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ON THE SPECIFICATION OF UNEMPLOYMENT AND INFLATION IN THE OBJECTIVE FUNCTION*

BY CARL J. PALASH**

This paper investigates the consequences of specifying in various ways objective functions whose arguments are unemployment and inflation. Of the objective functions examined, it is argued that most of those which are nonlinear cannot be properly specified without consideration of the constraints. It is further argued that the horizon problem is more likely to lead to unacceptable optimal solutions the closer an objective function is specified to be non-truncated linear. Rationales for parameters in the objective function are provided in terms of the short and long runs. The analysis is conducted through numerical experiments with the MPS model, a quarterly econometric model of the U.S., as constraints.

This paper attempts to put in perspective the question of proper specification of unemployment and inflation in the objective function. A criterion is set forth to judge the results of maximizing linear, truncated quadratic and exponential functions with the MIT-PENN-SSRC (MPS) model as constraints. This consists of bounds on unemployment and inflation, outside of which is considered unacceptable unless substantial improvement in the other variable is observed. A precondition for the specification of most of the objective functions examined is that knowledge of the short-run model constraints not influence specification. A dichotomy between preferences and constraints is a standard assumption of welfare theory.

The objective functions that are examined, examples from a wide spectrum of possible functions, can be categorized into those whose derivatives are functions of the target variables and those whose derivatives are independent of such variables. The quadratic and exponential functions fall into the first category while the linear function belongs to the second. The relative weights among targets, within a particular quarter and over time, implicit in the objective functions of the first group, are susceptible to the state of and exogenous shocks to the economy, while this is not true of the functions of the second group.

The results of maximizing the objective functions over two time periods, 1971–75 and 1965–69, indicate that the relative weights, either between targets or over time, implicit in the functions of the first category, become excessively biased in favor of one target or quarter when the economy experiences severe exogenous shocks such as in 1973–74. It is

*The views expressed here are those of the author and do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System.

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also shown that under more normal circumstances explicit recognition has to be given to initial conditions, such as inflationary expectations embodied in the wage-price mechanism[1] in order to ensure desirable results. Unfortunately, the procedures to eliminate these problems require destroying the dichotomy between preferences and constraints. When initial conditions are the primary focus for consideration of "exogenous" forces, it is suggested that the desired relationship between initial conditions and long-run targets be reflected in the weights assigned to the individual terms in the objective function. In this way, the specification that desired unemployment and inflation both be zero is maintained. For this reason, this approach appears more reasonable than truncating the quadratic or exponential function at some point.

The specification of the objective function that eliminates these considerations, and thus preserves dichotomy between preferences and constraints, is linear. It is shown, however, that penalizing deviations of unemployment and inflation linearly fails to adequately capture the full effects of particular policies because of the truncated horizon.[2] The bias appears to be possibly eliminated when no gain is permitted to be registered in the objective function for values of the unemployment rate less than the estimated long-run natural rate of the MPS model. The rationale for this procedure is based on the consideration that long-run acceleration or deceleration of prices is not desirable. This rationale is admittedly weak, though, because extended periods of unemployment below its natural rate value, may, in fact, be desirable. The solution to the horizon problem, however, remains to be discovered. While terminal conditions can be specified in the objective function, a reasonable specification of them is quite difficult to arrive at, and none are suggested here.

In Section I the framework of the analysis will be discussed. In Section II the effects of maximizing objective functions specified as linear, truncated quadratic or exponential, with unemployment and inflation as arguments, will be analyzed. The constraints will consist of the MPS quarterly econometric model.

An objective function should represent preferences accurately. There could be difficulties in doing so, however, in the case of a quadratic function with the unemployment rate as argument. The well-known symmetry property associated with a quadratic function implies that after some point lower values of unemployment are penalized just as heavily as

[1]These expectations are estimated to be extremely "sticky" in the short run. This implies that an objective function that calls for their elimination may entail extreme deflationary optimal control.

[2]The most likely case of a truncated horizon is the standard econometric forecast. This is usually restricted to eight to twelve quarters into the future, which places a physical limit on the horizon. This limit, however, may not equal that which is desired.

higher values. It has been argued by many, however, that low levels of unemployment are undesirable only to the extent that they lead to high rates of inflation. Therefore, it is further argued that the inflation rate should be included in the objective function, as well. However, if both unemployment and inflation are represented through quadratic terms, there would be, in a sense, double counting. If the unemployment rate is specified alone in the objective function with the rationale that symmetric penalization accomodates preferences concerning inflation because unemployment and inflation are to a large extent inversely related in the model, then weak dichotomy between preferences and constraints is destroyed. Consequently, the implicit relative weight on inflation may not correctly reflect true preferences, because it depends on system dynamics.

A common correction of the symmetry problem of a quadratic function is to truncate the latter at some point. Truncation means that below some point no gain is registered for pushing the unemployment or inflation rate further downward. If an arbitrary point is used to truncate the unemployment-inflation space than the question of rationale might again be raised. It could be argued that there is no reason to disallow a gain in unemployment below the truncation point unless it excessively exacerbates inflation, which, however, is already incorporated in the objective function. Furthermore, any argument in favor of truncation at an arbitrary point that is in terms of the unemployment-inflation model trade-off would violate the property of weak dichotomy between preferences and constraints.

