because only the net change in employment reflects the net improvement brought by creative destruction.

Table 9 looks at growth contributions, in percentage points per year, for firm age groups by subperiod. All age groups contributed to the acceleration of growth from 1982–1995 to 1996–2005, as well as the subsequent deceleration over 2006–2013. But, as displayed in Table 10, older firms drove the speedup (70%) and the slowdown (59%). These fractions are still smaller than their share of employment, but much larger than their share of growth in the slow-growth periods. The growth slowdown did not stem from a falling rate of innovation by entering and young firms, but rather by older established firms.

The modest contribution of new and young firms to the 1996–2005 growth acceleration may seem surprising in light of the IPO boom in the late 1990s. But such high valuations may have reflected a stream of expected future innovations by these firms, not just the innovations by these firms during this window.

5.5. Results: firm size groups

Table 11 displays the contribution of each size group to growth. Size is based on employment in the *previous* year. Size 0 firms are hence new firms. The Table shows four firm size groups, each with roughly one-fourth of all employment: those with 0–19 employees in the prior year, 20–249 workers, 250–4,999 workers, and 5,000+ workers. The smallest firms account for over 60% of growth on average. Each of the other size groups account for about one-half as much growth as they do employment.

In Table 12 we show that small firms play an outsized role in all subperiods. All size groups accelerated and decelerated along with aggregate TFP. In fact,

Table 8: Growth by firm age groups, 1982–2013

	Age 0	Age 1–5	Age 6–10	Age 11+
% of growth	30.3%	18.9%	9.7%	41.1%
% of employment	3.3%	13.4%	11.2%	72.1%
% of firms	10.7%	31.1%	18.5%	39.6%
% of new plant emp.	58.8%	5.4%	4.5%	31.2%

Source: Census LBD. Average growth rate over the entire period is 1.66%. Firm age is the difference between the year of observation minus the birth year of the firm's first plant.

Table 9: Growth by firm age groups, subperiods

Period	g	Age 0	Age 1–5	Age 6–10	Age 11+
1982–1995	1.15	0.45	0.25	0.10	0.35
1996–2005	2.82	0.52	0.47	0.31	1.51
2006–2013	1.03	0.31	0.17	0.09	0.46

Source: BLS Multifactor Productivity series. Percentage points of average yearly productivity growth within the specified period. Yearly growth is the sum of R&D and IP contributions to BLS MFP growth, converted into labor augmenting form. All other variables are calculated using the Census LBD. Firm age is the difference between the year of observation minus the birth year of the firm's first plant.

Table 10: Share of the *change* in TFP growth by age group

Period	g	Age 0	Age 1–5	Age 6–10	Age 11+
1982–1995 vs 1996–2005	1.67	4.4%	13.4%	12.6%	69.6%
1996–2005 vs 2006–2013	-1.79	11.8%	17.0%	12.3%	58.9%

Source: BLS Multifactor Productivity series. Percentage points of average yearly productivity growth within the specified period. Yearly growth is the sum of R&D and IP contributions to BLS MFP growth, converted into labor augmenting form. All other variables are calculated using the Census LBD. Firm age is the difference between the year of observation minus the birth year of the firm's first plant.

Table 11: Growth by firm size groups, 1982–2013

	Small (0–19)	Medium (20–249)	Large (250–4999)	Mega (5000+)
% of growth	62.2%	15.0%	12.2%	10.7%
% of employment	21.4%	26.3%	26.9%	25.4%
% of firms	88.0%	11.2%	0.8%	0.03%

Source: Census LBD. Average growth rate over the entire period is 1.66%. Firm size is the sum of employment across all of its plants.

each size group contributed more or less equally to the speedup and slowdown in Table 13. Thus the performance of mega firms does not directly explain the rise and fall of growth.

Tables 14 and 15 focus on the prominent role played by new firms vs. small young firms — and, for contrast, small old firms. Here we define young as ages 1–5 and small as having 1–19 employees. Small young firms contribute twice as much to growth relative to their share of employment. Older small firms also contribute more to growth than their share of employment, but the difference is much less pronounced. This is reminiscent of the finding by Haltiwanger et al. (2013) that, even conditional on size, young firms grow faster than old firms.

5.6. Results: innovation type for each age and size group

We know from Table 8 that new firms contribute disproportionately to growth relative to their employment share. And we know they do no own innovation, by definition. But they do contribute via creative destruction or new varieties? Table 16 says that new firms mostly enter with new varieties. This seems intuitive in that it might be hard for new firms to eclipse existing products. At the other extreme, older firms (age 11+) contribute 40% of average growth, and

Table 12: Growth by firm size groups, subperiods

Period	g	Small (0–19)	Medium (20–249)	Large (250–4999)	Mega (5000+)
1982–1995	1.15	0.85	0.14	0.08	0.08
1996–2005	2.82	1.30	0.55	0.51	0.46
2006–2013	1.03	0.63	0.15	0.14	0.10

Source: BLS Multifactor Productivity series. Percentage points of average yearly productivity growth within the specified period. Yearly growth is the sum of R&D and IP contributions to BLS MFP growth, converted into labor augmenting form. All other variables are calculated using the Census LBD. Firm size is the sum of employment across all of its plants.

Table 13: Share of the *change* in TFP growth by size group

Period	g	Small (0–19)	Medium (20–249)	Large (250–4999)	Mega (5000+)
1982–1995 vs 1996–2005	1.67	27.0%	24.7%	25.5%	22.7%
1996–2005 vs 2006–2013	-1.79	37.5%	22.2%	20.2%	20.1%

Source: Census LBD. Firm size is the sum of employment across all of its plants.

Table 14: Contribution of new, young-small and old-small firms, 1982-2013

	New	Young small	Old small
% of growth	29.9	14.1	11.6
% of employment	3.3	6.1	8.3
% of firms	10.7	28.8	32.0
% of new plant emp.	58.8	1.1	3.5

Source: Census LBD. “New” = age 0, “Young” = age 1–5 and “Small” = employment 1–19. Firm age is the difference between the year of observation minus the birth year of the firm’s first plant. Firm size is the sum of employment across all of its plants.

Table 15: Contribution of new, young-small, and old-small firms by period

Period	New	Young small	Old small
1982–1995	37.9	17.1	10.8
1996–2005	18.5	10.9	11.3
2006–2013	30.1	12.8	13.2

Source: Census LBD. “New” = age 0, “Young” = age 1–5 and “Small” = employment 1–19. Firm age is the difference between the year of observation minus the birth year of the firm’s first plant. Firm size is the sum of employment across all of its plants.

