On the Distribution of Estates and the Distribution of Wealth: Evidence from the Dead

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Abstract

Detailed information about the distribution of the estates left at death has commonly served for the estimation of wealth distribution among the living via the mortality multiplier method. The application of detailed mortality rates by demographics and other determinants of mortality is crucial for obtaining an unbiased representation of the wealth distribution of the living. Yet, in this paper we suggest that a simplified mortality multiplier method, derived using average mortality rates and aggregate tabulations by estate size, may be sufficient to derive compelling estimates of wealth concentration. We show that the application of homogeneous multipliers is equivalent to estimate the distribution of estates. The latter also appears sufficiently close in level and trend to the wealth distribution derived in the existing literature with the detailed mortality multiplier method for a variety of countries. The use of mortality rates graduated by estate size does not confute this finding. This paper shows that the application of mortality multipliers does not alter the distribution of estates substantially. We derive the general formal conditions for the similarity between the distributions of wealth of the living and estates at death and discuss the main caveats. We believe these findings may unlock a wide array of aggregate estate tabulations, previously thought to be unusable, for estimating historical trends of wealth concentration.

Keywords: Wealth inequality, mortality multiplier method, estates, mortality rates, economic history
JEL Codes: D3, H2, N3

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1 Introduction

There are traditionally five main sources of evidence about the distribution of personal wealth: (i) household surveys, such as the Survey of Consumer Finance conducted by the Flow of Funds Unit of the US Federal Reserve, the Household Finance and Consumption Surveys co-ordinated by the European Central Bank, or the UK Wealth and Asset Survey, conducted by the Office of National Statistics; (ii) administrative data on personal wealth derived from annual wealth taxes; (iii) administrative data on investment income, capitalized to yield estimates of the underlying wealth; (iv) lists of large wealth-holders, such as the annual Forbes Richest People in America list; (v) administrative data on individual estates at death, multiplied-up to yield estimates of the wealth of the living through the mortality multiplier method.

The information from estates (the net value of worldwide real and financial property of a deceased person) has commonly served for the estimation of the distribution of wealth among the living via the mortality multiplier method since the works by Mallet (1908) and Mallet and Strutt (1915) in the United Kingdom. The principle of the method is very basic, even if a number of important conceptual challenges are readily involved: the set of deceased people are taken as a sample of the living, and each estate is expanded by a multiplier (weight) equal to the inverse probability of death.

Death does not uniformly "sample" the population. Older individuals, as well as males and people from poorer backgrounds, have, other things being equal, higher mortality risk. Differential mortality multipliers can, however, be used to transform the estate data into estimates of wealth-holding. Under the assumption that death is random within specific cells of observed demographic and social strata, one can view death occurrence as an effective sampling of the living population. When the age and gender multipliers were first employed in the United Kingdom, it was seen as overcoming a "fatal" objection to the use of estate data with a constant multiplier. Mallet (1908, p. 67) argued that "the accumulated wealth of an individual increases with years, and is usually greatest when a man dies."

The distribution of wealth of the living is conceptually different from that of the decedents. Consequently, the application of detailed multipliers (that is, those taking into account the dimensions that affect the probability of dying) can increase or decrease top wealth shares as well as change the time patterns relative to estate shares, given that the two distributions make reference to different variables. Yet, the distribution of estates has never been under extensive scrutiny in and of itself. Recent research has highlighted that the concentration of estates and the derived concentration of wealth at the top through the mortality method seem to be very similar. The motivation for this work comes from an observation made in Alvaredo, Atkinson and Morelli (2018) – that the conclusions reached regarding the degree of concentration do not change radically when both distributions are compared in the United Kingdom, the United States, and 19th century Paris.

In this paper, we implement a simplified multiplier method which makes use of limited estate and mortality data, namely the average mortality rate in the population and the aggregate tabulations of estates by size. We show that the application of homogeneous multipliers is equivalent to estimating the distribution of estates, and that the latter is sufficiently close in level and trend to the distribution of wealth in countries where the mortality multiplier method has been applied in existing works, namely Australia, France, Italy, Korea, the United Kingdom, and the United States. We then investigate why the application of mortality multipliers does not significantly alter the picture when the distributions of wealth and estates are compared. We find that this relationship depends crucially on the covariance between mortality rates and estate amounts.

More specifically, this paper aims to better understand the following issues: What is the nature of the relationship between the distribution of estates at death and that of the wealth of the living through the mortality multiplier method? What are the general conditions under which the concentration of estates at death provides the same informative content as the concentration of the wealth of the living? What drives the relevance and the direction of the potential bias created by the lack of appropriate control for the growing longevity of wealthy individuals?

The formalization of the problem leads to several insights. First, we show that the top wealth shares are trivially equal to the top estate shares in the special case when homogeneous multiplies are applied to the population of decedents. Thus, for simplicity, we may refer interchangeably to the distribution of estates and the distribution of wealth when homogeneous multipliers are applied. When disaggregated multipliers are used, the multiplier choice may affect both the estimation of the estate concentration (when the estate data represent only a share of the decedent population) and of the wealth distribution, so the two distributions may not be identical.

We then show that the difference between top wealth shares of a specific q-percentile obtained with an homogeneous average multiplier and with demographic-differential multipliers is driven by two main factors: the covariance between mortality rates and the estate value of the $p\%$ richest decedents required to estimate the $q\%$ wealth holders and the difference between the mean wealth of the top $p\%$ estates and the $q\%$ top estate holders. A similar result is derived for the coefficient of variation (CV) as a measure of inequality, in an analogy to the derivation of the CV for the capitalization method in Atkinson and Harrison (1978). Comparing the CV of the wealth distribution in the mortality multiplier method between demographic-differential multipliers and the average multiplier suggests a close relationship between the two. These results further suggest that the empirical similarity in inequality measures between estates and wealths may not be limited to top shares, and may also apply to synthetic inequality indicators summarizing the full distributions.

In practice, estate data usually represent only a share of the decedent population, with substantial heterogeneity across countries. Thus, the natural focus of our empirical work rests on the analysis of the wealthiest brackets, using the wealth shares of the richest groups as the inequality measures under investigation.[1]

[1] Focusing on the overall estate distribution is challenging and requires estimating the estates of the missing population, not represented in the tax records. This is easier in countries with high coverage rate of inheritance/estate tax data such as Italy and the United Kingdom, but represents a greater challenge in countries with minimal coverage

As a first step we consider the simplified case of an average multiplier. It can be applied to aggregate estate tabulations by size, ignoring all demographic information (*e.g.* some historical tabulations can only be found in this form). The application of a homogeneous multiplier for all estates is equivalent to analyze the distribution of estates. The multiplier to be applied to the top of the estate distribution is, ideally, the multiplier that best corresponds to the decedents included in the tax records. In the absence of such information one could use the average multiplier of the adult population.

We then consider the more realistic case of differential mortality multipliers by demographic characteristics (the case of differential multipliers by socio-economic characteristics will be dealt separately). The latter is an important starting point as mortality rates by age and gender generally map most of the variability in mortality observed in a country in a given year. Moreover, these data are available throughout history for many countries.

As a final step, we consider the relevance of "unobserved" heterogeneity in mortality rates, such as a the potential wealth effect on mortality that is operating over and above the effect of demographic characteristics (*e.g.* age and gender). The work by Saez and Zucman (2016) brought to the fore old-standing concerns about the mortality multiplier method, suggesting that the failure to appropriately control for decreasing mortality of wealthy individuals may severely underestimate the top wealth shares. Saez and Zucman (2016) proposed that such an underestimation may play an important role in the United States for the reconciliation between the estate-based top wealth shares series and that derived through the capitalization of investment and capital incomes.

After controlling for realistic mortality-wealth gradients we find that the newly estimated top wealth share is closer to the one derived with the use of the average multiplier among adults. Although individuals at the top of the estate distribution have higher mortality rates as they are relatively older on average, this is counterbalanced by their higher economic status, which may lead to healthier lives and better medical care, reducing their probability to die, other things being equal. As a result, the differences between the mortality multipliers at the top of the estate distribution to the average mortality multiplier of the entire decedent population are small enough to create only a limited discrepancy between the two top wealth shares estimated with differential and average multipliers.

These findings may unlock a wide array of aggregate tabulations that were previously thought to be unreliable and unusable. Information about the distribution of wealth is scarce, for the recent period and more so for historical series. Yet, many countries have published detailed data on the distribution of estate taxes. These are only rarely accompanied by demographic characteristics such as age and gender. Thus, one cannot apply heterogeneous mortality rates to the estate tax data. The simplified multiplier method may be implemented in such cases for estimating historical trends of wealth concentration.

