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# Australian Government Balance Sheet Management

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## 7.1 Introduction

The Australian government has taken steps over the past decade to improve the sustainability of government finances. In particular, the government has significantly reduced its debt liabilities, avoiding many of the risks associated with high debt levels. In April 2006, the Australian government announced that it had eliminated general government net debt.

Having reduced net debt, the government's attention has turned to the financing of broader balance sheet liabilities, such as superannuation obligations to its employees. The government has announced that it will establish a "Future Fund" to finance public-sector superannuation liabilities.<sup>1</sup> This will assist in relieving future generations of some of the financing burden associated with other intergenerational fiscal pressures that are expected to emerge over the medium term.

The creation of the Future Fund raises the significant policy question of how best to structure the government's balance sheet to reduce overall finan-

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1. These liabilities relate to public sector employees only, not broader social insurance obligations found in many OECD countries.

cial risk. This paper sets out a framework for optimal government balance sheet management and presents some preliminary estimates of the types of financial assets and liabilities that would reduce overall financial risk.<sup>2</sup>

## 7.2 The Australian Government Balance Sheet

The Australian government's *Charter of Budget Honesty Act 1998* (the Charter) highlights the need for governments to manage balance sheet risks. The purpose of the Charter is to improve fiscal policy outcomes by requiring the fiscal strategy to be based on principles of sound fiscal management and by facilitating public scrutiny of fiscal policy and performance.

The Charter facilitates optimal balance sheet management in two ways. First, the Charter requires governments to make regular financial reports that comply with external reporting standards, including the Australian Bureau of Statistics (ABS), Government Finance Statistics (GFS), and Australian Accounting Standards (AAS). This means the government balance sheet is comparable across entities and jurisdictions. Second, the Charter requires the *prudent* management of financial risks, including those relating to the broader government balance sheet (such as risks relating to the tax base). By requiring transparent presentation of the balance sheet and effective management of financial risks, the Charter allows the community to hold the government accountable for its financial performance.

### 7.2.1 The Balance Sheet

The Australian government's general government sector has published a balance sheet in the budget papers since the 1999 to 2000 period, consistent with international reporting standards.<sup>3</sup> The balance sheet reported in the 2004–2005 Final Budget Outlook is reproduced in table 7.1.

The major assets on the government's balance sheet are: financial equity, mainly reflecting the government's remaining share in Australia's major telecommunication company Telstra (\$50 billion); nonequity assets, mainly taxes owed but not yet received by the government (\$17 billion); and investments, loans, and placements, largely deposits at the Reserve Bank (\$35 billion). The major liabilities are superannuation liabilities (\$91 billion) and gross debt issuance (\$62 billion).

2. The paper does not discuss the appropriate size of government expenditure, the level or composition of taxation necessary to fund it, or the optimal size of a net asset portfolio (Future Fund) through time.

3. The Charter requires a balance sheet to be published as part of the budget papers (usually produced in May), the mid-year economic and fiscal outlook (by the end of January in each year, or within six months after the last budget, whichever is later), and at the final budget outcome (up to three months after the end of the financial year). The Charter requires the balance sheet to be on the ABS, GFS, and AAS basis. However, the primary budget statements (and therefore all references in this paper) are on a GFS basis.

**Table 7.1 Australian government balance sheet, 2004–05 (\$ millions)**

	2004–05 estimate at 2005–06 budget	2004–05 outcome
<i>Assets</i>		
Financial assets		
Cash and deposits	927	1,808
Advances paid	19,314	20,199
Investments, loans, and placements	31,066	35,022
Other nonequity assets	17,147	16,772
Equity <sup>a</sup>	50,895	50,183
Total financial assets	119,351	123,984
Nonfinancial assets		
Land	4,863	6,140
Buildings	13,894	14,195
Plant, equipment, and infrastructure	8,411	8,209
Inventories	5,299	4,524
Heritage and cultural assets	6,698	7,275
Other nonfinancial assets	2,085	2,032
Total nonfinancial assets	41,250	42,374
Total assets	160,601	166,358
<i>Liabilities</i>		
Deposits held	365	403
Advances received	0	0
Government securities	61,452	62,331
Loans	5,595	5,648
Other borrowing	224	182
Superannuation liability	91,071	91,172
Other employee entitlements and provisions	7,605	8,178
Other nonequity liabilities	28,416	30,423
Total liabilities	194,727	198,337
Net worth <sup>b</sup>	−34,126	−31,979
Net debt <sup>c</sup>	16,328	11,534

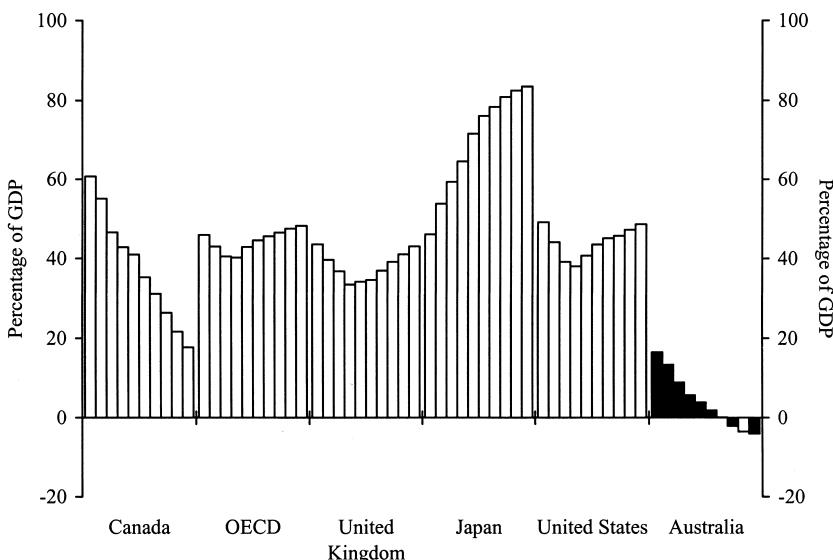
Source: Final Budget Outcome 2004–05, Australian Government.

<sup>a</sup>The 2004–05 equity and net worth outcomes include the Telstra shareholding valued at the closing share price on 30 June 2005.

<sup>b</sup>Net worth is calculated as total assets minus total liabilities.

<sup>c</sup>Net debt equals the sum of deposits held, advances received, government securities, loans, and other borrowing, minus the sum of cash and deposits, advances paid, and investments, loans, and placements.

There are two notable aspects to the Australian Government balance sheet. First, the government has reduced net debt to very low levels—net debt has fallen from \$96 billion (18.5 percent of GDP) in 1995 to 1996 to \$11.5 billion (1.3 percent of GDP) in 2004 to 2005. Having achieved further reductions in net debt the government announced, in April 2006, that it had eliminated general government net debt. This is in stark contrast with the net debt positions in nearly all other OECD countries (figure 7.1).



