On the Distribution of Estates and the Distribution of Wealth: Evidence from the Dead

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Abstract

The distribution of estates (the net value of real and financial property of a deceased person) has commonly served for the estimation of the distribution of wealth among the living via the estate multiplier method, but has never been under extensive scrutiny in and of itself. Theoretically, the application of detailed multipliers can increase or decrease top wealth shares relative to estate shares. This depends on the evolution of mortality rates with respect to age, gender, income and wealth. In this paper, we highlight that the concentration of estates and the derived concentration of wealth at the top through the multiplier method are actually very close to each other. We investigate why the application of mortality multipliers does not alter significantly the picture when both distributions are compared. We study the general conditions under which the concentration of estates at death provides the same informative content as the concentration of the wealth among the living, and identify that the relationship depends on the covariance between mortality rates and estates amounts. As a result, we provide novel historical series of wealth concentration in country-years where there is no information to apply the multiplier method, but enough data to make well-grounded inferences on the distribution of estates.

Keywords: Wealth inequality, estates, mortality rates, economic history
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1 Introduction

There are five main sources of evidence about the distribution of personal wealth:

- Household surveys of personal wealth, such as the Survey of Consumer Finance conducted by the Flow of Funds Unit of the US Federal Reserve, the Household Finance and Consumption Surveys co-ordinated by the European Central Bank, or the UK Wealth and Asset Survey, conducted by the Office of National Statistics;
- Administrative data on personal wealth derived from annual wealth taxes, as in the cases of Spain and France today;
- Administrative data on investment income, capitalized to yield estimates of the underlying wealth;
- Lists of large wealth-holders, such as the annual Forbes Richest People in America List;
- Administrative data on individual estates at death, multiplied-up to yield estimates of the wealth of the living through the estate multiplier method.

The distribution of estates (the net value of worldwide real and financial property of a deceased person) has commonly served for the estimation of the distribution of wealth among the living via the estate multiplier method, but has never been under extensive scrutiny in and of itself. Theoretically, the application of detailed multipliers can increase or decrease top wealth shares relative to estate shares. When the age and gender multipliers were first employed in the United Kingdom, it was seen as overcoming a “fatal” objection to the use of estate data, since “the accumulated wealth of an individual increases with years, and is usually greatest when a man dies” (Mallet, 1908, p. 67). However, recent research has highlighted that the concentration of estates and the derived concentration of wealth at the top through the mortality method are actually very close to each other (Alvaredo, Atkinson and Morelli, 2018).

In this paper, we investigate first why the application of mortality multipliers does not alter significantly the picture when both distributions are compared. We study the general conditions under which the concentration of estates at death provides the same informative content as the concentration of the wealth among the living, and identify that the relationship depends crucially on the covariance between mortality rates and estates amounts.

Second, we compare the distribution of estates and the distribution of wealth in those countries where the estate multiplier method can be applied, namely Australia, France, Italy, Korea, the United Kingdom and the United States.

Third, and most importantly, we produce novel long-run historical series of the distribution of wealth for Belgium Japan, the Netherlands, and South Africa, where data from the distribution of estates have not been exploited yet. If top estate shares are good indicators of top shares of wealth among the living, then one can make use of basic statistical information from the administration of inheritance and estate taxes to make well-grounded inferences about the concentration of wealth in those country-years where there is no information to apply the multiplier method. These are the vast majority of cases. This is additionally relevant for historical inequality studies, given that existing series of wealth inequality are very scarce and only cover a few developed countries (see The World Wealth and Income Database (2017)).

The work by Alvaredo, Atkinson and Morelli (2018) has shown that the concentration of estate at death and the derived concentration of wealth at the top following the application of the mortality multipliers (based on gender, age, social-class differentials or wealth differentials) are actually very close to one another. In their words, "the application of mortality multipliers does not alter the picture concerning the distribution of the wealth of the living, as commonly believed." As described by Cowell (1978), referring to Atkinson and Harrison (1978): "though the particular refinement of mortality multiplier that is used considerably affects the calculation of total wealth, the resultant effect on top wealth shares is not all that great." An implicit recognition of this similarity can be also found in Piketty, Postel-Vinay and Rosenthal (2006) and Moriguchi and Saez (2008), who treated the distribution of estates, estimated using estate tax records, as a de facto equivalent to the distribution of wealth.

Information about the distribution of wealth is scarce, for the recent period and more so for historical series. However, many countries have published detailed data on the distribution of estate taxes. These are only rarely accompanied by demographic characteristics such as age and gender. Therefore, one cannot apply heterogeneous mortality rates to the estate tax data. Better understanding of the effect of multipliers on the estimates of top wealth shares could mitigate this limitation. It may unlock a wide array of aggregate tabulations that were previously thought to be unreliable and unusable.

Recently, the work by Saez and Zucman (2016) also brought to the fore old-standing concerns about the estate multiplier method, suggesting that the failure to appropriately control for decreasing mortality of wealthy individuals may severely underestimate the top wealth shares. Saez and Zucman (2016) proposed that such an underestimation may play an important role in the United States for the reconciliation between the estate-based top wealth shares series and that derived through the capitalization of investment and capital incomes. In a recent working paper, Saez and Zucman (2019) suggested that correcting the estate multiplier top wealth share estimates using granular mortality rates by income percentiles, age, and year (Chetty et al., 2016) can indeed help reconciling between the different methods. Taking this issue seriously, the main analysis by Alvaredo, Atkinson and Morelli (2018) further simulated the effect of wider mortality differentials by wealth levels on UK data and highlighted that the "sensitivity of top wealth shares to steepening mortality-wealth gradients is an empirical matter, and that such elasticity is small." These issues concerning the application of mortality multipliers are not new to the literature (*cfr.* Atkinson and Harrison (1978)).

More specifically, this paper aims to better understand the following issues: What is the nature of the relationship between the distribution of estates at death and that of the wealth of the living through the estate multiplier method? What are the general conditions under which the concentration of estates at death provides the same informative content as the concentration of the wealth of the living? What drives the relevance and the direction of the bias created by the lack of appropriate control for the growing longevity of wealthy individuals? Do reasonable changes in the gender-age-wealth mortality rates affect the nature of these results?

To illustrate these points, we begin by considering the case of an average multiplier that can be applied to aggregate estate tabulations by size, ignoring all demographic information (*e.g.* some historical tabulations can only be found in this form). The multiplier to be applied to the top of the estate distribution is, ideally, the multiplier that best corresponds to the average age of the decedents reflected in the tax records. In the absence of such information one could use the average multiplier of the overall population. As a second step, we consider the more realistic case of differential mortality multipliers by demographic characteristics (the case of differential multipliers by socio-economic characteristics will be dealt separately). The difference between the derivation of top wealth shares between demographic-differential multipliers and average multipliers is formalized and discussed. The latter is an important starting point as mortality rates by age and gender generally map most of the variability in mortality rates observed in a country in a given year. Moreover, these data are available throughout history for many countries.

We show that the top wealth shares are exactly equal to the top estate shares in the special case of using the average multiplier of the overall population. We further show that the estimates of top wealth shares with and without heterogeneous mortality multipliers are found to strongly co-move and to be rather similar in level. This finding confirms what was found by Alvaredo, Atkinson and Morelli (2018).

We further discuss the relevance of "unobserved" heterogeneity in mortality rates, such as a the potential wealth effect on mortality that is operating over and above the effect of demographic characteristics (*e.g.* age and gender). Accounting for a mortality-wealth gradient would create a more accurate picture of mortality multipliers and hence lead to a more realistic estimation of top wealth shares. To the extent that the relevance of socio-economic status (proxied by level of wealth) as a driver of mortality have been changing over time, this might create an important trend effect on our estimates of top wealth shares which would remain otherwise unaccounted for.

After controlling for realistic mortality-wealth gradients, the newly estimated top wealth share is now closer to the one derived with the use of the average multiplier among adults. Although individuals at the top of the estate distribution have higher mortality rates as they are relatively older on average, this is counterbalanced by their higher economic status, which may lead to healthier lives and better medical care, reducing their probability to die, other things being equal. As a result, the differences between the mortality multipliers at the top of the estate distribution to the average mortality multiplier of the entire decedent population are small enough to create only a limited discrepancy between the two top wealth shares estimated with differential and average multipliers.

The formalization of the problem shows that the difference between top wealth shares of a specific q-percentile obtained with average multipliers and with demographic-differential multipliers is driven by two main factors: the covariance between average mortality rates and the estate value of the p% richest decedents required to estimate the q% wealth holders and the difference between the mean wealth of the top p% estates and the q% top estate holders.

A similar result is derived for the coefficient of variation (CV) as a measure of inequality, in an analogy to the derivation of the CV for the capitalization method in Atkinson and Harrison (1978). Comparing the CV of the wealth distribution in the estate multiplier method between demographic-differential multipliers and average multipliers suggests a close relationship between the two. It further demonstrates that the similarity in inequality measures between estates and wealths may not be limited to top shares, but also when complete distributions are taken into account.

We then move to the empirical evidence for Australia, France, Italy, Korea, the United Kingdom, and the United States. For each country, we compare the top wealth shares derived with an average multiplier to the top wealth share derived using disaggregated demographic data, as reported in the works of Katic and Leigh (2016), Garbinti, Goupille-Lebret and Piketty (2017), Acciari, Alvaredo and Morelli (2020), Kim (2018), Alvaredo, Atkinson and Morelli (2018), Kopczuk and Saez (2004) and Saez and Zucman (2016). For each country, we apply the average population multiplier to very basic information on the distribution of estates at death. This simple exercise turned out to provide estimates which mimic very well the dynamics of top wealth shares derived with disaggregated data and more sophisticated information on mortality. Their level is generally higher, but only to a rather small degree (the median difference in the top 1% share in all countries across all years was 1.2 percentage points). The main reason is that not controlling for the age composition of the decedents reported in the tax records, results in a generally higher multiplier for richer segments of the population compared to what can be estimated from the average age of decedent filers (the average age of rich decedents is somewhat higher).

Finally, we present new estimates for the top 10%, 1%, and 0.1% wealth shares in Belgium (between 1937 and 1994) and in Japan (between 1970 and 2015), based on estate tax tabulations and only aggregate mortality data. We apply the estate multiplier method with the same multiplier, the average mortality multiplier in the population (the number of living divided by the number of decedents), to all decedents. We find that wealth inequality in Belgium decreased dramatically during the course of the 20th century. The top 10% wealth share decreased from around 80% in the late 1930s to around 55% in the mid-1990s. Similar trends are found for the top 1% and 0.1%. The levels and the trend found closely follow those found in France (Garbinti, Goupille-Lebret and Piketty, 2017).

