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THE MEASUREMENT OF CONCENTRATED INDUSTRIAL STRUCTURE AND THE SIZE DISTRIBUTION OF FIRMS

BY JOHN C. HAUSE

The Herfindahl (H) concentration index is theoretically appropriate for studying price-cost margins of industries in Cournot-Nash equilibrium. This paper argues that the H index provides a lower bound for reasonable concentration measures, since the C-N equilibrium ignores possible large-firm collusion dependent on the size distribution. Strong restrictions on theoretically acceptable concentration indices are deduced. Two new indices satisfying these restrictions are proposed, and basic defects of current measures are identified. Swedish and Japanese data yield correlations between the widely used 4-firm concentration ratio, C_4, and H ranging from .13 to .79 for highly concentrated industries. This demonstrates C_4 is a poor proxy for theoretically superior measures in the empirical study of highly concentrated industries, despite widespread belief to the contrary.

I. INTRODUCTION

Two central assumptions of the competitive market model are (1) a *large number* of (2) *independently acting* producers and (buyers). These assumptions assure that no discrepancy between price and marginal receipts is perceived by individual producers in their attempt to maximize profits, and leads to the equality of price and marginal cost. The markets of the real world differ greatly in the number of firms and their relative size. American manufacturing includes such industries as primary aluminum with seven firms and a four-firm concentration ratio of .96 and wood furniture (not upholstered), with 2,927 firms and a concentration ratio of .11.[1] One of the ongoing embarrassments of economic theory (and there are several) is the absence of a persuasive model that links the number of firms and their relative size with the expected degree of competition in an industry. Nature and students of industrial organization (and others) abhor a theoretical vacuum, and a plethora of thoroughly *ad hoc* indices have been proposed for measuring industrial concentration.

The "degree of competition in an industry" has no widely accepted

*I am indebted to George Stigler's (1968) stimulating article, "The Measurement of Concentration," for my initial interest in this problem. Herbert Mohring's substantive and stylistic comments have been very beneficial. The generous cooperation of Gunnar Du Rietz, who painstakingly assembled the industrial data for his own studies, and of Lars Wohlin, director of the Industriens Utredningsinstitut in Stockholm, made possible the empirical results reported in Section IV of the paper. Edward Fagerlund efficiently carried out the main calculations. Reactions from seminar presentations of the paper at Texas A&M, Stanford, UCLA, and the University of Minnesota materially improved the study. Proprietary responsibility for any errors resides with the author.

[1]See U. S. Senate, Committee on the Judiciary, Subcommittee on Antitrust and Monopoly Report, *Concentration Ratios in Manufacturing Industry: 1963, Part I*, Washington, D. C., 1966.

meaning in the economics literature. It is defined in this study as a measure determined by persistent discrepancy between price, p, and marginal costs, MC, in an industry. Consider first a monopolized industry. The Lerner index, defined by the ratio $(p - MC)/p$, has been widely (but not universally) accepted by economists as a useful quantitative characterization of monopoly power. The index is sometimes defined as the ratio of monopoly rent, $(p - MC)q$, to the monopolist's total receipts, pq. The equivalence of the two definitions is apparent since the monopolist's output, q, cancel out in the latter ratio. For a single-price monopolist, it is well-known that profit maximization by the monopolist leads to a value of $-1/\eta$ for the Lerner index, where η is the elasticity of demand for the product.

The second definition of the Lerner index is readily extended to industries containing n firms by the definition $L = [\sum_{i=1}^{n} (p - MC_i)q_i]/[\sum_{i=1}^{n} pq_i]$, i.e., by the ratio of the sum of the monopoly rents to all firms in the industry to total industry receipts.[2] This definition assumes a homogeneous (single-product) industry, and a single price, p, at which the product is sold. In this equation, q_i and MC_i are the output and marginal cost, respectively, of the i^{th} firm.

An index of industrial concentration is a function of the size distribution of firms in an industry; the purpose of the index is to explain (in conjunction with other relevant variables characterizing an industry) an important dimension of industry performance.[3] A concentration index should be considered "theoretically reasonable" only if a plausible theoretical link has been established between the functional form of the index and the way in which it determines the industry performance characteristic of interest. This study considers only the degree of competition in an industry, as measured by the L index. The reader should bear this in mind in interpreting statements about "theoretically reasonable measures of concentration" in the sequel. In principle, a theoretically reasonable concentration index may depend on the specific measure of industry performance under study, although it seems plausible that such indices of concentration are highly correlated with each other.

Section II presents notation and some general considerations useful for the analysis of size distributions and industrial concentration. Section III argues that the theory of Cournot-Nash (C-N) equilibria provides strong restrictions on the class of theoretically acceptable measures of in-

[2]Equivalently, L can be defined as the weighted average of the individual firm ratio of price less marginal cost to price, where the weights are the firm shares of industry output. But the definition suggested in the text for L is related more transparently to the empirical price-cost margin literature.

[3]For a concentration index to make any sense in this context, the size distribution must correspond to some sort of industry equilibrium. If firm shares are essentially random and demonstrate extreme and rapid fluctuations in magnitude, it is difficult to see why a concentration index would be related to any phenomenon of interest to economists.

dustrial concentration. The argument establishes that the Herfindahl index (H) (sum of the squared shares of firms in an industry) is related in an extremely simple way to the Lerner index at the C-N equilibrium, and provides a useful *lower bound* on reasonable industrial concentration measures, since it ignores cooperation between the firms, based on their perceived interdependence. Several new concentration indices are proposed, based on criteria for reasonable measures that follow from this line of argument. Section IV presents cross-sectional correlations between several industrial concentration measures, along with intertemporal correlations of the indices for samples of Swedish and Japanese manufacturing industries. These calculations contradict the widespread belief that concentration indices are so highly correlated that they are likely to be indistinguishable in empirical studies. Previous work suggests that industrial concentration is most likely to be associated with departures from competition primarily at fairly high levels of concentration. Yet it is primarily at high levels of concentration that the measures differ most from each other. The results show that the correlation between the widely used four-firm concentration ratio (C_4) and the Herfindahl index (H) is only .614 when the sample is restricted to Swedish industries with H greater than .16.[4] Thus the study of the effects of industrial concentration on significant dimensions of industry performance may be substantially influenced by the specific concentration measure used, when attention is focussed on "highly concentrated" industries. This important conclusion is discussed in more detail in the following paragraph. The empirical results also suggest that H is likely to be a substantially better measure of concentration than C_4 for determining competitiveness of an industry. Finally, Appendix A discusses the main concentration measures that have been introduced into the literature, including C_4 and the entropy index, and the serious way in which they violate the theoretical criteria is demonstrated.[5] Appendix B describes briefly the concentration data and presents the various indices for 45 Swedish manufacturing industries.

Because of the important implications of this paper for empirical studies of industrial concentration, it is desirable to be more explicit about the current professional folklore. There seems to be a belief among some economists that further attempts to refine concentration measures are

[4]The appropriateness of the Herfindahl index for restricting the industry sample will become apparent in Section III. Here one simply notes that all industries with a C_4 of .8 or greater are included by this criterion. All industries in which the largest firm has a share greater than .4, where the largest two firms have a total share greater than .566, or where the largest three firms have a total share greater than .693 are also included by it.

[5]This study is concerned primarily with the measurement of economically relevant industrial concentration from the firm size distribution. See G. J. Stigler (1968, pages 29–38), for a broader critique of the defects of current measures including the arbitrary time period of one year generally used for measuring the firm size distribution. For other reviews of concentration measures, see G. Rosenbluth (1955), G. J. Stigler (1955), C. Marfels (1971), P. Hart (1971). See T. R. Saving (1970) for an earlier theoretical discussion.

likely to be an academic exercise, in the pejorative sense.[6] This belief is based on two sorts of evidence. First, a number of studies have reported very high correlations between alternative measures. Rosenbluth (1955) calculated Spearman rank correlation coefficients ranging from .979 to .981, between the three-firm concentration ratio, H, and the number of firms required to produce 80 percent of industry output, for 96 Canadian manufacturing industries. Scherer (1970) found a simple correlation coefficient of .936 between C_4 and H for a sample of 91 four-digit United States census industries, using data assembled by Ralph Nelson. Bailey and Boyle (1971) tabulated simple correlation coefficients ranging from .96 to .98 between C_4 and estimated H indices for all 417 census industries in 1963. Second, studies relating industrial concentration (usually measured by C_4) and measured profits or price behavior have generally reported a rather weak empirical association.[7] Given these findings of very high correlations between concentration indices, but much lower (simple and partial) correlations between the indices and price-cost differentials or profits, it might seem unlikely that any empirical tests will establish much justification for choosing between concentration measures. Hence, one might conclude, one should rely on the widely available C_4 measures without purist apologies for their theoretical demerits in carrying out empirical studies.

There are several reasons for rejecting this conclusion. Some empirical evidence suggests that C_4 has little association with profits until C_4 is fairly high, say .50 or .80.[8] But it is precisely in this range of high concentration levels that substantial differences in the relative size of the

[6]Scherer (1970, page 52) states, "Fortunately, the chances of making a grievous analytic error in the choice of a market structure measure are slender, for the principal concentration indicators all display similar patterns. For most interindustry comparison purposes, then, it is senseless to spend sleepless nights worrying about choosing the right concentration measure." D. Bailey and S. Boyle (1971, page 705) write, "Approximately the same results are obtained for both the Herfindahl Index and the simple concentration ratio ... Although researchers will seldom know the actual firm distribution in each industry, the results indicate that this information would generally be irrelevant."

[7]G. J. Stigler (1968, pages 33 34) comments, "A considerable history could be written on the search for high correlations between these concentration ratios and (purported measures of competitive performance). The main finding has been disappointment: seldom have good relationships been found between the concentration ratio and these potential indexes of monopoly." L. Weiss (1971, page 369) after reviewing some 32 studies relating to concentration and profits, concludes, "The typical result of concentration-profits studies, especially those based on firms, has been a significant but fairly weak positive relationship."