Table 16: Growth by innovation type for each age group, 1982–2013

Age	CD	NV	OI	Total	% of Emp
0	7.2	23.0	0.0	30.3	3.3
1–5	0.7	1.2	17.1	18.9	13.4
6–10	0.7	0.9	8.1	9.7	11.2
11+	4.4	2.1	34.6	41.1	72.1

Source: Census LBD. CD = Creative Destruction, NV = New Varieties, and OI = Own Innovation (incumbents improving on their own varieties). Firm age is the difference between the year of observation minus the birth year of the firm's first plant.

Table 17: Growth contribution by innovation type and age groups, subperiods

Period	Age	CD	NV	OI	Total	% of Emp
1982–1995	0	7.8	31.0	0.0	38.8	3.9
	1–5	0.9	1.9	19.0	21.8	15.5
	6–10	0.4	1.5	7.2	9.1	12.1
	11+	5.1	1.9	23.4	30.4	68.5
1996–2005	0	5.1	13.4	0.0	18.5	3.1
	1–5	0.8	0.1	15.9	16.8	12.7
	6–10	1.0	0.6	9.5	11.2	11.1
	11+	4.6	0.4	48.5	53.5	73.0
2006–2013	0	8.9	21.2	0.0	30.1	2.5
	1–5	0.1	1.2	15.3	16.6	10.6
	6–10	0.7	0.4	8.0	9.1	9.5
	11+	2.9	4.5	36.9	44.2	77.3

Source: Census LBD. CD = Creative Destruction, NV = New Varieties, and OI = Own Innovation (incumbents improving on their own varieties). Firm age is the difference between the year of observation minus the birth year of the firm's first plant.

more than 3/4 of this comes from their efforts to improve their own products. They do comparatively little creative destruction and variety creation. The picture that emerges is that old firms improve their own products and new firms introduce new varieties. Creative destruction is more evenly split among entrants and older firms.

Table 17 breaks the growth contribution of each age group into each innovation type *for each subperiod*. Old firms were evidently responsible for the bulk of the growth acceleration and slowdown. In Table 18 we assess the type of innovation pursued by each firm size group. For every group other than entrants — from firms with 1–19 up through 10,000 or more employees — their main source of growth is own innovation. Table 19, which gives contributions by innovation type and size groups in subperiods, reveals some interesting patterns. The surge in own innovation from 1996–2005 occurred in firms with 20 or more employees. Even though young firms typically do a lot of own innovation, this increased relatively little in this fast-growth subperiod.

Table 18: Growth contribution by innovation type for each size group, 1982–2013

Size	CD	NV	OI	Total	% of Emp
0	7.2	23.0	0.0	30.3	3.3
1–19	1.3	0.6	30.0	31.9	18.1
20–249	0.1	2.8	12.1	15.0	26.3
250–999	0.4	0.5	4.9	5.8	12.8
1k–4999	0.8	0.1	5.5	6.4	14.1
5k–9999	0.5	0.1	1.9	2.5	5.8
10k+	2.6	0.1	5.5	8.2	19.6

Source: Census LBD. CD = Creative Destruction, NV = New Varieties, and OI = Own Innovation (incumbents improving on their own varieties). Firm size is the sum of employment across all of its plants.

Table 19: Growth contribution by innovation type and size groups, subperiods

Period	Size	CD	NV	OI	Total	% of Emp
1982–1995	0	7.8	31.0	0.0	38.8	3.9
	1–19	0.5	1.2	33.5	35.2	19.0
	20–249	0.1	3.7	8.1	11.9	25.8
	250–999	0.5	0.0	2.5	3.0	12.0
	1k–4999	1.0	0.1	2.9	4.0	13.5
	5k–9999	0.7	0.1	0.8	1.5	5.8
	10k+	3.6	0.2	1.8	5.6	19.9
1996–2005	0	5.1	13.4	0.0	18.5	3.1
	1–19	1.8	0.1	25.9	27.8	17.8
	20–249	0.1	0.6	18.8	19.5	26.9
	250–999	0.6	0.0	7.7	8.4	13.2
	1k–4999	1.1	0.0	8.5	9.6	14.3
	5k–9999	0.3	0.3	3.0	3.6	5.8
	10k+	2.6	0.1	10.0	12.7	18.9
2006–2013	0	8.9	21.2	0.0	30.1	2.5
	1–19	2.2	0.2	28.9	31.4	17.0
	20–249	0.1	3.9	10.7	14.7	26.3
	250–999	0.1	1.8	5.4	7.3	13.4
	1k–4999	0.4	0.0	6.3	6.7	15.0
	5k–9999	0.2	0.0	2.5	2.7	5.8
	10k+	0.7	0.1	6.3	7.1	19.9

Source: Census LBD. CD = Creative Destruction, NV = New Varieties, and OI = Own Innovation (incumbents improving on their own varieties). Firm size is the sum of employment across all of its plants.

5.7. Obsolescence

As we described, our model features exit of the lowest quality products each period. In the data, we observe both the exit rate of plants and the average employment of exiting plants. We can calculate these moments separately by firm age and size, as well.

How do we distinguish the rate of creative destruction — which also produces plant exit — from obsolescence? The key is our assumption that creative destruction is undirected, or just as likely for low and high quality products. For this reason, creatively destroyed plants should be of average size. In the data, in contrast, exiting plants are smaller on average than plants operating in the previous year. As a result, we can infer the fraction of exiting plants from the size and number of plants in the left tail plus the difference between the average size of exiting plants and all plants.

From the point of view of our accounting, these are “lost varieties” which detract from growth.¹¹ We do not subtract this obsolescence from any particular innovation channel, but instead from all channels in equal proportion to their contribution to growth. Table 20 provides our estimates of the fraction of plants exiting due to obsolescence, and their share of employment. We infer that 3-4% of employment is at the 9-10% of plants who will exit due to obsolescence in the next year. These rates show no clear trend across subperiods.

Tables 22 and 23 in the appendix trace the obsolescence to plants of varying ages and sizes. There is no obsolescence among entering plants (age=0, size=0) by construction. Roughly one-half of obsolescence occurs at plants 6+ years old, and the other half at plants 1–5 years old. Not surprisingly, most of obsolescence involves the smallest plants, with 1–19 employees.

¹¹A structural model might feature overhead labor to explain such exit.

Table 20: Obsolescence

Period	% of Emp	% of Plants	contrib OB
1982–1995	4.15	10.34	-1.06
1996–2005	2.93	8.53	-0.29
2006–2013	2.83	9.01	-0.84

Source: Census LBD. Entries are percent of all employment or percent of the overall number of plants.

