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Volume Author/Editor: Mervyn A. King and Don Fullerton, eds.

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Chapter Author: Mervyn A. King, Don Fullerton

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Appendix D
Technical Aspects of the United States Tax System

This appendix describes a procedure to take A_z, the present value of depreciation allowances on a dollar of investment, and average it over different assets. Capital stocks by themselves do not provide correct weights, because short-lived assets require relatively more reinvestment that also qualifies for depreciation deductions. In particular, a dollar of asset type j can be maintained in real terms by reinvesting, at time u, a nominal amount equal to $\delta_j e^{\pi u}$, where δ_j is the jth asset's economic depreciation rate and π is the inflation rate. Each dollar of that reinvestment receives A_{zj} of depreciation allowances in present value terms at time u. Discounting those nominal amounts by ρ, the nominal discount rate, provides

$$
\begin{aligned}
PV(A_{zj}) &= A_{zj} + A_{zj} \int_0^\infty \delta_j \, e^{\pi u} e^{-\rho u} du \\
&= A_{zj}\left[1 + \frac{\delta_j}{\rho - \pi}\right] \qquad \text{(D.1)}
\end{aligned}
$$

as the present value of depreciation on a maintained real dollar of capital. Note that $[1 + \delta_j/(\rho - \pi)]$ is the present value of expenditures necessary to keep a real dollar of capital stock. These present values can be averaged over the twenty equipment types or fourteen structure types, weighting by capital stock in each type, K_j, to get the present value of depreciation for an aggregate asset in each industry:

$$\overline{PV}(A_z) = \sum_{j=1}^{N} K_j \, PV(A_{zj}) / \sum_{j=1}^{N} K_j, \qquad \text{(D.2)}$$

where N is twenty for equipment and fourteen for structures (industry subscripts are suppressed for simplicity, but K is actually a thirty-four-by-three matrix). We can now ask the following question: Suppose a maintained real dollar of a single aggregate asset (either equipment or structures) had an aggregate economic depreciation rate $\bar{\delta}$ and was allowed exponential tax depreciation at rate $\bar{a}$. What rate $\bar{a}$ would provide the same present value of allowances as (D.2) above?

The present value of depreciation for a current dollar of investment in this aggregate asset is

$$\text{(D.3)} \qquad \overline{A_z} = \int_0^\infty \bar{a}e^{-\bar{a}u}\, e^{-\rho u}\, du = \frac{\bar{a}}{\bar{a}+\rho}.$$

The present value of depreciation for a maintained real dollar of this aggregate asset is

$$\text{(D.4)} \qquad PV(\overline{A_z}) = \frac{\bar{a}}{\bar{a}+\rho}\left[1 + \frac{\bar{\delta}}{\rho - \pi}\right].$$

Setting (D.4) equal to (D.2) implies that

$$\text{(D.5)} \qquad \overline{A_z} = \frac{\bar{a}}{\bar{a}+\rho} = \frac{\sum_{j=1}^{N} K_j A_{zj}\left[1 + \frac{\delta_j}{\rho - \pi}\right]}{\sum_{j=1}^{N} K_j\left[1 + \frac{\bar{\delta}}{\rho - \pi}\right]}.$$

The right-hand side of (D.5) is used to calculate $\overline{A_z}$ from the disaggregate A_{zj} described in the text. Since $\bar{\delta}$, derived in section 6.2.4, is just the capital-weighted average of δ_j, the denominator of equation (D.5) can be written as $\Sigma K_j[1 + \delta_j/(\rho - \pi)]$. Thus the desired aggregate present value $\overline{A_z}$ *is* just a weighted average of the A_{zj}, but the weights are $K_j[1 + \delta_j/(\rho - \pi)]$ rather than K_j. These weights can be interpreted as capital plus the present value of economic depreciation. Assets that depreciate faster than the average are given more weight because they will require more than the average amount of reinvestment receiving A_{zj} in future years. Or, if depreciation rates δ_j were the same for all assets, then A_z would be a simple capital-weighted average of the A_{zj}.

Now consider the averaging of g_j, the rate of grant for the jth asset. For the same permanent one-dollar increase in the stock of this asset, nominal replacement investment at time u is $\delta_j e^{\pi u}$. These nominal amounts receive credits at rate g_j and can be discounted by the nominal rate ρ to obtain $PV(g_j)$, the present value of credits for the original investment as well as for the subsequent reinvestment:

$$\text{(D.6)} \qquad \begin{aligned} PV(g_j) &= g_j + g_j \int_0^\infty \delta_j\, e^{(\pi-\rho)u}\, du \\ &= g_j\left[1 + \frac{\delta_j}{\rho - \pi}\right]. \end{aligned}$$

These present values can be weighted by K_j, the stock of capital in the jth asset, to obtain,

$$\overline{PV}(g) = \sum_{j=1}^{N} K_j \, PV(g_j) / \sum_{j=1}^{N} K_j, \tag{D.7}$$

where N is twenty for equipment and fourteen for structures. If an aggregate asset of all equipment or of all structures were to depreciate at rate $\bar{\delta}$ and receive a credit at rate $\bar{g}$, then the present value of its credits would be:

$$\begin{aligned} PV(\bar{g}) &= \bar{g} + \bar{g} \int_0^{\infty} \bar{\delta} e^{(\pi - \rho)u} \, du \\ &= \bar{g}\left[1 + \frac{\bar{\delta}}{\rho - \pi}\right]. \end{aligned} \tag{D.8}$$

Set (D.7) equal to (D.8) and solve for $\bar{g}$ as:

$$\bar{g} = \frac{\sum K_j g_j \left[1 + \dfrac{\delta_j}{\rho - \pi}\right]}{\sum K_j \left[1 + \dfrac{\bar{\delta}}{\rho - \pi}\right]}. \tag{D.9}$$

Since $\bar{\delta}$ is the capital-weighted average of δ_j, equation (D.9) just weights g_j by $K_j[1 + \delta_j/(\rho - \pi)]$, the capital stock plus present value of actual depreciation.