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# Income-Distribution Concerns In Regulatory Policymaking 

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This paper presents and applies new methodological tools for relating concerns with income distribution to regulatory policymaking. The tools are based on the concept of social-welfare dominance, according to which a policy is said to social-welfare-dominate another if it is preferred by all social decisionmakers who subscribe to a certain set of compelling axioms. We present a variety of necessary and sufficient conditions for socialwelfare dominance of regulatory policies-conditions that can be tested by means of market data. We demonstrate the feasibility of this new approach to normative analysis by means of pilot studies of some policies toward electric-utility and telephone-service pricing.

Changes in regulatory policies generally benefit some economic agents, while they hurt others. The standard approach to evaluating such policies entails aggregating costs and benefits over all agents and using the algebraic sum as the decision criterion. This methodology views as equivalent dollars of net benefit, irrespective of to whom they accrue. It can only be justified by appeal to costless lump-sum transfers or to social indifference about the distribution of income. In the absence of these unrealistic conditions, state-of-the-art normative analysis conceptualizes possibly different welfare weights applied to the net benefits of different agents. The first section of the article sketches this theoretical development and argues that the numerical values of such welfare weights are arbitrary and can be critical for policy assessment.

Consequently, in the second section we develop qualitative relationships among the welfare weights that are equivalent to three axioms: the Pareto principle, anonymity, and the undesirability of regressive transfers of real income. These axioms underlie the social-welfaredominance relation between policies.

We show that a policy change is social-welfare-dominant if and only if a series of observable conditions hold. These conditions include the Hicksian dictum that the total net benefits of the change be positive, as well as the Rawlsian dictum that the poorest member of society be made better off. In addition, social-welfare dominance requires the positivity of the net benefits of each group formed by starting from the bottom and working up the scale of real income to any level.

This theory offers a concept of welfare improvement that is far more general than Pareto optimality. Social-welfare-dominating policies can have losers as well as gainers. A policy is social-welfare-dominant as long as losses are counterbalanced by gains to those who are poorer in real income. Such policies can be formed through political linkages between efficient and inefficient components. Such pervasive packaging of regulatory policies makes little sense when analyzed within the usual optimization framework.

The third section presents several additional sets of empirically useful conditions for social-welfare dominance of policies. For example, if the aggregate benefit-cost ratio exceeds unity and if the class-specific benefitcost ratios decline with income, then the policy change is social-welfaredominant. First-order necessary conditions for local non-social-welfare dominance are developed.

In the fourth section we discuss philosophical stances from which social-welfare-dominance tools can be utilized in normative and positive studies. The fifth and sixth sections detail pilot applications to seasonal peak-load electricity pricing and to the Ramsey pricing of long-distance telephone services.

## Social-Welfare Functions

The current state of the art, by our reading, incorporates income-distribution concerns into normative analysis by means of a Samuelson-Bergson social-welfare function (Samuelson 1963). Policy-setting power is viewed as vested in the hands of a social decisionmaker (SDM). It is assumed that the SDM acts as if he has a complete and continuous preference ordering over social states that obeys the Pareto principle. That is, the SDM respects individual preferences, and views as desirable any change that improves the lots of some people (by their own values) while leaving all others indifferent.

For the sake of concreteness, we limit the discussion to social states that differ only in static, certain allocations of private commodities among neoclassical consumers. Then, social states are represented by vectors
$\left(x_{11}, \ldots, x_{1 m}, x_{21}, \ldots, x_{2 m}, \ldots, x_{n 1}, \ldots, x_{n m}\right) \in \mathbf{R}_{+}^{n m}$,
where $x_{i j}$ is the consumption of commodity $j$ by consumer $i$. The SDM has a complete and continuous preference ordering over $\mathbf{R}_{+}^{n m}$ that is represent-
able by the ordinal real-valued function $V(x)$, where, for $x^{\prime}, x^{\prime \prime} \in \mathbf{R}_{+}^{n m}, x^{\prime}$ is preferred to $x^{\prime \prime}$ if and only if $V\left(\mathrm{x}^{\prime}\right)>V\left(x^{\prime \prime}\right)$.

If one assumes that each consumer is indifferent about the consumption of others (no extended sympathy), the $i$ th neoclassical consumer's preferences over his own consumptions can be represented by an ordinal, increasing real-valued utility function, $U^{i}\left(x_{i 1}, \ldots, x_{i m}\right)$. In accordance with the Pareto principle, the SDM's conditional preference ordering over vectors ( $x_{i 1}, \ldots, x_{i m}$ ), with the consumptions of other consumers held constant, must coincide with the $i$ th consumer's preference ordering and must be independent of the consumptions of others. It follows (Leontief 1947; Debreu 1960) that the variables $x_{i 1}, \ldots, x_{i m}$ are separable in $V(\cdot)$ from its other arguments, that $U^{i}\left(x_{i 1}, \ldots, x_{i m}\right)$ can serve as an aggregator of $x_{i 1}, \ldots$, $x_{i m}$ in $V(\cdot)$, and that, consequently, for $\phi(\cdot)$ which is some real-valued function over $\mathbf{R}^{n}$,

$$
\begin{equation*}
V\left(x_{i 1}, \ldots, x_{1 m}, x_{n 1}, \ldots, x_{n m}\right) \equiv \phi\left(U^{1}\left(x_{11}, \ldots, x_{1 m}\right), \ldots, U^{n}\left(x_{n 1}, \ldots, x_{n m}\right)\right) . \tag{1}
\end{equation*}
$$

Further, in view of the Pareto principle, $\phi(\cdot)$ is nondecreasing in each of its arguments and is increasing in some of them.

State-of-the-art normative theory proceeds by assuming that each consumer faces a common vector of prices, $\mathbf{p}=\left(p_{1}, \ldots, p_{m}\right)$, has income $m_{i}$, and chooses his own consumption bundle ( $x_{i 1}, \ldots, x_{i m}$ ) to maximize $U^{i}(\cdot)$ subject to the budget constraint $\Sigma_{j} x_{i j} p_{j} \leq m_{i}$. In such a free consumption market, the $i$ th consumer's maximized utility level is denoted by the indirect utility function
$l^{i}\left(\mathbf{p}, m_{i}\right) \equiv U^{i}\left(X^{i 1}\left(\mathbf{p}, m_{i}\right), \ldots, X^{i m}\left(\mathbf{p}, m_{i}\right)\right)$,
where $X^{i j}\left(\mathbf{p}, m_{i}\right)$ is the function giving the market demand for commodity $j$ by consumer $i$.

The combination of equations (2) and (1),

$$
\begin{equation*}
V(\cdot)=\phi\left(l^{1}\left(\mathbf{p}, m_{1}\right), \ldots, l^{n}\left(\mathbf{p}, m_{n}\right)\right) \equiv \psi\left(\mathbf{p} ; m_{1}, \ldots, m_{n}\right), \tag{3}
\end{equation*}
$$

gives the preferences of the SDM over price vectors, $\mathbf{p}$, and distributions of nominal income, $m_{1}, \ldots, m_{n}$, induced by his preferences over allocations of commodities, given the free consumption market. The representation of social preferences given in equation (3) is in a form theoretically applicable to the evaluation of microeconomic policy. ${ }^{1}$ For example, the attitudes of the SDM toward changes in regulated prices can be expressed by means of the tradeoffs between prices that are inherent in the function $\psi$. Such tradeoffs are invariant to ordinal transformations of $V(\cdot), \phi(\cdot)$, and $\psi(\cdot)$. Differentiation of equation (3) gives
$\frac{\partial \psi}{\partial p_{i}} / \frac{\partial \psi}{\partial p_{j}}=\sum_{k} \phi_{k} \frac{\partial l^{k}}{\partial p_{i}} / \sum_{k} \phi_{k} \frac{\partial k^{k}}{\partial p_{j}}$,
where
$\phi_{k}=\frac{\partial \phi}{\partial l^{k}}$.
This can be put in a more interpretable form by means of Roy's law: ${ }^{2}$
$\frac{\partial l^{k}}{\partial p_{j}}=-X^{k j} \frac{\partial l^{k}}{\partial m_{k}}$.
Roy's law says that a consumer's rate of substitution between income and the $j$ th price is that consumer's demand for the $j$ th commodity. Substituting (5) into (4) gives
$\frac{\partial \psi}{\partial p_{i}} / \frac{\partial \psi}{\partial p_{j}}=\sum_{k}\left(\phi_{k} \frac{\partial l^{k}}{\partial m_{k}}\right) X^{k i} / \sum_{k}\left(\phi_{k} \frac{\partial k^{k}}{\partial m_{k}}\right) X^{k j}$.
Equation (6) relates the SDM's tradeoff between prices $i$ and $j$ to two elements ${ }^{3}$ the distribution of demands for goods $i$ and $j$ among consumers and the collection of weighting factors on these demands,
$\omega_{k} \equiv \phi_{k} \frac{\partial l^{k}}{\partial m_{k}}=\frac{\partial \psi}{\partial m_{k}}$.
Each of the weighting factors measures the marginal social utility, in the view of the SDM, of income to a consumer.
If the SDM were indifferent to small lump-sum transfers of income between any two consumers, then all the $\omega_{k} \mathrm{~s}$ would be equal. This case would also occur if a costless mechanism for interpersonal lump-sum transfers were available to the SDM. With $\omega_{k}=\omega$ for all $k$, equation (6) would reduce to
$\frac{\partial \psi}{\partial p_{i}} / \frac{\partial \psi}{\partial p_{j}}=\sum_{k} X^{k i} / \sum_{k} X^{k j}$.
Here, the tradeoff between $p_{i}$ and $p_{j}$ does not depend on the distribution of demands among consumers. Instead, it is the ratio of total market demands for the goods, which is also the rate of substitution between the prices for market consumers' surplus.

Generally, however, the SDM will not be costlessly able to effect lumpsum transfers, will not be indifferent to transfers, and will not impute equal marginal social utility to each consumer's income. In this case, equation
(6) shows that the larger $\omega_{k}$ is relative to the other weights, the more important is the demand pattern of consumer $k$ for the SDM's view of price tradeoffs. If consumers differ in their demand patterns, then the relative sizes of the weights can become critical for the SDM's decisions on policies affecting prices.

