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Chapter Title: Adjustments to the Data Set and Specification of Parameters

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6 Adjustments to the Data Set and Specification of Parameters

6.1 Introduction

In chapters 4 and 5 we described the basic data that we use in our general equilibrium model. We must adjust these data in several ways, however, before we can use them. In particular, we always assume that the United States economy was in equilibrium in 1973, so the data must satisfy certain equilibrium conditions. Demands must equal supplies for all goods and factors. All industries must earn zero profits. Government receipts must equal government expenditures. In addition, we require certain conditions of consistency of the input-output matrix, of the matrix of goods consumption for consumers, and of the matrix of conversion between producer goods and consumer goods. In section 6.2 we describe these consistency adjustments.

Even after we make these adjustments, we are not yet ready to perform simulations with the model. In order to perform simulations we need parameter values that would describe the behavior of consumers, producers, the government, and our foreign trade partners.

We determine parameter values for the equations in the model using a nonstochastic *calibration* method. The model is calibrated to the base-year equilibrium, such that the adjusted data will be reproduced exactly as an equilibrium solution to the model. An alternative procedure would be to estimate the parameters of the model econometrically. Unfortunately, our model is much too large to be estimated as an econometric system of simultaneous equations. On the other hand, if we were to use single-equation methods to estimate the parameters and then calculate an equilibrium solution for the economy, the solution would not match the adjusted benchmark data.[1]

1. For a detailed discussion of these issues, see Mansur and Whalley 1984.

We describe our calibration procedures in section 6.3. When we assume that some type of activity can be characterized by a Cobb-Douglas function, we can calibrate the model merely by looking at the budget shares of consumers or the input purchases of producers. More often, however, we use the more complicated constant elasticity of substitution (CES) functional form. In this case we have to prespecify the value of the elasticity of substitution before we can proceed with the calibration. We describe our choices of elasticities in section 6.4.

Before we discuss our consistency adjustments, one other matter must be discussed. All of our data are in value terms, i.e., they are the products of prices and quantities. For several reasons we will want to deal separately with prices and quantities. To do this we adopt units conventions that tell us, for example, what constitutes a unit of labor and a unit of capital. These conventions allow us to translate data on factor payments by industry into observations on the physical quantities used. We use these observations directly when we determine the production function parameters.

Since we treat factors of production as being perfectly mobile among alternative uses, the allocation of factors by industry in equilibrium will equalize the returns received net of taxes in all industries. It is therefore convenient to adopt a definition of a physical unit of each factor as the amount that can earn in equilibrium a reward of $1 per period net of all taxes in any of the factor's alternative uses. In the case of capital, units are defined as net of both capital use taxes and personal factor taxes. Units for commodities are similarly defined as those amounts that in equilibrium sell for $1 net of all consumer taxes and subsidies. The observed benchmark equilibrium is characterized by an equilibrium price vector of unity for both goods and factors; ownership of a unit of labor or capital services yields a net income of $1.

We should stress the implications of these unit conventions. With labor services, for instance, the number of workers in an industry is an inappropriate measure of the amount of labor used by the industry. Our procedures implicitly assume that more productive individuals are endowed with a greater number of effective labor units. Similarly, we ignore the portfolio composition of capital ownership, since we assume that all assets yield the same real risk-free net rate of return.

6.2 Consistency Adjustments

6.2.1 Factor Payments and Factor Incomes

In order to illustrate our consistency adjustment procedures, it might be best to start with an example. One requirement of the general equilibrium model is that the total net-of-tax payments to labor by industry and

Table 6.1 **Total Labor Income by Consumer Group, before and after Adjustments (all figures in millions of 1973 dollars)**

Consumer Group	Labor Income before Adjustment	Labor Income after Adjustment
1	$8,596.9	$9,719.6
2	11,274.1	12,746.3
3	16,126.3	18,232.1
4	21,140.8	23,901.4
5	22,074.9	24,957.5
6	26,380.7	29,825.6
7	59,377.0	67,130.6
8	66,254.3	74,906.0
9	109,566.5	123,881.5
10	146,605.9	165,757.2
11	80,673.2	91,207.8
12	119,705.8	135,337.4
TOTAL	$687,776.4	$777,603.0

government must equal the total labor income of consumers. In the unadjusted data, the total payments to labor by the nineteen industries come to $643,040 million (see table 4.1). Government's payments to labor equal $134,563 million (see section 5.5.2), and the total of these payments is $777,603 million. On the other side of the ledger, we can multiply the number of households (shown in table 5.3) by the average labor income per household (shown in table 5.8) to get the total labor income of consumers. The total for all twelve consumers is about $687,776 million. In order to reconcile the two figures, we choose to accept the figures for payments to labor, and then scale up the income figures proportionally. The results are shown in table 6.1.

We follow a similar procedure in adjusting the data on net payments to capital and consumer capital income.[2] In table 4.2 we show payments to capital by industry. These figures are gross of the personal factor tax, however, and we subtract personal factor taxes before undertaking the adjustments. The total of personal factor taxes (PFT) paid by industry is $40,932.5 million (see table 4.7). When we subtract this from the $181,973 million of capital income gross of the PFT, we have $141,040.5 million. We then add in the government's use of capital net of the PFT ($5,557.9 million) to get our total of purchases of capital, net of all taxes. This total is $146,598.4 million, and we scale down consumer capital incomes to match it. To do so we first multiply the number of households (table 5.3) by the average capital income before adjustment, shown in

2. Payments for capital would not have to equal domestic capital income in a model with international capital flows. In the standard version of our model, with no international capital flows, we assume that domestic capital is owned by domestic consumers.

Table 6.2 Total Capital Income by Consumer Group, before and after Adjustments (all figures in millions of 1973 dollars)

Consumer Group	Capital Income before Adjustment	Capital Income after Adjustment
1	$5,965.1	$5,316.2
2	4,816.8	4,292.9
3	4.636.6	4,132.3
4	5.447.9	4,855.3
5	4.982.2	4,440.3
6	4.992.4	4,449.3
7	9,266.5	8,258.5
8	10.358.1	9,231.5
9	14,725.2	13,123.5
10	20.610.7	18,368.8
11	14.234.3	12,686.1
12	64.454.5	57,443.7
TOTAL	$164.490.3	$146,598.4

table 6.2. The total for all twelve consumer groups is $164,490.3 million, which must be multiplied by 0.891 to yield the figure for total net purchases of capital of $146,598.4 million. The capital incomes of consumers after adjustment are also shown in table 6.2.

Whenever we have to reconcile one vector with another, we use this procedure of accepting one vector and scaling the other vector up or down to match.[3] When we have to adjust a matrix, we use the RAS adjustment procedure developed by Michael Bacharach (1971). The RAS procedure adjusts a matrix so that the row sums and column sums simultaneously equal totals that have been estimated separately. We will discuss this procedure in detail below when we discuss the input-output matrix.

Figure 6.1 is a schematic representation of all of the consistency adjustments that are required of the data set. We will now describe the rest of the consistency adjustments. We accept the data on tax collections because they are recent and reliable. Since we have already performed our adjustments on factor returns by industry (and since factor returns plus factor taxes equal value added in each industry), we now have a final series for value added.

