Volume Title: Annals of Economic and Social Measurement, Volume 6, number 1

Volume Author/Editor: NBER

Volume Publisher:

Volume URL: http://www.nber.org/books/aesm77-1

Publication Date: 1977

Chapter Title: On Numerical Computation in Simultaneous Equation Systems

Chapter Author: Warren T. Dent

Chapter URL: http://www.nber.org/chapters/c10505

Chapter pages in book: (p. 123 - 125)

Annals of Economic and Social Measurement, 6/1, 1977

NOTES

ON NUMERICAL COMPUTATION IN SIMULTANEOUS EQUATION SYSTEMS

BY WARREN T. DENT

The introduction of sophisticated numerical analysis techniques into parameter estimation in econometric simultaneous equation systems was originally made by Dent and Golub (1972) at the Sixth Interface Symposium on Computer Science and Statistics, Berkeley, 1972, and in a consequent widely distributed Stanford report (1973). This foundation work involved the application of numerically stable and accuracy preserving Householder transformations and the Singular Value Decomposition to the determination of the Limited Information Maximum Likelihood (LIML) estimates of the parameters in a single equation of a simultaneous economic system. Recently, Belsley (1974) has incorporated these concepts into the construction of a comprehensive computerized package (NBER-GREMLIN) of accuracy-preserving estimation procedures for linear and nonlinear econometric systems. We take this opportunity to clarify some misunderstandings and misinterpretations that have arisen in this work, and to summarize some econometric insights available with the techniques.

In particular Belsley suggests (p. 564, eqn (1.12)) that the "LIML estimator is calculated as a k-class estimator with k equal to the minimum eigenvalue of the eigensystem

$$\left|(Y'Y)_{\perp x_1} - (Y'Y)_{\perp x}\right| = 0."$$

The correct definition involves the determinantal equation (in Belsley's notation)

$$\left|(W'W)_{\perp x_1} - (W'W)_{\perp x}\right| = 0$$

where $W = [Y\,y]$. This system has $G + 1$ eigenroots, not G.

As it stands the Householder transformation series applied in this case is inappropriate, as are equations (1.15) p. 565), and the comments (1.16) et seq. The necessary corrections to Belsley's procedure can be de-

**Editor's Note:* Belsley notes that Equation (1.12) *et seq.* can be made correct by a simple expedient. In (1.4) define $R^*_{33} = \begin{bmatrix} R_{33} & R_{34} \\ 0 & R_{44} \end{bmatrix}$, $R^*_{13} = [R_{13}\ R_{14}]$ and $R^*_{23} = [R_{23}\ R_{24}]$. Now in (1.12) *et seq.* read W in place of Y, and R^*_{ij} in place of R_{ij} $i,j = 3,4$. The resulting equations are correct and make sense.

rived.* Dent and Golub's algorithm for LIML estimation is criticized (note 13, p. 568) on the basis that it requires storage of the large orthogonal Householder transformation matrix. This is not so however, and the criticism is therefore unfounded. Dent and Golub's algorithm applies the Singular Value Decomposition to the correct Householder transformation, and takes notice of rank problems associated with the matrix $(W'W)_{\perp X}$. The insight gained here is important, as is knowledge of the relationship between the LIML and Two Stage Least Squares estimators in over-identified cases. Determination of all eigenvalues of the system above (and not just the smallest) allows one to apply Fisk's (1967) tests of identification for the equation in question, a process of considerable, but neglected, merit.

Econometric insight from application of numerical analysis techniques has also been gained in three other areas. The first concerns uncorrelated residuals in single equation models. Grossman and Styan (1972) show how application of Householder transformations in the sense described by Belsley (p. 558) readily yields uncorrelated residuals. These latter also may be conveniently used to prove in more classical contexts the existence of χ^2 distributions for estimators of disturbance variance under normality assumptions. Numerical and analytic properties of uncorrelated residuals are further advanced in Grossman and Styan (1972), Styan (1972) and Dent and Styan (1973).