6. Further results and future applications

6.1. Treating all 0–5 year old firms as entrants

In Table 16, we show that new firms mostly contribute by introducing new varieties, while young firms aged 1 to 5 contribute mostly through own-innovation. In the presence of adjustment costs, however, what we infer as own innovation by young firms may actually be the dynamics of accumulating inputs and acquiring customers. We therefore check the robustness of our findings to treating age 1 to 5 firms as new firm. Doing so reinterprets the growth of young firms to come from creative destruction and new varieties rather than own innovation. Table 21 displays the results. We find that new firms (aged 0 to 5) now account for 49 percent of all growth, up from 30 percent. The contribution of own innovation falls from 60 percent to 43 percent. That from creative destruction increases from 13 to 17 percent, while the contribution from new varieties increases from 27 to 40 percent.

6.2. Contribution of superstar firms

One advantage of our method is that it can be used to quantify the contribution of individual firms to aggregate growth. Since we cannot reveal the identity

Table 21: Treating all 0–5 year old firms as entrants

Age	CD	NV	OI	Total	% of Emp
0–5	12.0	37.2	0.0	49.2	16.7
6–10	0.7	0.9	8.1	9.7	11.2
11+	4.4	2.1	34.6	41.1	72.1
Total	17.1	40.2	42.7		

Source: Census LBD. CD = Creative Destruction, NV = New Varieties, and OI = Own Innovation (incumbents improving on their own varieties). Firm age is the difference between the year of observation minus the birth year of the firm's first plant. Entries are percent of all employment or percent of the overall number of plants.

of individual firms in the Census data, we apply our method to the National Establishment Time-Series (NETS) Database, which ranges from 1990 to 2014. The unit of observation is an establishment. As in the Census, we define entry of an establishment as the first time an establishment is observed in a location in the data. We define exit as when an establishment ceases to operate in a location. For each establishment, we observe its employment and, mostly importantly, the identity of its headquarter plant and therefore its firm identity.¹² Finally, we restrict the time period to 1996–2005 because this period is comparable to the high growth period in our calculations with the Census data, and the post-2005 NETS data displays large swings in the entry and exit of plants that are inconsistent with the Census data.

As a demonstration of our method, we used the NETS 1996–2005 data to calculate the contribution of Amazon and Walmart. We find that, while both companies grew during this window, their contribution to overall growth was quite modest. Amazon accounted for only 0.0041% of growth (less than 1%

¹²We do not use NETS sales data because it is imputed at the establishment level from establishment employment and firm-level sales. Also, like what we did for the Census data, we remove mechanical effects from mergers and acquisitions by defining the owner of an establishment as the firm that owns the establishment at the time of entry.

of all growth), while Walmart contributed 0.80% (closer to 1% of all growth, or over 2 basis points of annual TFP growth). Nonetheless, Amazon's contribution was three times its employment share, while Walmart's growth contribution was twice its employment share.

6.3. Future applications

Our innovation accounting is distinct from gross job creation, gross job destruction, or net job creation. It can therefore be helpful to contrast our results to these statistics, which have garnered substantial attention. For example, young fast-growing firms — so-called “gazelles” — are responsible for a notable fraction of gross and net job creation. How much growth do they generate on average? And how much of gross job destruction is due to creative destruction, as opposed to obsolescence?

We could carry out several important robustness checks. We could follow Aghion, Bergeaud, Boppart, Klenow and Li (2019) and distinguish between measured and true growth. They argue that measured growth mostly captures own innovation by incumbents, and misses most growth from new varieties and creative destruction. We could re-estimate parameters under this hypothesis.

For manufacturing the Census has data on revenue and capital as well as labor at the plant level. Thus for manufacturing we could back out underlying productivity, rather than assume sales are proportional to employment and the absence of adjustment costs and distortions. We could apply our innovation accounting on directly to this underlying productivity measure, and see how our inference compares to that with employment alone.

Future work could implement our approach to assess the contribution of locations (states, cities) to aggregate growth. It could also decompose growth within industries such ICT or retail. One could also look at which cohorts of entering firms have contributed the most to growth. Our method does not mechanically count firm employment growth due to mergers and acquisitions as

contributing to growth. But one could investigate whether targets of M&A activity become more innovative after they are taken over. This could obtain results at odd with existing analyses, which focus on revenue productivity.

7. Conclusion

We exploited U.S. Census data on employment at plants across all firms in the nonfarm business sector from 1982–2013. We traced aggregate TFP growth to the innovation efforts of firms by their size and age. We arrived at three main findings. First, young firms (ages 0 to 5) generate one-half of growth, roughly three times more than their share of employment might suggest. Second, large firms are at the other end of the spectrum, contributing notably less to growth than their share of employment. Third, a majority of growth takes the form of quality improvements by incumbents on their own products. New varieties and creative destruction contribute less, but are still important. Such own innovation accounts for the bulk of the growth speedup and slowdown in the middle of our sample.

Our study comes with many caveats that we hope future work, with better data, can address. We used a plant as a proxy for a product. With more detailed data on products sold within plants, this can be relaxed. See, for example, Bernard, Redding and Schott (2010) on manufacturing and Hottman et al. (2016) on packaged consumer goods.

We gauged a plant's size by its employment rather than its sales and combined plants with a CES aggregator. Though they are highly correlated in manufacturing, where we can see both, some innovation surely tilts toward capital and away from labor (automation). And a richer analysis could place industries into CES nests and entertain non-CES aggregation.

Even under CES aggregation, a plant's sales are a perfect reflection of its underlying productivity only in the absence of adjustment costs and distortions such as markups, markdowns, and financial frictions. Adjustment costs could

include those for inputs, or to acquiring a customer base. We could be overstating the importance of young firms for innovation (overall and by incumbents on their own products) if their fast growth is due, in part, to adjusting their inputs and stock of customers.

We assumed that creative destruction was untargeted. That is, higher quality products are just as likely to be creatively destroyed as lower quality products. One can think of *a priori* reasons why creative destruction would target higher quality products (more sales to gain) or lower quality products (perhaps easier to improve upon).

We made the arrival rates and step sizes of innovation depend only on the size and age of innovating firms. Studies such as Akcigit and Kerr (2018) have argued that innovation by new and young firms generates bigger spillovers. This would reinforce our conclusion that younger firms punch above their weight class in terms of growth.

Our innovation accounting is silent on the determinants of arrival rates and step sizes of innovation. Knowing the structural parameters that drive these variables is vital for drawing policy lessons. See Atkeson and Burstein (2019). One key question for future research is whether creative destruction is a strategic complement or substitute for own innovation by incumbents. If they are substitutes, as in Peters (2019) and Akcigit and Ates (2019), then a falling rate of creative destruction may be offset by a rising rate of own innovation. If they are complements, however, then perhaps the threat of creative destruction spurs innovative efforts by incumbents. This is the spirit of the model in Aghion, Bloom, Blundell, Griffith and Howitt (2005) in which the threat of being overtaken by followers or entrants leads to more innovation by leaders.

