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TOWARDS THE CONSTRUCTION OF AN OPTIMAL AGGREGATIVE MODEL OF INTERNATIONAL TRADE: WEST GERMANY, 1963–1975

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A mapping from international to domestic prices arising from Samuelson's 1953 model of international trade is fitted to German data. A method and a search procedure are described for constructing an aggregative, lower-dimensional, model for which an appropriately defined measure of aggregation bias is acceptably small, and partitions of the set of internationally traded commodities are obtained which fulfill this criterion for purposes of explaining the respective domestic price indices, one at a time. It is found that the optimal number of explanatory variables (import and export price indices) lies somewhere between 6 and 9.

I. INTRODUCTION

Almost all econometric models currently in use are macroeconomic in nature, dealing with variables such as national income, employment, the general price level, etc., and ignoring relative prices and resource allocation. The reason nations engage in trade, however, is that they differ in their relative endowments of labor, capital, and natural resources and therefore in their relative production costs in the absence of trade. One can therefore hardly ignore relative prices in a model of international trade.

It appears to have been generally overlooked that in 1953, Samuelson [23] presented a model of the relation between international and domestic prices that is very simple in structure and highly amenable to statistical treatment. Samuelson's paper is best known for its celebrated "factor price equalization theorem," to the effect that if certain stringent conditions are fulfilled, notably that all countries have identical production functions, that they all produce some amount of every traded commodity (this assumption can be relaxed somewhat), that their factor endowments are in a precise but complicated sense not too far apart,[1] or alternatively,

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[1]To be precise, the endowment vectors of the various countries should all lie in the same "diversification cone" (cf. Chipman [1, p. 24]).

that there is no "factor-intensity reversal" in a generalized sense,[2] then the rentals of factors of production will be equal across countries. Owing to the startling nature of this result, and to the obvious fact that it is not fulfilled in the real world (immigration barriers would not be needed if the result were true), attention has been diverted from the fact that the theorem is only a corollary of a much more basic proposition which does not require any of the stringent assumptions enumerated above. The basic proposition is that under the assumption of competitive markets, absence of joint production, and constant returns to scale, if there are at least as many "products" as "factors" then the rentals of the latter will depend only on the prices of the former, and in particular will be independent of factor endowments.

This proposition can be generalized still further, as shown in the next section, to state that as long as the number of internationally traded commodities also produced at home exceeds or equals the number of primary factors of production, all domestic prices will depend uniquely on international prices, independently of factor endowments and the balance of payments. In a way this result is even more startling than the factor price equalization theorem; and it cries out for statistical verification.

An immediate difficulty that presents itself at the outset is that the hypothesis that the number of traded commodities produced at home exceeds or equals the number of primary factors is not subject to empirical verification—at least directly; for the outcome depends entirely on how the products and factors are aggregated into groups. Tariff schedules of various countries distinguish at least 10,000 commodity groups, and each of these is, typically, itself a highly heterogeneous aggregate.[3] Allowing for the heterogeneity of capital, not to mention that of labor, it should not be hard to reach or exceed this number on the "factor" side. The theory itself provides no guide as to how one might recognize the number of products and of factors,[4] any more than it provides us with a guide as to the correct numerical magnitudes of the coefficients. This latter analogy provides us with the required clue: *we should let the data decide the question.*

The hypothesis that the number of traded commodities produced at

[2] The commonly accepted condition generalizing the notion of "non-reversal of factor intensity" to the case of n factors and n products is the Gale-Nikaido condition that the principal minors of the Jacobian matrix of the transformation from product prices to factor rentals be all positive. Cf. Chipman [1, p. 30].

[3] The German 7-digit code [11] covers 8678 commodity groups. The tariff schedules [6] require a further expansion of this list to a 9-digit code.

[4] One interesting criterion setting a lower limit on the number of "products" is that the subdivision should not be so fine as to seriously violate the assumption that there is no joint production. This criterion is recognized by compilers of industrial classification systems, e.g., those of the N.A.C.E. system who remark [20, p. 6]: "The finer the degree of subdivision the less chance there is of finding homogeneous units at the level chosen."

home exceeds or equals the number of primary factors can in principle be tested *indirectly* by reason of the fact that it implies that domestic product prices and factor rentals depend on prices of imports and exports alone, independently of factor endowments and of the balance of payments on current account. However, our logical conundrum remains: in carrying out such a test, should "labor" be treated as a single homogeneous aggregate, or as a set of "specific factors" immobile among industries? The treatment of capital and natural resources presents even greater difficulties, particularly that of finding reliable data. It seems not unreasonable, therefore, to proceed on the hypothesis that the number of traded commodities produced at home exceeds or equals the number of primary factors, and to try and construct an optimal aggregative model on the assumption that this hypothesis is true: having at that stage settled on the optimal (on the hypothesis) number of groups of products and factors and on their composition (these numbers satisfying the analogous hypothetical inequality in terms of the groups of traded commodities produced at home and of primary factors), one could then proceed with the required statistical test. Should we at that point decide to reject the hypothesized model as an unsatisfactory representation of reality, we will at least not have rejected it because we chose unsuitable modes of aggregation;[5] and we may have learned something along the way.

My approach in this paper will thus be to adopt the hypothesis that there exists some way of partitioning products and factors into groups of aggregate products and aggregate factors such that the number of aggregate products produced at home exceeds or equals the number of aggregate factors, and such that the aggregative model is a good approximation of reality. The object of the statistical analysis is then to decide, on the basis of the data, on the optimal number of aggregative groups of products and factors, on the optimal way of partitioning the given sets of products and factors into the required number of aggregative groups, and on the numerical magnitudes of the parameters — all simultaneously.

Owing, however, to some mathematical programming, statistical, and econometric problems that remain as yet unresolved, it has been possible in this paper to carry out the above objectives only to a partial extent. The first difficulty that must be faced is that whatever criterion is chosen for measuring aggregation bias, the number of partitions of k objects into k^* groups is, for any realistic values of k, so huge that computation of the optimal mode of aggregation is quite out of the question. This difficulty is inherent in the aggregation problem, and cannot be avoided. We must be content with finding modes of aggregation for which the aggregation bias

[5] See, for example, Johnson's [17] argument that the "Leontief paradox" can be reconciled with the Heckscher-Ohlin model if skilled labor is aggregated together with physical capital rather than with unskilled labor.

is acceptably small, and which therefore stand a good chance of being close to the optimal partition. The criterion of acceptability chosen in section 3 below is that the mean square error of the blown-up aggregative parameter estimator should be significantly less (at a prescribed significance level) than that of the corresponding direct least squares estimator. This leads directly to the second difficulty, however, which is that the appropriate test statistic has (under the null hypothesis) a non-central Hotelling T_0^2 distribution, the percentage points of which are as yet unknown except for the case of a single dependent variable. Precise results are therefore obtained only for the more limited problem of finding an optimal partition of the exogenous variables (the international prices) into aggregative price indices for purposes of forecasting a single endogenous variable (a domestic price or aggregate price index).

