Volume Title: Monetary Trends in the United States and United Kingdom: Their Relation to Income, Prices, and Interest Rates, 1867-1975

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9 Division of Change in Income between Prices and Output

On the average of the 102 years from 1873 to 1975, nominal income in the United States rose 4.9 percent per year, divided between a rise of 3.1 percent per year in output and 1.8 percent per year in prices. The corresponding figures for the United Kingdom are 4.0 percent per year in nominal income, 1.7 percent per year in output and 2.3 percent per year in prices. On the average, rising prices accounted for over one-third of the rise in nominal income in the United States, for over one-half in the United Kingdom.

These proportions varied greatly over time. For example, in both the United States and the United Kingdom, prices fell from 1880 to the mid-1890s while nominal income rose, so prices accounted for a negative fraction of the rise in nominal income. In the United Kingdom, output fell during World War I while nominal income rose sharply, so prices accounted for more than the whole of the rise in nominal income.

The theory sketched in chapter 2 to interpret the division of a change in nominal income between prices and output distinguishes between long-run forces that determine what are there called the anticipated or permanent levels of nominal income, prices, and output, and the shorter-run forces that determine the discrepancy between the actual and anticipated levels. It regards the long-run behavior of output as determined primarily by real factors and of prices primarily by monetary factors. The shorter-run discrepancies between actual and anticipated levels cannot be so sharply dichotomized. The shorter-run movements in both prices and output were treated in chapter 2 as depending on both monetary and real factors, and as reflecting an adjustment process involving a feedback from past observed values.

That chapter was silent about the chronological counterpart of "short run" and "long run"—as is economic theory in general, since the distinction is analytical and its chronological counterpart may vary from problem to problem or country to country or time to time. In applying the theory of chapter 2 to our phase-average data on prices and output, we implicitly treat the short run as corresponding to changes over periods longer than a phase (which averages two years in duration for the United States, 2.8 years for the United Kingdom). We are encouraged to proceed in this way by the results of earlier chapters, which show that the adjustment to monetary disturbances is fairly slow, even in terms of phases. That confidence is reinforced by the results of this chapter. Nonetheless, there is no assurance that the conclusions we derive for phases will apply, either in general or in detail, to the intracycle movements lasting a few months or a few quarters. Though this caveat is equally applicable to earlier chapters, it seems especially relevant to this one, since the interplay between movements in output and prices has been at the heart of much business-cycle analysis.

As we explained in chapter 2, the simple Keynesian and quantity theory hypotheses can be regarded as special cases of the more general theory. So also can a third simple theory: namely, that the division of a change in nominal income between prices and output depends only on the size of the change in nominal income (or what comes to the same thing, depends on the same variables as determine the change in nominal income, and no others).[1] This third simple theory can be regarded as a particular extension to prices and output of the monetary theory of nominal income.

Section 9.1 examines these three special cases in their most rigid form and shows that all can be rejected, though the simple quantity theory hypothesis conflicts with the evidence less than either of the others.

In the course of section 9.1, we uncover a tendency for observed price and output changes to be correlated negatively instead of positively, as is typically assumed in theories of general economic fluctuations. We explore this tendency in section 9.2, with special reference to purely statistical reasons for the observed phenomenon.

Section 9.3, like section 9.1, is a preliminary empirical exploration of one feature of the theoretical analysis, namely, the division between "long run" and "short run." It explores that feature by examining how the division of a change in nominal income between prices and output is affected by lengthening the period averaged in computing rates of change. The results are suggestive but inconclusive.

Section 9.4 constructs a framework for the further analysis of the more general theory embedded in equations (12) and (13) of chapter 2, which allows both anticipations about prices and deviations of actual output

1. In the linearized version in chapter 2, this special case is obtained by setting $\xi = 0$ in equations (12) and (13).

from potential output to affect the division of changes in nominal income between prices and output.

That framework links the analysis of the present chapter with that of chapter 8. Sections 9.5 and 9.6 apply to prices and output separately two special cases analyzed for nominal income, one that allows for the effect of the current variables affecting nominal income—money and yields (sec. 9.5); the other that expresses these variables in terms of current and prior money and prior income (sec. 9.6). These sections use the same variables for prices and output separately as were used in chapter 8 for nominal income.

Sections 9.7 and 9.8 bring into the analysis variables special to the division of nominal income between prices and output—in particular, anticipations about inflation and the degree of utilization of capacity. The results suggest that inflation anticipations are far more important in determining the rate of price change than the level of utilization of capacity.

9.1 Alternative Simple Explanations

The scatter diagram in chart 9.1 depicts the variable contribution of prices and output to changes in income. The chart plots phase rates of change of prices and output for both the United States (squares) and the United Kingdom (triangles), and the mean rates of change for the United Kingdom and the United States. War phases are circled.

The scatter diagram enables us to reject a number of possible simple explanations of the division of the change in nominal income between prices and output.

One possible explanation is that the same variables that determine the change in nominal income determine changes in prices and output and that they do so in such a way as to establish a one-to-one correspondence between the change in nominal income and in prices and output separately. In that case the changes in prices and output separately could be predicted from the change in nominal income and the points in chart 9.1 for each country would fall on a single curve—in the simplest case, that of strict proportionality between changes in prices and output, on a straight line through the origin and the point designating the mean rates of change. Clearly, the points do not conform to that explanation.

A second possible explanation is the simple Keynesian view that a change in nominal income is wholly absorbed by output until the "point of full employment" is reached, and by prices thereafter. In that case rapid rates of growth of output (in the limit, infinite) would be associated with low rates of growth of prices (in the limit, zero), and slow rates of growth of output (in the limit, zero) with high rates of growth of prices (in the limit, infinite). In terms of chart 9.1, the points would tend to cluster

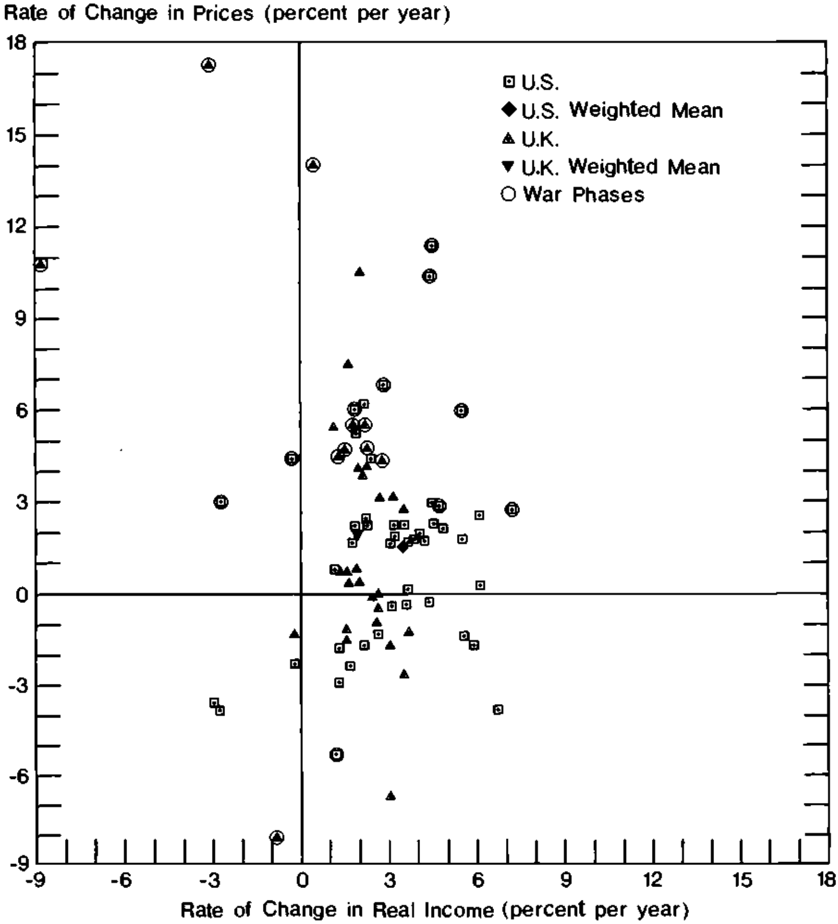


Chart 9.1 Scatter diagram of rates of change of real income and implicit prices, United States and United Kingdom, 1873–1975.

either in the lower right-hand corner (high output growth, low price growth), or in the upper left-hand corner (low output growth or decline, high price growth), with few or no points between the two clusters. Clearly, the points do not conform to that explanation either.

A third possible explanation is the simple quantity theory view that changes in output and prices are independent of one another; that changes in output respond to changes in resources and technology that alter the feasible output of the economy, while changes in prices respond to changes in the quantity of money per unit of output. In that case, changes in output would tend to be independent not only of changes in the price level but also in the quantity of money. The points in chart 9.1 conform more closely to this explanation than to either of the others, since the correlation between price change and output change is clearly small. Yet they do not conform fully to this explanation either, though for rather different reasons for the United States and the United Kingdom.

For the United States, the correlation between prices and output is positive for the period as a whole, including or excluding wartime phases, and for two out of three nonwar subperiods; though for the pre–World War I subperiod, the correlation is close to zero (.06) (table 9.1). The other three positive correlations differ significantly from zero at a .05 or .01 level. The one jarring note is for the period after World War II, when the correlation not only is negative, but differs significantly from zero. The correlation between money and output is consistently positive, though close to zero (.03) for the post–World War II period, and is consistently higher than that between prices and output. All in all, the hypothesis that output changes are independent of changes in prices and money must be rejected.

For the United Kingdom, the correlation between prices and output is consistently negative, though only the correlation for the period as a whole differs significantly from zero at a .05 level. The correlation between money and output is negative for the period as a whole and for the

Table 9.1 **Correlations between Phase Rates of Change in Money,[a] Nominal Income, Prices, and Output**

Period and Country	Rates of Change in Money and			Rates of Change in Prices and Output	Number of Observations
	Nominal Income	Prices	Output		
Full period					
United States	0.94**	0.88**	0.64**	0.36*	47
United Kingdom	0.91**	0.92**	−0.24	−0.33*	35
Full ex-wars					
United States	0.96**	0.83**	0.79**	0.43**	37
United Kingdom	0.92**	0.92**	0.03	−0.08	25
Pre–World War I					
United States	0.80**	0.78**	0.37	0.06	19
United Kingdom	0.90**	0.57	0.49	−0.33	9
Interwar					
United States	0.97**	0.88**	0.95**	0.78*	7
United Kingdom	0.80*	0.95**	−0.32	−0.28	7
Post–World War II					
United States	0.95**	0.76**	0.03	−0.60*	11
United Kingdom	0.98**	0.98**	−0.41	−0.53	9

[a]Money series adjusted for effect of postwar readjustment and upward demand shift.
*Significant at .05 level.
**Significant at .01 level.

interwar and postwar periods, essentially zero for the nonwar phases, and positive for the pre–World War I period. However, none of these correlations differs significantly from zero at the .05 level. All in all, the United Kingdom comes closer to conforming to the simple quantity theory than the United States, but the consistent negative relation between prices and output and occasionally even between money and output gives pause before that interpretation can be accepted.

Chart 9.2, which replots the points in chart 9.1 as time series rather than as a scatter diagram, reflects these correlations. For the United States there is a positive correlation between rates of change in prices and output up to Word War II, when it becomes equally clearly negative. For the United Kingdom there is a negative correlation throughout, though it is much more marked for the post–World War II period.

One other feature brought out by chart 9.2 is the greater stability of output growth in the United Kingdom than in the United States—certainly if World War I is excluded. As table 9.2 shows, the standard deviation of the rate of change of output is consistently higher for the United States than for the United Kingdom. The difference is least for the post–World War II period. For prices the situation is more complicated. Prices were decidedly more stable in the United Kingdom before World War I, somewhat less stable during the interwar period, and much less stable in the post–World War II period. The roughly equal standard deviations for the United States and the United Kingdom for the period as a whole simply average out these substantial differences between the subperiods.

For the United Kingdom, prices are consistently more variable than output; for the United States that is also true for the post–World War II period, trivially so for the pre–World War I period, and the reverse for the interwar period. The greater importance of price change relative to output change for the United Kingdom than for the United States is a phenomenon that we shall document repeatedly in later sections of this chapter.

The much stabler output for the United Kingdom reinforces the impression from chart 9.1 that the United Kingdom comes closer to conforming to the simple quantity theory than does the United States. The changing pattern of price stability presumably reflects the changed role of the United Kingdom in the world monetary system discussed in chapter 7. Before World War I, the United Kingdom was the center of the international monetary system; changes in internal prices generated a rapid response in its international payments, so it had little leeway for departing from the pattern of world prices in gold: the "law of one price" operated with full force. The United States was more isolated; it could have a more idiosyncratic pattern of price movement; hence prices were less stable in the United States than in the United Kingdom. The interwar

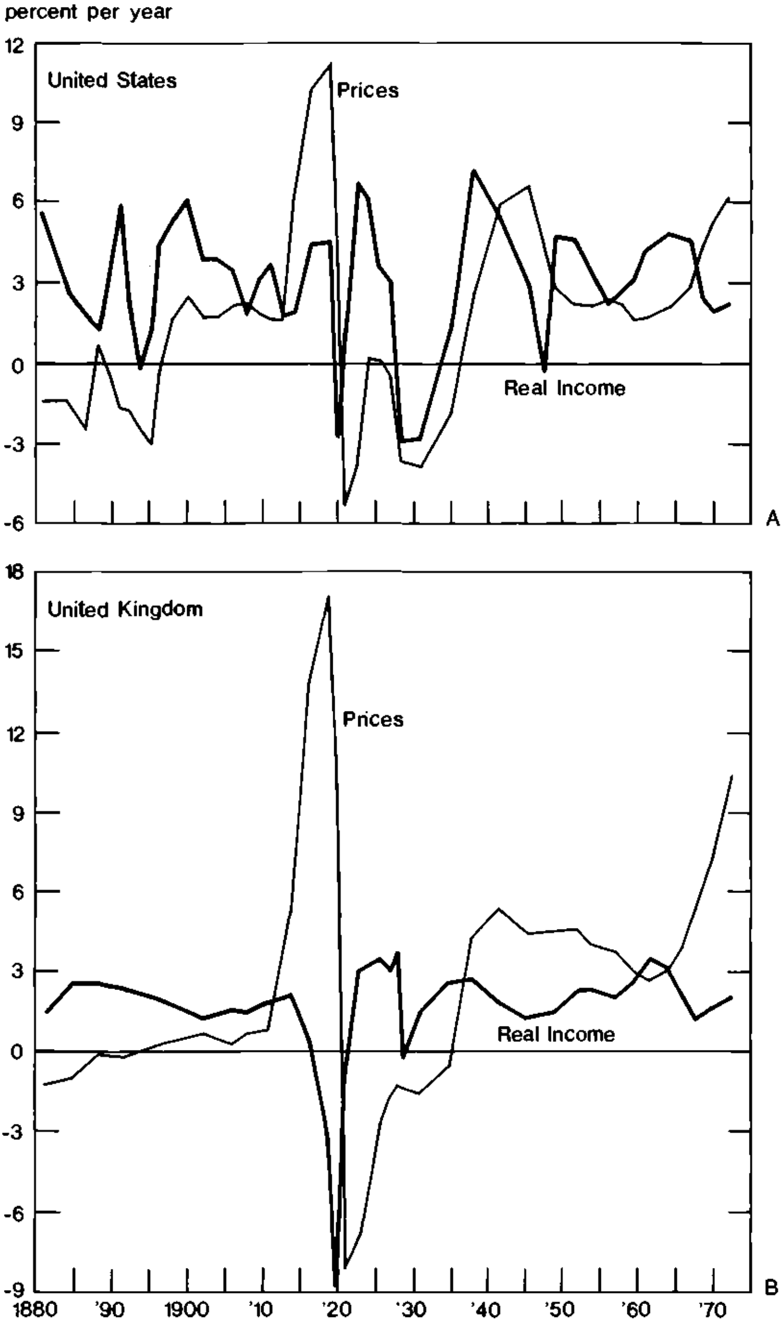


Chart 9.2 Time series of rates of change of real income and prices, United States and United Kingdom, 1880–1975. Points are plotted at the midpoint of central phase of the triplet from which rate of change is computed.

Table 9.2 Variability of Phase Rates of Change in Money,[a] Nominal Income, Prices, and Output

Period and Country	Standard Deviation of Rates of Change: Money	Nominal Income	Prices	Output	Number of Observations
Full period					
United States	4.95	5.10	3.53	2.61	47
United Kingdom	3.55	3.33	3.52	0.95	35
Full ex-wars					
United States	4.00	4.33	2.54	2.58	37
United Kingdom	2.55	2.52	2.49	0.63	25
Pre–WW I					
United States	2.12	2.53	1.75	1.73	19
United Kingdom	0.81	0.73	0.71	0.52	9
Interwar					
United States	4.40	4.56	1.51	3.28	7
United Kingdom	2.32	2.00	2.03	1.03	7
Post–WW II					
United States	1.51	1.18	1.44	1.10	11
United Kingdom	4.02	2.29	2.58	0.70	9

[a]Money series adjusted for effect of postwar readjustment and upward demand shift.

period was a period of transition, though the United States was definitely assuming a larger role. Equally important,while the United States stayed on the prewar gold standard until 1933, the United Kingdom was on the prewar gold standard only between 1925 and 1931. So price variability in the United Kingdom was moderately greater than in the United States. After World War II the United Kingdom's role in the international financial system continued to decline, while that of the United States grew, as the world—at least until 1971—essentially adopted a dollar standard. The United States was now more strongly affected by the "law of one price"; the United Kingdom had greater autonomy, reflected in occasional changes in official exchange rates before 1971, and subsequent floating thereafter, and in the control of foreign exchange transactions (until 1979). One result was that price variability increased in the United Kingdom and decreased in the United States.

9.2 Price and Output Correlations

Most students of cyclical fluctuations doubtless share our own initial expectation that on the whole price and output changes are positively correlated; yet the evidence for the two countries combined is that the phase rates of change more frequently show a negative than a positive correlation. What explains this result?

On analysis, the phenomenon that turns out to need explanation is less the empirical result than the initial expectation. There are strong statistical and economic reasons to expect a negative relation.

Consider first the statistical reason. The three magnitudes—nominal income, real income, and prices—are not statistically independent. In general, nominal income and one of the remaining two magnitudes are independently calculated, and the third is obtained by division. As a result, errors of estimate of real income and prices are negatively correlated.

The economic reasons are twofold. The first is essentially the same as the statistical reason. Any economic force that impinges autonomously on prices or output will tend to produce a negative relation between them. A good or bad harvest, for example, that raises or lowers real output without affecting, at least for the time being, the quantity of money or any other determinant of nominal income will tend to affect prices in the opposite direction. Only those autonomous influences that affect nominal income spill over to both prices and output and introduce a positive correlation.

But even the influences that affect nominal income have some effects that may reduce or reverse the positive correlation. This second economic reason, which is much more complex, arises from the difference in the temporal reaction pattern of output and prices to autonomous changes in nominal income produced by monetary, or indeed, other forces. In general, output is affected sooner than prices: an initial acceleration in nominal income, for example, leads to an acceleration in output after a brief lag (about six to nine months for the United States and the United Kingdom) and has little effect initially on prices. Later the impact shifts to prices (after about another fifteen to twenty months for the United States and the United Kingdom). As prices take over, output decelerates in response. The positive correlation between prices and output imparted by a change in nominal income thus tends to be offset by the temporal differences in response.

It follows from these considerations that a positive correlation is to be expected only when the autonomous forces affecting nominal income are sufficiently dominant to overcome both the statistical and the economic forces making for a negative relation. And that is precisely what our results show.

The only significant positive correlation between rates of change of prices and output for a subperiod is for the United States for the interwar period (0.78), and that is also the subperiod for which the standard deviation of the rate of change of nominal income (and also of money) is the highest: 4.56 percentage points, almost twice as high as the next highest, 2.53, which is for the United States pre–World War I, the only other subperiod for which the price-output correlation is positive(.06) (tables 9.1 and 9.2). At the other extreme, the negative price-output correlation that is largest in absolute value is for the United States for the post–World War II period (– .60), and that is the subperiod for which the standard deviation of nominal income (and also of money) is next to the lowest(1.18). The lowest for both money and income is for the United Kingdom pre–World War I.

A glance at chart 9.2 for the United States reinforces and amplifies this explanation. For the period before World War I, the positive correlation reflects the deep depression of the 1890s—an episode when there were exceptionally wide fluctuations in nominal income as a result of autonomous forces—in our opinion, predominantly monetary—affecting nominal income. For the rest of the period the correlation appears negative. The essentially zero correlation for the whole pre–World War I period reflects an averaging out of the positive correlation from the end of the nineteenth century through the beginning of the twentieth, and the negative correlation before and after that episode.

For the interwar period, two similar episodes—the sharp contractions of 1920–21 and of 1929–33 and their aftermath—are the source of the positive correlation. Again, these are episodes that were characterized by exceptionally wide fluctuations in nominal income as a result of autonomous forces—in our opinion, again predominantly monetary—affecting nominal income.

We suspect that these episodes—so dramatic and so important—explain our initial expectation that the correlation would be positive.

The one puzzle that remains from tables 9.1 and 9.2 is the relation between the variability of money and of income. As the preceding paragraphs indicate, we interpret fluctuations in the rate of change of nominal income as associated with fluctuations in the rate of change of money. As table 9.2 indicates, there is in general a close relation between the standard deviations of money and of income. However, the relation for the post–World War II period differs from that for the earlier periods. For the United States, income is more variable than money for the pre–World War I and interwar periods; money is more variable than income for the post–World World War II period. For the United Kingdom, money is uniformly more variable than income, but the difference is small for the first two periods—11 percent for the pre–World War I period, 16 percent for the interwar period—much larger for the post–World War II period—76 percent. For both countries, therefore, the post–World War II period displays enhanced variability of money relative to income.