If a point on the long-run Phillip's curve is used to truncate the quadratic, then the role of the control horizon must be examined. When the horizon is sufficiently long so that all the effects of the policy instruments are registered in the objective function then strong dichotomy between preferences and constraints is possible and such truncation is not necessary. However, when all the effects of the policy instruments are not captured by the objective function, then weak dichotomy between preferences and constrains as is embodied in such truncation might be necessary. If the effects beyond the horizon were not taken into account, then the optimal policy could be biased in favor of one target or the other.

Given that the horizon effects are acceptably accounted for, the question arises as to how to specify preferences over unemployment and inflation within the horizon, assuming at least weak dichotomy. In particular, how should deviations of unemployment or inflation from zero (in the case of strong dichotomy) or their long-run values (in the case of weak dichotomy) be penalized. The issue centers on whether the deviations should be penalized linearly, by a truncated quadratic, or exponentially. Generally, such deviations would be penalized most heavily through an exponential, less so by a truncated quadratic and least by a linear function. This is because in the exponential or quadratic cases the relative weights given to target variables (either among them or over time) are functions of the space at which the objective function is evaluated. Furthermore, the derivatives of an exponential with respect to target variables are respectively greater than those of a truncated quadratic everywhere. The relative weights given to target variables are independent of the state of the economy with a linear function.

Whether more or less penalty on deviations from, say, zero is desired depends on the aspects of the problems at hand. In [1] it was argued that huge exogenous shocks increased the rate of inflation in 1973–74, which if not offset in an exponential or quadratic objective function would lead to undue weight on inflation and an excessively deflationary optimal policy. The undesirably high, implicit weight given to inflation in the exponential was eliminated by applying multiplicative, time-varying weights, as in equation (1), that deemphasized the impact of the exogenous shocks in 1974. This procedure, however, violated the property of weak dichotomy between preferences and constraints. Alternative approaches to the problem will be explored below.

$$W = \sum_i \left[-a_i e^{u_i} - b_i e^{p_i}\right] \tag{1}$$

where u_i = unemployment rate in period i
p_i = inflation rate in period i

Under more normal circumstances some penalty on high values of unemployment or inflation is desirable. This is particularly true for high values of unemployment or inflation that result from policy actions. However, simple specification of such a consideration in the objective function, i.e., by penalizing deviations exponentially, linearly or by truncated quadratic, may yield some qualitative differences in the final result depending on the particular functional specification. For instance, the optimal sequence of unemployment may be oscillatory in one case but not in another.

The optimal sequences of unemployment and inflation result from the interaction of the objective function and model constraints. Examination of the constraints is necessary in order to analyze the results presented below. The MPS wage-price mechanism consists of a Phillips curve-type relationship to explain the rate of change of wages and a mark-up equation on expected minimized labor costs[3] to explain prices. The relationship between unemployment and inflation is most strongly

[3] Theoretically, the mark-up should be on minimized average total costs. However, the rental rate of capital did not figure significantly in estimation.

modeled through the wage equation, although the mark-up and productivity are dependent on demand conditions to some extent. The wage equation is basically specified as:

$$(2) \qquad \left(\frac{\Delta W}{W}\right)_t = a\frac{1}{u_t} - \frac{b(u_t - u_{t-1})}{u_{t-1}} + \sum_{i=t-1}^{t-18} c_i p_i$$

where W = wages $\quad a > 0$
u = unemployment $\quad b > 0$
p = inflation rate $\quad \Sigma c_i = 1$

A change in the unemployment rate will have an immediate effect on the rate of change of wages through the first two explanatory terms of equation (2). The effect of the rate of change of the unemployment rate on the rate of change of wages is temporary, however. Since prices are dependent on wages, though, the rate of change of prices will change and, in accordance with the distributed lag on the latter in equation (2), which proxies for inflationary expectations or as a "catch-up" term, will slowly influence wages over time. Thus, a permanent change in the unemployment rate will have a contemporaneous effect on wages through the first and second terms and a lasting effect through the first and third terms.[4] An additional characteristic of this equation is that since the sum of the coefficients in the distributed lag on past rates of change of prices equals unity, a natural rate of unemployment is implied. The implicit natural rate of unemployment is approximately 4.5 percent in the model. Unemployment rates in the long-run above the natural rate will cause continual deceleration of inflation, while those below the natural rate will cause continual acceleration of inflation.

The timing of the effects of unemployment on inflation[5] plays an important role in the determination of optimal policy. If the horizon is not long enough to account for all the impact of changes in unemployment on inflation, the optimal policy may be biased in favor of too much or too little stimulus. This will be called the horizon effect. Excluding this possibility, the lag between changes in unemployment and their full effect on inflation can still be a significant influence on the optimal solution. In particular, since the effects of a change in the unemployment rate are felt over time, the decision whether a given change in unemployment is desirable depends partly on how the future effects of such change are valued. With a linear specification of costs, the decision will be influenced by

[4]Because of the specification of the inverse of the unemployment rate in equation (2), the marginal effect on wages from changes in the unemployment rate is dependent on the level of unemployment. The marginal effect will decline as the unemployment rate rises.