6.2.2 Government Revenues and Expenditures

As we said, we accept the data on government tax revenues. We also accept the data on government endowments. We collect the totals for these government income sources from tables 4.1, 4.4, 4.13, and 5.4, and from the text of chapters 4 and 5, and present them in table 6.3.

3. As a check, we compare the adjusted data to the raw data to be sure that we have not unknowingly introduced dramatic changes.

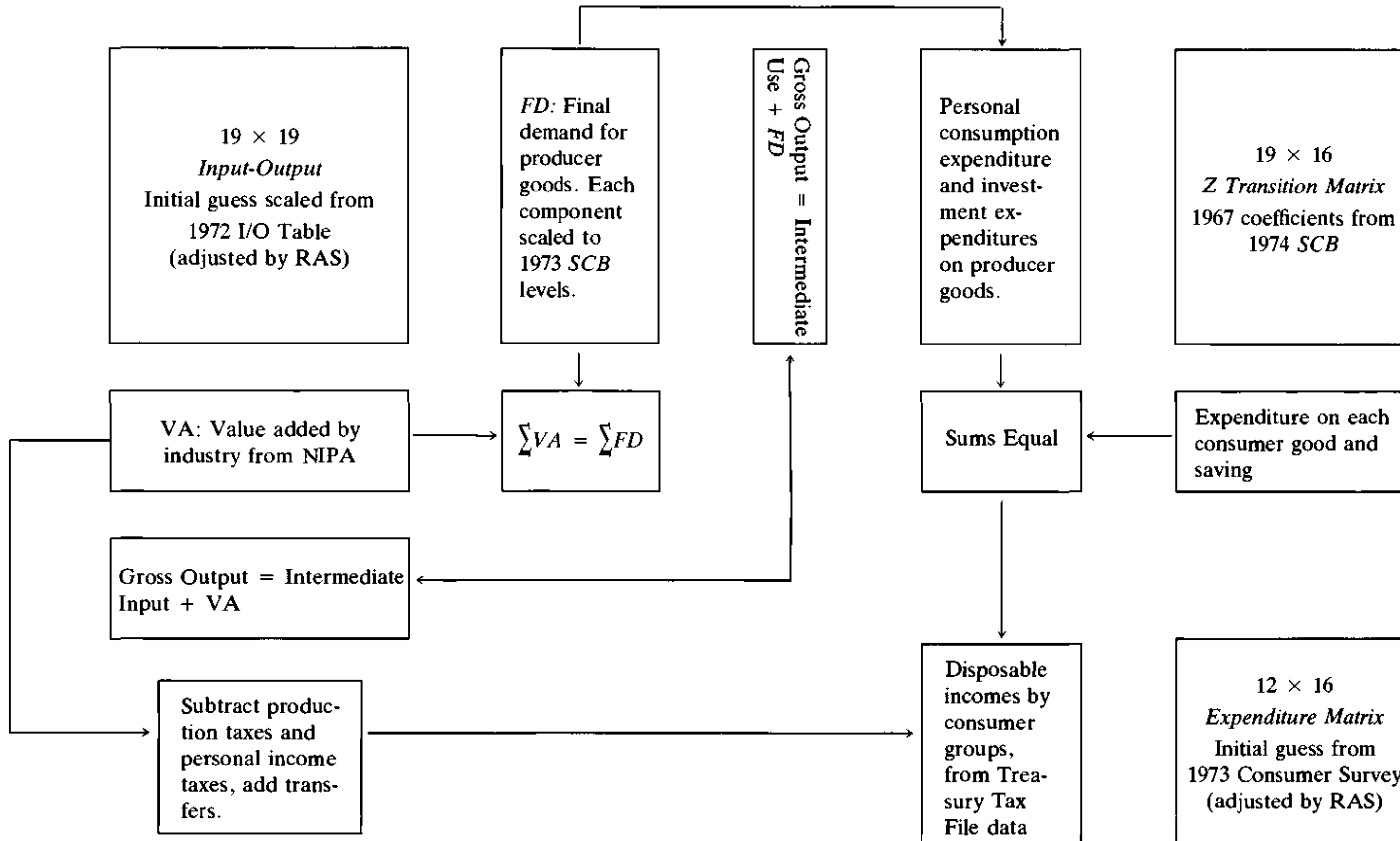

Figure 6.1 Data set modifications to produce overall static consistency.

Table 6.3 Sources of Funds for the Government (in millions of 1973 dollars)

Sources	Amount
Labor taxes from industry	$64,997
Government labor tax	14,499
Capital taxes from industry (including personal factor tax)	136,828
Output taxes	15,060
Intermediate taxes	1,656
Sales taxes	52,983
Personal income taxes (on labor)	78,248
TOTAL TAX REVENUE	$364,271
Income from sale of capital endowment	92,404
Income from sale of agricultural endowment	1,526
TOTAL ENDOWMENT INCOME	$93,930
TOTAL GOVERNMENT REVENUE	$458,201

The government spends its total revenue of $458,201 million on purchases of goods and factors and on transfer payments. It spends $97,961.9 million on purchases of capital,[4] and $149,062 million on labor services and labor taxes. If we multiply the number of households by transfer payments per household (table 5.9), we see that total transfer payments are equal to $106,057 million. This leaves $105,120.1 million for purchases of the eighteen commodities other than agriculture. However, table 5.1 indicates that the government spent $121,290 million on these eighteen commodities. We reconcile the two by scaling down the expenditure figures, multiplying each one by the ratio of 105,119 to 121,290. We show the figures for this adjustment in table 6.4.

6.2.3 Final and Intermediate Demands for Producer Goods

We have already adjusted the factor incomes of consumers, and we accepted the data on transfer incomes and income taxes without adjustment. As a result, we have all the ingredients of consumer disposable income. These are shown in table 6.5. The data in the column for the personal taxes are net of personal factor taxes. Consequently, the personal taxes shown here correspond to T_j^I in equation (3.38), including the tax on labor income for each group and the rebate or additional tax at the personal level for capital income of each group.

One of the requirements of the general equilibrium model is that

4. In chapter 5 we found that general government uses $7,700 million of privately owned capital. These interest and rental payments are fully taxed at the personal level, and they include personal factor taxes of $2,142.1 million. We subtract this figure from $100,104 million, the total use of capital by general government, to get $97,961.9 million for net purchases of capital.

Table 6.4 **Government Expenditures by Industry, before and after Adjustments (all figures in millions of 1973 dollars)**

Industry	Government Expenditures before Adjustment	Government Expenditures after Adjustment
Mining	$217	$188.1
Crude petroleum and gas	0	0.0
Contract construction	45,690	39,599.3
Food and tobacco	1,522	1,319.1
Textiles, apparel, and leather	699	579.8
Paper and printing	2,178	1,887.7
Petroleum refining	1,501	1,300.9
Chemicals, rubber, and plastics	4,141	3,589.0
Lumber, furniture, stone, clay and glass	927	803.4
Metals, machinery, instruments, and miscellaneous manufacturing	14,117	12,235.1
Transport equipment and ordnance	16,103	13,956.4
Motor vehicles	2,503	2,169.3
Transportation, communications, and utilities	9,961	8,633.2
Trade	2,121	1,838.3
Finance and insurance	884	766.2
Real estate	1,783	1,545.3
Services	16,154	14,000.6
Government enterprises	818	709.0
TOTAL	$121,290	$105,120.7