Chart 9.3 suggests that the explanation for this phenomenon is the length of time that it took for both countries to readjust to the major wartime disturbance of the monetary relations. In both countries that disturbance took the form of a sharp decline in the velocity of circulation

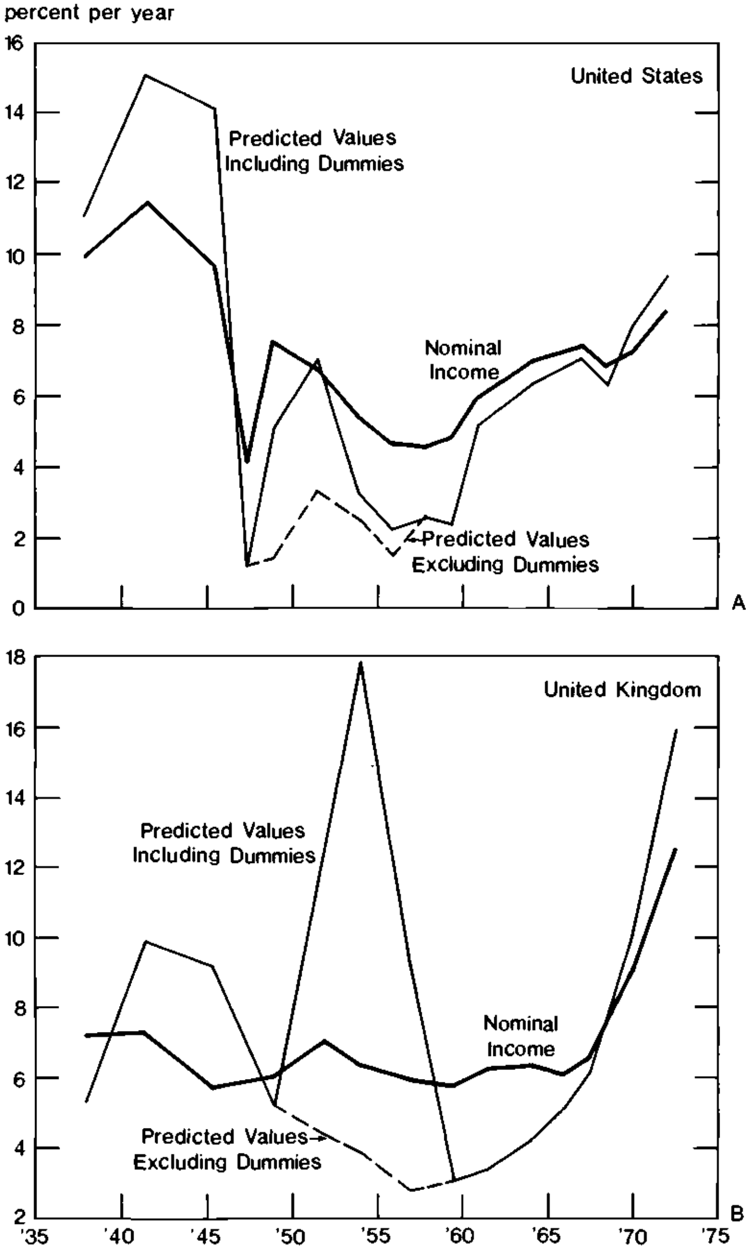


Chart 9.3 Actual and predicted rates of change of nominal income, post-1937 phases (predicted from regression of pre-1937 phases of rates of change of nominal income on rates of change of money in current and three prior phases).

of money—that is, a larger rise in money than in income. The two panels of the chart plot the actual rates of change of income and the rates of change predicted from a regression relating the rate of change of income to the rate of change of money in the current and three prior phases and based solely on data for the period before Word War II.[2]

Two sets of predicted values are plotted for the postwar phases affected by postwar readjustment and upward demand shift. The points connected by a solid line allow for these shifts on the basis of the effects of the corresponding shifts after World War I. Since those estimated effects are very unreliable, because based on very few observations, we also plot the points connected by a dashed line, which make no allowance for the shifts. Allowance for the shifts improves the predictions for the United States. For the United Kingdom the allowance is in the right direction but excessive in magnitude. For our present limited purpose, the predictions that neglect the shifts are the more illuminating.

For both countries the predicted pattern is similar to the actual pattern but differs in amplitude. The predicted is above the actual during wartime—the counterpart to the decline in velocity—and then below the actual after the war—the counterpart to the recovery in velocity. Income did not rise as much as might have been expected on the basis of monetary growth during the war, but it rose more after the war. The reaction took a long time in the United States and even longer in the United Kingdom. In the United States, the reaction was largely completed by the early 1960s, in the United Kingdom, not until the end of the 1960s.

Once the reaction was completed, the actual is remarkably close to the predicted, especially for an extrapolation of more than twenty-five years for the United States and more than thirty years for the United Kingdom. A particularly remarkable feature is that in both countries the agreement is closer at the end of the period of extrapolation than at the beginning—a rather impressive testimonial to the stability of the link between money and income. The wider fluctuation of the predicted rates of change of income than of the actual means that the variability of money relative to predicted income would have been decidedly less than its variability relative to actual income. It follows that the wider relative variability of

2. Whereas in most of chapter 8 and the rest of this chapter we have used rates of change of money adjusted for the postwar readjustment and upward demand shift, for the present purpose we did not, but rather included dummy variables for these shifts in the regression equation fitted to the pre–World War II data. The reason is that the parameters used in calculating the adjusted money figures are based on regressions for the period as a whole and hence the use of the adjusted money figures would mean that the predicted values were not based on completely independent observations.

The earliest rate of change observations we have for money are for phases centered on 1870 for the United States and 1881.5 for the United Kingdom. Accordingly, the period covered by the dependent variable in these regressions is 1878 to 1937 for the United States, 1890 to 1937 for the United Kingdom.

money after World War II reflects the war and its aftermath rather than a fundamental structural change.

9.3 The Effect of Lengthening the Period

In *A Monetary History* we concluded that, over long periods, differential rates of monetary growth are reflected primarily in differential rates of inflation and have little effect on output whereas, over brief periods, differential rates of monetary growth affect both prices and real income.[3] That is also the result implied by the theory of chapter 2. The question is, How brief is "brief"?

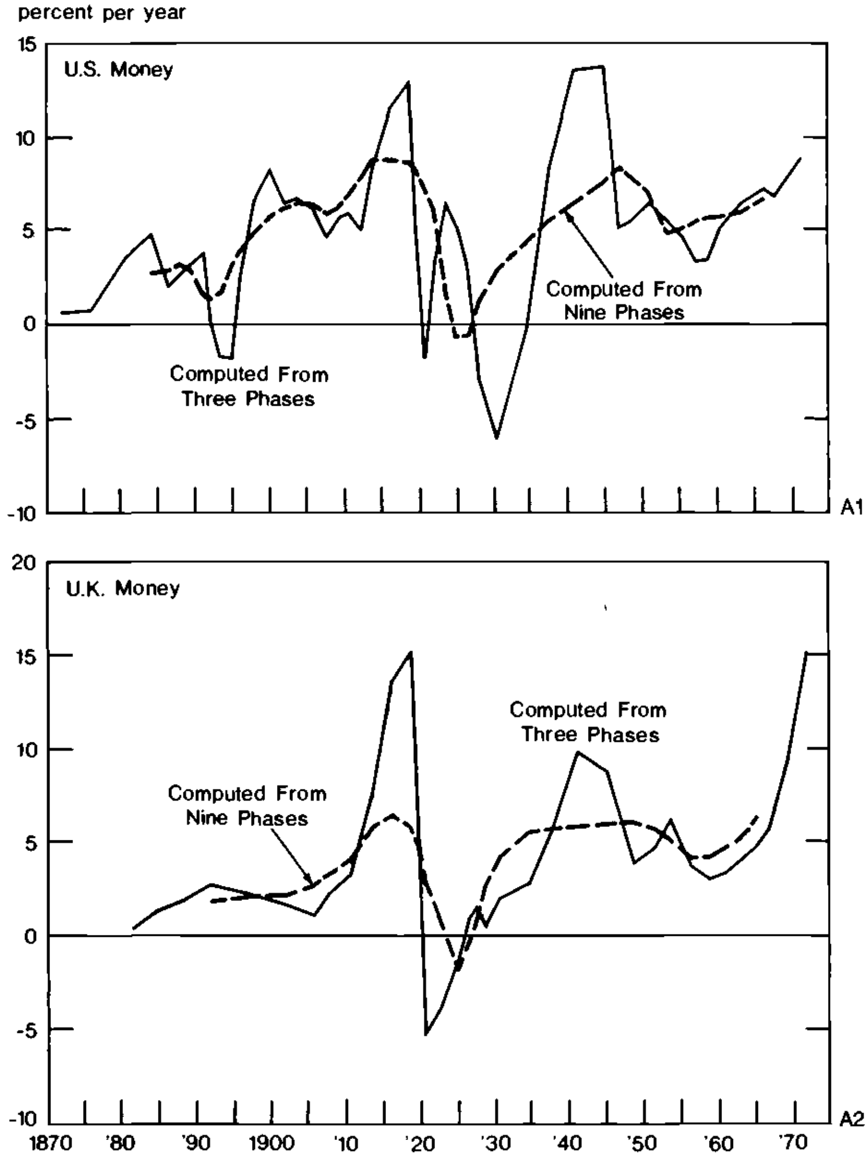
As a first step to answering that question, we have computed rates of change from groups of four, five, six, seven, eight, and nine successive phases, so moving from the average span of about four years for the United States or six years for the United Kingdom for our standard rates of change computed from triplets of phases to an average span of about seventeen (United States) or twenty-three (United Kingdom) years for rates of change computed from nonets of phases.[4] The rates of change for triplets and for the longest periods, nonets of phases, are plotted in chart 9.4—for money in A panels, for nominal income, real income, and prices in B, C, and D panels. Some numerical results are given in table 9.3 for all phases.

Lengthening the period smooths out the minor perturbations in the money and nominal income series and significantly raises the simple correlation between them: from .94 for triplets to .99 for nonets for the United States, from .91 to .94 for the United Kingdom. Of more interest for the present purpose, lengthening the period sharply reduces the amplitude of the movements in real income relative to those in both prices and money for the United States but not for the United Kingdom. For the United States, the standard deviation of output is 74 percent that of price for triplets, 39 percent for nonets; for the United Kingdom, the corresponding percentages are 27 and 29.[5] In this sense our generalization in *A Monetary History* is confirmed by the charts and the table for the United States but not for the United Kingdom—another manifestation of the difference we have been finding between the United States and the

3. *A Monetary History*, pp. 694–95.

4. The interval between the first and last phase in a rate of change computed from i phases is $(i-1)\bar{n}$, where $\bar{n}$ is the average length of a phase, which is 2.1 years for the United States, 2.8 for the United Kingdom.

5. An interesting detail is that, to the three decimal places given in the table, the standard deviation of prices is identical for the United States and the United Kingdom for each of the seven period lengths—differences appear only in the (unreported) fourth decimal place. This coincidence is a minor but impressive testimonial to the international character of price movements.


percent per year
15
10
5
0
-5
-10
U.S. Money
Computed From
Three Phases
Computed From
Nine Phases
A1
20
15
10
5
0
-5
-10
U.K. Money
Computed From
Nine Phases
Computed From
Three Phases
A2
1870
'80
'90
1900
'10
'20
'30
'40
'50
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'70

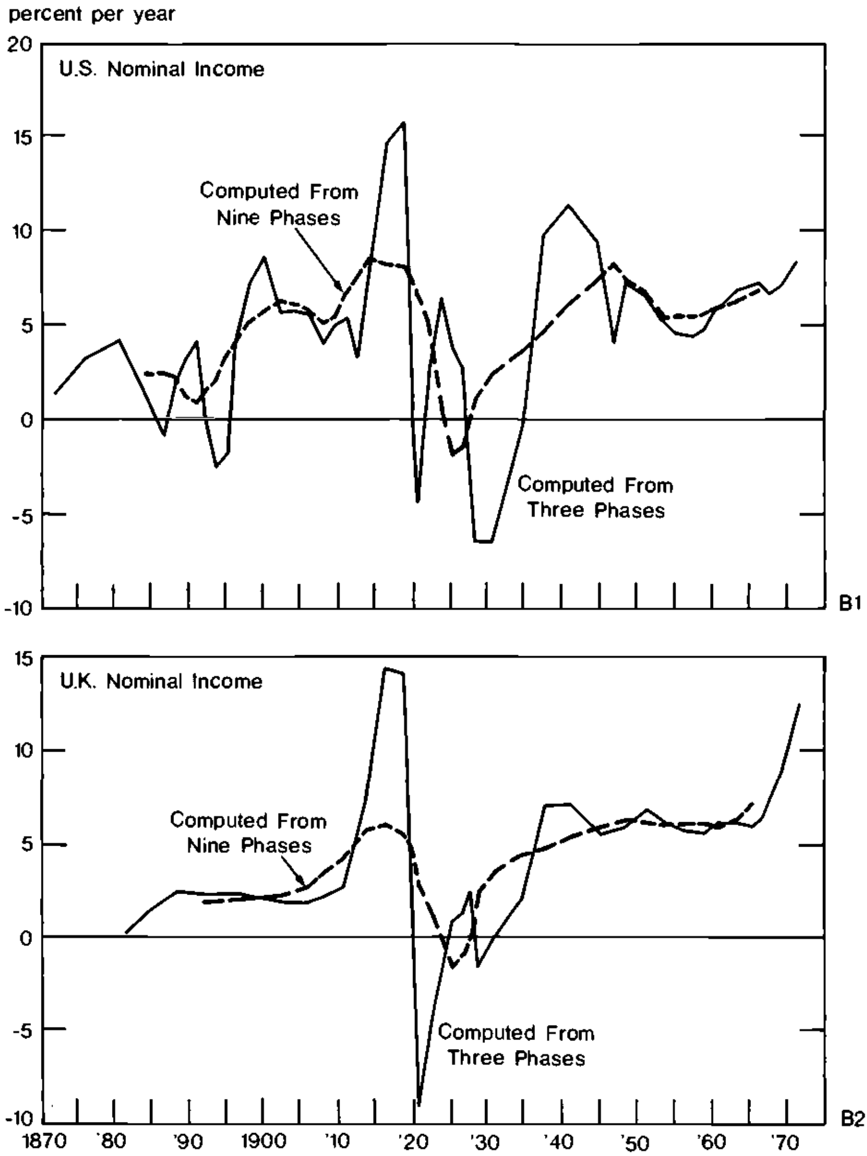
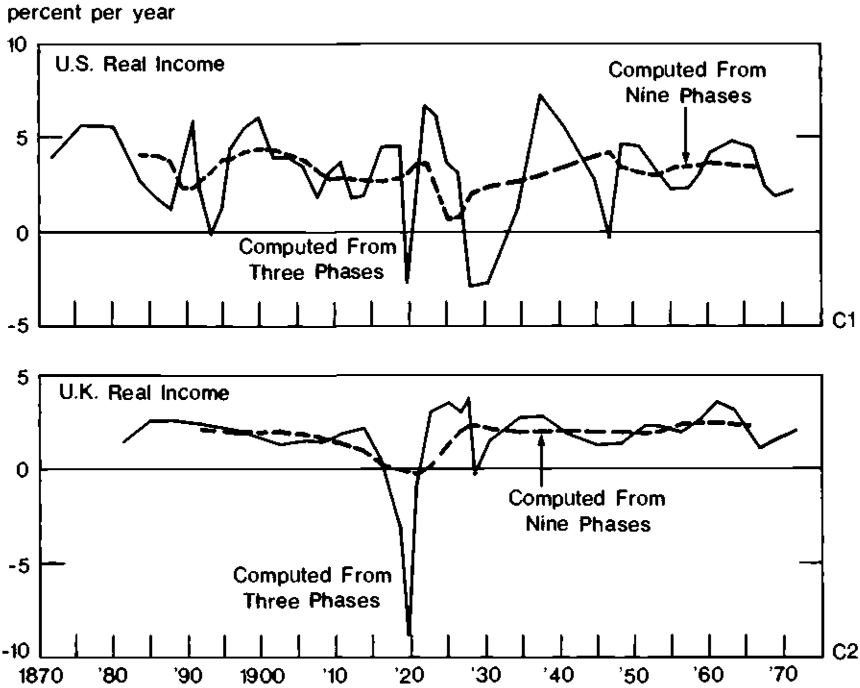


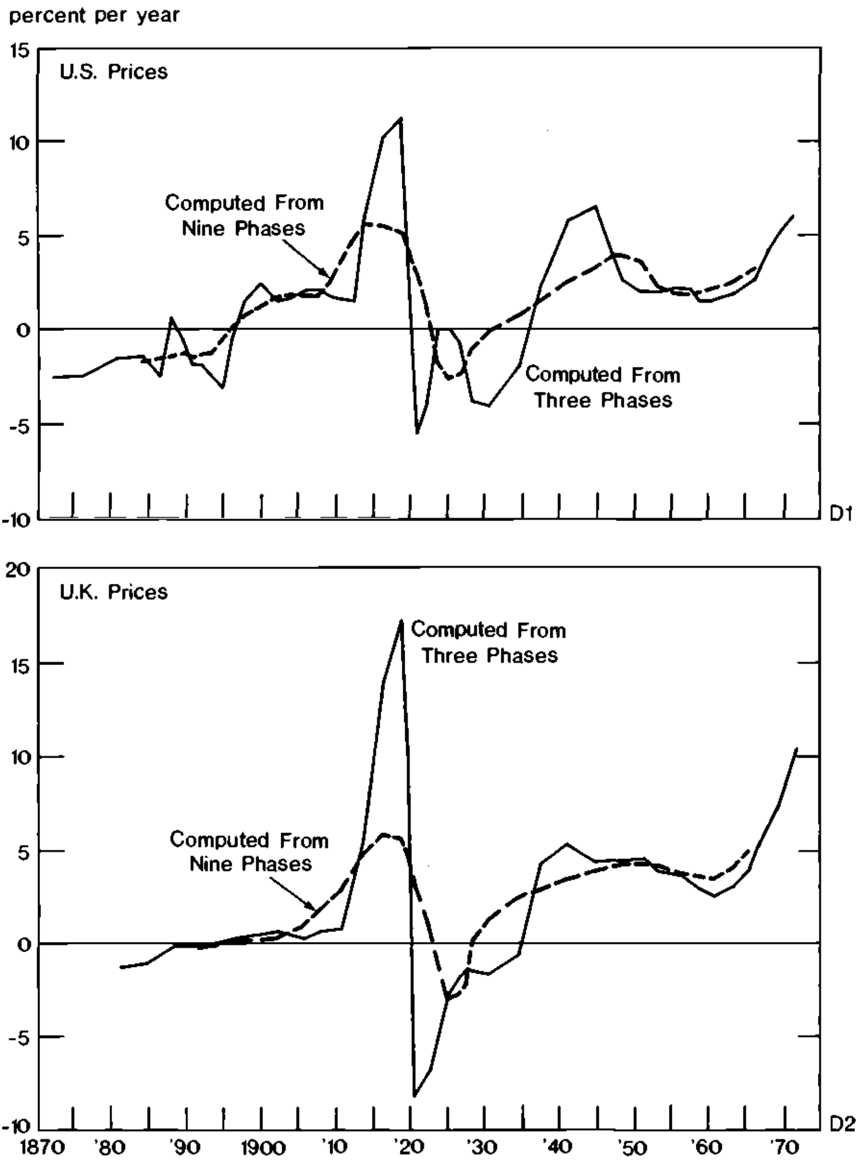
Chart 9.4 Rates of change computed from three- and nine-phase average values, for money, nominal income, real income, and prices, United States 1870–1975, United Kingdom 1881–1975.



United Kingdom in respect of the relative price and output reactions to monetary change. For the United States, the residual mild fluctuations in the rate of growth of real income remain positively correlated with those in money. The simple correlation for nonets is .70, the partial correlation is .80, holding prices constant. For the United Kingdom the simple correlation for nonets is negligible and negative (−.10), though the partial correlation, holding prices constant, is positive (.47), as in the United States.

These results seem to contradict our earlier generalization that over long periods rates of monetary growth have little effect on output. A reconciliation is suggested by the dating of the residual fluctuations in the rate of change of real income (see chart 9.4, C panels). For the United States, the residual fluctuations reflect primarily the impact of the deep depressions in the 1890s, 1920s, and 1930s; for the United Kingdom, the one major residual fluctuation reflects the effect of the First World War. In each case the smoothing process has spread the influence of the deep depressions and wars over a considerable period, has shifted the dating of the turning points, and, by averaging out the smaller fluctuations, has given a larger role to these major episodes.

For the United States, the positive correlation between monetary and real income changes for nonets of phases can therefore be regarded as a reflection of another of our major generalizations in *A Monetary His-*



tory—the one-to-one relation between severe economic contractions and severe monetary contractions, a cyclical relation of such importance that it takes more than a seventeen-year period to average it out.

The generalization that sustained monetary changes are reflected primarily in prices rather than in real income is strongly supported by the final columns of table 9.3, which give regression coefficients and hence measure quantitative effects. For the United States, a one percentage point change in the rate of monetary growth for a three-phase span is

Table 9.3 **Rates of Change Computed from Varying Number of Phases: Selected Statistical Characteristics of Money,[a] Nominal Income, Prices, and Real Income**

Number of Phases	Standard Deviations				Correlation Coefficients				Partial Correlation Coefficients			Regression Coefficients			
	σ_M	σ_P	$\sigma_{y'}$	σ_Y	r_{MY}	r_{MP}	$r_{My'}$	$r_{Py'}$	$r_{MP \cdot y'}$	$r_{My' \cdot P}$	$r_{Py' \cdot M}$	b_{PM}	$b_{y' M}$	$b_{PM \cdot y'}$	$b_{y' M \cdot P}$
							United States								
3	.050	.035	.026	.051	.937	.882	.638	.363	.906	.723	−.548	.627	.337	.780	.752
4	.043	.032	.021	.045	.944	.887	.650	.385	.907	.723	−.544	.658	.314	.817	.697
5	.038	.029	.016	.039	.966	.902	.695	.421	.934	.805	−.664	.688	.305	.900	.742
6	.033	.026	.013	.034	.978	.918	.722	.460	.954	.852	−.741	.725	.290	.967	.768
7	.029	.024	.011	.030	.982	.934	.724	.490	.963	.853	−.752	.758	.268	.987	.771
8	.026	.022	.009	.028	.984	.948	.716	.515	.968	.834	−7.36	.786	.248	.986	.779
9	.024	.021	.008	.026	.985	.960	.700	.530	.972	.803	−.706	.810	.233	.974	.806
							United Kingdom								
3	.036	.035	.010	.033	.906	.921	−.240	−.334	.919	.184	−.298	.914	−.064	.885	.120
4	.032	.032	.008	.030	.926	.930	−.269	−.396	.931	.296	−.413	.937	−.072	.894	.199
5	.028	.029	.008	.026	.944	.939	−.253	−.412	.947	.427	−.524	.977	−.071	.928	.319
6	.025	.026	.007	.024	.950	.942	−.238	−.406	.952	.466	−.554	1.003	−.068	.955	.360
7	.022	.024	.006	.022	.948	.942	−.211	−.374	.952	.450	−.531	1.019	−.061	.977	.359
8	.020	.022	.006	.021	.946	.940	−.168	−.331	.951	.447	−.517	1.032	−.050	.999	.374
9	.019	.021	.006	.020	.944	.937	−.102	−.278	.950	.470	−.523	1.040	−.033	1.019	.412

[a]Money series adjusted for effect of postwar readjustment and upward demand shift.

accompanied on the average by a 0.63 percentage point change in prices and a 0.34 percentage point change in real income (the simple regression coefficients), so that even for a span of four years prices account on the average for nearly two-thirds of the total effect. The sum of the two effects gradually rises from 0.96 for triplets to 1.04 for nonets, and the relative importance of the price effect rises, so that for nonets prices account for nearly four-fifths of the total effect. For the United Kingdom the results are even more clearly in line with the generalization. The output effect is throughout negative and small, while the price effect rises from 0.91 percentage points to over 1 percent, and, as in the United States, the sum rises from 0.85 for triplets to 1.01 for nonets. These results reflect the consistent finding that the United Kingdom tends to come closer to conforming to a simple quantity theory than does the United States.

