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OPTIMAL COORDINATION OF AGGREGATE STABILIZATION POLICY AND PRICE CONTROLS: SOME SIMULATION RESULTS

BY SURENDER K. GUPTA, LAURENCE H. MEYER, FREDRIC Q. RAINES, AND TZYH-JONG TARN*

In this paper we present a deterministic, discrete time macrodynamic model that allows for the introduction of varying degrees of price control as well as traditional stabilization policy, and that specifies the formation and implications of price expectations both when price controls are on and when they are off. We then solve by numerical methods for the stabilization and price control policy vectors that minimize a cost function over an eight period horizon giving weight both to departure from policy goals and to policy costs.

This simulation is carried out for three alternative versions of the inflation process and for two different sets of initial conditions. The three alternative theories of the inflation process (introduced by changing parameter values in the basic model) may be described as follows: (1) The Phillips-Lipsey (PL) "traditional" Phillips curve approach which ignores the feed-back of price expectations on actual prices; (2) The Friedman–Phelps–Mortensen (FPM) approach which denies the existence of a long-run trade-off; and (3) The "eclectic" (E) approach which accepts the importance of price expectations but permits a long-run trade-off. The different initial conditions refer to the economic environment we contrive at the outset and to which policy must respond. The two basic environments we consider are: (a) excess supply in the output market plus inherited inflation generated by previous excess demand and (b) inflation accompanied by excess demand in the commodity market.

The organization of the paper is as follows. In Section I, we present the basic model. Section II defines the cost function and Section III presents the assumed parameter values.[1] The base simulations which indicate the dynamic performance of the economy in the absence of controls, the optimal policy simulations, and the inferences we draw are presented in Section IV. The concluding section discusses some further useful avenues of research suggested by the present study.

I. DEVELOPMENT OF THE MODEL

The basic model consists of a multiplier accelerator approach to income determination and a Phillips curve approach to price determination. This type of approach to short-run dynamics can be found, for instance, in Laidler [4].

Demand for Output

Aggregate demand is the sum of consumption, investment, and government expenditures. The aggregate demand equation is

$$E_k = \alpha Y_{k-1} + w\left(\frac{\overline{W}}{P_{k-1}} + \overline{K}\right) + a(Y_{k-1} - Y_{k-2}) + A + G_k \tag{1}$$

where E_k = aggregate demand in period k, G_k = government expenditures in period k, A = autonomous component of private demand, and Y_k = total output

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[1] Due to space limitations we present a very condensed exposition of the model. For a more detailed discussion, see [6].

in period k, all in real terms; and where P_k = price level in period k, $\overline{W}$ is the nominal value of cash balances plus government securities, and $\overline{K}$ is the real value of equities and tangible assets.

Real consumption expenditures [represented by the first two terms in (1)] are seen to be a simple function of lagged income (equals output), a wealth effect, and an autonomous component. We include a wealth term in the consumption function in order to provide a link between the price level and aggregate demand. In a more complete model, the primary channel through which changes in the price level affect aggregate demand is via their influence on financial markets and the interest rate. Our model does not, however, include a financial sector. Real gross investment (represented by the third term in (1)) is determined by a simple accelerator plus an autonomous component. These specifications are admittedly simplistic. They ignore the role of permanent income on consumption and financial considerations on investment. Moreover, in an environment of excess supply, the accelerator mechanism may only be weakly operative, if at all. Nevertheless, they capture to a first approximation the same sorts of determinants that would be present in more refined specifications.

A, the autonomous component of private demand is initially set equal to zero. Changes in A are used to generate the initial conditions for our policy runs. G_k, government expenditures in period k is defined as

$$G_k = \overline{G} + u_{k-1} \tag{2}$$

where $\overline{G}$ is the "normal" value of government expenditures and u_{k-1} is the deviation from the normal level introduced for stabilization purposes; i.e. u_{k-1} is the aggregate stabilization policy instrument in our model.

Supply of Output

The position adopted in this paper is that output supplied is responsive to aggregate demand, though, in a dynamic context, not necessarily equal to demand. Thus we specify

$$Y_{k+1} - Y_k = g(E_k - Y_k), \qquad 0 < g < 1, \tag{3}$$

subject to the restriction that

$$Y_k \leq Y_k^* \quad \text{for all } k \tag{3'}$$

where Y_k^* is defined as potential output in period k.

The restriction on Y_k is self-explanatory. However, we postpone consideration of the short-run determinants of potential output until we have examined price determination and the role of price controls in the model.

The Price Level: Actual and Expected

Actual Price Level

Changes in the price level, in the absence of price controls, are determined by a "Phillips curve" type of relation, equation (4a).

$$
\dot{P}_{k+1} = \begin{cases} h_0 + h_1 \dfrac{E_k - Y_k^*}{Y_k^*} + f\dot{P}^e_{k+1}, & \text{if } \gamma_k = 0 \text{ or} \quad (4a) \\ & \text{if } \gamma_k \neq 0 \text{ and } h_0 + h_1 \dfrac{E_k - Y_k^*}{Y_k^*} + f\dot{P}^e_{k+1} \leq m_k \\ m_k, \quad \text{if } \gamma_k \neq 0 \text{ and } h_0 + h_1 \dfrac{E_k - Y_k^*}{Y_k^*} + f\dot{P}^e_{k+1} > m_k & (4b) \end{cases}
$$

where

$$
\gamma_k = \begin{cases} 0, & \text{if } m_k < 0 \text{ (price controls off)} \quad (5a) \\ 1, & \text{if } m_k \geq 0 \text{ (price controls on)} \quad (5b) \end{cases}
$$

The most general form of the Phillips curve identifies three influences on price change: (1) excess demand in the output market;[2] (2) price expectations; and (3) the tendency of prices to creep upward even in the presence of excess capacity and the absence of inflationary expectations, represented by the positive constant, h_0.