Second, the use of Householder techniques in determination of linearly estimable functions was developed by Golub and Styan (1973) and expanded by Dent (1973) and Dent and Foreschle (1973). Convenient g_{123} inverses are utilized to develop elegant and practical linearly estimable functions in the present of multicollinearity.

The third further instance of analytical insight suggested by applications of Singular Value and Householder transformation decompositions is in the broader k-class estimation. One is not guaranteed that Belsley's $G + K_1$ system (1.10) is of full rank and that the corresponding estimators exist. Dent shows (1975, eqns. (15) and (16)) that by applying partitioned inversion, a smaller system of order only $G \times G$ need be examined. The rank of this system may be checked by the Singular Value Decomposition, and indeed this process determines "inadmissable" values of k for the data in question. It is shown analytically from this latter decomposition that inadmissable k values are always greater than unity, giving impetus to the use of values less than unity, such as those prescribed by Fisk (p. 50) and Zellner (1975). The algorithm yields insight, but there is no intent here to imply that savings in time or computation are simultaneously possible.

The procedure for Three Stage Least Squares estimation outlined by Belsley (pp. 580–585) is superior to that of Dent (1975) in that smaller computer storage areas are required. The issue of large systems has also recently been taken up by Jennings (1974) and a package for standard estimation techniques is currently under implementation at the University of Illinois (Chicago Circle) and The University of Iowa. Special attention in this development is given to the case of non-full rank and the options of estimates of complete covariance matrices or diagonal elements only.

University of Iowa

References

Belsley, David A. (1974): "Estimation of Systems of Simultaneous Equations, and Computational Specifications of GREMLIN." *Ann. Ec. Soc. Meas. 3*, 4 pp. 551 564, October.

Dent, Warren T. (1973): "Information in Less Than Full Rank Regression Models." Working Paper Series No. 73-12. College of Business Administration, The University of Iowa, May.

Dent, Warren T. (1973): "Efficient Computation in Simultaneous Equations Estimation," Working Paper Series No. 73-13. College of Business Administration, The University of Iowa, May.

Dent, Warren T. (1976): "Information and Computation in Simultaneous Equations Estimation." *Journal of Econometrics*, Vol. 4, pp. 89 95.

Dent, Warren T., and James R. Foreschle (1973): "Best Linear Unbiased Estimates of Linearly Estimable Functions." Working Paper Series No. 73-20. College of Business Administration, The University of Iowa, July.

Dent, Warren T., and Gene H. Golub M 1972): "Computation of the Limited Information Maximum Likelihood Estimator." Proceedings of INTERFACE: Computer Science and Statistics, 6th Annual Symposium, Berkeley, October.

Dent, Warren T., and Gene H. Golub (1973): "Computation of the Limited Information Maximum Likelihood Estimator." STAN-CS-73-339. Computer Science Department, Stanford University, February.

Dent, Warren T., and George P. H. Styan (1973): "Uncorrelated Residuals from Linear Models." Technical Report 88, Institute for Mathematical Studies in the Social Sciences, Stanford University, January.

Fish, R. (1967): *Stochastically Dependent Equations*, Hafner, New York.

Grossman, Stanley I. and George P. H. Styan (1972): "Optimality Properties of Theil's BLUS Residuals." *Journal of the American Statistical Association, 67*, No. 339, pp. 672, 673, September.

Golub, Gene H. and George P. H. Styan (1973): "Numerical Computations and Univariate Linear Models." *Journal of Statistical Computation and Simulation, 3*.

Jennings, Les S. (1974): "A Computational Approach to Simultaneous Estimation." STAN-CS-74-454. Computer Science Department, Stanford University, September.

Styan, George P. H. (1973): "Properties of Discrepancy Covariance Matrices for Uncorrelated Residuals." *Proc. First Canadian Conference Applied Statist.*, August.

Zellner, Arnold (1975): "Estimation of Functions of Population Means and Regression Coefficients including Structural Coefficients: A Minimum Expected Loss (MELO) Approach." Unpublished manuscript, University of Chicago, June.