Appendix A. Revenue productivity vs employment growth

Figure 9: Microsoft

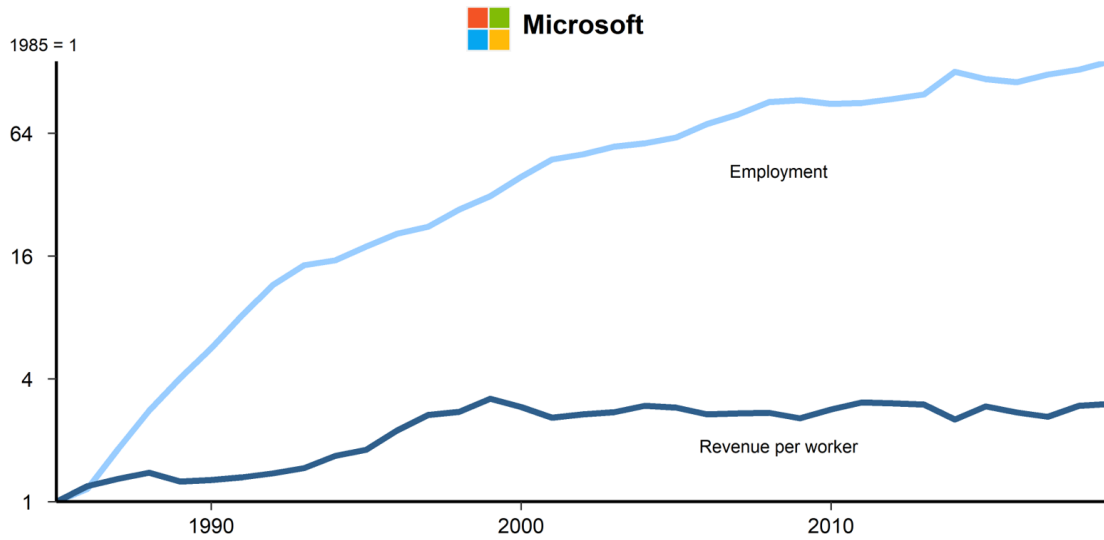
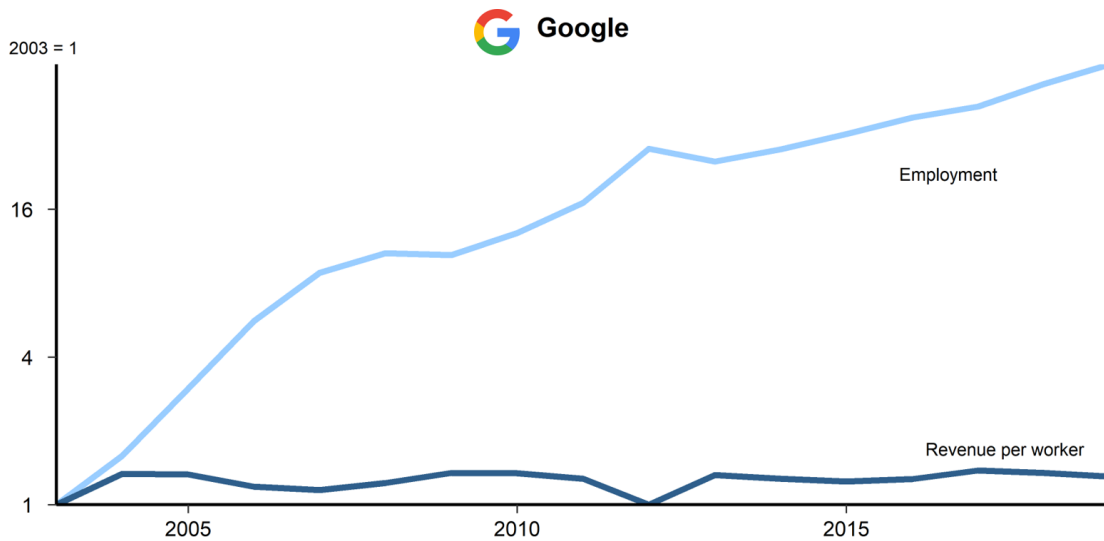


Figure 10: Google



Source: WRDS Compustat and BEA Table 1.1.9. Revenue per worker = “REVT / (GDP Deflator x EMP)”, Employment = “EMP” and Year = “FYEAR”. The graph displays revenue per worker and employment relative to the first year when both series are available for the firm.