In our policy simulations we employ three alternative versions of the Phillips curve: (1) the Phillips [9]–Lipsey [5] "traditional" version in which $h_0 > 0$ and $f = 0$; (2) the natural rate or accelerationist (FPM) version suggested by Friedman [2], Phelps [8], and Mortensen [7] in which $h_0 = 0$ and $f = 1$; and (3) an eclectic (E) version in which $h_0 > 0$ and $0 < f < 1$. Further, we assume it to be a characteristic of all three models that final output prices are more responsive upward to excess demand than they are downward to excess supply. Hence we have assumed throughout that h_1 takes on a larger value when $E_k - Y_k^* > 0$, (h_1'), than when $E_k - Y_k^* < 0$, (h_1'').

The variable γ_k indicates the status of price-controls: "on" ($\gamma_k = 1$), or "off" ($\gamma_k = 0$). Price controls are turned on or off by means of the value selected for m_k. If the program selects $m_k < 0$, γ_k is set equal to zero and no price controls are applied in period $k + 1$.[3] If the program selects $m_k \geq 0$, γ_k is set equal to one, activating controls. The actual inflation rate under controls is then given by either equation (4a) or (4b). If the actual inflation rate that would prevail under price controls in $k + 1$ is less than m_k, equation (4a) determines the inflation rate. Otherwise, equation (4b) holds and $\dot{P}_{k+1} = m_k$.

Expected price changes. The most common ex ante behavioral hypothesis concerning expectations is that of simple adaptive expectations, which, with respect to the rate of inflation, is given by equation (6a), where the weight of more remote inflation experience becomes increasingly important as λ approaches zero.

[2] The relevant supply concept in a measure of the excess demand gap is potential output rather than actual output. This is because any existing gap between Y_k and Y_k^* must be due to one or both of (1) a failure of demand; (2) a planned transitory adjustment lag, neither of which should put upward pressure on prices. Conversely, if E_k exceeds Y_k^*, prices should tend to rise even if actual output has not reached potential.

[3] The use of the γ_k variable is necessitated by the fact that the m_k variable cannot itself take on some value which implies no controls. Thus, $m_k = 0$ means a complete freeze, not the absence of controls.

Note also in conjunction with equation (4a), that, if $f > 0$, the larger is λ, the faster is the feedback of past inflation on current inflation.

$$\dot{P}^e_{k+1} = \begin{cases} (1-\lambda)\dot{P}^e_k + \lambda \dot{P}_k, & \text{if } \gamma_k = 0 \text{ and } \gamma_{k-1} = 0 \quad \text{(6a)} \\ m_k, & \text{if } \gamma_k \neq 0 \quad \text{(6b)} \\ \dot{P}^{enc}_{k+1} & \text{if } \gamma_k = 0 \text{ and } \gamma_{k-1} \neq 0, \quad \text{(6c)} \end{cases}$$

Equation (6a) holds if there are no price controls this period or last period and (6b) holds if there are price controls this period; i.e., if there are price controls this period, the expected inflation rate is assumed to be the maximum allowable rate. Note that since the expected rate of inflation enters concurrently into the determination of the actual inflation rate, it follows that price controls can influence the actual inflation rate in $k + 1$ even though $\dot{P}_{k+1} < m_k$; i.e., price controls can operate indirectly on the actual inflation rate via their influence on the expected inflation rate.

There is one more possibility—no controls this period, but controls last period. Equation (6a) is not suitable in this case because it assigns weight to the price controlled inflation rate last period, oblivious to the fact that price controls are no longer operative. To handle this case we define a "no-control" expected inflation rate, $\dot{P}^{enc}_{k+1}$, given by equation (7). If controls are inoperative for two consecutive periods $\dot{P}^{enc}_{k+1}$ becomes identical with $\dot{P}^e_{k+1}$, as defined by (6a). However if controls were applied last period, (but not this period) equation (7b) obtains. According to (7b), the expected

$$\dot{P}^{enc}_{k+1} = \begin{cases} \dot{P}^e_{k+1}, & \text{if } \gamma_{k=0} \text{ and } \gamma_{k-1} = 0 \quad \text{(7a)} \\ \rho[(1-\lambda)\dot{P}^{enc}_k + \lambda \dot{P}^{nc}_k] + (1-\rho)\dot{P}_k, & \text{otherwise} \quad \text{(7b)} \end{cases}$$

where

$$\dot{P}^{nc}_{k+1} = \begin{cases} \dot{P}_{k+1}, & \text{if } \gamma_k = 0 \text{ and } \gamma_{k-1} = 0 \quad \text{(8a)} \\ h_0 + h_1 \dfrac{E_k - Y^*_k}{Y^*_k} + f\dot{P}^{enc}_{k+1}, & \text{otherwise} \quad \text{(8b)} \end{cases}$$

inflation rate when controls are removed is a weighted average of actual price experience under controls and a "shadow" price expectations effect that reflects the rates of inflation that would have been expected in the absence of controls. This in turn depends on shadow inflation rate series given by equation (8b).

To clarify the specification of (7b), assume that $\rho = 1$. In this case, the expected inflation rate if price controls were removed would depend exclusively on the "shadow" inflation rate variable given by (8b) and the past history of that variable. This assumes that economic agents implicitly calculate a series of hypothetical actual rates assuming no price controls and use these to compute an expected rate next period if controls are removed. However, with $\rho = 1$, any direct impact of controls on expectations through its influence on the actual inflation rate would be precluded. To avoid such a narrow interpretation, we set $1 > \rho > 0$. Thus, the relative weight of actual experience under price controls varies inversely with ρ.