The changes recorded in the table proceed continuously so that they do not permit a sharp demarcation between "brief" and "long"—the question we started with. We shall explore this question further, and in a more sophisticated way, in later sections.

9.4 Framework for Further Analysis

The theory outlined in chapter 2 implies that a permanent one percentage point increase in the rate of monetary growth will ultimately be reflected in a one percentage point increase in the rates of growth of both nominal income and prices, leaving the rate of growth of output unchanged.[6]

The proposition is a special case of the more general view that economic actors are concerned with real variables and do not consistently err in their estimates of real variables because of purely monetary changes. Money illusion, to use the earlier terminology, is a transitory phenomenon, if it occurs at all. Expectations, to use more recent terminology, are rational.

On this view, the average growth of output over long periods is determined by real factors such as natural resource endowment, social institutions, human capacities, technology, invention, enterprise, and thrift. It is independent of *anticipated* changes in nominal magnitudes except as they affect real magnitudes—for example, the real interest rate or the real quantity of money.

Over shorter periods as well, output growth will be affected by real factors. However, over such periods output will also be affected by

6. As to level of output, the effect would be to reduce the level of output properly measured because the higher cost of holding money will lead to smaller real balances (higher velocity) and hence to a lower stream of productive and nonpecuniary services of money balances (see M. Friedman, *Optimum Quantity of Money*).

unanticipated changes in nominal magnitudes. Deviations of nominal income from its anticipated growth path, produced by deviations of monetary growth from its anticipated path, will produce deviations in output from the path that would be mandated by real factors alone.

The real factors that determine the potential output of an economy at any point in time generally change slowly and gradually. For both the United States and the United Kingdom, the absence of any secular trend in the rate of growth of output indicates that the effect of these trend factors on the growth of potential output has been roughly constant over the century we study.[7]

Occasionally real factors do affect output and output potential appreciably over shorter periods. For the period we study, the most important have been wars and, at the very end of the period, the emergence of the Organization of Petroleum Exporting Countries (OPEC) and the accompanying drastic alteration in the conditions of supply of crude oil. War affects output during active hostilities and, in addition, the destruction of physical and human capital reduces potential output for at least a time after the initial period of reconversion. The impact of OPEC is similarly capable of being long lasting. However, that episode came too late to affect our results appreciably.

Other real factors that affect output over short periods, such as weather conditions affecting agricultural output and construction, labor disputes and similar temporary disruptions of supply, or the bunching of new innovations, can almost surely be neglected for our purpose. Our data are for the United States and the United Kingdom as a whole, which averages out many local real elements, and for cycle phases as a whole, which further averages out transitory and cyclical phenomena. Of course the averaging out is far from complete. The Great Depression leaves an unmistakable impress on our data. But we are inclined to interpret that depression as an example of a major economic movement produced primarily by monetary rather than real phenomena.

Hence, in analyzing in more detail the division of a change in nominal income between price and output change, we shall neglect all real factors other than wars plus those that operate slowly or gradually. We shall allow for wars by presenting results only for nonwar phases. That restriction recommends itself also for a different reason—the questionable reliability of the price series during wars even after we adjust them for price controls.[8] We shall allow for the real factors that operate slowly by including a trend term in equations based on observations for levels and a constant term in equations based on rates of change.

7. The correlation coefficient between the phase rates of change of output and time is −.084 for the United States, .022 for the United Kingdom, values that would be exceeded by chance well over half the time if the true correlation were zero.

8. That problem also arises with respect to our post–World War II observations, but we regard it as less serious.

Although we neglect other real factors that affect the division of a change in nominal income between prices and output, we do not assume away changes in output. On the contrary, we neglect these real factors in order to concentrate attention on the effect of *unanticipated* changes in money and nominal income on fluctuations in output.

Unanticipated changes in nominal income alter the demand for particular products. They affect output because sellers and producers of these products have no way at the outset of knowing whether the change in demand for their products is a change relative to the demand for other products, to which it is in their interest to react by expanding or contracting output, or a change in general nominal demand, to which the appropriate response is an adjustment of prices.[9]

In chapter 2 this effect was embodied in equations (9) and (10), or in linearized form, in equation (12) and (13). These equations are:

$$(1) \qquad g_P = g_P^* + \eta\,(g_Y - g_Y^*) + \xi\,(\log y' - \log y'^*)$$

$$(2) \qquad g_{y'} = g_{y'}^* + (1 - \eta)\,(g_Y - g_Y^*) - \xi\,(\log y' - \log y'^*),$$

where the values with asterisks are anticipated or permanent values. The first term on the right-hand side of these equations relates price and

9. An important source of misunderstanding about the distinction between "anticipated" and "unanticipated" changes is the failure to specify the time unit involved. Many of the criticisms of "rational expectations models," and of their implications for policy, are really criticisms of the implicit assumption in some of these models that the same time unit, relatively brief, is relevant to anticipations for all variables. Longer or shorter time units may be relevant for different variables. Anticipations in active financial markets may be for very short periods, since commitments are for short periods, whereas anticipations are likely to be for much longer periods for construction or labor contracts. Once this point is recognized, much of the appeal of some simple rational expectations models disappears, but so also do many of the criticisms of a more sophisticated use of the concept of rational expectations.

A recent example is the criticism by George A. Akerlof of rational expectations models on the grounds that they are inconsistent with relatively long spells of unemployment, because they assume continuous market clearing. That is a valid criticism of models that assume that anticipations are for brief periods, and that errors are serially independent. It is not a valid criticism of models that allow for long contract periods. See Akerlof, "The Case against Conservative Macroeconomics: An Inagural Lecture," *Economica* 46 (August 1979): 219–37. On the effect of contract length, see J. A. Gray, "On Indexation and Contract Length," *Journal of Political Economy* 86 (February 1978): 1–18. See also note 33 below and sections 10.1.1 and 10.7.3.

Karl Brunner, Alex Cukierman, and Allan H. Meltzer have combined the distinction between anticipated and unanticipated with the distinction between permanent and transitory shocks to construct formal models in the spirit of the analysis of chapter 2 and this section but much more detailed. See "Stagflation, Persistent Unemployment, and the Permanence of Economic Shocks," *Journal of Monetary Economics* 6 (October 1980): 467–92; and "Money and Economic Activity, Inventories and Business Cycles" (March 980, unpublished).

output change to *anticipated* changes in the corresponding magnitudes; the second term relates them to *unanticipated* changes in nominal income; and the third term relates them to the deviation of actual output from permanent and potential output, itself the cumulated residue of earlier unanticipated changes in nominal income.[10]

Equation (15) of chapter 2 relates the unanticipated change in nominal income to monetary change, in linearized form, as

$$g_Y - g_Y^* = \Psi\ (g_{MS} - g_{MD}) + \Phi\ (\log M^S - \log M^D). \tag{3}$$

where the superscripts S and D refer to "supplied" and "demanded." In chapter 8 we explored two special cases of equation (3), one that assumed an instantaneous adjustment of M^S to M^D or of g_{MS} to g_{MD} and another that corresponded to the monetary theory of nominal income. The results were encouraging though by no means fully satisfactory. They gave equations for nominal income and the rate of change of nominal income in terms of current money and current yields for the first special case, and in terms of current money and past money and past income, or current and past money alone, for the second special case.

As a first step in investigating empirically equations (1) and (2), we shall in sections (9.5) and (9.6), resort to the same special cases for prices and output separately. This requires modifying equations (1) and (2) (and their counterparts for levels) by setting $\xi = 0$, replacing g_Y (or Y) by an equation relating g_Y (or Y) to current money and yields, or to current money and earlier money and income, or to current and earlier money, and replacing η by a vector of parameters multiplying the individual coefficients of the several independent variables used in the price and output regressions.

As a second step, we replace equation (3) by the simpler version:

$$g_Y - g_Y^* = \Psi\ (g_M - g_M^*). \tag{4}$$

Equation (4) simplifies equation (3) in three ways. It neglects the stock adjustment term on the ground that it is likely to be less important for output than the stock adjustment term in equations (1) and (2) and, in any event, is likely to be highly correlated with that term.[11] It replaces g_{MS} by g_M on the ground that M can be regarded as predominantly exogenous.

10. These equations suffer from the defect discussed in the preceding footnote that they do not explicitly allow for the time period that anticipations cover, or for the possibility that different time periods are relevant for different sectors of the economy. That defect is less serious for our empirical work, which uses as a time unit a phase, than for studies using, say, quarterly data, because a phase is long enough to allow for the bulk of the relevant contract periods, though as chapter 10 suggests, the defect is not completely eliminated.

11. Because, if past deviations of g_M from g_M^* have, for example, left M^S above M^D, they can also be expected to have left y' above y'^*.

It replaces g_{MD} by g_M^* on the ground that, wartime apart, the real factors affecting the demand for money can be regarded as changing slowly.

In section 9.7 we use the Phillips curve as a convenient device for examining the effect of the degree of utilization of capacity and of inflationary anticipations as approximated by a linear function of the rate of inflation in the prior phase. The results are intriguing, enough so that we are led in section 9.8 to introduce more sophisticated hypotheses about the formation of inflationary anticipations. None of those hypotheses proves nearly as satisfactory as the simpler formulation of section 9.7. However, the approach does yield some interesting results about the role of capacity utilization.

9.5 Effect of Money and Yields

By definition, the sum of the logarithms of price and output equals the logarithm of nominal income. It follows that, since current money and yields have a consistent relation to nominal income, they must also be related to prices or to output or to both.

For most of the previous chapter, we could separate the behavior of nominal income from its division into prices and output by treating the real income elasticity of demand for money as if it were unity. This assumption enabled us to write nominal income as a function of nominal money and yields alone, without also introducing real income, and it meant that the rate-of-change equations connecting nominal income with nominal money and yields would, in principle, have zero intercepts.

When we break nominal income down into its price and output components separately, even the assumption that the real income elasticity of demand is unity will not enable us to omit real income or to eliminate the constant term from the rate-of-change equations. Consider equation (6) of chapter 8, simplified by omitting the dummy variables.

(5) $$\log Y = -\log k + \zeta \log M + (1-\alpha)\log y - \delta R_N - \epsilon g_Y$$

plus the identity

(6) $$\log P = \log Y - \log y - \log N,$$

where y is per capita real income and N is population.

Substitute equation (5) into equation (6):

(7) $$\log P = -\log k - \log N + \zeta \log M - \alpha \log y - \delta R_N - \epsilon g_Y.$$

Similarly, the rate of change equation becomes:

(8) $$g_P = -g_N + \zeta g_M - \alpha g_y - \delta DR_N - \epsilon Dg_Y.$$

Setting $\alpha = 1$ eliminates the term in log y from equation (5) and reduces it to the simplified form of equation (6a) of chapter 8, but it does not eliminate the term in y from equations (7) and (8). The one simplification it introduces is to permit per capita real income and population to be combined in a single term corresponding to total output, and to fix the coefficient of that term at unity.

Unfortunately, it seems undesirable statistically to include either per capita output or total output in equations for prices (and conversely, prices in equations for output). The reason is that the two variables are generally not statistically independent. The estimates of nominal income are generally derived independently, and then a separate estimate is made either of output at constant prices or of prices, and the remaining variable derived by dividing the nominal income estimate by the output estimate or the price estimate. As a result, errors of measurement of the estimates of prices and output are negatively correlated. We decided in chapter 8 that this problem was significant empirically and seriously distorted the direct estimates of equation (6) of that chapter. We resolved it there by assuming a real income elasticity of unity.

We resolve it here by approximating population and real income by expressions in terms of other variables. We show in the appendix to this chapter (sec. 9.10) that if we replace population by an exponential trend and assume that deviations of real per capita income from an exponential trend (which can be regarded as an estimate of anticipated or permanent or potential per capita output) are related to M, R_N, and g_Y, we can express equations (7) and (8) as

$$\begin{aligned}(9)\qquad \log P(t) = &- \log k_P - \lambda_{3P}Z - [\bar{g}_P + \lambda_{5P}Z]T(t) \\ &+ \zeta_P \log M(t) - \delta_P R_N(t) - \epsilon_P g_Y(t),\end{aligned}$$

$$\begin{aligned}(10)\qquad g_P(t) = &-\bar{g}_P - \lambda_{5P}Z + \zeta_P g_M(t) - \delta_P D[R_N(t)] \\ &- \epsilon_P D[g_Y(t)],\end{aligned}$$

where

$$\begin{aligned}
\log k_P &= \log k + A_N + (\alpha + \lambda_4 Z)A_y,\\
\lambda_{3P} &= \lambda_3 + \lambda_{3N} + (\alpha + \lambda_4 Z)\lambda_{3y},\\
\bar{g}_P &= \bar{g}_N + (\alpha + \lambda_4 Z)\bar{g}_y,\\
\lambda_{5P} &= \lambda_{5N} + (\alpha + \lambda_4 Z)\lambda_{5y},\\
\zeta_P &= \zeta - (\alpha + \lambda_4 Z)\,\zeta_y,\\
\delta_P &= \delta - (\alpha + \lambda_4 Z)\,\delta_y,\\
\epsilon_P &= \epsilon - (\alpha + \lambda_4 Z)\,\epsilon_y,
\end{aligned}$$

$T(t)$ is the chronological date corresponding to phase t, and A_N, A_y, $\bar{g}_N$, and $\bar{g}_y$ are the constant and exponential trend terms for the United States in the equations used to approximate population and real income respectively; λ_{3N}, λ_{3y}, λ_{5N}, and λ_{5y} are the excess of the corresponding terms for the United Kingdom over those for the United States, and ζ_y, δ_y, and ϵ_y are the coefficients of log M, $R_N(t)$, and $g_Y(t)$ in an equation like equation (9) for log y. Similar equations hold for output (see appendix equations A14 and A17). In using these equations, and throughout the rest of this chapter, we replace R_N by R_S for reasons explained in chapter 8.

Table 9.4 gives multiple regressions for nonwar phases between nominal income, prices, and output as dependent variable, and (1) money and trend alone or (2) money, trend, and yields as independent variables. Only two out of each set of three regressions are independent, since the price and output regressions must add to the income regression, but we give all three because t values and standard errors of estimate are of interest for prices and output separately. We have computed similar regressions for all phases; also for three subperiods: before World War I, the interwar years, and after World War II; and also for the United States plus the United Kingdom, using dummy variables to allow for country effects. As already indicated, we present the results for nonwar phases to allow for possible wartime real effects. However, it is worth noting that the level equations for all phases differ only inconsequentially from those for nonwar phases. The rate-of-change equations differ more appreciably, yet none of our major results would be affected by using all phases instead of all nonwar phases.[12] We use in later tables some of the results of the regressions for separate periods. However, they are individually rather unreliable because of the small number of observations on which some of them are based.[13] Similarly, in some of the later tables we use results for the United States and United Kingdom combined.

12. The biggest difference is that for the United States, the coefficient of g_M is greater for prices, and smaller for output, for all phases than for all nonwar phases, a difference that turns out to be traceable to the heavier weight in the nonwar than in all phases of the interwar observations, with their two major contractions.

13. The maximum number of observations is as shown in table 9.N.1 (fewer when lagged variables are included).

Table 9.N.1 **Number of Level- and Rate-of-Change Observations, by Nonwar Subperiods, Available for Regressions for Separate Periods**

	Levels		Rates of Change	
Period	United States	United Kingdom	United States	United Kingdom
Pre–World War I	21	11	19	9
Interwar period	9	9	7	7
Post–World War II	13[a]	11[a]	11	9
All nonwar phases	43[a]	31[a]	37	25

[a]One fewer when rate of change of nominal income is an independent variable.

Table 9.4 **Relation of Nominal Income, Prices, and Output to Money and Yields, Nonwar Phases**

		Coefficient (and Absolute t Value)										
Country	Dependent Variable (1)	Intercept (Level Equation) (2)	t Value	Trend (Time or Intercept of Rate of Change) (3)	t Value	M or g_M (4)	t Value	R_s or DR_S (5)	t Value	g_Y or Dg_Y (6)	t Value	Standard Error of Estimate (7)
United States	Y	0.94	6.1	−0.007	3.2	1.06	26.9					.054
	Y	1.05	7.6	−0.004	1.9	1.01	24.3	1.07	1.7	0.77	4.1	.044
	P	−5.51	23.7	−0.027	8.6	0.85	14.2					.082
	P	−5.64	21.9	−0.031	7.3	0.92	11.8	−1.54	1.3	0.19	0.5	.083
	y'	6.44	26.5	0.021	6.2	0.20	3.3					.086
	y'	6.69	27.0	0.026	6.5	0.10	1.3	2.61	2.2	0.58	1.7	.080
United Kingdom	Y	0.02	0.1	−0.002	0.9	1.10	23.0					.062
	Y	0.51	3.9	0.002	1.2	0.97	24.1	3.28	4.8	1.10	4.1	.041
	P	−5.94	35.1	−0.016	7.7	1.03	19.5					.068
	P	−5.99	31.1	−0.015	7.4	1.02	17.4	0.94	0.9	−1.10	2.8	.060
	y'	5.96	27.0	0.014	5.3	0.08	1.1					.088
	y'	6.50	34.2	0.017	8.3	−0.05	0.9	2.34	2.4	2.20	5.7	.059

United States	g_Y	−0.0052	1.9	1.04	19.8					.0126
	g_Y	−0.0003	0.1	0.91	12.6	1.74	1.8	0.50	4.1	.0105
	g_P	−0.0172	5.4	0.53	8.9					.0143
	g_P	−0.0104	2.4	0.37	3.9	2.75	2.2	0.10	0.6	.0138
	$g_{y'}$	0.0120	3.4	0.51	7.6					.0161
	$g_{y'}$	0.0100	2.1	0.54'	5.3	−1.01	0.7	0.40	2.3	.0149
United Kingdom	g_Y	0.0044	1.5	0.90	11.0					.0103
	g_Y	0.0072	2.8	0.76	9.4	2.80	3.5	0.05	0.2	.0085
	g_P	−0.0164	5.7	0.90	11.2					.0100
	g_P	−0.0133	5.9	0.77	11.0	2.73	4.0	−0.41	1.7	.0074
	$g_{y'}$	0.0208	11.3	0.0075	0.1					.0065
	$g_{y'}$	0.0205	11.3	−0.01	0.2	0.06	0.1	0.47	2.3	.0060

Note: Number of observations is 42 for United States levels, 30 for United Kingdom levels, 37 for United States rates of change, 25 for United Kingdom rates of change.

Nominal income regressions in this table differ from those in chapter 8. Chapter 8 regressions do not exclude wartime phases and do not include time as a variable in the level equations and an intercept in the rate-of-change equation.

Table 9.4 confirms the quantity theory expectation that monetary change affects prices more than output. In seven out of eight comparisons, the *t* value for the coefficient of money is greater for prices than for output (the exception is for United States rates of change for the equation including yields) and, indeed, for five of the eight equations, including all four for the United Kingdom, the coefficient for output does not differ significantly from zero at a .05 level. These results are in accord with a more extreme form of the quantity theory for cycle phases than we outlined in chapter 2—what we called in section 9.1 the "simple" quantity theory—and tend to confirm our finding in section 9.1 that the "United Kingdom comes closer to conforming to the simple quantity theory than the United States."

Examination of similar equations for the subperiods shows that the statistically significant coefficients of money for three of the four United States output equations are produced by the interwar period. We have four parallel equations for each of three subperiods, or twelve in all. None of the eight coefficients for the pre–World War I and post–World War II periods differs significantly from zero; all four for the interwar period do. Similarly, the coefficient for a similar equation for the pre–World War I period and the post–World War II period combined does not differ significantly from zero. The different result for the interwar period reflects the two major contractions (1920–21 and 1929–33) during that period. The changes in output during those contractions were so large that, as noted in section 9.3, even averaging over as many as nine phases does not eliminate them.

The significant coefficients for the output equations for the interwar years represent a conflict with the simple quantity theory; but of course they do not conflict with the more sophisticated quantity theory outlined in chapter 2. As we have repeatedly emphasized, we regard these major contractions as a response to monetary disturbances that produced unanticipated changes in the quantity of money of sufficient size and duration to produce important effects on output.

Despite the lack of statistical significance of most of the separate coefficients on money in the output equations, the results as a whole give some evidence of a systematic influence both in the few significant coefficients and in the generally positive signs of the coefficients. Six out of the eight coefficients in table 9.4 are positive, and so are fifteen out of twenty-four of the coefficients for the subperiods.

Including yields in the equations reduces the standard error for three out of four price equations and for all four of the output equations. However, some of the reductions are trivial. Similarly, only about half of the coefficients of yields for the price and output equations are statistically significant at a .05 level. Nonetheless, it is interesting that some coefficients are significant not only for prices but also for output. Indeed, a few more of the coefficients are significant for output than for prices, and for six out of the eight comparisons the *t* values of the yield terms are greater for output than for prices. One other suggestive detail is that, whereas the coefficient of R_S has a higher *t* value than the coefficient of g_Y for three out of four price equations, the opposite is true for three out of four output equations. While obviously not very firmly based statistically, this result has considerable theoretical appeal. R_S is the nominal yield on nominal assets; g_Y is our proxy for the nominal yield on physical assets; it is therefore plausible that g_Y would have the greater effect on output, as a better indicator of the incentive to expand or contract output. All four of the coefficients of g_Y for output are positive, as this interpretation suggests, while two of the coefficients of g_Y for price are negative, which also fits this interpretation, suggesting that the output effect of a higher or lower yield on physical assets more than offsets the effect on prices via its impact on the demand for money. Note that the coefficient for income, which reflects only the impact through the demand function for money, is uniformly positive (though for one equation trivial in size), which is the sign to be expected.

The tendency for prices and output to be negatively correlated, discussed in section 9.2, is reflected in table 9.4 in the standard errors of estimate in column 7. If prices and output were statistically independent, the standard error for income would be larger than the standard errors for price and output separately.[14] Yet that is true for only two out of the eight comparisons in table 9.4 (both for United Kingdom rates of change). For the other six comparisons, the standard error of estimate is uniformly less for income than for prices, and also than for output.[15] This result must reflect a negative correlation between prices and output when the independent variables are held constant.