One important caveat remains, nonetheless. Although, as discussed, changes in the multipliers

rate, such as the United States (see Appendix A).

do not dramatically affect the shape of the wealth distribution, they may have a large impact on estimated aggregate variables, such as wealth totals and the ratio between the average estate and the average wealth among the living. These aggregate variables are usually of lesser interest if the goal is obtaining information on the distribution of wealth. However, the derivation of macroeconomic aggregate series are of direct interest to economists (see Piketty and Zucman (2014) and Alvaredo, Garbinti and Piketty (2017)), and, we argue, may well serve as an indirect test of the appropriateness of the multipliers used to derive distributional measures. Mortality multipliers thus matter in various aspects.

In this paper we show, for instance, that it is possible that the application of a very steep mortality-wealth gradient at the top of the estate distribution would correspond to total wealth levels that exceed the known total estimates. It is also possible that the application of graduated multipliers leads to very low levels of the ratio between average wealth at death to average wealth among the living, a parameter that has been shown to fall between 1 and 2 on a historical perspective (Alvaredo, Garbinti and Piketty, 2017).

We also note that the discussion in this paper assumes that the information provided by the value of estates at death is broadly valid. In some cases it could be argued that the estates recorded by the tax administration are particularly imperfect, due to high level of exemptions, evasion, or through the effects of tax planning. In such cases the concerns about the effect of mortality multipliers become less crucial compared to the inaccuracy of observed estates in describing the personal wealth of decedents. Yet, these discussions exceed the scope of this paper.

2 The mortality multiplier method

The mortality multiplier method makes use of the information on the wealth and the demographic characteristics of decedents reported to the tax authorities for the administration of inheritance or estate taxes. The decedent population can then be re-weighted and become representative of the living population to estimate the distribution of wealth among the living. Let's consider the population of N_E decedents and the total value of their estates, W_E:

$$W_E = \sum_{i=1}^{N_E} w_{E,i} \,. \tag{2.1}$$

The estates $w_{E,i}$ are arranged in descending order, *i.e.* $w_{E,i} \geq w_{E,j}$, if $i < j$. The following relationship holds:

$$W = \sum_{i=1}^{N_E} m_i w_{E,i} \,, \tag{2.2}$$

where W is the total wealth among the living population, and $m_i \equiv \frac{1}{p_i}$ is the mortality multiplier of individual i, equal to the inverse of the mortality rate, adjusted to take into account the added longevity of the top wealth holders.

The mortality rates vary across a set of socio-demographic characteristics. Therefore each multiplier represents the number of living individuals who share the same socio-demographic characteristics of decedent i. The average mortality rate of the population, $\bar{p}$, is defined as the ratio between the number of deceased, N_E, and the number of living, N. Similarly, we define the average multiplier, $\bar{m}$, which is equal to N/N_E.

We are interested in estimating the wealth share of the top quantile $0 < q < 1$, where $q = 0.1$ corresponds to 10%, $q = 0.01$ corresponds to 1%, etc. It is natural to think that the value of multipliers will affect the number of decedents that will be needed in order to account for the top q quantile among the living. For example, if the multipliers of the rich decedents are high, compared to the average multipliers in the population, less decedents would be required to account for the top q quantile among the living than when the multipliers of the rich decedents are lower. This number is represented by the index I_q such that[2]

$$\sum_{i=1}^{I_q} m_i = qN \,. \tag{2.3}$$

This way we can define the top q wealth share as

$$(1 - L_q)^W = \frac{\sum_{i=1}^{I_q} m_i w_{E,i}}{W} \,. \tag{2.4}$$

3 The concentration of wealth in a simplified mortality multiplier method: heterogeneous vs. average multipliers

The application of the full mortality multiplier method is conditioned on the availability of detailed mortality data as well as detailed estate data by demographic characteristics. However, such information may not be readily available for estate tabulations may not be disaggregated by demographic characteristics or because detailed mortality data may not exist in a particular country or year.

The work by Alvaredo, Atkinson and Morelli (2018) has shown that the concentration of estate at death and the derived concentration of wealth at the top following the application of the mortality multipliers (based on gender, age, social-class differentials or wealth differentials) are very close to one another. In their words, "the application of mortality multipliers does not alter the picture concerning the distribution of the wealth of the living, as commonly believed." As described by Cowell (1978), referring to Atkinson and Harrison (1978): "though the particular refinement of mortality multiplier that is used considerably affects the calculation of total wealth, the resultant effect on top wealth shares is not all that great." An implicit recognition of this similarity can

[2] If there is no equality, I_q is defined as the smallest index such that $\sum_{i=1}^{I_q} m_i > qN$.

be also found in Piketty, Postel-Vinay and Rosenthal (2006) and Moriguchi and Saez (2008), who treated the distribution of estates, estimated using estate tax records, as a de facto equivalent to the distribution of wealth.

Thus, an alternative and simplified solution when detailed mortality data are unavailable would be to rely on average mortality rates. Following the notation used above, and noticing that $I_q = qN_E$ when $m_i = \bar{m}$, the top wealth shares will take the following form

$$(1 - L_q)^W_{avg} = \bar{m} \frac{\sum_{i=1}^{qN_E} w_{E,i}}{W} . \tag{3.1}$$

We compare top wealth share series for several countries in which the full mortality multiplier method was performed in the existing literature, using disaggregated multipliers, to series derived using the simplified mortality multiplier method using average mortality data. Importantly, noting that $(1 - L_q)^W_{avg}$ is formally equivalent to the top q estate share, this exercise would be equivalent to comparing wealth and estate top shares as done in Alvaredo, Atkinson and Morelli (2018).[3]

Figure 1 presents these results for Australia, France, Italy, the Republic of Korea (South Korea), the United Kingdom, and the United States. For each country, the evolution of top wealth shares reported in the literature is compared to estimates of top wealth shares derived in this paper using average mortality multiplier. The empirical exercise highlights that in all countries the top wealth shares estimated with the simplified mortality multiplier method strongly co-move with those reported in the literature, and they are generally similar in level. The largest differences appear in the case of the United States and South Korea.

The quality and features of estate data in each of these countries differ substantially (see Appendix A). For example, in Italy the data cover roughly 60% of decedents every year, however only tabulations are publicly available. In France the data cover a much smaller share (about 10%) of the decedent population, yet micro data are available. In the United States, only a tiny share of the decedent population is covered (roughly 0.2% in recent years) and public aggregate tabulations as well as detailed micro data are available to researchers. Despite the specific differences between the data sources, the application of the mortality multiplier method is similar and requires the same information – the values of estates at the top of the estate distribution (or equivalent tabulations), the corresponding mortality multipliers at the top of the estate distribution, and the total personal wealth.

Different works make use of different adjustments to the data to allow for under-reporting, tax avoidance and evasion. In the United States, Kopczuk and Saez (2004) included estimates of wealth held in trusts and the cash surrender value of pensions and life insurance assets. In France,

[3]When $m_i = \bar{m}$, then $W = \sum_{i=1}^{N_E} m_i w_{E,i} = \bar{m} \sum_{i=1}^{N_E} w_{E,i} = \bar{m} W_E$. We obtain

$$(1 - L_q)^W_{avg} = \frac{\sum_{i=1}^{I_q} m_i w_{E,i}}{W} = \frac{\bar{m}}{\bar{m}} \frac{\sum_{i=1}^{qN_E} w_{E,i}}{W_E} = (1 - L_q)^E . \tag{3.2}$$

See also Appendix B for more details.

Garbinti, Goupille-Lebret and Piketty (2020) impute missing net wealth to provide consistency with official national balance sheet data for the household sector. Other works, such as Acciari, Alvaredo and Morelli (2020) for Italy, provide a full array of adjusted, unadjusted, and imputed series. As we are able to make use of the unadjusted series which is derived from the pure application of mortality multipliers here, we primarily use the case of Italy for our empirical analysis in the following sections.

To gain a better intuition for how sensitive the results can be to the choice of multipliers, we use equations Eq. (2.4) and Eq. (3.1) and derive the conditions for the equality of the top wealth shares with the average multiplier and with disaggregated multipliers:

$$\frac{\sum_{i=1}^{I_q} m_i w_{E,i}}{W} = \bar{m}\frac{\sum_{i=1}^{qN_E} w_{E,i}}{W} \iff \sum_{i=1}^{I_q} \frac{m_i}{\bar{m}} w_{E,i} = \sum_{i=1}^{qN_E} w_{E,i} \ , ^* \tag{3.3}$$

and the equality is trivially satisfied if multipliers do not vary across the population (*i.e.* $m_i = \bar{m}$).