[8]See G. J. Stigler (1964, page 59), "In general the data suggest that there is no relationship between profitability and concentration if H is less than 0.250 or the share of the four largest firms is less than about 80 percent." See L. Weiss (1971, pages 371 372) for a brief review on whether there is a "critical level of concentration" that leads to significantly stronger association of profits and concentration levels. L. Telser (1972, page 322) finds that four-firm concentration levels exceeding .5 have larger slopes when trying to determine the effect of concentration on profits.

largest firms are most likely to affect the degree of industry competitiveness. Disclosure rules govern most official statistics on industry concentration, and crude estimates of alternative concentration indices (other than C_4, C_8, and C_{20}) are likely to exaggerate the correlation between various indices. Among the correlations based on Canadian and U.S. data cited above, only the one reported by Scherer is based on the actual size distribution of the firms. And if one recomputes the correlation, restricting the sample alternatively to industries for which C_4 exceeds .50 and .70, the C_4 – H correlation coefficients fall to .866 and .705, which are, of course, substantially smaller than Scherer's correlations. Section IV presents for the first time the correlation matrices of all the concentration measures that have been widely discussed in the literature, based on the actual size distributions of firms, and allows the reader to assess for himself the modest correlations between some of the measures, especially in samples of highly concentrated industries. These findings confirm that if C_4 is used as a proxy for a theoretically more satisfactory concentration index in statistical work, the proxy is an extremely imperfect one.

II. Concentration Measures and the Size Distribution of Firms

This section develops some notation and general considerations useful for thinking about size distributions and industrial concentration. Assume there are n firms in a well-defined industry, let s_i denote the share of the i^{th} firm in industry sales (or some other measure of relative size), where the firms have been ordered by size so that $s_1 \geqq \ldots s_i \geqq \ldots s_n$. By definition, $\Sigma\, s_i = 1$. An index of industrial concentration is a function C of $\{s_i\}$, the set of shares that define the firm size distribution. Highly concentrated industries take on a higher value for the index, and are expected to be closer to the monopoly end of the spectrum from monopoly to competition than industries with low values for the index, taking due account of other industry characteristics that are expected to play a major role in affecting the "degree of competition" in the industry.

If an n-firm industry is made up of equal-size firms, then $s_i = 1/n$ for all firms. In this special case, the concentration index is denoted $C^=(n)$. It is assumed that $C^=(n)$ is a monotonically decreasing function of n, reflecting the belief that the structural competitiveness of an industry should increase as the number of firms increases from one (pure monopoly) to very large n. The expected behavior of $C^=(n)$ is discussed more precisely in Section III.

In general, the relative sizes of firms in an industry differ substantially. The important concept of *the equivalent number of equal-size firms*, n_e, is defined in the following way. Let the concentration index assume the value K_o for a specific size distribution $\{s_i\}$. i.e., $C(\{s_i\}) = K_o$.[9] If $C^*(n)$ is a strictly monotonically decreasing function of n. and is defined for all real numbers $n \geqq 1$ (not just the positive integers). C^* has an inverse. C^{*-1}, and $n_e = C^{*-1}(K_o)$, i.e., n_e is the value (generally not an integer) such that $C^*(n_e) = K_o$. Two distinct concentration indices C and C^* are said to be dispersion-isomorphic if the n_e corresponding to any specific size distribution is the same for C and C^*.

Most measures of industrial concentration considered in this paper give rise to a strictly decreasing monotonic function $C^*(n)$ for all real numbers $n \geqq 1$, not just the integers. Since n usually denotes the integer number of firms that belong to an industry in this paper. the argument n_e is used in the function $C^*(n_e)$ whenever the behavior of the function C^* is discussed. unrestricted to integer values. Thus $C^*(n_e)$ is identical to the original concentration function $C(\{s_i\})$. where n_e corresponds to the size distribution $\{s_i\}$. as defined by the concentration measure.

For example. consider n_e for the r-firm concentration ratio C_r. the fraction of industry sales accounted for by the largest r firms in the industry. This index can be written $C_r = (r\overline{X}_r)/(n\overline{X}_n)$. where $\overline{X}_r$ is the average sales of the largest r firms. and $\overline{X}_n$ is the corresponding mean for all firms in the industry. If the industry were actually made up of equal-size firms. then $\overline{X}_r = \overline{X}_n$: so $C_r = r/n$ and $n = r/C_r$. Thus for firms of unequal size. $n_e = r/C_r$. (This paragraph assumes $n \geq r$.)

This discussion indicates that a concentration index possesses two distinct. important characteristics. The first is the cardinal behavior of the function $C^*(n)$, i.e.. how rapidly the index decreases with increases in the number of equal-size firms.[10] The second is how the index maps arbitrary size distributions into n_e. If a proposed measure seems to behave reasonably with respect to this second property, but not the first. a modified concentration index could be obtained by using the original index to calculate n_e. and one could then take a monotonically decreasing function of n_e that provides more satisfactory behavior for the first characteristic. The

[9] For most size distributions, K_o will not correspond precisely to the value of the index for some integer number of equal-size firms, i.e., there is no positive integer n such that $C^*(n) = K_o$. Nevertheless. there will generally be a positive integer m such that $C^*(m) \geqq K_o > C^*(m+1)$. One then says that the n_e corresponding to the concentration index C and the firm size distribution $\{s_i\}$ lies between m and $(m+1)$.

[10] Some students of industrial organization have waived the cardinal significance of the concentration index in empirical research by introducing one or more dummy variables corresponding to intervals of the concentration index as independent variables to explain profits or other performance variables. For example. Bain (1951) divided his industries into two classes. depending on whether the eight-firm concentration ratio C_8 is larger or smaller than .7. Dichotomizing industry by concentration levels in this way is probably responsible for several empirical attempts to determine whether there is a "critical concentration" level required to make concentration a significant factor in determining industry profit rates. There is little theoretical basis for expecting a strong discontinuity in the effect a concentration measure has on any performance characteristic. and it is desirable to construct concentration measures in which the cardinal properties are taken seriously.

modified index is thus the composite function of the size distribution corresponding to these two operations. The idea of computing n_e for comparing alternative concentration measures is useful, because it clarifies whether the major differences between them lie in the values they give for n_e or in the way the equal-size function $C^*(n)$ behaves for the alternative indices.

Most measures of industry concentration that have been proposed can be expressed as a weighted sum of the shares of firms in the industry, where the weights are functions of the share and/or the firm rank, $\varphi(s_i, i)$. In this situation, $C(\{s_i\}) = \Sigma\ s_i\ \varphi(s_i, i)$. For example, $\varphi(s_i, i) = 1$ for $i \leq r$; $\varphi(s_i, i) = 0$ for $i > r$ for the r-firm concentration ratio, C_r. For H, $\varphi(s_i, i) = s_i$. If φ is only a function of s_i and is strictly monotonic in s_i, n_e is easily obtained. In this case, one immediately obtains $n_e = 1/\varphi^{-1}(K_o)$, where φ^{-1} is the inverse of the weight function φ, and K_o is the value of the index for the size distribution in question.[11]

III. Criteria for Theoretically Reasonable Industrial Concentration Measures and the Cournot-Nash Equilibrium

This section discusses first the Cournot-Nash (abbreviated C-N) equilibrium and shows how its use in the Lerner index of monopoly power leads quite naturally to the Herfindahl index as the measure of industrial concentration. Then it is argued that the C-N equilibrium provides a reasonable *upper bound* on the expected output of an oligopolistic industry, given the industry demand function and the firm cost functions. This conclusion suggests that the Herfindahl index provides a *lower bound* for "theoretically reasonable" industrial concentration indices that range from zero for pure competition to one for pure monopoly. From this result, one can deduce several properties that a theoretically acceptable measure of industrial concentration should possess. None of the concentration measures discussed in the literature satisfy these conditions, and two new families of concentration measures are proposed that do possess the desired characteristics.

The introduction pointed out that the competitive model assumes a large number of independently acting firms. Cournot's model for homogeneous oligopoly considers the equilibrium that would be obtained if the large numbers assumption is rejected, but a certain kind of independent (noncooperative) behavior is imposed that assumes each firm chooses its output as if other firms will hold their output constant. (This restriction is relaxed later in the discussion.) It is assumed that no entry takes place. There are n firms in the industry that are assumed to have strictly increasing marginal cost functions, which need not be identical.

[11]Proof: For equal-size firms, $\sum^n s_i\ \varphi(s_i) = n[1/n\varphi(1/n)] = K_o$, and so $n_e = 1/\varphi^{-1}(K_o)$.

Industry demand is represented by the strictly decreasing function $p = f(q)$, where p is price and total industry output is $q = \sum_{i=1}^{n} q_i$ (q_i is output of the i^{th} firm). The firms attempt noncooperatively to maximize their own profits. The profit function of the i^{th} firm is $[pq_i - c_i(q_i)]$, where $c_i(q_i)$ is the cost function of the firm. The equilibrium condition for the i^{th} firm is obtained by maximizing its profits with respect to q_i:

$$p + q_i(df/dq)[1 + \sum_{j \neq i} \partial q_j / \partial q_i] = MC_i \quad (1)$$

In this equation MC_i is the i^{th} firm's marginal cost. In the Cournot model, the i^{th} believes all other firms will hold their output constant when it changes its own output: hence the terms in the summation on the LHS of equation (1) are zero. With this assumption, equation (1) can be rewritten in the familiar form:

$$p(1 + s_i/\eta) = MC_i \quad (2)$$

where η is the elasticity of demand for the industry and $s_i = q_i/q$, the i^{th} firm's share of industry output at equilibrium. If the firms have identical marginal cost functions, $s_i = 1/n$ for all firms, i.e., the firms have equal shares in equilibrium and the same level of marginal cost. Note that if the firms have different shares in the C-N equilibrium, the equilibrium levels of marginal costs are highest for the firms with the smallest shares. This situation is consistent with empirical evidence that indicates the largest firms in an industry tend to have higher rates of return and thus provides a theoretical rationale for some findings by Demsetz (1974).