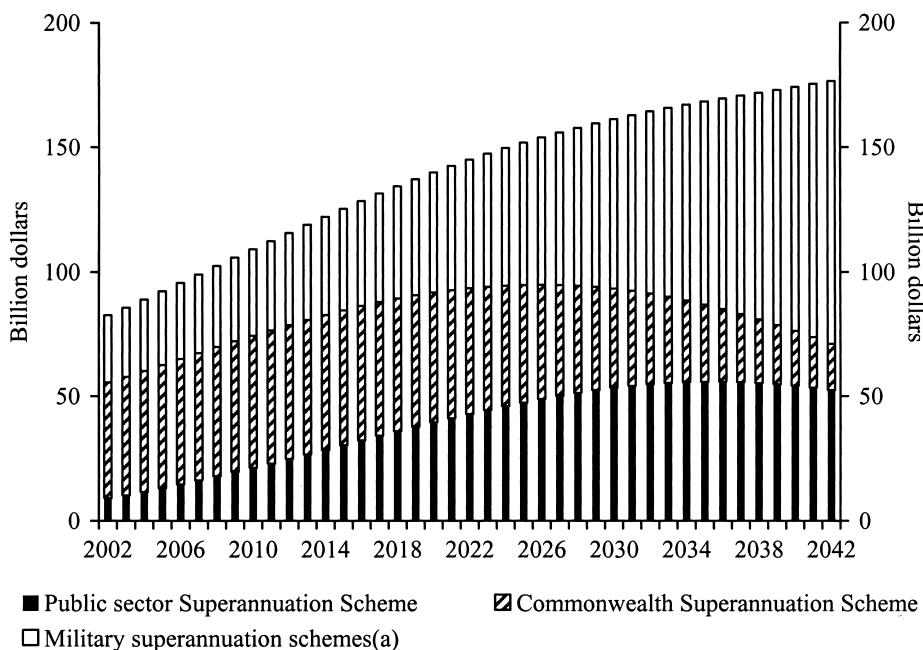
**Fig. 7.1 General government net debt in selected countries (1998–2007)**

Source: Budget Strategy and Outlook, 2006–07, Budget Paper No. 1, Australian Treasury.

The reduction in net debt reflects fiscal surpluses and asset sales over a number of years. It also reflects that these surpluses have been invested in debt assets. Following the *Review of the Commonwealth Government Securities Market 2002*, the government decided to maintain the domestic bond market to facilitate interest rate risk management by the private sector. The government therefore maintains a stock of around \$50 billion of mainly long-dated securities, while investing the proceeds of debt issuance in term deposits at the Reserve Bank of Australia.

The other notable feature about the balance sheet is that the government's most significant financial liability is public-sector superannuation, estimated at \$91 billion (10.2 percent of GDP) as of June 30, 2005. While this liability is expected to increase further in the future, a significant portion reflects liabilities to past government employees. The Australian government closed the main public-sector superannuation fund to new members from July 1, 2005. This means the government will pay the superannuation liability for new public servants employed after this date as they accrue, rather than growing the superannuation liability further. Also, in 2004 to 2005 the government paid \$4.6 billion to Telstra and Australia Post to extinguish remnant superannuation liabilities from the corporatization of these firms a decade or so ago.

Despite these policies, the existing superannuation liability is expected to remain sizable, reaching \$140 billion in 2020 (7.1 percent of GDP),



**Fig. 7.2 Public sector superannuation liability (2002–42)**

<sup>a</sup>Includes the Military Superannuation and Benefits Scheme and the Defence Force Retirement and Death Benefits Scheme.

Source: Australian Treasury.

largely due to growth in the superannuation schemes for military and defense employees (see figure 7.2).<sup>4</sup>

In response to these financial management challenges, the government has announced the creation of a “Future Fund” with the aim of offsetting the government’s unfunded superannuation liabilities by 2020. The fund will assist in increasing the government’s net worth and increasing national savings. Contributions will be made to the fund whenever the budget is in surplus. That is, rather than realized budget surpluses being used to retire debt or build up term deposits at the Reserve Bank as currently occurs, they will be invested in the fund.

The fund will be established using accumulated cash reserves currently on term deposit with the Reserve Bank. Additional contributions from realized surpluses and the reinvestment of returns on the fund’s assets will be needed to meet the government’s target.

4. Since the liability depends on the final salaries of public-sector employees, there are risks around this estimate. Revaluations of the liability are regularly reported in the budget papers.

### 7.2.2 Contingent Risks

The government balance sheet provides important information on the financial performance of the government from period to period. An increasing net worth means that a government is reducing rather than increasing net liabilities on future generations. However, there are many rights and obligations of government that are excluded from the balance sheet, mainly because of valuation problems. The most significant item missing from the balance sheet is the *primary asset* of the government—the power to tax.<sup>5</sup> While this power is limited by such factors as the constitution, international tax competition, the size and growth of the economy, the effects of tax rate and base changes on economic efficiency and equity—the taxing power provides strong assurance of the government's ability to meet its liabilities.

On the other side of the ledger, certain explicit government obligations that do not meet accounting standards tests for the recognition of liabilities are also not recorded.<sup>6</sup> Under the international IMF GFS framework, only obligations payable *in any event* are on the balance sheet, whereas those that occur only on *uncertain events* (even if they are probable) are not (International Monetary Fund 2001:34). Unless presented carefully, this can lead to misunderstanding of the underlying economic value of specific assets and liabilities on a government's balance sheet. For example, the Australian government departs from the GFS framework by recording provisions against expected defaults on student loans in the balance sheet.<sup>7</sup>

Probably the largest contingent liabilities not recorded on the balance sheet relate to future pensions and public health costs. However, these obligations to fund future expenses have an impact on the economy today, as well as on fiscal sustainability. So the Charter also requires the government to produce an intergenerational report every five years, which essentially captures those obligations not recorded on the balance sheet. The last report from 2002 to 2003 projected spending associated with an aging population to require a fiscal adjustment of 5.0 percent of GDP by 2041 to 2042, or \$87 billion in 2002 to 2003 dollars (Commonwealth of Australia, 1:2002a).

These conceptual and measurement problems mean the government

5. Problems also exist in valuing substantial heritage assets on the balance sheet, such as Parliament House and the Australian War Memorial.

6. This paper is concerned only with explicit financial risks, defined as rights or obligations on government established by law or contract. Implicit financial risks provide a different set of policy problems, such as policy issues, which are beyond the scope of this paper.

7. The Swedish government budgets by appropriating the anticipated loss from guarantees for individual risks, ensuring equivalence between traditional outlays and financial instruments that transfer risk to the government (Hagelin and Thor 2003). Similarly, the governments of the United States and the Netherlands explicitly appropriate the subsidy component of concessional loans and loan guarantees (Schick 2002:90). The Australian National Audit Office has valued the potential exposure from other selected financial instruments containing contingent risks—such as financial guarantees—at \$115 billion (ANAO 2003).

balance sheet is not directly comparable with similar private-sector financial statements.<sup>8</sup> Government balance sheet management therefore requires a different framework for determining whether investment strategies are optimal. In particular, contingent assets and liabilities are likely to have a significant influence on how best to structure the government balance sheet to reduce risk and improve fiscal sustainability.