The assumptions embodied in this formulation, even for ρ approaching 1.0, may still be overly optimistic about the ability of price controls to moderate inflation. One potentially important behavioral aspect that the model omits is the attempt to "catch-up" after controls are removed. To introduce this feature into our equation explaining the inflation rate, we could include an additional term specifying the price rise associated with "catching-up" as proportional to the gap between the actual price level and the price level that would have prevailed in the absence of controls.[4]

Potential Output

The level of potential output is given by

$$Y_k^* = [1 + \Omega(\dot{P}_k - \dot{P}_k^e) - \beta\gamma_{k-1}\phi(\dot{P}_k^{enc} - m_{k-1})]\bar{Y} \tag{9}$$

where

$$\phi(\dot{P}_k^{enc} - m_{k-1}) = \begin{cases} 0, & \text{if } \dot{P}_k^{enc} - m_{k-1} \leq 0 \quad \text{(10a)} \\ \dot{P}_k^{enc} - m_{k-1}, & \text{if } \dot{P}_k^{enc} - m_{k-1} > 0 \quad \text{(10b)} \end{cases}$$

The quantity $\bar{Y}$ is the maximum feasible level of output in the *PL* version in which $\Omega = 0$ and is the maximum level of output that can be sustained without accelerating inflation in the FPM and E versions in which $\Omega > 0$. Succinctly put, the Friedman–Phelps–Mortensen theory states that employees will tend to overestimate real wages during a period of accelerating inflation due to the lag in expected inflation adjusting to actual inflation. This overestimate (an underestimate would obtain in reverse circumstances) leads to a temporary outward shift in labor supply curves and hence in potential supply, and to a transitory reduction in unemployment rates as acceptance wages appear to be more readily met and search times are reduced. Thus, potential output, Y_k^*, will depend on the gap between actual and expected inflation. This effect is also included in the eclectic version.

The specification of the potential output equation also takes account of the potential decline in supply of output associated with the imposition of price controls. If $\gamma_{k-1} = 0$, the additional term drops out. If price controls are on, $\gamma_{k-1} = 1$ and the supply effect is assumed to be proportional to the reduction in the rate of inflation economic units attribute to the operation of price controls. If $\dot{P}_k^{enc} \leq m_{k-1}$, then economic units believe that price controls were inoperative; i.e., the maximum allowable rate was higher than the rate expected for that period. In this case, price controls do not affect the supply of output. On the other hand, if $\dot{P}_k^{enc} > m_{k-1}$, economic units find that price controls are biting with the result that the potential supply of output will decline. If there is excess supply of output, then output is not likely to be affected. If there is excess demand, on the other hand, a decline in potential output will carry actual output lower also. This specification, therefore, restricts the impact on the supply of output to situations in which there is no excess supply in the output market.

The decline in potential output could also be made to depend on the *cumulative* application of controls—for instance, on the difference between the actual price level and the price level that would have been expected to prevail in the absence of controls.

[4] We are indebted to the referee for drawing our attention to the possibility of a catch-up effect. We intend to explore the implications of this effect in subsequent research.

Additional Restrictions on the Use of Price Controls

We impose two restrictions on the use of price controls. First, price controls are only introduced if they will actually limit the inflation rate; i.e.,

(11) $$m_{k-1} \leq \dot{P}_k^{nc}$$

Secondly, price controls are not imposed if prices would be falling in the absence of controls: If

(12) $$h_0 + h_1 \frac{E_{k-1} - Y_{k-1}^*}{Y_{k-1}^*} + f\{\rho[(1-\lambda)\dot{P}_{k-1}^{enc} + \lambda \dot{P}_{k-1}^{nc}] + (1-\rho)\dot{P}_{k-1}\} \leq 0$$

then $\gamma_{k-1} = 0$.

II. The Cost Functional

In order to explore the optimal coordination of the two policy instruments, we define a cost functional to be minimized as given in equation (13).

(13) $$J = \sum_{k=1}^{N} \Bigg[q_1(k)\left(\frac{\bar{Y} - Y_k}{\bar{Y}} \times 100\right)^2 + q_2\left(4\frac{P_k - P_{k-1}}{P_{k-1}} \times 100\right)^2 + r_1\left(4\frac{u_{k-2} - u_{k-3}}{\bar{G}} \times 100\right)^2 + \gamma_{k-1}\{r_2 + r_3(k-1) + r_4[4(\dot{P}_k^{nc} - m_{k-1}) \times 100]^2\} \Bigg]$$

where N is the time horizon for the optimal policy,

(14a), (14b) $$q_1(k) = \begin{cases} q_1', & \text{if } \bar{Y} - Y_k > 0 \\ q_1'', & \text{if } \bar{Y} - Y_k < 0 \end{cases}$$

and

(15a), (15b) $$r_3(k-1) = \begin{cases} \bar{r}, & \text{if } m_{k-1} \neq m_{k-2} \\ 0, & \text{if } m_{k-1} = m_{k-2} \end{cases}$$

where the initial conditions for the policy instruments are $u_{-2} = u_{-1} = \gamma_{-1} = 0$.

Costs are imposed for deviating from target values of output and inflation and for the use of policy instruments. The target value of output is $\bar{Y}$, the maximum feasible output in the PL model. Although output can exceed $\bar{Y}$ in the FPM and E models where $\Omega > 0$, we assume $\bar{Y}$ is the target value of output in all three models because it is the maximum feasible level of output that can be achieved without imposing accelerating inflation rates necessary to sustain unanticipated inflation in our model. We impose a higher cost if output is below $\bar{Y}$,(q_1'), than if output exceeds $\bar{Y}$, (q_1'').

The target inflation rate is assumed to be zero. Costs are therefore imposed for any departure from price stability.