Table 6.5 **Consumer Disposable Incomes, after Consistency Adjustments (all figures in millions of 1973 dollars)**

Consumer	Labor Income	Capital Income	Transfers	Personal Taxes	Total Disposable Income
1	$9,719.6	$5,316.2	$12,918.0	$ −1,477.2	$29,431.0
2	12,746.3	4,292.9	11,193.6	−746.6	28,979.4
3	18,232.1	4,132.3	10,015.0	−210.2	32,589.6
4	23,901.4	4,855.3	9,166.6	179.5	37,743.8
5	24,957.5	4,440.3	7,411.7	601.2	36,208.3
6	29,825.6	4,449.3	5,691.7	1,260.4	38,706.2
7	67,130.6	8,258.5	9,590.6	4,047.0	80,932.7
8	74,906.0	9,231.5	7,928.1	5,245.9	86,819.7
9	123,881.5	13,123.5	8,610.9	10,541.0	135,074.9
10	165,757.2	18,368.8	9,617.0	17,301.4	176,441.6
11	91,207.8	12,686.1	5,281.9	11,315.2	97,860.6
12	135,337.4	57,443.7	8,631.6	30,190.1	171,222.6
TOTAL	$777,603.0	$146,598.4	$106,056.7	$78,247.8	$952,010.4

Table 6.6 **Personal Consumption Expenditures, before and after Adjustments (all figures in millions of 1973 dollars)**

Consumer Group	Personal Consumption before Adjustment	Personal Consumption after Adjustment
Food	$146,763	$150,140.4
Alcoholic beverages	21,302	21,792.2
Tobacco	13,134	13,436.3
Utilities	38,644	39,533.3
Housing	123,173	126,007.6
Furnishings	31,716	32,445.9
Appliances	26,836	27,453.6
Clothing and jewelry	68,062	69,628.3
Transportation	7,326	7,494.5
Motor vehicles, tires, and auto repairs	70,607	72,231.8
Services	117,219	119,916.5
Financial services	55,894	57,180.3
Reading, recreation, and miscellaneous	39,398	40,304.7
Nondurable, nonfood household items	31,916	32,650.5
Gasoline and other fuel	35,535	36,352.8
Savings	103,070	105,441.7
TOTAL	$930,595	$952,010.4

consumer disposable incomes must be exhausted by consumer expenditures on current consumption and saving. Table 5.4 provides a vector of consumer expenditures on the sixteen consumption goods. (The total for the sixteenth good, which is the savings good, is taken from the total of adjusted private fixed capital formation in the nineteen industries.)[5] We list these consumer expenditures in table 6.6. Their total is $930,594.8 million, which does not match the total for consumer disposable income of $952,010.4 million. As usual, our procedure is to scale up the expenditure totals by the ratio of 952,010.4 to 930,594.8, which equals about 1.023. The results of this scaling are shown in table 6.6.[6]

Now that we have a vector of consumption of the sixteen consumer goods, we can proceed to calculate the demands for the nineteen producer goods. We do this by using the Z matrix, which was shown in table 4.10. We do not adjust the Z matrix itself in any way. We premultiply the consumption vector (which is of length sixteen) by the (19×16) Z

5. The procedures whereby we produce these investment data are described in section 5.4.

6. Note that these consumption figures include sales taxes. We will return to this point shortly.

matrix, and the result is a nineteen-element vector for the amounts of producer goods used in consumption and investment.

We have now adjusted all the elements of the final demand for each industry output: consumption, investment, government demand, and net exports.[7] We also have all the elements of value added in each industry: payments to labor, payments to capital, and taxes. We have constructed our consistency adjustments such that the sum over all industries of the final demands equals the sum over all industries of the value added. We still must make one other major adjustment, however, before the production side of the economy is complete. One of the requirements of the general equilibrium model is the zero-profit condition. Receipts must equal expenditures for each of the nineteen industries. Receipts come from the various components of final demand as well as from other industries that pay for intermediate inputs. Expenditures are made for the elements of value added as well as for intermediate inputs from other industries. In terms of figure 6.1, the sum of any given row of input-output matrix plus the sum of the final demands for that industry must equal the corresponding column sum of the input-output matrix plus value added in that industry.

Since the interindustry transactions matrix (table 4.8) was compiled from sources different from either final demand or value added, this consistency condition is not met by the basic data. The first step, as described in section 4.5, is to scale up each column of the 1972 table by the ratio of 1973 value added to 1972 value added. Next we use Bacharach's RAS procedure, which we mentioned earlier. This procedure takes a total of each row plus final demand and compares it to the sum of the corresponding column plus value added. We then adjust upward or downward each element of the row in order to make the totals more nearly equal. (For technical reasons, we only adjust part of the way at each iteration.) The next step is to make the same sort of adjustment on the columns. The problem is that when we adjust the rows, the column totals change, and vice versa. It therefore takes several iterations before the RAS procedure converges to an acceptable degree of accuracy. We iterate until every row sum plus final demand is within $1,000 of the corresponding column sum plus value added.

6.2.4 Expenditures and Sales Taxes on Consumer Goods

We have already adjusted the aggregate vector of sixteen consumption values such that their sum equals the sum of the vector of the twelve groups' disposable incomes. To obtain the matrix of expenditures on each consumer good by each group, we multiply the numbers in table 5.2

7. Exports in table 5.1 were not adjusted. Imports were scaled to the same total, as described in section 5.6.

(which represent expenditures per household) by the number of households in each consumer group (table 5.3). The resulting matrix, however, is not consistent with either the aggregate consumption vector or the disposable income vector. The sum of each household's expenditures does not match its disposable income, and the sum of all groups' expenditures on a given product does not equal the appropriate total consumption for that good. We therefore apply the RAS procedure again to this matrix. Rows and columns are scaled successively until each row adds to the correct total consumption and, simultaneously, each column adds to the correct disposable income.

The final step is to divide the expenditure data between net expenditures and payments of sales taxes. We divide total consumer sales tax payments (table 5.4) by total consumer expenditures for each of the sixteen consumer goods, giving us a sales tax rate for each good.[8] We multiply these sales tax rates by the newly adjusted elements of the consumer expenditure matrix, and the result is a matrix of sales tax payments. We then subtract this sales tax matrix from the expenditure matrix to get a matrix of net expenditures.

6.3 Benchmark Calibration

As noted in the introduction to this chapter, we choose parameters for the model by using the calibration method. We assume that producers minimize cost and receive zero excess profits. These assumptions have certain implications that allow us to choose the values of the production function parameters. Similarly, the assumption that households maximize their utility subject to a budget constraint has implications that allow us to choose the values of the utility function parameters. Similar logic applies to the choice of parameters in the government's utility function. We describe these calculations in this section.

When we use CES utility functions or production functions, the assumptions of cost minimization and utility maximization leave us with one free parameter. To deal with this problem we specify the elasticity of substitution parameters on the basis of estimates from the econometric literature. The values of the other parameters then follow from the restrictions imposed by cost minimization and utility maximization. The choices of elasticity parameters are discussed in section 6.4.

Notation in the rest of this chapter corresponds to that of chapter 3 and to the notational appendix of chapter 3.

8. We believe it is reasonable to assume that the government does not differentiate among consumers when it levies sales taxes, such that each consumer faces the same sales tax rate.