Economic theory can shed little light on the sizes of the welfare weights. Some analysts (for example, Feldstein [1972a] and Atkinson [1970]) have specified functional forms for the behavior of $\psi(\cdot)$ with respect to $m_{1}, \ldots$, $m_{n}$, and have thus implicitly related the full set of weights to a small set of parameters. However, such procedures are inherently arbitrary. Conceptually, it may be empirically or experimentally possible to determine the $\psi$ function of an SDM, and to proceed to policy analysis on that basis. Such a tack is yet to be carried out successfully.

Here we shall proceed by using the fact that if the SDM would rather not see income transferred from consumer $i$ to $j$, then $\omega_{j} \leq \omega_{i}$. With $t$ representing such a transfer, the SDM's preferences are represented by
$\phi\left(\ldots, l^{i}\left(\mathbf{p}, m_{i}-t\right), \ldots, l^{j}\left(\mathbf{p}, m_{j}+t\right), \ldots\right)$.
Then
$0 \geq \frac{d \phi}{d t}=-\phi_{i} \frac{\partial l^{i}}{\partial m_{i}}+\phi_{j} \frac{\partial l^{j}}{\partial m_{j}}=-\omega_{i}+\omega_{j}$,
and, at $t=0, \omega_{i} \geq \omega_{j}$.
We specify intuitive conditions on the SDM's preferences that are equivalent to such inequalities among the welfare weights, and then ask whether a policy is preferred to another for all weights satisfying the qualitative relationships. Thus, we avoid the need for quantitatively specified weights, while we lose completeness of the preference ordering among policies. The result is a partial ordering that was named social-welfare dominance by Willig and McCabe (1977).

## Social-Welfare Dominance

In this section we characterize the social-welfare-dominance partial ordering over policies. One policy social-welfare-dominates another if it is preferred by all SDMs whose social preferences satisfy the three axioms below. ${ }^{4}$ (The axioms are expressed in terms of the premises described above: static, riskless allocations of private goods among neoclassical consumers who face a free market without extended sympathy.)

Axiom 1 (Pareto Principle) The SDM is indifferent to a change that leaves all consumers indifferent. The SDM does not view as desirable a change that no consumer finds desirable. A change that is beneficial to at least one consumer while leaving all others indifferent is viewed as desirable by the SDM.

Axiom 2 (Anonymity) At prices $\mathbf{p}^{0}$, the SDM is indifferent to a reversal of nominal incomes between any two consumers.

Axiom 3 (Regressive Transfer Aversion) At prices $\mathbf{p}^{0}$, the SDM does not find desirable any transfer of income from a nominally poorer to a richer consumer.

In our context, axiom 1 implies that the social preferences of the SDM can be represented by the function of indirect utility levels, $\phi$, defined in equation (3). Because of the forms of axioms 2 and 3 , it is convenient to work with the special indirect utility functions, $\mu^{k}\left(\mathbf{p}^{0} \mid \mathbf{p}, m_{k}\right)$, which are measured in terms of real income, base $\mathbf{p}^{0}$. These income-compensation functions, first studied rigorously by Hurwicz and Uzawa (1971), can be defined implicitly by
$l^{k}\left(\mathbf{p}^{0}, \mu^{k}\left(\mathbf{p}^{0} \mid \mathbf{p}, m_{k}\right)\right) \equiv l^{k}\left(\mathbf{p}, m_{k}\right)$.
They give the nominal income required by consumer $k$, in facing prices $\mathbf{p}^{0}$, to be indifferent to income $m_{k}$ at prices $\mathbf{p}$.

When viewed as a function of $\mathbf{p}$ and $m_{k}, \mu^{k}\left(\mathbf{p}^{0} \mid \mathbf{p}, m_{k}\right)$ is a proper indirect utility function for consumer $k$. This can be seen from equation (9). Because the function $l^{k}$ is increasing in its income argument for fixed $\mathbf{p}^{0}$, the left-hand side of $(9)$ is an ordinal transformation of the function $\mu^{k}$. Thus, $l^{k}(\cdot)$ is an ordinal transformation of the function $\mu^{k}$ and can be written for fixed $\mathbf{p}^{0}$ as
$l^{k}\left(\mathbf{p}, m_{k}\right)=T^{k}\left(\mu^{k}\left(\mathbf{p}^{0} \mid \mathbf{p}, m_{k}\right)\right)$.
Now, for each $k$, substitute (10) into (3) and define

$$
W\left(z_{1}, \ldots, z_{n}\right) \equiv \phi\left(T^{1}\left(z_{1}\right), \ldots, T^{n}\left(z_{n}\right)\right) .
$$

Then,
$\psi\left(\mathbf{p}: m_{1}, \ldots, m_{n}\right) \equiv W\left(\mu^{1}\left(\mathbf{p}^{0} \mid \mathbf{p}, m_{1}\right), \ldots, \mu^{n}\left(\mathbf{p}^{0} \mid \mathbf{p}, m_{n}\right)\right)$
yields a particularly convenient representation of the SDM's preferences.

Of course, by axiom $1, W(\cdot)$ is nondecreasing in all of its arguments and increasing in at least one.
Axiom 2 says that at prices $\mathbf{p}^{0}$ it is only the vector of nominal incomes, and not their assignment to particular consumers, that matters to the SDM. Thus, $\psi\left(\mathbf{p}^{0} ; m_{1}, \ldots, m_{n}\right)$ is symmetric in its income arguments. For example,
$\psi\left(\mathbf{p}^{\mathbf{0}} ; m_{1}, m_{2}, m_{3}, \ldots, m_{n}\right)=\psi\left(\mathbf{p}_{0} ; m_{2}, m_{1}, m_{3}, \ldots, m_{n}\right)$.
Note from (9) that
$\mu^{k}\left(\mathbf{p}^{\mathbf{0}} \mid \mathbf{p}^{0}, m_{k}\right)=m_{k}$.
It follows from (12) and (11) that

$$
\begin{align*}
\psi\left(\mathbf{p}^{0}, m_{1}, \ldots, m_{n}\right) & =W\left(\mu^{1}\left(\mathbf{p}^{0} \mid \mathbf{p}^{0}, m_{1}\right), \ldots, \mu^{n}\left(\mathbf{p}^{0} \mid \mathbf{p}^{0}, m_{n}\right)\right) \\
& =W\left(m_{1}, \ldots, m_{n}\right) \tag{13}
\end{align*}
$$

Consequently, axioms 1 and 2 together imply that $W(\cdot)$ is symmetric in its arguments. This has been established by reference to prices $\mathbf{p}^{0}$, where $\mu^{k}=m_{k} ;$ it holds as well for other prices where the arguments of $W(\cdot)$ are $\mu^{k}\left(\mathbf{p}^{0} \mid \mathbf{p}, m_{k}\right)$.

By axiom 3 and equation (8), ${ }^{5} m_{i}>m_{j}$ implies that, at prices $\mathbf{p}^{0}$,
$\frac{\partial \psi}{\partial m_{i}} \leq \frac{\partial \psi}{\partial m_{j}}$.
In view of (13), $m_{i}>m_{j}$ implies that
$\frac{\partial W\left(m_{1}, \ldots, m_{n}\right)}{\partial m_{i}} \leq \frac{\partial W\left(m_{1}, \ldots, m_{n}\right)}{\partial m_{j}}$.
This is a property of the function $W(\cdot)$, whatever the interpretation of its arguments. Thus, under axioms 1-3,
$\mu^{i}>\mu^{j}$ implies $\frac{\partial W}{\partial \mu^{i}} \leq \frac{\partial W}{\partial \mu^{j}}$.
Now, consider the social-welfare comparison between policies $\rho^{\prime}$ and $\rho^{\prime \prime}$ that result in ( $\mathbf{p}^{\prime} ; m_{1}^{\prime}, \ldots, m_{n}^{\prime}$ ) and ( $\mathbf{p}^{\prime \prime} ; m_{1}^{\prime \prime}, \ldots, m_{n}^{\prime \prime}$ ) respectively.

## Definition

$\rho^{\prime}$ social-welfare-dominates $\rho^{\prime \prime}\left(\rho^{\prime}>\rho^{\prime \prime}\right)$ if $\rho^{\prime}$ is preferred to $\rho^{\prime \prime}$ by all SDMs whose preferences satisfy axioms $1-3$.

## Theorem 1

$\rho^{\prime}>\rho^{\prime \prime}$ if and only if
$\sum_{i=1}^{k} \mu^{i}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime}, m_{i}^{\prime}\right)>\sum_{i=1}^{k} \mu^{i}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime \prime}, m_{i}^{\prime \prime}\right), \quad k=1, \ldots, n$,
where the indices $i$ are assigned to possibly different consumers under $\rho^{\prime}$ and $\rho^{\prime \prime}$ so that
$\mu^{1}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime}, m_{1}^{\prime}\right) \leq \mu^{2}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime}, m_{2}^{\prime}\right) \leq \cdots \leq \mu^{n}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime}, m_{n}^{\prime}\right)$
and
$\mu^{1}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime \prime}, m_{1}^{\prime \prime}\right) \leq \mu^{2}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime \prime}, m_{2}^{\prime \prime}\right) \leq \cdots \leq \mu^{2}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime \prime}, m_{2}^{\prime \prime}\right)$.
(This theorem, proved below, is mathematically analogous to results established by Rothschild and Stiglitz [1973]. A more general version was demonstrated by Willig and McCabe [1977].)