[5]The effect of a change in wages on prices is immediate, although it might be offset to some extent through a change in productivity or in the markup.

whether the cumulative effects on inflation resulting from the change in unemployment are greater or less than the cumulative change in the unemployment rate. If one moves to quadratic or exponential representation of costs, the comparison will be between the quadratic or exponential transformations of the changes in unemployment and inflation. Generally, exponential and quadratic functions penalize upward movements in their arguments more heavily than they reward downward movements, assuming that their arguments are initially equal in value (or equidistant from long-run desired targets). Thus, the delayed increases in inflation from a fall in unemployment should be valued relatively greater than they would be linearly. On the other hand, the delayed declines in inflation from a rise in unemployment should be valued relatively less than they would be linearly. Consequently, one should expect relatively less extreme trade-offs between unemployment and inflation over time in the case of quadratic and exponential objective functions compared to that of linear functions. In other words, one should expect oscillations of smaller amplitude over time of the unemployment rate when preferences are represented by quadratic or exponential functions compared to linear functions.

In practice, both the horizon and oscillation effects will co-exist. The horizon effect is undesirable. The oscillation effect may or may not be so; it is a characteristic of a particular objective function that should be weighed when deciding the latter's desired specification. The question of which effect dominates a control solution must be determined through experimentation. This will be conducted in the next section.

II

Empirical Results

In this section the implications of using linear, truncated quadratic and exponential objective functions will be examined with the MPS model as constraints.[6] The experiments will be conducted over two time periods. The first, 1971–75, was marked by huge exogenous shocks to prices in 1973–74.[7] The second, 1965–69, was less subject to major exogenous shocks.[8]

Some loose criteria will be used to judge the implications of the different objective functions. First, if the optimal solution calls for extreme worsening on the part of one target which does not lead to substantial

[6]The residuals of the equations were added back so that the model tracks history without error when the historical policies are simulated.

[7]See [1] for an analysis of these shocks.

[8]While both fiscal and monetary policy can be considered extraordinary in some respects during this period they are endogenous to the control problem.

improvements on the part of the other target, then that objective function which yields this result will be considered undesirable. Second, the long-run level of the unemployment rate that is consistent with a negligible rate of inflation (0 percent to 2 percent) has been estimated by many to have been between 4–4.5 percent in the 1960's and to have rise to between 4.5–5.5 percent in the 1970's. Therefore, in cognizance of the horizon effect, optimal policy that pushes the unemployment rate below 4 percent in 1965–69 period or 4.5 percent in the 1971–75 period, especially towards the end of the period under control, will be considered unacceptable. This, unfortunately, is a tenuous criterion. Although unemployment rates below these levels could be associated in the long-run with a higher than desirable or possibly even accelerating inflation, it is not necessarily undesirable to have such low unemployment rates for extended periods of time. However, this criterion will be used because the instances in which the unemployment rate is driven below these minimum levels appear to be associated with the horizon effect.

The first period under examination contained huge shocks to the economy.[9] As was mentioned above, the objective function used in [1], violated the property of weak dichotomy between preferences and constraints.[10] However, it was also shown in [1] that an objective function that was less guilty of violating weak dichotomy led to unacceptably high levels of unemployment with little improvement in inflation.[11] In other words, the first criterion was not satisfied. Therefore, to determine an acceptable objective function it is necessary to find one that satisfies at least weak dichotomy and the other two criteria just mentioned.

The first objective function to be maximized[12] is the linear function:

[9]See Table 1, column (a) for the historical values of unemployment and inflation. The rate of change of the GNP deflator is used throughout the paper to represent inflation.

[10]See Table 1, column (b) for these results.

[11]See Table 1, column (c) for these results.

[12]In all the problems below, unless otherwise mentioned, the Treasury bill rate and Federal Government nondefense, nonwage expenditures were used to maximize the objective function. For the 1971–75 period the following inequality constraints were in effect:

$$3 \leq RTB \leq 13$$

$$\Delta RTB \leq .5 * RTB_{-1}$$

$$55 \leq EGF \leq 70.$$

For the 1965–69 period the constraint on EGF was changed to:

$$55. \leq EGF \leq 85.$$

where RTB = Treasury bill rate
EGF = Federal Government spending (1958$)

These constraints are meant to ensure that the analysis will be pertinent to relevant application of optimal control to economic policy problems.

$$W = \Sigma - (u + p) \tag{3}$$

Since both desired u and p are specified to be as low as possible, this function satisfies strong dichotomy. Furthermore, as it is linear, its derivatives are independent of the state of the economy. Thus, it avoids the problem of having undue weight given to one of the targets because of exogenous shocks to the economy. The results of maximizing equation (3) are shown in Table 1 column (d). As can be seen, unemployment is driven to extraordinary low levels in 1974. This violates the minimum value criterion and vitiates the usefulness of this objective function as it stands. It is most likely that the horizon effect is responsible for the low values of unemployment in 1974.[13]