We define

$$\bar{w}_{qN_E} = \frac{\sum_{i=1}^{qN_E} w_{E,i}}{qN_E} \ ; \ \bar{w}_{I_q} = \frac{\sum_{i=1}^{I_q} w_{E,i}}{I_q} \tag{3.4}$$

and

$$\bar{w}_{qN_E} - \bar{w}_{I_q} = \frac{I_q}{\bar{m}qN_E}\mathrm{Cov}\left[m_i, w_{E,i}\right] \ , \tag{3.5}$$

where $\mathrm{Cov}\left[m_i, w_{E,i}\right] = \frac{1}{I_q}\sum_{i=1}^{I_q}\left(m_i - \frac{1}{I_q}\sum_{j=1}^{I_q} m_j\right)\left(w_{E,i} - \bar{w}_{I_q}\right)$.

Now, rearranging terms, it is possible to write down explicitly the difference between the top wealth shares and obtain via the same notation and using the same expansion:

$$\begin{aligned}(1-L_q)^W - (1-L_q)^W_{avg} &= \frac{\bar{m}qN_E}{W}\left(\bar{w}_{I_q} - \bar{w}_{qN_E}\right) + \frac{I_q}{W}\mathrm{Cov}\left[m_i, w_{E,i}\right] \\ &= \frac{I_q}{W}\left[\bar{m}_{I_q}\left(\bar{w}_{I_q} - \bar{w}_{qN_E}\right) + \mathrm{Cov}\left[m_i, w_{E,i}\right]\right] \ ,\end{aligned} \tag{3.6}$$

where $\bar{m}_{I_q}$ is the average multiplier at the top of the estate distribution $(\Sigma_{i=1}^{I_q} m_i / I_q)$.

The right hand side of Eq. (3.6) shows that the difference between top wealth shares depends on an average level effect of the multipliers, $\bar{m}_{I_q}\left(\bar{w}_{I_q} - \bar{w}_{qN_E}\right)$, and on the covariance, $\mathrm{Cov}\left[m_i, w_{E,i}\right]$. The average level effect is such that the closer the average of the multipliers at the top is to the average multiplier, the closer the index I_q is to qN_E, and hence, the closer the difference $\bar{w}_{I_q} - \bar{w}_{qN_E}$ would be to zero.

In practice, the average multiplier at the top tends to be on average lower than $\bar{m}$. This is a straightforward result of life cycle effects – mortality is predominantly determined by age, and older people tend to be richer, on average. Therefore, the top of the estate distribution is likely to be composed of people that are older than the average age among the adult population. Therefore, in order to account for the top qN living individuals, we would need more than qN_E decedents (note

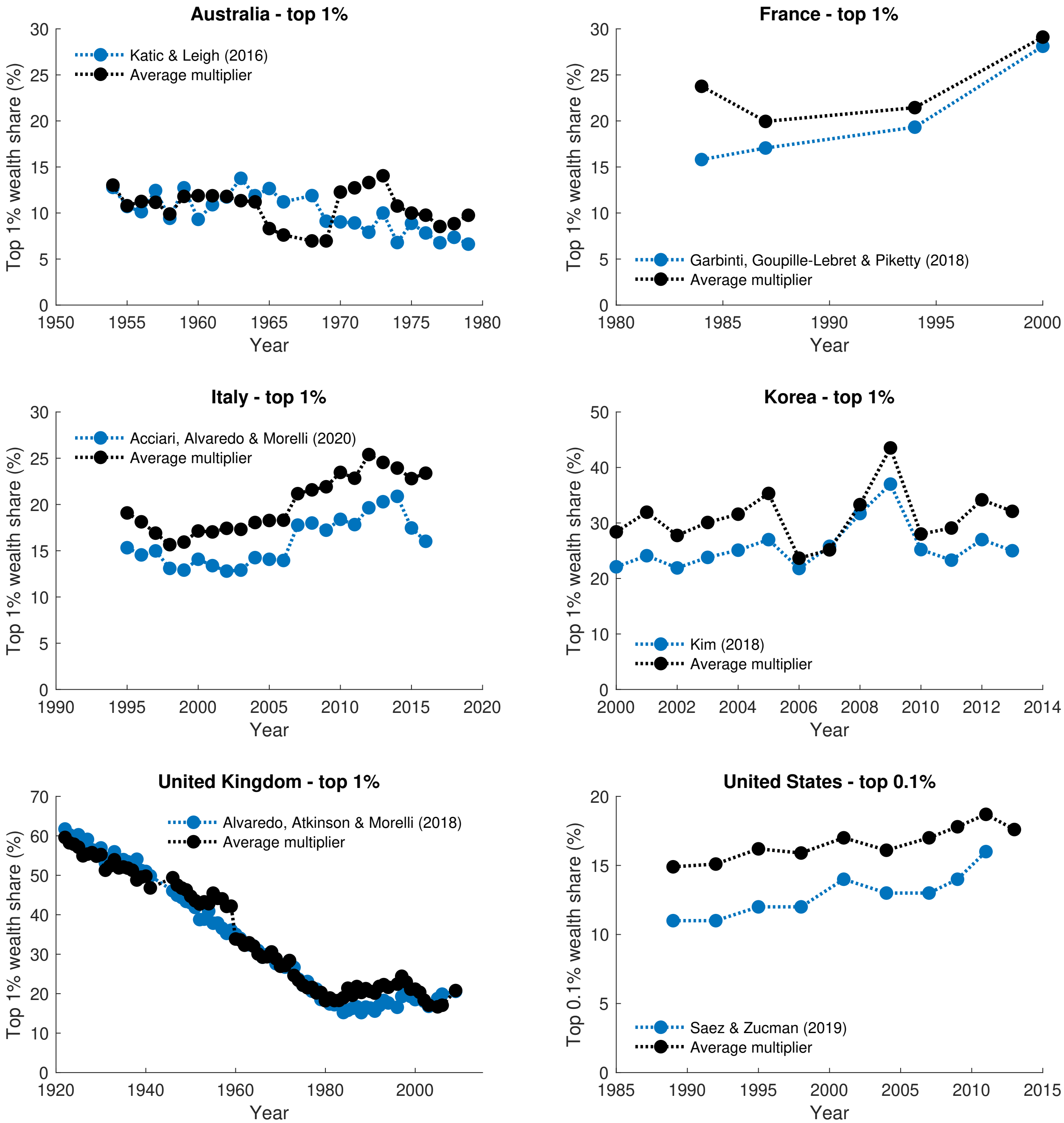

Figure 1: The top wealth shares in Australia, France, Italy, South Korea, the United Kingdom, and the United States. Tabulations and and the top wealth shares were taken from Katic and Leigh (2016), Garbinti, Goupille-Lebret and Piketty (2020), Acciari, Alvaredo and Morelli (2020), Kim (2018), Alvaredo, Atkinson and Morelli (2018), and Saez and Zucman (2016), respectively. The estimated top wealth shares were produced using the mortality multiplier method assuming the average multiplier for all observed decedents. The mortality data were taken from The Human Mortality Database (2018).

that $\bar{m} = \frac{N}{N_E}$). For this reason the difference $\bar{w}_{I_q} - \bar{w}_{qN_E}$ would tend to be negative. The covariance ($\text{Cov}\left[m_i, w_{E,i}\right]$) also tends to be negative in practice. Therefore, using the average multiplier will tend to lead to over-estimation of the top wealth shares, when compared to disaggregated demographic multipliers.

We note that a similar derivation can be used to compare the coefficient of variation (CV) of the wealth distribution with homogeneous and disaggregated multipliers (see Appendix C). It clarifies the intuition for the result obtained for top shares above. In particular, it shows that the difference between the CV of wealth and estates is mainly driven by the multipliers at the top of the estate distribution. This supports the observation that a similarity between the multiplier at the top of the estate distribution to the average mortality multiplier would result in a similarity between the estimated concentration of wealth and the concentration of estates.

Figure 2 shows the dependence of the difference between top 1% wealth shares in Italy with disaggregated multipliers and with the average multiplier on the covariance between multipliers and estates at the top, and on the difference between average estates at the top. The correlation between multipliers and estates at the top of the estate distribution appears to explain a large part of the small difference between top wealth shares. This correlation can be determined by various factors. First, a common factor that affects both mortality and wealth is age. Mortality is strongly determined by age, and above the age of 40, mortality rates tend to increase exponentially with age (see Appendix D). At the same time wealth is only weakly determined by age. Wealth increases with age, on average (Shorrocks, 1975; Modigliani, 1986). However, the variability of the average age within different wealth groups is very large.