The third difficulty that is skirted here but which must eventually be faced is that it is impossible to separate the problem of commodity aggregation from that of temporal aggregation, since the data reflect dynamic adjustment processes rather than equilibrium states. The decision concerning the appropriate lag structure and that concerning the appropriate commodity classification system should properly be made simultaneously. It goes without saying that the analysis of commodity aggregation presented here reflects some inevitable and unknown degree of distortion resulting from sub-optimal temporal aggregation.[6]

2. The Generalized Stolper-Samuelson Mapping

My starting-point is Samuelson's [23] model of production and trade, in which prices of commodities produced in positive amounts just cover unit costs, which in turn depend only on the input prices. I shall assume that there are four categories of commodities, and that over the period of time studied, commodities do not switch categories. Let p^1, p^2, and p^3 be row vectors of prices of k_1 internationally traded (exported or imported) products also produced domestically, k_2 imported products not produced domestically, and k_3 products produced domestically but not traded internationally, and let p^4 be a $1 \times k_4$ row vector of rentals of k_4 primary factors of production (including industry-specific immobile factors). Then we have

$$
\begin{aligned}
p^1 &= p^1A_{11} + p^2A_{21} + p^3A_{31} + p^4A_{41} \\
p^2 &< p^1A_{12} + p^2A_{22} + p^3A_{32} + p^4A_{42} \qquad (1) \\
p^3 &= p^1A_{13} + p^2A_{23} + p^3A_{33} + p^4A_{43}.
\end{aligned}
$$

[6] For a discussion of the temporal aggregation problem, see Zellner & Montmarquette [27].

where the A_{ji} are $k_j \times k_i$ input-output matrices (which will in general depend on the prices p^1, p^2, p^3, p^4); the strict inequality in (1) is required to ensure that a slight perturbation does not result in commodities switching categories. In this paper I shall confine the empirical analysis to the case of the Leontief technology in which the input-output matrices A_{ji} are fixed.[7]

The equations in (1) may be written in the form

$$(2)\qquad (p^1,p^2)\begin{bmatrix} I - A_{11} & A_{13} \\ -A_{21} & A_{23} \end{bmatrix} = (p^3,p^4)\begin{bmatrix} A_{31} & I - A_{33} \\ A_{41} & -A_{43} \end{bmatrix}.$$

For a given set of observable values of the external prices (p^1,p^2), the solution (p^3,p^4) of (2) (which exists by assumption) will in general not be unique, but will depend on factor endowments and the balance of payments on current account.[8] Formally, we may write the set of solutions of (2) as

(3)

$$(p^3,p^4) = (p^1,p^2)\begin{bmatrix} I - A_{11} & A_{13} \\ -A_{21} & A_{23} \end{bmatrix}\begin{bmatrix} A_{31} & I - A_{33} \\ A_{41} & -A_{43} \end{bmatrix}^{-}$$

$$+ z\left(\begin{bmatrix} I_{k_3} & 0 \\ 0 & I_{k_4} \end{bmatrix} - \begin{bmatrix} A_{31} & I - A_{33} \\ A_{41} & -A_{43} \end{bmatrix}\begin{bmatrix} A_{31} & I - A_{33} \\ A_{41} & -A_{43} \end{bmatrix}^{-}\right),$$

where z is an arbitrary $1 \times (k_3 + k_4)$ row vector and M^- denotes any generalized inverse of M.[9] If the $(k_3 + k_4) \times (k_1 + k_3)$ matrix on the right side of (2) has rank $k_3 + k_4$ (which implies that $k_1 \geqq k_4$, i.e., that the number of traded products produced at home exceeds or equals the number of primary factors), then the arbitrary term in (3) will vanish and the solution of (2) will be unique. This is the essential mathematical result

[7] Removal of this limitation would be one of the many steps one could take to improve upon the methods presented in this paper. Non-linear regression methods have recently been employed by Hudson & Jorgenson [16] to estimate variable input-output matrices in a model which, like the present one, exploits a "non-substitution theorem" arising out of Samuelson's work.

[8] The situation may be visualized as follows, for the case $k_1 = 2$, $k_2 = 0$, $k_3 = 1$, and $k_4 = 2$. The country's production possibility frontier will be a ruled surface (its shape depending on the factor endowments): if, starting from balanced trade, the country goes into deficit, it must shift resources out of the export and import-competing industries into production of the domestic, non-traded good in order to satisfy the increased demand for the latter (assuming it to be a superior good). This can be accomplished by moving along the linear segment of the ruled surface to which the price plane is tangent. If $k_4 = 3$, however, the production possibility surface will (except for singular cases) be strictly concave to the origin, and the price of the domestic good will necessarily rise as the deficit increases, to an extent depending on factor endowments and consumer preferences. Assuming consumer tastes to remain stable, the domestic price will then depend on factor endowments and the size of the deficit in addition to the international prices.

[9] I.e., any matrix M^- such that $MM^-M = M$. Cf. Chipman [3].

underlying Samuelson's factor-price equalization theorem.[10] Since empirical matrices can be expected always to have full rank, the condition $k_1 \geq k_4$ is also sufficient (with probability 1) for uniqueness of the solution of (2). Under these conditions, and assuming also that the matrix $I - A_{33}$ satisfies the Hawkins-Simon conditions [13] (i.e., has positive principal minors), it may be verified that we can express this solution as

(4)
$$(p^3, p^4) = (p^1, p^2)\begin{bmatrix} [A_{13} + (I - \bar{A}_{11})\bar{A}_{41}^R A_{43}](I - A_{33})^{-1} & (I - \bar{A}_{11})\bar{A}_{41}^R \\ (A_{23} - \bar{A}_{21}\bar{A}_{41}^R A_{43})(I - A_{33})^{-1} & -\bar{A}_{21}\bar{A}_{41}^R \end{bmatrix},$$

where

$$\bar{A}_{i1} = A_{i1} + A_{i3}(I - A_{33})^{-1}A_{31} \quad (i = 1, 2, 4),$$

and where $\bar{A}_{41}^R$ denotes any right inverse of $\bar{A}_{41}$.

Since (4) is a generalization of the relation first investigated by Stolper and Samuelson [24] between international prices and domestic factor rentals for the 2-product, 2-factor case, it may be described as the "generalized Stolper-Samuelson mapping." Our main object in the next section will be to obtain quantitative estimates of the parameters of this mapping, or rather of a simplified aggregative variant of it, by means of a regression model in which the external prices (p^1, p^2) are treated as exogenous variables and the internal prices (p^3, p^4) as endogenous variables. Some care must be taken, however, in order to justify the treatment of external prices as exogenous. In the first place, it must be noted that, in general, it is not logically possible for the external prices (p^1, p^2) in (4) to be exogenous except under certain conditions or within certain well-defined limits. For, the assumed existence of a solution to (2), together with the assumption that the matrix on the right in (2) has rank $k_3 + k_4$, implies that the vector on the left side of (2) must be contained in the $(k_3 + k_4)$-dimensional space spanned by the rows of the matrix on the right. If $k_1 + k_2 > k_3 + k_4$, this means that the price vectors (p^1, p^2) must themselves be so confined, and their exogenous variation assumed to be limited to a subspace of dimensionality $k_3 + k_4$.[11] If $k_1 + k_2 \leq$

[10] In general, with input-output coefficients depending upon input prices, the uniqueness need only be local in the absence of further conditions (see footnote 2 above). This means that the mapping (4) may be different for different countries even if production functions are identical as between countries. But this does not concern us here, since we are interested only in the mapping for a particular country.