TABLE 1

RESULTS OF MAXIMIZING OBJECTIVE FUNCTIONS OVER PERIOD MARKED BY HUGE EXOGENOUS SHOCKS

	u						p					
	(a)	(b)	(c)	(d)	(e)	(f)	(a)	(b)	(c)	(d)	(e)	(f)
1971.1	5.9	6.1	6.0	6.1	6.1	6.1	4.6	4.7	4.7	4.7	4.7	4.7
.2	5.9	6.2	6.2	6.3	6.2	6.2	4.6	4.4	4.6	4.4	4.4	4.3
.3	6.0	6.2	6.7	6.4	6.4	6.2	2.6	2.4	2.4	2.4	2.4	2.6
.4	6.0	6.2	7.1	6.3	6.7	6.4	2.0	1.8	1.7	1.7	1.8	1.7
1972.1	5.8	5.9	7.5	6.2	6.9	6.0	5.3	5.2	4.8	5.3	4.9	5.0
.2	5.7	5.7	7.9	6.2	7.2	5.7	1.9	2.0	1.3	1.6	1.4	2.2
.3	5.6	5.7	8.4	6.0	7.2	5.8	3.3	3.2	2.4	3.1	2.5	3.1
.4	5.3	5.5	8.8	5.8	7.1	5.5	4.1	3.2	3.0	3.8	3.2	3.8
1973.1	5.0	5.3	9.2	5.7	6.6	5.3	5.1	4.8	3.8	4.7	4.0	5.2
.2	4.9	5.3	9.7	5.5	6.2	5.3	7.2	6.2	5.9	6.6	6.4	6.8
.3	4.8	5.1	10.1	5.0	5.6	5.1	8.1	7.6	6.6	7.5	6.9	7.7
.4	4.7	4.8	10.5	4.6	5.2	4.9	8.2	7.9	6.1	8.0	7.8	7.9
1974.1	5.1	4.9	10.8	4.5	5.1	4.9	11.8	10.2	8.1	11.6	10.7	11.5
.2	5.2	4.4	10.7	3.6	4.0	4.4	9.0	7.9	4.5	9.5	9.1	9.1
.3	5.5	4.1	10.8	2.9	3.9	4.2	11.5	11.0	7.6	12.9	12.0	11.8
.4	6.6	4.7	11.6	3.3	4.4	4.5	13.7	13.3	10.3	15.3	13.4	13.9
1975.1	8.4	6.0	12.8	4.6	5.6	5.5	8.1	7.8	4.5	9.6	8.7	9.0
.2	8.9	6.0	12.8	4.5	5.9	5.4	5.1	4.6	1.9	6.3	4.8	5.5

u = unemployment rate
p = rate of change of GNP deflator
column (a) = historical values
objective function underlying column (b) = $\Sigma\,(-e^{(u-\bar{u})} - e^{(p-\bar{p})})$, where $\bar{u}$ and $\bar{p}$ were obtained from an ad hoc policy described in [1].
objective function underlying column (c) = $\Sigma\,(-e^{(u-5)} - e^{p})$
objective function underlying column (d) = $\Sigma(-u-p)$
objective function underlying column (e) = $\Sigma\,(-e^{(u+p)})$
objective function underlying column (f) = $\Sigma\,(-(u \geq 4)-p)$

[13] The unemployment rate increases after 1974.3 because the inequality constraints on the policy instruments were binding in those quarters.

In order to mitigate the horizon effect the following objective function was maximized:

(4) $$W = \Sigma[-e^{(u+p)}]$$

The acceleration of the penalty on p as p increases should prevent unemployment from being pushed as low as it was in equation (3). Furthermore, since the derivative of W with respect to u_i or p_i is $e^{(u_i+p_i)}$, the problem of undue weight given to one of the target variables because of exogenous shocks to the economy is eliminated. Finally, strong dichotomy is maintained.

The results are given in Table 1 column (e). While the unemployment rate does not decline as much in 1974 as it does in column (d), it is still too low. In addition, the unemployment rate is pushed to fairly high levels in 1972, 1973, 1975, causing a moderate reduction in the inflation rate. These results can be explained from the observation that the exogenous component of p in 1974, while it does not impose undue weight on p relative to u in 1974, does increase the weight on u and p in 1974 relative to their values in the other quarters. Thus, unemployment is pushed up in 1971–73 in order to reduce the contribution of the distributed lag on past rates of change of prices to the inflation rate in 1974. In other words, this objective function is biased in favor of unemployment and inflation in 1974.

Another possible way of taking account of post horizon considerations is to only penalize upward deviations of unemployment and inflation from their long-run desired or model consistent values. The idea here is that all the effects of decreasing unemployment are not registered in the objective function because of the truncated horizon. Therefore, reducing unemployment below its long-run value should not be considered gainful. However, for reasons mentioned above, this approach might be overly restrictive for correct representation of short-run preferences. Nevertheless, the following objective function was maximized:

(5) $$W = \Sigma[-(u \geq 4.5) - p]$$

This function satisfies weak dichotomy and its derivatives are independent of the state of economy when the unemployment rate does not equal 4.5 percent. The results are shown in Table 1 Column (f). They appear to be the most reasonable so far, although the horizon effect may be having too much influence after 1974:2. Remarkably, the results are extremely similar to those in Column (b), which were obtained through an objective function that violated weak dichotomy.[14]

[14] The policy assumptions that underly the results given in column (b) and (c) include a reduction in the OASI tax in 1974.1 and 1974.2. This helps to explain the smaller rate of inflation in these quarters compared to the other columns.