Proof of Theorem 1 Suppose that equation (15) is satisfied. We must prove that it follows that $\rho^{\prime}$ is preferred to $\rho^{\prime \prime}$ by any SDM who satisfies axioms $1-3$. Let the SDM's social preferences be represented as in (13). By axiom 2, the consumers can be relabeled to satisfy (16) without changing the relevant values of $W(\cdot)$. For notational convenience, let $Z_{i}^{\prime} \equiv$ $\mu^{i}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime}, m_{i}^{\prime}\right)$, let $Z_{i}^{\prime \prime} \equiv \mu^{i}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime \prime}, m_{i}^{\prime \prime}\right)$, and let $\mathbf{Z}^{\prime}$ and $\mathbf{Z}^{\prime \prime}$ be the corresponding vectors. Define, with $t$ a scalar, $F(t)=W\left((1-t) \mathbf{Z}^{\prime}+t \mathbf{Z}^{\prime \prime}\right)$. Then, using the mean-value theorem,

$$
\begin{aligned}
W\left(\mathbf{Z}^{\prime \prime}\right)-W\left(\mathbf{Z}^{\prime}\right)= & F(1)-F(0) \\
= & \frac{d F(\bar{t})}{d t} \\
= & \sum_{i}\left(\mathbf{Z}_{i}^{\prime \prime}-\mathbf{Z}_{i}^{\prime}\right) \frac{\partial W\left((1-\bar{t}) \mathbf{Z}^{\prime}+\bar{t} \mathbf{Z}^{\prime \prime}\right)}{\partial Z_{i}} \\
& \text { for some } \bar{t} \in[0,1] .
\end{aligned}
$$

Since the components of $\mathbf{Z}^{\prime}$ and $\mathbf{Z}^{\prime \prime}$ are ordered in an increasing fashion, so too are the components of $(1-\bar{t}) \mathbf{Z}^{\prime}+\bar{f} \mathbf{Z}^{\prime \prime}$. Then, by axioms 1 and 3 and equation (14),

$$
\omega_{1} \geq \omega_{2} \geq \cdots \geq \omega_{n} \geq 0
$$

where
$\omega_{i} \equiv \frac{\partial W\left((1-\tilde{t}) \mathbf{Z}^{\prime}+\tilde{t} \mathbf{Z}^{\prime \prime}\right)}{\partial Z_{i}}$.
Using this notation, it follows from algebraic manipulation that
$W\left(\mathbf{Z}^{\prime \prime}\right)-W\left(\mathbf{Z}^{\prime}\right)=\sum_{k=1}^{n-1}\left(\left(\omega_{k}-\omega_{k+1}\right) \sum_{i=1}^{k}\left(Z_{i}^{\prime \prime}-Z_{i}^{\prime}\right)\right)+\omega_{n} \sum_{i=1}^{n}\left(Z_{i}^{\prime \prime}-Z_{i}^{\prime}\right)$.
By equation (15), $\sum_{i=1}^{k}\left(Z_{i}^{\prime \prime}-Z_{i}^{\prime}\right)>0 \quad$ for $k=1, \ldots, n$. Also, $\left(\omega_{k}-\omega_{k+1}\right) \geq 0$. Let $j$ be the largest index such that $\omega_{1}=\omega_{2}=\ldots=\omega_{j}$. If $j=n$, then $\omega_{n}>0$ by axiom 1 and $W\left(\mathbf{Z}^{\prime \prime}\right)-W\left(\mathbf{Z}^{\prime}\right)=\omega_{n} \Sigma_{i=1}^{n}\left(Z_{i}^{\prime \prime}-Z_{i}^{\prime}\right)$ $>0$. If $j<n$, then $\omega_{j}-\omega_{j+1}>0$ and $W\left(\mathbf{Z}^{\prime \prime}\right)-W\left(\mathbf{Z}^{\prime}\right) \geq\left(\omega_{j}-\omega_{j+1}\right)$ $\Sigma_{i=1}^{j}\left(Z_{i}^{\prime \prime}-Z_{i}^{\prime}\right)>0$. Thus, $W\left(\mathbf{Z}^{\prime \prime}\right)-W\left(\mathbf{Z}^{\prime}\right)>0$ for any SDM in the class.

For the other direction of proof, assume that $\rho^{\prime}>\rho^{\prime \prime}$. With consumers ordered according to relations (16), let $W(Z) \equiv \sum_{i=1}^{k} Z_{i}$. Such preferences clearly satisfy axiom 1 . At $\mathbf{p}^{0}, Z_{i}=m_{i}$, and axiom 2 is immediate. Consider a transfer of $y>0$ from $i$ to $j$ with $i<j$, and let $G(y)$ be the resulting level of $W(\cdot)$. If $i \leq k$ and $j>k$, then $G(y)=G(0)-y<G(0)$. If $i>k, \quad j>k$, and $m_{i}-y \geq m_{k}$, then $G(y)=G(0)$, whereas if $m_{i}-y$ $<m_{k}$, then $G(y)=G(0)-\left(m_{k}-m_{i}+y\right)<G(0)$. If $i<j \leq k$ and $m_{j}+y \leq m_{k+1}$, then $G(y)=G(0)$. If $i<j \leq k$ and $m_{j}+y>m_{k+1}$, then $G(y)=G(0)-\left(m_{j}+y-m_{k+1}\right)<G(0)$. Thus, $W(Z)=\Sigma_{i=1}^{k} Z_{i}$ satisfies axioms $1-3$ for any $k$ and, for dominance, (15) must hold. Q.E.D.

To interpret theorem 1, note first that in the situations caused by each policy, consumers are relabeled so that consumer 1 is poorer in real income (base $\mathbf{p}^{0}$ ) than is 2 , who is poorer than 3 , and so on. This is permissible, loosely, by virtue of the anonymity axiom. If moving from $\rho^{\prime}$ to $\rho^{\prime \prime}$ does not change the order of consumers' real incomes, then the consumer indexed by $i$ is the same in both situations.

The inequalities in equation (15) say that the sum of the real incomes of the poorest $k$ consumers under $\rho^{\prime}$ must, for dominance, exceed that under $\rho^{\prime \prime}$ for all $k$. For $k=1$, equation (15) is the Rawlsian condition that the lot of the poorest consumer be improved. For $k=n$, equation (15) is the Hicksian condition that the sum of the changes in all consumers' real incomes be positive. The novelty and strength of equation (15) is that the condition is required for $k=1$, for $k=n$, and for all $k$ in between.

It is important to realize that the conditions given in (15) can be checked by means of market data. It was shown by Willig $(1973,1976)$ that
$\mu\left(\mathbf{p}^{0} \mid \mathbf{p}, m\right)$ is approximated closely by the sum of $m$ and the Marshallian consumer's surplus area (a line integral for multiple price changes) between $\mathbf{p}^{0}$ and $\mathbf{p}$, if that area is a relatively small portion of $m$ and if the income elasticities of demand are in the usual range. Under such conditions, (15) can be investigated with only small errors by means of sums of consumers' incomes and surpluses. In particular, if $m_{i}^{\prime}=m_{i}^{\prime \prime}$, then a sum of the form
$\sum_{i=1}^{k}\left[\mu^{i}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime}, m_{i}^{\prime}\right)-\mu^{i}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime \prime}, m_{i}^{\prime \prime}\right)\right]$
can be well approximated by the sum of the poorest (in real income) $k$ consumers' surpluses between $\mathbf{p}^{\prime \prime}$ and $\mathbf{p}^{\prime}$. This sum is independent of $\mathbf{p}^{0}$, if the identities of the relevant consumers are independent of $\mathbf{p}^{0}$.

Thus-loosely, or precisely in terms of approximations with wellunderstood bounds-equation (15) has the following interpretation: A necessary condition for social-welfare dominance is passage of the standard aggregate ( $k=n$ ) consumers' surplus test. The necessary and sufficient conditions for social-welfare dominance are that the policy change increase the aggregated surpluses of the poorest $k$ consumers, for all $k$.

Of course, in an actual economy, the number of conditions represented by equation (15) is staggeringly large. For practical analysis, it may suffice to partition consumers into a manageable number of income classes, to consider as identical the welfare weights of all consumers within a class, and to apply theorem 1 across classes rather than strictly across consumers. We present below several more easily verified conditions that are either necessary or sufficient for dominance. But let us momentarily digress on the subject of policy packages.

Consider a policy $\rho^{\prime}$ that does not dominate the status-quo policy, $\rho^{0}$. Denote

$$
\begin{equation*}
\Delta Z_{i}=\mu^{i}\left(\mathbf{p}^{0} \mid \mathbf{p}^{\prime}, m_{i}^{\prime}\right)-\mu^{i}\left(\mathbf{p}^{0} \mid \mathbf{p}^{0}, m_{i}^{0}\right) \tag{17}
\end{equation*}
$$

as the change in the $i$ th consumer's real income due to the policy, measured against the status-quo base. Then, for at least some $k$,

$$
\sum_{i=1}^{k} \Delta Z_{i} \leq 0
$$

Suppose that $\rho^{\prime}$ is efficient in the usual sense in that it increases total surplus:
$\sum_{t=1}^{n} \Delta Z_{i}>0$.
Then, there may exist an ancillary policy change that, when linked with $\rho^{\prime}$, makes the policy package dominate the status quo. Such an ancillary change need not itself dominate the status quo, and the policy package need not dominate $\rho^{\prime}$. Moreover, the ancillary change may be an inefficient addition to $\rho^{\prime}$ in that it decreases the gain in aggregate surplus that could have been achieved by $\rho^{\prime}$ alone. ${ }^{6}$

Thus, the concept of social-welfare dominance can provide a framework for understanding the common regulatory and political process of linking policies together into packages. Such practices have no rationale when analyzed within the usual optimization framework.

## Tests for Dominance

To make possible the use of differential analysis, suppose that there are $h$ continuous policy controls, $\theta_{1}, \ldots, \theta_{h}$, that are restricted by the feasibility condition that $F(\theta) \geq 0 .^{7}$ Let the status quo be represented by $\theta^{0}$, with $F\left(\theta^{0}\right)=0$. Further, let the real income (base $\mathbf{p}^{0}$ ) of consumer $i$ be given by $Z_{i}(\theta)$, where $Z_{i}\left(\theta^{0}\right) \leq Z_{i+1}\left(\theta^{0}\right)$.