6.3.1 Calibration of the Value-Added Functions

Consider the CES value-added function in equation (3.2), reproduced here:

$$VA = \phi \left[\delta L^{\frac{\sigma-1}{\sigma}} + (1-\delta) K^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}. \tag{6.1}$$

For expositional simplicity, we suppress the i subscripts of all variables and parameters. If producers minimize the cost of a unit of output, they will minimize the Lagrangean:

$$\mathcal{L} = P_K^* K + P_L^* L + \lambda \left[\phi \left\{ \delta L^{\frac{\sigma-1}{\sigma}} + (1-\delta) K^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}} - 1 \right], \tag{6.2}$$

where P_K^* is the cum-tax cost of capital, $P_K(1 + t_K)$, and P_L^* is the cum-tax cost of labor, $P_L(1 + t_L)$. The first order conditions with respect to L and K are given by:

$$\frac{\partial \mathcal{L}}{\partial K} = P_K^* + \lambda\phi\{\cdot\}^{\frac{1}{\sigma-1}}(1-\delta)K^{-\frac{1}{\sigma}} = 0 \tag{6.3}$$

and

$$\frac{\partial \mathcal{L}}{\partial L} = P_L^* + \lambda\phi\{\cdot\}^{\frac{1}{\sigma-1}}\delta L^{-\frac{1}{\sigma}} = 0, \tag{6.4}$$

where $\{\cdot\}$ is $\left\{\delta L^{\frac{\sigma-1}{\sigma}} + (1-\delta)K^{\frac{\sigma-1}{\sigma}}\right\}$.

If we divide (6.3) by (6.4), we get:

$$\frac{P_K^*}{P_L^*} = \frac{(1-\delta)K^{-\frac{1}{\sigma}}}{\delta L^{-\frac{1}{\sigma}}}. \tag{6.5}$$

With a little rearrangement we can solve for δ.

$$\delta = \frac{P_L^* L^{\frac{1}{\sigma}} / P_K^* K^{\frac{1}{\sigma}}}{1 + P_L^* L^{\frac{1}{\sigma}} / P_K^* K^{\frac{1}{\sigma}}}. \tag{6.6}$$

Recall that, by our unit conventions, the benchmark net-of-tax factor prices P_L and P_K equal one. As a result, equation (6.6) can be rewritten as:

$$(6.7)\qquad \delta = \frac{(1 + t_L)L^{\frac{1}{\sigma}}/(1 + t_K)K^{\frac{1}{\sigma}}}{1 + (1 + t_L)L^{\frac{1}{\sigma}}/(1 + t_K)K^{\frac{1}{\sigma}}}.$$

Our unit conventions further imply that the number of units of each factor equals the *value* of factor use net of tax. Thus, for each industry, L and t_L are available from table 4.1, and K and t_K are available from table 4.7. As a result, when we specify a value for σ (see section 6.4 below), we have all the information necessary to use equation (6.7) and calculate δ.

Once we know σ and δ for each industry, we can calculate ϕ, using the zero-profit condition. This condition implies that

$$(6.8)\qquad P_K^* K + P_L^* L = VA.$$

But since P_K and P_L are unity in the benchmark, we have

$$(6.9)\qquad \phi = \frac{(1 + t_K)K + (1 + t_L)L}{\delta L^{\frac{\sigma-1}{\sigma}} + (1 - \delta)K^{\frac{\sigma-1}{\sigma}}}.$$

In section (6.4) we choose $\sigma = 1$ for some of our industries. In this Cobb-Douglas case,

$$(6.10)\qquad VA = \phi L^{\delta} K^{1-\delta}.$$

The first-order conditions in this case are

$$(6.11)\qquad \frac{\partial \mathcal{L}}{\partial K} = P_K^* + \frac{\lambda\phi(1 + \delta)L^{\delta}K^{1-\delta}}{K} = 0$$

and

$$(6.12)\qquad \frac{\partial \mathcal{L}}{\partial L} = P^* + \frac{\lambda\phi\delta L^{\delta}K^{1-\delta}}{L} = 0.$$

If we follow through with the same of kind of manipulations represented by equations (6.5) and (6.6), we get

$$(6.13)\qquad \delta = \frac{P_L^* L/P_K^* K}{1 + P_L^* L/P_K^* K}.$$

It follows, in the Cobb-Douglas case, that

$$(6.14)\qquad \phi = \frac{(1 + t_K)K + (1 + t_L)L}{L^{\delta}K^{1-\delta}}.$$

Finally, the adjusted input-output transactions matrix discussed earlier is not in the final form we need. We need a matrix of input-output coefficients. To do this, we divide each column of the adjusted 1973 transactions table by gross output in that industry.

6.3.2 Calibration of Household Utility Functions

We set out the details of the structure of preferences for our twelve consumer groups in section 3.4. In order to perform equilibrium calculations with this structure, we need to specify the values of a large number of parameters for each consumer. These include the following: λ_1, λ_2, . . . , λ_{15}, the weighting parameters that determine the choice among the fifteen consumption goods other than saving; σ_1, the elasticity of substitution between present leisure and present consumption of goods; β, the weighting parameter between present leisure and present consumption of goods; σ_2, the elasticity of substitution between present and future consumption; and α, the weighting parameter between present and future consumption.

The first set of parameters that we specify is the set of λ_m ($m = 1, \ldots, 15$) budget share parameters for the fifteen consumer goods other than savings. Since the inner nest of the utility function is of the Cobb-Douglas type, we merely take each consumer's adjusted expenditure on each of the fifteen goods and divide by the consumer's total expenditure on all fifteen goods. The resulting values are shown in table 6.7. The trends in table 6.7 are rather unsurprising. For example, the proportion of income devoted to food (good 1), utilities (good 4), and housing (good 5) is larger among poor households than among rich ones. The richest group spends a higher proportion of its income on furnishings, clothing and jewelry, services, and financial services than does any other group.

In order to choose the other parameters, we first specify values for: γ, the after-tax rate of return in the benchmark; ξ, the uncompensated elasticity of labor supply with respect to the net wage rate; η, the uncompensated elasticity of saving with respect to the net rate of return; and ζ, the ratio of labor endowment to the labor supply (E/L where $L = E - \ell$).

We will discuss our choices of these latter parameters in section 6.4. For now let us say that our standard case is to set $\xi = 0.15$, $\eta = 0.4$, and $\zeta = 1.75$ for all consumers. Actual labor supply and ζ together imply knowledge of leisure and total endowment in the benchmark. (We will occasionally alter these parameters when we perform sensitivity analyses.) For γ we start with an average value of 0.04 and then make corrections for each household on the basis of marginal tax rates.