Now consider the relationship between this equilibrium and the generalized Lerner index of monopoly power L, defined in the introduction. The L index assumed the following value at the C-N equilibrium:

$$\left[\sum_i (p - MC_i)q_i\right] \Big/ \sum_i pq_i = -\sum_i (s_i/\eta)s_i = (-1/\eta)H \quad (3)$$

where H is the Herfindahl index, defined by the sum of the squared shares of the firms in the industry, and η is the elasticity of demand. Thus the L index for the C-N equilibrium is the product of two factors: The Herfindahl index (which depends only on the firm size distribution) and the absolute value of the reciprocal of industry demand elasticity.[12] This result shows that the H index is theoretically appropriate as an industrial concentration index for determining the degree of competition in an industry if the industry equilibrium corresponds to the Cournot model. This result also indicates the appropriate functional form for explaining price-

[12]Since writing the initial draft of this paper, I have discovered that a number of economists have recently become aware of the intimate relationship of H and L under Cournot equilibrium conditions. See T. Rader (1972, p. 221), R. Spann (1976), W. A. Helley, Jr. (1976), Dansby and Willig (1976).

cost differentials as a function of industry concentration and demand elasticity.

We now argue that the C-N equilibrium yields an upper bound on equilibrium industry output (assuming one exists) and a lower bound on the Lerner index for an oligopolistic industry. Since the H index is appropriate for measuring industrial concentration for the C-N equilibrium, this argument implies that the H index provides a lower bound for *theoretically reasonable* measures of industrial concentration that are scaled to have a maximum value of one for pure monopoly (one firm) and a minimum of zero for perfect competition.

In this discussion two interpretations of the Cournot model are distinguished: (a) it represents a single-decision, one-period game, and (b) it represents an iterated game. Consider first interpretation (a). Given the strategies available to the firms, a noncooperative Nash equilibrium exists if it is not profitable for any firm to change its strategy, given the equilibrium strategies of the other firms. In the one-period model, the strategies correspond to the selection of output by each firm. Nash (1951) has provided an important game-theoretic argument which implies that Cournot's solution is a noncooperative Nash equilibrium to the one-period Cournot model.[13] The completely cooperative solution, which would maximize joint profits to the firms in the industry, requires the marginal costs of all producing firms to be equal to marginal receipts to the industry at the equilibrium level, and would entail a reduction of industry output from the C-N equilibrium. To the extent that a cooperative solution is feasible, one expects the firms to achieve an outcome no worse than the noncooperative solution, for which the H index is theoretically appropriate.[14]

[13]See Telser (1972, pages 132–135) for a discussion suggesting that the Cournot model should be interpreted as a single-decision model.

[14]Two remarks are in order. First, if the firms have different marginal cost functions, the C-N equilibrium will generally correspond to firms with different shares of industry output, and hence different equilibrium marginal costs, as indicated in equation (2). Suppose the firms cooperated by reallocating production among themselves such that total output remained constant, but that each firm has the same marginal cost under the alternative arrangement. This form of cooperation would increase joint profits. Hence one can imagine a less cooperative arrangement in which joint profits are greater than at the C-N equilibrium, firms have the same marginal cost, but industry output is *greater* than in the original C-N equilibrium. Although this result contradicts the text assertion that incomplete cooperation should reduce output below the C-N equilibrium, such an outcome seems *extremely implausible*. The degree of profit-sharing that would be required to induce the firms to produce at equal marginal costs would surely make it feasible for them to obtain a superior outcome by reducing total output.

Second, if the terms $\partial q_j/\partial q_i$ in equation (1) are negative, the summation of these terms in equation (1) would make the lefthand side of the equation a larger positive number, and hence requires a higher output by the firm to reestablish equality. The term $\partial q_j/\partial q_i$ is called the "conjectural variation" (see Fellner (1949, pages 71–77)) in firm j's output per unit change in firm i's output anticipated by firm i, and which firm i takes into account when trying to choose its optimal output. Thus a negative conjectural variation $\partial q_j/\partial q_i$

We turn now to the interpretation of the Cournot model as an iterated game and show again the relevance of H as a lower bound for a theoretically reasonable concentration index. The iterated version of the model has been thoroughly denounced by Fellner (1949), Stigler (1968, page 36) and other prominent economists. This line of criticism maintains that joint maximization of industry profit is the appropriate theoretical goal for firms in an oligopolistic industry, and is concerned with identifying and elaborating on those factors that prevent this objective from being achieved. Since joint profit maximization requires cooperation, this view considers oligopoly a cooperative game. This criticism is persuasive. Unfortunately, there is no consensus on the appropriate model for characterizing cooperative oligopolistic equilibrium.[15] Hence one again considers the theoretical implications of noncooperative equilibrium for the unspecified cooperative model.

Noncooperative iterative models are still undergoing considerable theoretical elaboration, and it is premature to assert what consensus will be reached on them. The traditional dynamic mechanism that has been postulated to attain the C-N equilibrium point is an extremely myopic process, in which each firm chooses its output for period $(t + 1)$ so as to maximize its profits, under the assumption that the rest of the firms maintain the output level of period t. This is equivalent to saying that the firms all attach a probability of one that the current output of all other firms will be the same as the preceding period's output. Of course, if all firms adopt the same strategy and do not start out at the equilibrium point, they all discover in the next period that their forecast has been falsified. It is hard to imagine that even noncooperative louts would fail to learn that the constant output assumption is incorrect and inappropriate as the game is iterated. However, some recent theoretical work considers far more palatable adjustment processes. Robert Deschamps (1975) has applied the game theory algorithm "fictitious play" (F.P.), to homogeneous oliogopoly, and shows that under suitable restrictions on the industry demand

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would imply that firm i expects firm j to contract its output if firm i expands its output. It is readily shown that with perfect cooperation, the sum of the conjectural variations in equation (1) is positive, and equals $(1 - s_i)/s_i$, where s_i is the i^{th} firm's share in this situation. With any degree of cooperation, the conjectural variations should be positive. There is no satisfactory theoretical basis for deducing negative conjectural variations for each firm in a single-period noncooperative model, nor is there any justification for assuming them. Hence the Cournot assumption of zero conjectural variation provides the bounds on industry output and profits claimed in the text. Conjectural variations are discussed further when considering "dominant firm" models below.

[15] There is even debate about whether useful theoretical models can be produced that generate such equilibria. We do not examine this issue in the present study, and assume instead the existence of cooperative equilibria to which noncooperative equilibria provide one-sided bounds.

function and the firm cost functions, the firm output choices converge over time to the one-period C-N equilibrium point.[16] The F.P. process starts out with arbitrary output decisions by the two firms in period 1. In period $(t + 1)$ each firm considers the sequence of outputs of the other firm from period 1 through period t, and assumes that these empirical outputs define the probability distribution of output that the other firm will choose in period $(t + 1)$. Thus the optimizing strategy of firm 1 in period $(t + 1)$ (to maximize expected profits) is to choose an output $q_1(t + 1)$ that maximizes:

$$(1/t) \sum_{i=1}^{t} \pi_1(q_1(t + 1), \bar{q}_2(i))$$

where $q_2(i)$ is the output actually produced by firm 2 in period i, and $\pi_1(q_1, q_2)$ denotes profits of firm 1 as a function of the output of the two firms. Similarly, permuting the subscripts for firms 1 and 2 gives the decision rule for firm 2's output in period $(t + 1)$. This model converges (with appropriate restrictions) to the C-N equilibrium point by a simple dynamic learning process that is far more plausible than the traditional dynamic Cournot model.[17]

[16] I am indebted to David Schmeidler for calling my attention to Deschamps' study.

[17] Deschamps points out that it is not necessary to assume that firms take the unadjusted frequency distribution of the other firm's outputs as the probability distribution of the other firm's output in period $(t + 1)$. He gives an example of another set of weights that give heavier weight to recent outputs of the other firm which also lead to convergence at the C-N equilibrium point. Lester Telser (1972, pages 149-164) reports similar results for an n-firm oligopoly model, where each firm bases its expectations of industry output from the other firms on a weighted average (with fixed weights) of the observed outputs of firms from preceding periods. This work is primarily concerned with restrictions on the weights that would lead the process to converge to the C-N equilibrium point. A major limitation of this analysis is the assumption that the n firms have identical constant marginal costs. Further work is required to determine necessary and sufficient conditions on the demand function and the marginal cost functions of an n-firm oligopoly for the existence of a unique C-N equilibrium point in the one-period model, and on expectation-generating mechanisms that converge to this equilibrium.

Another argument by Luce and Raiffa (1957, pages 97-100) and applied by Telser (1972, page 141) to the noncooperative duopoly problem, concludes that iterating a nonzero sum game a finite number of times can lead to the choice of the same C-N equilibrium point each period. This counterintuitive result comes about essentially by showing that the choice in the last period will be the same as in a one-period model, and reasoning backwards, period by period, to the first period. This austere model contains no convergence mechanism —each firm must know the industry demand *and* the other firm's cost function in order to select the equilibrium point on the first move. It seems desirable to incorporate some learning mechanism into these models, since this would be an essential component of models with stochastic variations in demand and firm costs.

The discussion so far has assumed a basic symmetry in the behavior of firms. There is a class of models that rejects this assumption, and postulates that some firms act according to the Cournot hypothesis and take as given the output of other firms in choosing their output, while at least one firm takes this behavior into account in choosing its optimum output.[18] This class includes "dominant firm" models, and the relevance of such models for this study is now considered.[19] Given the industry demand function, a dominant firm realizes that other firms adapt their output in the current period as if the rest of the industry holds constant its output level from the preceding period. In formal terms, the non-dominant firms have a reaction function, which gives the output that would maximize their profit, *given* the industry demand function, the level of output of all other firms in the preceding period, and each firm's marginal cost function. The dominant firm then maximizes its profits, subject to its marginal costs and the reaction functions of the other firms. In general, the crucial feature of such models is that firms do not simultaneously assume that all firms but themselves choose output according to a reaction function, since no equilibrium usually exists under that assumption.[20]

A major objection by Fellner (1949, pages 66–69, 116–119) and others to dominant firm models in the context of static demand conditions lies in the arbitrary asymmetry between the dominant and subordinate firms. It is often assumed that the largest firm is the dominant firm, but this is an *ad hoc* specification that lacks theoretical support. If the largest firm is dominant, it usually (but not necessarily) turns out that in comparison with the C-N equilibrium, the dominant firm is larger and has higher profits, the subordinate firms are smaller and have lower profits, and industry output is greater (which implies industry price is lower) and the allocational loss from monopolistic restriction is lower.[21] But why should the smaller firms acquiesce to the largest firm if it lowers their profits? The claim that the dominant firm is "more powerful" does not

[18] I assume that one could impose the "fictitious play" mechanism instead of the traditional Cournot assumption if one interprets the dominant firm model as an iterated game. For brevity, discussion in the text uses the traditional statement of the Cournot hypothesis.