Figure 11: Facebook

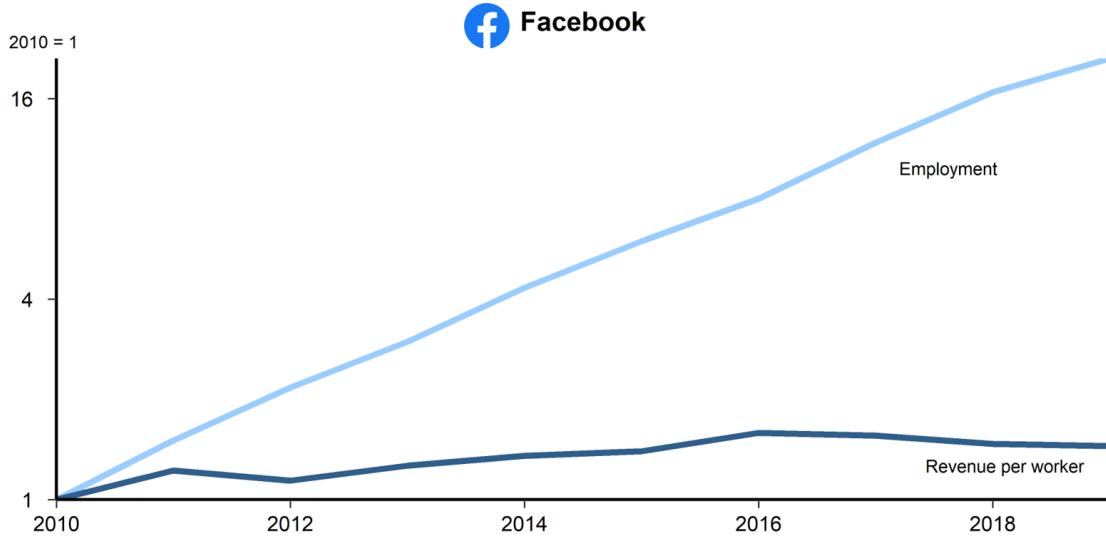
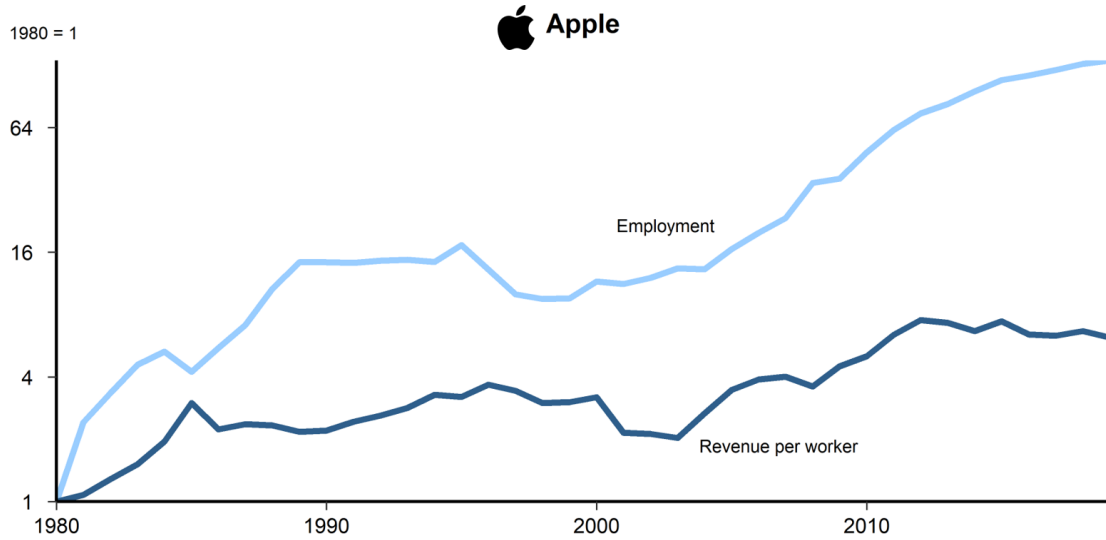
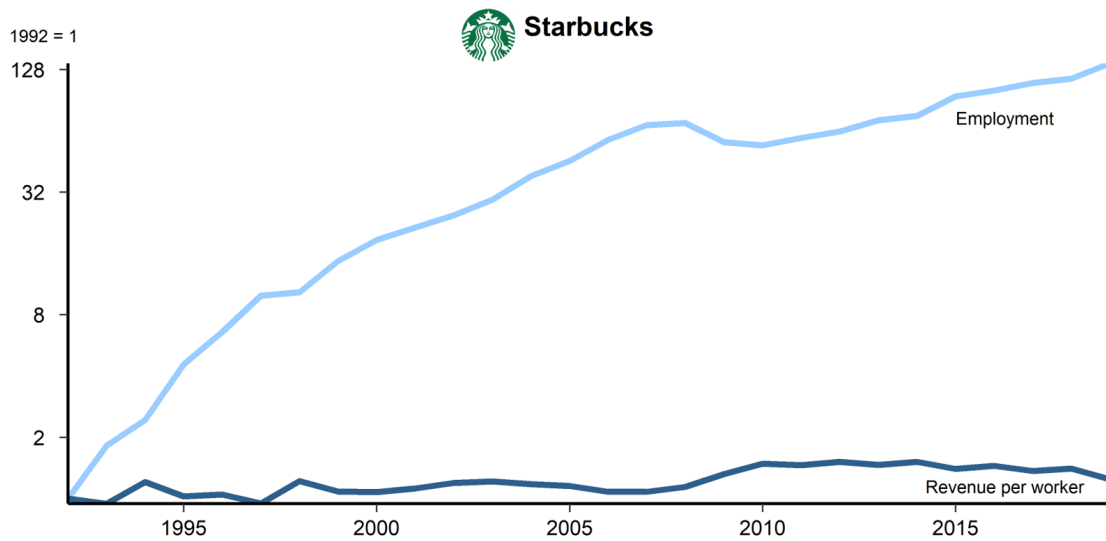


Figure 12: Apple



Source: WRDS Compustat and BEA Table 1.1.9. Revenue per worker = “REVT / (GDP Deflator x EMP)”, Employment = “EMP” and Year = “FYEAR”. The graph displays revenue per worker and employment relative to the first year when both series are available for the firm.

Figure 13: Starbucks



Source: WRDS Compustat and BEA Table 1.1.9. Revenue per worker = “REVT / (GDP Deflator x EMP)”, Employment = “EMP” and Year = “FYEAR”. The graph displays revenue per worker and employment relative to the first year when both series are available for the firm.

Appendix B. Extra tables

Table 22: Obsolescence by age groups

Period	Age	% of Emp	% of Plants	contrib OB
1982–1995	0	0.0	0.0	0.0
	1–5	2.1	5.8	-0.5
	6–10	0.7	1.7	-0.2
	11+	1.3	2.9	-0.3
1996–2005	0	0.0	0.0	0.0
	1–5	1.3	4.4	-0.1
	6–10	0.5	1.4	-0.0
	11+	1.1	2.8	-0.1
2006–2013	0	0.0	0.0	0.0
	1–5	1.5	4.6	-0.4
	6–10	0.6	1.5	-0.2
	11+	0.8	2.9	-0.2

Source: Census LBD. Firm age is the difference between the year of observation minus the birth year of the firm's first plant. Units are percent of total employment or total number plants

Table 23: Obsolescence by size groups

Period	Size	% of Emp	% of Plants	contrib OB
1982–1995	0	0.0	0.0	0.0
	1–19	2.6	9.2	-0.7
	20–249	0.9	0.3	-0.2
	250–999	0.1	0.1	-0.0
	1k–4999	0.1	0.2	-0.0
	5k–9999	0.1	0.1	-0.0
	10k+	0.4	0.4	-0.1
1996–2005	0	0.0	0.0	0.0
	1–19	1.5	7.4	-0.2
	20–249	0.6	0.2	-0.1
	250–999	0.1	0.1	-0.0
	1k–4999	0.1	0.2	-0.0
	5k–9999	0.1	0.1	-0.0
	10k+	0.5	0.6	-0.0
2006–2013	0	0.0	0.0	0.0
	1–19	2.0	8.3	-0.6
	20–249	0.7	0.1	-0.2
	250–999	0.0	0.0	0.0
	1k–4999	0.0	0.1	0.0
	5k–9999	0.0	0.1	0.0
	10k+	0.2	0.5	-0.0

Source: Census LBD. Firm size is the sum of employment across all of its plants. Units are percent of total employment or total number plants