The last two terms represent the costs associated with the use of the instruments. The parameter r_1 reflects the costs associated with changing the level of our aggregative stabilization instrument. The difficulties, delays, and potential wastes in the implementations of tax and expenditure changes for stabilization purposes are well recognized. Our specification allows for three components of the cost of price controls. Thus r_2 represents the fixed cost associated with the existence of price controls, r_3 represents the incremental costs associated with changes in price controls, and r_4 represents a variable cost associated with the use of controls. The "use" cost of price controls is assumed to vary with the amount of inflation the controls prevent as a measure of the extent to which controls interfere with private decision making and introduce distortions into the allocation of resources.

III. The Value of the Parameters

The complexity of the model precludes the derivation of analytical results. We have therefore selected values for the various parameters in the model and explored the properties of the models through a series of simulation experiments. It is convenient to divide the model parameters into three categories: behavioral parameters, exogenous variables, and cost parameters. The values of these model parameters are presented in Tables 1a, 1b, 2 and 3.

TABLE 1a
BEHAVIORAL PARAMETERS WITH THE SAME VALUE IN ALL MODELS

a	α	w	g	h_1'	h_1''	λ	β	ρ
1.0	0.7	0.05	0.5	.074	.037	0.2	0.5	0.9

TABLE 1b
BEHAVIORAL PARAMETERS WITH DIFFERENT VALUES

	h_0	f	Ω
PL	0.0074	0	0
FPM	0	1.0	0.25
E	0.0074	0.5	0.25

TABLE 2
EXOGENOUS VARIABLES

$\bar{W}$	$\bar{K}$	$\bar{A}$	$\bar{G}$
30,000	1,700	0	200

TABLE 3
COST PARAMETERS

q_1''	q_1'	q_2	r_1	r_2	r_3	r_4
0.1	1.0	1.0	0.025	7.0	2.0	0.8

IV. Simulation Results

A. *Initial Conditions*

We begin by contriving an inflationary environment accompanied by either excess supply or excess demand for output by altering the autonomous component in the aggregate demand equation. Initially, we set $Y = \bar{Y} = 1{,}000$ and $P = 100$. To generate initial conditions, which are identical for all three models, we specify a single set of parameter values.[5]

(1) *Excess supply and inflation*—To impose both inflation and excess supply on the model, we introduce a +35 disturbance in period 1, maintain this value through period 5, then reduce A by 60 and maintain this lower value throughout the base and policy simulations. By period 16 this generates an inflation rate of about $9\frac{3}{4}$ percent (at annual rate) and an output level 8.3 percent below $\bar{Y}$ (and declining). This is the economic environment at the start of the stabilization horizon in the excess supply case.

(2) *Excess demand and inflation*—To impose inflation and excess demand for output, we introduce a +25 disturbance in the aggregate demand equation in period 1 and maintain this value throughout the base and policy simulations. This generates an inflation rate of about $15\frac{1}{4}$ percent by period 17, with output at Y^*.

B. *Base Simulations*

Next we determine how the system would behave for the three alternative versions of the inflation process if we kept policy instruments at their initial values ($G_k = \bar{G}$ and $\gamma_k = 0$). The results of these "no policy" or base simulations are depicted in Figures 1 through 10.

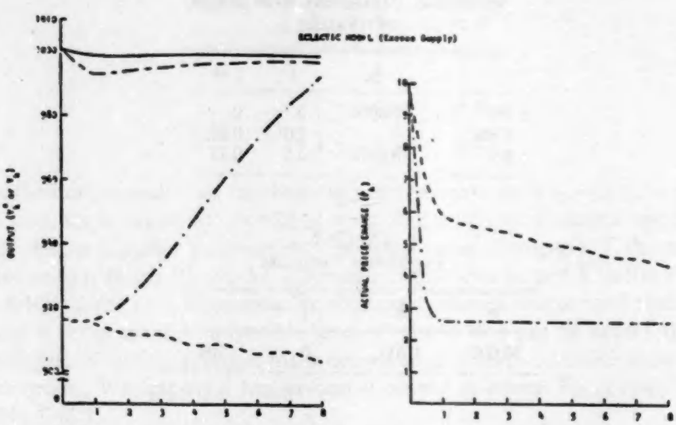


FIG 1(a) FIG 1(b)

[5] In particular, we set $h_0 = 0.0074$, $\Omega = 0.5$ and $f = 1.0$ for the excess supply run and we set $h_0 = 0.0074$, $\Omega = 0$, and $f = 1.0$ for the excess demand run.

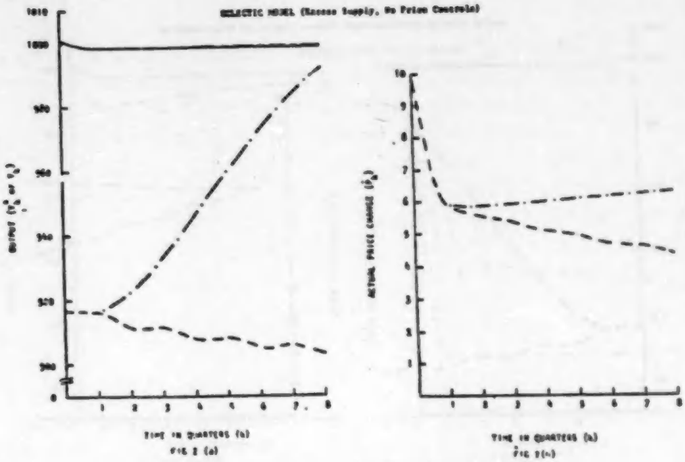


FIG 2 (a) FIG 2(b)

J_{min} = 392.75, (869.49) $J_{min}(Y)$ = 229.95, (461.62) $J_{min}(\dot{P})$ = 292.88, (207.87) $J_{min}(\dot{u})$ = 60.92 (0.0)