[11] Taking account of the dependence of input-output coefficients on input prices, the domain of the mapping (4) will be not a linear subspace but a submanifold of dimension $k_3 + k_4$. The idea of constrained or conditional exogeneity can also be explained in the following way. Let $k_1 + k_2 > k_3 + k_4$, and suppose that $r < k_3 + k_4$ of the international prices are unconditionally exogenous; suppose further that the country's imports and exports are functions of the international prices, and that some of the latter prices are in turn influenced by the country's imports and exports. How many of them must be so in-

$k_3 + k_4$, then since also $k_3 + k_4 \leqq k_1 + k_3$, we require $k_3 - k_2 \geqq k_1 - k_4 \geqq 0$. In the conventional case on which Samuelson [23] concentrated attention in which $k_2 = k_3 = 0$, this requires $k_1 = k_4$ (equal numbers of products and factors); it is apparent that the present, more general, formulation is substantially less restrictive than this.

There remains the empirical (as opposed to logical) question as to whether the international prices (p^1, p^2) may properly be specified as exogenous, given the above conditions and limitations. There are at least three possible grounds on which one might question this hypothesis in the case of West Germany. (1) It could be argued that Germany's share in world trade is large enough so that autonomous shifts in its imports and exports (or at least some of the latter) can be expected to exert a substantial influence on world prices. While this is no doubt true, the real issue is whether there have been significant autonomous shifts and whether they have been of importance relative to externally induced effects. For example, although Germany is an important exporter of coal, one could hardly argue that the 60% increase in the price index of its coal exports from October 1973 to October 1975 was a consequence of its own influence in the international coal market rather than of the rise in petroleum prices.[12] (2) One might argue that cost-push inflation induced by union pressure would lead to a devaluation of the mark and thus to a rise in the international prices (which are denominated in marks), so that the causation would be the reverse of that assumed. However, the value of the deutsche mark (in terms of U.S. dollars) rose quite steadily throughout most of the period, by over 50% in fact. (3) Domestic prices of those agricultural products subject to the variable levy under the European Community's common agricultural policy are insulated against changes in international prices. This cannot be denied; unfortunately, however, at the level of aggregation employed in the present study it was not possible to separate

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fluenced? The answer is: exactly $k_3 + k_4 - r$. This is not a fortuitous result, but is simply a logical requirement of the assumptions made. If we knew the magnitude of r and could identify the r unconditionally exogenous international prices, we could employ them as our exogenous variables. But we do not have such knowledge a priori.

[12]This does not mean that the domestic repercussions of the rise in oil prices could not have had a further influence on coal prices. Indeed, this was probably the case. The volume of Germany's coal exports was 30% higher (and of industrial production of coal, 10% higher) in the first quarter of 1974 than it had been in the third quarter of 1973, while the coal export and domestic prices had risen by 4% and 9%, respectively; by the third quarter of 1974 the coal export and domestic prices had risen by 30% and 28%, respectively. Thereafter the volume of coal exports dropped off sharply and continued declining until the third quarter of 1975, reflecting increasing domestic demand; and coal prices continued to rise significantly. The sharp rise in the volume of coal exports in 1974 could well have contributed to the sluggishness of the initial rise in price, and the sharp drop in the volume of coal exports in 1975 to the subsequent acceleration in price. Nevertheless, there is no question that the driving force was the petroleum price. The outcome would have been substantially the same if Germany had not been a significant coal exporter.

out the variable-levy commodities, which constituted about 40% of the value of agricultural imports from the U.S. (cf. Preeg [21, p. 33]).[13]

3. Construction of an Optimal Aggregative Model

On the basis of the mapping (4) we postulate the multivariate multiple regression model[14]

$$(5) \qquad Y = XB + E, \quad \mathcal{E}E = 0, \quad \mathcal{E}(\text{row } E)'(\text{row } E) = I_n \otimes \Sigma$$

where Y is an $n \times m$ matrix whose rows are consecutive observations of the $m = k_3 + k_4$ internal prices (p^3, p^4), X is an $n \times k$ matrix whose rows are consecutive observations of the $k = k_1 + k_2$ external prices (p^1, p^2), B is the $k \times m$ matrix of the generalized Stolper-Samuelson mapping (4), and E is an $n \times m$ matrix of random errors. Of course, (4) represents a theoretical relationship among *equilibrium* prices, and we have no formal dynamic theory of the adjustment process to describe the prices we can actually expect to observe. It was nevertheless decided, in order to concentrate on the commodity aggregation problem, to fit the model (5) directly; monthly data were averaged to quarterly data in the belief that this would minimize the specification error introduced by the neglect of lags.[15]

[13] Such a separation is planned in a more disaggregative study currently in progress.

[14] Here, "row E" denotes the row vector of rows of E, and $\otimes$ denotes the Kronecker product. "$\mathcal{E}$" denotes the expectation operator, assumed conditional on X.

[15] It should be noted first of all that, as is suggested by the discussion in footnote 12 above, information concerning quantities of imports and exports, and domestic production and consumption, could not very well be ignored in any satisfactory explanation of the dynamic adjustment process, even though these variables are absent from the equilibrium relationships among the prices. Even supposing, however, that they could be excluded as a first approximation, the introduction of lagged prices would of course not remove temporal specification error if the "true" dynamic process is considered to be a continuous-time one (cf. Telser [25]): at best it might reduce it. But the main practical and methodological difficulty involved in allowing for lags is that they use up degrees of freedom, making it necessary to increase the amount of aggregation over commodities; that is, we cannot reduce temporal aggregation error without aggravating the problem of aggregation over commodities. Moreover, it is clear that in deciding upon the optimal mode of aggregation over commodities, similarity in lag patterns would have to be taken into account in addition to the kinds of considerations (structural similarities and multicollinearity) that are involved in the static case (cf. Chipman [2], [3]). In short, a theory of optimal aggregation in dynamic models still needs to be developed.

In a study of price relationships in the case of the Japanese economy, involving monthly data on nine aggregative international price indices, Ho [15] found the average lag lengths to vary from two to six months, and to be concentrated around three and four months. Undoubtedly, such lags account for some of the anomalies found in the empirical results reported below; in particular, the high explanatory power of mining and quarrying import and export prices on domestic fuel prices found in some of the regressions may be ascribed to the fact that both import and export coal prices and domestic fuel prices lagged behind petroleum import prices.