These correlations between prices and output are given directly in table 9.5 both for all nonwar phases and for the subperiods. All but two are negative, the two being for the United Kingdom interwar period for levels and rates of change when money and trend are allowed for, and even these are converted to negative coefficients when yields are allowed for. This result is of course to be expected for the reasons outlined in section 9.2. If these equations accounted for all systematic influences on income, prices, and output, the only thing left would be measurement error; and, given the calculation of either price or output as a residual, any measurement error in one would be perfectly correlated negatively

14. Since $\sigma^2_{\log Y} = \sigma^2_{\log P} + \sigma^2_{\log y'} + 2r_{\log P, \log y'}\,\sigma_{\log P}\,\sigma_{\log y'}$.

15. For subperiods, the standard error is greater for income than for both prices and output for four out of twenty-four comparisons (two for the United States, two for the United Kingdom), less for income than for both prices and output for ten out of twenty-four comparisons, and less for income than one of the others for ten out of twenty-four comparisons.

Table 9.5 Correlation between Price and Output, after Allowing for Money and Trend and for Money, Trend, and Yields

Period and Country	Correlation Coefficient between Price and Output after Allowing for: Money and Trend	Money, Trend, and Yields
	Levels	
All nonwar phases		
United States	−0.80	−0.85
United Kingdom	−0.72	−0.76
Pre–World War I		
United States	−0.40	−0.50
United Kingdom	−0.85	−0.72
Interwar		
United States	−0.61	−0.71
United Kingdom	0.38	−0.46
Post–World War II		
United States	−0.29	−0.52
United Kingdom	−0.61	−0.58
	Rates of Change	
All nonwar phases		
United States	−0.66	−0.74
United Kingdom	−0.28	−0.21
Pre–World War I		
United States	−0.41	−0.50
United Kingdom	−0.84	−0.80
Interwar		
United States	−0.33	−0.92
United Kingdom	0.09	−0.54
Post–World War II		
United States	−0.95	−0.98
United Kingdom	−0.72	−0.90

Note: See note to table 9.4.

with the measurement error in the other. Of course, the equations do not account for all systematic influences, but comparing the standard errors with our estimates in the preceding chapter of the order of magnitude of measurement errors indicates that measurement errors may account for an appreciable fraction of residual variability.

Table 9.6 summarizes the evidence from the regressions in Table 9.4 and similar regressions for the subperiods on the division between prices and output of the change in nominal income accompanying a change in money. One striking feature is the difference between the United States and the United Kingdom. The percentage absorbed by prices in the United Kingdom exceeds that in the United States in fourteen out of sixteen comparisons. The two exceptions are for rates of change pre–World War I, when yields are not allowed for, and post–World War II,

Table 9.6 **Percentage of Nominal Income Change Associated with Monetary Change That Is Absorbed by Prices When Allowance Is Also Made for Trend, or for Trend and Yields**

	Percentage of Nominal Income Change Absorbed by Price Change in Regressions with	
Period and Country	Money and Trend	Money, Trend, and Yields
	Levels	
All nonwar phases		
United States	81	90
United Kingdom	93	106
Pre–World War I		
United States	84	81
United Kingdom	90	82
Interwar		
United States	30	20
United Kingdom	111	93
Post–World War II		
United States	102	104
United Kingdom	146	133
	Rates of Change	
All nonwar phases		
United States	51	40
United Kingdom	99	101
Pre–World War I		
United States	68	62
United Kingdom	61	73
Interwar		
United States	30	21
United Kingdom	120	86
Post–World War II		
United States	97	110
United Kingdom	113	95

Note: See note to table 9.4.

when yields are allowed for. This is further evidence of the difference between the United States and the United Kingdom remarked on in section 9.1.

The only significant difference we have hitherto found between the United States and the United Kingdom is in the elasticity of the demand for money with respect to real per capita income. At first it seems that we may have found another one here in the division of income change between prices and output. However, the situation is more complex and the evidence weaker than appears at first glance. As before, the major difference between the United States and the United Kingdom is for the interwar period, for which, according to the estimates in table 9.6, prices

Table 9.7 **Tests of the Significance of the Difference between the United States and the United Kingdom for Relations between Nominal Income, Prices, and Output, and Nominal Money and Yields**

	F Value			.05
Period	Income	Prices	Output	Value
Nonwar phases	1.17	4.61	6.73	2.38
Nonwar phases other than United States interwar	0.70	0.49	0.32	2.42
Pre–World War I	0.34	0.52	0.12	2.71
Interwar	2.63	5.79	3.94	4.39
Post–World War II	2.52	0.13	0.16	3.11

Note: All variables are rates of change.

In making these comparisons, a country dummy was included in the United States plus United Kingdom equations to allow for the difference over the period as a whole in average rates of growth in output arising from the differences between the two countries in population and per capita income trends.

account for 20 to 30 percent of the change in income for the United States, but for 86 to 120 percent in the United Kingdom. There is no doubt that the price-output response was very different for the United States in this period than for the United Kingdom.

For the remaining periods there is nothing like so clear a difference. True, for six out of eight comparisons in table 9.6 for the other subperiods, the percentage accounted for by price is greater for the United Kingdom than for the United States, but some of the differences are trivial and none anything like so large as for the interwar period. Table 9.7, which tests the significance of the difference between the rate-of-change equations for the United States and the United Kingdom, confirms this judgment.[16] Only the equations for price and output for all nonwar phases, and for prices for interwar phases, differ significantly at the .05 level. For the rest, the United States and the United Kingdom appear homogeneous.

The United States interwar period is clearly unique. Its dramatic character gave it far-reaching influence. It played an important role in leading Keynes himself, and his American followers even more, to regard prices as rigid and output as flexible, to interpret income change as corresponding primarily to output change and not at all, or hardly at all, to a change in prices. The percentages in table 9.6 for the United States for the interwar period are certainly consistent with that view—but they are the only ones that are. The rest of the data contradict the hypothesis Keynes was led to formulate. But that fact has not even yet been fully recognized.

16. These comparisons are restricted to rate of change equations for the reason given in note 27 of chapter 8.

9.6 Effect of Current and Prior Money and Prior Income

It is shown in the appendix (sec. 9.10) that, if we treat all phases as n years in length, equations (9) and (10) of the preceding sections can be expressed as:

$$\begin{aligned}\log P(t) = {} & a_P + f_P Z + (a'_P + f'_P Z)T(t) \\ & + b_P \log M(t) + c_P \log M(t-1) \\ & + d_P \log Y(t-1) \\ & + e_P \log Y(t-2)\end{aligned} \tag{11}$$

$$\begin{aligned}g_P(t) = {} & a'_P + b_P\, g_M(t) \\ & + c_P g_M(t-1) + d_P g_Y(t-1) + e_P g_Y(t-2),\end{aligned} \tag{12}$$

with similar equations for output. The relations connecting these coefficients with the structural parameters are much more complicated than the corresponding relations for the nominal income equation, because the structural parameters for both the nominal income equation and the price equation enter in.

Similarly, equations (11) and (12) can be converted into relations with prior money alone, but again the coefficients of the money relations are a function of both the coefficients of equations (11) and (12) and the nominal income equations that are the counterparts of equations (11) and (12). Equations (11) and (12) can be written as:

$$\log P(t) = a''_P + k''_P T(t) + \sum_{i=0}^{\infty} b_{P_i} \log M(t-i) \tag{13}$$

and

$$g_P(t) = k''_P + \sum_{i=0}^{\infty} b_{P_i}\, g_M(t-i), \tag{14}$$

where

$$\begin{cases} b_{P_0} = b_P \\ b_{P_1} = c_P + d_P b_0 \\ b_{P_i} = d_P b_{i-1} + e_P b_{i-2} \text{ for } i > 1, \end{cases} \tag{15}$$

and

$$\sum_{i=0}^{\infty} b_{P_i} = b_P + c_P + (d_P + e_P)\left[\frac{b+c}{1-d-e}\right], \tag{16}$$

where b_i in equation (15) are the coefficients of the nominal income counterpart of equations (13) and (14) and the other terms are defined in section 9.10. Similar equations are valid for output.

It will be recalled that

$$\sum_{i=0}^{\infty} b_i = \left[\frac{b+c}{1-d-e}\right]. \tag{17}$$

We concluded in the preceding chapter that we can regard the sum of the *b*'s for the nominal income equation as equal to unity. It follows from equation (16) that the sum of the *b*'s for the price equation is then an estimate of the fraction of the change in income that is absorbed by prices. Similarly, the sum of the *b*'s for the corresponding output equation is, under the same assumption, an estimate of the fraction of the change in income that is absorbed by output. However, unless the cross-equation restriction that the sum of the *b*'s for prices plus those for output add to unity is imposed on the price and output equations in computing them, there is no assurance that the two sums will add to unity. Rather, they will add to the sum of the *b* coefficients from the income equation calculated without imposing the restriction that those coefficients add to unity. In that case the estimated fraction of the change in income absorbed by prices is given by the ratio of the right-hand sides of equations (16) and (17), and that result will be consistent with the corresponding fraction for the output equations.

Table 9.8 summarizes both the sum of the *b*'s for the price equation and the percentage of the income change absorbed by price change calculated from the ratio of the right-hand sides of equations (16) and (17). The results are remarkably consistent for different methods of estimation.

The estimated cumulative percentage change in prices, like the percentages in table 9.6, are uniformly higher for the United Kingdom than for the United States for the period as a whole. However, that result is reversed for five of the eight comparisons for which the interwar period is excluded for the United States. For the estimated percentage of nominal income change absorbed by price change, the relation is reversed for levels, even for the period as a whole: the percentage change in income is so much higher in the United Kingdom than in the United States that the higher price effect constitutes a smaller percentage of the income effect.

We are puzzled by this result. The rate of change results seem much more reasonable. When the interwar period is excluded for the United States, the percentage absorbed by prices is consistently raised for the United States but still remains below the percentage for the United Kingdom. We found in table 9.6 that, for all nonwar phases, the various estimates of the percentage of the income change absorbed by prices varied from 40 percent to 90 percent for the United States, from 93 to 106 percent for the United Kingdom. The estimates in table 9.8 vary from 68 to 96 percent for the United States, from 84 to 114 percent for the United Kingdom. When interwar phases are excluded for the United States, the estimated percentage absorbed varies for the United States from 92 to 114, very closely matching the range for the United Kingdom.

The rate-of-change estimates for the United Kingdom in table 9.8, like those in table 9.6, conform almost precisely to the quantity theory implication that prices should ultimately absorb the whole of the change in

Table 9.8 **Cumulative Effect on Prices of a One Percent Increase in the Quantity of Money, and Percentage of Income Change Ultimately Absorbed by Price Change, Estimated from Equations Relating Price Change to Current and Prior Money (Direct) or Current and Prior Money and Prior Income (Indirect); Nonwar Phases, United States, and United Kingdom, and for United States also Excluding Interwar Phases**

	Cumulative Percentage Change in Prices						Percentage of Nominal Income Absorbed by Prices					
	Levels			Rates of Change			Levels			Rates of Change		
	U.S.			U.S.			U.S.			U.S.		
Basis of Estimates	All	Excluding Interwar	U.K.	All	Excluding Interwar	U.K.	All	Excluding Interwar	U.K.	All	Excluding Interwar	U.K.
Transient effects not allowed for												
Direct	98	120	108	88	104	124	96	114	92	79	94	100
Indirect	95	117	111	70	93	89	95	112	84	68	92	114
Transient effects allowed for												
Direct	96	119	104	79	95	124	93	108	85	73	94	100
Indirect	93	111	109	71	94	107	92	106	84	80	94	99

income associated with a monetary change. The estimated percentages clearly do not differ from 100. For the United States, the rate of change percentages are below 100 even when the interwar period is excluded. However, they do not differ significantly from 100.

All in all, we conclude that if the interwar period for the United States is treated as exceptional, the remaining evidence for both countries is remarkably consistent with the quantity theory implication that a monetary change should ultimately affect only prices.[17]

The evidence is also consistent with the absence of any significant difference between the United States and the United Kingdom in the relation between prices and earlier money and income, provided the United States relation is based on data excluding the interwar period.[18]

Our judgment that the interwar period in the United States is excep-

17. This conclusion is in general supported by F tests of the difference between Σb_{P_i} and unity. Table 9.N.2 shows the results of those tests for the sums based on the indirect approach. The only significant values are for the United States rate of change for all nonwar values—which we attribute to the interwar period—and for levels for the United States for all nonwar excluding interwar phases if transient effects are not allowed for—which reflects a sum of b's greater than unity.

Table 9.N.2 **F Values for Differences between Σb_{P_i} and Unity**

Period and Country	Transient Effects: Not Allowed for (1)	Transient Effects: Allowed for (2)	.05 Value Col. 1 (3)	.05 Value Col. 2 (4)
Levels				
United States				
All nonwar	0.47	0.98	4.11	4.13
All nonwar excluding interwar	7.19	4.04	4.21	4.24
United Kingdom, all nonwar	0.63	1.41	4.32	4.38
Rates of change				
United States				
All nonwar	12.48	8.25	4.15	4.17
All nonwar excluding interwar	0.37	0.22	4.24	4.28
United Kingdom				
All nonwar	0.00	0.19	4.45	4.54
United States and United Kingdom				
United States nonwar excluding interwar period plus all United Kingdom nonwar	0.01	0.18	4.05	4.06

18. The tests of significance are as shown in table 9.N.3. All but one of the F's for all nonwar phases are significant; none of the F's is when the interwar phases for the United States are excluded.

Table 9.N.3 ***F* Values for Differences between Countries**

	Transient Effects					
	Not Allowed for			Allowed for		
Period and Treatment of Σb_P's	Direct	Indirect	.05 Value	Direct	Indirect	.05 Value
Σ of b_P's not restricted						
All nonwar phases	5.17	5.60	2.30	3.87	4.02	2.15
United States interwar phases excluded	1.88	0.89	2.32	1.33	0.90	2.19
Σb_P's restricted to unity						
All nonwar phases	4.38	5.94	2.41	1.84	5.22	2.22
United States interwar phases excluded	1.48	0.98	2.43	0.76	1.01	2.25

tional and our exclusion of that period from some of the statistical calculations do not mean that we regard the period as unimportant or uninstructive. On the contrary, its very exceptional character reveals dramatically the power of monetary policy and monetary disturbances to influence the economy—which was why we were led to devote nearly half of the text of *A Monetary History* to these twenty years out of the ninety-three years our *History* covers. However, just as wartime periods are instructive yet may introduce undesirable heterogeneity and bias into statistical analysis, so the United States interwar period requires separate treatment rather than being lumped with the remaining episodes—and, we may add, we suspect that it is better treated by the episodic approach of *A Monetary History* than by the statistical approach of this book.

In chapter 8, we concluded that the cumulative effect of a 1 percent change in money can be taken to be a 1 percent change in nominal income (i.e., that $\Sigma b_i = 1$), and imposed that restriction in calculating the reaction pattern of nominal income in response to monetary change. It now seems that, the interwar period for the United States aside, the cumulative effect of a 1 percent change in money can be taken to be a 1 percent change in prices (i.e., that $\Sigma b_P = 1$). Accordingly, we shall impose that restriction as well in most of the rest of this section.

Table 9.9 compares the transient effect for prices with that for income. For levels, the sum of the transient effects for prices for the United States is in the wrong direction, negative instead of positive, yet statistically significant for two out of the four comparisons. For the United Kingdom the results are more nearly consistent with expectations. For rates of change, which might be expected to give more reliable estimates of transient effects, seven out of the eight sums of transient effects are positive, and the one negative sum is not statistically significant, whereas five of the seven positive sums are. The transient effect for prices is sometimes higher than for income, sometimes lower, as is consistent with our finding that output effects are small or nonexistent.

Table 9.10 summarizes the effect of allowing for alternative sets of variables on the residual variability of prices and output. For prices, money and trend are obviously far and away the most important variables, as has been true throughout earlier chapters as well. Allowing for yields generally makes an appreciable improvement. Substituting prior money for yields generally reduces the variability further, and allowing in addition for transient effects uniformly leaves a lower variability than adding yields to money and trend. This result is consistent with the theoretical model that we used to justify replacing yields by prior money. However, it cannot be regarded as strong confirmatory evidence for that theoretical model, since it could readily be that earlier money is replacing not yields but some other variable omitted in the money, trend, and yield regressions such as variables producing serial correlation in prices. Unfortunately our data, even though they stretch over a century, did not seem rich enough to enable us to explore such possibilities. The final pair of columns, which include income in the first and second prior phases instead of money in the second and third prior phases, give slightly poorer results for levels and for United States rates of change, mixed results for United Kingdom rates of change.

For output, the results for the United States are roughly the same as for prices. For the United Kingdom the most interesting result is for rates of change, for which all the measures are roughly the same. The rate of growth of output in Britain from cycle phase to cycle phase—at least as recorded in the imperfect figures we have available—has been highly stable and not affected significantly by any of the variables, including earlier income, implying essentially a series corresponding to a random walk or white noise. It is noteworthy that the total variability of output rates of change for all nonwar phases for the United Kingdom is not much more than half the lowest *residual* variability for United States nonwar phases. The situation is different for the rate of change of prices. Total variability is about the same in the two countries—reflecting the "law of one price." However, the variables we have considered had a greater influence on United Kingdom than on United States prices, so that *residual* variability is decidedly less for the United Kingdom than for the United States.

All in all, we regard as the most reliable and instructive relations those for rates of change for all nonwar phases for the United Kingdom and all nonwar phases excluding interwar phases for the United States, as well as the combination of these two.

Table 9.11 gives a different kind of summary of our results, namely, the estimated reaction of prices and output to a sustained 1 percent increase in the quantity of money. These estimates are based on the relations summarized in the final two columns of table 9.10, except that the restriction is imposed that the ultimate cumulative effect on prices be unity and on output zero, in accordance with our earlier finding that these theoretical expectations are not contradicted by our data. This table is the counterpart of table 8.9 for income except that it is restricted to all nonwar phases excluding interwar phases for the United States, and to all nonwar phases for the United Kingdom.

Chart 9.5 plots the reaction patterns based on rates of change. We regard these as most reliable not only for the reasons already cited but because technical considerations prevented us from estimating a transient effect from levels for the initial phase in which the monetary change is introduced. The price reaction obviously dominates. For both the United States and the United Kingdom it overshoots, but not until the second phase after the initial phase for the United States and the fourth phase for the United Kingdom, and then only modestly. The milder, yet

Table 9.9 **Estimates of Transient Effect on Income and Prices and Tests of Significance of Transient Effects on Prices (Σb's Restricted to Unity for both Income and Prices): Nonwar Phases, United States, and United Kingdom, and for United States also Excluding Interwar Phases**

Country, Method of Estimation, and Period	Sum of Transient Effects: Income		Sum of Transient Effects: Prices	Estimates of Separate Effects		F Ratio for Prices	
	Hypothetical	Calculated	Calculated	Initial	Second	Calculated	.05 Value
			Levels				
United States							
All nonwar phases							
Direct	1.84		−2.15	−1.57	−0.58	2.56	3.27
Indirect		−0.28	−2.38	−2.06	−0.32	5.41	
Pre–World War I, post–World War II							
Direct	1.62		−1.40	−1.51	0.11	1.81	3.37
Indirect		−0.27	−2.23	−1.78	−0.45	7.25	
United Kingdom							
All nonwar phases							
Direct	4.39		2.52	1.93	0.59	4.78	3.49
Indirect		0.46	0.93	1.40	−0.47	0.57	

			Rates of Change				
United States							
All nonwar phases							
Direct	2.24		1.56	0.26	1.30	6.10	3.31
Indirect		0.59	−0.86	−0.64	−0.22	1.44	
Pre–World War I, post–World War II							
Direct	1.82		1.39	0.68	0.71	4.06	3.40
Indirect		0.64	0.39	1.04	−0.65	3.22	
United Kingdom							
All nonwar phases							
Direct	2.84		3.65	2.49	1.16	7.24	3.63
Indirect		1.91	2.54	2.70	−0.16	2.47	
United States and United Kingdom							
Pre–World War I plus post–World War II, United States; all nonwar, United Kingdom[a]							
Direct	2.91		1.81	1.03	0.78	14.88	3.20
Indirect		0.78	1.09	1.25	−0.16	4.82	

[a]A country dummy was included in the United States plus United Kingdom equations.

Note: Transient effects from direct income calculations are not shown because there was no occasion to estimate them in chapter 8.

Table 9.10 **Variability of Prices and Output, before and after Allowing for Effects of Money; Money and Yields; Current and Prior Money; Current and Prior Money and Prior Income; and Trend**

	Measure of Variability[a]						
		After Allowing for Trend and Money					
				and Prior Money		and Prior Money and Prior Income	
Period and Country	Total[b,c] (1)	Only[c] (2)	and Yields[c] (3)	Only (4)	Plus Transient Effects (5)	Only (6)	Plus Transient Effects (7)
	Prices						
Levels							
United States							
All nonwar	57.16	8.24	8.26	7.52	7.20	7.98	7.13
All nonwar, excluding interwar	64.26	5.98	5.69	4.66	4.81	5.72	4.98
United Kingdom							
All nonwar	74.82	6.77	5.97	3.68	3.33	4.13	4.14
Rates of Change							
United States							
All nonwar	2.54	1.43	1.38	1.21	1.01	1.31	1.35
All nonwar, excluding interwar	2.21	1.35	1.33	1.04	0.93	1.24	1.15
United Kingdom							
All nonwar	2.49	1.00	0.74	0.82	0.57	0.71	0.66
United States and United Kingdom							
All nonwar, excluding interwar, United States	2.39	1.11	0.96	0.97	0.76	0.95	0.88

	Output						
Levels							
United States							
All nonwar	95.75	8.61	7.95	6.58	5.91	6.70	5.86
All nonwar, excluding interwar	107.62	6.18	5.76	4.66	4.39	4.66	4.01
United Kingdom							
All nonwar	52.44	8.85	5.90	7.59	5.96	6.25	6.27
Rates of Change							
United States							
All nonwar	2.58	1.61	1.49	1.33	1.35	1.19	1.15
All nonwar, excluding interwar	1.52	1.51	1.46	1.10	1.04	0.95	0.97
United Kingdom							
All nonwar	0.63	0.65	0.60	0.64	0.65	0.68	0.67
United States and United Kingdom							
All nonwar excluding interwar, United States	1.29	1.00	0.95	0.91	0.93	0.85	0.86

[a]For levels, measure of variability is standard deviation of logarithm times 100, which is equivalent to coefficient of variation of original observations in percent. For example, a measure of 10 means the standard deviation is 10 percent of the mean. For rates of change, measure is standard deviation of percentage rate of change. For example, a measure of 2 means that, if the residuals are normally distributed, two-thirds will be between +2 percent and −2 percent per year.

[b]Number of observations is as follows:

	Cols. 1–3	Cols. 4–7
Levels		
United States		
All nonwar	42	42
Nonwar excluding interwar	33	33
United Kingdom		
All nonwar	28	26

	Cols. 1–3	Cols. 4–7
Rates of Change		
United States		
All nonwar	37	37
Nonwar excluding interwar	30	30
United Kingdom		
All nonwar	25	22

[c]Note that the variability estimates for United Kingdom nonwar phases in cols. 1, 2, and 3 are not strictly comparable to those in columns 4 to 7 because they are based (as footnote b indicates) on more observations. However, the differences are small and would not affect our conclusions; hence we have given only one set of measurements.