### 7.3 Government Balance Sheet Management

The government balance sheet is a measure of the government's financial position at a point in time. Government balance sheet management is concerned with how the balance sheet may move through time. Managing the risks affecting the government balance sheet can assist in avoiding, or at least ameliorating, sharp changes in the financial position flowing from macroeconomic shocks. In particular, a government's balance sheet can be significantly affected by contingent risks affecting the tax base. This paper argues that a government financial portfolio—including financial assets, superannuation liabilities, and government securities—can be structured to reduce the financial impact of these risks.

Despite reportedly sound monetary and fiscal policies, as well as high domestic savings rates, many Asian economies suffered serious recessions in the late 1990s (for example World Bank 1993). These recessions were compounded, if not caused, by the crystallization of contingent liabilities, particularly around commitments to support exchange rates and banking systems. Public injections into the banking system after the Asian crisis more than doubled the size of government debt to GDP in Korea and Thailand (Wheeler 2004:105).<sup>9</sup> In emerging countries more generally over the 1990s, bail outs to public enterprises and banking systems have contributed more to the build up of government debt than recurrent deficits (Kharas and Mishra 2001). Indeed, the deterioration in the debt positions of emerging countries since the 1990s has been largely attributed to interest rate and exchange rate movements and the recognition of off-balance sheet and contingent liabilities (International Monetary Fund 2003:117).

For developed nations, managing balance sheet risks may not be as important in averting crises. However, balance sheet management can be used to improve the fiscal sustainability of government through time (and ultimately, avoid financial crises).<sup>10</sup> Since governments can rely on taxation to

8. Indeed, the *National Commission of Audit* (Commonwealth of Australia 1996) recommended that the term *balance sheet* be replaced with *Statement of Assets and Liabilities* to avoid misleading comparisons with the private sector.

9. Korea's government debt to GDP ratio went from 10.5 percent to 26.5 percent after the costs of bank recapitalization, while Thailand went from 14.6 percent to 46.6 percent.

10. There are other potential objectives of debt management policy, such as attempting to ameliorate the effects of incomplete or imperfect markets (e.g., improving market efficiency through improved risk sharing). However, alternative objectives have a less secure conceptual basis and some implementation problems (see Missale 1997).

finance themselves, the concept of fiscal sustainability must relate in some way to the expected path of taxation. Fiscal sustainability not only requires that governments are likely to remain solvent—in the sense that the anticipated path of taxation is reasonable—but that the volatility (or risks) around that path are not significant. The International Monetary Fund is incorporating country risk analyses into their fiscal sustainability assessments for some countries (for example, Barnhill and Kopits 2003). Indeed, the relationship between taxation and balance sheet assets and liabilities is central to the economics literature on balance sheet management.

### 7.3.1 A Framework for Analysis

#### *Intertemporal Budget Constraint*

An important conceptual tool for analyzing government balance sheet management is the intertemporal budget constraint. This budget constraint requires that at any date the sum of net worth and the net present value of taxation be equal to the net present value of government spending.

In this way, the intertemporal budget constraint relates the government balance sheet in any period to the contingent asset and liabilities that can affect the balance sheet.<sup>11</sup> If current period government spending is higher than current period taxation, the government can issue debt (or some other liability). However, this simply means taxes need to be higher sometime in the future. In this framework, debt (and other liabilities) passed onto the future are effectively *congealed taxation*. The intertemporal budget constraint requires taxes to rise from their current levels to finance future anticipated expenses.

#### *Tax Smoothing*

Once the limit to future taxation and spending is identified, the optimal path of taxation needs to be found. Barro (1979) uses the standard public finance assumption that the excess burden of taxation rises by more than any rise in the tax rate—a doubling of tax rates has more than twice as many costs. These costs are the loss in overall welfare caused by tax rates distorting individuals' and firms' consumption choices.

Given anticipated government expenditure, these costs are minimized through time if tax (defined as a proportion of GDP) is constant, with temporary macroeconomic shocks leading to deficit financing and surpluses. That is, for a given financing requirement, a constant tax rate through time will impose a smaller cost on the economy than would a low tax rate in one year and a high tax rate in the next. An important implication of tax

11. In a series of excellent papers (from Bradbury, et al. 1999 and Grimes 2001, onward) the New Zealand Treasury has used the intertemporal budget constraint to derive the concept of *comprehensive net worth* for balance sheet management purposes.

smoothing is that it is *anticipated* future tax rises, rather than simply current tax rates, which distort economic behavior. For example, if tax rates are expected to rise significantly in the future, investment (and therefore growth) is likely to be discouraged. This standard result on optimal long-run fiscal policy accords with the Australian government's commitment of no increase in the overall tax burden from 1996–97 levels.

This result depends critically on assumptions of the excess burden of taxation.<sup>12</sup> In the absence of these costs, there may be no role for government to smooth taxes through time, since individuals could adjust their own portfolios to account for the uncertainty in future tax liabilities. It is the presence of such costs that gives government balance sheet management its power.

### *Balance Sheet Risk*

For a macroeconomic shock that temporarily reduces economic growth, the government could resort to deficit financing by selling financial assets or issuing debt. However, if an unanticipated shock led to a permanent change in the resources available to government (for example, a fall in the present value of taxation revenue), the government would need to adjust fiscal policy because deficit financing would not be sustainable. Alternatively, governments could attempt to structure their financial portfolios to hedge against such risks.

Bohn (1990) extends the tax smoothing result to incorporate such uncertainty by imposing a budget constraint across all anticipated states of the world, as well as across time. Bohn shows that the government can reduce the expectation that tax rates will change by holding and issuing specific financial instruments.<sup>13</sup> In particular, an effective budget hedge would see the government's financial returns vary negatively with tax revenue during a macroeconomic shock. For example, the government's balance sheet is protected somewhat if it issues debt where repayments fall with economic growth and tax revenue.

This framework suggests that the optimal portfolio for a country depends on the structure of the economy. If an economy is susceptible to supply-side shocks, where inflation and growth move in opposite directions, then nominal debt issuance performs such a role. For example, the real value of government debt falls if an oil shock causes recession and inflation. Alternatively, if an economy is subject to demand-side shocks, where inflation and growth are positively correlated, then inflation-indexed and variable inter-

12. If the loss function is linear, then there is no need to minimize the variance in tax rates (Hansen 2003:9).

13. Bohn (1990) assumes risk-neutral individuals, so the costs relate to the expectation that taxes will change. For risk-averse individuals, the uncertainty that the government will raise taxes at times of low financial returns (i.e., high marginal utility of consumption) is an additional cost (see the Appendix in Hansen 2003).

est rate debt are better hedges. Of course, *ex ante*, it is extremely difficult to form an assessment of the types of shocks an economy will be subjected to in the future.