We find that wealth inequality rapidly increased in Japan between the late 1970s and the early

1990s, especially during the late 1980s. Inequality levels rapidly decreased since. During the Japanese asset price bubble in the late 1980s, inequality rose to levels comparable to US 2010s inequality levels, with the top 0.1% share being above 10%. Yet, during the 2000s and 2010s, the top 10% wealth share in Japan was between 40% to 50%, low in comparison to most developed countries.

Although, as discussed, changes in the multipliers do not dramatically affect the shape of the wealth distribution, they may have a large impact on estimated aggregate variables, such as wealth totals and the ratio between the average estate and the average wealth among the living. These aggregate variables are usually of lesser interest, if the goal is obtaining information on the distribution of wealth. Estate tax records rarely cover 100% of the decedent population. In particular, the derivation of total value of the estates unreported in tax records can be obtained by multiplying the average mortality rate of the population missing from tax records to the difference between total personal wealth (*e.g.* the aggregate personal wealth estimated from national balance sheet) and the identified wealth obtained via the application of multipliers to the full set of data on estate tax records. The use of different multipliers would change the estimates of total estate. The derivation of macroeconomic aggregate series are of direct interest to economists (see Piketty and Zucman (2014) and Alvaredo, Garbinti and Piketty (2017)), and, we argue, may well serve serve as a indirect test of the appropriateness of the multipliers used to derive distributional measures. Mortality multipliers thus matter in various aspects.

It is possible, for instance, that the application of a very steep mortality-wealth gradient at the top of the estate distribution would correspond to total wealth levels that exceed known levels of total wealth. It is also possible that the application of graduated multipliers lead to very low levels of the ratio between average wealth at death to average wealth among the living, a parameter that has been shown to fall between 1 and 2 on a historical perspective (Alvaredo, Garbinti and Piketty, 2017).

We also note that the discussion in this paper presupposes that the information provided by the value of estates at death is valid. In some cases it could be argued that the estates recorded by the tax administration are particularly imperfect, due to high level of exemptions, evasion, or through the effects of tax planning. In such cases the concerns about the effect of mortality multipliers become less crucial compared to the inaccuracy of observed estates in describing the personal wealth of decedents.

2 The estate multiplier method

One of the oldest methods to estimate wealth concentration makes use of information on the wealth and the demographic characteristics of decedents reported to the tax authorities for the administration of inheritance or estate taxes. The decedent population can then be re-weighted and become representative of the living population. This, in turn, allows estimating inequality indicators of the wealth holdings of the living. We follow the notation used in Atkinson and Harrison (1978), with some changes. The full nomenclature is given in Appendix A.

Consider the population of N_E decedents and the total value of their estates, W_E:

$$W_E = \sum_{i=1}^{N_E} w_{E,i} \,. \tag{2.1}$$

We also assume that the total personal wealth, W^{tot}, is given and that the estates $w_{E,i}$ are ordered in a descending order, *i.e.* $w_{E,i} \geq w_{E,j}$, if $i < j$. Assuming that W_E is the true representation of the total wealth holding of the deceased (*e.g.* no tax exemption, avoidance or evasion), then the following relationship must hold:

$$W^{tot} = \sum_{i=1}^{N_E} m_i w_{E,i} \,, \tag{2.2}$$

where we denote by $m_i \equiv \frac{1}{p_i}$ the mortality multiplier of individual i, equal to the inverse of her mortality rate.

The mortality rates vary across a set of socio-demographic characteristics. Therefore each multiplier represents the number of living individuals who share the same socio-demographic characteristics of decedent i. The average mortality rate of the population, $\bar{p}$, is defined as the ratio between the number of deceased, N_E, and the number of living, N. The average multiplier, $\bar{m}$, is then defined as the inverse of the average mortality rate (or N/N_E).

We are interested in estimating the wealth share of the top quantile $0 < q < 1$, where $q = 0.1$ corresponds to 10%, $q = 0.01$ corresponds to 1%, *etc.*. It is natural to think that the value of multipliers will affect the number of decedents that will be needed in order to account for the top q quantile among the living. For example, if the multipliers of the rich decedents are high, compared to the average multipliers in the population, less decedents would be required to account for the top q quantile among the living than when the multipliers of the rich decedents are lower. This number is represented by the index I_q such that[1]

$$\sum_{i=1}^{I_q} m_i = qN \,. \tag{2.3}$$

This way we can define the top q wealth share as

$$(1 - L_q)^W = \frac{\sum_{i=1}^{I_q} m_i w_{E,i}}{W^{tot}} \,. \tag{2.4}$$

In the absence of either detailed mortality data or detailed estate data by demographic characteristics, we would need to rely on the average mortality rates. In this case $m_i = \bar{m}$ and $I_q = qN_E$,

[1] If there is no equality, I_q is defined as the smallest index such that $\sum_{i=1}^{I_q} m_i > qN$.

and the top wealth shares will take the following form:

$$(1 - L_q)^W_{avg} = \bar{m} \frac{\sum_{i=1}^{qN_E} w_{E,i}}{W^{tot}} \,. \tag{2.5}$$

In practice, this simplified form of the top wealth shares $(1 - L_q)^W_{avg}$ is relevant especially for historical data, which are either found in aggregated tabulated form, or cannot be matched with disaggregated mortality data that are not always available. Whether or not this measure differs from the *true* top wealth share is the subject of the discussion in the following section. We also note that $(1 - L_q)^W_{avg}$ is also equivalent to the top q estate share (see Appendix B).

3 The concentration of wealth with and without multipliers

The validation of the empirical finding that top estate and wealth shares co-move and have similar levels (Alvaredo, Atkinson and Morelli, 2018) can be crucial for the expansion of severely sparse data series on wealth distribution, both across countries and over time. Indeed, in the case of the United Kingdom, the close relationship between estate distribution and wealth distribution provided a strong measurement benchmark in order to extend the wealth concentration series back in time to 1895. Similarly to what has been done for the United Kingdom, the construction of long series can become possible in other countries when the relevant information for the application of the estate multiplier method cannot be retrieved.

Here we present empirical estimates for the top wealth shares for Australia, France, Italy, Korea, the United Kingdom, and the United States. The availability of estate data in each of these countries differ substantially. For example, in Italy the data cover roughly 60% of decedents every year, however only tabulations are available. In France the data cover a much smaller share of the decedent population, yet micro data are available. Despite the specific differences between the data sources, the application of the estate multiplier method is similar and requires the same information – the values of estates at the top of the estate distribution (or equivalent tabulations), the corresponding mortality multipliers at the top of the estate distribution, and the total personal wealth.

Figure 1 presents top wealth share estimates for Australia, France, Italy, Korea, the United Kingdom, and the United States. In each, the top wealth shares reported in the literature, produced by the same estate record data, are compared to estimates of top wealth shares when assuming average mortality multipliers.

Figure 1 demonstrates a key point – in all countries we find that the top wealth shares estimated using the average multiplier and the top wealth shares in the literature co-move, and they are generally similar.

We note that the top wealth shares taken from Acciari, Alvaredo and Morelli (2020) for Italy and

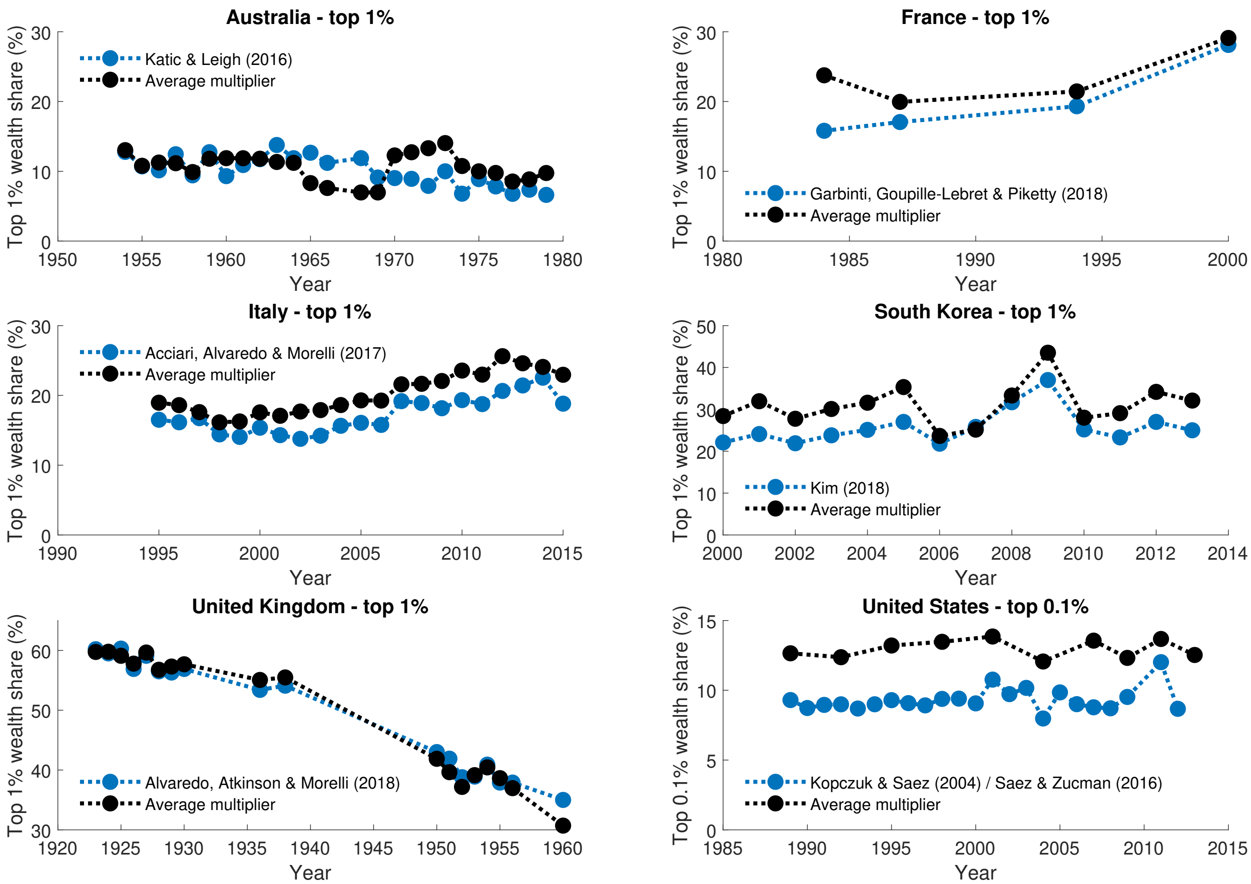

Figure 1: The top wealth shares in Australia, France, Italy, Korea, the United Kingdom, and the United States. Tabulations and and the top wealth shares were taken from Katic and Leigh (2016), Garbinti, Goupille-Lebret and Piketty (2017), Acciari, Alvaredo and Morelli (2020), Kim (2018), Alvaredo, Atkinson and Morelli (2018), and Saez and Zucman (2016), respectively. The estimated top wealth shares were produced using the estate multiplier method assuming the average multiplier for all observed decedents. The mortality data were taken from The Human Mortality Database (2018).

those for the United Kingdom taken from Alvaredo, Atkinson and Morelli (2018) are estimated from the inheritance tax records using the net wealth as reported, and without making any adjustments to the data to allow for under-reporting, tax avoidance and evasion.[2] Unlike the other countries they are based on the same data on which our estimates of top wealth shares with an average multiplier rely on. The results for the other countries include some adjustments such as the inclusion of trusts and pensions in the United States (Kopczuk and Saez, 2004), or requiring consistency with national accounts in France (Garbinti, Goupille-Lebret and Piketty, 2017). For this reason we will primarily use the case of Italy later on in our analysis.