Table 9.11 **Cumulative Effect on the Rate of Growth of Prices and Output of a Sustained One Percentage Point Increase in the Rate of Monetary Growth Initiated in Phase *t*; Eliminating and Including Transient Effects: United States, All Nonwar Phases Excluding Interwar; United Kingdom, All Nonwar Phases; United States Plus United Kingdom, All Nonwar Phases Excluding United States Interwar**

Country	Cumulative Effect at Phase							
	t	$t+1$	$t+2$	$t+3$	$t+4$	$t+5$	$t+6$	∞
PRICES								
Levels, Eliminating Transient Effects								
United States	1.501	1.015	0.971	0.991	0.999	1.000	1.000	1.000
United Kingdom	0.304	0.784	0.880	0.944	0.978	0.995	1.007	1.000
Levels, Including Transient Effects								
United States	1.501	0.161	0.753	0.991	0.999	1.000	1.000	1.000
United Kingdom	0.304	1.314	0.703	0.944	0.978	0.995	1.007	1.000
Rates of Change, Eliminating Transient Effects								
United States	0.118	0.992	1.005	1.005	1.003	1.001	1.000	1.000
United Kingdom	−0.540	0.348	0.668	0.925	1.037	1.052	1.031	1.000
United States and United Kingdom	−0.053	0.664	0.819	0.930	0.986	1.005	1.008	1.000
Rates of Change, Including Transient Effects								
United States	0.617	0.678	1.005	1.005	1.003	1.001	1.000	1.000
United Kingdom	0.484	0.289	0.668	0.925	1.037	1.052	1.031	1.000
United States and United Kingdom	−0.490	0.590	0.819	0.930	0.986	1.005	1.008	1.000

				OUTPUT				
				Levels, Eliminating Transient Effects				
United States	−0.443	0.060	0.045	0.009	0.000	0.000	0.000	0.000
United Kingdom	0.167	−0.019	0.038	0.041	0.030	0.016	0.002	0.000
				Levels, Including Transient Effects				
United States	−0.443	1.018	0.030	0.009	0.000	0.000	0.000	0.000
United Kingdom	0.167	−0.003	−0.155	0.041	0.030	0.016	0.002	0.000
				Rates of Change, Eliminating Transient Effects				
United States	0.360	−0.226	−0.100	−0.041	−0.015	−0.005	−0.001	0.000
United Kingdom	0.176	−0.152	0.086	0.113	0.066	0.020	−0.003	0.000
United States and United Kingdom	0.235	−0.102	0.014	0.036	0.026	0.013	0.004	0.000
				Rates of Change, Including Transient Effects				
United States	0.295	−0.036	−0.100	−0.041	−0.015	−0.005	−0.001	0.000
United Kingdom	0.061	−0.278	0.086	0.113	0.066	0.020	−0.003	0.000
United States and United Kingdom	0.222	−0.226	0.014	0.036	0.026	0.013	0.004	0.000

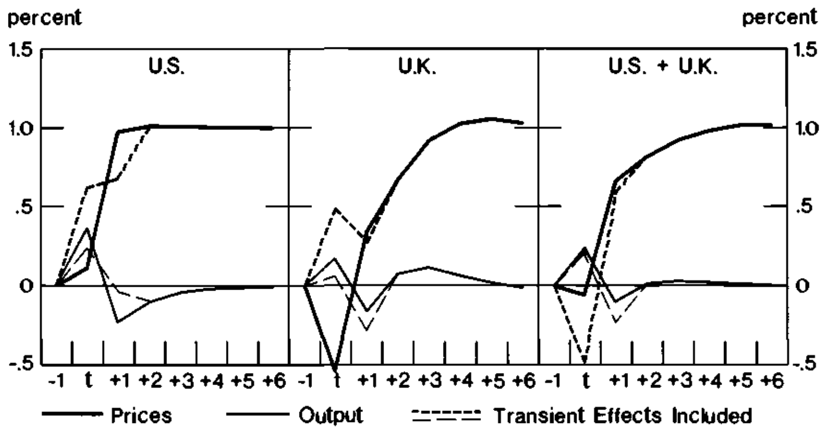


Chart 9.5 Reaction patterns of price and output to monetary change: United States, United Kingdom, and United States and United Kingdom (based on rates of change for all nonwar phases excluding United States interwar)

appreciable, output reaction is cyclical—first positive, reflecting the delay in the reaction of prices, then negative, as prices take over, then again positive.

These reaction patterns are suggestive, but we do not regard them as at all firmly established. They are a highly manufactured product and hence sensitive to small errors in the coefficients of the equations from which they are derived. For example, the patterns for the United States and the United Kingdom differ substantially, yet there is no statistically significant difference between the equations from which they are derived.

For the two countries together, the price pattern is very smooth, almost approximating an exponential approach to equilibrium. The output pattern remains cyclical as for each country separately and is quite sharply damped. The transient component is small, as it is for each country separately.

Though none of these patterns is statistically very reliable, they do correspond to the kind of pattern of reaction that is suggested by the theoretical analysis of chapter 2 and therefore give no reason to modify that analysis. Accordingly, we proceed to a more sophisticated application of it.

9.7 Effect of Output Capacity and Anticipations: The Phillips Curve Approach

By assuming up to now that $\xi = 0$ in equations (1) and (2), we have so far ruled out the effect that A. W. Phillips[19] incorporated in his famous

19. "The Relation between Unemployment and the Rate of Change of Money Wage Rates in the United Kingdom, 1861–1957," *Economica* 25 (November 1958): 283–99.

curve and that has played such an important part in the post–World War II economic literature bearing on the relation between inflation and unemployment. This literature has almost all been concerned with cyclical fluctuations rather than the longer-term movements with which we deal, yet it offers a convenient and familiar framework for investigating the factors affecting the division from phase to phase of changes in nominal income between prices and output. Moreover, as we have seen, in these matters the "short run" may not be very brief chronologically.

In his original article, Phillips related the level of utilization of capacity, which he measured inversely by the unemployment rate, to the rate of change of nominal wages. He argued that a high level of unemployment (a low level of utilization of capacity) was a sign that the quantity of labor offered at the going rate exceeded the quantity of labor demanded and hence exerted downward pressure on wage rates and conversely. Later writers extended the relationship from wages to prices, on the ground that prices move with costs and that labor costs dominate total costs. The theoretical argument is unexceptionable if "wages" are interpreted as "real wages."[20] However, Phillips himself, and for a long time most of his followers, interpreted "wages" as nominal wages.

It is interesting to note that in 1926 Irving Fisher studied precisely the same statistical relationship but justified it theoretically in a very different way, as an effect running from the rate of change of prices to employment rather than the other way.[21]

In terms of our framework, the Phillips curve explanation, if expressed in linear form, can be regarded as a special case of equations (1) and (2) of this chapter. If we replace $g_Y - g_Y^*$ in equations (1) and (2) by its equivalent from equation (4), they become

$$g_P = g_P^* + \eta\Psi(g_M - g_M^*) + \xi(\log y' - \log y'^*) \tag{18}$$

$$g_{y'} = g_{y'}^* + (I - \eta)\ \Psi(g_M - g_M^*) - \xi(\log y' - \log y'^*), \tag{19}$$

where η is the fraction of an unanticipated change in nominal income that is absorbed by unanticipated price change and Ψ is the multiplier relating unanticipated change in nominal income to unanticipated change in money.

In the initial version of the Phillips curve, it was implicitly assumed that $g_P^* = 0$, though nothing essential is altered if g_P^* is assumed equal to a constant other than zero. In addition, its strictest form, which regards the level of unemployment as the only factor affecting the rate of change of

20. See Milton Friedman, "The Role of Monetary Policy," *American Economic Review* 58 (March 1968): 1–17, reprinted in *The Optimum Quantity of Money*, pp. 95–110, especially p. 102.

21. "A Statistical Relation between Unemployment and Price Changes," *International Labour Review* 6 (June 1926): 785–92, reprinted in the *Journal of Political Economy* 81 (March/April 1973): 496–502.

prices, is equivalent to assuming $\eta = 0$. On these assumptions, equations (18) and (19) become:

(20) $$g_P = g_P^* + \xi \log (y'/y'^*), \text{ and}$$

(21) $$g_{y'} = g_{y'}^* + \Psi(g_M - g_M^*) - \xi \log (y'/y'^*),$$

or, since $\Psi\, g_M^* = g_Y^*$ and $g_Y^* = g_P^* + g_{y'}^*$,

(22) $$g_{y'} = -g_P^* + \Psi\, g_M - \xi \log (y'/y'^*).$$

This version is the obvious counterpart of the simple Keynesian hypothesis. Monetary change impinges in the first instance entirely on output, and affects prices only through the ratio of actual to potential output.

"Unfortunately for this hypothesis, . . . evidence failed to confirm it. Empirical estimates of the Phillips curve relation were unsatisfactory. More important, the inflation rate that appeared to be consistent with a specified level of unemployment did not remain fixed: in the circumstances of the post–World War II period, when governments everywhere were seeking to promote 'full employment,' it tended in any one country to rise over time and to vary sharply among countries. Looked at the other way, rates of inflation that had earlier been associated with low levels of unemployment were experienced along with high levels of unemployment. The phenomenon of simultaneous high inflation and high unemployment increasingly forced itself on public and professional notice, receiving the unlovely label of 'stagflation.' "[22]

Our data for cycle phases confirm this generally negative verdict.[23] To fit equations (20) and (22), we need an empirical counterpart to y'/y'^* in addition to our rates of change of money, prices, and output. One approach would be to treat y'^* as an unobservable anticipated magnitude, as we have treated Y^*. We have not followed that approach but rather have interpreted y'^* as corresponding to long-run potential output, and hence $\log y'/y'^*$ as the logarithm of the ratio of actual to potential output, capable of being directly apprehended by the economic actors. We have simply tried to find an empirical proxy for this concept. After much experimentation, we settled on the logarithm of the ratio of real income per capita for each phase to an exponential trend fitted to the phase averages of real income per capita. The trend allows for slow changes in productive capacity, the ratio to trend for shorter period

22. See Milton Friedman, "Inflation and Unemployment" (Nobel Lecture, 1976), reprinted in *Journal of Political Economy* 85 (June 1977): 451–72; quotation from p. 455.

23. For the rest of this chapter, we restrict the empirical analysis to rates of change instead, as hitherto, of making dual calculations for levels and rates of change. This economy is justified by the earlier sections of this chapter. When the results differed for levels and rates of change, we have generally decided in favor of the rate of change results.

changes in the rate of utilization of capacity. We term this measure the output ratio and denote it by $R_{y'}(t)$.[24]

Table 9.12 summarizes the estimates of equations (20) and (22), with g_P^* treated as a constant. For all nonwar phases in the equation for g_P, the coefficient of $R_{y'}$ is positive for both the United States and the United Kingdom. However, it does not differ significantly from zero for the United States at the .05 level, and barely does so for the United Kingdom. Moreover, for the United States, the sign of the coefficient is reversed if the idiosyncratic interwar period is excluded. For all three

24. The chief alternative we considered was the average fraction of the labor force employed during each phase. For the United States, we use the complement of the unemployment rate for 1890–1939 constructed by Stanley Lebergott (*Manpower in Economic Growth* [New York: McGraw-Hill, 1964], pp. 43–512) that he linked to the Census Bureau's Current Population Survey P-50 reports for 1940–46; thereafter we work with the figures given in *Economic Report of the President Transmitted to the Congress January 1979*, table B-27, p. 21. For the United Kingdom, we use the complement of the unemployment rate for 1870–1965 in C. H. Feinstein (*National Income, Expenditure and Output of the United Kingdom, 1855–1965* [Cambridge: Cambridge University Press, 1972], table 57, pp. T125–27), extended by us for 1966–75 by adding figures for Great Britain (*Annual Abstract of Statistics* 113 [1976]: 150) and for Northern Ireland (*Digest of Statistics* 47 [March 1977]: 8, 13) for civilian working population and unemployment to derive the United Kingdom unemployment rate. We interpret a high fraction of the complement as meaning a high ratio of utilization of capacity and conversely. We shall refer to this measure as the employment ratio.

Neither measure is satisfactory—the output ratio, because a single exponential trend is an unduly crude representation of the long-run change in productive potential, the employment ratio, because it refers to only one category of productive resources and, in addition, is not very reliable statistically.

For the United States there seems a clear case for preferring the output ratio: first, the output ratio is available for the whole of our period, the employment ratio, only since 1890: second, the two ratios are highly correlated (for phase averages, .72 from 1890–91 to 1973–75; for annual observations, .90 from 1890 to 1975); third, a considerable number of test correlations using the two measures as alternatives generally yielded more reasonable and statistically more reliable results when the output ratio served as the measure of capacity utilization.

For the United Kingdom, the choice is less clear. First, both measures are available for the whole of our period. Second, the correlation between them, while positive, is much lower than for the United States (for phase averages, .59 from 1874–79 to 1973–75; for annual observations, .61 from 1874 to 1975). Third, in test correlations, sometimes one measure yields more reasonable and statistically more reliable results, sometimes the other. On the whole, the output ratio perhaps has a slight margin in its favor, but that margin could be readily reversed by a different implicit weighting of the test correlations.

In the absence of a clear case for the United Kingdom, we have used the output ratio for both countries in order to maintain comparability. We have checked to make sure that our major conclusions would not be reversed for the United Kingdom by the substitution of the employment ratio.

The output ratio has recently been used in related contexts in two articles in a single issue of the *American Economic Review* 70 (March 1980): Vito Tanzi, "Inflationary Expectations, Economic Activity, Taxes, and Interest Rates," pp. 12–21; and Jeffrey Sachs, "The Changing Cyclical Behavior of Wages and Prices," pp. 78–90.

Table 9.12 Estimates of Simple Phillips Curve, Nonwar Phases

Equations Fitted $g_P = a + c R_{y'}$

$g_{y'} = a' + b' g_M + c' R_{y'}$

Country and Period	Dependent Variable	Coefficient (and Absolute t Value): Constant Term	Monetary Growth (g_M)	Output Ratio ($R_{y'}$)	Number of of Observations	Variability[a]: Total	Residual
United States							
All nonwar	g_P	0.001 (0.3)		0.053 (1.9)	37	2.54	2.45
	$g_{y'}$	0.014 (3.5)	0.464 (5.8)	0.022 (1.0)		2.58	1.61
All nonwar excluding interwar	g_P	0.016 (3.2)		−0.117 (2.0)	30	2.21	2.10
	$g_{y'}$	0.026 (3.4)	0.160 (1.2)	0.084 (2.1)		1.52	1.43
United Kingdom							
All nonwar	g_P	0.003 (0.6)		0.118 (2.2)	24	2.54	2.35
	$g_{y'}$	0.022 (11.1)	0.002 (0.04)	−0.015 (1.0)		0.64	0.65

[a]Standard deviation of percentage rates of change.

equations, the reduction in variability by taking $R_{y'}$ into account is trivial in size. In addition, while equations (20) and (22) have constant terms and coefficients of $R_{y'}$ that are equal in size and opposite in sign for the g_P and $g_{y'}$ equations, all constant terms in table 9.12 are positive, and the coefficients of $R_{y'}$ for the United States for all nonwar phases are of the same sign for g_P and $g_{y'}$. For the other pair of United States equations, the two coefficients are opposite in sign and do not differ significantly in size. For the United Kingdom pair, the coefficients are opposite in sign but differ significantly in size. As always for United Kingdom output, the series looks like white noise. At most, the calculations show only a trace of a simple Phillips curve effect.[25]

One reaction to the failure of the simple Phillips curve was an attempt by many economists to retain the basic idea, by keeping $\eta = 0$, but to eliminate the conflict with observation by treating the anticipated rate of inflation g_P^*, in equation (20) as a variable adapting to experience rather than a constant. Such an expectations-adjusted Phillips curve is indirectly a way to move from nominal wages to real wages.[26]

This modification leaves a "short-run" trade off between the level of unemployment and the rate of inflation because g_P^* is taken to be relatively sluggish, adapting only gradually to experience. However, taken as it stands, equation (20) would deny any "long-run" trade off, since once expectations adapt to experience, $(g_P - g_P^*) = 0$ and hence, with $\eta = 0$, $\log y' = \log y'^*$, which means that there is no relation between the absolute rate of inflation and the observed level of utilization of capacity. (This has come to be known as the "accelerationist hypothesis.")

To retain the possibility of a long-run trade off, investigators have rewritten equation (20) by attaching a coefficient to g_P^* that can differ from unity, but still assuming $\eta = 0$.

In addition, they have kept the idea of a constant element in inflationary expectations by adding a constant term. This gives[27]

$$g_P = a_0 + a_1 g_P^* + \xi R_{y'}, \quad (23)$$

$$g_{y'} = -a_1 g_P^* + \psi g_M - \xi R_{y'} . \quad (24)$$

25. The results for subperiods are consistent with this general conclusion.

26. For a fuller discussion, see Milton Friedman, *Price Theory* (Chicago: Aldine, 1976), pp. 221–29.

27. Stephen J. Turnovsky, "On the Role of Inflationary Expectations in a Short-Run Macro-economic Model," *Economic Journal* 84 (June 1974): 317–37, gives an excellent analysis of this approach and summarizes some of the statistical studies using it. His equation (11) is, except for notation and a linear approximation to the final term, identical with equation (23).

Investigators have then explored whether $a_1 = 1$ in the long run. Most have concluded that a_1 is less than unity, implying that there remains a long-run trade off. However, this conclusion has been questioned as reflecting both statistical bias and mistaken specification of the appropriate test.[28] The essential criticisms are twofold: (1) That the statistical studies have stressed exclusively the influence running from employment to prices, neglecting entirely the reverse influence that is given primary pride of place by Irving Fisher in his 1926 article. Essentially this criticism is that it is inappropriate to treat η as equal to zero in deriving equation (20).[29] (2) That the studies have used models for the formation of

28. For the criticisms, see Thomas J. Sargent, "A Note on the 'Accelerationist' Controversy," *Journal of Money, Credit and Banking* 3 (August 1971): 721–25, and his "Rational Expectations, the Real Rate of Interest, and the 'Natural' Rate of Unemployment," *Brookings Economic Papers*, no. 2 (1973), pp. 429–72. Robert E. Lucas, Jr., "Econometric Testing of the Natural Rate Hypothesis," in *The Econometrics of Price Determination Conference*, ed. Otto Eckstein (Washington: Board of Governors of the Federal Reserve System and Social Science Research Council, 1972); Robert E. Lucas, Jr., "Some International Evidence on Output-Inflation Trade-offs," *American Economic Review* 63 (June 1973): 326–34; and R. Auerbach and R. Moses, "A Comment on Rothschild's 'The Phillips Curve and All That,'" *Scottish Journal of Political Economy* 21 (November 1974): 299–301.

For attempts to estimate a_1 in equation (23) concluding that it is less than unity, see R. M. Solow, "Recent Controversy in the Theory of Inflation: An Eclectic View," in *Inflation: Its Causes, Consequences and Control*, ed. S. W. Rousseas (Wilton, Conn.: Kazanjian Economics Foundation, 1968), and idem, *Price Expectations and the Behavior of the Price Level* (Manchester: Manchester University Press, 1969); R. J. Gordon, "Inflation in Recession and Recovery," *Brookings Papers on Economic Activity*, no. 1 (1971), pp. 105–58; S. J. Turnovsky, "The Expectations Hypothesis and the Aggregate Wage Equation: Some Empirical Evidence for Canada," *Economica* 39 (February 1972): 1–17; S. J. Turnovsky and M. L. Wachter, "A Test of the 'Expectations Hypothesis' Using Directly Observed Wage and Price Change Expectations," *Review of Economics and Statistics* 54 (January 1972): 47–54.

Three recent review articles contain additional references: R. J. Barro and S. Fischer, "Recent Developments in Monetary Theory," and R. J. Gordon, "Recent Developments in the Theory of Inflation and Unemployment," both in *Journal of Monetary Economics* 2 (April 1976): 133–67, 185–219; and A. M. Santomero and J. J. Seater, "The Inflation-Unemployment Trade-off: A Critique of the Literature," *Journal of Economic Literature* 16 (June 1978): 499–544.

See also the papers and references in Bennett T. McCallum, ed., "Rational Expectations: A Seminar Sponsored by the American Enterprise Institute," *Journal of Money, Credit and Banking* 12 (November 1980, part 2): 691–836.

29. Some of the studies are subject to still other criticism. For example, Solow, in *Price Expectations and the Behavior of the Price Level*, fits an equation like equation (23) except that, to allow for costs as well as demand, he includes on the right-hand side the rate of change of wages. There is no reason to expect a_1 to be unity in such an equation, which is concerned with the margin between prices and wages rather than the behavior of either separately. Let the anticipated rate of inflation go up by one percentage point but the rate of change of wages be unchanged. Any resulting rise in prices would stimulate output by increasing selling prices relative to costs, which implies that, even if a_1 in equation (23) were unity, Solow's counterpart would be less than unity.

expectations(g_P^*) that may not be "rational," in the sense that they do not involve the full utilization of the data available to participants in forming their expectations.[30] This criticism applies equally to the adaptive expectations models that we have used in previous chapters and that we use in the next section.

In a series of very interesting and important papers, Lucas and Sargent have explored the implication of the rational expectations hypothesis and have tried to derive empirical tests of the slope of the long-run Phillips curve without the possibly misleading assumption of adaptive expectations.[31]

Their empirical tests use a different kind of information. For example, one implication of a rational expectations hypothesis is that, in a country in which prices have fluctuated a great deal, expectations will respond to changes in the current rate of inflation much more rapidly than in a country in which prices have been relatively stable. It follows that the observed short-run Phillips curve will be steeper in the first country than in the second. (We explore this effect with our data below.) Comparisons among countries in this way, as well as other tests, seem so far entirely consistent with what any reasonable person must surely expect: that, *since you can't fool all the people all the time, the true long-run Phillips curve is vertical*. However, the evidence is by no means all in, so we cannot regard the matter as settled.

On the more limited issue of whether it is appropriate to regard $\eta = 0$, and the output ratio as the major determinant of the deviation of the rate of inflation from its anticipated value, we can alter equation (20) by including a term in g_M. For this purpose, a crude and yet simple way of allowing for both inflationary anticipations (g_P^*) and anticipations about monetary growth (g_M^*) seems adequate. Accordingly, we do so in equation (18) by approximating ($g_P^* - \eta\psi g_M^*$) by $g_P(t-1)$ times a coefficient.[32]

30. Note the words "may not be." The adaptive expectations models that we and other investigators have used may or may not be "rational," depending on the assumed stochastic structure of disturbances. Brunner, Cukierman, and Meltzer, in their formal models constructed in the papers referred to in footnote 9 above, assume a stochastic structure under which adaptive expectations are "rational."