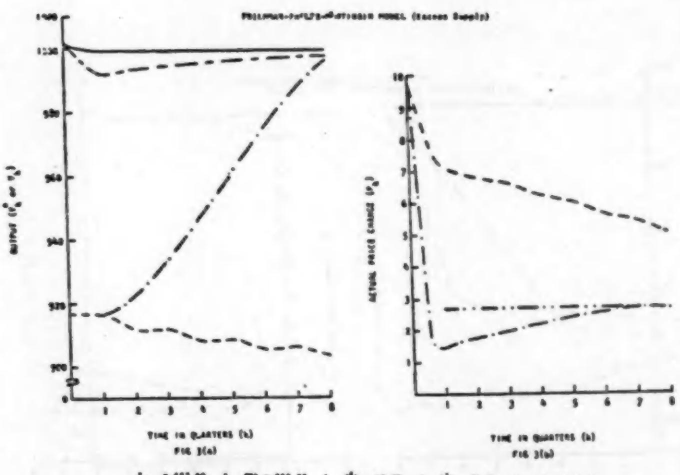


FIG 3(a) FIG 3(b)

J_{min} = 452.00, (967.61) $J_{min}(Y)$ = 227.87, (662.98) $J_{min}(\dot{P})$ = 40.72, (304.43) $J_{min}(\dot{u})$ = 39.32, (0.0) $J_{min}(u)$ = 126.09 (0.0)

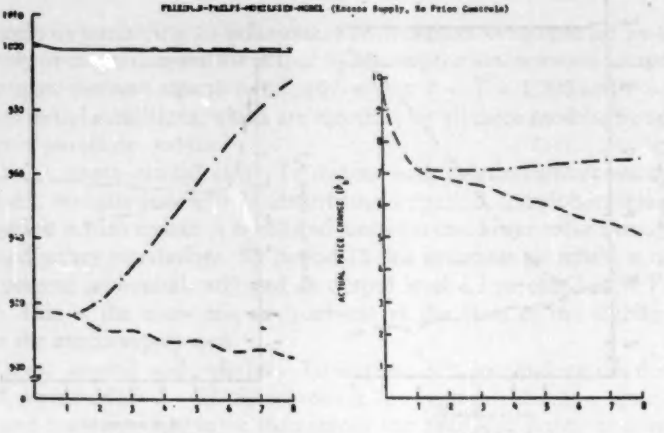


FIG 4(a) FIG 4(b)

J_{min} = [illegible], $J_{min}(Y)$ = [illegible], $J_{min}(\dot{P})$ = [illegible], $J_{min}(\dot{u})$ = 53.78 (0.0)

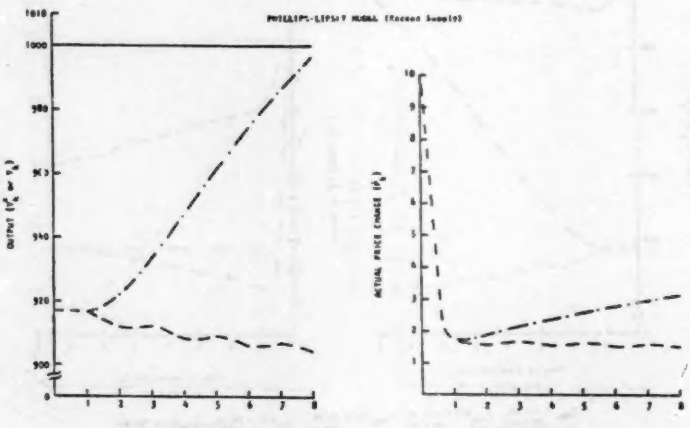


FIG 5(a) FIG 5(b)

J_{min} = 336.34 (677.32), $J_{min}(Y)$ = 225.60 (657.13), $J_{min}(\dot{P})$ = 49.78 (20.39), $J_{min}(\dot{u})$ = 60.96 (0.0), $J_{min}(m)$ = 0.0 (0.0)

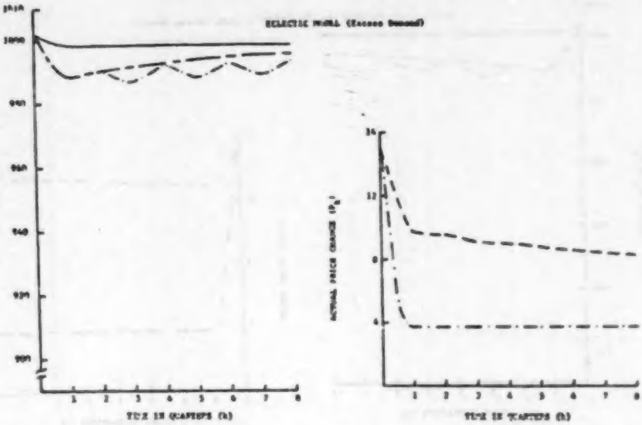


FIG 6(a) FIG 6(b)

$J_{min} = 303.03$, $J_{min}(Y) = 7.52$, $J_{min}(\dot P) = 111.29$, $J_{min}(\dot u) = 17.23$, $J_{min}(e) = 166.90$
(646.28) (0.14) (646.16) (0.0) (0.0)