### 7.3.2 Formal Presentation of the Model

Using the model developed by Bohn (1990) we can express more formally the intuition previously outlined.

Individuals are assumed to be infinitely lived and risk neutral, and maximize the expected utility derived from all future consumption:

$$(1) \quad U_t = E_t \sum_{j \geq 0} \rho^j c_{t+j}$$

where  $\rho$  is a discount factor, and  $c_{t+j}$  is consumption in period  $t + j$ .

Individuals receive a stream of endowments  $Y_{t+j}$  and pay taxes on endowments at a rate  $\tau_t$ . As taxes are distortionary there is an excess burden of taxation denoted by a convex loss function  $h(\tau_t)$ . Individuals are also able to trade a given set of assets, so that the individual budget constraint is given by

$$(2) \quad c_t + \sum_k p_{t,k} A_{t,k} = Y_t [1 - \tau_t - h(\tau_t)] + \sum_k (p_{t,k} + f_{t,k}) A_{t-1,k}$$

where  $A_{t,k}$  is the quantity of asset  $k$  held at the end of period  $t$ ;  $p_{t,k}$  is the price of asset  $k$  (denoted in terms of consumption goods); and  $f_{t+k,k}$  is the stream of cash flows derived from holding asset  $k$ . Individual optimization implies that expected returns across assets are equal, that is:  $E_t(1 + r_{t+1,k}) = 1/\rho$  for all  $k$ , where  $r_{t+1,k} = (p_{t+1,k} + f_{t+1,k})/p_{t,k} - 1$ . This assumption is nontrivial, particularly so when we introduce equities into our analysis.

The government can use tax revenues  $\tau_t Y_t$  and issue debt,  $B_{t,k}$ , to finance government expenditure,  $G_t$  (which we treat as exogenous in this model), and to meet outstanding debt obligations. The government budget constraint is given by:

$$(3) \quad \tau_t Y_t + \sum_k p_{t,k} B_{t,k} = G_t + \sum_k (p_{t,k} + f_{t,k}) B_{t-1,k}$$

The government can choose the type of debt instrument,  $k$ , and may be a net lender or net borrower in any security, as such  $B_t$  should be interpreted as the government's net liabilities. Following Bohn, we recast the objective function in terms of government policy by substituting equations (2) and (3) into equation (1), which gives<sup>14</sup>

$$(4) \quad U_t = E_t \sum_{j \geq 0} \rho^j \{ Y_{t+j} [1 - h(\tau_{t+j})] \}$$

14. As in Bohn (1990) we drop exogenous terms for simplicity, as they are irrelevant for deriving the first-order conditions for optimality.

The government chooses an optimal tax rate and debt portfolio to maximize individual utility, equation (4), subject to its own budget constraint, equation (3). In effect, the government's objective is to choose the structure of taxes and debt that minimize the expected present value of the excess burden of  $h(\tau_t)$ . The first-order conditions are<sup>15</sup>

$$(5) \quad \left. \begin{aligned} E_t[h'(\tau_{t+1})] &= h'(\tau_t) \text{ for } k = 0 \\ \rho E_t[h'(\tau_{t+1})(1 + r_{t+1,k})] &= h'(\tau_t) \text{ for } k > 0 \end{aligned} \right\}$$

where  $k = 0$  is the risk-free asset. That is, optimality requires that the expected marginal excess burden of taxation is constant through time.

As in Bohn, we assume a quadratic excess burden, so that the deadweight loss of a tax rate,  $\tau_t$ , is  $h(\tau_t) = (h/2)\tau_t^2$ . It follows then from the first-order conditions that an optimal policy requires

$$(6) \quad \text{Cov}_t(\hat{\tau}_{t+1}, \hat{r}_{t+1,k}) = 0$$

where  $\hat{\tau}_{t+1} = \tau_{t+1} - E\tau_{t+1}$  is the innovation in the tax rate, and  $\hat{r}_{t+1,k}$  is the innovation in the return to asset  $k$ . These innovations reflect the unanticipated components of changes in tax rates or returns.

Equation (6) implies zero conditional covariance between taxes and returns on available securities. That is, if the covariance between innovations in the tax rate and returns, for a specific debt, is negative then the government could improve tax smoothing by issuing more of this form of debt. The converse is also true: if the covariance is positive then the government could improve tax smoothing by purchasing more of this form of debt. This is the principal conclusion of Bohn—the government should smooth tax rates across different states of the world, as well as over time.

### 7.3.3 The Optimal Structure of the Government's Balance Sheet

To estimate the government's optimal portfolio an expression for the innovation in tax rates is required. The innovation in the tax rate determined by the government's budget constraint is:

$$(7) \quad \hat{\tau}_{t+1} = (1 - \psi)e^{-\bar{y}} \left( \sum_k \hat{r}_{t+1,k} d_{t,k} + \sum_{j \geq 0} \rho^j \hat{g}_{t+1+j} \right) - \tau_t \sum_{j \geq 0} \psi^j \hat{y}_{t+1+j}$$

where  $y_t$  is the growth rate of real output and  $\bar{y}$  is its mean. The term  $\sum_{j \geq 0} \psi^j \hat{y}_{t+1+j}$  is the present value of innovations in future growth rates of real output, where  $\psi$  is the discount factor and  $\hat{y}_{t+1+j} = E_{t+1} y_{t+1+j}$ . That is, it captures unexpected permanent changes in output and therefore in the government's ability to raise tax revenues at a constant tax rate. Similarly,  $\sum_{j \geq 0} \rho^j \hat{g}_{t+1+j}$  is the present value of innovations in government spending relative to output, where  $\hat{g}_{t+1+j} = (E_{t+1} G_{t+1+j} - E_t G_{t+1+j})/Y_t$ . The ratio of security  $k$  debt to output is denoted by  $d_{t,k}$ .

15. See Appendix 7.A for derivation of the first-order conditions.

The intuition behind equation (7) is that the present value of tax revenues must cover initial debt plus the present value of government spending. That is, tax rates will need to adjust whenever there are *unexpected* changes in the value of government debt, government spending, or output growth. For a government that is already optimally managing the balance sheet, the current tax rate already incorporates anticipated obligations.

Substituting the previous into equation (6) gives the optimality condition for each government security:

$$(8) \quad \sum_l cov_t(\hat{r}_{t+1,k}, \hat{r}_{t+1,l}) d_{t,l} + cov_t(\hat{r}_{t+1,k}, \sum_{j \geq 0} \rho^j \hat{g}_{t+1+j}) \\ - w_t cov_t(\hat{r}_{t+1,k}, \sum_{j \geq 0} \psi^j \hat{y}_{t+1+j}) = 0$$

where  $w_t = [e^{\bar{y}}/(1-\psi)]\tau_t$  is a weighting factor.<sup>16</sup>

That is, the government can smooth taxes to offset unexpected shocks in the present value of government spending and output through the issuance or purchase of state-contingent securities.