[19] I am indebted to Herbert Mohring and Michael Darby for expressing dissatisfaction at the neglect of dominant firm considerations in an earlier version of this paper.

[20] However, it is possible to construct models with a consistent hierarchy of dominance, possessing well-defined equilibria. Suppose the industry can be partitioned into m sets of firms (which may consist of single firms) with the following property: each firm in set i makes the Cournot assumption about any other firm belonging to set i and to all firms belonging to sets $1, 2, \ldots, i-1$. It chooses its output to maximize its profits, given the reaction functions of firms in sets $i+1, \ldots, m$, and its own marginal cost function. Finn Kydland (1976) alludes to this extension of dominant firm models in a dynamic setting.

[21] The reader familiar with the duopoly theory developed by Stackelberg and discussed by Fellner (1949) is aware of the uncomfortable variety of results that can be produced by

bear scrutiny. Even if true (whatever it means), the relevant question is whether the present value of the dominant firm would be greater by investing to teach other firms it intends to maintain output at the dominant firm equilibrium level or by accepting the C-N equilibrium.

Despite these serious doubts about the relevance of the dominant firm equilibrium, some economists find one version of the dominant firm model more persuasive when there is a competitive fringe of quite small firms that might passively adjust to whatever output level the large firms choose. Suppose the firms in the competitive fringe are so small that they regard price and marginal revenue as equivalent. How should this assumption be incorporated into the expected ratio of monopoly profits to industry revenue? If all other firms in the industry correctly perceive the supply curve of the competitive fringe, they can obtain the derived demand net of the amount supplied by the competitive fringe by the function $q(p) - q_c(p) = q^*(p)$, where $q(p)$ is the industry demand function, $q_c(p)$ is the quantity supplied by the competitive fringe, and $q^*(p)$ is the net demand for the rest of the industry. Suppose the rest of the firms in the in-

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the dominant firm model. As an illustration, consider a duopoly model with firms a and b possessing unit elastic marginal cost functions $MC_a(q_a) = q_a/.8$ and $MC_b(q_b) = q_b/.2$, and assume the industry demand function $q = (q_a + q_b) = 1/p$. The following table shows equilibrium outputs and profits for each firm, and equilibrium industry price under three alternative equilibria: C-N, firm a dominant, and firm b dominant.

	Equilibrium Values						
Solution	q_a	q_b	π_a	π_b	$\pi^*_{m.a}$	$\pi^*_{m.b}$	P
Cournot	.4216	.2108	.5556	.2222	.4444	.1111	1.581
Firm a Dominant	.4595	.2069	.5575	.2035	.4256	.0964	1.500
Firm b Dominant	.4154	.1908	.5774	.2237	.4696	.1327	1.650

$^*\pi_{m.i}$ is defined by monopoly rent of firm i: $(p - MC_i)q_i$

Relative to the C-N equilibrium, if firm a is dominant, a's output and profit is increased, and b's output and profit is decreased, total output increases, and industry price falls. This result conforms to the claim in the text. But if b is dominant, then compared with the C-N equilibrium, both a and b have lower output and higher profit; indeed, in this case the subordinate firm a has higher profits than it attains if it is the dominant firm. A condition that assures the commonly expected result that expansion by the dominant firm induces an output contraction by the subordinate firm is the following: The subordinate firm's marginal receipts curve falls when the dominant firm's output increases. R.G.D. Allen (1942, pages 345–347) expresses the condition as $dp/dq + q_b(d^2p/dq^2) < 0$, where a is the dominant firm and b the subordinate firm. An alternative condition that may be more convenient is $(dMR/dq)q_b - (q_a - q_b)(dp/dq) < 0$.

dustry attain the C-N equilibrium, taking into account the supply by the competitive fringe. One readily determines that the L index for the industry, given these assumptions, is:

$$-[1/(\eta - (1 - k_1)\xi)]H$$

where η is the industry demand elasticity, ξ is the competitive fringe supply elasticity, $k_1 = q^*/q$, the fraction of equilibrium industry output from the non-fringe firms, and H is the Herfindahl index for the industry.[22] Thus taking into account the competitive fringe modifies the relevant elasticity that appears in the L index, but does not alter the use of the H index as the relevant industrial concentration measure.[23] To simplify exposition in the rest of the paper, the competitive fringe model will not be considered again. However, the reader should keep in mind the adjustment that should be made to the industry demand elasticity when the fringe model is relevant.

The chief conclusion from this analysis is that the Herfindahl index provides a *lower bound* for theoretically reasonable measures of industrial concentration intended to explain the L index of an industry. It is a lower bound because it does not allow for explicit or tacit cooperation by the

[22] Proof: First one determines the demand elasticity collectively facing the firms not in the competitive fringe. Let $q(p) = q_c(p) + q^*(p)$ denote industry output as the sum of competitive fringe supply and output by the nonfringe firms. Then the demand elasticity collectively facing the nonfringe part of the industry $\eta^* = (p/q^*)d(q - q_c)/dp = (1/k_1)(\eta - (1 - k_1)\xi)$, where $k_1 = q^*/q$. Now let $s_i^* = q_i/q^*$, the output of a nonfringe firm as a fraction of output by all nonfringe firms. The equilibrium condition for the i^{th} firm is the analogue of equation (2): $p(1 + s_i^*/\eta^*) = MC_i$. From equation (4) monopoly rent as a fraction of total receipts of the nonfringe firms is $(-1/\eta^*)H^*$, where $H^* = \Sigma s_i^{*2}$. Hence monopoly rent as a fraction of total industry receipts is $(-k_1/\eta^*)H^*$. Let $s_i = q_i/q$ the output of the i^{th} nonfringe firm as a fraction of *industry* output. Since $s_i^* = s_i/k_1$, $H^* = H/k_1^2$, where H is the ordinary Herfindahl index for the industry. (This last step assumes the individual firms in the competitive fringe are so small that they make a negligible contribution to H.) Substituting the expressions for H^* and η^* into the Lerner index $(-k_1/\eta^*)H^*$ yields the expression given in the text. It may be that the fringe model is useful for analyzing industries exposed to significant foreign competition in their home market. The H function could either be based on the domestic share of domestic producers and foreign supply would be incorporated in the demand curve perceived by the domestic producers as a group. Or the share of foreign producers could be included in H, and foreign producers could be treated as a competitive fringe, as in the text.

[23] Of course, the equilibrium and allocational properties of the equilibrium do depend on whether the industry equilibrium with a competitive fringe is C-N, or whether the competitive fringe reacts passively to output decisions by nonfringe firms (and the nonfringe firms realize it). For example, consider an industry with demand function $q = 1/p$ (hence $\eta = -1$), and suppose there is one large firm with marginal cost function $MC^* = (2q^*)^{1/\xi}$ and a competitive fringe with *aggregate* marginal cost function $MC_c = (2q_c)^{1/\xi}$. The equilibrium share of the large firm, industry price, Lerner index, and Herfindahl index are shown for the different marginal cost elasticities ξ in the following table for the C-N equilibrium and the dominant firm equilibrium models.

additional restrictions on reasonable concentration indices follow from this conclusion. An index should approach the H index in the limit as the number of firms increases indefinitely, and the share of the largest firm tends to zero. Indeed, an even stronger restriction on reasonable indices seems appropriate. Suppose that the largest firm's share, s_1, is held constant, and the rest of the industry output is supplied by an extremely large number of firms, all very small. In the limit, the H index is s_1^2, the squared share of the largest firm.[24] It seems reasonable that departures from the C-N equilibrium (in the direction of greater monopoly) should occur only if there are two or more "large" firms in the industry. It has been widely argued that the costs of collusion rise with the number of (equal-size) firms. Since many of the little firms would have to collude with the large firm for there to be any feasibility of higher profits for the colluders, it appears that the H index is again the correct limit for a satisfactory concentration index for one "large" firm, and an indefinitely large number of small ones. A corollary to this argument is that the index should *not* approach zero as the number of firms in the industry increases indefinitely, if there are one or more "large" firms in the industry. This condition also implies that the n_e of a theoretically reasonable concentration index will

Model	ξ	s^* (large firm share)	p (industry prices)	L Index	H Index
Cournot-Nash	1	.3820	1.1118	.1459	.1459
	2	.3177	1.1092	.1009	.1009
	3	.2755	1.0971	.0759	.0759
	5	.2219	1.0765	.0492	.0492
Dominant Firm	1	.4227	1.0622	.1132	.1786
	2	.4007	1.0508	.0730	.1606
	3	.3904	1.0425	.0539	.1524
	5	.3805	1.0210	.0353	.1448

This table indicates the significant fall in the Lerner index as the supply elasticity of the competitive fringe increases for both the C-N and the dominant firm equilibria. The share of the large firm declines much more slowly with increases in the marginal cost elasticity in the dominant firm model than in the Cournot model. It is worth noting that even though the H index is larger for the dominant firm model than the Cournot case (given ξ), the opposite is true for the Lerner index. This result shows the importance of taking into account both the H index and the relevant demand elasticity for predicting the L index.

[24]Whether the industry equilibrium with a competitive fringe should be expected to conform to the Cournot model or the dominant firm model is moot. If one believes the latter is relevant, one can adjust the demand elasticity in the Lerner index, as indicated in the text, without modifying H. There is a technical difficulty in relating the two models to handle sequences of industries which subdivide a group of firms until they are treated as a passive competitive fringe by the rest of the industry. This problem is not explored in this study.

be finite if the share of the largest firm does not go to zero, regardless of the number of firms in the industry.