Genuine price indices (as opposed to unit value indices) for imported and exported products are available for very few countries.[16] For West Germany, monthly data are available back to January 1958, in terms of three Laspeyres series with bases 1958, 1962, and 1970, for some 200 commodity groups [9, Reihe 1]. At the time the study to be reported here was carried out, data had been acquired going back to 1963 for 37 import and 37 export categories, furnishing 52 quarterly observations; these were aggregated into 12 categories of each as indicated in Table 1a, furnishing $k = 24$ exogenous variables.[17] Monthly consumer and wholesale price indices, and quarterly wage and salary indices, were employed as indicated in Tables 1b–1d;[18] these furnished a total of $m = 39$ endogenous variables.

The export price indices are f.o.b., and the import price indices c.i.f., exclusive of tariffs. Tariff-inclusive import price indices are available only for certain basic materials.[19] Customs duties as a percentage of total import value are shown in Table 2, indicating a marked decline from 1963 to 1964 from 7.16% to 4.27%, followed by a gradual decline to 1.68% in 1975. The general import price index rose by 54.6% over the same period, with some substantial fluctuations including the four-fold increase in petroleum prices in 1973–74. The neglect of tariffs, while unfortunate, is probably not too serious, especially since non-tariff barriers have not been taken into account.

Let us now formulate the problem of constructing an optimal aggregative model. Let us assume that the price indices have been multiplied by a single set of weights (the more complicated problem of dealing with

[16]Namely, West Germany, Sweden, Finland, Japan, and South Korea (cf. Rostin [22, p. 393]). A U.S. series of gradually increasing coverage has been issued by the Bureau of Labor Statistics since 1974, the most recent accounting for 15% and 54% of the value of U.S. merchandise imports and exports respectively (mainly manufactures).

[17]This preliminary aggregation doubtless introduced some distortion in the subsequent analysis, and in retrospect could have been improved upon in some ways. Of course, the original price index data are not free from such defects, being themselves aggregates.

[18]Cf. [9], Reihe 6, 8, and 15 respectively. Since the study reported here was completed, a breakdown of the wage and salary data into industries roughly comparable in classification to that of the international prices has been obtained, as have producers' price indices for industrial and agricultural products (Reihe 3 and 4) which employ the same method of classification as the international price series. Annual profit data by industrial categories are also available (cf. [7]). A more comprehensive study employing these series, back to 1958, is currently under way.

[19][9], Reihe 2. Annual tariff revenue data are published in the annual supplements of [8], Reihe 2 (and unpublished monthly data have recently been acquired), classified according to the 2-digit Brussels Nomenclature, which forms neither a finer nor a coarser partition than the industrial commodity classification system [10] used for the price index series. Work is currently under way to convert the one to the other by regression and other approximation methods, with the help of import statistics classified by the two methods (in [8], Reihe 2, 1. 7), as well as of average tariff rates computed according to the industrial classification system from tariff schedules by Hiemenz and Rabenau [5, pp. 23–6], [14] separately for the European Community and other countries, for the years 1958, 1964, 1970, and 1972.

TABLE 1a

CLASSIFICATION OF INTERNATIONAL COMMODITIES ENTERING GERMAN TRADE, AGGREGATED FROM CLASSIFICATION ACCORDING TO INTERRELATIONS OF PRODUCTION, TOGETHER WITH WEIGHTS PER 1000 IN IMPORT AND EXPORT PRICE INDEX RESPECTIVELY

Classification	Import Weight	Export Weight
1. Agricultural, forestry, & fishery raw produce	143.88	13.96
Agricultural	135.30	13.10
Forestry	5.68	0.50
Fishery	2.90	0.36
2. Mining & quarrying	50.13	29.98
Coal mining products	6.25	20.34
Iron ore, non-ferrous metal ores, pyrites	29.50	
Potash & salts		2.06
Quarrying products	14.38	7.58
3. Petroleum & petroleum products, & other mining products	88.26	10.51
Petroleum, gas, & bituminous rock	62.97	
Other mining products, incl. peat		1.15
Petroleum products	25.29	9.36
4. Iron, steel and their products	86.66	135.13
Iron & steel	56.29	59.64
Foundry products	1.69	2.96
Products of drawing plants, cold rolling mills, & steel shaping	8.68	20.99
Products of structural engineering	3.87	8.50
Iron, steel, sheet & metal goods	16.13	43.04
5. Non-ferrous metals (incl. precious metals) & semi-finishes	79.09	22.74
6. Machinery[a]	81.77	217.64
Products of mechanical engineering (incl. locomotives & agricultural tractors)	62.78	198.50
Office machines and data processing equipment	18.99	19.14
7. Road vehicles (excl. agr. tractors & electr. driven vehicles)	46.98	150.01
8. Electrical & precision goods, etc.	75.29	130.37
Electrical goods	55.22	98.90
Precision & optical goods	11.04	23.38
Musical instruments, toys, athletic goods, jewelery, etc.	9.03	8.09
9. Chemical products	78.22	143.31
10. Wood, glass, plastics & rubber products	83.94	70.18
Fine ceramics	3.22	6.99
Glass & glass products	5.93	7.35
Sawn wood, plywood, other worked wood	13.79	3.30
Wood products	6.17	8.21
Wood pulp, cellulose, paper	28.82	6.67
Paper & paperboard	3.06	4.27
Printed products	3.65	7.50
Plastics products	9.21	15.74
Rubber & asbestos products	10.09	10.15
11. Leather & textile products	98.28	50.73
Leather	3.89	2.07
Leather products & footwear	9.93	3.94
Textiles	62.92	35.88
Clothing	21.54	8.84
12. Foodstuffs, beverages, & tobacco	87.52	25.44
Foodstuffs & beverages, etc.	87.13	24.63
Tobacco products	0.39	0.81

[a]The two groups in this category were combined in the price index statistics in the 1958- and 1962-based series, and first broken apart in the 1970-based series.

TABLE 1b

CONSUMER PRICE INDEX FOR 4-PERSON MIDDLE-INCOME EMPLOYEE HOUSEHOLDS IN GERMANY, WITH WEIGHTS PER THOUSAND

Classification	Weight
1. Food, beverages & tobacco (incl. restaurants)	439.83
2. Clothing & footwear	119.98
3. Lodging	93.63
4. Electricity, gas, and fuel	45.85
5. Other household goods & services	109.78
6. Transportation and communication	61.98
7. Personal care & health	30.97
8. Education & entertainment	62.97
9. Personal effects, other goods & services	35.01

TABLE 1c

WHOLESALE PRICE INDEX, GERMANY, WITH WEIGHTS PER THOUSAND

Classification	Weight
1. Grain, seeds, fertilizers & live animals	105.8
2. Textile raw materials, intermediate products, hides & skins	5.9
3. Chemicals for technical use, drugs	9.6
4. Coal, other solid fuels, & petroleum products	208.8
5. Iron, steel, non-ferrous metals & their semi-manufactures	110.8
6. Wood, wood products, & building & plumbing materials	88.8
7. Scrap and other waste products	19.5
8. Foodstuffs, beverages & tobacco	217.0
9. Clothing & footwear	33.9
10. Hardware, art materials, etc.	39.2
11. Electrical, precision & optical products, jewelery, etc.	19.4
12. Transportation equipment, machinery	78.8
13. Miscellaneous supplies	11.2
14. Pharmaceutical, cosmetic, dental, and medicinal articles	36.6
15. Paper & paper products, printed products, school & office supplies	14.7

