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Appendix C

Methods Used to Determine Effects of Changes in the Size Distributions of Receipts upon the Size Distribution of Total Income

To illustrate the methods of estimating the effects of changes in the distribution of wages, interest, and dividends on the distribution of total income (see Ch. 5), the methods of transforming the distribution of total income on the hypothesis that the Lorenz curve for wages is shifed 25 percent closer to the line of equal distribution are described. Similar procedures were followed to calculate the effect of redistributing dividends and interest.

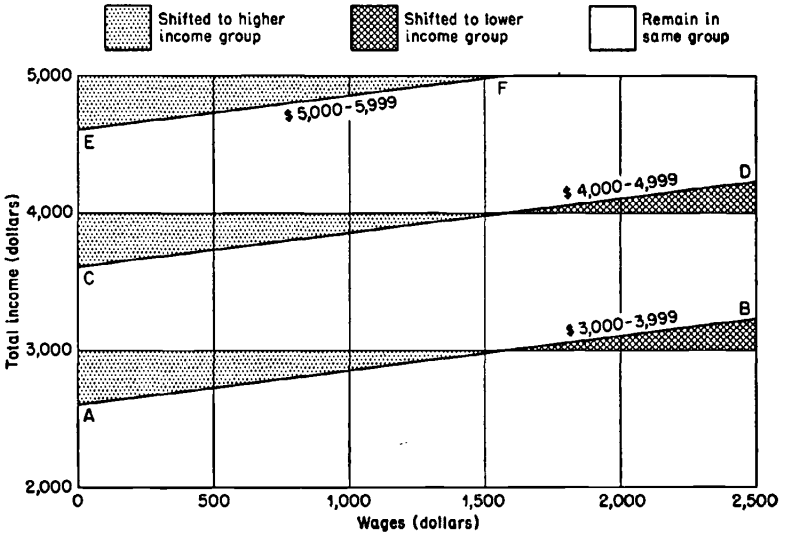
The basic data used in these computations, all from income tax returns filed in Wisconsin in 1936, are: (1) the distribution of total income by size; [1] (2) the distribution, by wage groups, of wages of individuals who receive wages only; [2] (3) the distribution, by wage groups cross-classified by total income groups, of wages and of the total income of individuals who received incomes other than wages.[3]

The first step was to subtract the distribution of the total income of persons who received some income in the form of wages from the distribution of the total income of all recipients. The residual was a distribution of the total income of persons who received no income in the form of wages, and consequently would be unaffected by any change in the distribution of wages.

1 *Wisconsin Individual Income Tax Returns, 1936,* IVA, Table A, p. 12.
2 *Ibid.,* Table 28, p. 38.
3 The distribution was computed from *ibid.,* IVA and B.

CHART A1

Reranking of Individuals when the Lorenz Curve for Wages Is Moved 25 Percent closer to the Line of Equal Distribution



The second step was to transform the total income distribution of those who received wages on the assumption that the wage distribution is shifted 25 percent closer to the line of equal distribution. This was done separately for individuals receiving wages only and for individuals receiving income also from other sources. For the former, each individual wage income was shifted 25 percent toward the mean wage in the manner described in Appendix B. For the latter, each individual wage income was shifted 25 percent toward the mean wage, then the other incomes of these wage recipients was added to their new wages. This was accomplished by the method illustrated in Charts A1 and 2.

Any individual whose total income is Y_0, of which X_0 is received in the form of wages, will receive, after the distribution of wages is shifted 25 percent closer to the line of equal distribution, a total income Y_1, given by

$$Y_1 = Y_0 + .25(M - X_0),$$

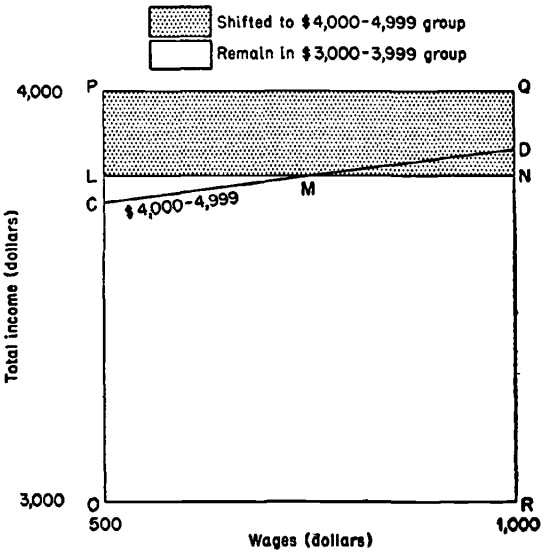
where M is the mean wage. Since M is $1,572 [4]

$$Y_1 = Y_0 + .25(\$1{,}572 - X_0).$$

[4] *Ibid.* IVA, Table B, p. 17.