The industrial concentration function is defined as the cumulative distribution function of industry output, with firms ordered from the largest to the smallest in the industry, i.e., $F_a(j) = \sum_{i=1}^{j} s_i$, for $1 \leq j \leq n$, where n is the number of firms in the industry, and the a subscript denotes a specific industry. It is convenient to define industry a as *strictly less concentrated* than industry b if $F_a(j) \leq F_b(j)$ for all j, and $F_a(j) < F_b(j)$ for at least one j.[25] In words, the concentration curve of a must lie strictly below the concentration curve of b at one point or more, but can never lie above the concentration curve of b. It follows immediately from this definition that an n-firm industry with equal-size firms is strictly less concentrated than any other size distribution with n firms. It also follows from the definition that if two (or more) firms in an industry merge, and no change takes place in the individual shares of the remaining firms, the size distribution of the pre-merger industry is strictly less concentrated than the post-merger distribution.[26] It is readily verified that if one distribution is strictly less concentrated than another, it necessarily has a smaller value for its Herfindahl index. There is nothing in the Cournot model, nor in feasibility of collusion considerations, to suggest that any reasonably defined measure of industrial concentration should not take a smaller value for strictly less concentrated size distributions, and this property should be verified when examining any newly proposed concentration measures.

The H index considered as a function of n_e is $1/n_e$, a convex function that declines with n_e, but at a declining rate. In the absence of a plausible theory of the relationship between collusion and the size distribution that implies a concentration index that violates this condition, it also seems a reasonable feature to expect from a satisfactory index.[27]

The most unsatisfactory aspect of this entire development is the lack

[25] The definition of "strictly less concentrated" corresponds to the concept of "stochastic dominance" that has been widely used by economists in the analysis of risk. Jack Meyer's comments on an earlier draft have greatly simplified the discussion in this paragraph. See Hanoch and Levi (1969) and Rothschild and Stiglitz (1970) for extensive discussion of stochastic dominance and several equivalent definitions.

[26] Of course, *if* the marginal cost functions of the merging firms are unaltered, the share of the merged firms would be smaller in the new equilibrium than the sum of the premerger shares in the initial equilibrium. This occurs because the marginal revenue perceived by the merged firm at the premerger level of output is lower than the marginal revenue for either firm in the premerged equilibrium.

[27] F. M. Scherer (1970, page 183) has suggested, "As a very crude general rule, if evenly matched firms supply homogeneous products in a well-defined market, they are likely to begin ignoring their influence on price when their number exceeds ten or twelve. It is more difficult to generalize when the size distribution is highly skewed." This claim might be interpreted as implying that $C^*(n_e)$ may have a nonconvex region over an interval of small values of n_e, since it suggests an abrupt decline in $C^*(n_e)$ when the firms begin to ignore their interdependence. To my knowledge, no theoretical argument or empirical evidence is available that would support the first sentence in this claim.

of a convincing cooperative model that puts significant restrictions on the departures that may be expected from the C-N equilibrium, and of making these departures a function of the firm size distribution, $\{s_i\}$. Stigler's important article (1964) on the theory of oligopoly indicates an interesting approach for attacking this problem, although the effect of inequality of firm size on the feasibility of collusion has hardly been touched upon.

In summary, the criteria for theoretically reasonable industrial concentration measures that have been deduced from consideration of the C-N equilibrium, the corresponding Lerner index implied by it, and a few extremely natural assumptions about the feasibility of collusion and the size distribution of firms are as follows.

1. If the two largest firms in an industry size distribution each have shares greater than a strictly positive constant, the concentration index should take on a value greater than H. This simply reflects tacit or explicit collusion, which leads to a larger value of the index than indicated by the C-N equilibrium.
2. If the largest firm is greater than a strictly positive constant, and one considers a sequence of industries where the rest of industry output is produced by an increasing number of firms such that the share of the second largest firm in the industry approaches zero, the concentration measure should converge to s_1^2, the squared share of the largest firm.
3. As the number of firms in the industry increases, and the share of the largest firm approaches zero, the concentration measure should converge to the Herfindahl index. For the special case of equal-size firms, this implies the index approaches the function $1/n$ as n becomes large.
4. If the cumulative size distribution of one industry is *strictly less concentrated* than another, the former industry should have a lower value for its concentration index.[28] As corollaries, the index should attain its minimum, given n, when firms are equal-sized; and the merger of two or more firms should increase the value of the index if the share of all other firms remains the same.
5. The index should be a decreasing, convex function of n_e, the equivalent number of equal-size firms implied by the index.[29]

Finally, the following normalization convention is adopted:

6. The index should equal unity if $n_e = 1$, and should approach zero as n_e increases without bound. This convention follows naturally

[28] The definition of "strictly less concentrated" is given earlier in the text. Essentially, it requires the cumulative size distribution of the industry to lie strictly below the other industry size distribution at least at one point, and never to lie above the other distribution.

[29] The decrease in the index as a function of n_e follows immediately from the theoretical arguments; the convexity condition does not, but seems highly plausible.

if one considers that a theoretically satisfactory index should essentially be an H-index, adjusted to allow for collusion. This normalization facilitates comparison of alternative measures; its only theoretical significance is that it suggests that concentration indices should have a bounded range.

These criteria do not define a unique index, which is hardly surprising, since an adequate theory of collusion has not yet been devised. But they are much more restrictive than others that have been proposed in the literature, and they follow immediately from the analysis of the C-N equilibrium, modified to allow for collusion.[30]

As far as I know, the measures of industrial concentration that have appeared in the literature all fail to satisfy one or more of these criteria. The appendix indicates the ways in which the most widely discussed measures violate them. This section concludes with the introduction and brief discussion of two new measures that are theoretically acceptable in terms of the analysis. Both depend on a single parameter that allows for the effects of collusion; both measures converge to the H index as the parameter increases without bound. These measures were devised primarily by considering simple functions of $\{s_i\}$ that satisfy the first four criteria, then restricting them to satisfy the fifth.

1. The multiplicatively-modified Cournot measure of industrial concentration (with parameter α) is defined by the equation

$$H_m(\alpha; \{s_i\}) = \sum s_i^{[2 - \{s_i(H - s_i^2)\}^{\alpha}]} \qquad (\alpha \geqq .15) \tag{4}$$

In this formula, H is the ordinary Herfindahl index for the size distribution. The expression raised to the α power always lies between zero and $2(1/3)^{1.5}$ (<1), and so the exponent on s_i always exceeds one.[31] If the firms are equal-size, the index takes on the value $(1/n)(1/n)^{-[(n-1)/n^3]\alpha}$, which converges to the H index, $1/n$ for large n. It is straightforward to verify that this index satisfies conditions 1–6 in the text, except for the convexity

[30] M. Hall and N. Tideman (1967) proposed the following properties for an index. It should be a (1) scalar function of (2) all the firm shares that (3) should decrease if part of a share is transferred from a larger to a smaller firm, (4) should decrease by the factor $1/k$ if there is a k-fold increase in the number of firms while holding constant the *relative* size distribution, and (5) should decrease with n for distributions of equal-size firms. Property 4 is rejected by our theory, which modifies the Cournot theory to allow for collusion. The rest of the Hall-Tideman criteria follow from our analysis. C. Marfels (1972) reports an attempted axiomatic approach by Jöhnk (1970), which proposes (1) symmetry of the index with respect to firm shares, (2) continuity with respect to share changes, (3) an increase if part of a share is transferred from a smaller to a larger firm, (4) if K is the index and $(w - v) = (v - u)$, $[K(w) - K(v)] \geqq [K(v) - K(u)]$, and (5) the index should range over the $0 - 1$ interval. The arguments of K are vectors of firm shares. This condition is apparently intended to imply convexity of the index although the formulation is unclear. These conditions are all implied by the criteria developed in the main text, and are much less restrictive in defining permissible indices than the arguments in this paper.

[31] The upper bound of $2(1/3)^{1.5}$ is obtained in a two-firm industry

TABLE I

THE INDEX $H_m(\alpha; \{s_i\})$ FOR EQUAL-SIZE FIRMS

n	α = .15	.25	.5	1	2	∞*
1	1	1	1	1	1	1
2	.830	.755	.638	.545	.505	.500
3	.701	.591	.449	.362	.335	.333
4	.603	.475	.338	.267	.251	.250
5	.522	.395	.267	.211	.200	.200
10	.311	.203	.124	.102	.100	.100
20	.168	.097	.058	.050	.050	.050
100	.032	.016	.0105	.010	.010	.010
1,000	.0023	.0012	.0010	.001	.001	.001

*Herfindahl

condition in 5. The restriction $\alpha \geqq .15$ was found necessary to assure convexity. Table I compares this index for several values of α, and with the H index ($\alpha = \infty$) for distributions with equal-size firms. It is clear from the table that the tendency toward competition as n increases is much slower for low values of α than the H index implies.

2. The additively-adjusted Cournot measure of industrial concentration (with parameter β) is defined by the equation

$$(5) \qquad H_a(\beta; \{s_i\}) = \sum s_i^2 + [s_i(H - s_i^2)]^\beta \qquad (\beta > 1)$$

Again, H denotes the ordinary Herfindahl index for $\{s_i\}$. If the firms are equal-size, the measure becomes $[n^{-1} + n^{1-2\beta}(1 - n^{-1})^\beta]$. The restriction $\beta > 1$ is required to obtain the required convergence behavior for large n to satisfy condition 3.[32] It can be verified that with this restriction, the index $H_a(\beta; \{s_i\})$ satisfies conditions 1–6. Table II compares H_a for alternative values of β and the H index ($\beta = \infty$) for equal-size firm distributions.

Further theoretical and empirical work is required to determine where H_m or some other index is clearly better than H, as well as the value for the parameter α that seems most appropriate.[33] In principle, it seems plausible that the parameter α should be industry-specific, since it reflects the feasibility of collusion which surely depends on industry characteristics besides the size distribution. If it is desired to create an index that uses the same parameter for all industries, the parameter value should presumably

[32] If $\beta = 1$, the measure converges to $2/n$, instead of $1/n$, and converges as $1/n^\beta$ for $\beta < 1$.

[33] Although the values of the H_a index differ substantially from the H index for small β, the empirical results in Section IV indicate that $H_a(\beta)$ and H have very high correlation even in this case. Thus the statistical results using H_a would presumably differ little from those obtained with H.