Our calibration procedures begin with the derivation of $\hat{\xi}$, defined as the elasticity of demand for *leisure* with respect to its price P_ℓ, the net wage rate. This derivation proceeds as follows. We know that the labor supply elasticity is

Table 6.7 **Consumer Cobb-Douglas Preference Parameters for the Fifteen Consumer Goods Other Than Savings**

Good/Consumer	1	2	3	4	5	6	7	8	9	10	11	12
1. Food	0.197	0.202	0.198	0.201	0.189	0.182	0.185	0.185	0.180	0.177	0.169	0.146
2. Alcoholic beverages	0.018	0.023	0.024	0.025	0.025	0.028	0.025	0.029	0.024	0.027	0.024	0.028
3. Tobacco	0.022	0.019	0.018	0.018	0.019	0.019	0.020	0.019	0.017	0.016	0.013	0.008
4. Utilities	0.062	0.059	0.059	0.056	0.056	0.051	0.050	0.048	0.048	0.044	0.040	0.035
5. Housing	0.218	0.200	0.180	0.173	0.162	0.155	0.153	0.149	0.141	0.141	0.137	0.125
6. Furnishings	0.026	0.030	0.031	0.028	0.030	0.032	0.037	0.033	0.037	0.043	0.046	0.046
7. Appliances	0.028	0.029	0.030	0.031	0.032	0.038	0.037	0.035	0.037	0.033	0.031	0.025
8. Clothing and jewelry	0.058	0.067	0.065	0.074	0.072	0.076	0.077	0.082	0.081	0.087	0.090	0.095
9. Transportation	0.011	0.012	0.011	0.010	0.008	0.009	0.009	0.009	0.006	0.007	0.008	0.012
10. Motor vehicles, tires, and auto repairs	0.059	0.065	0.064	0.062	0.086	0.083	0.081	0.097	0.095	0.092	0.091	0.083
11. Services	0.137	0.124	0.133	0.137	0.132	0.133	0.131	0.122	0.127	0.134	0.145	0.195
12. Financial services	0.052	0.060	0.074	0.066	0.070	0.068	0.067	0.062	0.069	0.061	0.066	0.082
13. Reading, recreation, and miscellaneous	0.029	0.033	0.028	0.035	0.032	0.037	0.042	0.045	0.048	0.054	0.062	0.058
14. Nondurable, nonfood household items	0.046	0.042	0.042	0.045	0.044	0.041	0.040	0.041	0.041	0.038	0.036	0.029
15. Gasoline and other fuel	0.038	0.037	0.042	0.040	0.043	0.047	0.046	0.047	0.049	0.045	0.042	0.033

$$(6.15)\qquad \xi = \frac{\partial(E-\ell)}{\partial P_\ell}\cdot\frac{P_\ell}{(E-\ell)} = \left(\frac{\partial E}{\partial P_\ell} - \frac{\partial \ell}{\partial P_\ell}\right)\cdot\frac{P_\ell}{(E-\ell)}.$$

Since $\partial E/\partial P_\ell = 0$, we can define the elasticity of leisure demand as:

$$(6.16)\qquad \hat{\xi} = \frac{\partial \ell}{\partial P_\ell}\cdot\frac{P_\ell}{\ell} = -\xi\cdot\frac{(E-\ell)}{\ell} = -\xi\cdot\frac{1}{(\zeta-1)}.$$

In the central case where $\xi = 0.15$ and $\zeta = 1.75$, this equation implies that $\hat{\xi} = -0.20$.

The next step is to solve for σ_1, the elasticity of substitution between present consumption and present leisure. We start with equation (3.18), which represents the demand for leisure, reproduced here as:

$$(6.17)\qquad \ell = \frac{\beta(I - SP_S)}{P_\ell^{\sigma_1}\Delta_1}.$$

Notice that β and P_S are the only elements of equation (6.17) that do not depend on P_ℓ. Taking the derivative of this equation with respect to P_ℓ, we get

$$(6.18)\qquad \frac{\partial \ell}{\partial P_\ell} = \frac{-\beta(I - SP_S)\sigma_1}{\Delta_1 P_\ell^{(\sigma_1+1)}} + \frac{\beta}{P_\ell^{\sigma_1}\Delta_1}\left[\frac{\partial I}{\partial P_\ell} - P_S\frac{\partial S}{\partial P_\ell}\right] - \frac{\beta(I - SP_S)}{P_\ell^{\sigma_1}\Delta_1^2}\cdot\left(\frac{\partial \Delta_1}{\partial P_\ell}\right).$$

Let us take a closer look at $\partial S/\partial P_\ell$, which appears in the middle term of equation (6.18). First, we reproduce equation (3.14), which gave the demand for savings:

$$(6.19)\qquad S = \frac{(1-\alpha)I}{P_S^{\sigma_2}\left[\frac{\bar{P}}{P_K^{\gamma}}\right]^{\sigma_2-1}\Delta_2}$$

The net wage P_ℓ affects S in two ways. First, there is a $P_\ell E$ term in I which corresponds to the income effect of P_ℓ on S. Consequently, $\partial I/\partial P_\ell = E$. Secondly, there is a P_ℓ term in P_H which is in Δ_2, corresponding to the cross-price effect. This latter effect is indirect and can be shown to be very small in this case. Incorporating this effect would require advance knowledge of Δ_2, β, and σ_2, whose derivation in turn depends on σ_1, α, and Δ_1. While the system of nonlinear simultaneous equations in these variables is, in principle, soluble, we ignore the indirect cross-price effect and only consider the income effect. We thus use the approximation

$$\frac{\partial S}{\partial P_\ell} \approx \frac{(1-\alpha)E}{P_S^{\sigma 2}\left(\dfrac{\bar{P}}{P_K\gamma}\right)^{\sigma_2 - 1}\Delta_2} = \frac{SE}{I}. \tag{6.20}$$

If we substitute the results of the preceding paragraph into equation (6.18), we get

$$\frac{\partial \ell}{\partial P_\ell} = \frac{-\beta(I - SP_S)\sigma_1}{\Delta_1 P_\ell^{(\sigma_1+1)}} + \frac{\beta}{P_\ell^{\sigma 1}\Delta_1}\left[E - \frac{P_S SE}{I}\right] - \frac{\beta(I - SP_S)}{P_\ell^{\sigma 1}\Delta_1^2}\left(\frac{\partial \Delta_1}{\partial P_\ell}\right). \tag{6.21}$$

Next, we use the equation for leisure, (6.17), to factor this expression.

$$\frac{\partial \ell}{\partial P_\ell} = \frac{-\ell\sigma_1}{P_\ell} + \frac{\beta}{P_\ell^{\sigma 1}\Delta_1}\left[E - \frac{P_S SE}{I}\right] - \frac{\ell}{\Delta_1}\left(\frac{\partial \Delta_1}{\partial P_\ell}\right). \tag{6.22}$$

Finally, we must evaluate $\partial\Delta_1/\partial P_\ell$. We rewrite equation (3.19), which specifies Δ_1:

$$\Delta_1 = \left[(1-\beta)\bar{P}^{(1-\sigma_1)} + \beta P_\ell^{(1-\sigma_1)}\right]. \tag{6.23}$$

This implies that $\dfrac{\partial \Delta_1}{\partial P_\ell} = \dfrac{\beta(1-\sigma_1)}{P_\ell^{\sigma 1}}$, and, therefore:

$$\frac{\partial \ell}{\partial P_\ell} = \frac{-\ell\sigma_1}{P_\ell} + \frac{\beta}{P_\ell^{\sigma 1}\Delta_1}\left[E - \frac{P_S SE}{I}\right] - \frac{\ell\beta(1-\sigma_1)}{P_\ell^{\sigma 1}\Delta_1}. \tag{6.24}$$

The elasticity definition implies that

$$\hat{\xi} = \frac{\partial \ell}{\partial P_\ell}\cdot\frac{P_\ell}{\ell} = -\sigma_1 + \frac{\beta P_\ell^{(1-\sigma_1)}}{\ell\Delta_1}\left[E - \frac{P_S SE}{I}\right] - \frac{\beta P_\ell^{(1-\sigma_1)}(1-\sigma_1)}{\Delta_1}. \tag{6.25}$$