Proposition 1 There exists a feasible policy in the direction $V$ from $\theta^{0}$ that social-welfare-dominates the status quo $\theta^{\circ}$ if
$\nabla F\left(\theta^{0}\right) \cdot V>0 \quad$ and $\quad\left[\sum_{i=1}^{k} \nabla Z_{i}\left(\theta^{0}\right)\right] \cdot V>0, \quad k=1, \ldots, n$,
where either $Z_{i}\left(\theta^{0}\right)<Z_{i+1}\left(\theta^{0}\right)$ or, if $Z_{i}\left(\theta^{0}\right)=Z_{i+1}\left(\theta^{0}\right),{ }^{8}$ there is a $\hat{t}$ such that $Z_{i}\left(\theta^{0}+t V\right) \leq Z_{i+1}\left(\theta^{0}+t V\right)$ for $0<t \leq \hat{t}$.

Proof of Proposition 1 The functions of $t, F\left(\theta^{0}+t V\right)$, and $\sum_{i=1}^{k} Z_{i}\left(\theta^{0}+t V\right)$ all have positive derivatives with respect to $t$, at $t=0$, if relations (18) hold. Then, there is a $\hat{t}$ such that for $0<t<\hat{t}$,
$F\left(\theta^{0}+t V\right)>F\left(\theta^{0}\right)=0$
and
$\sum_{i=1}^{k} Z_{i}\left(\theta^{0}+t V\right)>\sum_{i=1}^{k} Z_{i}\left(\theta^{0}\right)$.

For positive $t$ sufficiently small,
$Z_{i}\left(\theta^{0}+t V\right) \leq Z_{i+1}\left(\theta^{0}+t V\right)$
and $\theta=\theta^{0}+t V$ is a feasible policy that dominates the status quo.
Proposition 1 shows that if local analysis uncovers a feasible dominating direction of policy movement, then there is a feasible dominating policy change that is a finite step in that direction.

Gordon's theorem (Mangasarian 1969) provides a condition that is equivalent to (18) and that is computationally efficient by linear programing methods.

Proposition 2 Relations (18) hold if and only if there is no non-negative, nonzero vector $\mathbf{y}$ with

$$
\begin{equation*}
\sum_{k=1}^{n} \mathbf{y}_{k} \sum_{i=1}^{k} \frac{\partial Z_{i}\left(\theta^{0}\right)}{\partial \theta_{j}}+\mathbf{y}_{n+1} \frac{\partial F\left(\theta^{0}\right)}{\partial \theta_{j}}=0, \quad j=1, \ldots, h . \tag{19}
\end{equation*}
$$

Equation (19) has the same form as the first-order conditions for $\theta^{0}$ to be Pareto-optimal for the $n$ "pseudoconsumers," each of whom is the aggregate of the $k$ poorest actual consumers ( $k=1,2, \ldots, n$ ). Such an approach could be used to derive necessary and sufficient conditions for $\theta^{0}$ to be locally nondominated. However, this tack would require curvature conditions on the functions that are not compelling at this level of generality.
The following mathematical result, proved in Willig and McCabe 1977, is useful in conjunction with Proposition 1 or, directly, with Theorem 1 .

Lemma Let $F(m)$ be an increasing function for $0 \leq m \leq \bar{m}$. Suppose
$\int_{0}^{\bar{m}} g(m) d F(m)>0$.
Let $g(m) \equiv b(m)-c(m)$, with $b(m) \geq(0), \quad c(m)>0$, and with $b(m) / c(m)$ a decreasing function of $m$ for $0<m<\bar{m}$. Then

$$
\begin{equation*}
\int_{0}^{m} g(m) d F(m)>0 \quad \text { for } 0<m \leq \bar{m} . \tag{21}
\end{equation*}
$$

To apply this lemma to theorem 1, consider a policy move that generates the changes in real income $\Delta Z_{i}$, where the indexes of consumers increase with real income. We can interpret $F(i)$ as the cumulative (step) density function of consumers, and $g(i)$ as $\Delta Z_{i}$. Then, $g(i) \equiv b(i)-c(i)$ is a benefit-
cost decomposition of the net change in real income, and the lemma establishes the result given in theorem 2.

Theorem 2 A policy move that does not alter the ranking of consumers by real income is social-welfare-dominating if its aggregate net benefit is positive and if it has benefit-cost ratios specific to real income classes that decrease with real income.

Combining the lemma with proposition 1 yields the local test for a dominating policy given as theorem 3 .

Theorem 3 With consumers indexed as in proposition 1, there exists a feasible policy in the direction $V$ from $\theta^{0}$ that social-welfare-dominates the status quo $\theta^{0}$ if
$\nabla F\left(\theta^{0}\right) \cdot V>0$,
if
$\sum_{i=1}^{n} \nabla Z_{i}\left(\theta^{0}\right) \cdot V>0$,
and if there exist $b_{i} \geq 0$ and $c_{i}>0$ such that
$\nabla Z_{i}\left(\theta^{0}\right) \cdot V=b_{i}-c_{i}$
and $b_{i} / c_{i}$ decreases with $i$.
Theorems 2 and 3 focus attention on benefit-cost ratios that are specific to real income classes of consumers. If these ratios decline with real income for a policy change that leaves intact the ordering of consumers by real income, then the move is social-welfare-dominant if and only if it is efficient in the standard aggregate sense. This result can be useful when data are insufficient for the calculations required by theorem 1 but qualitative information indicates the relationship between income levels and income-class-specific benefit-cost ratios.

Additional tests that are more useful in a different context are discussed in McCabe and Willig 1977.

## Positive and Normative Views

Suppose an analyst observes that an SDM has effected a policy move that has resulted in real-income changes, $\Delta Z_{i}$, that fail to satisfy equations (15)
and (16). The move can certainly not be logically criticized on those grounds, even by believers in axioms l-3. It is possible that the chosen policy was optimal for the SDM's particular preferences, and that those preferences satisfy the axioms. However, a believer in axioms $1-3$ would have grounds for protest if the SDM's choice revealed a decision criterion in conflict with the axioms. The inverse of theorem l yields the means for such revelation.

Proposition 3 A policy change is weakly preferred by no social preferences that obey axioms $1-3$, if and only if the $-\Delta Z_{i}$ satisfy (15) and (16); that is, if and only if the original policy social-welfare-dominates the new one.

What cogent aspersions may be cast at an SDM who has, with full information, effected a dominated policy? The pros and cons of such arguments are best organized around the defining axioms.

It can be argued that public policy regularly violates the Pareto principle via promotion of merit goods, legally mandatory insurance programs, paternalistic safety rules, legal restrictions on private conduct, and the like. Counterarguments assert that such violations of axiom 1 are in fact Pareto-optimal policies in the face of overlooked externalities, imperfect information, and extended sympathy. The only conclusion that can be drawn here is that without such potentially important complications, imposition on consumers of an SDM's personal tastes is socially counterproductive.

Violations of axiom 2 seem to indicate favoritism based on factors omitted from the explicit normative analysis. The deliberately simple model utilized here does omit many characteristics of individual consumers that might well be included in the evaluation of real income. Among these characteristics could be labor ability, wealth, physical and human capital, health, and family size. Thus, at prices $\mathbf{p}^{0}$, an SDM might reasonably fail to be indifferent to an interchange of nominal incomes between people whose states of health do not coincide. These matters are given a detailed treatment in Willig 1981, where it is shown that the conditions (15) and (16) can remain robust to such interpersonal differences when the axioms are suitably broadened.

Favoritism in violation of axiom 2 can also be motivated by reciprocal opportunities for the personal financial or political enrichment of the SDM. It is hoped that such violations can be uncovered by the social-welfare-dominance methodology.

The content of axiom 3 depends on the definition of real income. In a more general model than that presented here, real income could include adjustments for abnormal levels of selected characteristics and endowments. Then, for example, in theorem 1, consumer 1 would have the smallest adjusted real income, and the real-income ordering of consumers would depend partially on the set of characteristics for which adjustments are permitted. In this way, the conditions for social-welfare dominance would become less mechanical, less scientific, and more responsive to difficult ethical judgments. Revealed violations of axiom 3 must be viewed in this light. However, there remain wide classes of social-decisionmaking behavior that would fail to satisfy any ethically reasonable version of the axiom. For example, an SDM who acted to maximize his political support or personal finances (see Peltzman 1976; Stigler 1971) would undoubtedly exhibit many instances of preference for policies that are inefficient in the aggregate and that cause regressive transfers of real income. Such behavior can be exposed by analysis based on proposition 3 .

In our view, the methodologies presented here can systematically reflect income-distribution concerns in regulatory policymaking. Past regulatory decisions can be reviewed for efficiency and distribution effects within the framework provided by theorem 1 and proposition 3. To narrow the present sets of options, social-welfare-dominated policies can be excluded from consideration with analyses based on proposition 1 or 2 or on theorem 2 or 3 . These same methods can underlie a discovery that a regulatory status quo is social-welfare-dominated. Finally, the distributional effects of an innovative policy can be tested and summarized by means of the conceptual tools forged here.
The remaining sections of this article present pilot applications of social-welfare-dominance methods to regulatory policymaking.

## Peak and Off-Peak Pricing by Electric Utilities

Until recently, electric utilities in the United States rarely charged differential rates by time of day or by season of the year. Yet, these utilities face demands that exhibit definite peaks and troughs. Electric utilities in the southern and southwestern states, for example, face peaks during the summer months, caused in large part by summer demand for air conditioning. Under seasonal pricing, such an electric utility can cut down the need for additional capacity by instituting a summer surcharge, and can encourage a more efficient utilization of capacity throughout the year by also offering a winter discount. These well-known advantages of peak-
load pricing policies have proved to be of substantial economic importance in Europe (Mitchell and Acton 1977).

There is an additional aspect of peak-load pricing that is important in the United States. Public-utility commissioners, who are responsible for approving such policies, often have an interest in the income-distribution consequences of these policies. If seasonal pricing were to have equalizing distributional effects (as may be expected in the case of summer-peaking electric utilities), then commissioners might be particularly disposed to encourage this policy. In contrast, if seasonal pricing were likely to have an adverse effect on low-income families (as may occur in the case of winter-peaking electrics), there would then be hesitation about approval of the policy unless other benefits could be shown to be substantial.