TABLE II

THE INDEX $H_n(\beta; \{s_i\})$ FOR EQUAL-SIZE FIRMS

	β						
n	1.01	1.10	1.25	1.50	2	3	∞*
1	1	1	1	1	1	1	1
2	.745	.703	.649	.588	.531	.504	.500
3	.550	.505	.449	.394	.349	.335	.333
4	.432	.388	.337	.291	.259	.250	.250
5	.355	.313	.268	.229	.205	.200	.200
10	.186	.156	.128	.109	.101	.100	.100
20	.095	.076	.060	.052	.050	.050	.050
100	.019	.014	.011	.010	.010	.010	.010
1,000	.0019	.0013	.001	.001	.001	.001	.001

*Herfindahl

reflect in some sense the average level of interdependence that is present in the individual industries. Appendix A demonstrates the serious theoretical deficiencies of the four-firm concentration index, entropy, and several other indices that have been introduced into the literature.

A very recent article by Cowling and Waterson (1976) suggests an interesting way of analyzing the relationship of price-cost margins and concentration which doesn't require an *a priori* choice between the Cournot model and the dominant firm class of models. Cowling and Waterson propose the use of time series and cross-sectional data in this approach. They attempt to generalize the Cournot model by assuming that firms in an industry have stable, but possibly nonzero, conjectural variations about output responses of other firms to their own output changes. The equilibrium condition for the individual firms is

$$p[1 + s_i(1 + \lambda_i)/\eta] = MC_i, \tag{2'}$$

where $\lambda_i = \sum_j \partial q_j / \partial q_i$, the conjectural variations expected by the ith firm in response to its change in output. It is readily verified that the Lerner index corresponding to the industry equilibrium is given by

$$L = H(1 + \bar{\lambda})/(-\eta), \tag{3'}$$

where $\bar{\lambda} = \sum_i \lambda_i s_i^2 / H$, a weighted average of the expected conjectural variations, where the weights are the squared shares of the firms.

If the *across time* coefficients of variation of the industry demand elasticity, η, and of the adjustment factor for conjectural variation, $(1 + \bar{\lambda})$, are both substantially smaller than the across time coefficient of variation of the H index, then the ratio of the Lerner indices for an industry at two points in time is approximately equal to the ratio of the H indices for the same two points in time. Cowling and Waterson thus suggest taking the ratio of the price-cost margin in an industry (which is equivalent to the ratio of the Lerner indices) at two points in time as a way of getting rid of the difficult-to-measure parameters η and $\bar{\lambda}$. They report some regressions of the log of the price-cost margin ratio on the log of the H index ratio and several other variables, and compare the results with those obtained with the log of the C_4 ratio replacing the H index ratio. As the theory suggests, the H index ratio appears to give a better (although low) statistical fit.

Although Cowling-Waterson do not refer to any models that make a specific, nonzero assumption for the expected conjectural variations, $\bar{\lambda}$, the dominant firm models obviously belong to this class. If there is a significant difference in *a priori* beliefs about the relative plausibility of the Cournot-Nash equilibrium and a dominant firm equilibrium, the Cowling-Waterson paper suggests that some empirical results of interest may still be obtained by using the ratio of price-cost margins at two points in time as the dependent variable and the corresponding H ratio as an independent variable. If the assumption about the relative magnitude of the coefficients of variation of H, η, and $(1 + \bar{\lambda})$ holds, this procedure enables us to get rid of the latter two parameters.

IV. Cross-Section and Intertemporal Correlations of Industrial Concentration Indices—Some Empirical Results

This section presents and discusses correlations between various concentration indices based on employment for 45 Swedish manufacturing industries in 1968, as well as the correlation of the individual indices between 1954 and 1968. The industries contain the metal, metal-working, and plastic sectors, and include 4247 firms with about 405,000 employees in 1968 (about 48% of manufacturing employment). The indices for the individual industries are given in Appendix B.

The indices that have been discussed in the literature, their definitions, and their values for equal-size firms are as follows:

1. Four-firm concentration ratio $C_4 = \sum^4 s_i$, $C_4^*(n) = 1$ if $n \leq 4$, $C_4^*(n) = 4/n$ if $n \geq 4$;
2. Herfindahl index $H = \sum^n s_i^2$, $H^*(n) = 1/n$;
3. Entropy index[34] $E = \Pi^n s_i^{s_i}$, $E^*(n) = 1/n$;

[34] Entropy, as defined in physics and information theory is $-\ln(E)$, and is obviously not normed properly for a concentration index even if the sign is changed so that it is ordered in the correct direction. The entropy index as a measure of industry concentration is mentioned by Stigler (1968) and Marfels (1971). See M. O. Finkelstein and R. M. Friedberg (1967) for an early discussion of using entropy as a concentration index. Although the entropy index has serious theoretical deficiencies in terms of our criteria, this specific transformation of entropy at least has the merit of yielding a measure of concentration that is consistent with the Cournot model for industries with equal-size firms.

TABLE III

1968 Means, Standard Deviations, and Correlations of Alternative Concentration Indices and 1968 1954 Correlations

Part A.	Mean	Standard Deviation	C_4	H	E	R	CICI	$H_m(.15)$	$H_m(.25)$	Correlations $H_m(.5)$	$H_m(.75)$	$H_a(1.1)$
C_4	.869	.117	*.898*									
H	.396	.190	.614	*.810*								
E	.275	.191	.710	.931	*.826*							
R	.300	.206	.756	.832	.964	*.893*						
CICI	.664	.134	.811	.943	.920	.865	*.886*					
$H_m(.15)$	.618	.152	.887	.899	.925	.896	.986	*.905*				
$H_m(.25)$	.536	.168	.825	.945	.947	.897	.995	.992	*.891*			
$H_m(.50)$	.450	.182	.724	.986	.854	.877	.983	.958	.986	*.862*		
$H_m(.75)$	.419	.187	.672	.996	.947	.859	.967	.933	.970	.997	*.842*	
$H_a(1.1)$	.509	.180	.740	.980	.930	.857	.989	.961	.986	.996	.992	*.875*

A. 22 Industries, H ≥ .16

Part B.	Mean	Standard Deviation	C_4	H	E	R	CICI	$H_m(.15)$	$H_m(.25)$	$H_m(.5)$	$H_m(.75)$	$H_a(1.1)$
C_4	.773	.177	*.914*									
H	.306	.207	.795	*.855*								
E	.209	.187	.773	.944	*.866*							
R	.229	.201	.791	.877	.974	*.910*						
CICI	.558	.194	.933	.944	.881	.848	*.906*					
$H_m(.15)$	.514	.203	.933	.933	.905	.887	.990	*.918*				
$H_m(.25)$	.430	.212	.922	.962	.928	.897	.991	.995	*.910*			
$H_m(.5)$	.350	.213	.865	.990	.947	.898	.976	.973	.991	*.891*		
$H_m(.75)$	.324	.210	.832	.997	.949	.891	.963	.956	.979	.998	*.877*	
$H_a(1.1)$	.400	.222	.885	.982	.924	.878	.987	.980	.993	.997	.992	*.897*

B. 32 Industries, H < .09

Part C.												
C_4	.651	.253	*.963*									
H	.232	.211	.831	*.901*								
E	.155	.179	.774	.954	*.898*							
R	.170	.193	.787	.907	.980	*.932*						
CICI	.452	.239	.963	.941	.873	.856	*.950*					
$H_m(.15)$	.404	.246	.966	.939	.893	.885	.994	*.958*				
$H_m(.25)$	.331	.239	.935	.968	.926	.908	.989	.994	*.948*			
$H_m(.5)$	.265	.224	.883	.992	.952	.919	.969	.971	.991	*.931*		
$H_m(.75)$	.245	.217	.857	.998	.956	.916	.956	.956	.981	.998	*.919*	
$H_a(1.1)$	.305	.240	.905	.985	.931	.902	.981	.981	.995	.996	.996	*.937*

C. 45 Industries

Source: Data from Gunnar Du Rietz (1975).
In each part, italicized diagonal entries are correlations between the measures in 1954 and 1968.

4. Rosenbluth index[35] $R = [(2\sum^n is_i) - 1]^{-1}$, $R^*(n) = 1/n$

5. "Comprehensive industrial concentration index"[36] $CICI = s_1 + \sum_{i=2}^{n} s_i^2 (2 - s_i)$. $CICI^*(n) = (3n^2 - 3n - 1)/n^3$.

The two new concentration indices $H_m(\alpha; \{s_i\})$ and $H_a(\beta; \{s_i\})$ defined in Section III are also included in the correlation matrices for α = .15, .25, .50, and .75, and for β = 1.1.

Table III shows the correlations between these ten indices. The off-diagonal entries give the intercorrelations for 1968, while the italicized diagonal entries give the intertemporal correlation for each index between 1954 and 1968. The first matrix is restricted to the subsample of 22 industries with H ≧ .16; the second matrix is restricted to the 32 industries with H ≥ .09; and the third matrix is for the entire set of 45 industries. The mean and standard deviation of each index are given in columns adjacent to the corresponding correlation matrix.

Consider first the sample of 22 highly concentrated industries with H ≥ .16. If an industry is in C-N equilibrium, this restriction includes only those industries for which the ratio of monopoly rent to total receipts is 16 percent or more of the ratio that would exist under pure monopoly (assuming the demand function has constant elasticity). Four features of this table are particularly worth noting. First, the correlation between C_4 and H is .614, a quite modest value when compared with the professional folklore mentioned in the introductory section. The fact that C_4 accounts for only 38 percent of the variance of H contradicts the assumption that C_4 is a good proxy for H for the study of highly concentrated industries. $H_m(.15)$ is the theoretically acceptable index having the highest correlation with C_4, .887. Hence C_4 accounts for about 79 percent of the variance in $H_m(.15)$, still considerably less than the level of association between concentration measures widely assumed. It should be remembered that α = .15 is nearly the extreme permissible value for the H_m measure. Second, $H_a(1.1)$, the additively adjusted Cournot measure, has a correlation of .98 with H. Since β = 1.1 is near the extreme permissible value for H_a to be a theoretically acceptable measure, this empirical result suggests that H is a statistically adequate proxy for the whole family of $H_a(\beta)$ concentration indices. Third, if α ≧ .5, H and $H_m(\alpha)$ have a correlation ≧ .986. The lowest correlation between H and $H_m(\alpha)$ in the table is .899, and occurs for the extreme value α = .15. This result suggests that only the parameter range .15 ≦ α ≦ .50 for $H_m(\alpha)$ has much chance of producing statistical results that can be distinguished from those that would be ob-

[35]The Rosenbluth index is mentioned in Marfels (1971a) who credits G. Rosenbluth (1961) with its introduction into the literature. Hall and Tideman (1967) independently rediscovered it and proposed the measure in a publication more readily accessible to American economists.