As the paper focuses on shocks that affect the present value of output growth, we assume that the covariance between innovations in the present value of government spending and returns on assets (the second term in equation [8]), is zero. The following equation provides us with a solution to the government's optimal portfolio:<sup>17</sup>

$$(9) \quad \mathbf{d}_t = w_t \sum_r^{-1} \cdot \sum_{y,r}$$

where  $\Sigma_r$  is the variance-covariance matrix of returns (assumed to be non-singular) and  $\Sigma_{y,r}$  is the covariance vector matrix between returns and the present value of unexpected innovations in real output growth.

### Methodology

In order to solve equation (9) and evaluate the optimality of various portfolios of government assets, we need to calculate innovations in returns and the present value of future rates of growth in real output.

We limit our analysis to a bivariate comparison. For our first analysis we are interested in the optimal share of long-term domestic debt and long-term foreign debt. We then extend this analysis to consider alternative asset classes, such as equities.

The real return on long-term domestic debt,  $r_{t+1,d}$ , is influenced by the domestic nominal long-term interest rate  $l_{t+1}$ , changes in the current long-

16. For the purposes of the empirical analysis, we assume a discount factor of 0.98 (which equates to a 2 percent per quarter discount), an average tax rate of 24 percent, and an average real growth rate of 0.75 percent per quarter. The value of the weighting factor does not affect any of the qualitative conclusions.

17. For derivation of equation (9) see Appendix 7.B.

term market interest rate (which is used as an approximation of the capital gain component), and domestic inflation,  $\pi_{t+1}$ .<sup>18</sup> Foreign long-term debt returns,  $r_{t+1,f}$ , will in addition be influenced by the change in the exchange rate  $\Delta s_{t+1}$ . Innovations in returns are therefore given by:

$$\begin{aligned}\hat{r}_{t+1,d} &= -(\ell_{t+1,d} - E_t \ell_{t+1,d}) - (\pi_{t+1} - E_t \pi_{t+1}) \\ \hat{r}_{t+1,f} &= -(\ell_{t+1,f} - E_t \ell_{t+1,f}) - (\pi_{t+1} - E_t \pi_{t+1}) + (\Delta s_{t+1} - E_t \Delta s_{t+1})\end{aligned}$$

To calculate these innovations in real returns, vector autoregressions (VARs) are used to formulate expectations for the inflation rate, the percentage change in the exchange rate, the long-term domestic interest rate, and the long-term foreign interest rate. Following Hawkesby and Wright (1997), expectations are formed for each variable ( $y_{t+1}$ ,  $\pi_{t+1}$ ,  $l_{t+1,d}$ ,  $\Delta s_{t+1}$ ,  $l_{t+1,f}$ ) by regressing that variable on a constant and one lag of the variable, together with long lag of all other variables.<sup>19</sup> As expectations at time  $t$  depend only on information available up to time  $t$ , we must run a new VAR for each time period.<sup>20</sup>

The same method is used to calculate innovations in the growth of real output:  $\hat{y}_{t+1+j} = E_{t+1} y_{t+1+j} - E_t y_{t+1+j}$ . Expectations for real output growth need to be formed at each time period for rates of growth in all future time periods. That is an expectation that is formed at time  $t$ , for  $(y_{t+1}, y_{t+2}, \dots, y_{t+n})$  and at time  $t+1$  for  $(y_{t+1}, y_{t+2}, \dots, y_{t+n})$  given the additional information. The differences in expectations are then discounted at a rate,  $\psi$  (assumed to be 0.98).<sup>21</sup> This process is repeated for each time period to derive a time series for innovations in the present value of output.

The methodology used to derive innovations in equity returns and output is the same as that outlined previously, with innovations in equity returns given by:

$$\hat{r}_{t+1,e} = (p_{t+1,f} - E_t p_{t+1,f}) - (\pi_{t+1} - E_t \pi_{t+1}) + (\Delta s_{t+1} - E_t \Delta s_{t+1})$$

where  $\hat{p}_{t+1} = (p_{t+1}^* - E_t p_{t+1}^*)$  is the unanticipated component of capital gains. Capital gains are calculated using accumulation share indices for each country, which incorporate both share price growth and dividend growth.

18. The proxy used for capital gains may lead to an understatement of this component in the innovation of real returns since it does not take into account the time to maturity. The longer the time to maturity the larger will be the capital gain (or loss) associated with changes in interest rates.

19. The lag specification was chosen based on the lag length that minimized the Akaike and Schwarz information criteria. The estimated model does not capture the full range of variables that could be expected to determine output and inflation. For a more complete model of the Australian economy see Dungey and pagan (2000).

20. This essentially involves growing the sample size with each estimation. We also investigated an alternative approach of rolling the sample, thereby keeping the sample size constant. However, this did not have a material impact on the results.

21. We also estimated the results using a lower discount factor, which did not change our broad conclusions. However, the case for investing abroad was slightly weaker under this scenario.

### Data

All data are quarterly data for the post-float period 1983:4 to 2004:3. Long-term interest rates are the long-term government bond yields converted into quarterly returns. We take the first difference in bond yields, as we cannot reject nonstationarity over the sample period (based on the Augmented Dickey-Fuller and Phillips-Perron tests). Expectations for bond yields are then derived from summing the expectations of the first difference. Equity returns are derived using accumulation indices when these are available and for the periods when they are not, capital weighted share indices are used (this effectively assumes that dividend growth is constant over this period). The exchange-rate data are the log difference of the spot exchange rate expressed as the Australian dollar price of foreign currency. Inflation is estimated by taking the log difference of the GDP deflator. Growth rates in output are the log differences of real GDP.

### Results

Table 7.2 shows the variance-covariance matrix for innovations series using domestic and foreign debt. The results show that innovations in domestic returns and foreign returns vary negatively with innovations in output. This suggests that it is optimal for the Australian government to purchase securities denominated in both domestic and foreign currency. These results are consistent with the findings of Hawkesby and Wright (1997) and Missale (1999).

We also report the optimal portfolio of domestic and foreign debt as a ratio to quarterly GDP, calculated by solving equation (9). These shares should be interpreted with some caution, as the magnitudes are sensitive to the estimation methodology. We discuss some of the key qualifications in detail in the following. With this in mind, the results show that it is optimal for the government to invest a relatively larger amount in domestic rather than foreign debt. This is largely driven by the volatility in the exchange rate, which acts as a *penalty* on foreign investment. Volatility in the exchange rate (and therefore in foreign returns) is not necessarily bad, provided innovations in the exchange vary negatively with innovations in output. While this is the case for Japan and Germany, our results show a positive covariance between innovations in the exchange rate and output for the United States.<sup>22</sup>

The previous results can be disaggregated into the various elements that make up innovations (or unexpected changes) in returns (see Appendix 7.C). Doing so reveals that there is a positive covariance between innova-

22. The volatility in the exchange may be partly driven by the method used to derive exchange rate innovations. Meese and Rogoff (1983) have shown that models used to explain exchange rate movements over short intervals generally perform worse than a simple random walk.