CHART A2

Reranking of Individuals with $500-999 Wages and $3,000-3,999 Total Income when the Lorenz Curve for Wages Is Moved 25 Percent closer to the Line of Equal Distribution



The equation of the line showing the lower limits of the new $3,000-3,999 total income group expressed in terms of the total income, Y_0, and the former wage income, X_0, is therefore:

$$\$3,000 = Y_0' + .25(\$1,572 - X_0);$$

and, the lines showing the lower limits of the new $4,000-4,999 and $5,000-5,999 total income groups are:

$$\$4,000 = Y_0'' + .25(\$1,572 - X_0) \text{ and}$$
$$\$5,000 = Y_0''' + .25(\$1,572 = X_0).$$

These equations, shown in Chart A1 as lines AB, CD, and EF, cross the limits of the former group intervals at $X_0 = \$1,572$, for the total income of individuals who received the mean wage will remain unchanged. Individuals who receive less than $1,572 in wages are shifted to a higher income group; those who receive more, to a lower.

The method used to shift the proper number of individuals can be explained best by taking one cell as an example. The cell shown in enlarged form in Chart A2 has the following limits:

$500-999 wages and $3,000-3,999 total income. Line CD in Chart A2 shows the lower limit of the new $4,000-4,999 total income group in terms of the former income: all individuals above this line (i.e., those in the quadrangle CDQP) are shifted to the $4,000-4,999 total income group and all individuals below this line (i.e., those in the quadrangle CDRO) remain in the $3,000-3,999 group. However, to facilitate the computation, a line was drawn through the middle of the Line CD at point M, and the individuals in the rectangle PLNQ were shifted to the $4,000-4,999 total income group, while the individuals in the rectangle LNRO were left in the $3,000-3,999 total income group. This procedure shifts individuals in triangle MND rather than those in triangle LMC. Since it is assumed that all individuals in a single cell are distributed evenly in that cell, the proper number of individuals are shifted but a slightly higher total income and higher wage are attributed to the individuals shifted. The error is small, however; it is compensated by a similar error in the opposite direction for cells in which individuals are shifted to a lower income group.[5]

The computations of the number and new total income of individuals shifted to the $4,000-4,999 total income group and of those who remain in the $3,000-3,999 group are based upon the preceding equations. Total income in Chart A2 is

at point C, $\$4{,}000 = Y''_0 + .25(\$1{,}572 - \$500)$,

$Y''_0 = \$3{,}732$;

at point D, $\$4{,}000 = Y'''_0 + .25(\$1{,}572 - \$1{,}000)$,

$Y'''_0 = \$3{,}857$.

The total income at point M is the mean of the total incomes at points C and D: $3,794.50. Since it is assumed that the individuals in each cell are distributed evenly, 20.55 percent of the individuals are shifted to the $4,000-4,999 total income group while 79.45 percent remain in the $3,000-3,999 group. Thus, of the 144 individuals in the cell, 30 individuals are shifted and 114 are not.

The new aggregate incomes of the two groups of individuals are computed from the original formula:

$$Y_1 = Y_0 + .25(\$1{,}572 - X_0)$$

[5] The error in aggregate income due to the assumption that the individuals in a single cell are distributed evenly was less than one-tenth of one percent.

by adding for all individuals in each group. Hence

$$\Sigma Y_1 = N\overline{Y}_0 + (.25N \times \$1{,}572) - .25N\overline{X}_0$$

where N is the number of individuals, and $\overline{Y}_0$ and $\overline{X}_0$ are the mean original total and wage incomes for the group. For the 30 individuals shifted to the \$4,000-4,999 total income group

$$\Sigma Y_1 = (30 \times \$3{,}897.25) + (.25 \times 30 \times \$1{,}572) - (.25 \times 30 \times \$750) = \$123{,}083$$

where \$3,897.25 is the mean of the original total incomes of these individuals (i.e., the average of \$3,794.50 and \$4,000) and \$750 is the mean of the original wages of these individuals. The new aggregate income of the individuals not shifted is obtained in the same manner:

$$\Sigma Y_1 = (114 \times \$3{,}397.25) + (.25 \times 114 \times \$1{,}572) - (.25 \times 114 \times \$750) = \$410{,}714.$$

The tabulation summarizes these results.

Number of individuals in the \$500-999 wage group and \$3,000-3,999 total income group	144	
Number of individuals shifted to the \$4,000-4,999 total income group	30	
New aggregate income of individuals shifted to \$4,000-4,999 total income group		\$123,083
Number of individuals remaining in the \$3,000-3,999 total income group	114	
New aggregate income of individuals remaining in the \$3,000-3,999 income group		\$410,714

Similar procedures were followed for all other groups, except the '\$100,000 and over' total income group. The new total income for each individual in this group was computed directly from unpublished data.

By the above method, the number and new total incomes of the individuals who had income from other sources in each new total income group were computed. This distribution was then added to the new distribution of total income of individuals who received only wages and to the distribution of the total income of individuals who received no wages. The aggregate of these three distributions gives the distribution of total income after the wage distribution is shifted 25 percent closer to the line of equal distribution.