31. This and the following paragraph are from Milton Friedman, *Price Theory*, p. 231. The papers alluded to are: Robert E. Lucas, Jr., "Econometric Testing of the Natural Rate Hypothesis," in *The Econometrics of Price Determination Conference*, ed. Otto Eckstein, and "Econometric Policy Evaluation: A Critique," in *The Phillips Curve and Labor Markets*, ed. K. Brunner and A. H. Meltzer (Carnegie-Rochester Conference Series on Public Policy, vol. 1, 1976), pp. 19–46; Thomas J. Sargent, "Rational Expectations, the Real Rate of Interest, and the 'Natural' Rate of Unemployment," *Brookings Papers on Economic Activity*, no. 2, pp. 429–72.

32. One way of rationalizing this approach is by supposing that $g_P(t)$ is a random walk, so that the best estimate of $g_P^*(t)$ is $g_P(t-1)$, and that $g_M^*(t)$ can be regarded as a roughly constant fraction of $g_P^*(t)$, so that a proxy for $g_P^*(t)$ is also a proxy for $g_P^*(t) - \eta\psi g_M^*(t)$. Of course, there are many other possible rationalizations.

The resulting equations are:

$$g_P(t) = a_o + a_1 g_P(t-1) + \eta\psi g_M(t) + \xi R_{y'}(t) \quad (25)$$

$$g_{y'}(t) = a_o' - a_1 g_P(t-1) + (1-\eta)\psi g_M(t) - \xi R_{y'}(t) \,. \quad (26)$$

Table 9.13 reports estimates of these equations computed without the cross-equation restrictions on their coefficients embedded in equations (25) and (26).

These results make it clear that monetary growth is far more important than the output ratio—in our terms, if either η or ξ is to be set equal to zero, far better to set $\xi = 0$, certainly for the price equations. The only statistically significant coefficient of $R_{y'}$ in any of the price equations is for all United States nonwar phases, and for that equation the effect is in the wrong direction, the estimated ξ being negative rather than positive—a point to which we shall return.

For both price and output equations, the t value for the coefficient of g_M is consistently higher than for the coefficient of $R_{y'}$, and all but one of the coefficients of g_M is statistically significant. Not surprisingly, that one is for United Kingdom output—a series we have consistently been unable to explain. It appears, so far as we can tell, to be a purely random series.

The prior rate of price change has a statistically significant effect in all equations other than the United Kingdom output equation. For all three price equations, it is the single most important variable; for the two United States output equations, monetary growth is.

For the United States, the coefficients of $g_P(t-1)$ and g_M are not inconsistent with the restrictions embedded in equations (25) and (26): the coefficients of prior price change for the price and output equations are opposite in sign and not significantly different in size; the coefficients of monetary growth add up to 1.07 and 0.94, respectively, which are consistent with a theoretically anticipated value of ψ of about unity. For the United Kingdom, neither restriction is well satisfied—another example of our inability to interpret real income movements in the United Kingdom.

The residual variability is consistently lower, except again for the United Kingdom output equation, than the lowest variability obtained in table 9.10. This result may simply reflect the high serial correlation of g_P. However, it may also be evidence in favor of the validity of the approach incorporated in equations (18) and (19), except for the condition that ξ is either zero or not well identified. To illustrate, consider the price equations for the United States, excluding the interwar period, and for the United Kingdom for all nonwar phases, omitting, for both countries, the statistically not significant term in $R_{y'}$:

(27) United States $g_P(t) = -0.009 + 0.686 g_P(t-1) + 0.296\, g_M(t)$

Table 9.13 **Estimates of Relations between Rates of Change of Prices and Output, and Prior Price Change and Output Ratio, Nonwar Phases**

Equations Fitted: $g_P = a + b\, g_P(t-1) + c g_M(t) + d\, R_{y'}(t)$

$g_{y'} = a' + b' g_P(t-1) + c' g_M(t) + d' R_{y'}(t)$

Country and Period	Dependent Variable	Coefficient (and Absolute t Value) of: Constant Term	Prior Price Change $g_P(t\text{-}1)$	Monetary Growth g_M	Output Ratio $R_{y'}$	Number of Observations	Variability: Total	Variability: Residual
United States								
All nonwar	$g_P(t)$	−0.008 (3.6)	0.614 (9.1)	0.314 (6.4)	−0.036 (3.6)	37	2.54	0.75
	$g_{y'}(t)$	0.001 (0.4)	−0.627 (6.3)	0.760 (10.5)	0.022 (1.5)		2.58	1.10
All nonwar excluding interwar	$g_P(t)$	−0.009 (1.8)	0.686 (6.5)	0.296 (2.8)	−0.014 (0.5)	30	2.21	0.74
	$g_{y'}(t)$	0.009 (1.2)	−0.615 (3.6)	0.645 (3.7)	−0.001 (0.0)		1.52	1.20
United Kingdom								
All nonwar	$g_P(t)$	−0.006 (2.5)	0.582 (6.1)	0.422 (4.9)	0.014 (0.9)	24	2.54	0.58
	$g_{y'}(t)$	0.020 (7.0)	−0.113 (1.1)	0.086 (0.9)	−0.008 (0.5)		0.64	0.65

(28) United Kingdom $g_P(t) = -0.006 + 0.582 g_P(t-1) + 0.422 g_M(t)$.

The first observation suggested by these results is that the two equations are very similar. They clearly do not differ significantly.

A second observation is that both equations imply that in the long run (when $g_P(t) = g_P(t-1)$), the difference between g_M and g_P is roughly a constant, the relevant equations being:

(29) United States $g_P(t) = -0.029 + 0.94 g_M$

(30) United Kingdom $g_P(t) = -0.014 + 1.01 g_M$.

The constant term in principle is equal in numerical value but opposite in sign to the long-term rate of output growth times the income elasticity of demand for real balances, which is close to unity for both countries. The computed constant term does approximate the long-term rate of output growth, which was 3.1 percent per year for the United States, 1.7 percent per year for the United Kingdom. The coefficient of g_M is not significantly different from unity for either equation—the strict quantity theory conclusion.

A third observation is that, if we can take Ψ to be close to unity, the coefficient of g_M in equations (27) and (28) is an estimate of η—the fraction of unanticipated change in nominal income (i.e., the difference between actual and anticipated nominal income growth) absorbed by unanticipated price change (i.e., the difference between actual and anticipated inflation). The estimate is in the neighborhood of one-third for both countries. Other studies have shown a strong and systematic effect of unanticipated monetary growth on output but most such studies have been for quarterly or annual data.[33] Our results suggest that the averaging

33. Robert J. Barro, "Unanticipated Money Growth and Unemployment in the United States," *American Economic Review* 67 (March 1977): 101–15: "Unanticipated Money, Output, and the Price Level in the United States," *Journal of Political Economy* 86 (August 1978): 549–80; Robert J. Barro and Mark Rush, "Unanticipated Money and Economic Activity," in *Rational Expectations and Economic Policy*, ed. Stanley Fischer (Chicago: University of Chicago Press, 1980), pp. 23–48.

Robert J. Gordon, in his comment on the Barro-Rush paper (ibid. pp. 55–63) points out that the evidence in these papers does not reject the hypothesis that anticipated as well as unanticipated monetary changes affect real output, and he presents statistical evidence to support his skepticism. Gordon's analysis and evidence are persuasive. We suspect, however, that the key issue is not "anticipated" versus "unanticipated" but the time period discussed in footnote 9 above. Two time points are relevant: the date at which anticipations are formed; the date to which they refer. A change at time t anticipated at time $t-1$ may have been unanticipated at time $t-2$. It will presumably not affect real output decisions made at time $t-1$; it will affect those made at time $t-2$.

Barro's approach defines "unanticipated" entirely in terms of a one-year time span. Given the existence of longer term contracts, what Gordon designates as "anticipated" may involved in calculating cycle phases does not eliminate such an effect. As we also noted in section 9.3, the period of adjustment to unanticipated changes is surprisingly long.

One implication of the theory of rational expectations that was pointed out by Lucas and mentioned above is that the response to monetary change might well depend on anticipations not only about the level of inflation but also about the variability of inflation. In an economy in which the rate of change in the general level of prices has been highly stable, anticipations about that rate of change might be expected to be held with considerable confidence. Much new evidence will be required before participants in the economy come to interpret changes in the nominal demand for their products and services as reflecting changes in the rate of inflation rather than in relative demand. Conversely, in an economy that has experienced frequent and substantial changes in the rate of inflation, participants will readily alter their inflationary anticipations. We would expect a changed rate of monetary growth to be reflected initially more in output and less in prices in the first economy than in the second (that is, we would expect η to be smaller), and a longer period to elapse before the change was fully reflected in prices.

Lucas explored this implication in terms of differences among countries. In an interesting paper, Benjamin Klein has done so for time-series data for the United States.[34] As a measure of what he calls "short-term price unpredictability," Klein initially used a moving variance of rates of inflation. In his calculations for the United States with annual data, he related this variable to "long-term price unpredictability" to show an upward shift in the amount of long-term relative to short-term price uncertainty dating from the mid-1950s.

well have been unanticipated at the time the contracts were entered into. An attempt by Stanley Fischer to test the effect of including forecast errors for a two-year as well as a one-year time span yielded a lower residual variability. However, the effect was small (ibid., pp. 234–35).

Our own results suffer from the same difficulty as Barro's but to a lesser extent because of the longer and variable time unit.

In a recent article, Gordon provides independent confirmation, based on annual data for the United States, of our estimate that about one-third of the unanticipated change in nominal income is absorbed by unanticipated prices change. He writes: "nominal GNP changes have been divided consistently, with two-thirds taking the form of output change and the remaining one-third the form of price change. ("A Consistent Characterization of a Near-Century of Price Behavior," *American Economic Review Papers and Proceedings* 70 [May 1980]: 243–49; quotation from p. 243).

34. "Our New Monetary Standard: The Measurement and Effects of Price Uncertainty, 1880–1973," *Economic Inquiry* 13 (April 1975): 461–84. Klein included his "short-term price unpredictability" variable in demand for money regressions. The variable produced an increase in voluntarily held money balances ("The Demand for Quality-Adjusted Cash Balances: Price Uncertainty in the U. S. Demand for Money Function," *Journal of Political Economy* 85 (August 1977): 691–715).

Subsequently, in response to criticisms from Ibrahim and Williams, Klein developed what he regarded as superior measures of expected price variability.[35] We have calculated phase averages for both Klein's initial variable and his subsequent revised version of short-term price unpredictability (which we designate EVP), for both our United States and our United Kingdom data, and have added EVP to the regressions for $g_{y'}$ summarized in table 9.13 to test its usefulness.[36] Unfortunately the results were not encouraging. The coefficient of EVP does not differ significantly from zero for any of the three regressions corresponding to those in table 9.13.

These results do not invalidate the theoretical expectation that a high expected variability of prices will make for a fuller and more prompt adjustment of prices and a smaller adjustment of output in response to changes in monetary growth. We suspect that our results reflect much more the defects of the particular measure of expected variability of price plus the small number of degrees of freedom available to test its role.

One bit of evidence that our negative results may reflect the defect of our measure of uncertainty comes from a study by Donald Mullineaux of the effect of inflation uncertainty on employment and output in the United States during the post–World War II period. He uses two measures: "a moving-standard deviation of the observed inflation rate"—similar in kind though not in detail to the Klein measure—and "a time-series of standard deviations calculated from cross-section surveys of inflation anticipations." He found that the second measure "significantly affected unemployment and production" for both a longer (1950–75) and a shorter (1958–75) period he examined but that the first affected them only for the shorter period. Unfortunately, the cross-section measure of uncertainty that he used is available only for the post–World War II period, so we could not test it with our data.[37]

A more promising body of evidence for a test of the role of uncertainty over a longer period might be obtained from cross-country comparisons, such as those suggested by Lucas.

35. See B. Ibrahim and R. Williams, "Price Unpredictability and Monetary Standards: A Comment on Klein's Measure of Price Uncertainty," *Economic Inquiry* 16 (July 1978): 413–37; Benjamin Klein, "The Measurement of Long- and Short-Term Price Uncertainty: A Moving Regression Time Series Analysis," *Economic Inquiry* 16 (July 1978): 438–52.

36. We are grateful to Klein and to Michael Melvin for constructing the relevant United Kingdom series for us.

37. Donald J. Mullineaux, "Unemployment, Industrial Production, and Inflation Uncertainty in the United States," *Review of Economics and Statistics* 62 (May 1980): 163–69.

9.8 Effect of Output Capacity and Anticipations: The Approach through Alternative Models of the Formation of Anticipations

The results of the previous section seemed sufficiently encouraging to justify trying to adopt a more sophisticated approach to the formation of anticipations about g_P^*, $g_{y'}^*$, and g_M^* than the assumption that $g_P(t-1)$ is a satisfactory proxy for $g_P^* - \eta\psi g_M^*$. Accordingly, we have considered a number of alternative hypotheses in an attempt to approximate these unobservable magnitudes. Though our experiment unfortunately turned out to be unsuccessful, we did have one rather interesting by-product, not about anticipations but about the role of capacity utilization.

9.8.1 Alternative Hypotheses

Since

$$g_P^* + g_{y'}^* = g_Y^* ,$$

only two of three anticipated magnitudes are independent. We have earlier assumed that g_Y^* can be regarded as given by the Cagan-Koyck adaptive expectations mechanism (see equation A11 in sec. 8.4). That assumption yielded reasonably satisfactory results. We have therefore retained it and concentrated here on $g_{y'}^*$, treating g_P^* as given by $g_Y^* - g_{y'}^*$.

We list in the text below the alternative hypotheses we have considered, relegating the details about the equations that embody them to section 9.10.5.

Unchanging Anticipations

The simplest approach is to suppose that economic actors treat both $g_{y'}^*$ and g_M^* as constants.

In view of the absence of any long-period trends in $g_{y'}$ and the dependence of its secular movements primarily on basic institutional elements, it would not have been unreasonable for participants to have regarded $g_{y'}^*$ as a constant, except perhaps during the wars. The situation with respect to g_M^* is very different. In the first place, g_M has been much more variable than $g_{y'}$ (see table 9.2), so there is more incentive to take its fluctuations into account. In the second place, g_M is much more subject to direct government control than is $g_{y'}$ and is known to be so subject. This consideration is important for both the United States and the United Kingdom after World War II, when both countries were explicitly committed to using monetary and fiscal policy to promote full employment. It is least important for both the United States and the United Kingdom before World War I. The United States had no central bank—though the

Treasury Department acted in many ways like one—and was firmly committed to an international gold standard, so that the quantity of money was not a policy variable. The United Kingdom had a central bank, but it too was firmly committed to an international gold standard.

For both countries, the interwar period is intermediate. In the United States a central bank (the Federal Reserve) had been established in 1914, but it was new and untried; it proclaimed its dedication to the international gold standard until 1933; and it adopted a passive role thereafter. Nonetheless, the existence of the Federal Reserve as an intermediary in the transmission of monetary influences from abroad with the capacity to sterilize or reinforce gold movements did significantly alter the determination of the quantity of money.

In the United Kingdom, despite the post–World War I determination to return to gold at the prewar parity, the delay of the return to gold until 1925 and the subsequent departure in 1931 meant that throughout most of the period after the outbreak of World War I, the United Kingdom was on a fiduciary standard operated primarily to promote domestic objectives.

We conclude that the assumption that $g^*_{y'}$ and g^*_M were treated by economic actors as constants is plausible for both countries in the pre–World War I period; more plausible for the United States than for the United Kingdom in the interwar period; highly implausible for both in the post–World War II period; and highly implausible for both during the two wars.

Unchanging Anticipations for Output: Adaptive Anticipations for Money

A more complex approach is to assume that the economic actors regarded $g^*_{y'}$ as a constant but estimated g^*_M on the basis of earlier values of g_M, the kind of adaptive expectations we used for g^*_Y in chapter 8. One justification of this approach is that serial correlations of g_M tend to be higher than of $g_{y'}$; that is, earlier values of g_M contain more information about current values of g_M than earlier values of $g_{y'}$ contain about current values of $g_{y'}$ (the only exception is for the aberrant United States interwar period).[38]

38. The serial correlations are as follows:

	g_M	$g_{y'}$
Pre–World War I		
United States	.62	.34
United Kingdom	.49	.34
Interwar		
United States	.47	.50
United Kingdom	.90	.46

This approach leads to equations that are not linear in the parameters. Hence we have had to resort to an iterative procedure to estimate them.

Adaptive Anticipations for both Output and Money: Rate of Adaptation Equal ($w = w'$)

Though $g_{y'}$ has no long-term trend, it does have considerable fluctuations, and there is positive serial correlation between its successive values (see footnote 38 above). Hence, a further complication is to assume that the economic actors also estimate $g_{y'}^*$ on the basis of earlier values of $g_{y'}$. We again do so by Cagan-Koyck adaptive anticipations.

The simplest case arises if the weight (w') attached to the current value of $g_{y'}$ in deriving $g_{y'}^*$ is equal to the weight (w) attached to the current value of g_M in deriving g_M^*.

Adaptive Anticipations for both Output and Money: Rates of Adaptation Not Equal ($w \neq w'$)

If $w \neq w'$, the final equations are much more complex. Like those for the second hypothesis above, they are not linear in the parameters.

9.8.2 Comparison of Alternative Hypotheses

Although we have derived in the Appendix (sec. 9.10) pairs of equations—one for price, one for output—we need estimate only one to test the various hypotheses. Both contain the same unknown parameters, and both will give the same numerical estimates because of the identities connecting price, output, and income. We have chosen to concentrate on the output equation both because it is generally the simpler of the two and because the simple quantity theory implies that none of the variables included in these equations should have any significant influence on output (that is, on the simple quantity theory hypothesis, $\eta = \xi = 0$). And we have found repeatedly that the simple quantity theory hypothesis comes closer to being satisfied than the other simple hypotheses we have considered.

Table 9.14 indicates that for the United Kingdom none of the hypotheses contributes anything to the explanation of the rate of change of output for all nonwar phases. The simple quantity theory is confirmed again—as we have repeatedly found with respect to output for the United Kingdom. The total variability of the rate of change of output is 0.64

Post–World War II		
United States	.74	.54
United Kingdom	.88	.57
All nonwar phases		
United States	.61	.35
United Kingdom	.81	.47

percentage points; the lowest residual variability in table 9.10, which summarizes the effects of allowing for money and yields, prior money, prior income, and trend, is 0.60; the residual variability in table 9.13, which allows for prior price change and the output ratio, is 0.65; and in table 9.14 it is 0.62. Five out of six measures of residual variability in table 9.10, the residual variability in table 9.13, and six out of the eight measures in Table 9.14 are higher than the total variability. The situation is the same for interwar and post–World War II periods. For the pre–World War I period, all the measures of residual variability in table 9.14 are less than the total variability and a number are less than half the total variability. But these measures are based on only four to six degrees of freedom and hence are at best suggestive. All in all, it appears for the United Kingdom that neither the deviation of monetary growth from its anticipated value nor the ratio of actual output to anticipated output has any significant influence on the deviation of the rate of growth of output from its anticipated value—or alternatively, that none of our hypotheses about the formation of anticipated values provides a satisfactory approximation. The only hint of a different conclusion is for the pre–World War I period.

For the United States the situation is different—as is indicated at the outset by the much greater variability of output growth (more precisely, estimated output growth) for the United States than for the United Kingdom. All but two of the thirty-five estimates of residual variability in table 9.14 are less than the corresponding total variability, and many are much less. However, it is noteworthy that only two of the measures of residual variability for all nonwar phases and two for the period excluding interwar phases are as low as the residual variability for the simple Phillips-curve type of hypothesis in table 9.13. On the whole, the hypothesis of adaptive anticipations for money (cols. 5 through 10) gives appreciably lower variability than the hypothesis of constant anticipations (cols. 3 and 4) regardless of the hypothesis about output or the effect of capacity utilization with which it is combined.

To go beyond that statement, it is necessary to separate the results for an assumed zero and nonzero effects of the output ratio. For a zero effect ($\xi = 0$), adaptive anticipations for money, constant for output (column 5), is clearly the most satisfactory hypothesis: it gives the lowest residual variability of any of the $\xi = 0$ results except only for the idiosyncratic interwar period. Moreover, the residual variability for both all nonwar phases and for nonwar phases excluding interwar phases is lower than the lowest measure in table 9.10 (0.98 vs. 1.15 for all phases; 0.91 vs. 0.95, excluding interwar), and both are lower than the residual variability in table 9.13 (0.98 vs. 1.10 and 0.91 vs. 1.20).

For an assumed nonzero effect of the output ratio ($\xi \neq 0$), the hypothesis of adaptive anticipations for money, constant for output, requires

Table 9.14 Comparison of Alternative Hypotheses about Anticipations: Residual Variability of Output after Allowing for Anticipations

			Standard Error of Estimate (in Percentage Points) of Rate of Change of Output Hypothesis about Anticipated Rates of Money and Output Growth and Effect of Output Ratio							
			Constant		Adaptive for Money, Constant for Output		Adaptive for Both, Rates of Adaptation Equal		Unequal	
Period and Country	Number of Phases (1)	Total (2)	$\xi = 0$ (3)	$\xi \neq 0$ (4)	$\xi = 0$ (5)	$\xi \neq 0$ (6)	$\xi = 0$ (7)	$\xi \neq 0$ (8)	$\xi = 0$ (9)	$\xi \neq 0$ (10)
Number of parameters estimated		1	2	3	3	4	1	3	3	4
Nonwar phases										
United States										
All	37	2.58	1.61	1.61	0.98	1.01	1.53	1.12	1.30	Δ
Excluding interwar	30	1.52	1.51	1.43	0.91	Δ	1.35	0.81	1.28	Δ
United Kingdom										
All	24	0.64	0.67	0.67	0.62	0.63	0.89	0.73	0.74	0.68
Pre-World War I										
United States	19	1.73	1.65	1.49	1.08	Δ	1.62	0.92	1.57	1.09
United Kingdom	8	0.53	0.32	0.36	0.36	0.31	0.43	0.40	0.49	0.36
Interwar										
United States	7	3.28	1.17	1.23	1.29	0.65	1.86	0.86	1.12	0.35
United Kingdom	7	1.03	1.07	1.15	0.98	Δ	1.55	1.50	1.37	1.53
Post–World War II										
United States	11	1.10	1.16	1.16	0.57	Δ	0.73	0.45	0.68	0.69
United Kingdom	9	0.70	0.68	0.74	0.65	Δ	0.60	0.58	0.58	0.57

Note: Δ Nonlinear estimates did not converge.

nonlinear estimation (column 6), which failed to converge for three out of five sets of United States observations. Needless to say, we do not know whether this result reflects a defect of the nonlinear program or of our initial approximate parameter estimates, or whether it is a valid judgment on the economic validity of the hypothesis.