Figure 3 illustrate this point. It shows the lack of systematic correlations between age and wealth rank in France in different years, using a sample of the richest decedents in France obtained from the estate tax records. Thus, in practice, the covariance between multipliers and estates is very small and tends to be negative. The reason for the small covariance is the dominance of age in determining mortality, combined with the small positive correlation between age and estates.

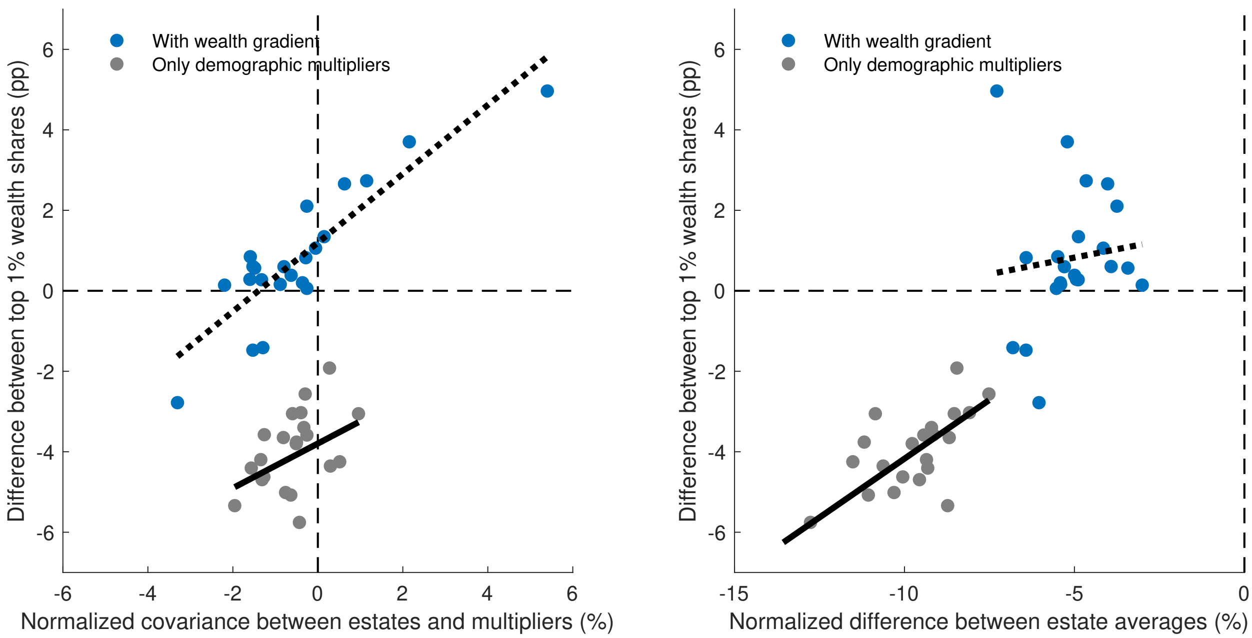

Figure 2: The dependence of the difference between top 1% wealth shares in Italy with disaggregated multipliers and with the average multiplier on the covariance between multipliers and estates at the top (see also Eq. (3.6)) $-\frac{I_q}{W}\text{Cov}\left[m_i, w_{E,i}\right]$ (left) and on the difference between average estates at the top $-\frac{I_q}{W}\bar{m}_{I_q}\left(\bar{w}_{I_q}-\bar{w}_{qN_E}\right)$ (right). The dashed and solid lines are linear fits after adding the mortality-wealth gradient and assuming only demographic multipliers, respectively (with $R^2=0.78$, $R^2=0.11$ in the left chart, respectively, and $R^2=0.01$, $R^2=0.54$ in the right chart, respectively).

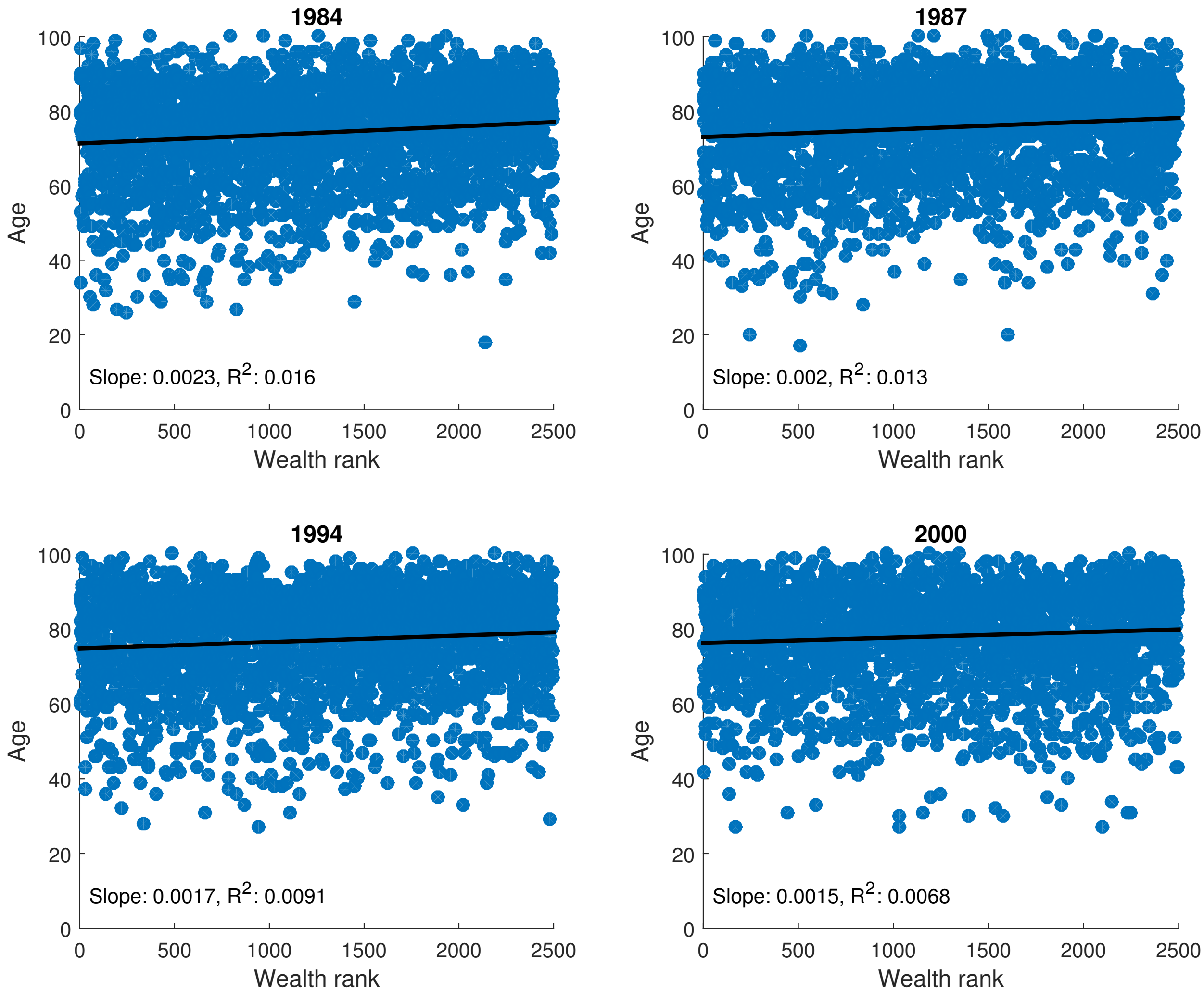

Figure 3: The age of a sample of the richest decedents in France according to their wealth rank (1 – least wealthy). The slope and R^2 are given for a linear fit of the data in each year.

3.1 Accounting for multipliers graduated by wealth levels

Mortality rates are clearly influenced by demographic factors, such as gender and age. However, social and economic conditions can also exert a substantial influence on the longevity of individuals. In particular, higher wealth levels may be systematically associated with lower mortality rates, over and above the effect of demographics and other factors. Failure to account for this additional source of heterogeneity in mortality rates may lead to systematic biases in the application of the mortality multiplier method (Atkinson and Harrison, 1978; Saez and Zucman, 2016, 2019). To account for the contribution of wealth to lower mortality over and above the effect of age, we use Italian estate tabulations from Acciari, Alvaredo and Morelli (2020) and apply mortality rate adjustment factors for wealth used by Garbinti, Goupille-Lebret and Piketty (2020).