TABLE 1d

WAGES AND SALARIES BY INDUSTRY, GERMANY, WITH WEIGHTS (PER 1000) IN WAGE AND SALARY INDEX RESPECTIVELY

Classification	Wage Weight	Salary Weight
1. Mining	49.07	22.47
2. Electricity, gas, and water supply	17.33	25.13
3. Primary & producers' goods industries	188.98	129.71
4. Investment goods industries	351.14	285.54
5. Consumer goods industries	195.74	88.17
6. Food, beverage, & tobacco industries	51.59	39.36
7. Construction industries	146.15	50.17
8. Trade, credit institutions, & insurance		359.45

TABLE 2

AVERAGE PERCENTAGE TARIFF RATE ON W. GERMAN IMPORTS

1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975
7.16	4.27	3.95	3.82	3.49	3.03	2.89	2.79	2.52	2.45	2.29	1.78	1.68

chain indices will not be taken up here), so that aggregation takes the form of addition. Let G and H be $k \times k^*$ and $m \times m^*$ *grouping matrices* having at most one unit element in each row and the remaining elements zero, G being a *proper* grouping matrix with exactly one unit element in each row, where $k^* < k$ and $m^* \leq m$. Define $X^* = XG$ and $Y^* = YH$. The aggregative model then takes the form

$$(6)\quad Y^* = X^*B^* + E^*, \quad \mathcal{E}^*E^* = 0, \quad \mathcal{E}^*(\text{row } E^*)'(\text{row } E^*) = I_n \otimes \Sigma^*,$$

where $\mathcal{E}^*$ is the assumed but not "true" expectation operator. The discrepancy between the "true" and "false" expectations of Y^* being $\mathcal{E}Y^* - \mathcal{E}^*Y^* = X(BH - GB^*)$, the aggregation bias may be defined as an appropriate "distance" between the transformations BH and GB^*. In [2] I have argued in favor of the Mahalanobis distance, so that we may define the distance of any $k \times m^*$ matrix C from BH to be

$$(7)\qquad d(BH, C) = m^{*-1}\,\mathrm{tr}(BH - C)'X'X(BH - C)(H'\Sigma H)^{-1}.$$

We define $d(BH, GB^*)$ to be the *aggregation bias* associated with the model (6). It was proved in [3, p. 668] that for given G and H, this aggregation bias attains a minimum with respect to B^* when $B^* = G^{\#}BH$, where $G^{\#} = (G'X'XG)^{-}G'X'X = (X^{*\prime}X^*)^{-}X^{*\prime}X$.[20] The *minimum aggregation bias* is then

$$(8)\quad \lambda(G, H) \equiv \inf_{B^*} d(BH, GB^*) = m^{*-1}\,\mathrm{tr}\, H'B'X'X(I - GG^{\#})BH.$$

The problem of *optimal aggregation* is then that of selecting G and H out of a certain class of pairs of matrices (G, H) so as to minimize (8).

The problem as just posed is, however, quite intractable. Suppose H and k^* are fixed. Then a proper grouping matrix G, considered as a set of columns without regard to their order, completely defines a partition of k elements into k^* subsets. Now, there are a total of

$$\frac{1}{k^*}\sum_{i=0}^{k^*}(-1)^i\binom{k^*}{i}(k^* - i)^k$$

[20]See footnote 9. For empirical matrices X we can expect $X^* = XG$ to have full rank k^*, so that $G^{\#} = (G'X'XG)^{-1}G'X'X$. $G^{\#}$ is called in [3] a "generalized quasi-inverse" of G

such partitions (cf. Chipman [2, p. 151]). Taking, for example, $k = 24$ and $k^* = 9$, the number of partitions is approximately 1.206×10^{17}. There would be no hope of finding the optimal partition even if B were completely known. We must be content to accept a partition for which the minimum aggregation bias is acceptably small. And, of course, we must be content with *estimates* of B and B^*.

In the case at hand, our 52×24 matrix X of quarterly observations of international prices, while quite ill-conditioned,[21] is of full rank, as is every $X^* = XG$ that arises. The least-squares estimators of B and B^* are then given uniquely by $\tilde{B} = (X'X)^{-1}X'Y$ and $\tilde{B}^* = (X^{*\prime}X^*)^{-1}X^{*\prime}Y^*$. It was proved in [4, Theorem 1] that

$$(9) \qquad \mathcal{E}d(BH, G\tilde{B}^*) \leqq \mathcal{E}d(BH, \tilde{B}H) \Longleftrightarrow \lambda(G, H) \leqq k - k^*.$$

In words: the "blown-up" aggregative least squares estimator $G\tilde{B}^*$ of BH has lower mean square error than the direct least squares estimator $\tilde{B}H$ if and only if the aggregation bias associated with G and H is less than the reduction of dimensionality from the original to the aggregative model. The latter hypothesis can in principle be tested, by means of the statistic

$$(10) \qquad v = m^{*-1}\,\mathrm{tr}[(H'S^*H - H'SH)(H'SH)^{-1}],$$

where $S = Y'[I - X(X'X)^{-1}X']Y$, $S^* = Y'[I - X^*(X^{*\prime}X^*)^{-1}X^*]Y$. When the residuals in (5) are normally distributed, $m^*(n - k)v$ has (under the null hypothesis) a non-central Hotelling T_0^2 distribution. Calculation of its percentage points remains a difficult task: I therefore have concentrated in the present study on the case $m^* = m = 1$ and $H = 1$, in which $(n - k)v/(k - k^*)$ has (under the null hypothesis $\lambda(G, 1) = k - k^*$) a non-central F distribution with $k - k^*$ and $n - k$ degrees of freedom and non-centrality parameter $k - k^*$. If the hypothesis $\lambda(G, 1) \leqq 24 - k^*$ is not rejected (say at the 5% level), G is considered to be acceptable.

It remains to find a search procedure to discover partitions that have a good chance of passing the aforementioned test. To this end, Marquardt's [19] estimation procedure has been applied.