**Table 7.2 Variance-covariance matrix for innovations series: Debt securities**

	U.S.	Japan	Germany
Variance-covariance matrix			
Var( $r_d$ )	0.32	0.37	0.37
Var( $r_j$ )	10.11	41.81	36.95
Cov( $r_d, r_j$ )	-0.21	0.28	0.11
Cov( $r_d, y$ )	-0.12	-0.10	-0.04
Cov( $r_j, y$ )	-0.02	-0.38	-0.26
Optimal portfolios			
Domestic	-4.59	-3.17	-1.22
Foreign	-0.12	-0.09	-0.08

*Notes:* The results are for pair-wise comparisons between Australia and the reported country. This means that a separate VAR is calculated for each country including five variables: output, inflation, nominal domestic bond yield, nominal foreign bond yield, and the percentage change in the exchange rate.

tions in output and inflation, which is a key driver of our results. This implies that periods of unexpectedly low inflation (and therefore high returns) have tended to occur during periods of unexpectedly low output. This may largely result from the early 1990s recession, when inflation and domestic interest rates fell substantially.

Turning now to equities, *a priori*, we might expect that it would be unlikely that domestic equity investment would provide an effective hedge against macroeconomic shocks, given the high correlation between company profits and output. This is confirmed by our results, which show that the covariance between innovations in domestic equities and output is positive (table 7.3).

In contrast, the covariance between innovations in foreign equities and output is negative, suggesting that an optimal portfolio would include some investment in foreign equities for the countries considered.

### 7.3.4 Qualifications

There are some significant qualifications to our results that require further investigative effort. First, the paper focuses on unconstrained portfolios for only a select number of countries. Further work to determine the optimal constrained portfolio involving multivariate financial assets needs to be undertaken before using the model for policy purposes. For example, the optimal portfolio suggests that 2 percent of quarterly GDP should be invested in German shares. This is clearly unrealistic. However, in the absence of obvious constraints on the portfolio at the time of writing this paper, we decided to report the unconstrained case only. Similarly, Japan, Germany, and the United States were chosen since they represent the largest economies in their respective geographical regions of Asia, Europe, and the Americas.

**Table 7.3 Variance-covariance matrix for innovations series: Equities**

	Australia	U.S.	Japan	Germany
Variance-covariance matrix				
Var( $r_d$ )	0.31	0.32	0.34	0.33
Var( $r_e$ )	206.98	60.30	91.40	204.61
Cov( $r_d, r_e$ )	0.56	0.35	0.50	-0.10
Cov( $r_d, y$ )	-0.04	-0.07	-0.08	-0.05
Cov( $r_e, y$ )	0.21	-0.10	-0.18	-0.25
Optimal portfolios				
Domestic debt	-1.52	-2.49	-2.85	-1.75
Equities	0.02	-0.01	-0.01	-0.02

*Notes:* The results are for pair-wise comparisons between domestic debt securities and equities in the reported country. This means that a separate VAR is calculated for each country including five variables: output, inflation, nominal domestic bond yield, equity prices (to proxy capital gains), and the percentage change in the exchange rate.

Second, we have ignored the relationship between innovations in output and government spending. That is, government expenditure is also likely to be linked to macroeconomic shocks and potentially able to be offset by government financial investment policy. It is likely that the effects of a macroeconomic shock on spending reinforce the impact on taxation. However, Bohn (1990) suggests that this impact is likely to be small and insignificant. In theory it should be possible to determine the present value risk characteristics associated with major expense obligations (such as health care) and invest in assets to offset these risks.

Third, the results are based on agents forming expectations on future financial risks based on past financial events. This expectations formulation defies the rational expectations (Lucas) critique that economic agents should use all available information when forming expectations—past returns (or risks) are no guide to future returns (or risks) in the presence of significant policy change (Lucas 1976). In particular, Australia has been subject to significant economic reforms over the past two decades, which may make relying on certain systematic relationships to form expectations difficult. However, nearly all financial models in regular use rely on past data to measure risks (for example, the capital asset pricing model). Further, our results use data from 1983, after the floating of the Australian dollar, which was perhaps the largest structural change in the financial sector.

Finally, the model does not explicitly incorporate the existing stock of assets and liabilities held by other levels of government and by the private sector. In Australia, the state governments are in net asset positions (2 percent of GDP in 2004 to 2005), however the private sector has significant net external debt (48 percent of GDP in 2004 to 2005). The current model assumes that the obligations of the private sector and other governments are independent of the fiscal position of the Australian government. By focus-

ing only on the *explicit* assets and liabilities of the central government (i.e., rights or obligations established by law or contract), the paper ignores those that may be *implicit* (i.e., rights or obligations dependent on moral suasion). For example, the community may expect the government to take over some of the financial obligations of large financial institutions that would otherwise be subject to failure. These expectations make the distinction between private- and public-sector debt in the paper (and by investors) less clear.

One potential way around this caveat is to include the net external obligations of the private sector when determining the optimal government portfolio.<sup>23</sup> For example, roughly half of net Australian private-sector debt is held in U.S. dollars, implying that an optimal portfolio would hold less of these liabilities if the government were also considering implicit risks on the government balance sheet. Of course, there may be significant moral hazard type problems for any government that makes this an explicit portfolio objective (see the discussion on policy endogeneity in section 7.4).

## 7.4 Discussion

### 7.4.1 Policy Implications

The key conclusion from the debt management literature is that the optimal financial investment strategy depends on the types of shocks affecting the macroeconomy. That is, the optimal way to structure debt (short or long, nominal or price-indexed, domestic or foreign currency) or invest in financial assets is an empirical question. Hansen (2003; following Missale 1997) summarized the main results from this literature:

1. Buy (short-sell) assets whose returns have a positive (negative) correlation to public spending and negative (positive) correlation to the tax base; and
2. Issue
  - nominal debt for government spending and productivity shocks;
  - price-indexed debt for monetary and real demand shocks causing inflation;
  - foreign currency debt when output and inflation shocks are correlated internationally;
  - maturity structure of debt to match structure of planned fiscal surpluses; and
  - short maturity debt when positive correlation between output and real interest rates.

23. The model incorporates implicit liabilities to the extent they are incorporated in  $G_t$ . Managing the government balance sheet against implicit obligations is problematic for the moral hazard reasons discussed in section 7.4.2.1.

Our results suggest that the Australian economy has been subject to more demand shocks than supply-side shocks over the sample period. If this were to continue to be the case going forward, then the tax-smoothing framework adopted in this paper suggests that the government should purchase domestic nominal bonds. During high-output periods in Australia, inflation and tax revenues would increase offset by lower real returns on domestic nominal bonds, such as government (including state government debt) or high-grade corporate debt. However, during low-output periods, inflation and tax revenues will fall and be offset by higher real returns to domestic nominal bonds. These results support the government's policy of reducing net debt—both by reducing the size of gross debt issuance and investing in debt assets. The results also suggest that balance sheet risk could be further reduced by issuing price-indexed bonds (such as treasury indexed bonds), rather than nominal bonds.