[2]In the case of Italy, real-estate values have been transformed from cadastral to “market” valuation.

4 Formal comparison of top wealth shares with different choice of multipliers

It is possible to analyze how different choices of multipliers affect the estimates of top wealth shares. This is motivated by the need to obtain a better intuition for the sensitivity of the estate multiplier method to the choice of multipliers.

As described above, the top wealth share of quantile q is

$$(1-L_q)^W = \frac{\sum_{i=1}^{I_q} m_i w_{E,i}}{W^{tot}} . \tag{4.1}$$

We saw that the top wealth in the average multiplier case is

$$(1-L_q)^W_{avg} = \bar{m}\frac{\sum_{i=1}^{qN_E} w_{E,i}}{W^{tot}} . \tag{4.2}$$

It is possible to determine the conditions for equality of the top wealth shares with the average multiplier and with disaggregated multipliers:

$$(1-L_q)^W = (1-L_q)^W_{avg} \iff \frac{\sum_{i=1}^{I_q} m_i w_{E,i}}{W^{tot}} = \bar{m}\frac{\sum_{i=1}^{qN_E} w_{E,i}}{W^{tot}} \iff \sum_{i=1}^{I_q} \frac{m_i}{\bar{m}} w_{E,i} = \sum_{i=1}^{qN_E} w_{E,i} \quad * \tag{4.3}$$

Denoting

$$\bar{w}_{qN_E} = \frac{\sum_{i=1}^{qN_E} w_{E,i}}{qN_E} \; ; \; \bar{w}_{I_q} = \frac{\sum_{i=1}^{I_q} w_{E,i}}{I_q}; \tag{4.4}$$

we can rewrite the equality condition (*) as[3]

$$\bar{w}_{qN_E} - \bar{w}_{I_q} = \frac{I_q}{\bar{m} q N_E} \mathrm{Cov}\left[m_i, w_{E,i}\right] . \tag{4.5}$$

Alternatively, it is possible to write down explicitly the difference between the top wealth shares and obtain via the same notation and using the same expansion:

$$\begin{aligned}(1-L_q)^W - (1-L_q)^W_{avg} &= \frac{\bar{m} q N_E}{W^{tot}}\left(\bar{w}_{I_q} - \bar{w}_{qN_E}\right) + \frac{I_q}{W^{tot}} \mathrm{Cov}\left[m_i, w_{E,i}\right] \\ &= \frac{I_q}{W^{tot}}\left[\bar{m}_{I_q}\left(\bar{w}_{I_q} - \bar{w}_{qN_E}\right) + \mathrm{Cov}\left[m_i, w_{E,i}\right]\right] ,\end{aligned} \tag{4.6}$$

where $\bar{m}_{I_q}$ is the average multiplier at the top of the estate distribution ($\Sigma_{i=1}^{I_q} m_i/I_q$).

The right hand side of Eq. (4.6) shows that the difference between top wealth shares depends on an average level effect of the multipliers ($\bar{w}_{I_q} - \bar{w}_{qN_E}$) and on the covariance ($\mathrm{Cov}\left[m_i, w_{E,i}\right]$). The

[3] $\mathrm{Cov}\left[m_i, w_{E,i}\right] = \frac{1}{I_q}\sum_{i=1}^{I_q}\left(m_i - \frac{1}{I_q}\sum_{j=1}^{I_q} m_j\right)\left(w_{E,i} - \bar{w}_{I_q}\right)$

effect reflected in the covariance term in Eq. (4.6) is nuanced, and will be further discussed in the next sections.

The average level effect, however, is simpler – the closer the average of the multipliers at the top is to the average multiplier, the closer the difference $\bar{w}_{I_q} - \bar{w}_{qN_E}$ would be to zero. The closer the average of the multipliers at the top is to the average multiplier, the closer the index I_q is to qN_E. As we will see, when disaggregated multipliers are considered, the average multiplier at the top tends to be lower than $\bar{m}$. This is a straightforward result of life cycle effects – mortality is predominantly determined by age, and older people tend to be richer, on average. Therefore, the top of the estate distribution is likely to be composed of people that are older on average than the adult population. Therefore, in order to account for the top qN living individuals, we would need more than qN_E decedents (note that $\bar{m} = \frac{N}{N_E}$). For this reason the difference $\bar{w}_{I_q} - \bar{w}_{qN_E}$ would tend to be negative.

As we will also see, the covariance ($\mathrm{Cov}\left[m_i, w_{E,i}\right]$) tends to be negative if we only account for demographic multipliers, ignoring the mortality-wealth gradient. Therefore, using the average multiplier will tend to lead to over-estimation of the top wealth shares, compared to disaggregated demographic multipliers. Accounting for the mortality-wealth gradient will slightly increase the covariance, creating possibly a positive association between estate values and mortality multipliers at the top of the estate distribution. At the same time, the gradient will increase the average multiplier at the top, which will, in turn, increase the difference $\bar{w}_{I_q} - \bar{w}_{qN_E}$. Together, this may lead to equality between the estimated top wealth share when assuming disaggregated multipliers with a mortality-wealth gradient, and when assuming the average multiplier.

4.1 The coefficient of variation with and without multipliers

A similar derivation can be used to compare the coefficient of variation (CV) of the wealth distribution with and without multipliers. The derivation is inspired by a derivation presented in Atkinson and Harrison (1978), comparing the CV between capital income and wealth, for the capitalization method. It clarifies the intuition for the result obtained for top shares above. Yet, it is conceptually simpler, since the index I_q does not play a role in the CV. It is also not limited to a specific quantile q, but involves the complete distributions.

The CV of estates, denoted Y_E, follows

$$Y_E^2 = \frac{\sigma_E^2}{\bar{w}_E^2}\,. \tag{4.7}$$

The wealth CV, denoted Y_E, follows

$$Y_W^2 = \frac{\sigma_W^2}{\bar{w}_W^2}\,, \tag{4.8}$$

where σ_E^2 is the variance of estates, σ_W^2 is the variance of wealths, $\bar{w}_E$ is the average estate, and $\bar{w}_W$ is the average wealth.

Given the estate multiplier method, there is a simple relationship between Y_W^2 and Y_E^2 (see Appendix C for the full derivation):

$$Y_W^2 = Y_E^2 \left(1 + \frac{\frac{1}{N_E} \sum_{i=1}^{N_E} \left(\frac{\mu^2 m_i}{\bar{m}} - 1 \right) w_{E,i}^2}{\sigma_E^2} \right) . \tag{4.9}$$

This rather simple result leads to several important observations that clarify the similarity between inequality of estates and of wealths. First, the difference between the CV of wealth and estates is mainly driven by the multipliers at the top of the distribution. This is because the difference $\left(\frac{\mu^2 m_i}{\bar{m}} - 1 \right)$ is weighted by the level of estates. Thus, the similarity between Y_W and Y_E, like the top shares, mainly depends on the interaction between estates and multipliers among the richest decedents.

Second, there is a dampening effect that limits the extent to which Y_W and Y_E are distant from one another. If the multipliers at the top are high compared to the average multiplier then $m_i/\bar{m} > 1$. μ is then likely to be lower than 1. The inverse is true if $m_i/\bar{m} < 1$. This creates a dampening effect that makes the expressions $\left(\frac{\mu^2 m_i}{\bar{m}} - 1 \right)$ in Eq. (4.9) generally closer to 0.

Third, comparing the coefficients of variation further demonstrates that the similarity in inequality measures between estates and wealths may not be limited to top shares, but also when complete distributions are taken into account.

Our aim in the next section is to further establish the intuition for the similarity between top wealth shares with and without heterogeneous multipliers and to present evidence for the relevance of our observations. We will also discuss the conditions under which substantial discrepancy between different estimates of top wealth shares should be observed and whether these conditions our plausible.

5 The relationship between mortality multipliers and socio-demographic characteristics

Eq. (4.6) shows that the differences between top wealth shares for different choices of multipliers are a function of the difference between the average estates $\left(\bar{w}_{I_q} - \bar{w}_{qN_E} \right)$, and the covariance of multipliers and estates at the top of the estate distribution. Fig. 2 demonstrates this relationship empirically for Italy, when considering a mortality-wealth gradient and when assuming disaggregated demographic multipliers only.

The correlation of multipliers and estates at the top of the estate distribution can therefore explain a large part of the small difference between top wealth shares. This correlation can be determined by various factors.