The formalization described by equation Eq. (3.6) is well suited to take this issue into account and explain the main findings. Accounting for the mortality-wealth gradient does, indeed, increase the covariance, other things being equal, creating possibly a positive association between estate values and mortality multipliers at the top of the estate distribution. At the same time, the gradient increases the average multiplier at the top, which would, in turn, increase the difference $\bar{w}_{I_q} - \bar{w}_{qN_E}$. We should, therefore, expect the top wealth shares derived via wealth-gradient multipliers to be higher than those derived through demographic multipliers.

The results are presented in Fig. 4 where the derived series of top wealth shares using a wealth gradient in mortality rate is compared to those derived with average multiplier as well as heterogeneous multipliers by demographic characteristics. The results show that a steep mortality-wealth gradient can create a salient effect on the top wealth shares.[4]

Nevertheless, the wealth effect on mortality can counterbalance the small negative correlation between multipliers and estates at the top. Combined, the wealth and age effects on mortality may lead to correlation that is very close to 0 (see Fig. 2). If, indeed, the decreasing mortality of wealthy individuals is not accounted for, the correlation would be underestimated. At the same time, decreasing mortality by wealth acts to increase the life expectancy of older, wealthy individuals. This, in turn, leads to the decrease of the covariance between multipliers and estates at the top. For these reasons a large positive covariance between estate and multipliers at the top, which will lead to large positive differences between the top wealth shares with and without disaggregated multipliers, is implausible.

More surprisingly, Fig. 4 shows that the top wealth series derived using the simplified mortality multiplier method using average multipliers provide very similar results to those obtained by applying detailed multipliers by demographic and wealth status. The wealth gradient of mortality rates reduced the mortality rates of richest individuals, increasing multipliers. This means that wealth provides an ‘age premium’ to older rich individuals. In turn, this leads to mortality multipliers at the top of the estate distribution that are close to the average multiplier in the overall adult

[4] We note that it is possible that the mortality-wealth gradient described in Garbinti, Goupille-Lebret and Piketty (2020) may not be representative of Italy.

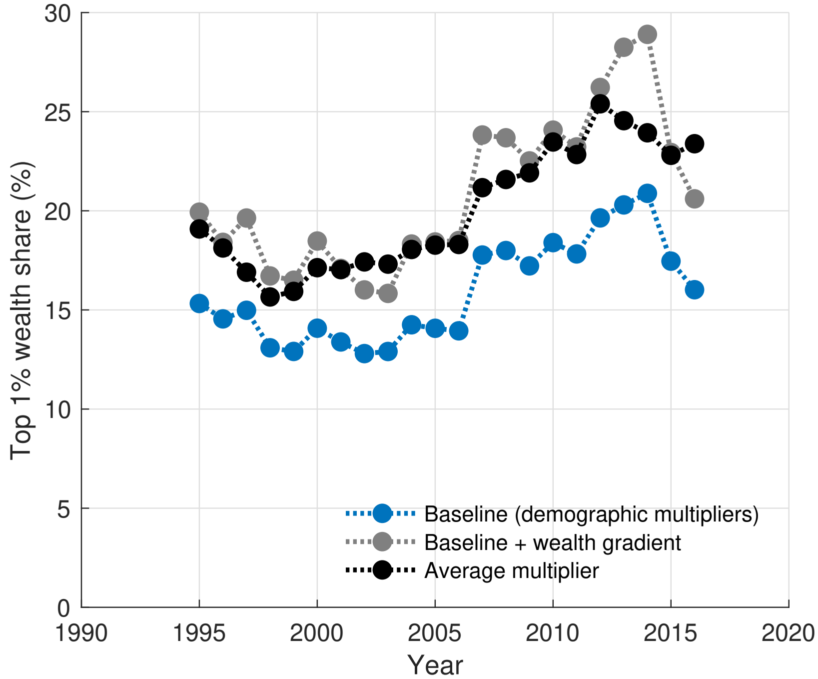

Figure 4: The top 1% wealth shares in Italy estimated using different multiplier choices – disaggregated demographic multipliers (blue); wealth-adjusted disaggregated demographic multipliers (gray); average multiplier (black).

population.

4 Collateral effects by using average multiplier and graduated multipliers

Figure 5 presents the evolution of various variables in Italy under different multiplier choices. It demonstrates that the mortality-wealth gradient used might be too steep in the Italian case, as it implies $\mu < 1$ for almost the entire period. $\mu < 1$ is a very unlikely case, implying that the decedents are poorer, on average, than the living. This is possible, in theory, if the rich are very unlikely to die, but that is an extreme case, undocumented so far (see, for example, Alvaredo, Garbinti and Piketty (2017); Alvaredo, Atkinson and Morelli (2018)).

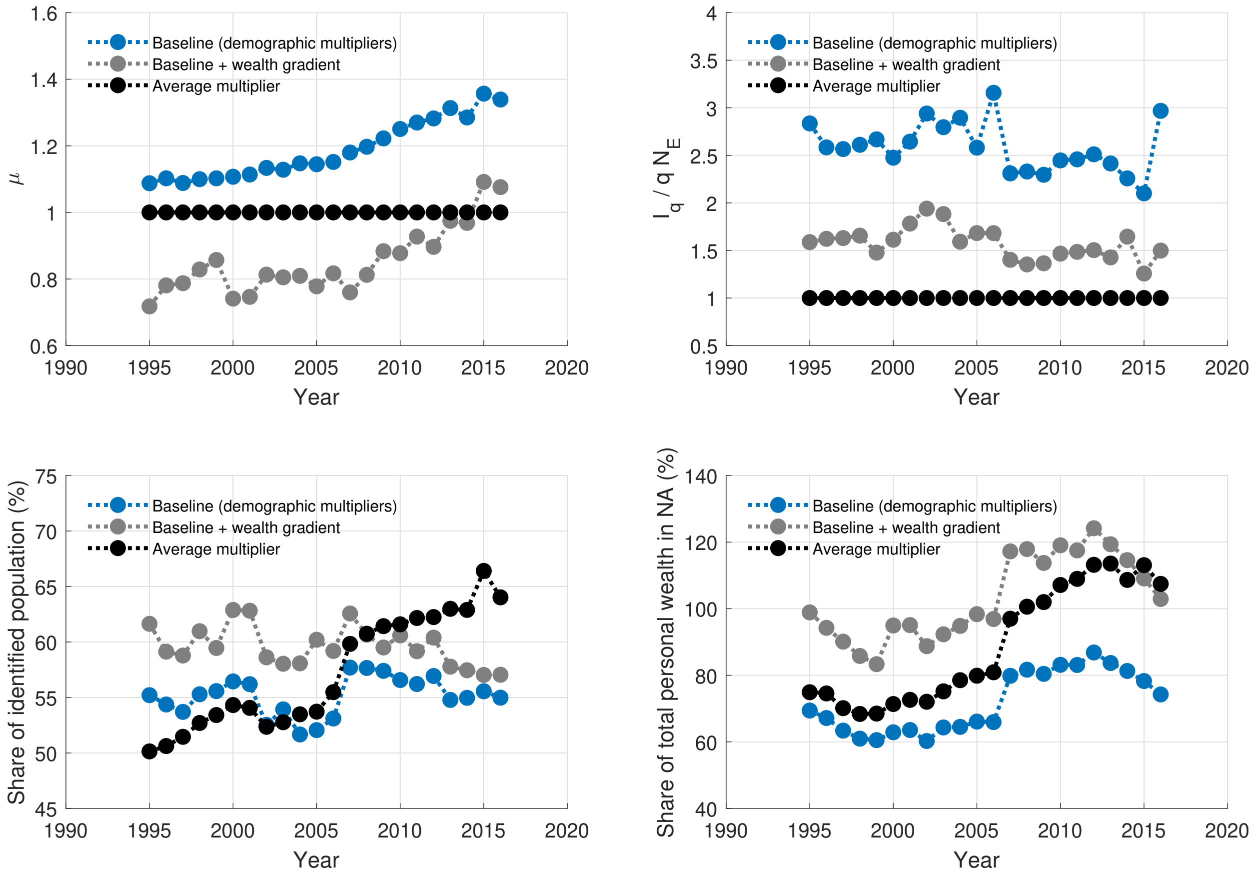

Figure 5: The evolution of various variables in Italy using different multiplier choices. Top left) μ; Top right) $\frac{I_q}{qN_E}$; Bottom left) The share of identified population from total population; Bottom right) The identified wealth as share of total personal wealth from the national accounts. Mortality data are taken from The Human Mortality Database (2018), the estate tabulations and demographic data as well as the total personal wealth are taken from Acciari, Alvaredo and Morelli (2020). The mortality-wealth gradients used were those used for France in Garbinti, Goupille-Lebret and Piketty (2020).