Our results also suggest that investing in foreign equities is likely to reduce overall balance sheet risk. In effect, the government already has a significant stake in Australian equities because of the tax revenue earned from domestic capital income (not to mention the presumably highly correlated flow on effects through taxation of domestic labor income). Auerbach (2004) notes that, even though the U.S. government does not hold much equity directly, it has significant exposure to variations in stock prices through its claims to future tax revenues. Indeed, Auerbach argues that the U.S. government's implicit equity position is larger than the stock market itself, consistent with the fact that revenues from all sources are responsive to stock market returns.

#### 7.4.2 Potential Criticisms

##### *Policy Endogeneity*

There are some important criticisms of the tax-smoothing approach that can affect our policy conclusions. First, there is the potential problem of *policy endogeneity*. If the government's improved financial asset performance encourages greater government spending then the independence between government spending and taxing is violated. Similarly, there may be risks that if the government takes a controlling interest in a domestic company, some sections of the community may expect increased assistance for that company. In such circumstances, the optimal balance sheet strategy for government might be to avoid accruing a financial asset portfolio altogether and simply eliminate all risk, balancing the budget through the cycle (see Pinfield 1998). Another potential solution is to restrict the degree of controlling interest a government investment fund can maintain in specific domestic companies. Further, it is unlikely that the Future Fund will increase the incentive of future governments to spend more on public-

sector superannuation expenses. The bulk of the super liability relates to previously accrued entitlements that are reasonably well defined. Finally, the government reports its underlying cash surplus exclusive of fund earnings so that they cannot be used for recurrent expenditure.

*Moral hazard* is a particularly severe form of policy endogeneity that appears to have limited the use of some financial instruments to manage government balance sheets. Traditionally, the economics literature on optimal debt management has focused on *state-contingent* debt. As early as 1941, inflation-indexed bonds were seen as a means for removing the incentive of governments to inflate the economy and reduce the real value of their obligations (Bach and Musgrave 1941). However, in the real world there is little evidence of state-contingent debt instruments being issued by governments.<sup>24</sup> This may primarily be due to some state-contingent debt instruments being subject to moral hazard problems (sometimes referred to as *time inconsistency*) if governments can affect the states (Calvo and Guidotti 1990; Bohn 1990). For example, bond returns that fall when an index of government expenditure rises would hedge the balance sheet against economic downturns. However, governments would also have an incentive to increase expenditure. This risk would then be priced in the value of such bonds, making them unattractive even for well-intentioned governments to issue.

More recently, the literature has focused on hedging the balance sheet by optimal design of the maturity and denomination of conventional debt securities. For example, a shorter average debt maturity increases the exposure to short-term interest rate rises. Long maturities can avoid exposure to *roll-over* (Barro 1995). It is not necessary, as our results show, to issue state-contingent debt to offset specific balance sheet risks. Investing in a broad and diverse range of financial assets effectively eliminates moral hazard type problems.

#### *Agency Costs*

Second, there are significant *agency costs* associated with government management of financial assets. In the tax-smoothing model, the government is assumed to maximize the welfare of all individuals in the community. However, in practice the incentives of government and the agents used by government may not be so aligned. This can lead to poor investment decisions. The solution to the agency cost problem is to ensure that the governance of the Future Fund is clear and transparent and investments are made on a commercial basis within the investment guidelines set by gov-

24. Real-world examples include Mexico issuing bonds tied to oil prices and Costa Rica, Bulgaria, and Bosnia issuing bonds containing an element of indexation to GDP (Borenstein and Mauro 2002). Even in Australia, the pool of inflation-indexed bonds is relatively small, with outstanding Treasury Indexed Bonds around 10 percent (or \$6.4 billion) of total Commonwealth debt (Commonwealth of Australia 2002b).

ernment. Indeed, applying best corporate practice would allow individual government financial entities to set their own strategic asset allocation, after taking into account the nature of their liabilities (Grimes 2001). This is likely to improve governance, accountability, and entity performance. In the case of the Future Fund, this would involve directing the Fund to invest in assets of a similar risk to the government's superannuation liabilities.

In dealing with agency costs, the government imposes constraints on the optimal portfolio. The significance of these constraints has been highlighted by the results of Fowlie and Wright (1997) for New Zealand. They found that the optimal financial portfolio incorporated foreign-currency denominated debt when taxes are included, but only domestic debt when taxes are excluded. This means that a narrow focus only on balance sheet assets can lead to financial investments that increase the chances of volatile tax changes.

However, this does not mean that financial investment decisions should not be decentralized (down to an agency level) or linked to narrow portfolio benchmarks (such as matching financial assets to future superannuation liabilities). Rather, a single central agency needs to be aware of how individual elements of the balance sheet interact with each other during macroeconomic shocks. Some commentators see the centralization of broader balance sheet risk management with debt management as a *logical step* (Currie and Velandia-Rubiano 2002). For example, the Swedish Debt Management Office advises government on the costs of contingent liabilities and the government debt portfolio (Hörngren 2003).<sup>25</sup> Such a structure allows natural hedges in the balance sheet to be identified, reducing the need (and costs) from individual agencies hedging. Alternatively, such a balance sheet perspective allows for large cumulative risks to be identified and brought to the attention of government (Wheeler 2004:67).

### *Imperfect Capital Markets*

The model discussed in this paper assumes that capital markets are not perfect; or at least that certain restrictions exist that stop governments from using financial instruments to perfectly hedge balance sheet risk. While governments can use some existing financial instruments to reduce balance sheet risk, certain types of risks are still likely to remain unhedged. In particular, incomplete capital markets may mean governments are unable to hedge against certain types of risks (such as catastrophic risk). There may be no private sector substitutes for government bonds (Arrow and Lind 1970, Stiglitz 1983). If capital markets are incomplete, there may be gains from governments issuing standardized products that can out-

25. Other countries where debt managers are integrated with contingent liability management include New Zealand, South Africa, and Colombia (Wheeler 2004:24).

weigh benefits from state-contingent products (Missale 1997). Alternatively, governments may have other policy objectives, such as maintaining some debt to allow the development of important financial products (Comley and Turvey 2005). There may also be other reasons for investing domestically. For example, informational, governance, or tax advantages may lead to a home country bias for equities (for a review, see Karolyi and Stulz 2002). Such constraints can limit the ability of government's movement toward the optimal portfolio outlined in this paper.