Letting $X = Q\Gamma P'$ be the singular value decomposition (cf. [12]) of X, where $Q'Q = P'P = I_k$ and Γ is the $k \times k$ diagonal matrix of singular values of X (in descending order), we define Γ_r to be the diagonal matrix obtained from Γ by replacing its $k - r$ smallest diagonal elements by zeros, and $\Gamma_r^\dagger$ to be the generalized inverse of Γ_r (positive diagonal ele-

[21] The condition number $\kappa(X)$ of X, defined as the ratio of the largest to the smallest non-zero singular value of X (cf. [12]), is in the present case $203.9296/.0612 = 3332.18$. For all variables other than petroleum and non-ferrous metal import prices, when X is rescaled so that $X'X$ is a correlation matrix, 17 of the 23 correlation coefficients exceed .99.

ments replaced by reciprocals, zero elements left unchanged). The matrix $X_r = Q\Gamma_r P'$ is then the best approximation of X by an $n \times k$ matrix of rank r.[22] Its generalized inverse is $X_r^\dagger = P\Gamma_r^\dagger Q'$, and Marquardt's estimator of B in (5), of rank r, is

$$\tilde{B}_r = X_r^\dagger Y. \tag{11}$$

Marquardt has shown [19, p. 601] that if certain a priori bounds are placed on the elements of B, (11) will have lower mean square error than the least squares estimator $\hat{B} = \tilde{B}_k$.[23]

These estimates were calculated for $r = 24, 23, \ldots, 1$. In Figure 1 is shown the "rank chart" corresponding to the fourth column of $\tilde{B}_r$, containing the estimated regression coefficients (measured on the vertical axis) for the dependent variable CPI 4 (consumer price index component for electricity, gas, and fuel), for each value of r (measured on the horizontal axis). The chart has a remarkable feature (which it has in common with those for most of the remaining dependent variables): for $24 \geqq r > 16$, the regression coefficients oscillate wildly, with implausibly large magnitudes in absolute value, but they become quite stable for $16 \geqq r > 3$. In the stable regions, the estimates provided visual clusterings which were used for trial grouping matrices G. The same process was repeated with X replaced by the original price index series (and the estimates scaled back to the weighted form), and again with X normalized to have columns of unit length.[24]

With the trial G matrices so obtained, the hypothesis test of (9) was carried out for each dependent variable separately (the case $m^* = m = 1$). The results are summarized in Table 3. At $k^* = 12$, 71.03% of the trial modes of aggregation passed the test; but the Marquardt estimates were judged to have somewhat implausible economic values. For $k^* = 4$, only 0.85% of the trial modes passed the test. It was concluded that the optimal k^* was somewhere in the region $9 \geqq k^* \geqq 6$. While this argues in favor

[22] In the sense of minimizing the Frobenius norm

$$\|X - X_r\|_F = \mathrm{tr}[(X - X_r)'(X - X_r)]^{1/2} \quad \text{(cf. [12]).}$$

[23] This method also furnishes a means of effecting the requirement of constrained exogeneity described in section 2 and footnote 11 above, the discrepancy between X and X_r being attributed to errors of measurement, or rounding, or of dynamic adjustment in X, or to the approximation of an r-dimensional submanifold by an r-dimensional linear subspace.

[24] If X_0, Y_0 are the data matrices with the original price indices, and W_X, W_Y the diagonal matrices of weights, then $X = X_0 W_X$, $Y = Y_0 W_Y$. The estimate (11) converted to "elasticity form" is $W_X \tilde{B}_r W_Y^{-1}$. The Marquardt estimate in original form, scaled back to weighted form, is $W_X^{-1}(X_0)_r^\dagger Y_0 W_Y = W_X^{-1}(X W_X^{-1})_r^\dagger Y$. Likewise, normalization entails replacing X_0 by $X_N = X_0 N^{-1}$ where N is the diagonal matrix whose diagonal elements are the square roots of the diagonal elements of $X_0' X_0$. For an analysis of the sensitivity of $X_r^\dagger$ to such scale transformations see Wedin [26].

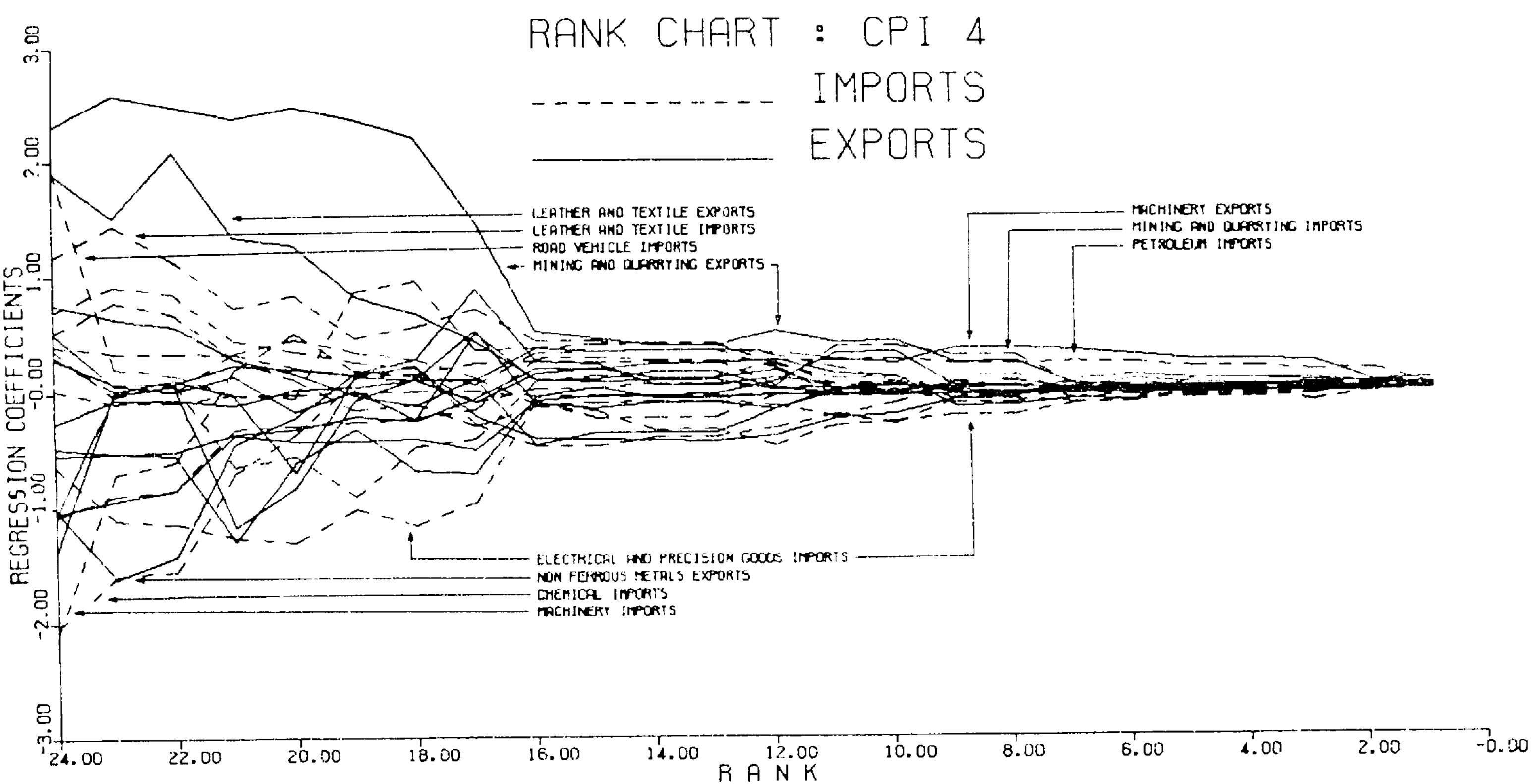
RANK CHART : CPI 4
IMPORTS
EXPORTS
LEATHER AND TEXTILE EXPORTS
LEATHER AND TEXTILE IMPORTS
ROAD VEHICLE IMPORTS
MINING AND QUARRYING EXPORTS
MACHINERY EXPORTS
MINING AND QUARRYING IMPORTS
PETROLEUM IMPORTS
ELECTRICAL AND PRECISION GOODS IMPORTS
NON FERROUS METALS EXPORTS
CHEMICAL IMPORTS
MACHINERY IMPORTS
REGRESSION COEFFICIENTS
3.00
2.00
1.00
-0.00
-1.00
-2.00
-3.00
24.00
22.00
20.00
18.00
16.00
14.00
12.00
10.00
8.00
6.00
4.00
2.00
-0.00
RANK

Figure 1.