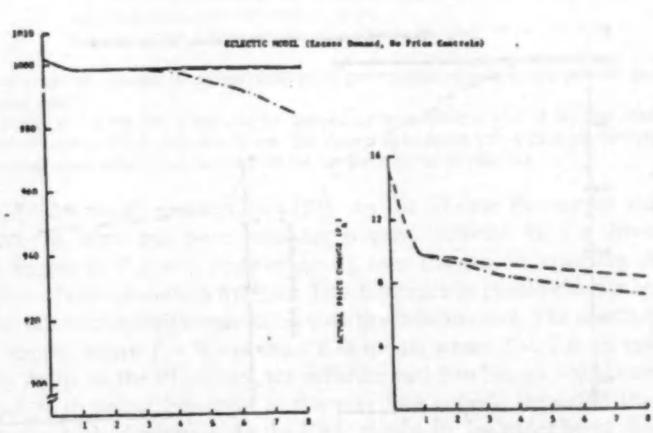


FIG 7(a) FIG 7(b)

$J_{min} = 397.46$, $J_{min}(Y) = 3.10$, $J_{min}(\dot P) = 354.90$, $J_{min}(\dot u) = 27.66$
(648.29) (0.26) (648.16) (0.0)

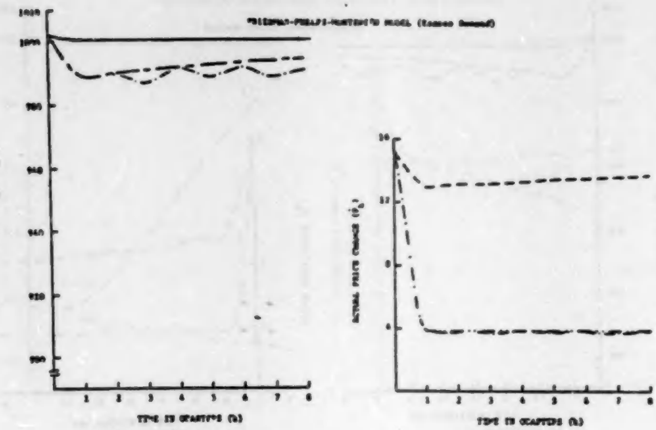


FIG 8(a) FIG 8(b)

$J_{min} = 494.82$, $J_{min}(Y) = 8.65$, $J_{min}(\dot{p}) = 117.88$, $J_{min}(\dot{u}) = 17.90$, $J_{min}(u) = 351.31$
(1431.73) (0.0) (1431.73) (0.0) (0.0)

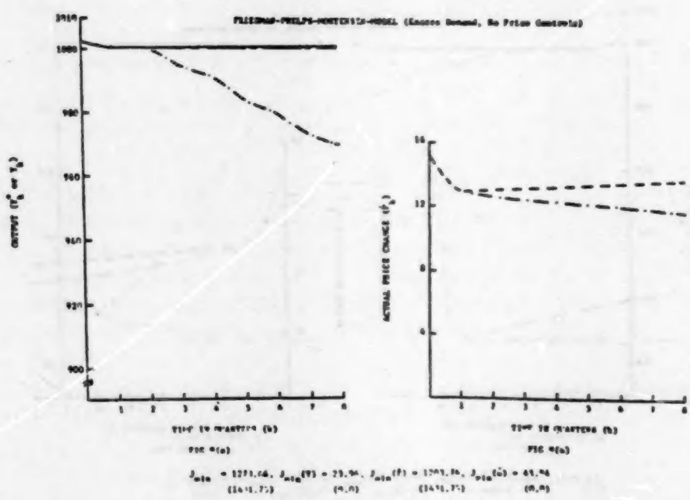
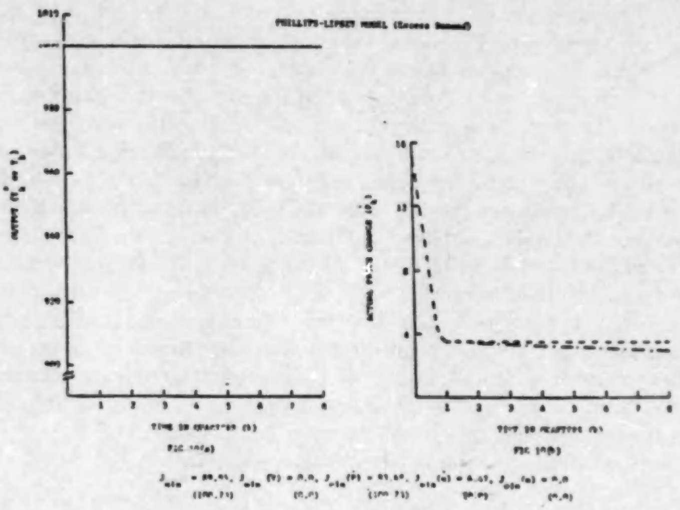


FIG 9(a) FIG 9(b)

$J_{min} = 1271.64$, $J_{min}(Y) = 23.94$, $J_{min}(\dot{p}) = 1203.36$, $J_{min}(\dot{u}) = 43.94$
(1431.73) (0.0) (1431.73) (0.0)



KEY TO FIGURES

Type of curves	Quantity they represent
– – – – –	No-policy trajectories
————	No-policy responses of potential output
—·—·—	Optimal responses under the corresponding optimal policy (policies)
— — —	Optimal responses of potential output
–··–··–	Guidelines for the maximum price change allowed

Period to period changes in output, additional government spending, and percent price changes are in annual rates.

J_{min} values at the bottom of each figure represent the minimum value of the cost functional and its components obtained for each simulation. The figures in brackets below each number represent the corresponding values when no policy is used for the given initial conditions.