First, a common factor that affects both mortality and wealth is age. Mortality is strongly deter-

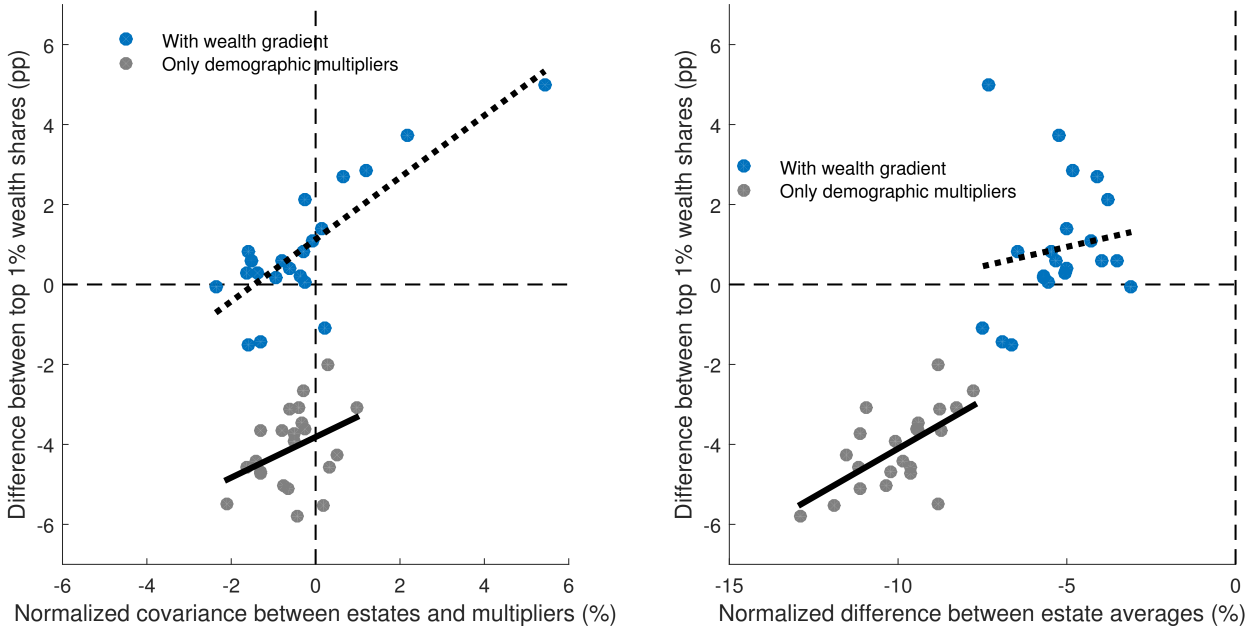

Figure 2: The dependence of the difference between top 1% wealth shares with disaggregated multipliers and with the average multiplier on the covariance between multipliers and estates at the top (see also Eq. (4.6)) $-\frac{I_q}{W^{tot}}\text{Cov}\left[m_i, w_{E,i}\right]$ (left) and on the difference between average estates at the top $-\frac{I_q}{W^{tot}}\bar{m}_{I_q}\left(\bar{w}_{I_q} - \bar{w}_{qN_E}\right)$ (right). The dashed and solid lines are linear fits after adding the mortality-wealth gradient and assuming only demographic multipliers, respectively (with $R^2 = 0.66$, $R^2 = 0.14$ in the left chart, respectively, and $R^2 = 0.02$, $R^2 = 0.39$ in the right chart, respectively).

mined by age, and above the age of 40, mortality rates tend to increase exponentially with age (see Appendix D). At the same time wealth is only weakly determined by age. Wealth increases with age, on average (Shorrocks, 1975; Modigliani, 1986). However, the variability of the age within different wealth groups is very large. To illustrate this point, Fig. 3 shows the lack of systematic correlation between age and wealth rank among the wealthiest individuals in the United States, as reported in the Forbes rich lists. Therefore, in practice, the covariance between multipliers and estates is very small and tends to be negative. The reason for the small covariance is the dominance of age in determining mortality. Using a sample of the richest decedents in France obtained from estate tax records, Fig. 4 shows a similar relationship.

Wealth can also lead to lower mortality rates that is over and above the effect of age on mortality. This is what has been suggested by Saez and Zucman (2016) and Saez and Zucman (2019) as an effect that has not been accounted for appropriately. However, based on our observations so far, the effect is likely to be small in practice. That has been suggested already by Daniels and Campion (1936). They describe two types of mortality multipliers – number multipliers, similar to the multipliers that depend only on age and gender, and estate multipliers, that take into account the effect the estate values have on mortality over and above that of the number multipliers. They argue that “the general estate multiplier will always be less than the general number multiplier.”

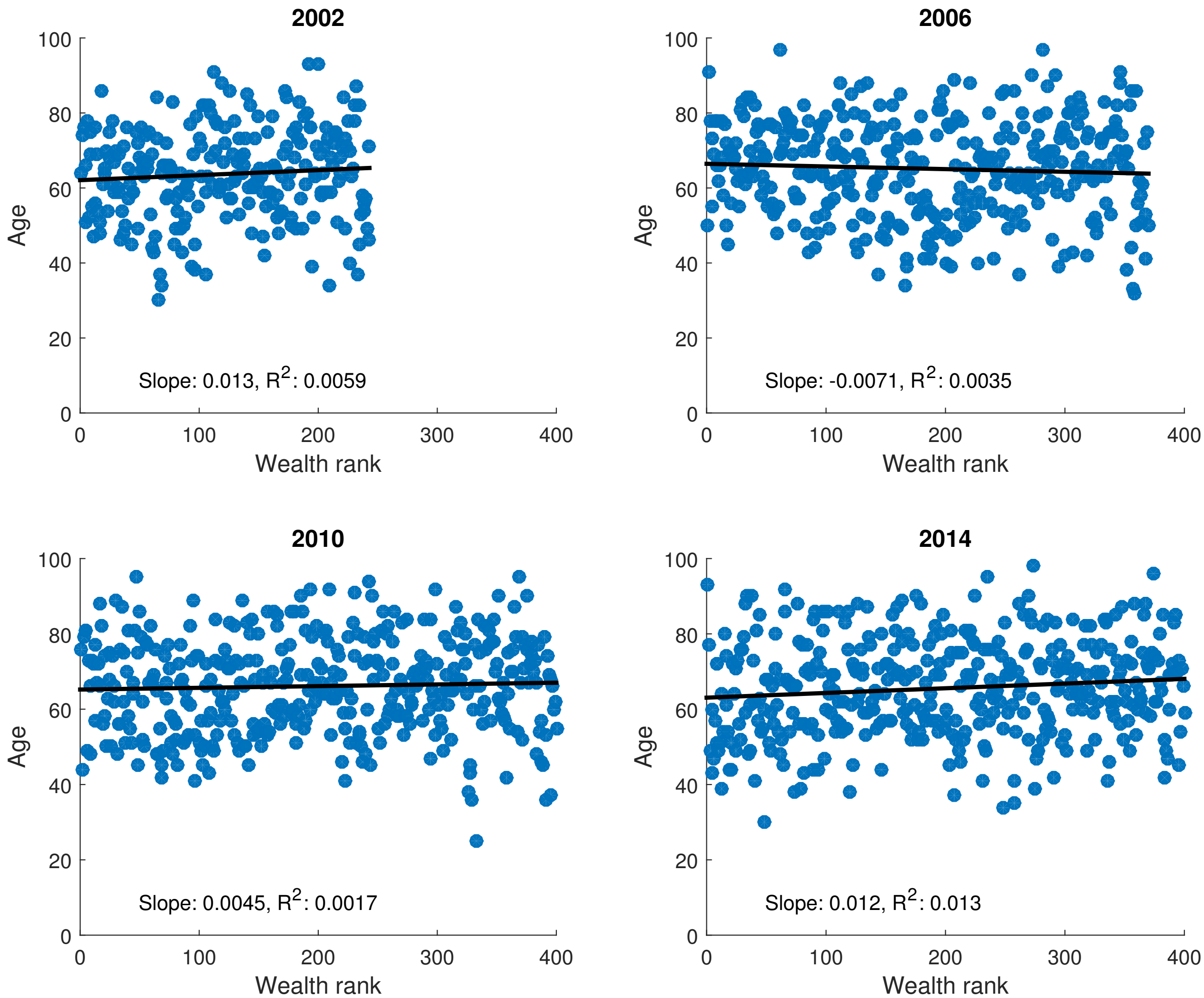

Figure 3: The age of Forbes listed individuals in the United States according to their wealth rank (1 – least wealthy). The slope and R^2 are given for a linear fit of the data in each year.

Nevertheless, the wealth effect on mortality can counterbalance the small negative correlation between between multipliers and estates at the top. Combined, the wealth and age effects on mortality may lead to correlation that is very close to 0. If, indeed, the decreasing mortality of wealthy individuals is not accounted for, the correlation would be underestimated. At the same time, decreasing mortality by wealth acts to increase the life expectancy of older, wealthy individuals. This, in turn, leads to the decrease of the covariance between multipliers and estates at the top. For these reasons a large positive covariance between estate and multipliers at the top, which will lead to large positive differences between the top wealth shares with and without disaggregated multipliers, is implausible.

Figure 5 demonstrates these observations using microdata for France. Based on the same mortality rate adjustment factors for wealth used by Garbinti, Goupille-Lebret and Piketty (2017), it is possible to account for the contribution of wealth to lower mortality over and above the effect of

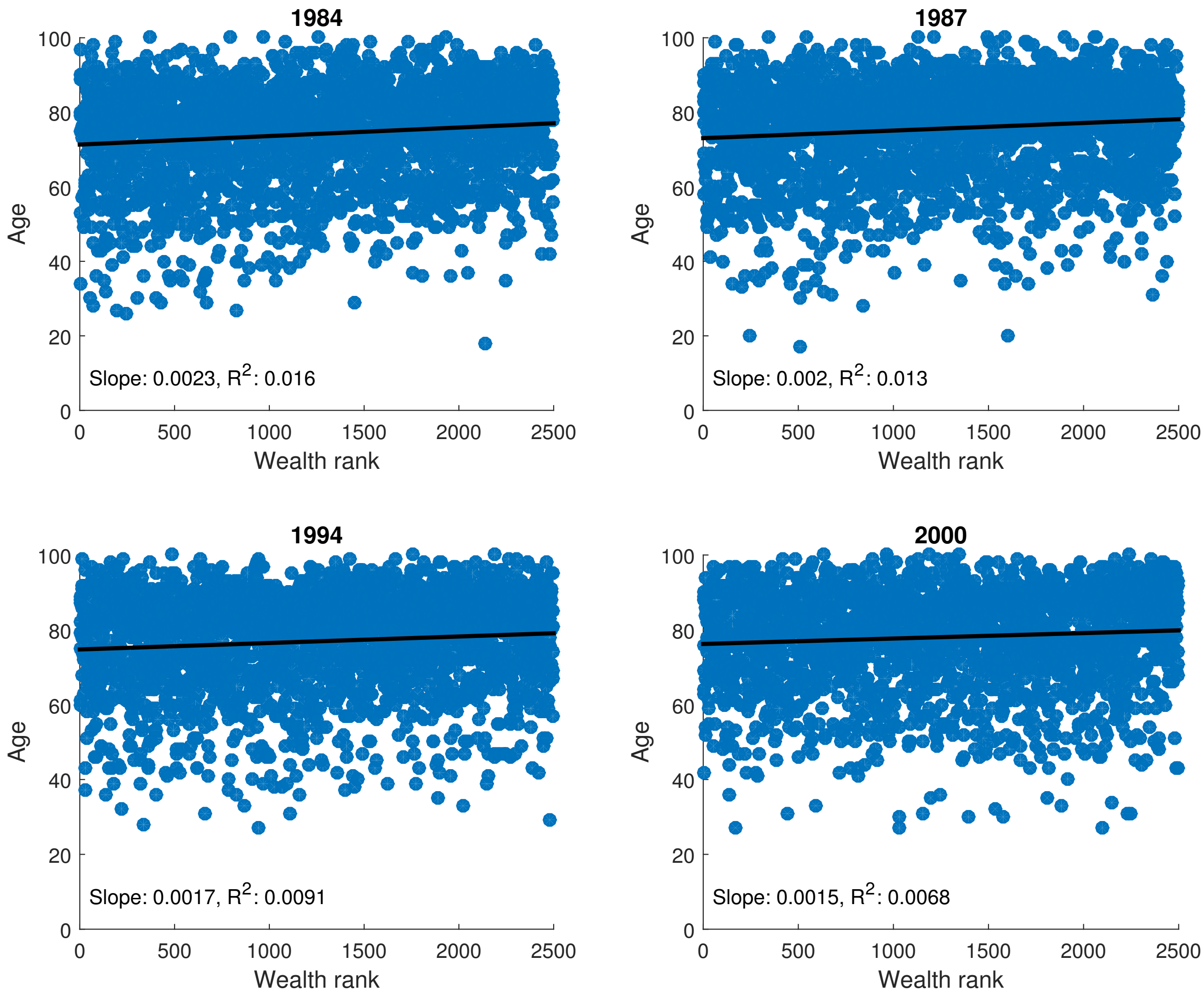

Figure 4: The age of a sample of the richest decedents in France according to their wealth rank (1 – least wealthy). The slope and R^2 are given for a linear fit of the data in each year.

age. In addition, Fig. 5 shows how mortality multipliers vary across age and wealth (taking also into account gender differences). It is hard to detect any signal in the dependence of multipliers on wealth, before or after adjustment by wealth group. Yet, there is a clear exponential dependence on age, where the differences by age and gender only create limited variability of multipliers.