These results also show that when including the mortality-wealth gradient (as well as for the average multiplier case), it is possible for the identified wealth to be higher than the total personal wealth among the living from the national accounts. This is possible if the unobserved population has negative net wealth, which is possible, if it consists of the poorest individuals. Yet, this is also a rather extreme case, which requires verifying the validity of the mortality-wealth gradient applied. In particular, this may serve as a warning sign. In some cases we might think that the mortality-wealth gradient applied is not steep enough, resulting in lower top wealth shares. However, there is a plausible constraint to the steepness of this gradient, which can be derived by comparing the identified wealth to an external estimated total personal wealth.

5 Which homogeneous multiplier?

For implementing the simplified mortality multiplier method one has to choose the homogeneous multiplier to be applied. The choice in the average multiplier of the adult population, $\bar{m}$, is only one possible choice. In the absence of detailed demographic data for the decedents included in the estate tax records, it is possible to use an approximation for their mortality multiplier to estimate the top wealth shares. Even when demographic data are available, homogeneous multipliers can simplify the estimation process. We list below several possible choices of a homogeneous multiplier and present the differences between them and the different resulting top wealth shares:

- m_1: A simple and natural choice of such a multiplier is the average multiplier $\bar{m}$, which is the ratio between the population size of the living and the dead. Considering such an multiplier makes an implicit assumption that the mortality rate of the observed decedents is similar to that of the unobserved decedents.

- m_2: If detailed demographic data are available, it is possible to take the arithmetic average of the disaggregated individual multiplier m_i. m_2 is expected to be lower than m_1, since the average multiplier among the observed decedents tends to be lower than the average multiplier, however this is not always the case.

- m_3: m_2 changes the identified wealth compared to the case in which the disaggregated individual multiplier m_i are considered, because $\Sigma_i^{N_E} m_i w_{E,i} \neq \frac{\Sigma_i^{N_E} m_i}{N_E} \Sigma_i^{N_E} w_{E,i}$. Another possible choice of homogeneous multiplier would be a multiplier that is consistent with the identified wealth – $m_3 = \frac{\Sigma_i^{N_E} m_i w_{E,i}}{\Sigma_i^{N_E} w_{E,i}}$.

- m_4: If no demographic data are available, but mortality data are, it is possible to assume that the representative multiplier of the observed decedents is the multiplier that corresponds to an individual whose age is the average age at death, based on the mortality data. Typically, since this age is higher than the average age of decedents in the tax records, this multiplier will be substantially lower than the other choices of multiplier.

- m_5: The same m_2 but after adding a mortality-wealth gradient to the demographic data for obtaining disaggregated individual multipliers.

The evolution of these multipliers in time and the resulting top 1% wealth shares are presented in Fig. 6. The choice of a homogeneous multiplier matters for the estimated top shares. Yet, almost all the options considered lead to levels of inequality that closely follow the results when disaggregated multipliers are used.

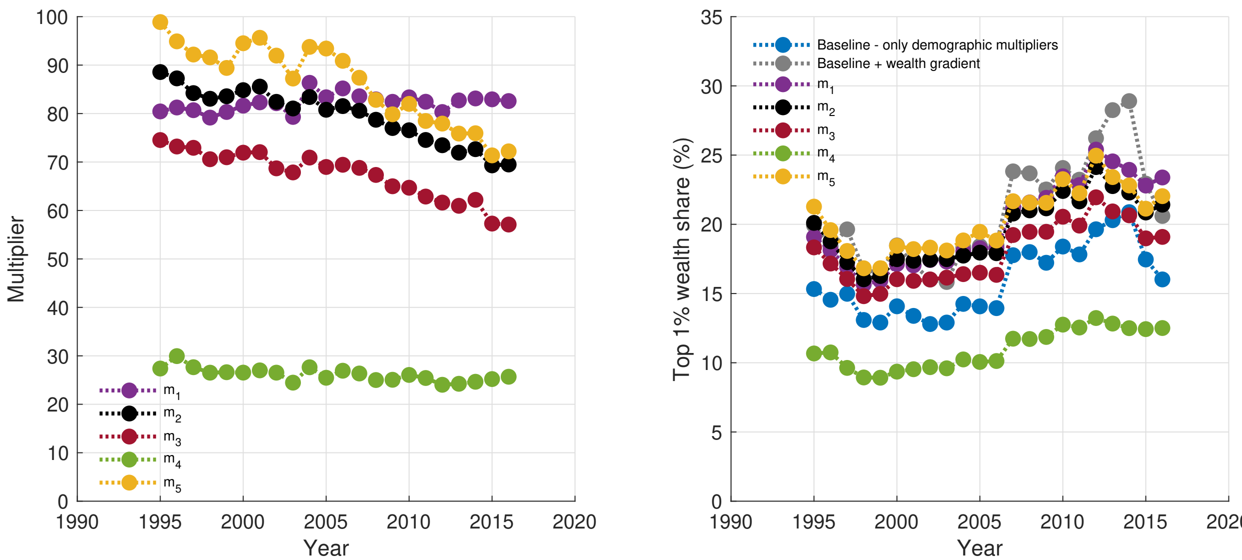

Figure 6: Homogeneous mortality multipliers in Italy 1995–2016. Mortality data were taken from The Human Mortality Database (2018), and the estate tabulations and demographic data were taken from Acciari, Alvaredo and Morelli (2020). The mortality-wealth gradients used were those used for France in Garbinti, Goupille-Lebret and Piketty (2020). The homogeneous multipliers used are: m_1 – the average multiplier $\bar{m}$, the ratio between the population size of the living and the dead; m_2 – the arithmetic average of the disaggregated individual multiplier m_i; $m_3 = \frac{\Sigma_i^{N_E} m_i w_{E,i}}{\Sigma_i^{N_E} w_{E,i}}$; m_4 – the multiplier that corresponds to an individual whose age is the average age at death, based on the mortality data; m_5 – the same m_2 but after adding a mortality-wealth gradient to the demographic data.

The major exception is m_4, the multiplier that corresponds to the average age of decedents in a given year. m_4 is much lower than the other suggested choices, as it effectively ignores the presence of younger decedents among the wealthiest decedents. As seen previously for France in Fig. 3, top wealth groups include a significant presence of younger individuals. Since mortality rates are approximately exponential in age, the impact of these younger individuals on the most representative multiplier for decedents is, in fact, substantial.[5]

[5]This is a direct implication of Jensen's inequality for the exponential function:

$$\mathrm{Exp}\left[E\left[a\right]\right] < \mathrm{E}\left[\mathrm{Exp}\left[a\right]\right] , \tag{5.1}$$

where a is the decedents' age. Because the multipliers depend exponentially on age, the multiplier corresponding to the average age at death is much lower than the average multiplier. Had the dependence of the mortality rate on age been linear, for example, the two quantities would have been equal.

6 Conclusion

By clarifying the functioning of the mortality multiplier method and its structural limitations, this paper contributes to the evolving literature on wealth distribution estimation as well as on the important ongoing methodological debate surrounding the mortality multiplier method itself.

On the one hand, the validation of the empirical finding that top estate and wealth shares co-move and have similar levels (Alvaredo, Atkinson and Morelli, 2018) can be crucial for the expansion of severely sparse data series on wealth distribution, both across countries and over time. Indeed, in the case of the United Kingdom, the close relationship between estate distribution and wealth distribution provided a strong measurement benchmark in order to extend the wealth concentration series back in time to 1895, and to fill in missing years. Similarly, construction of long series can become possible in other countries when the relevant information for the application of the mortality multiplier (*i.e.* detailed estate tabulations or detailed mortality rates) method cannot be retrieved.

On the other hand, the answers to the main questions raised in this paper are crucial for the reliability of the mortality multiplier method. The mortality multiplier method is one of the few viable benchmark methods to estimate wealth concentration, also in a historical perspective. This is important as the use of different methodologies and sources of data for the estimation of wealth distribution remains essential for illuminating the limitations of each source of data and methodology, and to inform us about the levels and trends of wealth concentration. Moreover, and as a matter of fact, the mortality multiplier method is often the only one available to yield estimates of wealth distribution and concentration for specific countries or time periods.