(1) *Excess supply and inflation* (*SI*)—In the SI case the output trajectories are much the same but price behavior is quite different for the three models. Output begins at 918 and declines gently over the period, reaching about 905 by the end of the simulation horizon. The difference in price behavior reflects the powerful influence of price expectations on the inflation rate. The contrast between the PL model where $f = 0$ and the FPM model where $f = 1$ is, of course, particularly sharp. In the PL model, the inflation rate (see Figure 5b) plummets from its initial $9\frac{3}{4}$ to below 2 percent in the very first period; thereafter the inflation rate remains almost constant. In the FPM model, on the other hand, the inflation rate (see Figure 3b) declines from $9\frac{3}{4}$ to only 7 percent in the first period, then gradually declines over the next seven periods, reaching 6 percent by the eighth period. The results using the E model more closely resembles the FPM than the PL results. The inflation rate (see Figure 1b) declines from $9\frac{3}{4}$ to 6 percent in period one, declines gradually thereafter, reaching 5 percent by period eight.

(2) *Excess demand and inflation* (*DI*)—Similarly, in the DI case the no policy trajectories differ significantly only with respect to inflation. Although prices are rising in all three cases, the rise is markedly greater in the FPM and E models as compared to the PL model, again reflecting the prominant role of expectations in the FPM and E versions. In the PL model, the inflation rate declines from its initial $15\frac{1}{4}$ percent value to 4 percent in the first period and remains at that level throughout the next seven periods. Inherited inflation has no effect on actual inflation in this model; therefore, the initial conditions with respect to the inflation rate have no influence on the actual inflation rate. In the FPM model, where inherited inflation plays a powerful role, the inflation rate dips initially from $15\frac{1}{4}$ to 13 percent, then rises gradually. The initial decline reflects the fact that the initial inflation rate was generated using a model that set $h_0 = 0.0074$ as well as $f = 1$. When the FPM model takes over h_0 is set to zero and this results in an initial decline in the inflation rate. The powerful influence of expectations takes over and generates modest increases in the inflation rate thereafter. In the E model, the inflation rate drops from $15\frac{1}{4}$ to 10 percent in the first quarter and then gradually declines reaching 9 percent by the eighth period. The initial decline in this case reflects the fact that the $15\frac{1}{4}$ percent rate was generated assuming $f = 1.0$; when f was reset at 0.5 the inflation rate immediately dropped.

C. *Optimal Policy Simulations*

We ran two types of optimal policy runs. In one run we permit the use of both aggregate stabilization policy and price controls while in the other we allow only aggregate stabilization policy. The benefits of using price controls are better judged by comparing these two policy simulations rather than comparing the base simulations and the first policy simulation.

The model is highly nonlinear, involves numerous constraints, uses multiple controls and has a cost function which is not differentiable. As a result, optimization using analytical techniques was not possible. A nongradient direct search method was therefore developed to permit optimization by numerical method. The algorithm is a modification of the Complex Method developed by Box [1]. It can be shown that the algorithm will converge to a locally optimal control solution under the assumption of a convex feasible set. While it cannot be proven that the algorithm converges to the global optimum, the probability of such convergence can be shown to increase with the number of initial points randomly chosen in the solution space. Since computer costs increase with the number of points chosen, this factor must be taken into account.[6] Some experimentation revealed that the algorithm converged to the same solution values given the same problem but different initial points. Moreover, the algorithm was successfully applied to a variety of non-linear non-analytic test problems with known solution values. Thus it is likely that the policy solutions we present are globally optimal.[7]

[6] Following the recommendation of Box, the number of initial points chosen was set equal to $n + 2$ where n is the dimension of the solution space.

[7] See [3] for a full description of the algorithm.

Excess Supply—Inflation (SI)

The trajectory followed by our aggregate stabilization instrument is quite similar for all three models, whether or not price controls are available. Government expenditures are immediately increased by about $20 billion and further raised to approximately $40 billion by time period 3, and thereafter tend to level off and decline slightly. The strategy clearly is to eliminate excess supply as quickly as is feasibly possible, a choice made obvious by the fact that the reduction of excess supply adds little to inflationary pressures (for the very reasons that the existence of excess supply contributes little toward downward price flexibility). The success of this policy is seen by the fact that in each case actual output has returned by period 8 to within less than 1 percent of potential output (see Figs. 1(a)–5(a)).

The real differences among the models involve the price control instrument. Both the E and FPM models start out with a good deal of inherited expectational inflation—annual rates of 6 percent and 7 percent respectively (see for instance Figs. 1(b), 3(b)). In both cases, maximum allowable price increases of less than 3 percent per annum are imposed at the outset, and *retained* at that level for the duration of the simulation. In the E version, price controls are binding throughout, while in the FPM variant, actual price changes are below the maximum allowable until the last two time periods (Fig. 3(b)). This does not mean that price controls are redundant until period 7 of the FPM simulation. Quite the contrary: the genius of price controls in the FPM framework is that by delimiting expected inflation, price controls rapidly and powerfully reduce actual inflation, even below permissible limits. In contrast, since there is no inherited inflation in the PL model, there is little role for price controls to play, and their costs in fact determine that they are never used! Thus Fig. 5 shows the optimal paths whether or not price control is an admissible policy tool. Note in Fig. 5(b) that the optimal inflation rate is allowed to creep upward from less than 2 to more than 3 percent as excess supply is eliminated: it is simply too expensive to try to control this magnitude of inflation by either sacrificing output or imposing regulations.

The E and FPM simulations show rather poor inflation records in the absence of price controls, with annual inflation rates of 6 and 7 percent respectively, and rising over time in both cases. The overall effect of allowing or prohibiting price controls can be seen in Table 4 which shows the net reductions in total costs

TABLE 4
NET COST REDUCTIONS
(percent, relative to base run costs)

Model	Stabilization Policy Only	Stabilization Plus Price Control Policy
Excess Supply—Inflation (SI)		
E	33.0%	47.1%
FPM	26.2	53.2
PL	50.4	50.4
Excess Demand—Inflation (DI)		
E	9.7%	53.1%
FPM	11.0	64.4
PL	10.7	10.7

relative to the base runs. Note that in the SI simulations, total costs are reduced by about half under each variant of the model when both policy instruments are available, whereas the reduction is only a quarter under the FPM variant, and a third under the E variant, when only stabilization policy is used. Thus the efficiency gains due to price controls are 27 percent for FPM, 14 percent for E, and 0 percent for the PL variant.