Table 24: Contribution by age-size groups, 1982–1995

Size	Age	CD	NV	OI	Total	% of Emp
0	0	7.6	30.3	0.0	37.9	3.9
1–19	1–5	0.1	0.7	16.4	17.1	7.2
1–19	6–10	0.3	0.4	6.1	6.8	4.2
1–19	11+	0.4	0.0	10.4	10.8	7.7
20–249	1–5	0.0	1.1	2.2	3.4	5.6
20–249	6–10	0.0	0.8	1.3	2.1	4.6
20–249	11+	0.1	1.6	4.4	6.1	15.5
250–999	1–5	0.3	0.0	0.0	0.3	1.3
250–999	6–10	0.2	0.0	0.0	0.2	1.4
250–999	11+	0.1	0.0	2.5	2.6	9.3
1k–4999	1–5	0.2	0.0	0.0	0.2	0.9
1k–4999	6–10	0.4	0.0	0.2	0.6	1.1
1k–4999	11+	0.6	0.1	2.6	3.4	11.5
5k–9999	1–5	0.2	0.0	0.0	0.2	0.5
5k–9999	6–10	0.2	0.0	0.0	0.2	0.3
5k–9999	11+	0.4	0.0	0.7	1.2	5.1
10k+	6–10	1.4	0.0	0.0	1.4	0.4
10k+	11+	3.5	0.2	1.7	5.4	19.5

Source: Census LBD. CD = Creative Destruction, NV = New Varieties, and OI = Own Innovation (incumbents improving on their own varieties). Firm size is the sum of employment across all of its plants. Firm age is the difference between the year of observation minus the birth year of the firm's first plant.

Table 25: Contribution by age-size groups, 1996–2005

Size	Age	CD	NV	OI	Total	% of Emp
0	0	5.1	13.4	0.0	18.5	3.1
1–19	1–5	0.0	0.1	10.8	10.9	5.6
1–19	6–10	0.4	0.0	5.1	5.5	3.6
1–19	11+	1.3	0.0	10.0	11.3	8.5
20–249	1–5	0.1	0.0	4.1	4.2	4.6
20–249	6–10	0.0	0.6	2.9	3.5	4.2
20–249	11+	0.0	0.0	11.8	11.8	18.0
250–999	1–5	0.2	0.0	0.7	0.9	1.2
250–999	6–10	0.1	0.0	0.7	0.8	1.3
250–999	11+	0.3	0.0	6.4	6.6	10.7
1k–4999	1–5	0.3	0.0	0.3	0.6	0.8
1k–4999	6–10	0.2	0.0	0.6	0.8	1.1
1k–4999	11+	0.6	0.0	7.7	8.2	12.4
5k–9999	1–5	0.2	0.0	0.0	0.2	0.4
5k–9999	6–10	0.1	0.0	0.1	0.2	0.3
5k–9999	11+	0.1	0.3	2.9	3.3	5.0
10k+	6–10	0.2	0.0	0.2	0.4	0.5
10k+	11+	2.4	0.1	9.8	12.3	18.4

Source: Census LBD. CD = Creative Destruction, NV = New Varieties, and OI = Own Innovation (incumbents improving on their own varieties). Firm size is the sum of employment across all of its plants. Firm age is the difference between the year of observation minus the birth year of the firm's first plant.

Table 26: Contribution by age-size groups, 2006–2013

Size	Age	CD	NV	OI	Total	% of Emp
0	0	8.9	21.2	0.0	30.1	2.5
1–19	1–5	0.0	0.2	12.6	12.8	4.8
1–19	6–10	0.2	0.0	5.2	5.4	3.2
1–19	11+	2.1	0.0	11.1	13.2	9.0
20–249	1–5	0.0	0.9	2.4	3.3	4.0
20–249	6–10	0.1	0.4	1.8	2.3	3.7
20–249	11+	0.0	2.5	6.6	9.1	18.6
250–999	1–5	0.0	0.0	0.3	0.3	1.0
250–999	6–10	0.1	0.0	0.5	0.6	1.2
250–999	11+	0.0	1.8	4.5	6.3	11.2
1k–4999	1–5	0.1	0.0	0.0	0.1	0.6
1k–4999	6–10	0.1	0.0	0.4	0.4	0.9
1k–4999	11+	0.2	0.0	5.9	6.2	13.6
5k–9999	1–5	0.0	0.0	0.0	0.0	0.3
5k–9999	6–10	0.1	0.0	0.0	0.1	0.2
5k–9999	11+	0.1	0.0	2.5	2.6	5.3
10k+	6–10	0.2	0.0	0.1	0.2	0.3
10k+	11+	0.5	0.1	6.3	6.9	19.6

Source: Census LBD. CD = Creative Destruction, NV = New Varieties, and OI = Own Innovation (incumbents improving on their own varieties). Firm size is the sum of employment across all of its plants. Firm age is the difference between the year of observation minus the birth year of the firm's first plant.

Appendix C. Directed creative destruction

The benchmark model assumes creative destruction innovation is random in that all existing products have the same probability of being creatively destroyed. This appendix lays out an extension of the model where creative destruction can be directed to certain ranges of quality. That is, we allow probability of being creatively destroyed to depend on whether the quality of the incumbent product is above a quality cutoff. Creative destruction is random conditional on whether the quality is above or below the cutoff. The benchmark model is a special case of this extension with the cutoff equal to the obsolescence cutoff. We use one cutoff here for illustration. The model can be extended in the same way for multiple cutoffs.

Suppose new producers of a product take over $\delta_{ft}^H N_{t-1}$ non-obsolete products with quality above q_t^H by improving on the quality of those products by step size $\frac{H}{f_t} > 1$ and $\delta_{ft}^L N_{t-1}$ non-obsolete products with quality below q_t^H by improving on the quality of those products by step size $\frac{L}{f_t} > 1$. Thus, the share of products in $t-1$ that is subject to creative destruction is

$$\delta_t := \int_{f \in \mathcal{M}_t} \delta_{ft}^H + \delta_{ft}^L df$$

We need to modify how firms lose their products to creative destruction. Let $x_{f,t-1}^{CDH} N_{f,t-1}$ and $x_{f,t-1}^{CDL} N_{f,t-1}$ be the number of products a firm active in $t-1$ loses between $t-1$ and t due to high and low quality creative destruction, respectively. In equilibrium, the arrival rate of creative destruction equals the sum of the rate of producers exit from a product due to creative destruction. That is,

$$\delta_t = \int_{f \in \mathcal{M}_{t-1}} (x_{f,t-1}^{CDH} + x_{f,t-1}^{CDL}) \frac{N_{f,t-1}}{N_{t-1}} df = \int_{f \in \mathcal{M}_{t-1}} x_{f,t-1}^{CD} df$$