Therefore, as a first application, we examine the income-distribution consequences of a switch from a pricing structure that is uniform throughout the year to one that takes account of seasonal peaks. Since our intent is to illustrate how to test for the existence of a social-welfare-dominant policy, we confine the discussion to a very simple structure in which a summer-peaking electric utility increases its summer block prices by $s$ and decreases its winter block prices by $w .{ }^{9}$ We presume that, under the status quo, the seasonally uniform prices exceed marginal cost during the offpeak winter months and are below marginal cost during the summer peak. Thus, we assume that implementation of small positive changes $s$ and $w$ is a move toward marginal-cost pricing that increases aggregate con-sumers'-plus-producer's surplus.
In the absence of data on demand elasticities, marginal costs, and the ownership of the electric utility, we cannot quantify the effects of price changes on the utility's profit and on the incomes of its owners. Thus, we focus on the welfare effects of various price changes on the consumers of the utility's outputs. We proceed with this pilot application by investigating the income-distribution effects of price changes as if they held the utility's profit constant and had welfare effects on only electricity consumers.
Our data consist of the monthly 1973 electricity usages of the 49 representative households in Gainesville, Florida studied by Roth (1976). Twenty of the households were classified as low-income, 18 as middieincome, and 11 as high-income. The data for households in each income class are summarized in table 2.1 for the summer months (June-September) and for the rest of the year. Note that, while total summer usage is less than total winter usage, monthly output is higher during the peak summer period than it is during the winter.

Table 2.1 Income-distribution consequences of seasonal pricing of electricity, based on data of Roth (1976).

|  | Income Group |  |  |
| :---: | :---: | :---: | :---: |
|  | Low | Middle | High |
| Number of households | 20 | 18 | 11 |
| Average 1970 income | \$6,150 | \$11,125 | \$19,757 |
| Average 1973 usage, June-Sept. | $2,140 \mathrm{kWh}$ | $6,389 \mathrm{kWh}$ | $13,645 \mathrm{kWh}$ |
| Average 1973 usage, Oct.-May | $4,180 \mathrm{kWh}$ | 6,206 kWh | $15,318 \mathrm{kWh}$ |
| Total 1973 usage, June-Sept. | $42,800 \mathrm{kWh}$ | $115,000 \mathrm{kWh}$ | $150,100 \mathrm{kWh}$ |
| Total 1973 usage, Oct.-May | $83,600 \mathrm{kWh}$ | $111,700 \mathrm{kWh}$ | $168,500 \mathrm{kWh}$ |
| $r=0.51{ }^{\text {a }}$ |  |  |  |
| Benefit-cost ratio | 1.00 | . 50 | . 57 |
| $r=0.846$ : |  |  |  |
| Benefit-cost ratio | 1.68 | . 82 | . 95 |
| Class net benefit | \$279 | -\$204 | -\$75 |
| Cumulative net benefit | \$279 | +\$75 | \$0 |
| $r=1.00$ <br> Benefit-cost ratio | 1.95 | . 97 | 1.12 |
| $\begin{aligned} & r=1.03: \\ & \text { Benefit-cost ratio } \end{aligned}$ | 2.01 | 1.00 | 1.16 |

a. $r=$ Winter price decrease per $\mathrm{kWh} /$ Summer price increase per kWh .

The welfare effects of specific changes of $w$ and $s$ can be analyzed by means of consumers'-surplus methods. However, we lack the requisite monthly demand function for each income class. Consequently, guided by proposition 1, we proceed with local analysis of directions of change in $w$ and $s$ from 0 .

If we take the then-current set of prices as the base for real incomes, the derivative of a consumer's 1973 real income with respect to $w$, evaluated at 0 , is his winter 1973 electricity consumption, and that with respect to $s$ is minus his summer 1973 consumption. Thus, with the relative winter and summer price changes denoted by $r=d w / d s$, the rate of change in real income for a particular consumer is

$$
\begin{align*}
\frac{d Z}{d s} & =r \times(\text { Winter consumption })-(\text { Summer consumption }) \\
& =(\text { Winter consumption })\left(r-\frac{\text { Summer consumption }}{\text { Winter consumption }}\right) \tag{22}
\end{align*}
$$

The ratio of the benefits from the winter price decrease to the costs from the summer price increase is

$$
\begin{equation*}
-\frac{\partial Z / \partial w}{\partial Z / \partial s} \frac{d w}{d s}=r \times \frac{\text { Winter consumption }}{\text { Summer consumption }} \tag{23}
\end{equation*}
$$

Because Roth aggregated his sample of households into three income classes, we can only take the point of view that the welfare weights are the same for all households within the same class. We then consider that axioms 1-3 apply to the different income classes.

Equations (22) and (23) show that the income-distribution effects of price changes in the ratio $r$ depend upon the relative winter and summer consumption of electricity by each of the three income classes. If, for example, relative seasonal consumption were the same for all classes, then, regardless of the absolute levels of usage, the price changes would be distributionally neutral. In particular, the benefit-cost ratio would be identical for each income group.

However, the data displayed in table 2.1 reveal patterns of electricity usage that are very different among income classes. The low-income households have rather equal usage throughout the year, and hence their ratio of consumption during the four summer months to that during the eight winter months is 0.51 . The middle-income households use about 50 percent more electricity than the low-income consumers during the winter, but almost three times more in the summer period, presumably because they employ air conditioning. Their ratio of summer to winter usage is 1.03. The high-income households use substantially more electricity than the middle-income consumers throughout the year. Perhaps because they use electric heating in addition to air conditioning, their ratio of summer to winter consumption ( 0.89 ) is smaller than that of the middle-income group.

Combining equation (22) with the summer-winter usage ratios shows that if $r$ (the ratio of winter price decrease to summer price increase) exceeds 1.03 , then all income classes of consumers will benefit. However, such a large relative price decrease may cause infeasible losses for the utility company.

The normative methodology presented here shows that more moderate decreases in the winter price, relative to the summer price, would social-welfare-dominate the status quo, provided that they did not decrease vendor profit. Following the prescription of proposition 1, we can sum equation (22) over the low and middle income classes and over all three classes. The corresponding aggregated summer-winter usage ratios are 0.808 and 0.846 , respectively. For $r>0.846$, the low-income group, the low-income together with the middle-income group, and all groups of consumers aggregated together gain. ${ }^{10}$ Thus, by proposition 1, for any feasible $r$ greater than 0.846 there is a set of price changes in that ratio
whose effects on consumers would be desirable in the view of any SDM whose preferences satisfy axioms $1-3$.

Of course, for $1.03>r>0.846$, a move toward peak-load pricing that would leave profit constant would social-welfare-dominate but would not Pareto-dominate the status quo, because the middle- and high-income groups would lose. However, for such a policy, their losses would be smaller than the gains to the low-income households, and the move would be desirable under axiom 3. Table 2.1 displays these gains and losses for $r=0.846$, scaled by setting $s=\$ 0.01$.

To illustrate the discussion of the preceding section, we note that for $0.846>r>0.51$ the peak-load pricing policy decreases the aggregate real income of electricity consumers and fails to social-welfare-dominate the status quo. Yet, such changes benefit the low-income group and may be desirable to some SDMs who satisfy axioms 1-3. In contrast, for $r<0.51$, the price changes are social-welfare-dominated by the status quo. The analysis suggests that if such a policy was carried out under stable production costs, the SDM was either misinformed or deliberately ignoring consumers' interests.

## Welfare-Optimal Pricing of Long-Distance Telephone Services

The next policy we evaluate is a set of price changes in the direction that yields the best local improvement in aggregate welfare, given that the net revenue ${ }^{11}$ from the studied group of services remains unchanged. The specific example involves 1973 long-distance telephone prices in the United States.

Our work with this example began with a pilot study of the 1973 price structure of direct-distance-dialed (DDD) telephone services (Willig and Bailey 1977b). The study sought to determine whether these prices satisfied first-order conditions for welfare optimization under the constraint that net revenues be held at the then-current level.

The relevant first-order conditions with equal welfare weights for all consumers, no external effects, and no cross-elasticities are given by the inverse elasticity rule:
$\alpha_{i} \equiv \frac{P_{i}-M C_{i}}{P_{i}} \times e_{i}=\alpha, \quad$ for all $i$,
where $e_{i}, P_{i}$, and $M C_{i}$ are the elasticity, price, and marginal cost of service $i$, respectively. The $\alpha_{i}$ are Ramsey numbers. ${ }^{12}$ The Ramsey rule states that, from service to service, the percentage deviation of price from marginal


Figure 2.1 Rough experimental data on price, marginal cost, and elasticity for 1973 DDD services.
cost times the price elasticity of demand should be constant. The more unresponsive the demand for a product to changes in its price, the less welfare is lost if that price is increased, and so the larger the optimal deviation between price and marginal cost. Thus, the overhead expenses for services produced with economies of scale are best covered with revenues from the more inelastic of the services.
The two components of the inverse elasticity formula are displayed in figure 2.1 for day, evening, and night/weekend calls for each of 21 mileage bands (short-haul to long-haul). The price variable is the revenue from a call of average duration. The price elasticity and marginal cost for each distance and time-of-day element of the DDD schedule are calculated from rough, experimental pre-1973 data, ${ }^{13}$ on which we performed a number of interpolations.
It is readily seen that day calls had the largest relative markup of price


Figure 2.2 Critical Ramsey numbers for direct-distance dialing in 1973, determined from experimental data by the economic-gradient method with the Euclidean metric.
over marginal cost for any given mileage band, and the lowest elasticities. Inversely, evening calls had the lowest relative deviation of price from marginal cost and the highest elasticities. These qualitative relationships are in accord with equation (24). However, across mileage bands, the longhaul calls had both the highest relative markup of price over marginal cost and the highest elasticities. These qualitative directions violate the dictum of equation (24).

Figure 2.2 shows a plot of Ramsey numbers calculated from the experimental data for the DDD services. It is evident that these data suggest that the 1973 prices of day, evening, and night/weekend telephone services classified by length of haul do not satisfy equation (24), as the Ramsey numbers do not all coincide. Instead there is a systematic bias, with low Ramsey numbers in short mileage bands and large Ramsey numbers in the longer mileage bands, and with larger Ramsey numbers for evening and night calls than for day calls for almost.all of the mileage bands.