TABLE 3

CRITICAL UPPER 5% POINTS FOR TESTING WHETHER BLOWN-UP AGGREGATIVE ESTIMATORS HAVE LOWER MEAN SQUARE ERROR THAN LEAST SQUARES ESTIMATORS, AND SOME SUMMARY RESULTS

$k^* = k - p = 24 - p$	12	11	10	9	8	7	6	5	4
$p = k - k^* = 24 - k^*$	12	13	14	15	16	17	18	19	20
Critical .05 point	1.74	1.86	1.98	2.09	2.20	2.33	2.46	2.57	2.70
Highest t observed	6.602			12.314	16.838		51.383		72.464
Category	WPI 2			WPI 15	WPI 2		WAG 7		SAL 6
Lowest t observed	0.116			0.608	0.584		0.793		1.819
Category	WAG 2			SAL 1	WPI 7		WAG 2		WPI 10
Percentage of observed values below critical point	71.03			36.26	41.67		16.60		0.85
Number of modes of aggregation tested	10			7	12		36		3

of simple models, it suggests that the 2-dimensional models that have been the favorite of trade theorists are not adequate to represent reality.[25]

In Table 4 are displayed the three sets of Marquardt rank 9 estimates for the dependent variable CPI 4 (i.e., the fourth column of the 24×39 matrix $\tilde{B}_9$), scaled to weighted form. Three clusterings were tried on the basis of the estimates in column 1, and the corresponding blown-up aggregative estimates are displayed in columns 4–6; the clustering can be inferred from the values of the coefficients. Columns 7 and 8 display the blown-up aggregative estimates derived from clusterings suggested by the estimates of columns 2 and 3. Table 5 shows the same results, but with the regression coefficients rescaled to the form corresponding to the case in which the observations are the original price indices; this is called the "elasticity form," and is useful for checking the economic plausibility of the magnitudes. The clustering can be inferred from the common values of the t-ratios within groups. For example, we read from column 7 that in partition 2, import prices of mining and quarrying and of petroleum have been aggregated together, and the estimate implies that a ten per cent rise in petroleum import prices will lead (roughly) to a 2.5 per cent rise in the domestic consumer price index component for electricity, gas, and fuel.

From Tables 4 and 5 we read that partition 2 had the lowest aggregation bias for prediction of CPI 4, as measured by the t-statistic (namely 1.312). On the other hand, for purposes of predicting CPI 1 (price of

[25] Cf. Jones & Scheinkman [18], who defend the traditional 2×2 model by showing that certain of its propositions, when appropriately reformulated or weakened, carry over to higher-dimensional cases provided all commodities are produced and traded and that their number does not exceed the number of factors (the opposite of the situation being considered here). Note that even if this is true it does not follow that a 2-dimensional model can adequately represent a higher-dimensional situation.

TABLE 4

RANK 9 MARQUARDT ESTIMATES, AND RELATED BLOWN-UP AGGREGATIVE ESTIMATES, EXPRESSED IN WEIGHTED FORM

(Critical upper 5% point for test for aggregation bias: 2.09.)

	Int'l Price	Marquardt Rank 9 Estimates: 1 Weighted Data	2 Original Data	3 Normalized Data	Blown-Up Aggregative Estimates: 4 Partition 1a	5 Partition 1b	6 Partition 1c	7 Partition 2	8 Partition 3	Least Squares: 9 OLS Estimate
IMPORTS	1 AGR	−.056	.053	.002	−.029	−.085	−.021	.071	.090	.035
	2 M&Q	.299	.291	.338	.284	.482	.195	.261	−.038	−1.076
	3 PETR	.235	.286	.106	.260	245	.195	.261	.146	.405
	4 I&S	−.213	−.172	−.147	−.511	−.502	−.432	−.346	−.721	.506
	5 NFM	.083	−.025	.029	.052	.073	.125	.071	.090	.694
	6 MACH	−.098	−.027	−.033	.108	.153	.197	.071	.090	−2.083
	7 RD V	−.029	.174	.061	.042	.025	.065	.095	.146	1.296
	8 ELEC	−.014	−.122	−.105	.042	.025	.065	−.346	.555	−.639
	9 CHEM	−.074	−.276	−.244	−.092	−.085	−.021	−.060	.138	−2.031
	10 WD.GL	.122	−.109	.009	−.101	−.316	−.940	−.346	.090	.375
	11 L.TEX	.006	.016	−.002	.042	.025	.065	.071	.090	1.167
	12 FOOD	.066	−.007	−.015	.052	.073	.125	.071	.090	.318
EXPORTS	1 AGR	−.022	−.406	.244	.042	.025	.065	−.566	−.088	−1.072
	2 M&Q	.242	.783	1.003	.260	.482	1.535	1.230	1.047	2.297
	3 PETR	.025	−.488	.777	.052	.073	.125	−.745	−.170	−.493
	4 I&S	−.125	.031	.008	.108	.153	.197	.071	.090	−.546
	5 NFM	.029	.373	.312	.052	.073	.125	.177	−.038	−1.024
	6 MACH	.363	.149	.090	.305	.253	.074	.095	.146	−.256
	7 RD V	−.005	.040	.015	.042	.025	.065	.071	.090	.759
	8 ELEC	−.067	.098	.076	−.092	−.085	−.021	.095	.146	.304
	9 CHEM	.075	−.004	.034	.052	.073	.125	.071	.090	.514
	10 WD.GL	.042	.094	.202	.052	.073	.125	.095	−.088	−1.045
	11 L.TEX	−.029	.001	−.076	.042	.025	.065	.071	.090	1.902
	12 FOOD	.053	.401	.189	.052	.073	.125	.247	−.088	−1.382
	z(CPI 4)				2.156	2.024	1.464	1.312	1.588	
	z(all CPI's)				3.449	3.493	4.028	3.727	3.861	

foodstuffs), partition 1c would have been the best of these. The bottom row of each table furnishes the value of the z-statistic for the case in which $H = [I_9, 0, 0]$; this provides a general-purpose measure of aggregation bias for the 9 CPI components simultaneously, and the best partition on this criterion is partition 1a, with $z = 3.449$. Partition 1c turned out to be the best of these for prediction of wages and salaries, i.e., for the case $H = [0, 0, I_{15}]$.