[36]The CICI index was originally proposed by J. Horvath (1970).

tained by using the Herfindahl index as an explanatory variable.[37] Fourth, the intertemporal correlations between 1954 and 1968 for the alternative concentration indices range from .905 for $H_m(.15)$ to .810 for H. C_4 has the second highest intertemporal correlation, .898. These results confirm the widespread belief that concentration indices measure an aspect of the firm size distribution that tends to change slowly over significant periods of time. However, it may be that the C_4 index exaggerates the intertemporal stability, since the family of $H_m(\alpha)$ indices have lower intertemporal correlations than C_4 for $\alpha \geqq .25$.

The correlation matrices for the less restricted sample of 32 industries with H $\geqq$ .09, and for the entire sample of 45 industries, show similar patterns. Most (not all) of the correlations become larger, as one would expect. Even so, it is interesting to note that C_4 accounts for only 63 percent of the variance in H in the 33-industry sample, and 69 percent in the full sample, providing further evidence of the substantial statistical differences in these indices.[38]

Some empirical studies have been more interested in the relationship between the change in industrial concentration and the change in some other industry characteristic than in level of concentration and the industry characteristic. Table IV shows the simple correlations between the changes of the various concentration indices between 1954 and 1968. The bottom line shows the correlation between the various concentration index changes and the industry growth rates in employment between 1954 and 1968, defined by ρ = [log(1968 employment) − log(1954 employment)]/14. The correlations between the various index changes are substantially smaller than the index level correlations reported in Table III. For example, the correlation between ΔC_4 and ΔH is .417, thus only 17% of the variance of changes in the H index is accounted for by changes in the C_4 index. The correlation between the H index change and average industry growth rate is small (−.22), but twice the size of the C_4 change and growth rate. It is not surprising that the change correlations are lower than the level correlations, but these results provide further evidence on the substantial differences in the statistical characteristics of many of the concentration indices.

[37]This assertion assumes that the concentration index is not highly correlated with other independent variables in regression analyses intended to estimate the partial effect of concentration on the dependent performance variable.

[38]There are three reasons why the correlation between C_4 and H is so much smaller for the Swedish unrestricted sample than for the unrestricted calculation reported by Scherer (1970) for American manufacturing. First, Census disclosure rules prevented reporting of H (and thus its use) for industries where H is very large. Second, the absolute size of the American market is much larger than the Swedish market, and concentration indices for most industries are significantly higher in the Swedish market and have less dispersion. Third, the sample of Swedish industries tended to include those industries with above-average levels of concentration.

TABLE IV

CORRELATIONS OF CHANGES IN INDUSTRIAL CONCENTRATION INDICES WITH EACH OTHER AND WITH INDUSTRY GROWTH RATE,* ρ, 1954 1968 ENTIRE SAMPLE (45 INDUSTRIES)

	ΔC_4	ΔH	ΔE	ΔR	ΔCICI	$\Delta H_m(.15)$	$\Delta H_m(.25)$	$\Delta H_m(.5)$	$\Delta H_a(1.1)$
ΔH	.417								
ΔE	.303	.906							
ΔR	.375	.929	.982						
ΔCICI	.779	.796	.671	.703					
$\Delta H_m(.15)$	.794	.804	.722	.760	.973				
$\Delta H_m(.25)$	.698	.884	.809	.838	.962	.985			
$\Delta H_m(.5)$	.552	.968	.892	.914	.901	.918	.970		
$\Delta H_a(1.1)$	.612	.940	.912	.848	.830	.940	.978	.959	
ρ	−.111	−.220	−.270	−.223	−.286	−.301	−.295	−.242	−.281

*Industry growth rate, ρ, = log (1968 employment) − log(1954 employment)/14.

Source: Data from Gunnar Du Rietz (1975)

TABLE V

CORRELATION, MEANS, AND STANDARD DEVIATIONS OF C_4 AND H INDEX FOR HIGHLY CONCENTRATED MANUFACTURING INDUSTRIES IN JAPAN*

Level of H Index	Number of Industries	Correlation between H and C_4	Mean		Standard Deviation	
			H	C_4	H	C_4
$H \geqq .36$	31	.127	.484	.975	.1100	.0485
$H \geqq .25$	87	.449	.366	.932	.1129	.0705
$H \geqq .16$	165	.701	.287	.863	.1183	.1014
$H \geqq .09$	250	.882	.230	.774	.1247	.1533

*1972 data from Japanese Fair Trade Commission (1975).

The Japanese Fair Trade Commission has recently published detailed data on the H index and concentration ratios for the largest 3, 4, 5, 8, and 10 firms for some 350 manufacturing industries. These data make it possible to compute the simple correlation coefficient of H and C_4 for highly concentrated industries, using H as a cut-off criterion for defining different levels of "highly concentrated" industries. Table V shows the means, standard deviations, and correlation between H and C_4 for four levels of high concentration. It is worth noting that C_4 accounts for less than 2 percent of the variance in H for very highly concentrated industries ($H \geqq .36$), while C_4 still accounts for slightly less than half the variance in H with a much lower cut-off criterion ($H \geqq .16$). These calculations clearly reveal that C_4 is a very poor statistical proxy for H in samples of highly concentrated industries.

Both the Swedish and Japanese data strongly support the conclusion that there is little justification for the bland, almost monolithic reliance on the widely available C_4 concentration measure for the empirical study of concentration and industry performance, particularly in the analysis of highly concentrated industries.

Concentration indices, of course, are only part of the story of the conditions that lead to competitive behavior. The simple Cournot-Nash model explored in this paper attaches high importance to the industry elasticity of demand, and much more remains to be done in establishing the empirical significance of entry. But it does seem clear that economists should be able to do better than the casual theorizing about concentration and the equally casual choice of a concentration index for empirical work that dominates the literature.

APPENDIX A

Theoretical Defects of Current Measures of Industrial Concentration

This appendix considers five measures of industrial concentration that have been discussed in the literature and estimated in Section IV:

TABLE VI

Performance of Alternative Industrial Concentration Measures in Terms of Criteria for Theoretically Adequate Measures

Does Index Satisfy Criterion:	1? (Index exceeds H if two or more "large" shares)	2? (Index equals H if only one "large" share)	3a? (Strict case-index converges to H index as maximum share s_1 goes to zero)*	3b? (Special case-index converges to $1/n$ for equal-size firms)	4a? (Strictly less concentrated industries should have smaller index values)	5 (Index a decreasing convex function of n_e)	6 (Index is one if $n_e = 1$, and approaches zero as n_e increases without bound)
Index							
1. 4-firm Concentration Ratio C_4	yes	no	no	no (converges to $4/n$)	sometimes	yes	yes
2. Herfindahl Index H	no	yes	yes	yes	yes	yes	yes
3. Entropy Index E	no	no (severely)	no	yes	yes	yes	yes
4. Rosenbluth Index R	no	no (severely)	no	yes	yes	yes	yes
5. "Comprehensive Industrial Concentration Index" CICI	yes	no	no	no	usually (anomalous case for $n = 2$: {2/3: 1/3} minimizes)	no (concave for $1 \leq n_e \leq 2$)	yes

*The convergence relevant here assumes an initial distribution $\{s_i\}$ with s_1 strictly nonzero and is concerned with the limit as $m \to \infty$ of m-fold increases in the distribution as the individual shares are reduced by $1/m$.

C_4, H, E, R, and CICI. It determines the extent to which they satisfy the criteria developed in the text for theoretically acceptable concentration indices. Table VI assesses these indices in terms of the six theoretical criteria developed in the text.

Table VI shows that none of the measures satisfy all three of the especially important theoretical criteria 1, 2, and 3a. The implications of these failures for each of the measures are discussed below.

The concentration ratio C_4 is by far the most widely used index of industrial concentration, and dominates cross-sectional studies of American manufacturing, since it has been computed and made available by the Census Bureau for a large number of industries and product classes since 1947. A major theoretical defect is that changes in the relative size of the largest four firms have no effect on the index, so long as their total share remains constant. It seems implausible that it makes no difference for industry behavior whether the top four firms each have a 20 percent share, or whether one firm has a 78.5 percent share and the next three firms each have only .5 percent. Its value does exceed the H index if there are two or more large firms (criterion 1), but does not behave reasonably for many different configurations of relative size of individual firms, including possible mergers of moderate size.

The Herfindahl index satisfies all of the criteria except the first, which it obviously must fail. This generally favorable performance is hardly surprising, since the text argued that the H index arises naturally out of the Cournot model. Its major theoretical defect is that it makes no allowance for tacit or overt collusion if several firms have large shares, a situation which makes a C-N equilibrium rather implausible. Whether such collusion would be expected to obtain high returns depends, of course, on other industry characteristics not captured in the size distribution data, such as the demand elasticity, the ease of entry, and the feasibility of high rates of expansion by other firms in the industry. And in a study of firm or industry profit rates these other factors should be taken into account, as well as the concentration index.

The entropy index, E, fails all three of the important criteria 1, 2, and 3a. It is always *less* than the H index, except for equality if the industry is composed of equal-size firms. Thus it does not in general yield an index as large as that produced by the Cournot model which ignores the additional effect of collusion. An equally serious defect is the overwhelming influence that a large number of very small firms can have on the index. It is easily demonstrated that if one firm has a 99.99 percent share of industry sales, if the remaining sales are from an indefinitely large number of small firms, the equivalent number of equal-size firms tends to infinity, i.e., industry structure becomes perfectly competitive.[39] For the entropy index to be a

[39]Proof: Let the largest firm have share s_1, let the remaining m firms have equal share k/m, where $k = (1 - s_1)$. Then

reasonable measure of industrial concentration for studying the degree of competition in a sample of industries, one would have to discard completely the Cournot theory, modified to allow for collusion. Perhaps one could argue that if an industry includes an extremely large number of small firms, this indicates that entry is very easy, and so even if industry sales are largely due to one or several large firms, the potential entry will make the industry completely competitive in price and output. There may be some truth to the ease of entry argument, but this alternative theory essentially says that the size distribution is irrelevant; the only relevant dimension to the problem is the ease of entry. But this alternative theory has nothing to say about the relevance of the size distribution if entry is *not* extremely easy. Hence the entropy index seems to have highly unreasonable theoretical properties as a measure of industry concentration.