We specifically discuss the relevance of unobserved heterogeneity in mortality rates, such as a the potential wealth effect on mortality that is operating over and above the effect of demographic characteristics. Accounting for a mortality-wealth gradient would create a more accurate picture of mortality multipliers and hence lead to a more realistic estimation of top wealth shares. We find that the difference between the top wealth shares obtained with or without mortality-wealth gradients cannot be large under realistic assumptions and given the observed regularities of the interrelation between the wealth distribution and demographic characteristics. While the mortality-wealth gradient can be steep for younger age groups, it is not as steep for older age groups as economic status does not counterbalance the biological limitations to human longevity. Therefore, adjusting the multipliers at the top of the distribution and taking into account the mortality-wealth gradient is muted by the fact that relatively older people are more represented among the richest decedents. Also, within the top of the estate distribution, there is only a weak dependence of age on wealth rank. As a result, the multipliers at the top may continue to be poorly correlated with wealth ranks, and may continue to be close to the average multiplier of the overall population.

This leads to the important finding that taking into account both demographic multipliers and mortality-wealth gradient yields very similar top wealth shares to those obtained using the average multiplier. Although individuals at the top of the estate distribution have higher mortality rates as they are relatively older on average, this is counterbalanced by their higher economic status, which may lead to healthier lives and better medical care, reducing their probability to die, other things being equal. As a result, the differences between the mortality multipliers at the top of the estate distribution to the average mortality multiplier of the entire decedent population are small enough to create only a limited discrepancy between the two top wealth shares estimated with refined multipliers and the average multiplier. These results are of particular relevance for the estimation of historical series of wealth concentration. They would allow using a wide array of aggregate estate tabulations that were previously thought to be unreliable and unusable.

We end with a practical important remark. Information about the wealth gradient of mortality rates is scarce, and we only know little about how this gradient has evolved over time. In very few cases, such as France and the United States during the last several decades, we have some information about the income gradient of mortality and its trend. Hence, in practice, the application of a mortality-wealth gradient is surrounded with considerable uncertainty. Thus, applying such gradients may not necessarily be satisfactory. They may also create a problem with the total wealth recovered, which, as explained, can be above the known total personal wealth if the gradient applied is too steep. At the same time, applying an average multiplier to the entire decedent population, as we suggest, can also create a similar problem. For these reasons we highlight the need to be careful and transparent when using the mortality multiplier method, and making use of as much data as possible for consistency. Applying the population average multiplier to all decedents may indeed provide reliable estimates of top wealth shares, especially in a historical context. Yet, they still need to be taken with the necessary caution. We also note that the discussion in this paper presupposes that the information provided by the value of estates at death is valid. In some cases it could be argued that the estates recorded by the tax administration are particularly imperfect, due to high level of exemptions, evasion, or through the effects of tax planning. In such cases the concerns about the effect of mortality multipliers become less crucial compared to the inaccuracy of observed estates in describing the personal wealth of decedents. Yet, these discussions exceed the scope of this paper.

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A Estate data coverage

Estate data usually represent only a share of the decedent population, with substantial heterogeneity across countries. Fig. 7 shows the share of the decedent population represented in the data in the group of countries analyzed in Fig. 1.

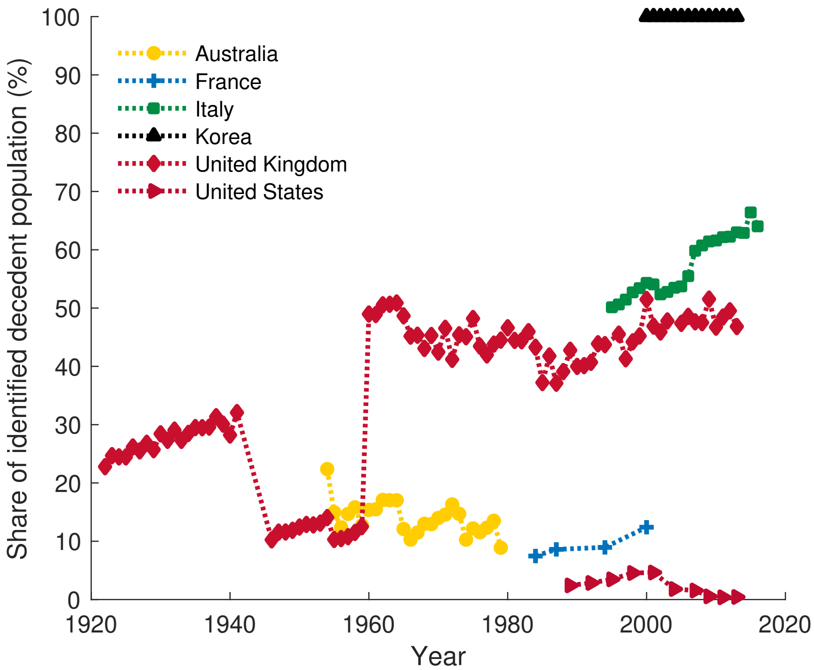

Figure 7: The share of decedents covered in the estate data in Australia, France, Italy, Korea, the United Kingdom and the United States. The data were taken from Katic and Leigh (2016), Garbinti, Goupille-Lebret and Piketty (2020), Acciari, Alvaredo and Morelli (2020), Kim (2018), Alvaredo, Atkinson and Morelli (2018), and Saez and Zucman (2016), respectively, combined with mortality data from The Human Mortality Database (2018).

B The concentration of estates at the top

As shown by Alvaredo, Atkinson and Morelli (2018), relying on unadjusted tax data on estate value can also be informative about the concentration of wealth at the top. To show that we first need to define the top estate share of quantile q

$$(1 - L_q)^E = \frac{\sum_{i=1}^{qN_E} w_{E,i}}{W_E}. \tag{B.1}$$

This requires summing the estates of the richest qN_E decedents and the estimation of the total value of estates of the full decedent population, W_E. However, the estimation of the latter is not a trivial exercise. It requires the estimation of the total value of unobserved estate of the deceased excluded from the tax records, W_E^{exc}. This creates uncertainty in the top estate share estimates.

In practice, estimating W_E^{exc} can be done using the total wealth of the living population not represented by the re-weighted tax records (excluded population), $N^{exc} = N - \sum_{i=1}^{N_E} m_i$. The latter can be directly estimated from external sources of data, such as surveys or other administrative records, if the general identity of the excluded population could be inferred.

The total identified wealth is known through the multipliers and observed estate values:

$$W^{iden} = \sum_{i=1}^{N_E^{tax}} m_i w_{E,i}. \tag{B.2}$$

In the absence of disaggregated multipliers this becomes

$$W^{iden} = \sum_{i=1}^{N_E^{tax}} \bar{m} w_{E,i} = \bar{m} W_E^{iden}. \tag{B.3}$$

The total excluded wealth is then

$$W^{exc} = W - W^{iden}. \tag{B.4}$$

At the same time

$$W^{exc} = \bar{m}^{exc} W_E^{exc}, \tag{B.5}$$

where $\bar{m}^{exc}$ is the average multiplier of the excluded decedents. $\bar{m}^{exc}$ can be estimated depending on how refined are the demographic data and mortality data available. If mortality by age and gender is available, it is possible to define a different multiplier for the excluded decedents in each age and gender:

$$m_{a,g}^{exc} = \frac{N_{a,g}^{exc}}{N_{E,a,g}^{exc}}, \tag{B.6}$$

where $N_{a,g}^{exc}$ is the number of living with age a and gender g not observed by the tax records, and $N_{E,a,g}^{exc}$ is the number of decedents with age a and gender g not observed by the tax records. In this case $\bar{m}^{exc}$ would be the average of all multipliers $m^{exc}_{a,g}$. Alternatively, in the absence of such data, $\bar{m}^{exc}$ can be defined as the ratio between the excluded living population and the excluded decedent population:

$$\bar{m}^{exc} = \frac{N^{exc}}{N_E - N^{tax}_E} \,. \tag{B.7}$$

It is clear that different sets of multipliers would lead to different estimates of W^{exc}_E. This leads to different total value of estates, which, in turn, leads to different top estate share estimates. In Sec. 3 we use this calculation to provide different estimates of top estate shares and compare them to top wealth shares reported in the literature.

C The coefficient of variation of estates and of wealths

To illustrate the similarity between the concentration of wealth and of estates it is possible to compare the coefficient of variation (CV) of the wealth distribution with and without multipliers. The derivation is inspired by a derivation presented in Atkinson and Harrison (1978), comparing the CV between capital income and wealth, for the capitalization method. It clarifies the intuition for the result obtained for top shares discussed above. Yet, it is conceptually simpler, since the index I_q does not play a role in the CV. It is also not limited to a specific quantile q, but involves the entire distribution.

The coefficient of variation of estates, denoted Y_E, follows

$$Y_E^2 = \frac{\sigma_E^2}{\bar{w}_E^2} \,. \tag{C.1}$$

The coefficient of variation of wealths, denoted Y_E, follows

$$Y_W^2 = \frac{\sigma_W^2}{\bar{w}_W^2} \,, \tag{C.2}$$

where σ_E^2 is the variance of estates, σ_W^2 is the variance of wealths, $\bar{w}_E$ is the average estate, and $\bar{w}_W$ is the average wealth.