We can rewrite equation (6.17) as:

$$\frac{\beta P_\ell^{(1-\sigma_1)}}{\Delta_1} = \frac{P_\ell \ell}{I - SP_S}. \tag{6.26}$$

With this arrangement, we can derive a new expression for $\hat{\xi}$:

$$\hat{\xi} = -\sigma_1 + \frac{P_\ell E}{I} - \frac{P_\ell \ell(1-\sigma_1)}{I - SP_S}. \tag{6.27}$$

Finally, solving for σ_1, we get:

$$\sigma_1 = \left[-\xi + \frac{P_\ell E}{I} - \frac{P_\ell \ell}{I - SP_S}\right] \Bigg/ \left[1 - \frac{P_\ell \ell}{I - SP_S}\right]. \tag{6.28}$$

The parameter ξ is derived above, P_ℓ is obtained from the consumer's marginal tax rate, and the other values appear in the benchmark data set, including E, I, and S. The price of savings, P_S, is less than unity because the U.S. tax system allows deductions for certain kinds of savings as described in chapter 9.

Once σ_1 is obtained from (6.28), we can solve for β. First, we reproduce equation (3.17), which gives us the amount of current consumption on goods other than leisure.

$$\bar{X} = \frac{(1-\beta)(I - SP_S)}{\bar{P}^{\sigma_1}\Delta_1}. \tag{6.29}$$

Taking the ratio of equation (6.17) to equation (6.29), we have

$$\frac{\ell}{\bar{X}} = \frac{\beta(I - SP_S)/P_\ell^{\sigma_1}\Delta_1}{(1-\beta)(I - SP_S)/\bar{P}^{\sigma_1}\Delta_1} = \frac{\beta}{(1-\beta)} \cdot \frac{\bar{P}^{\sigma_1}}{P_\ell^{\sigma_1}}. \tag{6.30}$$

Solving for β yields

$$\beta = \frac{\ell P_\ell^{\sigma_1}/\bar{X}\bar{P}^{\sigma_1}}{1 + \ell P_\ell^{\sigma_1}/\bar{X}\bar{P}^{\sigma_1}}. \tag{6.31}$$

We know the λ_m expenditure shares on the fifteen consumer goods, and we know cum-tax prices P_m^* from the unit convention and tax rates, so we can calculate $\bar{X}$ from equation (3.23) and $\bar{P}$ from equation (3.26). Other right-hand parameters have been discussed, so β is now available. In table 6.8 we present the values of σ_1 and β for the twelve consumers.

Our next task is to find values for σ_2, the elasticity of substitution between present and future consumption, and the weighting parameter, α. We first specify a value for $\eta = (\partial S/\partial r)\,(r/S)$—the elasticity of saving with respect to the real after-tax rate of return. This rate of return r is given by $r = P_K\gamma/P_S$, as discussed in section 3.4. To find σ_2 as a function of η we could, in principle, follow procedures very similar to those above. To find σ_1 as a function of ξ, we differentiated ℓ with respect to P_ℓ. Here we would differentiate the demand for S, equation (3.14), with respect to the rate of return r. Reproduced here,

$$S = \frac{(1-\alpha)I}{P_S^{\sigma_2}\left(\dfrac{\bar{P}}{P_K\gamma}\right)^{\sigma_2 - 1}\Delta_2}. \tag{6.32}$$

Table 6.8 **Values for the Elasticity of Substitution between Current Leisure and Current Consumption, σ_1, and the Weighting Parameter, β (assuming the labor supply elasticity is 0.15)**

Consumer Group	σ_1	β
1	0.569	0.201
2	0.674	0.248
3	0.777	0.287
4	0.838	0.309
5	0.886	0.325
6	0.948	0.343
7	0.983	0.354
8	0.990	0.359
9	1.027	0.371
10	1.005	0.369
11	0.969	0.366
12	0.738	0.350

As a practical matter, it is exceedingly difficult to evaluate $\partial S/\partial r$ analytically. The $\partial \bar{P}/\partial P_K$ and $\partial P_S/\partial P_K$ terms depend on the capital/labor ratios of particular outputs, and there are many other complex interactions as well. Consequently, we evaluate $\partial S/\partial r$ numerically, using an iterative procedure. The object of the procedure is to choose a value of σ_2 that implies a given value for the saving elasticity.[9] Basically, the iterative procedure involves a numerical differentiation. First, we calculate the values of S and r when all prices are equal to one. Then, we arbitrarily increase the value of P_K by 1 percent and recalculate all of the prices, incomes, and demands. The 1 percent change in P_K results in particular changes for r—the rate of return—and for S—the value of saving. These changes are used to obtain $\eta = \Delta S/\Delta r \cdot r/S$ for each consumer. If this η is greater than the desired value, then σ_2 is adjusted downward, and conversely. After few iterations we obtain the vector shown in table 6.9, resulting in $\eta = 0.4$ for every consumer. (A similar procedure of varying the price of labor by 1 percent was used to verify the 0.15 uncompensated wage elasticity of labor supply.)

Once we have values of σ_2, we can calculate values for α for each household. To do so, we need the expressions for H (present consumption of goods and leisure) and C_F (future consumption), which first appeared as equations (3.11) and (3.12). These are reproduced here as:

$$H = \frac{\alpha I}{P_H^{\sigma_2}\Delta_2} \tag{6.33}$$

9. We usually choose a savings elasticity of $\eta = 0.4$, although we sometimes perform sensitivity analyses with respect to this parameter. For any alternative η, we must iterate again to find the corresponding σ_2.

Table 6.9 **Values for the Elasticity of Substitution between Present and Future Consumption, σ_2, and the Weighting Parameter, α (assuming the saving elasticity is 0.4)**

Consumer Group	σ_2	α
1	1.319	0.980
2	1.531	0.937
3	1.585	0.900
4	1.600	0.857
5	1.618	0.823
6	1.641	0.787
7	1.668	0.739
8	1.673	0.689
9	1.697	0.625
10	1.698	0.580
11	1.675	0.560
12	1.500	0.488

and

(6.34) $$C_F = \frac{(1-\alpha)I}{P_{CF}^{\sigma_2}\Delta_2},$$

where P_{CF} is the "price" of future consumption, $P_S\bar{P}/P_K\gamma$. Taking the ratio of the two, we get

(6.35) $$\frac{H}{C_F} = \frac{\alpha I/P_H^{\sigma_2}\Delta_2}{(1-\alpha)I/P_{CF}^{\sigma_2}\Delta_2} = \frac{\alpha}{1-\alpha}\cdot\frac{P_{CF}^{\sigma_2}}{P_H^{\sigma_2}}.$$

Solving for α yields

(6.36) $$\alpha = \frac{HP_H^{\sigma_2}/C_F P_{CF}^{\sigma_2}}{1 + HP_H^{\sigma_2}/C_F P_{CF}^{\sigma_2}}.$$

From equation (3.7), we can see that

(6.37) $$C_F = SP_S/P_{CF}.$$

It follows that

(6.38) $$\alpha = \frac{HP_H^{\sigma_2}/SP_S P_{CF}^{(\sigma_2-1)}}{1 + HP_H^{\sigma_2}/SP_S P_{CF}^{(\sigma_2-1)}}.$$

Since σ_1 and β are already available, H can be obtained from equation (3.15) and P_H from equation (3.27). Savings S and prices are also available for the right-hand side. The resulting values for σ_2 and α are shown in table 6.9.