The above results indicate that linear functions of unemployment and inflation that penalize deviations above long-run values yield satisfactory optimal solutions according to the criteria set forth while weak dichotomy is not violated. Only by violating weak dichotomy could quadratic or exponential functions yield satisfactory results. The implications of maximizing these functions under more normal conditions remain to be seen. The period 1965:1–1969:4 was chosen for this purpose.[15]

The first question to be examined in the context of this earlier time period is whether any of the functions yield satisfactory results under strong dichotomy. For this purpose the linear, quadratic (truncated at zero unemployment and inflation) and exponential functions that penalized positive deviations of unemployment and inflation from zero were maximized. Specifically, the functions are:

$$W = \Sigma[-u - p] \tag{6}$$

$$W = \Sigma[-(u \geq 0)^2 - (p \geq 0)^2] \tag{7}$$

$$W = \Sigma[-e^u - e^p] \tag{8}$$

The results are given in Table 2, Columns (b), (c), (d), respectively. They display several characteristics. First, the average optimal unemployment rate for the period is lower as one moves from the linear to the quadratic to the exponential case. This phenomenon follows from the observation that the unemployment rate was higher on average than the inflation rate for the period and that such relatively extreme deviations from zero increase the relative weight on unemployment in the quadratic and exponential cases. Unemployment was higher than inflation, on average, for the period, because of initial conditions (i.e., low inflationary expectations) in conjunction with the lag in the full response of inflation to changes in unemployment. The relationship between initial levels of inflation and desired levels is far from obvious. It is debatable whether unemployment should be kept low relative to its long-run level to take advantage of low initial levels of inflation or kept high relative to its long-run value to offset high initial values.[16] This issue will be further explored below. Second, the horizon effect appears to dominate most in the linear case and next in the quadratic case. This is related to the fact that contemporaneous increases in inflation in response to declines in unemployment are penalized relatively most heavily in the exponential case followed by the quadratic and

[15] See Table 2, column (A) for the historical values of unemployment and inflation. It can be seen that the historical values violate the criteria set forth. Namely, the unemployment rate is below 4 percent for much of the period. However, it is believed by many that the economy was pushed too hard in the mid-to-late 1960's, so that the criteria are in fact reasonable.

[16] This relates to the issue of whether the criterion that unemployment be above 4 percent is arbitrary.

TABLE 2

HISTORICAL AND MAXIMIZATION RESULTS FOR A "NORMAL" PERIOD

	u				p			
	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(d)
1965.1	4.9	5.0	5.0	4.7	2.1	2.1	2.1	2.2
.2	4.7	5.0	4.9	4.4	1.9	1.7	1.7	2.1
.3	4.4	5.1	4.8	4.3	1.1	.6	.8	1.2
.4	4.1	5.4	4.8	4.3	1.8	1.0	1.3	1.7
1966.1	3.9	5.6	4.8	4.3	3.1	2.0	2.4	2.9
.2	3.8	5.7	4.9	4.5	4.0	2.6	3.1	3.4
.3	3.8	5.6	4.8	4.4	3.6	2.1	2.6	3.0
.4	3.7	5.4	4.6	4.3	3.0	1.5	2.0	2.5
1967.1	3.8	5.3	4.6	4.0	2.8	1.4	2.0	2.3
.2	3.8	5.3	4.5	3.9	2.2	.7	1.4	2.2
.3	3.8	5.2	4.4	4.6	4.0	2.4	3.1	3.2
.4	3.9	5.3	4.6	4.4	4.6	2.9	3.7	3.7
1968.1	3.7	5.1	4.3	3.7	3.6	1.6	2.4	3.4
.2	3.5	4.8	4.1	3.7	3.8	1.7	2.6	3.4
.3	3.5	4.5	4.0	3.9	4.3	2.1	3.0	3.4
.4	3.4	4.0	3.8	3.6	4.5	2.4	3.2	3.7
1969.1	3.4	3.7	3.7	3.7	4.1	2.0	2.8	3.6
.2	3.4	3.4	3.6	4.2	5.5	3.7	4.2	4.2
.3	3.6	3.3	3.7	4.3	5.9	4.3	4.7	4.5
.4	3.6	3.0	3.5	3.7	5.4	4.0	4.2	4.4

column (a) = historical values
objective function underlying column (b) = $\Sigma(-u-p)$
objective function underlying column (c) = $\Sigma[-(u \geq 0)^2 - (p \geq 0)^2]$
objective function underlying column (d) = $\Sigma(-e^u - e^p)$

lastly linear. Thus, the net gain from decreasing unemployment at the end of the period is relatively lowest in the exponential case.

An approach to specification of the objective function that takes explicit cognizance of differences in the levels of the initial inflation rate and unemployment is to alter the implicit weights on the unemployment and inflation (nonlinear) terms to reflect long-run values.[17] For exponential and quadratic functions this could appear as:[18]

(9) $$W = \Sigma[-e^{(u-\bar{u})} - e^{(p-\bar{p})}]$$

(10) $$W = \Sigma[-e^{u/\bar{u}} - e^{p/\bar{p}}]$$

(11) $$W = \sum \left[-\frac{1}{\bar{u}} * 2 - \frac{1}{\bar{p}} * p2 \right]$$

[17] It should be remembered that the following specifications of the objective function would have yielded unacceptable results in the 1971–75 period.