To calculate candidate Ramsey-optimal prices, we made assumptions


Figure 2.3 Best directions of price change, with greatest price change equal to 100 percent (based on experimental data).
about the functional forms for demands and costs. In particular, we supposed that the marginal cost of a unit of any service remains unchanged throughout the relevant range, and that demands either were linear or had constant elasticity. Though we had no assurance that these functional forms spanned the likely cases, we felt they might yield information concerning the closeness of the 1973 prices to those that would have been optimal at that time. Unfortunately, we found that these assumptions about demand and cost curves led to calculated price increases in the shortest mileage bands of day calls that were so large in percentage terms that they undermined the credibility of our assumptions for the range of price changes indicated. We did, however, calculate the size of the welfare improvement under these assumptions, and found that the change in total welfare was small when viewed as a percentage of the total consumers' surplus from, or the total consumers' expenditures on, these services.
To circumvent the difficulties in determining global functional forms for demands and costs, we applied the economic-gradient method (Willig and Bailey 1977a). This method determines the feasible direction of price change that is locally optimal from data on demands, costs, elasticities,
and marginal costs at the current point of operation. It also requires the specification of a metric to measure the sizes of sets of price changes.

Figure 2.2 displays the critical Ramsey number, $\alpha^{*}$, determined by the economic-gradient method with the Euclidean metric. The best feasible direction entails decreases in the prices of services with Ramsey numbers above the critical line, and inversely. Figure 2.3 shows the price changes that result from a specific move in the locally best direction, ${ }^{14}$ a move that is normalized to make the largest percentage change in a single price equal to 100 percent. All short-haul prices increase, long-haul prices decrease, night/weekend prices remain relatively unchanged, day prices generally rise, and evening prices generally fall. The largest percentage change is in the price of the shortest-distance day calls.

Welfare analysis showed that, while not substantial, the rate of change in total consumers' surplus with respect to the size of price changes in this best local direction was significantly positive. Our initial conjecture about the income-distribution consequences of these price changes was that they would produce regressive transfers of real income. We supposed this because it seemed to us that richer households would place a higher proportion of their calls on the long-haul routes. However, the data did not confirm this conjecture.

Figure 2.4 displays data from AT\&T's Market Research Information System (MRIS) on relative usage patterns of the lowest income class and a middle-income group. Surprisingly, the patterns are very similar, and thus it appears on the basis of this evidence that pricing of these services might be distributionally neutral. Table 2.2 summarizes usage patterns by time of day for all the income groups. It is clear that the richer households placed a larger proportion of their calls during the day rate period and a smaller proportion during the night/weekend rate period. Consequently,

Table 2.2 Distribution of telephone traffic by income class and rate period: 1973 MRIS data for long-distance service.

|  | Percentage of Calls |  |  |
| :--- | :--- | :--- | :--- |
| Income Class | Days | Evenings | Nights. Weekends |
| $<\$ 3.000$ | 23.10 | 45.59 | 31.31 |
| $\$ 3.000-\$ 4.999$ | 23.93 | 45.40 | 30.66 |
| $\$ 5,000-\$ 7.499$ | 23.84 | 46.47 | 29.69 |
| $\$ 7.500-\$ 9.999$ | 26.90 | 45.05 | 28.05 |
| $\$ 10.000-\$ 14.999$ | 26.84 | 46.86 | 26.31 |
| $\$ 15.000-\$ 19.999$ | 29.33 | 46.97 | 23.70 |
| $\$ 20.000-\$ 29.999$ | 29.87 | 45.44 | 24.69 |
| $\geqslant \$ 30.000$ | 32.20 | 46.07 | 21.73 |



Figure 2.4 Distribution of telephone traffic by income class, mileage band, and time of day. Solid line represents $<\$ 3,000$ income class; dotted line represents $\$ 10,000-\$ 14,999$ class.
a pricing policy that would increase the off-peak discount for nights and weekends would yield relatively larger benefits to poorer households.

Making use of the MRIS data on demand patterns by income class, we applied the method suggested by Theorem 3 to price changes in the locally best direction. By construction, this direction is feasible in that it maintains the status-quo level of net revenues. Also, local movement of prices in this optimal direction is guaranteed to increase total surplus. Thus, the first two conditions of theorem 3 are satisfied. To check the third, we defined the benefits to be the relative price decreases times initial quantities, and the costs to be the relative price increases times initial quantities. Table 2.3 summarizes the resulting benefit-cost ratios for the sample households. The differences in the rate-period-specific ratios among income classes reflect the differing distributions of calls among mileage bands. Surprisingly, these ratios show that the very poorest households place a greater proportion of their calls on the longest routes and a smaller proportion on the shortest routes than do other households. This pattern is most pronounced during the discounted evening and night/weekend rate periods.

All classes suffer from the increases in day rates and benefit from the general increase in the offpeak discounts. The total benefit-cost ratios generally decrease with income, owing to the fact (documented in table 2.2) that poorer consumers place a smaller fraction of their calls during the day rate period, which bears the brunt of the price increases. Most striking, however, is the result that all the total benefit-cost ratios substantially exceed 1 . This suggests that the policy of changing prices in the locally best direction results in a significant Pareto improvement for residential consumers by income class.

There is an apparent contradiction between this finding and our earlier observation that the total net benefits from large price changes to a calculated Ramsey optimum were not substantial. This contradiction can be resolved by recognizing that the data underlying the Ramsey analysis include both business and residential calls. Although we do not have explicit information on relative proportions of business and residential calls over the various mileage and time-of-day categories, we do know that business calls are concentrated during the day. Thus, the substantial price increases for this category of calls probably mean that the business class as a whole will lose from the policy. As these losses are passed on to consumers in the form of increased product prices or declines in the values of securities, much of the immediate welfare gain to consumers will be dissipated.

Table 2.3 Benefits and costs of price changes in the locally best direction for social welfare: 1973 MRIS data for long-distance telephone service.

|  | Benefit/Cost Ratio |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| Income | Days | Evenings | Nights, <br> Weekends | Total |
| $<\$ 3.000$ | 0.45 | 23 | 40 | 5.2 |
| $\$ 3.000-\$ 4,999$ | 0.39 | 14 | 24 | 3.8 |
| $\$ 5,000-\$ 7,499$ | 0.45 | 13 | 23 | 3.9 |
| $\$ 7,500-\$ 9,999$ | 0.21 | 12 | 27 | 3.1 |
| $\$ 10.000-\$ 14,999$ | 0.30 | 14 | 25 | 3.3 |
| $\$ 15,000-\$ 19,999$ | 0.25 | 14 | 23 | 2.7 |
| $\$ 20,000-\$ 29,999$ | 0.33 | 14 | 30 | 3.3 |
| $\geqslant 30,000$ | 0.42 | 13 | 23 | 3.0 |

It seems plausible, moreover, that these losses will fall more heavily on richer households than on poor ones. To the extent that business outlays for telephone calls are overhead costs rather than marginal costs, and to the extent that consumer-goods prices move with marginal rather than with average costs, the losses in business telephone consumers' surplus will decrease profits rather than be passed through to households in the form of higher consumer-goods prices. Such declines in profit would undoubtedly affect richer individuals more than proportionately.

If this were the case, then the income-class-specific benefit-cost ratios resulting from the full effects of the price changes would be smaller than those shown in table 2.3, but it is plausible that they would be declining with income. Further, the aggregate net benefits would be positive by the construction of the price changes. Then, theorem 3 would enable us to infer from our crude data and assumptions that there existed a set of regulated price movements in the locally best direction that social-welfaredominated the then-current status quo.

## Conclusion

We have presented a variety of necessary and sufficient conditions for social-welfare dominance of regulatory policies-mconditions that can be tested by means of market data. In our study of electricity pricing, we showed that the entire gamut of policy conclusions could be spanned by varying the relative levels of winter and summer prices. In the study of long-distance-telephone pricing we showed that, contrary to our expectations, a move in the locally best direction might have satisfied the monotonicity requirement of theorem 3 and might thus have been social-
welfare-dominant. Our discussion of these specific pricing policies has served to demonstrate the workability of our new approach, and to highlight the usefulness of the concept of social-welfare dominance.

It gives us pleasure to thank Jim McCabe for his critical contributions, Glenn Loury for many stimulating conversations on this subject, and W. J. Baumol and R. Schmalensee for their helpful editorial suggestions.

## Notes

1. This approach was taken most recently by Broadway (1976).
2. Katzner 1970 gives a clear treatment and the original references.
3. This approach to normative analysis of pricing is due to Feldstein (1972b).
4. The choice of these axioms was inspired by the Rothschild-Stiglitz (1973) analysis of nominal income inequality. Their axioms are extended here to carefully defined measures of real incomes. Yet more general versions of the axioms are studied in Willig and McCabe 1977.
5. Here, for convenience, we assume that $\psi$ and $W$ are differentiable. The development in Willig and McCabe 1977 dispenses with this assumption.
6. These possibilities are analogous to the case of a risk-a verse investor who seeks securities with returns that are negatively correlated with the returns on his portfolio. Such an investor will gladly give up some aggregate mean return in order to reduce the risk of his total holdings.
7. This scalar constraint can be easily replaced, as is shown below, by a vector of constraints.
8. We are grateful to R. Schmalensee for pointing out that the case of ties in real income requires special treatment.
9. This illustration is far simpler than the policies being studied in a number of current electricity-pricing experiments (see, for example, CPUCA 1977 and Manning et al. 1976), but the principles developed here can be readily adapted to more complex cases.
10. These numerical results are sensitive to the numbers of households in each income class. For example, had there been 5 low-income, 72 middle-income, and 11 high-income households, the policy for which $r=1$ would have been beneficial in the aggregate but would have failed to social-welfare-dominate the status quo.
11. Net revenue $=$ total revenue - (sum of marginal costs $\times$ quantities).
12. These are named for Frank Ramsey, who first derived this rule in the context of optimal taxation. See Baumol and Bradford 1970 for a discussion of the rule and its history.
13. Consequently, the elasticities and marginal costs do not reflect post-1973 price levels, production technologies, and potentially competitive market structures. Further, to facilitate this pilot study, we assume that there are zero cross-elasticities among the studied categories of calls, that changes in the prices under consideration do not affect the net revenues from other categories of calls, and that the durations of calls do not change endogenously. Without these somewhat unrealistic assumptions, the conditions for Ramsey optimality are more complicated than equation (24).
14. Of course, the calculated locally best feasible direction depends crucially on the use of the inverse elasticity rule and on the magnitudes assigned to its variables. Thus, the quantitative and qualitative price movements displayed in figure 2.3 have no more validity than do the underlying data and the assumptions detailed in n .13 .