Since the estate data for France are available in a small number of years, we apply the same mortality-wealth gradient described in Garbinti, Goupille-Lebret and Piketty (2017) to Italy, in which the data cover 20 years and 50%–60% of the decedent population. We consider three estimates of top wealth shares:

- Assuming the average multiplier for all decedents
- Assuming disaggregated multipliers by age and gender
- Assuming disaggregated multipliers by age and gender and adding a mortality-wealth gradient

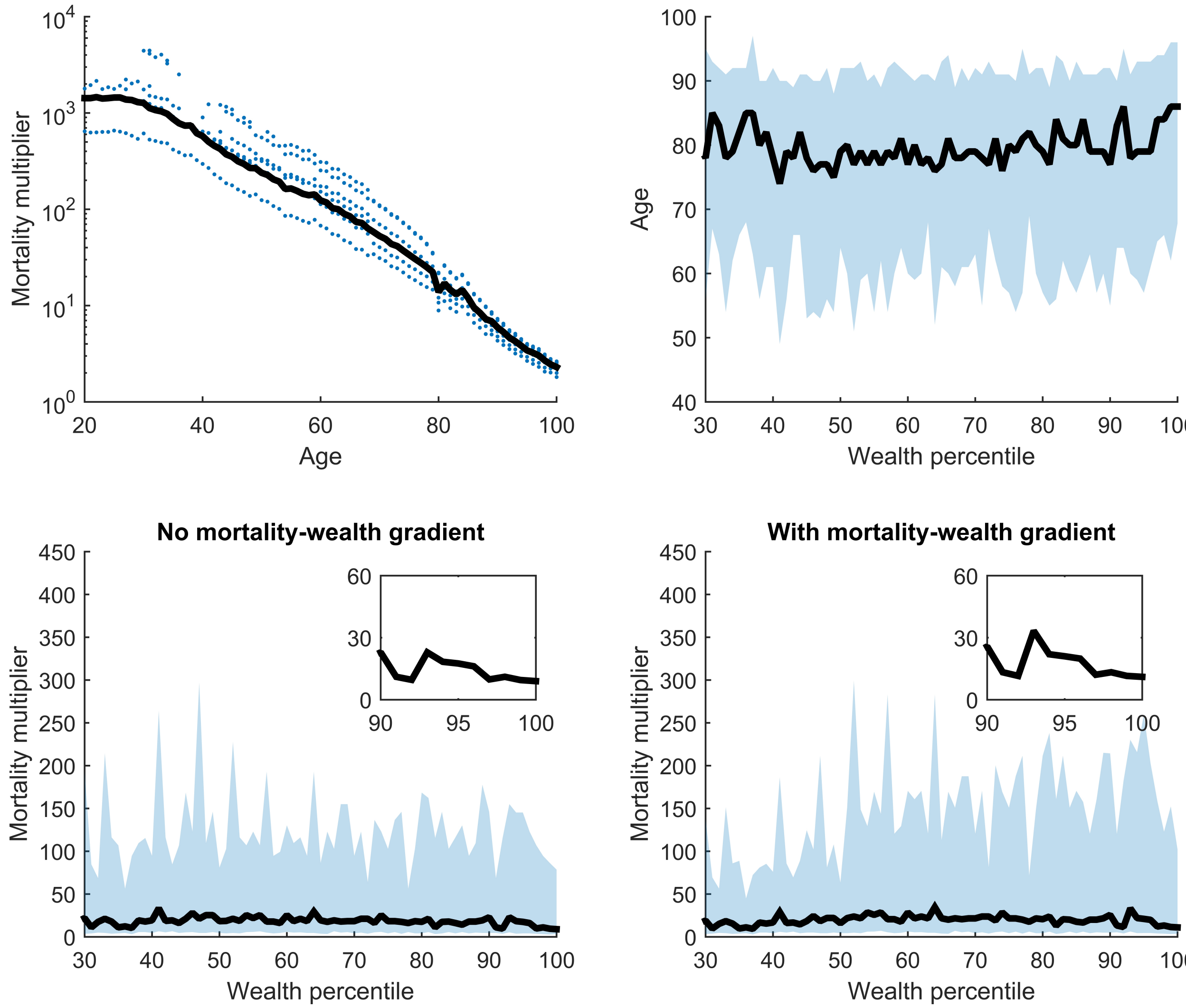

Figure 5: The mortality multipliers for France in 2000. Top left) The mortality multipliers by age according to death and population records (black). The blue dots are a scatter plot of the disaggregated mortality multipliers by age and gender and adjusted for wealth group according to Garbinti, Goupille-Lebret and Piketty (2017); Top right) The median age of decedents by wealth rank (black). The shaded blue area stands for 10%–90% confidence interval for each wealth rank; Bottom left) The median mortality multiplier of decedents by wealth rank (black), before adjusting for wealth group. The shaded blue area stands for 10%–90% confidence interval for each wealth rank; Bottom right) The median mortality multiplier of decedents by wealth rank (black), after adjusting for wealth group. The shaded blue area stands for 10%–90% confidence interval for each wealth rank.

(adjustment by wealth rank, age and gender following Garbinti, Goupille-Lebret and Piketty (2017))

The results are presented in Fig. 6. As expected, and as discussed above, the different top wealth share estimates tend to co-move. The results also show that a steep mortality-wealth gradient can create a large effect on the top wealth shares. We note that it is possible that the mortality-wealth gradient described in Garbinti, Goupille-Lebret and Piketty (2017) may not be representative of Italy, and is perhaps over-estimated. Yet, it demonstrates that large mortality-wealth gradients can create a sizable difference from top wealth shares estimated using only disaggregated demographic data.

At the same time, Fig. 6 demonstrates that the top wealth shares estimated using the average multiplier are very close to the results after adding the mortality-wealth gradient. As suggested above, the reason is that the gradient counteracts the effect of the disaggregated demographic multipliers by increasing the multipliers of the richest decedents. This leads to average multipliers at the top of the estate distribution that are close to the average multiplier in the overall population. We note that the choice in the average multiplier $\bar{m}$ as the fixed multiplier at the top of the estate distribution is only one possible choice. Appendix E describes different possible choices for the fixed mortality multiplier of the observed decedents and compares their values, and the top wealth shares they imply, also in comparison to the baseline estimates of top wealth shares in Italy.

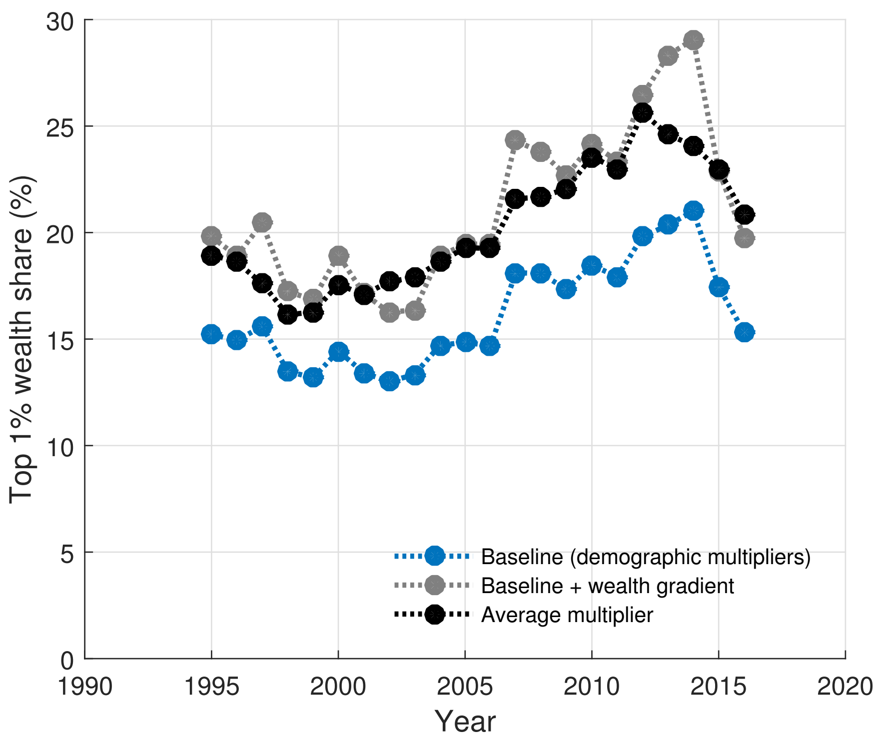

Figure 6: The top 1% wealth shares in Italy estimated using different multiplier choices – disaggregated demographic multipliers (blue); wealth-adjusted disaggregated demographic multipliers (gray); average multiplier (black).

Specifically, it is possible to show that the effect of the mortality-wealth gradient on the value of I_q, the number of decedents that will be needed in order to account for the top q quantile among the living is strong. This can explain, in part, the similarity between the top wealth shares estimated using the average multiplier and the results obtained after adding the mortality-wealth gradient in Fig. 6. Fig. 7 presents the evolution of various variables in Italy under different multiplier choices.