[18] The introduction of $\bar{u}$ and $\bar{p}$ in equations (9) and (11) serves to shift the curves. In equation (10) it acts to rotate the curves.

where $\bar{u}$ = long run unemployment rate (natural rate)

$\bar{p}$ = long-run inflation rate

The relative weights between unemployment and inflation equal unity at the point $(\bar{u}, \bar{p})$ in equations (9) and (11). With the long-run Phillips curve estimated to be vertical, any level of inflation is model consistent with the natural rate of unemployment in the long-run, given the appropriate monetary policy. Therefore, by setting $\bar{u}$ at the natural rate, the decision whether to live with or attempt to change the effects of initial conditions can be resolved by setting $\bar{p}$ high or low relative to initial conditions.[19] For instance, setting $\bar{p}$ high relative to initial conditions would permit unemployment to be pushed down fairly far, as long as inflation does not rise excessively relative to $\bar{p}$, and vice versa. Weak dichotomy is not preserved, however, in this objective function, but the extent to which unemployment can decline without excessively exacerbating inflation can be left to the interaction between the objective function and model constraints and is not arbitrarily limited by a truncation point.

The results of maximizing equations (9), (10), and (11) are given in Table 3, columns (a) through (c), respectively. They indicate that inflation, relative to a long run value, becomes more important than unempolyment, relative to its long-run value, as one moves from equation (9) to equation (11). This can be explained by the observation that the second derivatives become smaller, in absolute value, as one moves from equation (9) to (11). Thus, it is less costly to increase unemployment as one moves from equation (9) to (11).[20]

In the extreme, the initial values of inflation could be considered the only level of inflation worth maintaining.[21] In this case, first differences of inflation or deviations from the value of inflation (or some average) just prior to the period under control could be penalized. The following two functions were maximized to illustrate this case:

$$W = \Sigma[-u^2 - 10{,}000*(p - p_{-1})^2] \tag{12}$$

$$W = \Sigma[-u^2 - 10*(p - p^*)^2] \tag{13}$$

where p^* = 1.725, the value of inflation in 1964:4

[19] If the long-run Phillips curve is not vertical, then after the desired p is chosen, $\bar{u}$ should be selected that is consistent with $\bar{p}$ according to the long-run Phillips curve.

[20] It should be noted that the optimal policy would be more stimulative if the weight between short-run and long-run inflation was smaller than that between short-run and long-run unemployment relative to the values implicit in these problems.

[21] This position could be taken by those who feel it is irrelevant to consider a long-run desired value of inflation because in the long-run the economy is homogenous of degree zero with respect to the rate of change in prices given a natural rate of unemployment. However, short-run variability of inflation may be costly. Moreover, the underlying spirit of this position should be taken as the desire to see as small a variance of the inflation rate as possible. This position implies that weak dichotomy is not necessary for specifying an acceptable objective function.

TABLE 3

RESULTS OF MAXIMIZING OBJECTIVE FUNCTIONS THAT TAKE EXPLICIT COGNIZANCE OF INITIAL CONDITIONS

	u			p		
	(a)	(b)	(c)	(a)	(b)	(c)
1965.1	5.0	5.0	4.0	2.1	2.1	2.4
.2	5.0	5.1	4.5	1.7	1.7	2.6
.3	5.1	5.3	5.8	.6	.6	.06
.4	5.4	5.7	6.2	1.0	.9	.7
1966.1	5.7	6.2	6.5	2.0	1.8	1.7
.2	6.2	6.8	6.8	2.5	2.2	2.3
.3	5.7	6.9	7.1	1.8	1.4	.7
.4	5.6	6.6	7.1	1.8	1.0	.7
1967.1	5.5	6.3	7.3	.7	.7	.7
.2	5.2	6.6	7.6	1.1	.5	−.2
.3	6.2	7.7	7.7	2.0	1.3	1.4
.4	6.1	7.7	7.9	2.0	1.3	1.9
1968.1	5.2	7.0	8.0	1.5	.6	.3
.2	5.1	6.9	8.0	1.4	.4	.06
.3	5.3	6.8	8.1	1.5	.4	−0.1
.4	4.9	6.5	8.0	1.4	.5	.03
1969.1	5.1	6.5	8.0	1.3	−.2	−.9
.2	5.9	6.8	7.9	1.6	.8	.4
.3	5.9	6.4	7.8	1.7	.9	.8
.4	4.8	5.5	7.7	1.3	.6	−.1

objective function underlying column (a) = $\Sigma\,(-e^{(u-4.5)} - e^{p})$
objective function underlying column (b) = $\Sigma\,(-e^{u/4.5} - e^{p})$
objective function underlying column (c) = $\Sigma\,(-1/4.5\,u^2 - p^2)$

Multiplicative parameters on the inflation terms are greater than unity in absolute value in these problems because smaller values that were tried indicated solutions outside the feasible space, i.e., supply constraints were hit and the model failed to converge to a solution.[22] In other words, if insufficient weight is placed on the inflation terms a zero desired unemployment rate dominates, especially since the horizon is truncated.[23] The results are given in Table 4, columns (a) and (b), respectively. While the equations are not comparable because of the different constraints on the policy instruments between them, it is clear that equation (13) comes closer to satisfying the underlying spirit of the preferences it represents by yielding a sequence of inflation with the smaller variance. This is not especially surprising because equation (13) tends to fix the value around which it penalizes deviations more concretely than does equation (12).