The estimates obtained furnished a number of interesting results along with some puzzles. Among the interesting results it was found that, almost without exception, the regression coefficients for wages and salaries had, for each import or export price, the same sign for all industries and comparable magnitudes when expressed in weighted form. While no

TABLE 5

MARQUARDT RANK 9 AND BLOWN-UP AGGREGATIVE ESTIMATES, EXPRESSED IN ELASTICITY FORM. PARTITIONS IDENTIFIED BY IDENTICAL t-RATIOS IN PARENTHESES

	Int'l Price	Marquardt Estimates 1 Weighted Data	2 Original Data	3 Normalized Data	Blown-up Aggregative Estimates 4 Partition 1a	5 Partition 1b	6 Partition 1c	7 Partition 2	8 Partition 3
IMPORTS	1 AGR	−.088 (−0.50)	.083 (2.75)	.002 (0.11)	.046 (−1.32)	−.134 (−1.27)	−.033 (−0.35)	.111 (2.59)	.141 (2.49)
	2 M&Q	.163 (3.15)	.159 (3.34)	.337 (2.72)	.158 (1.60)	.264 (3.59)	.107 (7.32)	.143 (5.38)	−.021 (−0.24)
	3 PETR	.226 (6.62)	.275 (5.13)	.106 (4.13)	.250 (9.46)	.236 (8.75)	.188 (7.32)	.251 (5.38)	.141 (5.10)
	4 I&S	−.201 (−4.45)	−.163 (−4.66)	−.148 (−3.31)	−.483 (−2.30)	.474 (−2.33)	−.408 (−2.34)	−.327 (−3.52)	−.682 (−4.27)
	5 NFM	.072 (2.37)	−.022 (−1.07)	.029 (1.40)	.045 (1.63)	.063 (2.20)	.108 (4.08)	.061 (2.59)	.078 (2.49)
	6 MACH	.087 (−2.26)	−.024 (−0.88)	−.033 (−0.93)	.096 (0.80)	.136 (1.16)	.176 (1.77)	.063 (2.59)	.080 (2.49)
	7 RD V	.015 (−0.89)	.089 (2.97)	.061 (1.32)	.021 (0.78)	.013 (0.50)	.033 (1.56)	.049 (3.35)	.075 (5.10)
	8 ELEC	−.011 (−0.28)	−.100 (−5.60)	−.105 (−4.12)	.034 (0.78)	.021 (0.50)	.054 (1.56)	−.284 (−3.52)	−.456 (−3.69)
	9 CHEM	.063 (−3.40)	−.235 (−8.66)	−.244 (−7.85)	.079 (−1.32)	.073 (−1.27)	−.018 (−0.35)	.052 (−0.23)	.117 (0.60)
	10 WD, GL	.112 (1.57)	−.100 (−3.59)	.009 (0.80)	−.092 (−0.44)	.289 (−1.26)	.860 (−3.60)	−.317 (−3.52)	.082 (2.49)
	11 L, TEX	.006 (0.07)	.017 (0.90)	−.002 (−0.11)	.044 (0.78)	.027 (0.50)	.070 (1.56)	.076 (2.59)	.096 (2.49)
	12 FOOD	.063 (0.62)	−.007 (−0.27)	−.015 (−0.55)	.050 (1.63)	.069 (2.20)	.119 (4.08)	.067 (2.59)	.086 (2.49)
EXPORTS	1 AGR	.003 (−1.50)	−.062 (−0.95)	.244 (0.53)	.006 (0.78)	.004 (0.50)	.010 (1.56)	.086 (1.31)	−.013 (−0.75)
	2 M&Q	.080 (2.93)	.256 (5.07)	1.003 (9.34)	.085 (9.46)	.158 (3.59)	.502 (5.42)	.402 (6.11)	.342 (6.04)
	3 PETR	.003 (0.97)	.056 (−1.14)	.777 (2.98)	.006 (1.63)	.008 (2.20)	.014 (4.08)	−.085 (−1.95)	−.019 (−0.41)
	4 I&S	.180 (2.34)	.045 (1.12)	.008 (0.30)	.159 (0.80)	.225 (1.16)	.290 (1.77)	.104 (2.59)	.133 (2.49)
	5 NFM	.007 (1.74)	.093 (6.78)	.312 (5.58)	.013 (1.63)	.018 (2.20)	.031 (4.08)	.044 (1.23)	−.009 (−0.24)
	6 MACH	.862 (8.76)	.355 (8.49)	.090 (15.54)	.725 (9.53)	.600 (5.79)	.175 (1.29)	.225 (3.35)	.347 (5.10)
	7 RD V	.007 (−0.06)	.066 (3.83)	.015 (3.44)	.068 (0.78)	.041 (0.50)	.107 (1.56)	.115 (2.59)	.147 (2.49)
	8 ELEC	.096 (0.93)	.140 (5.48)	.076 (5.42)	.131 (−1.32)	−.121 (1.27)	−.030 (−0.35)	.135 (3.35)	.208 (5.10)
	9 CHEM	.118 (0.81)	.012 (0.71)	.034 (4.37)	.082 (1.63)	.113 (2.20)	.195 (4.08)	.110 (2.59)	.141 (2.49)
	10 WD, GL	.032 (0.55)	.072 (2.18)	.202 (5.42)	.040 (1.63)	.056 (2.20)	.095 (4.08)	.073 (3.35)	−.067 (−0.75)
	11 L, TEX	.016 (−0.87)	.001 (0.05)	−.706 (−3.09)	.023 (0.78)	.014 (0.50)	.036 (1.56)	.039 (2.59)	.050 (2.49)
	12 FOOD	.014 (2.05)	.111 (3.07)	.189 (1.36)	.014 (1.63)	.020 (2.20)	.035 (4.08)	.069 (2.01)	−.024 (0.75)
	Σ(CPI 4)				2.156	2.024	1.464	1.312	1.588
	Σ(all 9 CPIs)				3.449	3.493	4.028	3.727	3.867

formal test was applied, the result could be interpreted as justifying the treatment of "labor" as a single factor, mobile among industries. Of course, other interpretations would be possible, e.g., that union policies to equalize wage rates among industries keep wage and salary movements in line across industries; but a deeper analysis of such union policies might reveal that they are not unrelated to potential labor mobility. Another interesting result (combined with a puzzle) was that the elasticities of wages and salaries with respect to international prices were positive for nearly all export prices (the exceptions being categories 7, 11, and 12), and negative for nearly all import prices (the exceptions being categories 3, 4, and 12). The result suggests that there is something to the common practice of treating exports and imports as natural categories for purposes of aggregation; but it also suggests that there are very important exceptions to this rule. One might be tempted to interpret the general result as providing evidence of a "Leontief paradox" for West Germany, though independent calculations with returns to capital (as well as more detailed dynamic analysis) would be needed to confirm this as well as to determine whether one can justify treatment of "capital" as an aggregate factor.

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