It does mean that, if the output ratio is allowed for ($\xi \neq 0$), our best results are for the hypothesis of adaptive anticipations for both money and output, with equal rates of adaptation (column 8). The residual variability for this hypothesis for the United States is slightly lower for all nonwar phases combined than the lowest measure in table 9.10 (1.12 vs. 1.15) and appreciably lower for nonwar phases excluding the interwar phases (0.81 vs. 0.95), and both are lower than the residual variability in table 9.13.

As between the hypothesis that is best when the output ratio is omitted ($\xi = 0$) and the one that is best when the output ratio is allowed for ($\xi \neq 0$), the evidence is conflicting. For the separate periods, the residual variability is consistently less for the United States when the output ratio is allowed for (column 8) than when it is not (column 5). That is also true for nonwar phases excluding interwar phases. However, for all nonwar phases combined the situation is reversed, implying that not allowing for the output ratio gives more homogeneous results for different periods than allowing for it. For the United Kingdom, the residual variability is higher when the output ratio is allowed for (col. 8) than when it is not (col. 5) for all nonwar phases and for interwar phases, lower for pre–World War I and post–World War II. One further minor bit of evidence for the hypothesis of column 5 is from the United Kingdom: it is the only hypothesis, excluding those for which some estimates are missing, for which the residual variability is consistently less than the total variability.

As further evidence, table 9.15 gives the parameter estimates for these two alternative hypotheses. The estimate of ψ (the response of nominal income to nominal money) is the same for both hypotheses. For combined phases, the values of ψ do not differ significantly from unity—the result we have repeatedly observed. For separate periods, with small numbers of observations, the estimates vary widely.[39]

39. The estimates differ significantly from unity for both countries for the post–World War II period and for the United Kingdom for the pre–World War I period. The low postwar values are somewhat paradoxical and reflect the particular pattern of the postwar reaction to the wartime decline in velocity.

The postwar rise in velocity at first suggests that the estimated ψ should be greater rather than less than unity. However, equation (A44) of the appendix is based on second derivatives, the acceleration of income and money, rather than the first derivatives, the rates of change. The postwar pattern was that the rate of monetary growth fell sharply in the early postwar period, then rose sharply, whereas the rate of income growth fell less, was fairly flat

Table 9.15 **Estimates of Parameters on Two Alternative Hypotheses**

Period and Country	Number of Phases	Estimates of Parameters for Hypotheses about Anticipated Rates of Money and Output Growth and Effect of Output Ratio							
		Adaptive for Money, Constant for Output, $\xi = 0$				Adaptive for Both Money and Output, Rates of Adaptation Equal, $\xi \neq 0$			
		ψ (1)	$g_{y'}^*$ (2)	η (3)	w (4)	ψ (5)	η (6)	w (7)	ξ (8)
Nonwar phases									
United States									
All	37	1.05	.0182	0.19	0.38	1.05	0.74	2.71	−0.08
Excluding interwar	30	0.97	.0281	−0.28	0.58	0.97	0.65	2.11	−0.10
United Kingdom									
All	24	0.81	.0208	0.14	0.77	0.81	0.68	1.36	−0.01
Pre–World War I									
United States	19	1.03	.0279	−0.44	0.63	1.03	0.73	2.37	−0.12
United Kingdom	8	0.49	.0174	−1.05	0.49	0.49	0.13	3.17	−0.02
Interwar									
United States	7	1.10	.0098	0.20	0.39	1.10	−0.16	−3.36	0.54
United Kingdom	7	1.87	.0147	−1.62	0.84	1.87	0.60	0.10	0.15
Post–World War II									
United States	11	0.63	.0229	−0.33	0.29	0.63	−0.07	0.89	0.01
United Kingdom	9	0.51	.0241	0.95	0.36	0.51	0.62	−0.70	−0.16

Note: Calculated from output regressions.

With respect to the remaining parameters, which differ between the two hypotheses, neither set of estimates is very appealing, though the estimates for the hypothesis of column 5 of table 9.14 seem preferable. For that hypothesis, the estimated permanent rate of real output growth is of the right order of magnitude—though perhaps a little low for the United States, a little high for the United Kingdom. Similarly, w, the estimated weight attached to current monetary growth in computing anticipated growth, is reasonable, being consistently between zero and unity and implying that the current phase receives from one-third to four-fifths of the weight in forming anticipations about monetary growth. However, the estimated values of η, the fraction of an unanticipated change in nominal income absorbed by an unanticipated change in prices, seem much too low, only 19 percent for all nonwar phases for the United States, and actually negative for pre–World War I and post–World War II phases. That negative result implies that a monetary growth higher than was anticipated would mean an inflation rate *lower* than was anticipated. Though not necessarily inconsistent with the observed positive relation between measured monetary growth and measured inflation, the result is certainly implausible.[40]

For the hypothesis of column 8 of table 9.14, the estimated values of η are more in line with expectations, but those of w and ξ are wholly out of line. Only two out of the nine estimates of w are within the anticipated zero to unity range; and only three of the estimates of ξ have the anticipated positive sign. The relatively good fit of the underlying equation for the United States clearly cannot be regarded as evidence in favor of the hypothesis from which it was derived.

All in all, the results from the much simpler relations of section 9.7 seem much more in line with expectations. They too yield a $\psi = 1$, but a value of η of roughly a third.

We have here a special case of a general proposition: the same empirical relation can be generated by more than one alternative hypothesis. As

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for a time, and then rose, though less sharply than money. This meant that acceleration of both money and income was first negative, then positive, but the swing in money had a greater amplitude than in income, thereby producing an estimated ψ less than unity.

40. The reason a negative η is not necessarily inconsistent with a positive correlation between g_M and g_P is the assumed positive relation (indeed, equality) between g_M^* and g_P^* and the heavy weight of $g_M(t)$ in estimating $g_M^*(t)$. For example, for the parameters $\eta = -0.28$, $\psi = 0.97$, and $w = 0.58$, a one percentage point jump in g_M would increase $g_M^*(t)$ by 0.58 percentage points and mean a deviation of 0.42 percentage points between g_M and g_M^*. That would mean introducing a deviation of $0.97 \cdot 0.42 \cdot (-0.28)$ equal to -0.11 percentage points between g_P and g_P^*. If, before the disturbance, $g_P^* = g_Y^* - g_{y'}^* = g_M^* - 0.0281 = 0$, say, then subsequent to the disturbance, g_M^* would become $.0281 + .0058 = .0339$, and g_P^* would become .0058. Since $g_P - g_P^* = -0.0011$, the final result would be that g_P would be 0.0048, i.e., a one percentage point jump in g_M would be accompanied by a 0.48 percentage point immediate jump in inflation, compared with the 0.62 percentage point change in table 9.11 in the initial period.

it happens, the hypotheses underlying columns 5 and 8 of table 9.14 yield equations in observable variables very similar to some of those considered in section 9.6, namely, relating current output growth to earlier growth in money and income. The key difference for column 5 is the inclusion of an earlier value of output growth rather than of nominal income growth. The minor improvement in results presumably reflects the serial correlation of output growth from phase to phase. The key difference for column 8 is the inclusion of the current and lagged output ratio in addition to earlier monetary growth. In both cases, the variables that make a difference are only very loosely related to our alternative assumptions about the formation of anticipations.

We conclude that, while our results may give some evidence on the role of the output ratio, they do not provide any reliable evidence on the process of the formation of anticipations. From that point of view we must simply record an unsuccessful experiment.

The evidence on the role of the output ratio is provided by the estimated values of ξ—which measure the response of output and prices to the output ratio. A positive ξ, the sign to be expected on Phillips-curve lines, means that a high ratio of output stimulates inflation relative to output growth; a low ratio means the reverse. The fascinating feature of the values given in table 9.15 is that they are generally negative rather than positive. Instead of a high ratio of output to capacity tending to increase the price response and reduce the output response to a change in monetary growth, it appears to have precisely the opposite effect. For the United States, the negative values for combined periods (the first two lines of table 9.15) differ significantly from zero; for the United Kingdom the negative value does not.[41] The one appreciable exception is for the United States interwar period, for which the estimated value of ξ is positive and significantly different from zero at a .05 level.[42]

These are fascinating results. First, they reinforce the conclusion of section 9.5 that the United States interwar period is both idiosyncratic and the only period that is consistent with the Keynesian analysis—in this case, the post-Keynesian analysis embodied in the so-called Phillips curve. In the second place, they suggest that, for the other periods, the Phillips curve, insofar as it applies, is positively rather than negatively sloped.[43]

41. The absolute value of t for these three estimates of ξ are 2.4, 3.7, and 0.6; the first is significant at a .05 level, the second at a .01 level.

42. The t value is 3.8 and there are 4 degrees of freedom.

43. This conclusion, based on the equation underlying column 8 of table 9.14, is also consistent with the estimates of ξ based on the equations corresponding to columns 4, 6, and 10. Out of twenty such estimates, 13 are negative, seven positive; and of the four that are statistically significantly different from zero at a .05 level, all are negative. For the United Kingdom separately, the evidence is weaker, as is to be expected from the lack of statistical significance of most of the regressions. Of the ten estimates for the United Kingdom for

This result, though wholly out of tune with professional views in the 1950s and 1960s, seems far less so from the perspective of the late 1970s. As the postwar period unfolded, higher inflation tended to be accompanied by higher, not lower, unemployment, leading to the hypothesis of a positively sloped Phillips curve.[44] Our results suggest that that hypothesis may apply over a much longer time span than the post–World War II period alone. However, we hasten to repeat that our evidence on this particular point is weak and is suggestive rather than conclusive.

9.9 Conclusion

The results of this chapter are at the same time disappointing and most informative.

They are disappointing because we have not succeeded, as we had hoped we would, in giving satisfactory empirical content to the sophisticated theoretical analysis in chapter 2 bearing on the division over short periods of a change in nominal income between prices and output. The empirical analysis in section 9.8 comes closest in sophistication to the theoretical analysis—yet that section, while it yields some fascinating results, also records an unsuccessful experiment.

One possible interpretation of our failure to give satisfactory empirical content to the theoretical analysis is a point noted at the outset of this chapter—that we have used too lengthy a unit of observation, and that a similar empirical analysis using monthly or quarterly data might uncover empirical intracycle relations corresponding to our theoretical hypothesis. The results of section 9.3, on the effect of lengthening the period, as well as the many regressions in this and previous chapters using observations for earlier phases, rather argue against that interpretation. We have repeatedly found that the adjustment time is long, to be measured in cycle phases, not in months. Nonetheless, it may be that, for the particu-

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equations corresponding to columns 4, 6, and 10, five are positive, five negative. However, the only one significantly different from zero at the .05 level is negative.

The only bit of contradictory evidence comes from equations for the United Kingdom corresponding to columns 4 and 8 of table 9.14, but using the employment ratio rather than the output ratio. While for the output ratio four of the eight estimates are positive, four are negative, and none is statistically significantly different from zero, for the employment ratio, five of the eight estimates are positive, three are negative, and one differs significantly from zero; that one is positive, and is for all nonwar phases and the equation corresponding to column 8. (The estimated value of ξ is 0.31, and the t value is 2.3, with 19 degrees of freedom.) However, the equation involved has a residual standard error higher than the initial standard deviation, so this result does not deserve much confidence; presumably, it reflects intercorrelations among the independent variables rather than a relation to the dependent variable.

44. For a fuller discussion of these issues, see Milton Friedman, "Inflation and Unemployment," referred to in note 22 above.

lar problem of the relative roles of prices and output, timing differences measured in months are vital, and that these are obliterated by our phase bases.

The results are informative in three rather different ways: (1) they narrow sharply the phenomena requiring explanation; (2) they suggest that much "conventional wisdom" reflects overgeneralization of a special case; and (3) they suggest that what "conventional wisdom" currently regards as a special case may be the norm.

1. For the United Kingdom there seems little if any relation between monetary change and output: a simple quantity theory that regards price change as determined primarily by monetary change and output by independent other factors fits the evidence for the period as a whole (excluding wars, which we have largely omitted from our analysis). The whole of a change in the quantity of money is absorbed sooner or later by prices and, in the early stages, more by changes in velocity than by changes in output. The output series bears little or no relation to any monetary factors, either the quantity of money, in the current or prior phases, or yields. From that point of view, the rate of change of output appears to be a random series (though not one displaying complete serial independence). Finally, the rate of change of output is much less variable over time than the rate of change of prices. Its variability is of the same order of magnitude as that which would be produced simply by measurement error.

For the United States, for the period as a whole, there is a positive relation between monetary change and output change, and price change absorbs a smaller percentage than in the United Kingdom of the change in the quantity of money, and of the related change in nominal income. In sharp contrast to the United Kingdom, the rate of change of output is more variable than the rate of change of prices.

On further examination, and this leads on to point 2, it turns out that much of the difference between the United States and the United Kingdom reflects the United States interwar period—a twenty-year span containing three major contractions. That period seems idiosyncratic. If it is omitted, and the pre–World War I and post–World War II periods combined, the two periods together conform to a simple quantity theory about as well as the United Kingdom data for the period as a whole. Indeed, we cannot find any statistically significant difference between the United States and the United Kingdom relations—though it does remain true that the rate of change of output varies more than the rate of change of prices in the United States and less in the United Kingdom.

We must confess to being surprised at our failure—if the United States interwar period is excluded—to find a positive relation between price change and output change. Misled by the "conventional wisdom," we expected a positive relation between price and output change and between both and monetary change, and we searched long and diligently—and we believe, not simple-mindedly—to uncover such a relation. On the contrary, a negative relation between price and output change is more typical, for both statistical and economic reasons (section 9.2). We are reminded of Keynes's dictum expressed in another connection:

> I find myself moved, not for the first time, to remind contemporary economists that the classical teaching embodied some permanent truths of great significance, which we are liable today to overlook because we associate them with other doctrines which we cannot now accept without much qualification. There are in these matters deep undercurrents at work, natural forces, or even the invisible hand, which are operating towards equilibrium.[45]

2. We have concluded that the widely held belief in a positive relation between price and output change and in the inadequacy of the quantity theory stems largely from a tendency to regard the United States interwar period as the norm rather than, as we found it, idiosyncratic. That period generated Keynes's *General Theory* and sparked the Keynesian revolution. It appears that Keynes's theory, far from being general, is highly special, a view that has often been expressed but seldom documented as fully as we believe we have been able to.

3. One of the most important of the by-products of the Keynesian revolution was the "Phillips curve"—the notion that there is a stable tradeoff between inflation and unemployment and conversely; or, equivalently, that a high level of output relative to capacity (the output ratio) will be reflected in a high level of inflation, a low output ratio in a low level of inflation.

This relation has clearly broken down in the postwar period as country after country has experienced stagflation—higher inflation accompanied by higher unemployment, not lower. However, this empirical positively sloped Phillips curve has been regarded as an exception, and the postwar period as idiosyncratic.

Sections 9.7 and 9.8 indicate that this conclusion is not justified. Section 9.7 demonstrates rather decisively that past price change is far more important than the output ratio as a determinant of inflation. Section 9.8 demonstrates that, insofar as there is any relation between the output ratio and inflation, it is in the direction called for by a positively sloped Phillips curve. This is true not solely for the post–World War II period but throughout—the idiosyncratic interwar period for the United States alone excepted.

45. J. M. Keynes, "The Balance of Payments of the United States," *Economic Journal* 56 (June 1946): 172–87. Quotation is from p. 185.

9.10 Appendix: Derivation of Price and Output Relations

9.10.1 Relation of Prices and Output to Real per Capita Income and Yields

To convert the equations in the appendix to chapter 8 into equations for prices and output separately, we make use of the identities:

(A1) $$\log Y(t) = \log P(t) + \log y(t) + \log N(t),$$

where N is population,
and

(A2) $$g_Y(t) = g_P(t) + g_y(t) + g_N(t).$$

Start with equation (A5) of chapter 8, which is

(A3) $$\log Y(t) = -[\log k + \lambda_3 Z] + \zeta \log M(t) + (1 - \alpha - \lambda_4 Z) \log y(t) - \delta R_N(t) - \epsilon g_Y(t),$$

and subtract $\log y(t) + \log N(t)$ from both sides. This gives

(A4) $$\log P(t) = -[\log k + \lambda_3 Z] - \log N(t) + \zeta \log M(t) - (\alpha + \lambda_4 Z) \log y(t) - \delta R_N(t) - \epsilon g_Y(t).$$

Subtract $\log P(t)$ from both sides of equation (A1). This gives

(A5) $$\log y'(t) = \log y(t) + \log N(t).$$

Similarly for rate of change equations, we start with equation (*A*8) of the appendix to chapter 8, or

(A6) $$g_Y(t) = \zeta g_M(t) + (1 - \alpha - \lambda_4 Z) g_y(t) - \delta D[R_N(t)] - \epsilon D[g_Y(t)],$$

and subtract $g_y(t) + g_N(t)$ to get

(A7) $$g_P(t) = \zeta g_M(t) - (\alpha + \lambda_4 Z) g_y(t) - g_N(t) - \delta D[R_N(t)] - \epsilon D[g_Y(t)].$$

Subtract $g_P(t)$ from both sides of equation (A2). This gives

(A8) $$g_{y'}(t) = g_y(t) + g_N(t).$$

These equations make one point explicit: even if $\alpha = 1$, the behavior of real output per capita and of population cannot be neglected in discussing the breakdown of nominal income between prices and output.

To determine the influence of the variables so far discussed on the breakdown between prices and output, we must express $\log y$ and $\log N$ as functions of these variables.

With respect to population, we found in chapter 5 that population contributed a trivial fraction of the variation of money and income about their trend. Accordingly, we shall assume that we can represent population by

$$\text{(A9)} \qquad \log N(t) = A_N + \lambda_{3N}Z + [\bar{g}_N + \lambda_{5N}Z]T(t),$$

where $T(t)$is the chronological date corresponding to the midpoint of phase t. The parameter λ_{3N} is the excess of the constant term of the trend equation for the United Kingdom over that for the United States, λ_{5N} is the excess of the slope term, and similarly with subsequent coefficients of Z.

By differentiating equation (A9) with respect to T, we can represent the rate of change of population by

$$\text{(A10)} \qquad g_N(t) = \bar{g}_N + \lambda_{5N}Z.$$

With respect to real income per capita, we cannot neglect the deviations from trend, but we can assume that they are related to variables affecting nominal income, namely, $M(t)$, $R_N(t)$, and $g_Y(t)$, and that the relation is the same in the United States and the United Kingdom. On that assumption, we can represent per capita income by

$$\text{(A11)} \qquad \log y(t) = A_y + \lambda_{3y}Z + (\bar{g}_y + \lambda_{5y}Z)\ T(t) + \zeta_y \log M(t) - \delta_y R_N(t) - \epsilon_y g_Y(t),$$

and the rate of change of real income per capita by

$$\text{(A12)} \qquad g_y(t) = (\bar{g}_y + \lambda_{5y}Z) + \zeta_y g_M(t) - \delta_y D[R_N(t)] - \epsilon_y D[g_Y(t)].$$

Substitute equations (A9) and (A11) in equations (A4) and (A5) to get:

$$\text{(A13)} \qquad \log P(t) = -\log k_P - \lambda_{3P}Z - [\bar{g}_P + \lambda_{5P}Z]T(t) + \zeta_P \log M(t) - \delta_p R_N(t) - \epsilon_P g_Y(t),$$

and

$$\text{(A14)} \qquad \log y'(t) = A_{y'} + \lambda_{3y'}Z + [\bar{g}_{y'} + \lambda_{5y'}Z]T(t) + \zeta_y \log M(t) - \delta_y R_N(t) - \epsilon_y g_Y(t)$$

where

$$\text{(A15a)} \qquad \left\{ \begin{array}{rclcll} \log k_p & = & \log k & + & A_N & + (\alpha + \lambda_4 Z)A_y \\ \lambda_{3P} & = & \lambda_3 & + & \lambda_{3N} & + (\alpha + \lambda_4 Z)\lambda_{3y} \\ \bar{g}_P & = & \bar{g}_N & + & & (\alpha + \lambda_4 Z)\bar{g}_y \\ \lambda_{5p} & = & \lambda_{5N} & + & & (\alpha + \lambda_4 Z)\lambda_{5y} \\ \zeta_p & = & \zeta & - & & (\alpha + \lambda_4 Z)\zeta_y \\ \delta_P & = & \delta & - & & (\alpha + \lambda_4 Z)\delta_y \\ \epsilon_P & = & \epsilon & - & & (\alpha + \lambda_4 Z)\epsilon_y \end{array} \right.$$

and

(A15b)
$$\begin{cases} A_{y'} = A_N + A_y \\ \lambda_{3y'} = \lambda_{3N} + \lambda_{3y} \\ \bar{g}_{y'} = \bar{g}_N + \bar{g}_y \\ \lambda_{5y'} = \lambda_{5N} + \lambda_{5y}. \end{cases}$$

Similarly, for rate of change equations, substitute equations (A10) and (A12) into equations (A7) and (A8) to get:

(A16)
$$g_P(t) = -\bar{g}_P - \lambda_{5P}Z + \zeta_P g_M(t) - \delta_P D[R_N(t)] - \epsilon_P D[g_Y(t)],$$

and

(A17)
$$g_{y'}(t) = \bar{g}_{y'} + \lambda_{5y'}Z + \zeta_y g_M(t) - \delta_y D[R_N(t)] - \epsilon_y D[g_Y(t)].$$

So far, we have made no assumptions about income elasticity. Real income per capita (y) does not enter on the right-hand side of the equations, but α does in equation (A13) though not equation (A14) (see equations A15a and A15b). One consequence is that fitting equation (A13) for the United States and the United Kingdom combined in its most general form (i.e., $\alpha \neq 1$ and $\lambda_4 \neq 0$) would require using three country parameters ($\lambda_{3P}, \lambda_4, \lambda_{5P}$). This is feasible, since values of α and λ_4 could be taken from the estimated demand equation for the two countries, but it would be complex and, in view of our reliance on the assumption of $\alpha = 1$ and $\lambda_4 = 0$ in chapter 8, hardly worthwhile. We have introduced the approximations of equations (A11) and (A12) not to avoid that assumption but because, even with that assumption, real income per capita enters the price and output equations. Accordingly, in what follows we shall, in combining the two countries, proceed as if $\alpha = 1$, $\lambda_4 = 0$. The assumption of $\alpha = 1$ is not implicit in the equations for each country separately.