It demonstrates that the mortality-wealth gradient used might be too steep in the Italian case, as it implies $\mu < 1$ for almost the entire period. $\mu < 1$ is a very unlikely case, implying that the decedents are poorer, on average, than the living. This is possible, in theory, if the rich are very unlikely to die, but that is an extreme case, undocumented so far (see, for example, Alvaredo, Garbinti and Piketty (2017); Alvaredo, Atkinson and Morelli (2018)). In addition, these results also show that under including the mortality-wealth gradient implies that the identified wealth is higher than the total personal wealth of the living. This is possible if the unobserved population has negative net wealth, which is possible, if it consists of the poorest individuals. Yet, this is also a rather extreme case, which requires verifying the validity of the mortality-wealth gradient applied.

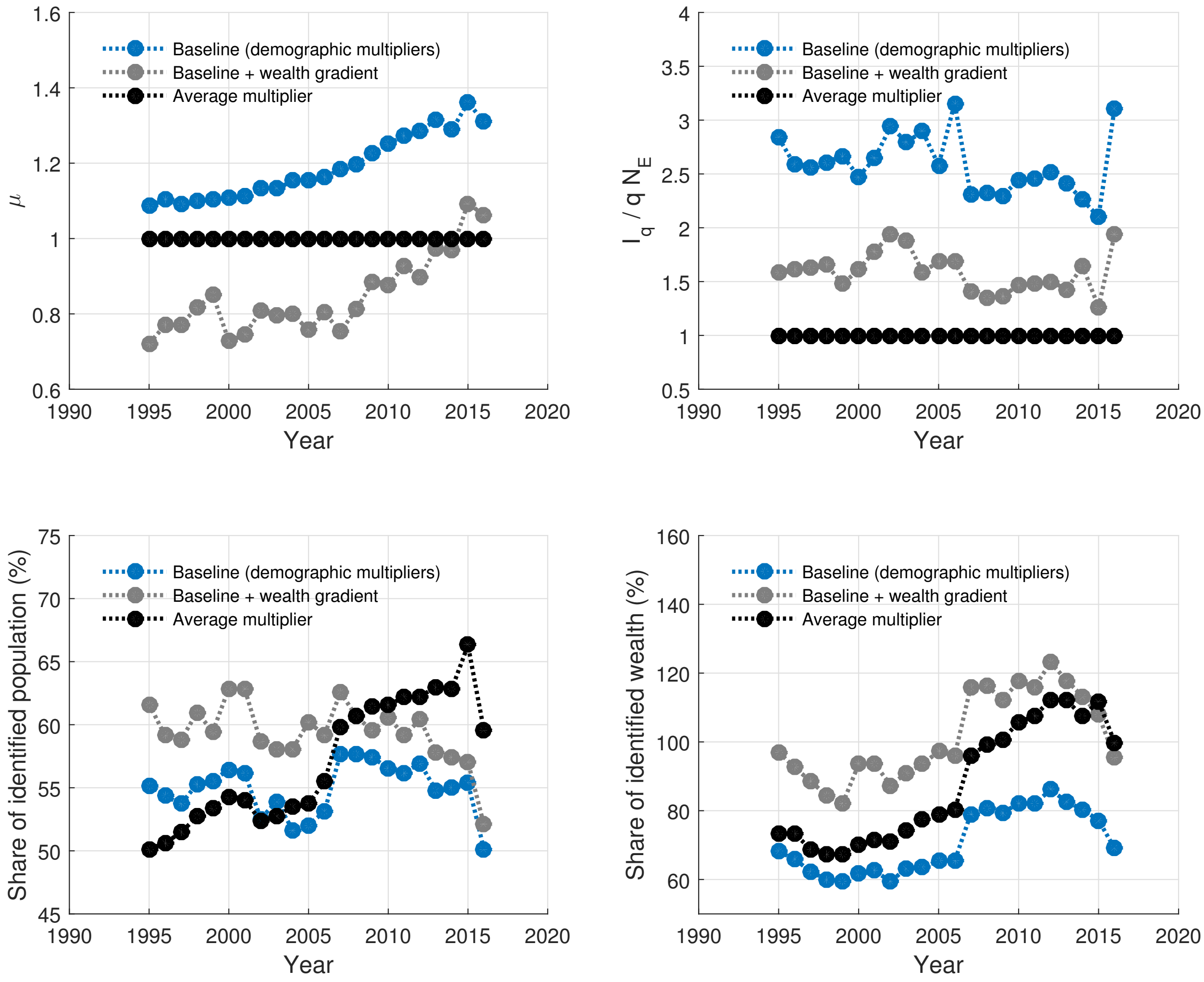

Figure 7: The evolution of various variables in Italy using different multiplier choices. Top left) μ; Top right) $\frac{I_q}{qN_E}$; Bottom left) The share of identified population from total population; Bottom right) The share of identified wealth from total personal wealth. Mortality data are taken from The Human Mortality Database (2018), and the estate tabulations and demographic data are taken from Acciari, Alvaredo and Morelli (2020). The mortality-wealth gradients used were those used for France in Garbinti, Goupille-Lebret and Piketty (2017).

6 New estimates of historical top wealth shares

The final step in our analysis is the application of the estate multiplier method in cases for which disaggregated mortality multipliers cannot be used. For example, historical estate tax tabulations for Belgium can be found in archives of its central statistics office. However, historical mortality data by age, gender, and other variables, may not be available. Similarly, the Japanese tax administration annually publishes estate tax return statistics since 1905.

We use these data to produce new series for the top 10%, 1%, and 0.1% wealth shares in Belgium and Japan. In both cases we apply a fixed mortality multiplier – the average multiplier in the population – to all observed decedents. We then apply the estate multiplier method in the two countries: in Belgium for the years 1937–1994 and in Japan for the 1970–2015. The results are presented in Fig. 8.

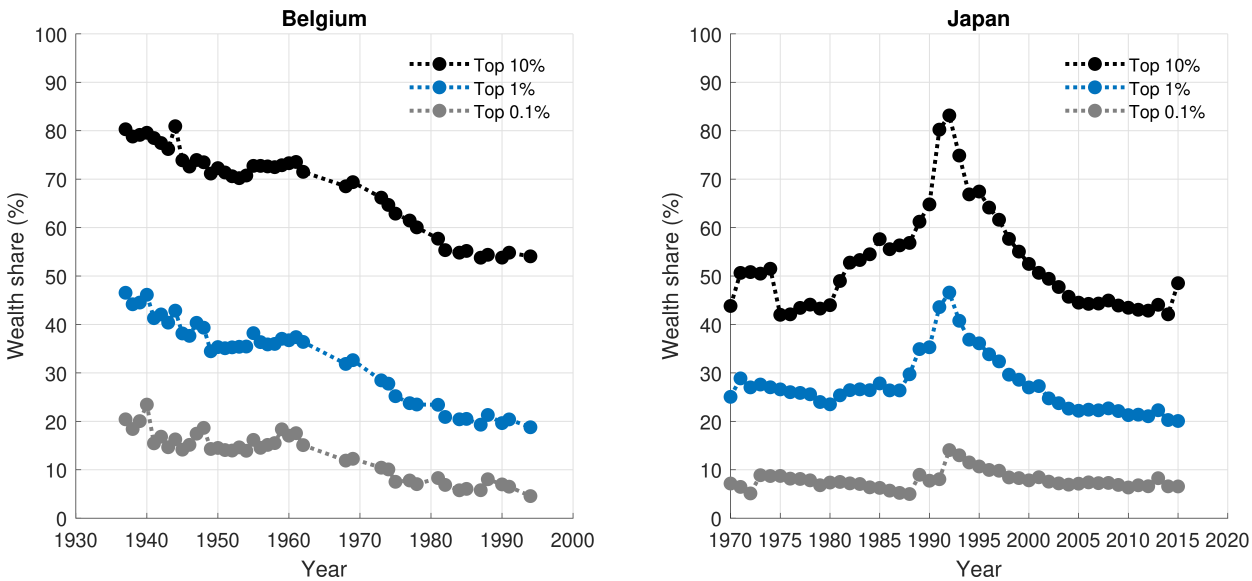

Figure 8: Top wealth shares in Belgium (left) and Japan (right). The results were obtained by applying the estate multiplier method with a fixed multiplier to all decedents. The data for Belgium are taken from the archives of the Belgian central statistical office, and the mortality data are taken from The Human Mortality Database (2018). The data for Japan are taken from the published records of the Japanese tax administration, and the mortality data are taken from The Human Mortality Database (2018). The analysis for Japan uses an external total personal wealth, taken from Piketty and Zucman (2014) (updated in The World Wealth and Income Database (2017)).

In Belgium, wealth inequality dramatically decreased during the course of the 20th century. The top 10% wealth share decreased from around 80% in the late 1930s to around 55% in the mid-1990s. Similar trends are found for the top 1% and 0.1%. These levels and trends closely follow those found in France (Garbinti, Goupille-Lebret and Piketty, 2017), which further improves our confidence in the relevance of the estimates.[4]

[4]We note that these estimates assume that decedents whose wealth is not reported in the estate tax data had no

In Japan wealth inequality rapidly increased between the late 1970s and the early 1990s, especially during the late 1980s. Inequality levels rapidly decreased in the years that followed. During the Japanese asset price bubble in the late 1980s, inequality rose to levels comparable to inequality levels that characterize the United States nowadays, with the top 0.1% share being above 10%. Yet, during the 2000s and 2010s, the top 10% wealth share in Japan was between 40% to 50%, *i.e.* low in comparison to most developed countries.

These results are clearly 'rough' estimates, and may be refined if more data were used. However, they demonstrate that in some cases, where estate tax is the only available data source, it is still possible to use the estate multiplier method to obtain reliable estimates of wealth inequality. This analysis is planned to be further extended to additional countries and years for which estate tax records can be used, even if without disaggregated mortality multipliers.

7 Conclusion

By clarifying the functioning of the estate multiplier methods and its structural limitations, this paper contributes to the evolving literature on wealth distribution estimation as well as on the important ongoing methodological debate surrounding the estate multiplier method itself. It also contributes to the analysis of historical trends in wealth inequality by applying the estate multiplier method to estate tax data in countries and years for which evidence on top wealth shares was very limited so far – Belgium since 1937 and Japan since 1970.

On the one hand, the validation of the empirical finding that top estate and wealth shares co-move and have similar levels (Alvaredo, Atkinson and Morelli, 2018) can be crucial for the expansion of severely sparse data series on wealth distribution, both across countries and over time. Indeed, in the case of the United Kingdom, the close relationship between estate distribution and wealth distribution provided a strong measurement benchmark in order to extend the wealth concentration series back in time to 1895, and to fill in missing years. Similarly, construction of long series can become possible in other countries when the relevant information for the application of the estate multiplier (*i.e.* detailed estate tabulations or detailed mortality rates) method cannot be retrieved.