where $x_{f,t-1}^{CD} := x_{f,t-1}^{CDH} + x_{f,t-1}^{CDL}$. For simplicity we assume both high and low quality creative destruction innovation are random conditional on the type of destruction. This implies all firms lose the same fraction of high-quality and low qual-

ity non-obsolete products, denoted by $\hat{\delta}_t^H$ and $\hat{\delta}_t^L$, respectively. However, unlike the benchmark model, firms with different composition of high and low quality products will have different average probability of losing a product if $\hat{\delta}_t^H \neq \hat{\delta}_t^L$. Hence for all firms in $t - 1$

$$\mathbf{x}_{f,t-1}^{CDH} = \hat{\delta}_t^H (1 - G_{f,t-1}(q_t^H)), \quad \mathbf{x}_{f,t-1}^{CDL} = \hat{\delta}_t^L (G_{f,t-1}(q_t^H) - G_{f,t-1}(\kappa_t))$$

and the share of products of a firm in $t - 1$ that are not produced by the firm in period t is

$$\mathbf{x}_{f,t-1} = \mathbf{x}_{f,t-1}^O + \mathbf{x}_{f,t-1}^{CD} = G_{f,t-1}(\kappa_t) + \hat{\delta}_t^H (1 - G_{f,t-1}(q_t^H)) + \hat{\delta}_t^L (G_{f,t-1}(q_t^H) - G_{f,t-1}(\kappa_t))$$

and the aggregate exit rate of products from producers at the end of $t - 1$ is

$$x_{t-1} = \int_{f \in \mathcal{M}_{t-1}} \mathbf{x}_{f,t-1} \frac{N_{f,t-1}}{N_{t-1}} df = G_{t-1}(\kappa_t) + \hat{\delta}_t^H (1 - G_{t-1}(q_t^H)) + \hat{\delta}_t^L (G_{t-1}(q_t^H) - G_{t-1}(\kappa_t)).$$

The extension affects the quality distribution of products experiencing own-innovation. Recall θ_{ft} fraction of the products that survive both creative destruction and obsolescence experience own innovation by the incumbent produce. With the extension, the share of products of firm f that experience own innovation is

$$\theta_{ft} \cdot (1 - \mathbf{x}_{f,t-1}^{CD} - \mathbf{x}_{f,t-1}^O) = \theta_{ft} \cdot [1 - G_{f,t-1}(\kappa_t) - \hat{\delta}_t^H (1 - G_{f,t-1}(q_t^H)) - \hat{\delta}_t^L (G_{f,t-1}(q_t^H) - G_{f,t-1}(\kappa_t))]$$

and the share of all products in $t - 1$ that experience own innovation is

$$\int_{f \in \mathcal{M}_{t-1}} \theta_{ft} \cdot [1 - G_{f,t-1}(\kappa_t) - \hat{\delta}_t^H (1 - G_{f,t-1}(q_t^H)) - \hat{\delta}_t^L (G_{f,t-1}(q_t^H) - G_{f,t-1}(\kappa_t))] \frac{N_{f,t-1}}{N_{t-1}} df$$

Having shown how the extension modifies the innovation process, we next show how the extension affects the expression for aggregate productivity growth.

Recall that we can decompose growth into innovation types

$$\begin{aligned} e^g &= \left(\frac{\int_{i \in 2N_t} q_t(i)^{\sigma-1} di}{\int_{i \in 2N_{t-1}} q_{t-1}(i)^{\sigma-1} di} \right)^{\frac{1}{\sigma-1}} \\ &= \left(\frac{\int_{i \in 2NV_t} q_t(i)^{\sigma-1} di + \int_{i \in 2CD_t} q_t(i)^{\sigma-1} di + \int_{i \in 2OI_t} q_t(i)^{\sigma-1} di + \int_{i \in 2S_t} q_t(i)^{\sigma-1} di}{\int_{i \in 2N_{t-1}} q_{t-1}(i)^{\sigma-1} di} \right)^{\frac{1}{\sigma-1}} \end{aligned}$$

The first term is not affected. The second term becomes

$$\left(\int_{f \in 2M_t} \delta_f^H \frac{H^{\sigma-1}}{f} df \right) \left(\frac{\hat{Q}^H}{Q} \right)^{\sigma-1} + \left(\int_{f \in 2M_t} \delta_f^L \frac{L^{\sigma-1}}{f} df \right) \left(\frac{\hat{Q}^L}{Q} \right)^{\sigma-1}$$

where

$$\hat{Q}^H := N_{t-1} \cdot E_{G_{t-1}}(q^{\sigma-1} | q \geq q_t^H) \quad (15)$$

$$\hat{Q}^L := N_{t-1} \cdot E_{G_{t-1}}(q^{\sigma-1} | \kappa_t \leq q < q_t^H) \quad (16)$$

The third term changes to

$$\begin{aligned} & \int_{f \in 2C_{t,t-1}} \theta_{ft} O_f^{\sigma-1} \left[(1 - \hat{\delta}_t^L) \int_{i \in 2N_{ft-1}, q_t^H \leq q_t(i) < \kappa_t} \frac{q_t^{\sigma-1}(i)}{Q_t^{\sigma-1}} di \right. \\ & + \left. (1 - \hat{\delta}_t^H) \int_{i \in 2N_{ft-1}, q_t^H < q_t(i)} \frac{q_t^{\sigma-1}(i)}{Q_t^{\sigma-1}} di \right] df \\ & = \left(\frac{\hat{Q}^H}{Q} \right)^{\sigma-1} \int_{f \in 2C_{t,t-1}} \theta_{ft}^H O_f^{\sigma-1} \frac{E_{G_{f,t-1}}(q^{\sigma-1} | q \geq q_t^H)}{E_{G_{t-1}}(q^{\sigma-1} | q \geq q_t^H)} df \\ & + \left(\frac{\hat{Q}^L}{Q} \right)^{\sigma-1} \int_{f \in 2C_{t,t-1}} \theta_{ft}^L O_f^{\sigma-1} \frac{E_{G_{f,t-1}}(q^{\sigma-1} | \kappa_t \leq q \leq q_t^H)}{E_{G_{t-1}}(q^{\sigma-1} | \kappa_t \leq q \leq q_t^H)} df \end{aligned}$$