A second and more subtle consequence of an $\alpha \neq 1$, affecting the equations for the separate countries as well as for the two combined, has to do with the comparability of the equation for nominal income with the equations for price and output separately. If $\alpha = 1$ and $\lambda_4 = 0$, then the sum of equations (A13) and (A14) will, as a mathematical matter, give equation (A3), for $\alpha = 1$ and $\lambda_4 = 0$. However, suppose we estimate equations (A13) and (A14) by straightforward multiple regression. Their sum will equal the corresponding regression estimate of equation (A3) with the term in log $y(t)$ omitted only if we impose the cross-equation conditions that

(A18)
$$\begin{aligned} \log k_P - A_{y'} &= \log k \\ \lambda_{3P} - \lambda_{3y'} &= \lambda_3 \\ \bar{g}_P &= \bar{g}_{y'} \\ \lambda_{5P} &= \lambda_{5y'}, \end{aligned}$$

that is, conditions on the estimated constant terms and the coefficients of Z, T, and ZT in equations (A13) and (A14).

If we do not impose any cross-equation conditions, then the sum of the regressions corresponding to equations (A13) and (A14) will equal

$$\text{(A19)} \qquad \log Y(t) = -\log k - \lambda_3 Z + \bar{g}_Y T(t) + \lambda_5 T(t) + \zeta \log M(t) - \delta R_N(t) - \epsilon g_Y(t).$$

If, empirically, α is close to unity and λ_4 to zero, then the estimated $\bar{g}_Y$ and λ_5 will be close to zero. However, there is nothing that requires that result. Accordingly, to avoid the complexity of introducing cross-equation conditions, we have introduced terms in T in the equations for Y in tables 9.4, 9.5, and 9.6, and in T and ZT for corresponding equations for the combined United States and United Kingdom (not reported). As a result, they do not correspond precisely to those presented in chapter 8. In effect, this involves indirectly allowing for an $\alpha \neq 1$.

The same problems arise for the rate of change equations (A16) and (A17) and mean that in computing these equations for the two countries combined, we implicitly assume $\alpha = 1$, $\lambda_4 = 0$; and that in computing the corresponding equations for the rate of change of nominal income, we include a constant term plus, for the two countries combined, a term in Z.

9.10.2 Relation of Prices and Output to Current and Prior Money and Income

If we replace R_N by R_S, we can express R_S and $D(R_S)$ and g_Y and $D(g_Y)$ in terms of earlier money and income by the same procedure as was used in the appendix to chapter 8. The final result, however, is much more complicated because, in replacing $R_S(t)$ by $k_0 + g_Y{}^*(t)$ and $g_Y{}^*(t)$ in turn by its value in terms of earlier money and income, it is necessary to use the equations for $\log Y(t)$, not for $\log P(t)$, or $\log y'(t)$. Hence the final equations cannot be obtained, as might at first appear, by simply replacing ζ by ζ_P or $\zeta_{y'}$, δ by δ_P or $\delta_{y'}$, and so on. Both sets of structural parameters enter in. However, the final results, on the assumption that all phases are equal in length, can still be expressed in the same form as equations (A23) and (A42) of the appendix to chapter 8.[46] The results are

46. The steps in the derivation of the equation for $\log P$ are as follows:

(i) In equation (A13) of this appendix, with R_S substituted for R_N, replace $R_S(t)$ by $k_0 + wg_Y(t) + (1 - w)g_Y^*(t-1)$.
[Denote the resulting equation by (i), and similarly for later steps.]

(ii) Do the same in equation (A6) of the appendix to chapter 8, modified by substituting R_S for R_N.

(iii) Write modified equation (A6) of the appendix to chapter 8 for $(t-1)$, and replace $R_S(t-1)$ by $k_0 + g_Y^*(t-1)$.

(iv) Multiply both sides of equation (iii) by $(1-w)\delta_P/\delta$

(v) Subtract equation (iv) from equation (i).

(A20)
$$\log P(t) = a_P + a'_P T(t) + f_P Z + f'_P ZT(t) + b_P \log M(t) + c_P \log M(t-1) + d_P \log Y(t-1) + e_P \log Y(t-2),$$

(A21)
$$\log y'(t) = a_{y'} + a'_{y'} T(t) + f_{y'} Z + f'_{y'} ZT(t) + b_{y'} \log M(t) + c_{y'} \log M(t-1) + d_{y'} \log Y(t-1) + e_{y'} \log Y(t-2),$$

where

$$a_P = -\log k_P - w\delta_P k_0 + (1-w)\log k\ \delta_P/\delta + \frac{w(w\delta_P + \epsilon_P)(\log k + \delta k_0)}{nQ}$$

(A22a)
$$\begin{cases} a'_P = -\bar{g}_P \\ f_P = -\lambda_{3P} + \lambda_3 (1-w)\,\delta_P/\delta + \dfrac{w(w\delta_P + \epsilon_P)}{nQ}\lambda_3 \\ f'_P = -\lambda_{5P} \\ b_P = \zeta \left(\dfrac{\zeta_P}{\zeta} - \dfrac{w\delta_P + \epsilon_P}{nQ} \right) \\ c_P = -(1-w)\,\zeta \left(\dfrac{\zeta_P}{\zeta} - \dfrac{w\delta_P + \epsilon_P}{nQ} \right) \\ d_P = \dfrac{\delta_P}{\delta}(1-w)\left(1 + \dfrac{\epsilon}{n}\right) + \dfrac{w\delta_P + \epsilon_P}{nQ}\left(w - \dfrac{(1-w)\epsilon}{n} \right) \\ e_P = -(1-w)\dfrac{\epsilon}{n}\left(\dfrac{\delta_P}{\delta} - \dfrac{w\delta_P + \epsilon_P}{nQ} \right) \end{cases}$$

and the corresponding coefficients for equation (A21) are

(vi) In equation (v), replace $g_Y(t)$ by $[\log Y(t) - \log Y(t-1)]/n$, and $g_Y(t-1)$ by the corresponding expression.

(vii) In equation (vi), replace $\log Y(t)$ by its equivalent from equation (A23) of the appendix to chapter 8.

The procedure for deriving the equation for $\log y'$ is the same except starting with equation (A14) of this appendix instead of equation (A13).

$$a_{y'} = A_{y'} - w\delta_y k_0 + (1-w) \log k \ \delta_y/\delta + \frac{w(w\delta_y + \epsilon_y)(\log k + \delta k_0)}{nQ}$$

$$a'_{y'} = \bar{g}_{y'}$$

$$f_{y'} = \lambda_{3y'} + \lambda_3(1-w)\delta_y/\delta + \frac{w(w\delta_y + \epsilon_y)}{nQ}\lambda_3$$

$$f'_{y'} = \lambda_{5y'}$$

(A22b)
$$b_{y'} = \zeta\left(\frac{\zeta_y}{\zeta} - \frac{w\delta_y + \epsilon_y}{nQ}\right)$$

$$c_{y'} = -(1-w)\ \zeta\left[\frac{\delta_y}{\delta} - \frac{w\delta_y + \epsilon_y}{nQ}\right]$$

$$d_{y'} = (1-w)\frac{\delta_y}{\delta}(1 + \frac{\epsilon}{n}) + \frac{w\delta_y + \epsilon_y}{nQ}(w - \frac{(1-w)\epsilon}{n})$$

$$e_{y'} = -(1-w)\ \frac{\epsilon}{n}\left(\frac{\delta_y}{\delta} - \frac{w\delta_y + \epsilon_y}{nQ}\right).$$

The corresponding rate of change equations are

(A23)
$$g_P(t) = a'_P + f'_P Z + b_P g_M(t) + c_P g_M(t-1) + d_P g_Y(t-1) + e_P g_Y(t-2),$$

(A24)
$$g_{y'}(t) = a'_{y'} + f'_{y'} Z + b_{y'} g_M(t) + c_{y'} g_M(t-1) + d_{y'} g_Y(t-1) + e_{y'} g_Y(t-2).$$

As with equations (A13) and (A14) and (A16) and (A17), so with equations (A20) and (A21) and (A23) and (A24), the equations for the countries combined implicitly assume $\alpha = 1$, $\lambda_4 = 0$. For the rate of change of nominal income for each country separately, the equations obtained by summing (A23) and (A24) are the same as those computed in chapter 8 with a constant term (to allow for trend). For the two countries combined, a term in Z is also required.

9.10.3 Relation of Prices and Output to Current and Prior Money Only

To express equations (A20) and (A21), and (A23) and (A24) in terms of earlier money alone, we make use of equations (A70) and (A71) of the appendix to chapter 8. These are for each country separately; that is, $Z = 0$, to which we restrict our analysis in this section, as we did in the corresponding section of the appendix to chapter 8.

They can be written as

(A25) $$\log Y(t) = a'' + \frac{k'}{1-d-e} nt + \sum_{i=0}^{\infty} b_i M(t-i),$$

(A26) $$g_Y(t) = \frac{k'}{1-d-e} + \sum_{i=0}^{\infty} b_i g_M(t-i),$$

where

(A27) $$\begin{cases} b_0 & = b \\ b_1 & = c + bd \\ b_i & = db_{i-1} + eb_{i-2} \text{ for } i > 1. \end{cases}$$

Replacing t by $t-1$ and $t-2$ successively, we may substitute the results for $\log Y(t-1)$ and $\log Y(t-2)$ in equations (A20) and (A21), and for $g_Y(t-1)$ and $g_Y(t-2)$ in equations (A23) and (A24). The results for prices are:

(A28) $$\begin{aligned} \log P(t) = & \left[a_P + (d_P + e_P)a'' - (d_P + 2e_P) \frac{k'n}{1-d-e} \right] \\ & + \left[a'_P + (d_P + e_P) \frac{k'}{1-d-e} \right] nt \\ & + \sum_{i=0}^{\infty} b_{P_i} \log M(t-i), \end{aligned}$$

and

(A29) $$g_P(t) = a'_P + (d_P + e_P) \frac{k'}{1-d-e} + \sum_{i=0}^{\infty} b_{P_i}\, g_M(t-i),$$

where

(A30) $$\begin{cases} b_{P_0} & = b_P \\ b_{P_1} & = c_P + d_P b_0 \\ b_{P_i} & = d_P b_{i-1} + e_P b_{i-2} \text{ for } i > 1. \end{cases}$$

From equation (A30) it follows that

(A31) $$\begin{aligned} \sum_{i=0}^{\infty} b_{P_i} & = b_P + c_P + (d_P + e_P) \sum_{i=0}^{\infty} b_i \\ & = b_P + c_P + (d_P + e_P) \frac{b+c}{1-d-e}. \end{aligned}$$

If $\Sigma b_i = 1$, which has been imposed on the final equations in chapter 8, then

(A32) $$\sum_{i=0}^{\infty} b_{P_i} = b_P + c_P + d_P + e_P \text{ for } \Sigma b_i = 1,$$

so that the sum of the coefficients of the money and income terms is an estimate of the fraction of the change in nominal income that ultimately takes the form of price change. More generally, that fraction is given by

(A33) $$\frac{\Sigma b_{P_i}}{\Sigma b_i} = \frac{(b_P + c_P)(1-d-e)}{b+c} + d_P + e_P.$$

The condition that this fraction equal unity, that is, that the whole of a change in monetary growth ultimately be reflected in prices, is then

(A34) $$\frac{1-d_P-e_P}{b_P+c_P} = \frac{1-d-e}{b+c}.$$

The equations for output are identical with those for prices, except that y' replaces P wherever P appears as a subscript.

The sum of the price and output equations estimated separately from the regressions on current and prior money and prior income will equal the corresponding equation for income if the latter is estimated without imposing the restriction that $\Sigma b_i = 1$. If that restriction is imposed, then the separate equations will sum to the income equation only if cross-equation restrictions are imposed assuring that outcome.

However, the condition that $\Sigma b_{P_i} = 1$ can be imposed along with the condition that $\Sigma b_i = 1$, by imposing the condition on the price equation that $b_P + c_P + d_P + e_P = 1$.

9.10.4 Transient Effects

The treatment of transient effects in the appendix to chapter 8 carries over directly to prices and output separately, except only that the theoretical value given by equation (A82) of that section applies to the sum of price and output transient effects.

9.10.5 Alternative Hypotheses about Anticipations

Unchanging Anticipations

If $g^*_{y'}$ and g^*_M are treated as constants, equations (18) and (19) of the text reduce to

(A35) $$g_P(t) = -(g^*_{y'} + \eta\psi g^*_M) + g^*_Y(t) + \eta\psi g_M(t) + \xi \log R_{y'}(t),$$

(A36) $$g_{y'}(t) = [g^*_{y'} - (1-\eta)\psi g^*_M] + (1-\eta)\psi g_M(t) - \xi \log R_{y'}(t).$$

As written, equation (A36) makes $g_{y'}$ a function of current g_M only. In chapter 8 we introduced prior monetary change by making the demand for money a function of nominal interest rates and nominal interest rates in turn a function of expected rate of change of nominal income. However, a constant g^*_M would also imply, in the thoretical model of chapter 2, a constant g_Y^*, and, together with a constant $g_{y'}^*$, a constant g_P^*. Hence that analysis gives no reason here to introduce directly prior values of g_M in either equation (A35) or equation (A36). The only justification for doing so on these assumptions would be to allow for discrepancies between actual and desired cash balances. Such discrepancies may well be extremely important for monthly, quarterly, or even annual time units, but we have so far neglected them for periods as long as our phases, and we shall continue to do so.

Unchanging Anticipations for Output:
Adaptive Anticipations for Money

On this approach, we supplement equations (18) and (19) of the text with

$$\text{(A37)} \qquad \frac{dg_M^*}{dt} = D(g_M^*) = w(g_M - g_M^*),$$

the solution of which, as usual, can be approximated, for discrete data, by

$$\text{(A38)} \qquad g_M^*(t) = wg_M(t) + (1 - w)g_M^*(t - 1),$$

where, for reasons explained in chapter 8, we treat the phase as a time unit, even though phases differ in chronological duration, and where we regard w, the rate of adaptation, as being the same for g_M^* as for g_Y^*.

Substitute equation (A38) and its counterpart for g_Y^*, in equations (18) and (19) of the text, which gives

$$\text{(A39)} \qquad \begin{aligned} g_P(t) = {} & g_Y(t) - g_{y'}^* - (1 - w)\,[g_Y(t) - g_Y^*(t - 1)] \\ & + \eta\psi(1 - w)[g_M(t) - g_M^*(t - 1)] + \xi \log R_{y'}, \end{aligned}$$

$$\text{(A40)} \qquad \begin{aligned} g_{y'}(t) = {} & g_{y'}^* + (1 - \eta)\psi(1 - w)[g_M(t) - g_M^*(t - 1)] \\ & - \xi \log R_{y'}\,. \end{aligned}$$

Write equations (18) and (19) of the text for $(t - 1)$, multiply by $(1 - w)$, subtract the results from equations (A39) and (A40), and simplify to get

$$\text{(A41)} \qquad \begin{aligned} g_P(t) = {} & -wg_{y'}^* + (1 - w)g_P(t - 1) + wg_Y(t) \\ & + \eta\psi(1 - w)[g_M(t) - g_M(t - 1)] \\ & + \xi[\log R_{y'}(t) - (1 - w) \log R_{y'}(t - 1)\,], \end{aligned}$$

$$\text{(A42)} \qquad \begin{aligned} g_{y'}(t) = {} & wg_{y'}^* + (1 - w)g_{y'}(t - 1) \\ & + (1 - \eta)\psi(1 - w)\,[g_M(t) - g_M(t - 1)\,] \\ & - \xi\,[\log R_{y'}(t) - (1 - w) \log R_{y'}(t - 1)\,]. \end{aligned}$$

These equations now contain only observable variables but are nonlinear in the parameters.[47,48]

The sum of equations (A41; and A42) does not contain the output ratio. If we divide the sum by $(1 - w)$, the result is

(A43) $$g_Y(t) - g_Y(t-1) = \psi[g_M(t) - g_M(t-1)],$$

or, dividing by $\bar{n}(t)$, the interval between the midpoints of phases t and $t-1$,

(A44) $$D[g_Y(t)] = \psi D[g_M(t)].$$

We can use this equation to estimate ψ.

Adaptive Anticipations for both Output and Money: Rates of Adaptation Equal

The counterpart for output of equation (A38) is:

(A45) $$g^*_{y'} = w' g_{y'}(t) + (1 - w') g^*_{y'}(t-1),$$

where w' is the weight attached to current output growth.

The case considered here is one in which the rates of adaptation are equal for money and output $(w = w')$.

Substitute equations (A38) and (A45), and their counterpart for g^*_Y, into equations (18) and (19) of the text to get:

(A46) $$\begin{aligned} g_P(t) = {} & w g_Y(t) + (1-w) g^*_Y(t-1) - w' g_{y'}(t) \\ & - (1-w') g^*_{y'}(t-1) \\ & + \eta\psi(1-w)\,[g_M(t) - g^*_M(t-1)] + \xi \log R_{y'}(t). \end{aligned}$$

(A47) $$\begin{aligned} (1-w') g_{y'}(t) = {} & (1-w') g^*_{y'}(t-1) \\ & + (1-\eta)\psi(1-w)[g_M(t) - g^*_M(t-1)] \\ & - \xi \log R_{y'}(t). \end{aligned}$$

For $w = w'$, write equations (18) and (19) of the text for $t - 1$, multiply by $(1 - w)$, and subtract from equations (A46) and (A47). After simplification, the result is

(A48) $$\begin{aligned} g_P(t) - g_P(t-1) = {} & \eta\psi[g_M(t) - g_M(t-1)] \\ & + \xi\left[\frac{\log R_{y'}(t)}{1-w} - \log R_{y'}(t-1)\right], \end{aligned}$$

47. If equation (A41) is expressed in a series of terms, there will appear to be six regression coefficients, but there are only five independent parameters: w, $g^*_{y'}$, η, ψ, and ξ, and η and ψ only appear as a product, so there are really only four independent parameters. Similarly, equation (A42) yields five linear regression coefficients, but there are only four independent parameters, w, $g^*d2y_{'}$, $(1-\eta)\psi$, and ξ.

48. A less rigid approach would be to regard g^*_M as a weighted average of current and earlier values of g_M but without restricting the weights to the exponential form implied by equation (A38). However, without imposing additional restrictions on the weights, it becomes difficult or arbitrary to identify ψ separately from the weights.

(A49)
$$g_{y'}(t) - g_{y'}(t-1) = (1-\eta)\psi[g_M(t) - g_M(t-1)] - \xi\left[\frac{\log R_{y'}(t)}{1-w} - \log R_{y'}(t-1)\right].$$

Unlike equations (A41) and (A42), these equations are linear in the parameters and can be estimated directly.

If ξ is set equal to zero, and the equations are divided through by $\bar{n}(t)$, equations (A48) and (A49) reduce to

(A50)
$$D[g_P(t)] = \eta\psi D[g_M(t)] \ ,$$

(A51)
$$D[g_{y'}(t)] = (1-\eta)\psi D[g_M(t)],$$

and their sum to equation (A44), the same as the sum of equations (A41) and (A42), implying the same estimate of ψ.

Adaptive Anticipations for both Output and Money: Rates of Adaptation Not Equal ($w \neq w'$)

Write equations (18) and (19) of the text for $(t-1)$, multiply by $(1-w')$ and subtract from equations (A46) and (A47) of the preceding section to get, after some rearrangement,

(A52)
$$\begin{aligned}(1-w')[g_P(t) - g_P(t-1)] &= (w-w')[g_Y(t) - g_Y^*(t+1)] \\ &+ \eta\psi(1-w)[g_M(t) - g_M(t-1)] \\ &- \eta\psi(w-w')[g_M(t-1) - g_M^*(t-1)] \\ &+ \xi[\log R_{y'}(t) - (1-w')\log R_{y'}(t-1)],\end{aligned}$$

(A53)
$$\begin{aligned}(1-w')[g_{y'}(t) - g_{y'}(t-1)] \\ &= (1-\eta)\psi(1-w)[g_M(t) - g_M(t-1)] \\ &- (1-\eta)\psi(w-w')[g_M(t-1) \\ &- g_M^*(t-1)] \\ &- \xi\,[\log R_{y'}(t) - (1-w')\log R_{y'}(t-1)].\end{aligned}$$

Replace $g_M^*(t-1)$ and $g_Y^*(t-1)$ by their equivalents for $(t-1)$ from equation (A38) and the corresponding equation for $g_Y^*(t)$. Subtract equations (A52) and (A53) rewritten for $(t-2)$ and multiplied by $(1-w)$ from the resulting equations, The final results are:

(A54)
$$\begin{aligned}g_P(t) = g_P(t-1) &+ (1-w)[g_P(t-1) - g_P(t-2)] \\ &+ \frac{(1-w)(w-w')}{(1-w')}[g_Y(t) - g_Y(t-1)] \\ &+ \eta\psi\frac{(1-w)}{(1-w')}[g_M(t) - g_M(t-1)] \\ &- \eta\psi(1-w)[g_M(t-1) - g_M(t-2)] \\ &+ \frac{\xi}{(1-w')}[\log R_{y'}(t) - (1-w)\log R_{y'}(t-1)] \\ &- \xi[\log R_{y'}(t-1) - (1-w)\log R_{y'}(t-2)],\end{aligned}$$

$$\text{(A55)} \qquad \begin{aligned} g_{y'}(t) = {} & g_{y'}(t-1) + (1-w)[g_{y'}(t-1) - g_{y'}(t-2)] \\ & + (1-\eta)\psi \frac{(1-w)}{(1-w')}[g_M(t) - g_M(t-1)] \\ & - (1-\eta)\psi(1-w)[g_M(t-1) - g_M(t-2)] \\ & - \frac{\xi}{(1-w')}[\log R_{y'}(t) - (1-w)\log R_{y'}(t-1)] \\ & + \xi[\log R_{y'}(t-1) - (1-w)\log R_{y'}(t-2)]. \end{aligned}$$

Like equations (A41) and (A42), these equations are nonlinear in the parameters. The sum of equations (A54) and (A55) is

$$\text{(A56)} \qquad \begin{aligned} g_Y(t) - g_Y(t-1) = {} & \frac{(1-w)(1-w')}{(1-w)^2 + w(1-w')}[g_Y(t-1) \\ & - g_Y(t-2)] \\ & + \frac{\psi(1-w)}{(1-w)^2 + w(1-w')}[g_M(t) - g_M(t-1)] \\ & - \frac{\psi(1-w)(1-w')}{(1-w)^2 + w(1-w')}[g_M(t-1) - g_M(t-2)]. \end{aligned}$$

Divide both sides by $\bar{n}(t)$ to get

$$\text{(A57)} \qquad \begin{aligned} D[g_Y(t)] = {} & \frac{(1-w)(1-w')}{(1-w)^2 + w(1-w')} D[g_Y(t-1)] \frac{\bar{n}(t-1)}{\bar{n}(t)} \\ & + \frac{\psi(1-w)}{(1-w)^2 + w(1-w')} D[g_M(t)] \\ & - \frac{\psi(1-w)(1-w')}{(1-w)^2 + w(1-w')} D[g_M(t-1)] \frac{\bar{n}(t-1)}{\bar{n}(t)}, \end{aligned}$$

from which, in principle, it is possible to estimate ψ, w, and w'. In practice, the estimates are wholly unacceptable.