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## Comment

Alvin K. Klevorick

Students of public regulation have developed a number of competing theories about the interactions between regulated firms and regulatory agencies and about the social effects of those interactions. The paper I am to discuss represents an output of regulator/regulatee cooperation that differs in kind from the outputs discussed by most of those theories. (Admittedly, though, the interaction that generated this paper does not fit the typical situation considered in these theories, since the regulator from the CAB does not regulate Bell Labs!) Willig and Bailey have produced an interesting, carefully done piece of theoretical and applied work on an important subject.

The authors present new methodological tools for taking incomedistribution concerns into account in regulatory policymaking. They also provide pilot applications of these new "social welfare dominance" methods to regulatory policymaking. The illustrative applications are interesting in and of themselves, and also indicate how the theoretical methods developed in the first part of the article might be put to use. At the same time, the electricity and telephone pricing examples serve to highlight several basic problems facing any attempt to apply the theoretical development in deciding real issues. I shall discuss two of these problems of implementation and then turn to some questions about the theoretical results themselves.

First, two of the axioms that underlie the social-welfare-dominance relation-axiom A2 (anonymity) and axiom A3 (which I shall call "regressive-transfer aversion")-are stated in the form "At prices $\mathbf{p}^{0}$, the social decisionmaker. ..." However, the prices $\mathbf{p}^{0}$ are not defined. These prices constitute the basis for computing real income. In the seasonal electricity pricing example, Willig and Bailey use the thencurrent set of prices as the base for real incomes. Though they do not say so explicitly, I take it they also use the then-current set of prices as $\mathbf{p}^{0}$ in the long-distance-telephone pricing example. At only one point in the paper is there a caveat about these base prices: In discussing theorem 1 , which provides necessary and sufficient conditions for one policy to social-welfare-dominate another, the authors note that the partial sum of the aggregated surpluses of the poorest $k$ consumers is independent of
$\mathbf{p}^{0}$ "if the identities of the relevant consumers are independent of $\mathbf{p}^{0}$." But why should those identities be independent of $\mathbf{p}^{0}$ ?

If the ranking of consumers by real income does depend on the base prices, we face a substantial (even if traditional) index-number problem. It would seem to be a serious problem for a methodology put forth to treat income-distribution concerns that its conclusions are sensitive to the choice of base prices $\mathbf{p}^{0}$. Application of the tools presented in the WilligBailey paper could indicate that one policy social-welfare-dominates a second under one set of base prices but that the second policy social welfare dominates the first under an alternative set of base prices. What criterion are we to use in choosing the base prices?

A second difficulty an analyst faces in applying the paper's theoretical results to actual policy issues derives from the level at which the theoretical development proceeds. The tests for social-welfare dominance presented in the paper's theorems and propositions are stated in terms of realincome changes (and benefit-cost ratios) of individual consumers. Bailey and Willig realize, however, that in practical analysis considerations of data availability will generally require that consumers be grouped into a manageable number of income classes; indeed, they use such classes in their pilot applications. This involves considering as identical the welfare weights of all consumers within a class and applying the paper's theoretical results across classes rather than across consumers. But then a finding that one policy social-welfare-dominates a second may well depend on the income classes one chooses to use or is forced to use because of data limitations. At a minimum, some sensitivity analysis of any policy conclusion should be performed to see whether alternative partitionings of consumers into income classes yield different conclusions. If grouping consumers differently does change the ranking of policies according to the social-welfare-dominance criterion, what policy conclusion should we draw? How are we to choose the "correct" income-class partition?

Let us turn now to the authors' interpretation of their basic result (theorem 1) and, specifically, to condition (15), which it imposes on the real-income changes of the $k$ poorest consumers. Willig and Bailey suggest that their condition incorporates the Rawlsian dictum that the poorest member of society be made better off. If we identify Rawls's view with his difference principle or a maximin social-welfare strategy, then in a pairwise comparison of policies a Rawlsian would prefer the policy that improved the lot of the worst-off person. (Philosophers would object to such a narrow view of Rawls's theory of justice.) Hence, for pairwise comparisons, the authors' interpretation is strictly correct. But, in the
context of choosing one policy from a set of many alternatives, there is a difficulty with viewing the authors' condition as incorporating Rawls's dictum: From the set of all feasible policies, someone following Rawls's maximin principle would choose that course which maximized the wellbeing of the worst-off person. And that policy need not be a social-welfare-dominating one; it need not satisfy the conditions in the authors' theorem 1. This, in fact, provides an illustration of the point Willig and Bailey make when they present "positive and normative views" of their theorems and approach. They write: "Suppose an analyst observes that an SDM has effected a policy move that has resulted in real-income changes ... that fail to satisfy equations (15) and (16) [the conditions of their fundamental theorem]. The move can certainly not be logically criticized on those grounds, even by believers in axioms 1-3. It is possible that the chosen policy was optimal for the SDM's particular preferences, and that those preferences satisfy the axioms."
Since we have mentioned Rawls's maximin criterion, it is worth noting that Willig and Bailey are subject to a question (with an implied criticism) which has frequently been directed at Rawls. It concerns the lexicographic nature of their conditions. How large would the gains of the best-off $n-k$ people in the society have to be before one would be willing to tolerate a very small loss (an $\varepsilon$ loss) in the well-being of the lowest-placed $k$ individuals? To put this another way, is there no conceivable ratio of benefits to costs for society as a whole that would induce one to accept an $\varepsilon$ loss in the partial sum of real-income changes for some group of low-ranking consumers, where the ranking is, as throughout, by real income? Perhaps the answer is that there is no such total benefit-cost ratio that one would accept. But surely this question merits reflection, for if such an overall benefit-cost ratio does exist, we should have some doubts about how readily we can accept the Willig-Bailey axioms.

More generally, the article implicitly raises, but does not address, one fundamental question. The analysis proceeds on the basis of an axiom that sounds weak but is, in fact, quite strong (in the sense that it does a lot of work in generating the results): axiom 3, which stipulates that the social decisionmaker displays "regressive-transfer aversion." It states: "At prices $\mathbf{p}^{0}$, the SDM does not find desirable any transfer of income from a nominally poorer to a richer consumer." But the supposition that the SDM has this view must be based on some conception on his part of what the just distribution of income is and how the distribution at prices $\mathbf{p}^{0}$ compares with that just distribution. The paper provides us with no underlying theory of the justice of the income distribution. To
ask that it provide one is, to be sure, to give the authors a large order to fill. But without an appeal to some theory (be it an existing one or a new one) of what ought to be, and how this compares with what is at any time, it is difficult to evaluate how compelling the Willig-Bailey axioms are. And, as Willig and Bailey recognize, any policy interest their results have must ultimately depend on how compelling one finds the axioms that lead to those results.

One should also ask whether the methodology presented in this paper is, in fact, well suited for application to piecemeal regulatory policy actions, or whether it is not better suited for evaluating complete social structures and states of the world. A decision chosen because it is social-welfare-dominating will generate a new set of nominal incomes and a new set of prices $\mathbf{p}^{0}$, which will, in turn, serve as the basis for the next piecemeal policy decision. The result of all these piecemeal policy steps need not social-welfare-dominate the position from which we begin. Alternatively, consider independent policy decisions being made simultaneously by different regulatory agencies, none of which takes into account the changes any of the others is making. The fact that each agency chooses a policy that social-welfare-dominates the status quo does not guarantee that the sum total of these policy choices will social-welfaredominate the starting position.

While I think that both the implementation issues and the broader questions I have raised about the Willig-Bailey analysis need attention before their results can have substantial impact on policy decisions, the authors have given us a very stimulating and thought-provoking paper.

## Comment

Richard Schmalensee

In this very interesting and well-written paper, Willig and Bailey propose, defend, and apply a general test for the desirability of economic policies that takes into account both the magnitude of net benefits and their incidence. Their basic approach follows that used by Atkinson, by Rothschild and Stiglitz, and by others in analysis of income-inequality measures. (Thus, what is called "social-welfare dominance" here has been called "Lorenz dominance" by Dasgupta, Sen, and Starrett.)

Willig and Bailey consider a class of social-welfare functions that are nondecreasing and symmetric in real incomes and have the property that regressive transfers do not increase welfare. (It follows from work by Rothschild and Stiglitz that this latter condition is implied by quasiconcavity of the welfare function, but it does not imply quasiconcavity.) They show that in a society with $n$ individuals, if a proposed policy would lead to a new income distribution in which the total of the lowest $k$ incomes exceeds the same total in the existing distribution, for all $k$ between 1 and $n$, all social-welfare functions in this class would be increased by adoption of the policy. The policy (or the income distribution to which it gives rise) is said to dominate the status quo. If all these sums would be lower with the policy in effect than in the status quo, all welfare functions in their class would be lowered by adoption of the policy, and the status quo dominates. If some sums are higher and some are lower, neither situation dominates, and the desirability of the policy cannot be determined without more information about the welfare function.