6.3.3 Calibration of Government Expenditures

As noted in section 3.6, the government has a Cobb-Douglas utility function, which is defined over the nineteen producer goods, plus labor and capital. The adjusted data for government expenditure on the nineteen producer goods were shown in table 6.4. We discussed the government's payments to labor and capital in section 5.5. When we combine all these expenditure data and calculate the expenditure shares, we get the Cobb-Douglas parameters that are listed in table 6.10.

6.4 Elasticities

6.4.1 Value-Added Elasticities

We rely on a literature search to provide estimates of production function substitution elasticities. Since the introduction of the CES function, many economists have estimated the values of elasticities. Estimates have been obtained using a variety of econometric procedures and for various industrial classifications, although in the process some seemingly contradictory estimates have been produced.

Table 6.10 **General Government Cobb-Douglas Preference Parameters for the Nineteen Producer Goods, Capital, and Labor**

Commodity	Expenditure Share
Agriculture, forestry, fisheries	0.0
Mining	0.004
Crude petroleum and gas	0.0
Contract construction	0.0886
Food and tobacco	0.0029
Textiles, apparel, and leather	0.0013
Paper and printing	0.0042
Petroleum refining	0.0029
Chemicals, rubber, and plastics	0.0080
Lumber, furniture, stone, clay. and glass	0.0018
Metals, machinery, instruments, and miscellaneous manufacturing	0.0274
Transport equipment and ordnance	0.0312
Motor vehicles	0.0049
Transportation, communications, and utilities	0.0193
Trade	0.0041
Finance and insurance	0.0017
Real estate	0.0035
Services	0.0313
Government enterprises	0.0016
Capital	0.3333
Labor	0.4316

Many researchers have attempted to estimate substitution elasticities between capital and labor in U.S. manufacturing. Ernst Berndt paraphrases the disagreements among estimates in the following terms:

> Studies based on cross-sectional data provide estimates which are close to unity, but time-series generally report lower estimates. Furthermore, estimates of σ seem to vary systematically with the choice of functional form; regressions based on the marginal product of capital relation generally produce lower estimates of σ than regressions based on the marginal product of labor relation. (1976, p. 59)

Several hypotheses have been advanced to rationalize these discrepancies, but none of the hypotheses has been wholly accepted. The most plausible explanation for the discrepancy between the time-series and cross-section results is that adjustments take place with a lag.

Our procedure is to use the hundreds of estimates reviewed by Vern Caddy (1976) and arrange these estimates on the producer good classification used in this study. For each group of estimates (for one elasticity parameter), we calculate the mean and variance of the group. These are reported in table 6.11, with a further partition into cross-section and time-series estimates.

In either the cross-section or time-series estimates of table 6.11, agriculture and food have elasticities somewhat lower than the manufacturing industries. Because of the difference between the two sets of estimates, we use the "overall" elasticities in the last columns of table 6.11. For those industries for which estimates are not available, we set the elasticities at unity, meaning that we employ a Cobb-Douglas production function.

6.4.2 Labor Supply Elasticities

In the general equilibrium model in its present form, consumers balance the competing objectives of increasing leisure by working less and of increasing consumption opportunities by working more. The uncompensated net-of-tax wage rate elasticity of labor supply, ξ, measures how this work choice is affected by changes in the net-of-tax wage. In subsection 6.3.2 we described the way in which a prespecified value of ξ is converted into the relevant parameters of the consumer's utility function.

Once again we appeal to the econometric literature in our search for the value of ξ. The econometric literature gives many estimates for population subgroups, since different individuals will typically have different rates of response to a new net-of-tax wage. Finegan's (1962) occupational study found managers, craftsmen, and clerical workers varying from a $-.29$ to a $+.42$ labor supply elasticity, while Boskin's (1973) division by sex, race, and age found estimates from $-.07$ (for prime-age white males) to $+1.60$ (for elderly black women). In table 6.12

Table 6.11 **Central Tendency for Estimates of Elasticity of Substitution by Industry Reported by Caddy (1976)**

	Cross-Section			Time-Series			Overall		
	a	b	c	a	b	c	a	b	c
Agriculture, forestry, fisheries	.8312	(.1078	27)	.3971	(.0688	15)	.6759	(.1369	42)
Mining	No information								
Crude petroleum and gas	No information								
Contract construction	No information								
Food and tobacco	.8912	(.1160	24)	.5322	(.1072	24)	.7117	(.1439	48)
Textiles, apparel, and leather	1.0530	(.1198	65)	.5773	(.1063	30)	.9025	(.1643	95)
Paper and printing	1.0468	(.0857	49)	.4895	(.0795	17)	.9033	(.1435	66)
Petroleum refining	.9342	(.1000	14)	.5479	(.1227	9)	.7830	(.1444	23)
Chemicals, rubber, and plastics	1.1482	(.2321	64)	.5086	(.1194	27)	.9603	(.2808	91)
Lumber, furniture, stone, clay, and glass	1.0218	(.1142	88)	.5585	(.1800	31)	.9123	(.1656	118)
Metals, machinery, instruments, and miscellaneous manufacturing	.8719	(.1183	109)	.4844	(.1057	58)	.7373	(.1479	167)
Transport equipment and ordnance	1.0534	(.3470	14)	.3407	(.0388	7)	.8159	(.3572	21)
Motor vehicles	1.1782	(.3755	18)	.4631	(.0770	10)	.9228	(.3863	28)
Transportation, communications, and utilities	No information								
Trade	No information								
Finance and insurance	No information								
Real estate	No information								
Services	No information								
Government enterprises	No information								

Notes: For each column the numbers reported are a) the mean of the estimates, b) the variance among the estimates, and c) the number of estimates available.

we list the results of a number of econometric studies. Table 6.12 is based primarily on the review by Mark Killingsworth (1983).

A certain injustice is perpetrated against these authors by reporting their results in such summary fashion. Each study has its own mesure of the wage, its own data year or time period, and its own functional form. Also, the studies differ as to how they account for participation rates. The numbers in table 6.12 are provided only to give the reader a framework for choosing a plausible aggregate labor supply elasticity.

Elasticity estimates for males are mostly small and negative, ranging from $-.40$ to 0. Borjas and Heckman (1978) review the econometrics of these studies and reduce the bounds to $-.19$ and $-.07$. The estimates for females are more often positive, and can be large in absolute value. Killingsworth finds that females' elasticity estimates are mostly between .20 and .90 in cross-section studies. To obtain the model's aggregate labor supply elasticity of 0.15, which we use for each of the twelve consumer groups, we perform a rough numerical calculation. The *Statistical Abstract* (U.S. Department of Commerce, Bureau of the Census, 1973) shows that the median money income of male employed civilians has consistently been twice that of females. It also shows about a 1.7 ratio of males to females in the labor force—a ratio that is decreasing with time. In any case, the ratio of male to female income should be at least 3.0 (though decreasing). Taking a male elasticity of $-.10$ and a female elasticity of $+.90$, the three-to-one weighted average is a 0.15 aggregate elasticity.