On the other hand, the answers to the main questions raised by this paper are crucial for the reliability of the estate multiplier method. The estate multiplier method is one of the few viable benchmark methods to estimate wealth concentration, also in a historical perspective. This is important as the use of different methodologies and sources of data for the estimation of wealth distribution remains essential for illuminating the limitations of each source of data and methodology, and to inform us about the levels and trends of wealth concentration. Moreover, and as a

wealth at the time of their death. This is important because the total personal wealth in Belgium is not available from an external source, so an assumption on the wealth of the unobserved decedent population is essential for the application of the estate multiplier method. Robustness tests assuming that the unobserved decedents had wealth that is equal to 50% of the poorest reported decedent, and to 10% of the average reported estate, had only a negligible impact on our estimates.

matter of fact, the estate multiplier method is often the only one available to yield estimates of wealth distribution and concentration for specific countries or time periods.

We specifically discuss the relevance of unobserved heterogeneity in mortality rates, such as a the potential wealth effect on mortality that is operating over and above the effect of demographic characteristics. Accounting for a mortality-wealth gradient would create a more accurate picture of mortality multipliers and hence lead to a more realistic estimation of top wealth shares. We find that the difference between the top wealth shares obtained with or without mortality-wealth gradients cannot be large under realistic assumptions and given the observed regularities of the interrelation between the wealth distribution and demographic characteristics. While the mortality-wealth gradient can be steep for younger age groups, it is not as steep for older age groups as economic status does not counterbalance the biological limitations to human longevity. Therefore, adjusting the multipliers at the top of the distribution and taking into account the mortality-wealth gradient is muted by the fact that relatively older people are more represented among the richest decedents. Also, within the top of the estate distribution, there is only a weak dependence of age on wealth rank. As a result, the multipliers at the top may continue to be poorly correlated with wealth ranks, and may continue to be close to the average multiplier of the overall population.

This leads to the important finding that taking into account both demographic multipliers and mortality-wealth gradient yields very similar top wealth shares to those obtained using the average multiplier. Although individuals at the top of the estate distribution have higher mortality rates as they are relatively older on average, this is counterbalanced by their higher economic status, which may lead to healthier lives and better medical care, reducing their probability to die, other things being equal. As a result, the differences between the mortality multipliers at the top of the estate distribution to the average mortality multiplier of the entire decedent population are small enough to create only a limited discrepancy between the two top wealth shares estimated with refined multipliers and the average multiplier. These results are of particular relevance for the estimation of historical series of wealth concentration. They allow using a wide array of aggregate estate tabulations that were previously thought to be unreliable and unusable.

We also present new estimates for the top 10%, 1%, and 0.1% wealth shares in Belgium (between 1937 and 1994) and in Japan (between 1970 and 2015), based on estate tax tabulations and only aggregate mortality data. We find that wealth inequality in Belgium decreased dramatically during the course of the 20th century. The top 10% wealth share decreased from around 80% in the late 1930s to around 55% in the mid-1990s. Similar trends are found for the top 1% and 0.1%. We find that wealth inequality rapidly increased in Japan between the late 1970s and the early 1990s, especially during the late 1980s. Inequality levels rapidly decreased since. During the Japanese asset price bubble in the late 1980s, inequality rose to levels comparable to US 2010s inequality levels, with the top 0.1% share being above 10%. Yet, during the 2000s and 2010s, the top 10% wealth share in Japan was between 40% to 50%, low in comparison to most developed countries.

We end with a practical important remark. Information about the wealth gradient of mortality rates is scarce, and we only know little about how this gradient has evolved over time. In very few cases, such as the United States during the last several decades, we have some information about the income gradient of mortality and its trend. Hence, in practice, the application of a mortality-wealth gradient is surrounded with considerable uncertainty. Thus, applying such gradients may not necessarily be satisfactory. They may also create a problem with the total wealth recovered, which, as explained, can be above the total personal wealth if the gradient applied is too steep. At the same time, applying an average multiplier to the entire decedent population, as we suggest, can also create a similar problem. For these reasons we highlight the need to be careful and transparent when using the estate multiplier method, and making use of as much data as possible for consistency. Applying the population average multiplier to all decedents may indeed provide reliable estimates of top wealth shares, especially in a historical context. Yet, they still need to be taken with the necessary caution.

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A Nomenclature

W^{tot} – total wealth of the entire population

W^E – total wealth of decedents

N – total population size

N_E – total number of estates in sample

i – index of decedents (between 1 and N_E)

$w_{E,i}$ – the estate of decedent i

p_i – the probability of death (mortality rate) of decedent i

m_i – the estate multiplier corresponding to decedent i ($m_i = 1/p_i$)

q – the top quantile corresponding to the wealth share of interest (*e.g.* $q = 0.01$ for the top 1%)

I_q – number of decedents that will be needed in order to account for the top q quantile among the living

$\bar{m}$ – the average estate multiplier ($= N/N_E$)

$\bar{m}_{I_q}$ – the average multiplier at the top of the estate distribution ($\Sigma_{i=1}^{I_q} m_i/I_q$)

μ – the ratio of average estate to average wealth

$(1 - L_q)^E$ – the estate share of quantile q

$(1 - L_q)^W$ – the wealth share of quantile q

B The concentration of estates at the top

As shown by Alvaredo, Atkinson and Morelli (2018), relying on unadjusted tax data on estate value can also be informative about the concentration of wealth at the top. To show that we first need to define the top estate share of quantile q

$$(1-L_q)^E = \frac{\sum_{i=1}^{qN_E} w_{E,i}^{tax}}{W^E} \,. \tag{B.1}$$

This requires summing the estates of the richest qN_E decedents and the estimation of the total value of estates of the full decedent population, W_E. However, the estimation of the latter is not a trivial exercise. It requires the estimation of the total value of unobserved estate of the deceased excluded from the tax records, W_E^{exc}. This creates uncertainty in the top estate share estimates.

In practice, estimating W_E^{exc} can be done using the total wealth of the living population not represented by the re-weighted tax records (excluded population), $N^{exc} = N - \sum_{i=1}^{N_E} m_i$. The latter can be directly estimated from external sources of data, such as surveys or other administrative records, if the general identity of the excluded population could be inferred.

The total identified wealth is known through the multipliers and observed estate values:

$$W^{iden} = \sum_{i=1}^{N_E^{tax}} m_i w_{E,i}^{tax} \,. \tag{B.2}$$

In the absence of disaggregated multipliers this becomes

$$W^{iden} = \sum_{i=1}^{N_E^{tax}} \bar{m} w_{E,i}^{tax} = \bar{m} W_E^{iden} \,. \tag{B.3}$$

The total excluded wealth is then

$$W^{exc} = W^{tot} - W^{iden} \,. \tag{B.4}$$

At the same time

$$W^{exc} = \bar{m}^{exc} W_E^{exc} \,, \tag{B.5}$$

where $\bar{m}^{exc}$ is the average multiplier of the excluded decedents. $\bar{m}^{exc}$ can be estimated depending on how refined are the demographic data and mortality data available. If mortality by age and gender is available, it is possible to define a different multiplier for the excluded decedents in each age and gender:

$$m_{a,g}^{exc} = \frac{N_{a,g}^{exc}}{N_{E,a,g}^{exc}} \,, \tag{B.6}$$

where $N_{a,g}^{exc}$ is the number of living with age a and gender g not observed by the tax records, and

$N^{exc}_{E,a,g}$ is the number of decedents with age a and gender g not observed by the tax records. In this case $\bar{m}^{exc}$ would be the average of all multipliers $m^{exc}_{a,g}$. Alternatively, in the absence of such data, $\bar{m}^{exc}$ can be defined as the ratio between the excluded living population and the excluded decedent population:

$$\bar{m}^{exc} = \frac{N^{exc}}{N_E - N_E^{tax}} \,. \tag{B.7}$$

It is clear that different sets of multipliers would lead to different estimates of W_E^{exc}. This leads to different total value of estates, which, in turn, leads to different top estate share estimates. In Sec. 3 we use this calculation to provide different estimates of top estate shares and compare them to top wealth shares reported in the literature.

C The coefficient of variation of estates and of wealths

A similar derivation can be used to compare the coefficient of variation (CV) of the wealth distribution with and without multipliers. The derivation is inspired by a derivation presented in Atkinson and Harrison (1978), comparing the CV between capital income and wealth, for the capitalization method.

The coefficient of variation of estates, denoted Y_E, follows

$$Y_E^2 = \frac{\sigma_E^2}{\bar{w}_E^2} \,. \tag{C.1}$$

The coefficient of variation of wealths, denoted Y_E, follows

$$Y_W^2 = \frac{\sigma_W^2}{\bar{w}_W^2} \,, \tag{C.2}$$

where σ_E^2 is the variance of estates, σ_W^2 is the variance of wealths, $\bar{w}_E$ is the average estate, and $\bar{w}_W$ is the average wealth.

We begin by writing down expressions for the variance estates and wealths:

$$\sigma_E^2 \quad = \tfrac{1}{N_E} \textstyle\sum_{i=1}^{N_E} w_{E,i}^2 - \tfrac{1}{N_E^2} \left(\textstyle\sum_{i=1}^{N_E} w_{E,i} \right)^2 \,; \tag{C.3}$$

$$\sigma_W^2 \quad = \tfrac{1}{N} \textstyle\sum_{i=1}^{N_E} m_i w_{E,i}^2 - \tfrac{1}{N^2} \left(\textstyle\sum_{i=1}^{N_E} m_i w_{E,i} \right)^2 \,. \tag{C.4}$$

Therefore we get

$$Y_E^2 = \frac{\tfrac{1}{N_E} \sum_{i=1}^{N_E} w_{E,i}^2 - \tfrac{1}{N_E^2} \left(\sum_{i=1}^{N_E} w_{E,i} \right)^2}{\tfrac{1}{N_E^2} \left(\sum_{i=1}^{N_E} w_{E,i} \right)^2} \,, \tag{C.5}$$

and

$$Y_W^2 = \frac{\tfrac{1}{N} \sum_{i=1}^{N_E} m_i w_{E,i}^2 - \tfrac{1}{N^2} \left(\sum_{i=1}^{N_E} m_i w_{E,i} \right)^2}{\tfrac{1}{N^2} \left(\sum_{i=1}^{N_E} m_i w_{E,i} \right)^2} \,. \tag{C.6}$$

μ is the ratio between the average estate and the average wealth

$$\mu = \frac{\tfrac{1}{N_E} \sum_{i=1}^{N_E} w_{E,i}}{\tfrac{1}{N} \sum_{i=1}^{N_E} m_i w_{E,i}} = \bar{m} \frac{\sum_{i=1}^{N_E} w_{E,i}}{\sum_{i=1}^{N_E} m_i w_{E,i}} \,, \tag{C.7}$$

so

$$\frac{1}{N^2} \left(\sum_{i=1}^{N_E} m_i w_{E,i} \right)^2 = \frac{1}{\mu^2} \cdot \frac{1}{N_E^2} \left(\sum_{i=1}^{N_E} w_{E,i} \right)^2 \,, \tag{C.8}$$

and therefore

$$Y_W^2 = \frac{\frac{1}{N}\sum_{i=1}^{N_E} \mu^2 m_i w_{E,i}^2 - \frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2}{\frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2} . \tag{C.9}$$

We can then rearrange Y_W^2 and get

$$Y_W^2 = Y_E^2 - \frac{\frac{1}{N_E}\sum_{i=1}^{N_E} w_{E,i}^2}{\frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2} + \frac{\frac{1}{N}\sum_{i=1}^{N_E} \mu^2 m_i w_{E,i}^2}{\frac{1}{N_E^2}\left(\sum_{i=1}^{N_E} w_{E,i}\right)^2} . \tag{C.10}$$

Taking $N = \bar{m} N_E$ we get

$$Y_W^2 = Y_E^2 \left(1 + \frac{\frac{1}{N_E}\sum_{i=1}^{N_E}\left(\frac{\mu^2 m_i}{\bar{m}} - 1\right) w_{E,i}^2}{\sigma_E^2}\right) . \tag{C.11}$$

We can now use this result to further study the conditions for which similarity between wealth and estate inequalities is to be expected (see Sec. 4.1).