The Rosenbluth index R suffers all of the theoretical defects of the entropy index, except that its behavior seems to be dominated even more strongly by small firms in summarizing empirical size distributions. Its usefulness in the analysis of highly concentrated industries for departures from competition seems highly dubious.

The CICI was developed in a highly intuitive way by Horvath (1970), in order to obtain a measure that refined what he considered to be the desirable characteristics of the H index. The CICI does indeed satisfy the first criterion of being larger than the H index if there are two or more firms, since it is larger term-by-term than H. However, it fails to satisfy the important conditions 2, 3a, and 3b. It exaggerates the role of the first firm if only one of the firms is "large" relative to the Cournot theory, since it takes the share of the firm, and not its square (condition 2). Its order behaves according to the Cournot theory, although it converges to a value three times the size of the H index as the share of the largest firm approaches zero, and as the number of firms increases without bound. The asymmetry of its leading term generates several bizarre characteristics, since in a two-firm industry, the index minimum occurs when the shares are (2/3, 1/3) and not when the firms are of equal size, each with sales of 1/2. Furthermore, the index is decreasing, but *concave*, for values of n_e in the interval from one to two.[40] There is no underlying economic theory

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$$E = \prod_{i=1}^{m+1} s_i^{s_i} = s_1^{s_1} (k/m)^k,$$

$$n_e = \frac{1}{E} = \frac{1}{s_1^{s_1} (k/m)^k}$$

and

$$\lim_{m \to \infty} \frac{1}{s_1^{s_1} (k/m)^k} = \infty .$$

[40]Marfels (1972) also notes the nonconvexity and odd behavior of this index.

to justify these odd properties, they just happen to be among the implications of the formal definition of the index. They do suggest the logical hazards of an intuitive approach to the definition of measures, since it is easy to include by accident anomalous properties.

On balance, all of the indices of industrial concentration discussed in this appendix have significant theoretical deficiencies. The H index seems most satisfactory from most standpoints, except for the way it ignores collusion. The concentration ratio C_4 is larger than H, as seems reasonable, but completely ignores the relative size of the large firms. The multiplicatively-modified Cournot measure $H_m(\alpha; \{s_i\})$ and the additively-adjusted Cournot measure $H_a(\beta; \{s_i\})$ defined in the main text satisfy all of the theoretical criteria presented here. Section IV shows that the statistical behavior of the H_a index differs very little from the H index in a sample of Swedish manufacturing industries. However the $H_m(\alpha)$ family may prove to be a useful point of departure for theorizing about the size distribution and collusion and for empirical experimentation, to see whether it performs significantly better than C_4 or H in studies of industrial concentration and the degree of competition.

Appendix B

Alternative Concentration Measures for Swedish Manufacturing Industries

Table VII presents various concentration measures for 45 Swedish manufacturing industries. A more detailed description of the data on which they are based is provided in Du Rietz (1975). Several industries were modified somewhat from the official industry classifications used for Swedish manufacturing statistics.

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TABLE VII

1968 Concentration Measures for 45 Swedish Manufacturing Industries

	Industry Code Number	Number of Firms	C_4	H	E	R	CICI	$H_m(.15)$	$H_m(.25)$	$H_m(.5)$	$H_m(1.0)$	$H_a(1.1)$
1	351310	13	.86227	.41094	.24237	.28647	.67166	.61568	.53319	.45149	.41674	.52723
2	351320	51	.62629	.13041	.06685	.07645	.37768	.31405	.22582	.15585	.13302	.19018
3	356000	234	.27108	.02854	.01166	.01230	.13622	.08279	.04872	.03154	.02865	.04041
4	371010	19	.54219	.10594	.08941	.11643	.31300	.31181	.20892	.13155	.10835	.16321
5	371020	6	.94976	.31826	.26686	.33546	.66402	.64080	.53547	.41061	.33896	.46644
6	371030	73	.53737	.09072	.04523	.05026	.29697	.24255	.16478	.10847	.09225	.13375
7	372030	18	.90633	.41535	.25897	.30843	.69055	.64800	.56417	.47175	.42556	.54265
8	372040	75	.46169	.07811	.03443	.03725	.27813	.19818	.13303	.08986	.07892	.11259
9	381100	164	.44404	.07151	.02256	.02030	.26427	.17406	.11827	.08137	.07217	.10119
10	381200	87	.47172	.07860	.03501	.03805	.27638	.20460	.13721	.09156	.07956	.11404
11	381300	722	.09431	.00578	.00276	.00323	.03957	.01622	.00852	.00598	.00578	.00767
12	381910	34	.65979	.22106	.09578	.09929	.48907	.39713	.31388	.24589	.22347	.30030
13	381920	46	.57645	.09740	.06376	.08237	.29625	.27605	.18576	.11906	.09937	.14714
14	381930	59	.66343	.25024	.09218	.08071	.51702	.41526	.33817	.27391	.25253	.33170
15	381940	149	.37264	.04867	.02219	.02516	.19670	.13820	.08603	.05552	.04904	.07042
16	381950	83	.42312	.06482	.03265	.03827	.24264	.17966	.11566	.07514	.06548	.09474
17	381990	628	.15646	.01063	.00391	.00417	.07077	.02999	.01650	.01122	.01064	.01437
18	382100	3	1.00000	.98413	.94971	.97916	.99209	.98618	.98485	.98420	.98413	.98869
19	382200	81	.60682	.17189	.06494	.05945	.43124	.33764	.25744	.19395	.17392	.23764
20	382310	119	.34642	.04140	.02224	.02718	.16217	.12387	.07431	.04699	.04166	.06025
21	382320	38	.57106	.09838	.06230	.08018	.30571	.27160	.18404	.11950	.10032	.14757
22	382410	27	.72490	.25500	.12282	.12749	.53569	.46107	.37353	.29263	.26007	.34854
23	382420	74	.50612	.08500	.04748	.05645	.27514	.23782	.15795	.10184	.08638	.12652
24	382490	276	.33348	.04023	.01271	.01182	.18317	.10804	.06771	.04495	.04045	.05673
25	382510	4	1.00000	.66086	.51171	.61905	.83562	.80668	.75128	.69192	.66560	.76669
26	382590	37	.87285	.31028	.18391	.20575	.61320	.57103	.47592	.37265	.32149	.43104
27	382991	153	.41002	.05813	.02314	.02405	.22520	.15660	.10124	.06671	.05865	.08367
28	382992	24	.70675	.14812	.10465	.13734	.40554	.38439	.27866	.18692	.15302	.22445

29	382993	32	.93019	.51880	.31311	.30431	.75437	.70946	.64096	.56436	.52678	.63841
30	382999	212	.48961	.09574	.03389	.02722	.31441	.23192	.16091	.11063	.09688	.13684
31	383100	58	.92286	.70568	.40456	.23291	.84630	.78482	.74934	.71704	.70674	.77842
32	383200	61	.83830	.36847	.19588	.19920	.63656	.57586	.48986	.40712	.37358	.47922
33	383300	49	.79602	.27188	.14392	.13699	.59157	.53446	.44327	.34094	.28581	.38781
34	383910	8	.93756	.37026	.27843	.36056	.68705	.65890	.56455	.45275	.38857	.51549
35	383920	5	.99765	.34955	.32006	.38147	.69809	.69214	.58687	.45412	.37405	.51288
36	383930	16	.88417	.36875	.22148	.26360	.64561	.59674	.50735	.41634	.37598	.48726
37	383990	70	.55160	.10012	.04929	.05515	.31648	.25973	.17999	.12023	.10201	.14725
38	384110	36	.84551	.22039	.15430	.19056	.52898	.50692	.39739	.28425	.23136	.32998
39	384120	106	.24827	.02660	.01593	.01946	.11689	.08133	.04610	.02931	.02669	.03815
40	384130	6	.89153	.22425	.19991	.24866	.53832	.53454	.41470	.29195	.23569	.34142
41	384210	4	1.00000	.50071	.45877	.53395	.83120	.80700	.73171	.62093	.53717	.68615
42	384310	7	.99847	.38247	.34877	.40591	.71885	.71535	.61401	.48487	.40650	.54810
43	384320	244	.53881	.12366	.03160	.02001	.36188	.27314	.20434	.14646	.12617	.17383
44	384400	16	.82136	.44075	.23028	.23828	.68024	.60240	.53007	.46581	.44330	.54590
45	384900	20	.49141	.09021	.07338	.09622	.28470	.26922	.17563	.10986	.09182	.13787

The industries in this table are:

1 Basic Plastics
2 Semi-Fabricated Plastics
3 Plastic Goods
4 Iron and Steel Works
5 Ferro-Alloy Works
6 Iron and Steel Castings
7 Non-Ferrous Products
8 Non-Ferrous Castings
9 Tools
10 Metal Furniture
11 Metal Construction
12 Metal Containers
13 Metal Wire, Screen, Lines and Cables
14 Nails, Screws and Bolts
15 Other Metal Goods for Construction
16 Household Metal Goods
17 Misc. Metal Goods
18 Stationary Motors & Turbines
19 Agricultural Machinery
20 Metal-Working Machinery
21 Wood-Working Machinery
22 Pulp and Paper Machinery
23 Cranes & Excavation Machinery
24 Other Production Machinery
25 Computers
26 Other Office Machines
27 Lifts and Hoists
28 Fluid Pumps
29 Machinery Parts
30 Other Machinery
31 Electric Motors, Generators, Electric Apparatus
32 Telephone Products
33 Electric Household Appliances
34 Electric Filaments & Cables
35 Batteries (including Storage)
36 Incandescent and Fluorescent Lamps
37 Misc. Electric Industry
38 Shipyards
39 Boat Construction
40 Boat and Ship Motors
41 Railway Cars
42 Automobiles
43 Auto Motor Parts, Trailers
44 Bicycles and Motorcycles
45 Other Transportation

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