[22]The upper bound on EGF had to be lowered to 65. as well, for equation (12).

[23]In equation (12), since the level of p is relatively unimportant, with small multiplicative weight on the inflation terms, decreasing the unemployment at one point during the period under control could lead to a desired higher inflation, and therefore lower unemployment, at another point in the period in order to minimize first differences of inflation.

TABLE 4

RESULTS OF MAXIMIZING OBJECTIVE FUNCTIONS THAT PLACE LARGE WEIGHTS ON SMALL CHANGES IN INFLATION FROM INITIAL VALUES

	u		p	
	(a)	(b)	(a)	(b)
1965.1	5.0	5.0	2.1	2.1
.2	4.6	4.3	1.8	1.8
.3	4.2	3.9	1.3	1.7
.4	4.7	4.2	1.6	1.8
1966.1	5.3	4.9	2.1	2.6
.2	5.7	5.6	2.7	2.7
.3	5.6	5.2	2.1	2.3
.4	5.7	4.6	1.7	2.2
1967.1	6.1	4.3	.9	2.2
.2	6.5	4.6	.6	1.6
.3	7.3	5.8	1.6	2.6
.4	7.4	5.8	2.0	2.5
1968.1	7.5	4.7	.9	2.4
.2	8.0	4.5	.4	2.4
.3	8.2	4.6	.4	2.6
.4	7.8	4.3	.3	2.6
1969.1	7.7	4.6	−0.2	2.2
.2	7.9	5.3	.8	2.8
.3	7.5	5.3	1.0	2.8
.4	7.2	4.4	.7	2.7
Variance	1.61	2.85	.54	.13

objective function underlying column (a) = $\Sigma\,(-u^2 - 10{,}000\,(p - p_{-1})^2)$
objective function underlying column (b) = $\Sigma\,(-u^2 - 10^*(p - p^*)^2)$ where $p^* = 1.725$, the value of p in 1964.4.

The symmetry property of the inflation terms imply that policy should take advantage of low initial levels or downward shocks to prices and become stimulative. Consequently, these equations should yield optimal sequences of unemployment that have high variances. This is suggested by the results in Table 4 and should *a fortiori* be the case as the weight on the inflation term increases in absolute value, *ceteris paribus.*

Initial conditions need not be of concern for some functional forms. The relative weights of a truncated linear function, such as equation (5) above, for instance, are independent of initial conditions, so that weak dichotomy is maintained. Initial conditions would also affect the relative weights equally in an exponential function such as equation (4) above. It is of interest, then, to see whether the results of maximizing equations (5) and (4) over the 1965–69 period are acceptable.

These are found in Table 5, columns (a) and (b), respectively. The results of maximizing equation (5) are basically acceptable, although the truncation point may be exerting excessive influence. This is especially true for 1969. It is of interest to note that compared to the results in Table

TABLE 5

RESULTS OF MAXIMIZING OBJECTIVE FUNCTIONS WHOSE RELATIVE WEIGHTS ARE INDEPENDENT OF INITIAL CONDITIONS

	u		p	
	(a)	(b)	(a)	(b)
1965.1	5.0	5.0	2.1	2.1
.2	4.9	5.0	1.8	1.7
.3	4.8	5.0	.8	.7
.4	4.8	5.0	1.3	1.2
1966.1	4.8	5.2	2.5	2.2
.2	4.9	5.2	3.1	2.9
.3	4.9	5.2	2.6	2.5
.4	4.8	5.2	1.9	1.6
1967.1	4.8	5.1	1.8	1.5
.2	4.9	5.3	1.1	.9
.3	4.9	5.5	2.6	2.2
.4	5.0	5.0	3.2	2.9
1968.1	4.8	4.5	2.1	2.3
.2	4.7	4.8	2.0	1.8
.3	4.8	5.0	2.2	1.8
.4	4.6	4.5	2.3	2.1
1969.1	4.5	4.2	1.7	1.7
.2	4.5	3.9	3.0	3.1
.3	4.5	3.4	3.4	4.1
.4	4.5	2.8	2.8	4.2

objective function underlying column (a) = $\Sigma\,(-(u \geq 4.5) - p)$
objective function underlying column (b) = $\Sigma\, - e^{(u+p)}$

2, column (b), the optimal sequence of unemployment is much flatter. This may be indicative of a possible general feature of the constrained maximization solutions with the MPS model: when more stimulus is called for at the end of the control period, more deflationary policy is called for at the beginning of the period.[24] The results of maximizing equation (4), on the other hand, are far from satisfactory. The horizon effect appears to dominate, although why this should be so for this particular functional form is not clear.

Conclusion

The specification of the objective function is a fairly intricate endeavor. It appears that short-run constraints must be taken into account for many functional forms whose second derivatives are not zero. When a

[24]This observation is confirmed in all the results given in Tables 2 and 5. The relationship between early optimal high unemployment and later optimal low unemployment has been shown to result from the distributed lag on past rates of change of prices in the wage equation; see [3].

linear function is used, the horizon effect appears to be an important problem that must be dealt with. Truncation of the objective function is only a rough solution. Much work remains to be done for the solution of the horizon problem.