D Mortality rates by age

Age is the most important statistical determinant of mortality. Fig. 9 shows the mortality rates in France, Italy, the United Kingdom, and the United States in 1950, 1970, 1990, and 2010, based on The Human Mortality Database (2018) data. It illustrates that mortality rates increase exponentially with age above the age of 40.

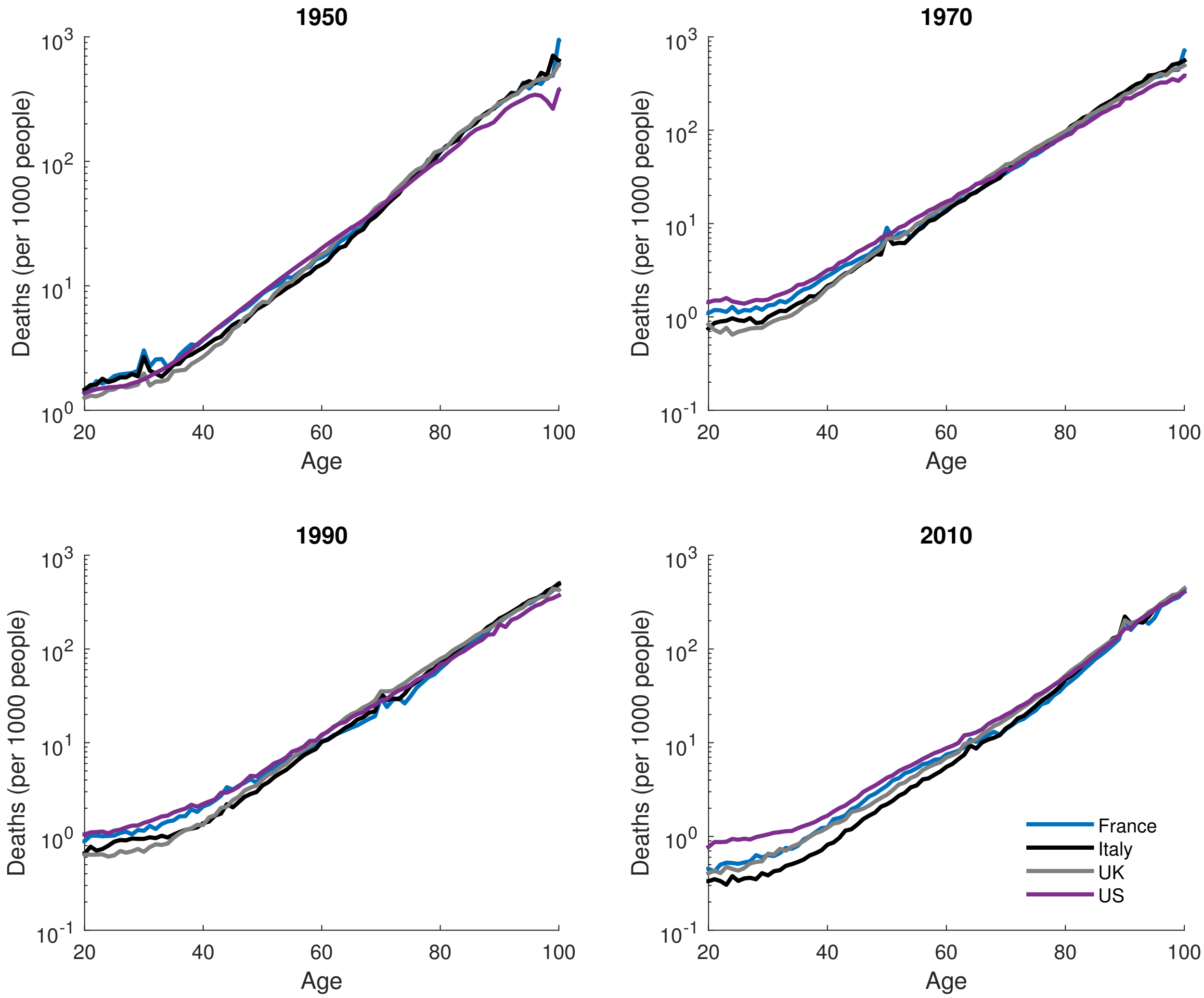

Figure 9: Mortality rates in France, Italy, the United Kingdom, and the United States in 1950, 1970, 1990, and 2010. Source: The Human Mortality Database (2018).

E Fixed mortality multipliers in Italy

In theory, in the absence of detailed demographic data for the decedents included in the estate tax records, it is possible to use an approximation for their mortality multiplier in order to use the estate multiplier method for obtaining an estimate of top wealth shares. Even when demographic data are available, fixed multipliers can simplify the estimation process. We list below several choices of such a fixed multiplier and present the differences between them and the different resulting top wealth shares:

- m_1: A simple and natural choice of such a multiplier is the average multiplier $\bar{m}$, which is the ratio between the population size of the living and the dead. Considering such an multiplier makes an implicit assumption that the mortality rate of the observed decedents is similar to that of the unobserved decedents.

- m_2: If detailed demographic data are available, it is possible to take the arithmetic average of the disaggregated individual multiplier m_i. m_2 is expected to be lower than m_1, since the average multiplier among the observed decedents tends to be lower than the average multiplier, however this is not always the case.

- m_3: m_2 changes the identified wealth compared to the case in which the disaggregated individual multiplier m_i are considered, because $\Sigma_i^{N_E} m_i w_{E,i} \neq \frac{\Sigma_i^{N_E} m_i}{N_E} \Sigma_i^{N_E} w_{E,i}$. Another possible choice of fixed multiplier would a multiplier which will be consistent with the identified wealth – $m_3 = \frac{\Sigma_i^{N_E} m_i w_{E,i}}{\Sigma_i^{N_E} w_{E,i}}$.

- m_4: If no demographic data are available, but mortality data are, it is possible to assume that the representative multiplier of the observed decedents is the multiplier that corresponds to an individual whose age is the average age at death, based on the mortality data. Typically, since this age is higher than the average age of decedents in the tax records, this multiplier will be substantially lower than the other choices of multiplier.

- m_5: The same m_2 but after adding a mortality-wealth gradient to the demographic data for obtaining disaggregated individual multipliers.

The evolution of the fixed multipliers in time and the resulting top 1% wealth shares are presented in Fig. 10.

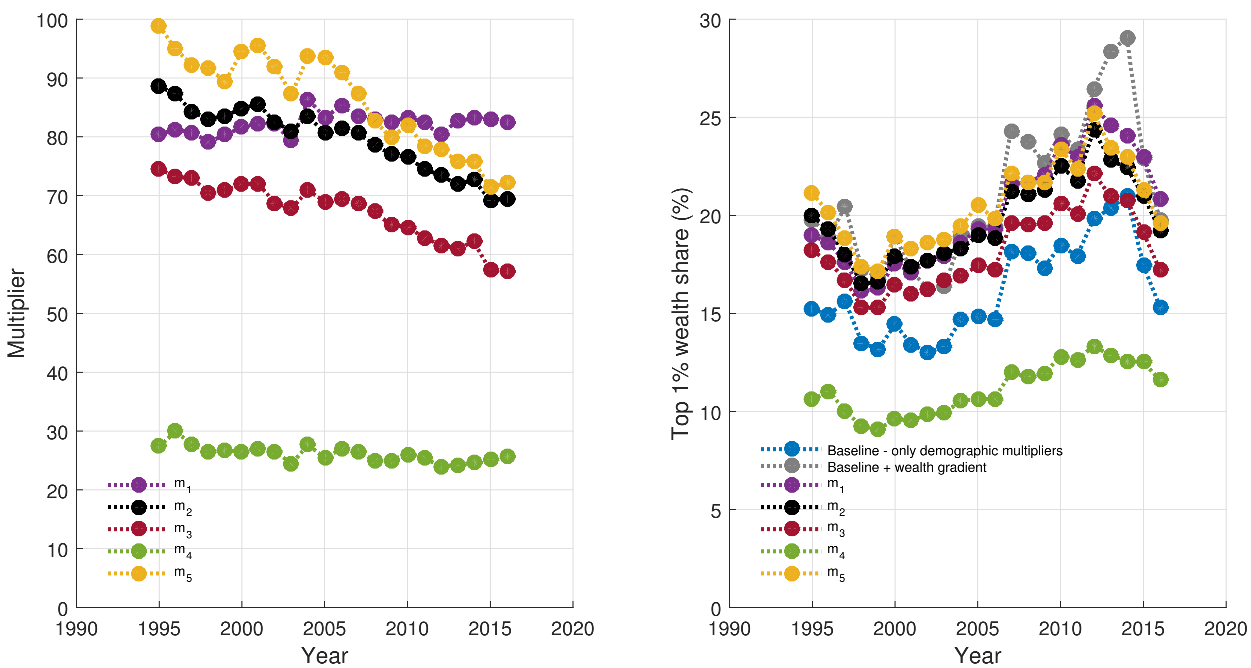

Figure 10: Fixed mortality multipliers in Italy 1995–2016. Mortality data were taken from The Human Mortality Database (2018), and the estate tabulations and demographic data were taken from Acciari, Alvaredo and Morelli (2020). The mortality-wealth gradients used were those used for France in Garbinti, Goupille-Lebret and Piketty (2017).