The fourth term contains products that survive creative destruction and ob-

solescence but do not experience own innovation. This term changes to

$$\begin{aligned}
& \int_{f \in \mathcal{C}_{t,t-1}} (1 - \theta_{ft}) \left[(1 - \hat{\delta}_t^L) \int_{i \in 2N_{ft-1}, q_t^H \geq q_{t-1}(i) \geq \kappa_t} \frac{q_{t-1}^\sigma(i)}{Q_t^{\sigma-1}} di \right. \\
& + \left. (1 - \hat{\delta}_t^H) \int_{i \in 2N_{ft-1}, q_t^H < q_{t-1}(i)} \frac{q_{t-1}^\sigma(i)}{Q_t^{\sigma-1}} di \right] df \\
& = (1 - \hat{\delta}_t^H) \left(\frac{\hat{Q}^H}{Q} \right)^{\sigma-1} \int_{f \in \mathcal{C}_{t,t-1}} (1 - \theta_{ft}) (1 - G_{f,t-1}(q_t^H)) \frac{N_{f,t-1} \mathbb{E}_{G_{f,t-1}}(q^{\sigma-1} | q \geq q_t^H)}{\hat{Q}_t^{H, \sigma-1}} df \\
& + (1 - \hat{\delta}_t^L) \left(\frac{\hat{Q}^L}{Q} \right)^{\sigma-1} \int_{f \in \mathcal{C}_{t,t-1}} (1 - \theta_{ft}) (G_{f,t-1}(q_t^H) - G_{f,t-1}(\kappa_t)) \frac{N_{f,t-1} \mathbb{E}_{G_{f,t-1}}(q^{\sigma-1} | q_t^H \geq q \geq \kappa_t)}{\hat{Q}_t^{L, \sigma-1}} df
\end{aligned}$$

Combining the terms we derived so far, we can write the aggregate productivity growth rate as

$$\begin{aligned}
e^{g(\sigma-1)} &= 1 + \int_{f \in 2M_t} \nu_f V_f^{\sigma-1} df + \int_{f \in 2M_t} \delta_f^H \left(\frac{H}{f}^{\sigma-1} - 1 \right) \left(\frac{\hat{Q}^H}{Q} \right)^{\sigma-1} df \\
&+ \int_{f \in 2M_t} \delta_f^L \left(\frac{L}{f}^{\sigma-1} - 1 \right) \left(\frac{\hat{Q}^{LL}}{Q} \right)^{\sigma-1} df \\
&+ \left(\frac{\hat{Q}^H}{Q} \right)^{\sigma-1} \int_{f \in \mathcal{C}_{t,t-1}} o_{ft}^H (O_f^{\sigma-1} - 1) \frac{\mathbb{E}_{G_{f,t-1}}(q^{\sigma-1} | q \geq q_t^H)}{\mathbb{E}_{G_{t-1}}(q^{\sigma-1} | q \geq q_t^H)} df \\
&+ \left(\frac{\hat{Q}^{LL}}{Q} \right)^{\sigma-1} \int_{f \in \mathcal{C}_{t,t-1}} o_{ft}^L (O_f^{\sigma-1} - 1) \frac{\mathbb{E}_{G_{f,t-1}}(q^{\sigma-1} | q \leq q_t^H)}{\mathbb{E}_{G_{t-1}}(q^{\sigma-1} | q \leq q_t^H)} df \\
&+ \left(\int_{f \in 2M_t} \nu_f V_f^{\sigma-1} df \right) \left(\left(\frac{\hat{Q}}{Q} \right)^{\sigma-1} - 1 \right) \\
&+ \left(\int_{f \in 2M_t} \delta_f^L \left(\frac{L}{f}^{\sigma-1} - 1 \right) df \right) \left(\left(\frac{\hat{Q}^L}{Q} \right)^{\sigma-1} - \left(\frac{\hat{Q}^{LL}}{Q} \right)^{\sigma-1} \right) \\
&+ \int_{f \in \mathcal{C}_{t,t-1}} o_{ft}^L (O_f^{\sigma-1} - 1) \frac{\mathbb{E}_{G_{f,t-1}}(q^{\sigma-1} | \kappa_t \leq q \leq q_t^H) - \mathbb{E}_{G_{f,t-1}}(q^{\sigma-1} | q \leq q_t^H)}{\mathbb{E}_{G_{t-1}}(q^{\sigma-1})} df \\
&- \left(1 - \left(\frac{\hat{Q}}{Q} \right)^{\sigma-1} (1 - G_{t-1}(\kappa_t)) \right)
\end{aligned}$$

where

$$\hat{Q}^{LL} := N_{t-1} \cdot E_{G_{t-1}}(q^{\sigma-1} | q < q_t^H)$$

is the quality distribution low quality creative destruction would have drawn from had there been no obsolescence.

Finally the expression for firm contribution to growth becomes

$$\begin{aligned} g_f = & \nu_f V_f^{\sigma-1} + \delta_f^H (O_f^{H\sigma-1} - 1) \left(\frac{\hat{Q}^H}{Q} \right)^{\sigma-1} + \delta_f^L (O_f^{L\sigma-1} - 1) \left(\frac{\hat{Q}^{LL}}{Q} \right)^{\sigma-1} \\ & + \left(\frac{\hat{Q}^H}{Q} \right)^{\sigma-1} o_{ft}^H (O_f^{\sigma-1} - 1) \frac{E_{G_{f,t-1}}(q^{\sigma-1} | q \geq q_t^H)}{E_{G_{t-1}}(q^{\sigma-1} | q \geq q_t^H)} \\ & + \left(\frac{\hat{Q}^{LL}}{Q} \right)^{\sigma-1} o_{ft}^L (O_f^{\sigma-1} - 1) \frac{E_{G_{f,t-1}}(q^{\sigma-1} | q \leq q_t^H)}{E_{G_{t-1}}(q^{\sigma-1} | q \leq q_t^H)} \\ & + o_{ft}^L (O_f^{\sigma-1} - 1) \frac{E_{G_{f,t-1}}(q^{\sigma-1} | \kappa_t \leq q \leq q_t^H) - E_{G_{f,t-1}}(q^{\sigma-1} | q \leq q_t^H)}{E_{G_{t-1}}(q^{\sigma-1})} \\ & - E_{G_{f,t-1}}[s_{t-1} | q < \kappa_t] G_{f,t-1}(\kappa_t) N_{f,t-1} \\ & + \left\{ \left(\int_{f \in \mathcal{M}_t} \nu_f V_f^{\sigma-1} df \right) \left(\left(\frac{\hat{Q}}{Q} \right)^{\sigma-1} - 1 \right) + \right. \\ & \left. \left(\int_{f \in \mathcal{M}_t} \delta_f^L (O_f^{L\sigma-1} - 1) df \right) \left(\left(\frac{\hat{Q}^L}{Q} \right)^{\sigma-1} - \left(\frac{\hat{Q}^{LL}}{Q} \right)^{\sigma-1} \right) \right\} \frac{G_{f,t-1}(\kappa_t) N_{f,t-1}}{G_{t-1}(\kappa_t) N_{t-1}} \end{aligned}$$

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