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"ON THE SPECIFICATION OF UNEMPLOYMENT AND INFLATION IN THE OBJECTIVE FUNCTION"

SOME COMMENTS

BY D. A. LIVESEY

This paper is fascinating for the issues which it raises rather than for the questions which it answers. It focuses upon the problems of specifying a suitable objective function. In particular, it concentrates upon the trade-offs between unemployment and inflation in an econometric model solved over a time horizon of five years. Specifying objective functions is a normative exercise and yet the discussion in this paper tries to be as positive as possible. This may be for one of two reasons; either economists have been implicitly taught to avoid welfare economics because of the fundamental difficulties which it brings to light; or, it could be that this work has evolved naturally out of the earlier simulation exercises which were a standard feature of econometric model building. Even if neither of these explanations is entirely correct, there is no escaping the fact that this paper does not squarely face up to the issues which it addresses. Many of the choices which the paper discusses have to be set firmly in a broader context; some examples of how this can be done are given below. At the same time, it has to be recognised that this paper signals that a significant stage has been reached in the application of optimal control theory to econometric models. At last those involved are addressing some of the difficult issues involved in formulating economic policy. They are realising that they cannot escape some of the fundamental choices simply by applying control theory. For a long time, I think some were under the impression that many of the dilemas which arise in short-term economic policy could be avoided by better planning and foresight. From this and other papers it clearly emerges that there is no unambiguous golden age to which all optimal policy is steered regardless of its implicit criterion function.

The beginning of section I is devoted to a discussion of the dichotomy between preferences and constraints. It is argued that the short time-horizon of many econometric models forces the policy maker to abandon the symmetry of the quadratic criterion. The imposition of one-sided constraints weakens the dichotomy between preferences and constraints. Whilst attempting to answer the criticism of the quadratic criterion, Palash does not positively state why in a perfect world he would expect preferences to be quadratic. It may be that he has in mind to argue that a quadratic criterion is the correct one for regulation problems. Whilst this would be a satisfactory argument, it would not solve all the problems. We would still need to know by which process the desired target path, about which the optimal controller is to regulate the economy, is chosen. Palash opts for the obvious targets of zero unemployment and zero inflation. It is reasonable to ask whether these targets are feasible given the constraints implied by the econometric model. If they are not, then the outcome of any policy making exercise is going to crucially depend upon the relative weights given to the deviations from target of unemployment and inflation. Since, in the first instance, the process of assigning these weights can only be arbitrary, the policy making exercise is going to involve many computational runs the outcomes of which will be judged qualitatively by the policy maker. The difficulties arise because the policy maker is concerned with two problems. Firstly, he is concerned with establishing the feasible region within which his policy options lie. Secondly, and workers in this field may soon be forced to admit that this is the minor of the two problems, the policy maker wishes to regulate the behaviour of the economy and steer it towards the desired feasible path. In this context, the trade-offs between unemployment and inflation seem to be more a question of feasibility than a matter of regulation. Given the present state of the art, econometric models have equations which describe what we might loosely term the Phillips Curve. The task of the policy optimiser is in this context to spell out the feasible region implied by these equations.

At the end of section I Palash discusses the relationship between the timing of inflationary shocks and the length of the time horizon. Everything which he says is perfectly correct but he does not discuss the crucial role which terminal constraints have to play in optimal control theory. From optimal growth theory we know that terminal constraints are required in finite horizon problems to compensate for the myopic nature of the plan. The specification of suitable terminal constraints have proved to be no easier, for those who have attempted it, than the specification of a suitable objective function.

At the beginning of section II, we come to the heart of the matter. The sixth sentence reads: "First, if the optimal solution calls for extreme worsening on the part of one target which does not lead to substantial improvements on the part of the other target, then that objective function which yields this result will be considered undesirable." What Palash is saying at this point is that he has a criterion function by which he judges the results of optimal control exercises and that this criterion function is not the objective function with respect to which the model's solutions are optimal. Clearly, if the solutions are optimal, then the improvement in one target, even if it is not substantial, which has been bought at the expense of a worsening of the other target must, by the standards of the objective function, be a better outcome. In rejecting the results, what Palash is telling us is that his implicit criterion function differs from the objective function specified. This is perfectly reasonable if one holds, as I have always maintained, that the derivation of optimal policy is an iterative procedure. Until we have seen the results of one exercise we cannot be sure how we would wish to specify the exact conditions under which the next exercise will be carried out. I would, however, have liked to have seen some recognition that an implicit criterion function was used in analyzing the results.

At this point, Palash returns to the issue of terminal constraints. If it is indeed true that the level of unemployment which was compatible in the long run with a negligible rate of inflation was somewhere between 4 and $4\frac{1}{2}$ percent in the 1960s then we would expect this result to emerge from the model. However, towards the end of the planning period the economy can sustain low levels of unemployment because it will not pay the price, in the years beyond the end of the time horizon, of inflation rising as a result of these low unemployment levels. It is a problem which has to be settled using terminal constraints. Once again, we have the problem of a blurring of the trade-off between unemployment and inflation. If 4 percent unemployment is really the rate below which the economy cannot go without generating inflation then why is it not the desired target for unemployment? Presumably, Palash would answer that we cannot be certain that 4 percent is the correct figure and that the figure would change from one planning horizon to another. The great danger with all of this is that so many implicit constraints will have been imposed by the policy maker that when we have the results we are unable to clearly distinguish between what is feasible and what we have imposed upon the solution.

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