The stochastic dominance literature, to which this approach is intimately formally related, suggests a simple illustration of this test. Suppose that incomes are initially normally distributed across individuals, with standard deviation $\sigma_{m}$. A policy is proposed that would yield a normal distribution of real net benefits across individuals, with mean $\bar{b}$, standard deviation $\sigma_{b}$, and correlation $\rho_{m b}$ with initial incomes. Real incomes after adoption of the policy would then also be normal. It is easy to show that adotion of the proposed policy dominates the status quo in the WilligBailey sense if and only if (a) $\bar{b}>0$ and (b) $\rho_{m b} \leq-\sigma_{b} / 2 \sigma_{m}$. If both these inequalities are reversed, the status quo dominates. Condition (a) always requires positive total net benefits. Condition (b) requires that inequality,
as measured by the standard deviation of real incomes, not be increased. If all individuals receive $b>0$, condition (a) is satisfied, and both sides of the inequality in (b) are zero. If different individuals receive different benefits, so that $\sigma_{b}$ is positive, benefits and initial income must be negatively correlated if the policy is to dominate the status quo. Distaste for risk in the stochastic dominance literature is formally equivalent to distaste for inequality here.

Axiomatic welfare theory of the sort propounded in this paper is potentially useful only to the extent that the axioms chosen reflect widely held beliefs. If this is the case, it is at least conceivable that most policymakers could be persuaded to agree with the axioms, even if in advance of such persuasion their behavior violated those axioms. (The analogy with the axioms of expected utility theory is close but not exact, since, as Willig and Bailey note, actors in the political process may have selfish incentives to take actions that lower social welfare as they perceive it.)

From this perspective, axioms 2 and 3, though certainly as reasonable as most in the literature, seem less than totally compelling. Since Professor Klevorick examined axiom 3 at length, I will concentrate on axiom 2.

Suppose we are choosing between two proposed policies that affect only two people: person $R$, who has initial income of $\$ 20,000$, and person $P$, who has initial income of $\$ 5,000$. Policy 1 would give each $\$ 5,000$ in net benefits, while policy 2 would lower $R$ 's real income by $\$ 10,000$ and raise $P$ 's real income by $\$ 20,000$. Since the two policies lead to the same $e x$ post income distribution, axiom 2 says that the social decisionmaker must be indifferent between them. I would expect most policymakers to prefer policy 1 , however, thereby exhibiting what might be called change aversion. In a real world where habits are not easily altered, there are costs to imposing large shocks on society, but axiom 2 assumes those costs away. Axiom 2 is compelling when one is evaluating hypothetical longrun equilibria; it is less so when one is considering the actual short-run impacts of proposed policies.

In order to apply the basic notions discussed above in a world of many commodities, in which prices change, one needs a definition of real income. As is well known, this concept has a unique meaning only under special assumptions; there is a classic index-number problem here. Willig and Bailey deal with this by choosing a single reference price vector, $\mathbf{p}^{0}$. If the actual price vector is $\mathbf{p}_{1}$, they define a household's real income as its money income plus the Hicksian equivalent variation associated with a price change from $\mathbf{p}^{0}$ to $\mathbf{p}_{1}$. In much of the paper, $\mathbf{p}^{0}$ is taken to be the actual initial price vector, so that net benefit from the price change be-
comes exactly the ordinary equivalent variation. As Willig has shown elsewhere, the choice of a reference price level and the choice between the equivalent and compensating variations do not matter much when policies with small income effects are considered. But if income effects are important enough that the Marshallian surplus approximation is questionable, these initial choices may affect the policy choices implied by the analysis. Willig and Bailey take a reasonable route around the indexnumber problem; they cannot eliminate it.

In the illustrative applications of their test, Willig and Bailey consider total net gains to all individuals originally in various income classes, rather than looking directly at distributions of individual incomes. That is, the operational version of the test requires that the net gain of the lowest income class be positive, that the total net gain of the lowest two income classes be positive, and so on. This provides only an approximate test, since policy may cause people to change income classes. To take an extreme example, consider a society composed only of individuals $R$ and $P$, as described above. Suppose a policy would give $\$ 17,000$ to $P$ and take $\$ 16,000$ from $R$. This passes the operational test as defined above; the gains to $P$ are positive, and the gains to $R$ and $P$ together are positive. But since $R$ is now the low-income individual, with an income of $\$ 4,000$, it is clear that the new income distribution does not dominate the old. For small changes, this sort of switching seems likely to be unimportant, and there is no obvious way to deal with it in applications in which individuals cannot be considered directly. But a problem does remain in principle when broad income classes are treated as individuals.

A few words on the potential applications of theorems 2 and 3 seem to be in order. These theorems state that if aggregate net benefits are positive, a sufficient condition for dominance (as defined above) is that there exist some division of net benefits received by each income class into gross benefits and costs such that the class-specific benefit-cost ratios decline with income. But this is only a sufficient condition, and the division of net benefits that does the trick need have no relation at all to the economic meanings of the terms "benefit" and "cost."

In a two-person society, suppose that a number of prices are changed on commodities consumed mainly by the poorer person, $P$. Suppose $P$ receives $\$ 10$ in benefits from price reductions but loses $\$ 9$ from price increases. The richer person, $R$, gains $\$ 2$ from the price cuts but loses $\$ 1$ from price rises. The policy has produced a dominating income distribution, but the benefit-cost ratio is larger for the high-income $R$ than for the low-income $P$. This is not a curiosum; in the last two rows of table 2.1,
electricity pricing policies are presented that dominate the status quo but do not have declining benefit-cost ratios. Table 2.3 shows a telephone pricing policy with the same properties. If net benefits can be computed in any application, information can only be lost by converting the results to benefit-cost ratios. On the other hand, theorems 2 and 3 may be of considerable value in situations in which qualitative information is available but a complete quantitative analysis is impossible.

Willig and Bailey clearly feel that their dominance criterion can be used to improve actual regulatory policymaking. The pilot applications to electricity and telephone pricing that they present shed some light on the merit of this position. In testing for dominance, both studies employ data on residential customers only. Income classes, rather than individuals, are the focus of the analysis, and class-specific demand elasticities are unavailable. The key data give consumption of the commodities considered by income class at initial prices. Data sets of this sort can presumably be routinely constructed at moderate cost.

Using their proposition 1, Willig and Bailey show that these data can support interesting analyses of the direct effects of price changes on residential customers. The basic tool amounts to a first-order approximation to consumer surplus changes. Suppose that it is proposed to change a vector of prices from $\mathbf{p}$ to $\mathbf{p}+\Delta \mathbf{p}$. Let $\mathbf{X}_{i}$ be the vector of purchases of these commodities by individuals in income class $i$ at prices $p$. Then a first-order approximation to the net benefits received by class $i$ is simply $B_{i}=-\left(\mathbf{X}_{i}\right)^{\prime}(\Delta \mathbf{p})$. As the Willig-Bailey paper shows, if the $B_{i}$ pass the fundamental test for dominance (and if it can be assumed that households do not switch income classes and that households with equal initial incomes satisfy the conditions of proposition 1), then there exists a positive constant, $\hat{t}$, such that the actual net benefits of a change from $\mathbf{p}$ to $\mathbf{p}+t(\Delta \mathbf{p})$ pass this same test as long as $t$ is less than $\hat{t}$. There is no way to know if the original proposal passes this test (that is, to know if $\hat{t} \geq 1$ ) without income-class-specific demand elasticity information. Still, this sort of local analysis can make at least a strong prima facie case for or against proposed changes in prices paid by households.

However, in both pilot applications in the paper the proposed price changes would also affect business customers. The tools presented by Willig and Bailey do not help one to translate changes in firms' costs to changes in real incomes of households in different income classes. These latter changes, which may be important in many applications, must depend on households' various roles as input suppliers and output demanders, and on conditions in the directly and indirectly affected markets. It
may seem plausible, for instance, that increased business telephone rates ultimately fall more heavily on rich households than on poor ones. But to demonstrate this quantitatively or even qualitatively in any particular case would require an elaborate and expensive general equilibrium analysis. Thus, while it is comparatively simple to analyze in a useful way the direct incidence of changes in prices that households pay, it is much harder to assess the total or ultimate incidence of price changes that also affect firms. It appears unlikely that the ultimate incidence of such changes could be considered on a routine basis by regulatory agencies or regulated firms unless new techniques of analysis are devised.

Difficulties of this same sort arise in principle even if one is concerned only with efficiency, of course. In a second-best world, with distortions elsewhere in the economy, precise measurement of the efficiency implications of any proposed price change becomes a very complex undertaking. But, efficiency analysis is greatly simplified if the rest of the economy is plausibly assumed to be approximately competitive. Simple pricing formulas, such as the Ramsey rule that Willig and Bailey employ, follow from this assumption. In analysis of incidence, however, the competitive assumption does not seem to have similar power. Without tractable methods for analyzing the ultimate incidence of price increases affecting firms, the value of the Willig-Bailey methodology to actual decisionmakers is hard to judge.

Even if such methods are devised, problems arise when one considers the consequences of requiring a number of regulatory (or other policymaking) agencies to employ the Willig-Bailey tests. If a large number of agencies attempt to make progressive redistributions, their actions may have non-negligible effects on incentives to supply labor, to invest in skill acquisition, and to supply capital. As the optimal income tax literature has shown, incentive effects of this sort can substantially raise the optimal level of inequality. While such second-order indirect effects of regulatory policies might conceivably be well considered in especially creative academic analysis, regulatory agencies or regulated firms obviously could not do so on a routine basis. Further, as the paper notes, it is possible for two policies, neither of which dominates the status quo, to dominate it if imposed as a package. But individual agencies can only make packages that employ the instruments at their disposal. Suppose for the sake of argument that it is possible to apply the Willig-Bailey test correctly, and that regulators are instructed to adopt only dominant policies. Then two potential policies of the sort described above, in the hands of separate agencies, would likely fail to be adopted without much more coordination
of decisionmaking than seems imaginable. On the other hand, if agencies are instructed only not to adopt dominated policies, a great deal of analytical effort (beyond that necessary to compute total net benefits) might be required per dominated policy detected. There is a case, I think, for centralizing responsibility for equity and decentralizing responsibility for efficiency.

The general message of this paper, that distributional effects deserve consideration in policymaking, is hard to quarrel with. Willig and Bailey propose an interesting and potentially usable test that considers such effects. 1 give them very high marks both for topic selection and for quality of output. Their tools permit one to assess the merit of changes in prices paid by households in a relatively convincing fashion. The value of their contribution would be greatly enhanced if similarly powerful tools were forged for analysis of changes in prices paid by firms.