We need to specify one other parameter dealing with the labor/leisure choice, and that is ζ, the ratio of labor endowment to actual labor supply in the benchmark. This is the parameter that we use to convert from a labor supply elasticity to a leisure demand elasticity (see section 6.3.2). In the absence of concrete data, we choose 1.75 for ζ, to reflect that individuals typically work a forty-hour, out of a possible seventy-hour week. This parameter is surprisingly important, however, since its value affects the difference between the compensated and uncompensated labor supply elasticities. Consider equations (6.28) and (6.16), which together show how ξ and ζ determine σ_1, the elasticity of substitution between consumption goods and leisure. With other parameters given, a higher ζ raises E in equation (6.28). It therefore raises σ_1, a crucial parameter in determining the distorting effects of taxes on labor. Without empirical estimates of ζ, then, it is particularly important to perform sensitivity analyses.[10]

10. Welfare costs of distorting labor taxes increase with σ_1 and therefore with ζ. When we integrate corporate and personal taxes, as in chapter 8, and replace lost revenue with additional taxes on labor, the net gains are inversely related to ζ. Fullerton, Henderson, and Shoven (1984) report results when ζ is set to 1.25, the standard 1.75, and a final value of 2.25. The present value of net welfare gains are \$512.5 billion, \$344.4 billion, and \$244.6 billion, respectively.

Table 6.12 **Estimates of the Uncompensated Labor Supply Elasticity**

A. Males

Authors	Data Subset	Type of Data	Range of Estimates
Finegan (1962)	Male family heads	Interoccupational	−.35 to −.25
Rosen (1969)	Male family heads	Interindustry	−.30 to −.07
Kalachek-Raines (1970)	Male family heads	U.S. cross-section	+.05 to +.30
Owen (1971)	Male family heads	U.S. time-series	−.24 to −.11
Greenberg-Kosters (1973)	Poor male family heads	U.S. cross-section	−.16 to −.05
Boskin (1973)	Different male subgroups	U.S. cross-section	−.07 to +.18
Hill (1973)	Poor male family heads	U.S. cross-section	−.32 to −.07
Ashenfelter-Heckman (1973)	Male family heads	U.S. cross-section	−.15
Fleisher-Parsons-Porter (1973)	Males age 45–59	U.S. cross-section	−.25 to −.10
Ashenfelter-Heckman (1974)	Married males	U.S. cross-section	0
Burtless-Hausman (1978)	Low-income males	Gary NIT cross-section	0
Hausman (1981)	Married males	U.S. cross-section	0

B. For Females

Authors	Data Subset	Type of Data	Range of Estimates
Finegan (1962)	Females	Interoccupational	−.095
Leuthold (1968)	Females	U.S. cross-section	−.067
Kalachek-Raines (1970)	Females	U.S. cross-section	+.20 to +.90
Boskin (1973)	Different female subgroups	U.S. cross-section	−.04 to +1.60
Ashenfelter-Heckman (1974)	Married females	U.S. cross-section	.87
Hausman (1981)	Married females	U.S. cross-section	.9
Hausman (1981)	Female household heads	U.S. cross-section	.5

C. Aggregate

Authors	Data Subset	Type of Data	Range of Estimates
Winston (1966)	Aggregate	International cross-section	−.11 to −.05
Lucas-Rapping (1970)	Short-run aggregate	Time-series	1.35 to 1.58
Lucas-Rapping (1970)	Long-run aggregate	Time-series	0 to 1.12

6.4.3 The Saving Elasticity

Our parameter η represents the uncompensated elasticity of saving with respect to changes in the real after-tax rate of return. To see what economic theory tells us about η, consider saving as an expenditure on future consumption. An increase in the net rate of return lowers the price of future consumption. The compensated quantity demanded must rise, but the percentage increases may exceed or fall short of the percentage decrease in price. The resulting expenditure on future consumption (saving) may rise or fall, so the sign of η is ambiguous.

Empirical estimates of η have hardly narrowed the range of plausible values. Denison's law states that η is zero, following Edward Denison's (1958) observation that saving as a fraction of income in the United States has been a historical constant. Econometric estimates by Michael Boskin (1978) suggest that η is significantly positive. Using eight different regressions, Boskin finds values for η that range from 0.2 to 0.6, but that cluster between 0.3 and 0.4. Howrey and Hymans (1978) use Boskin's data but find that estimates of η are sensitive to (1) the measure of expected inflation, (2) the sample period, (3) the definition of saving, and (4) the interest rate variable chosen for the regression. They cannot reject the hypothesis that η is zero.

More recently, Lawrence Summers (1981) builds a model in which lifetime consumption plans depend upon several factors, including parameters for intertemporal substitution in utility, time until retirement and death, the rate of time preference, rates of growth, and the rate of return to saving. The model is then solved for the saving elasticity. Plausible values for these other parameters imply values for η that range from 1.5 to 3.0, much higher than those of the econometric estimates described above. Finally, David Starrett (1982) and Owen Evans (1983) show how amendments to Summers's model could widen these bounds still further, but they argue for values of η that are lower than those found by Summers.

Given the wide range of estimates, the saving elasticity is a particularly important candidate for sensitivity analyses. In our standard set of parameters, used for calculations in later chapters, we employ Boskin's central estimate of 0.4 for η. We then report additional results for alternative values of this saving elasticity.

6.4.4 Commodity Demand Elasticities

We use Cobb-Douglas forms for the subutility function that determines the allocation of consumption expenditures among consumer good categories. As a result, there is no need to specify substitution elasticities. All own-price elasticities are -1, all income elasticities are unity, and all cross-price elasticities are zero. Cobb-Douglas exponents are given from data on expenditure shares (see table 6.7).

6.4.5 External Sector Parameters

Specification of parameter values for our foreign trade functions (3.46) involves the unit terms M_i^0, E_i^0, and the elasticity parameters μ and ν.

Since the benchmark equilibrium is characterized by prices of unity and overall trade balance, the unit terms M_i^0 and E_i^0 are equal to the benchmark trade values shown in table 5.1.

Equations (3.50) and (3.51) define the foreign price elasticities of export demand (ϵ_E^{FD}) and import supply (ϵ_M^{FS}). We reproduce these here as:

$$\epsilon_E^{FD} = \frac{\nu(1 + \mu)}{(\mu - \nu)} \tag{6.39}$$

and

$$\epsilon_M^{FS} = \frac{-\mu(1 + \nu)}{(\mu - \nu)}. \tag{6.40}$$

Once we have values for ϵ_E^{FD} and ϵ_M^{FS}, therefore, we can calculate implied values for ν and μ. An approximate central case value for ϵ_E^{FD} is -1.4, as seen in the compendium of trade elasticities provided by Stern, Francis, and Schumacher (1976). If we accept this value for ϵ_E^{FD}, we can rewrite equation (6.39) as:

$$\mu = \frac{0.4\nu}{1.4 + \nu}. \tag{6.41}$$

Of course, this equation is satisfied by an infinite number of combinations of μ and ν. Because the demand for U.S. exports in equation (3.41) should be highly sensitive to price, we postulate a high negative value for ν. As a practical matter, we set $\nu = -10$, so μ must be 0.465. Together, these figures imply 0.40 for the import supply elasticity ϵ_M^{FS}.