Excess Demand—Inflation (DI)

The aggregate stabilization instrument in the DI environment is used as might be expected to promote a substantial reduction in aggregate demand. The cutback in government spending is much sharper in the E and FPM models, starting at between minus $12 billion and minus $18 billion in the initial time period compared to minus $4 billion for the PL model, and is particularly steep in the E and FPM runs without price controls, rising to −$31 to −$34 billion in the final effective time period compared to a maximum of −$24 billion when price controls are available. The effect on output is typically small since most of the reduction is in excess demand.[8] However, when price controls are not available, the E and FPM versions show reduction from potential output of 2 and 3 percent respectively.

The pattern of price control usage across models was similar in the DI runs to that of the SI runs. Price controls of just under a 4 percent permissible annual inflation were applied at the outset in both the E and FPM models, and these controls were neither varied nor suspended over the duration, while price controls were never used in the PL version. The underlying logic is similar to the SI cases. Price controls prove an effective and relatively efficient means of dealing with the expectational inflation that substantially augments the direct excess demand inflation in the E and FPM models, whereas the absence of expectational inflation in the PL model implies that stabilization policy is most efficient. However, the results are far more extreme in the present instance. The performance of the stabilization instrument alone with respect to inflation is pathetic in the E and FPM models. The inflation rate hovers around 12 percent in the FPM model while falling only to about 7 percent in the E version. This compares to about a 3 percent inflation rate in the PL model. Thus, the potential gains in the use of price controls vary greatly across the models (base run costs for FPM were 14 times as great as those for PL), and these are reflected in the DI tabulations of Table 4. The gain in efficiency due to the use of price controls is 53 percent for the FPM version and 43 percent for the E model, and again is zero for PL.

Summary

The results of a simulation study are usually critically dependent on the values assumed for the parameters, and we have little reason to believe this will not be the case here. Moreover, we are still at an early stage of the analysis and a number of additional experiments that seem fruitful to us remain to be done. Some of these

[8] The use of price controls in the E and FPM versions results in a reduction in Y^* which effectively constrains actual output during the initial time periods. Similar reductions in Y^* also occurred in the SI simulations, but in those cases there was abundant excess supply.

are discussed in the concluding section of this paper. Nevertheless, even at this juncture, certain conclusions seem sufficiently robust as to warrant emphasis here.

1. If expectations play a significant role in inflation then price controls can make an important contribution towards efficiently achieving macroeconomic goals, *whether or not* the inflation is accompanied by excess aggregate supply or demand.
2. Indeed, somewhat counter-intuitively, our results suggest a much larger efficiency gain for price controls in the excess demand compared to the excess supply situation.
3. Conversely, if there is no expectational element to the inflation, optimal policy goals can be achieved without price controls.
4. The most favorable case for price controls is precisely a structure in which there is no long run inflation-unemployment tradeoff. For it is here that any inflation initiated by policy error or exogenous shock will stubbornly persist unless the economy is willing to accept extremely toxic dosages of contractionary fiscal-monetary policy.
5. Our simulation results provide little substantiation for the belief that the use of price controls need only be sporadic and temporary. In our expectational inflation models, price controls were applied at the first opportunity and retained for the duration of the run. It is not clear that a longer time horizon would have made much difference. However, we may have loaded the game against the temporary use of price controls by assuming in effect that price controls will only dampen expectations after a prolonged period of successful use. The introduction of the "catching-up" effect, moreover, would be likely to accentuate the tendencies of the present model with respect to price control usage; i.e. to reinforce a decision not to use controls and to prolong the duration of controls once introduced or possibly even to prevent their elimination. Note also that the inclusion of the cumulative impact of controls on potential output would be expected to increase the output loss associated with a given price control regime, and hence would be expected to restrict the application of controls.

V. Further Research

As noted above, the results of a simulation study are typically sensitive to the parameter values assumed. In the present study, we ran simulations with some alternative values of h_0, f, and Ω. One direction for further study using this model is to explore the sensitivity of the simulation results to alternative parameter values. In particular, we plan to carry out simulations with alternative values of the parameters that appear to be most critical to the effectiveness of controls; e.g., ρ, λ, and β.

Another direction for future research concerns expansion and elaboration of the basic model. We intend to respecify the inflation rate equation to incorporate the "catching-up" phenomenon and the potential output equation to include the *cumulative* effect of controls on potential output. In addition, we plan to incorporate a monetary sector and to connect the level of potential output to the change in the capital stock as determined by the investment equation. Introducing the monetary sector will permit us to introduce the (real) interest rate as a determinant of investment, provide an additional policy instrument, and permit us to capture with more precision the impact of changes in both the price level and the inflation rate on the demand for output. Relating changes in the capital stock to potential output will make the composition of output as well as the level of aggregate demand relevant to the determination of the inflation rate. Moreover, the aggregate supply sector could be further elaborated by specifying labor supply and demand equations and a wage determination equation. The advantage of so doing would be the ability to differentiate between wage and price control programs and their separate effects on the model.

Another possibility is to introduce some exogenous component of inflation into the Phillips curve equation to represent sectoral influences such as the farm and fuel price increases in the recent inflation experience. Assuming that the exogenous component of inflation is not subject to control, we could determine the implications of the modification for the optimal use of policy.

Finally, more work needs to be done exploring alternative values for the parameters of the cost function. In addition, we intend to rerun some simulations with a modified cost function which penalizes only *unanticipated* inflation.

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