We begin by writing down expressions for the variance estates and wealths:

$$\sigma_E^2 = \tfrac{1}{N_E} \textstyle\sum_{i=1}^{N_E} w_{E,i}^2 - \tfrac{1}{N_E^2} \left(\textstyle\sum_{i=1}^{N_E} w_{E,i} \right)^2 ; \tag{C.3}$$

$$\sigma_W^2 = \tfrac{1}{N} \textstyle\sum_{i=1}^{N_E} m_i w_{E,i}^2 - \tfrac{1}{N^2} \left(\textstyle\sum_{i=1}^{N_E} m_i w_{E,i} \right)^2 . \tag{C.4}$$

Therefore we get

$$Y_E^2 = \frac{\frac{1}{N_E} \sum_{i=1}^{N_E} w_{E,i}^2 - \frac{1}{N_E^2} \left(\sum_{i=1}^{N_E} w_{E,i} \right)^2}{\frac{1}{N_E^2} \left(\sum_{i=1}^{N_E} w_{E,i} \right)^2} \,, \tag{C.5}$$

and

$$Y_W^2 = \frac{\frac{1}{N} \sum_{i=1}^{N_E} m_i w_{E,i}^2 - \frac{1}{N^2} \left(\sum_{i=1}^{N_E} m_i w_{E,i} \right)^2}{\frac{1}{N^2} \left(\sum_{i=1}^{N_E} m_i w_{E,i} \right)^2} \,. \tag{C.6}$$

μ is the ratio between the average estate and the average wealth

$$\mu = \frac{\frac{1}{N_E} \sum_{i=1}^{N_E} w_{E,i}}{\frac{1}{N} \sum_{i=1}^{N_E} m_i w_{E,i}} = \bar{m} \frac{\sum_{i=1}^{N_E} w_{E,i}}{\sum_{i=1}^{N_E} m_i w_{E,i}} \,, \tag{C.7}$$

so

$$\frac{1}{N^2}\left(\sum_{i=1}^{N_E} m_i w_{E,i}\right)^2 = \frac{1}{\mu^2}\cdot\frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2 , \tag{C.8}$$

and therefore

$$Y_W^2 = \frac{\frac{1}{N}\sum_{i=1}^{N_E}\mu^2 m_i w_{E,i}^2 - \frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2}{\frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2} . \tag{C.9}$$

We can then rearrange Y_W^2 and get

$$Y_W^2 = Y_E^2 - \frac{\frac{1}{N_E}\sum_{i=1}^{N_E} w_{E,i}^2}{\frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2} + \frac{\frac{1}{N}\sum_{i=1}^{N_E}\mu^2 m_i w_{E,i}^2}{\frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2} . \tag{C.10}$$

Taking $N = \bar{m}N_E$ we get

$$Y_W^2 = Y_E^2\left(1 + \frac{\frac{1}{N_E}\sum_{i=1}^{N_E}\left(\frac{\mu^2 m_i}{\bar{m}} - 1\right) w_{E,i}^2}{\sigma_E^2}\right) . \tag{C.11}$$

This result leads to several important observations that clarify the similarity between inequality of estates and of wealths. First, the difference between the CV of wealth and estates is mainly driven by the multipliers at the top of the distribution. This is because the difference $\left(\frac{\mu^2 m_i}{\bar{m}} - 1\right)$ is weighted by the level of estates. Thus, the similarity between Y_W and Y_E, like the top shares, mainly depends on the interaction between estates and multipliers among the richest decedents.

Second, there is a dampening effect that limits the extent to which Y_W and Y_E are distant from one another. If the multipliers at the top are high in comparison to the average multiplier then $m_i/\bar{m} > 1$. μ is then likely to be lower than 1. The inverse is true if $m_i/\bar{m} < 1$. This creates a dampening effect that makes the expressions $\left(\frac{\mu^2 m_i}{\bar{m}} - 1\right)$ in Eq. (C.11) generally close to 0.

Third, comparing the coefficients of variation further demonstrates that the similarity in inequality measures between estates and wealths may not be limited to top shares, but also when full distributions are taken into account.

D Mortality rates by age

Age is the most important statistical determinant of mortality. Fig. 8 shows the mortality rates in France, Italy, the United Kingdom, and the United States in 1950, 1970, 1990, and 2010, based on The Human Mortality Database (2018) data. It illustrates that mortality rates increase exponentially with age above the age of 40.

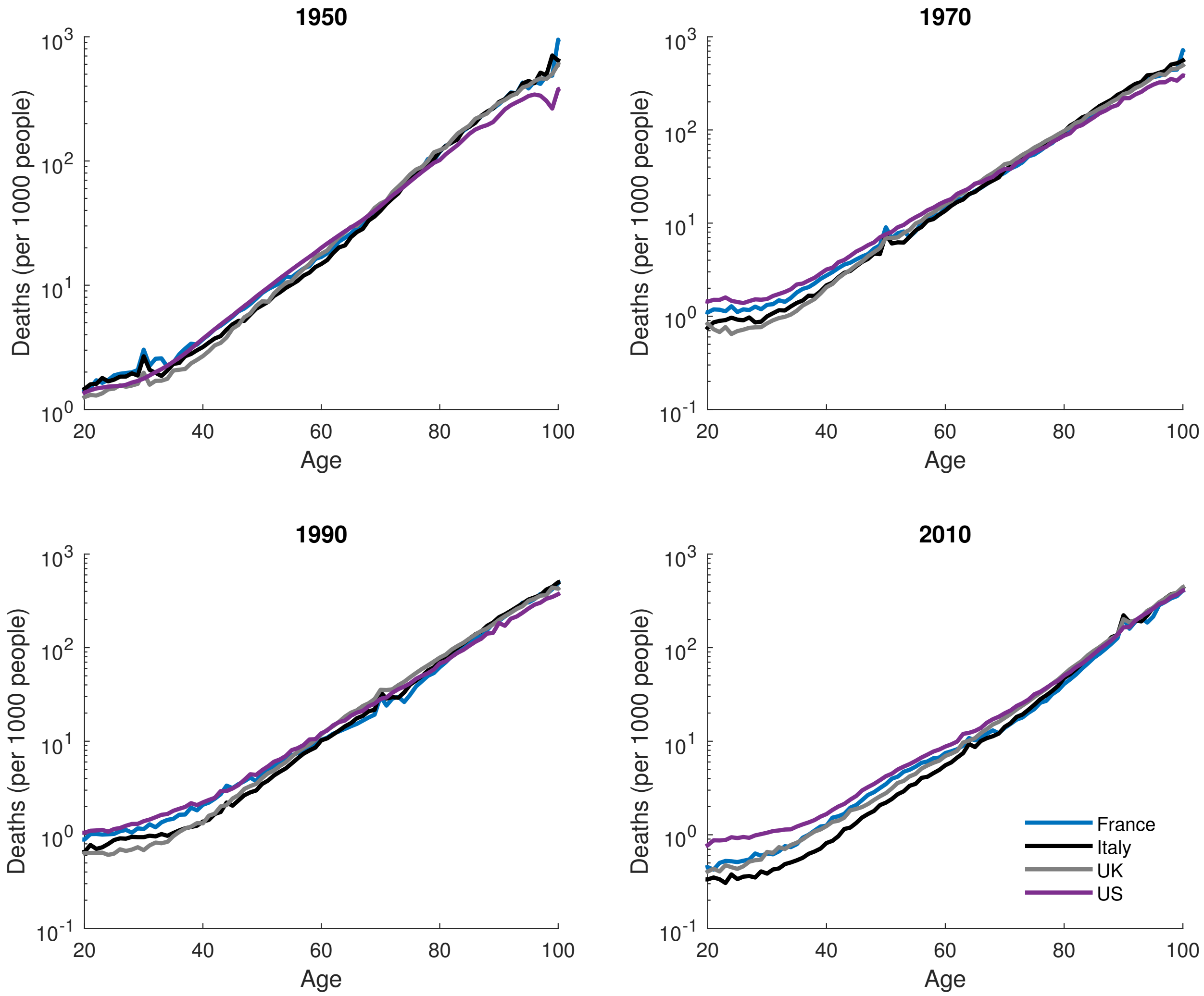

Figure 8: Mortality rates in France, Italy, the United Kingdom, and the United States in 1950, 1970, 1990, and 2010. Source: The Human Mortality Database (2018).