Even if capital markets are imperfect, governments can still invest in financial assets to the fullest extent possible using available securities. For the remnant unhedged risks, the government should consider building and maintaining a positive balance of net worth as self-insurance against large rare events.<sup>26</sup>

## 7.5 Conclusion

The economics literature relating to balance sheet management suggests that the government's financial portfolio should be structured to reduce the budget impacts of macroeconomic shocks. More specifically, an optimally structured balance sheet can reduce the risk that a major macroeconomic shock will see large changes in tax rates. This not only reduces the distortions caused by volatile tax rates, but increases the flexibility of governments to respond to unexpected fiscal pressures. A government that invests well has less need to significantly raise taxes or cut spending to finance itself. In most countries, this has meant structuring the debt portfolio so that liabilities do not become overly burdensome during recessions. However, Australia is amongst a small number of countries determining how best to structure a financial portfolio that includes positive net financial assets.<sup>27</sup>

This paper has shown that it is not only the budget position that is important for sustainability, but how the financial assets and liabilities of government are allocated. Our results support the Future Fund investing in a broad range of financial assets that includes nominal domestic debt and equities from selected countries. Indeed, by investing optimally the government is likely to reduce risks on the budget and improve growth prospects.

26. Hansen (2003:11) notes that building a *precautionary balance* is worthwhile if and only if the unhedged risks would otherwise result in a negative correlation between tax rates and consumption.

27. Other OECD countries with significant financial asset funds (including pension funds) are New Zealand, Norway, Ireland, Finland, and Denmark (see Comley and McKissack 2005).

## Appendix A

### *Derivation of First-Order Conditions*

The government chooses taxes and debt to maximize the individual objective function (A1) subject to its budget constraint (A2).

$$(7.A1) \quad U_t = E_t \sum_{j \geq 0} \rho^j \{ Y_{t+j} [1 - h(\tau_{t+j})] \} \\ = E_t Y_t [1 - h(\tau_t)] + \rho E_t \{ Y_{t+1} [1 - h(\tau_{t+1})] \} + \dots$$

$$(7.A2) \quad \tau_t Y_t + \sum_k p_{t,k} B_{t,k} = G_t + \sum_k (p_{t,k} + f_{t,k}) B_{t-1,k}$$

From the budget constraint, we can solve for  $\tau_1, \tau_{t+1}$ , and so on, and substitute into the objective function,  $U_t$ , which can then be maximized with respect to  $B_{t,k}, B_{t+1,k}$ , and so on.

The first-order condition with respect to  $B_{t,k}$  is:

$$(7.A3) \quad \frac{\partial U_t}{\partial B_{t,k}} = E_t \left\{ Y_t \left[ -h'(\tau_t) \frac{\partial \tau_t}{\partial B_{t,k}} \right] \right\} + \rho E_t \left\{ Y_{t+1} \left[ h'(\tau_{t+1}) \frac{\partial \tau_{t+1}}{\partial B_{t,k}} \right] \right\} = 0$$

Now, from the constraints:

$$(7.A4) \quad \frac{\partial \tau_t}{\partial B_{t,k}} = -\frac{p_{t,k}}{Y_t} \text{ and, } \frac{\partial \tau_{t+1}}{\partial B_{t,k}} = \frac{(p_{t+1,k} + f_{t+1,k})}{Y_{t+1}}$$

Substituting into equation (A3) gives:

$$E_t [-h'(\tau_t)p_{t,k}] + \rho E_t [h'(\tau_{t+1})(p_{t+1,k} + f_{t+1,k})] = 0$$

Since  $p_{t,k}$  is known at time  $t$ , and  $\tau_t$  is chosen at time  $t$ , equation (A4) can be written as:

$$h'(\tau_t) = \rho E_t \left[ \frac{h'(\tau_{t+1})(p_{t+1,k} + f_{t+1,k})}{p_{t,k}} \right]$$

Recalling that  $r_{t+1,k} = (p_{t+1,k} + f_{t+1,k})/p_{t,k} - 1$ , and noting also that the condition that expected returns must be equal implies that for the risk-free asset (defined as  $k = 0$ ),  $r \equiv 1/\rho - 1$ , then the previous expression will yield the first-order condition obtained in equation (5) of section 7.3.

## Appendix B

### ***Derivation of Equation (9)***

The optimality condition for each government security  $k$  ( $k = 1, \dots, K$ ) given in equation (8) is:

$$(7.B1) \quad \sum_l cov_t(\hat{r}_{t+1,k}, \hat{r}_{t+1,l}) d_{t,l} + cov_t(\hat{r}_{t+1,k}, \sum_{j \geq 0} p^j \hat{s}_{t+1+j}) - w_t cov_t(\hat{r}_{t+1,k}, \sum_{j \geq 0} \psi^j \hat{y}_{t+1+j}) = 0$$

As discussed in section 7.3 we assume that the second term is equal to zero and so the previous equation can be reduced to:

$$(7.B2) \quad \sum_l cov_t(\hat{r}_{t+1,k}, \hat{r}_{t+1,l}) d_{t,l} - w_t cov_t(\hat{r}_{t+1,k}, \sum_{j \geq 0} \psi^j \hat{y}_{t+1+j}) = 0$$

$$(7.B3) \quad \begin{bmatrix} cov_t(\hat{r}_{t+1,l}, \hat{r}_{t+1,l}) & \dots & cov_t(\hat{r}_{t+1,l}, \hat{r}_{t+1,l}) \\ \vdots & \ddots & \vdots \\ cov_t(\hat{r}_{t+1,k}, \hat{r}_{t+1,l}) & \dots & cov_t(\hat{r}_{t+1,K}, \hat{r}_{t+1,l}) \end{bmatrix} \begin{pmatrix} d_{t,1} \\ \vdots \\ d_{t,l} \end{pmatrix} = w_t \begin{bmatrix} cov_t\left(\hat{r}_{t+1,l}, \sum_{j \geq 0} \psi^j \hat{y}_{t+1+j}\right) \\ \vdots \\ cov_t\left(\hat{r}_{t+1,K}, \sum_{j \geq 0} \psi^j \hat{y}_{t+1+j}\right) \end{bmatrix}$$

Rearranging this gives:

$$(7.B4) \quad \begin{pmatrix} d_{t,1} \\ \vdots \\ d_{t,l} \end{pmatrix} = w_t \begin{bmatrix} cov_t(\hat{r}_{t+1,l}, \hat{r}_{t+1,l}) & \dots & cov_t(\hat{r}_{t+1,l}, \hat{r}_{t+1,l}) \\ \vdots & \ddots & \vdots \\ cov_t(\hat{r}_{t+1,K}, \hat{r}_{t+1,l}) & \dots & cov_t(\hat{r}_{t+1,K}, \hat{r}_{t+1,l}) \end{bmatrix}^{-1} \cdot \begin{bmatrix} cov_t\left(\hat{r}_{t+1,l}, \sum_{j \geq 0} \psi^j \hat{y}_{t+1+j}\right) \\ \vdots \\ cov_t\left(\hat{r}_{t+1,K}, \sum_{j \geq 0} \psi^j \hat{y}_{t+1+j}\right) \end{bmatrix}$$

This can be simplified with the following notation:

$$(7.B5) \quad \mathbf{d} = w_t \sum_r^{-1} \cdot \sum_{y,r}$$

In our first estimations, we restrict ourselves to the analysis of domestic and foreign currency debt.

$$(7.B6) \quad \begin{pmatrix} d_d \\ d_f \end{pmatrix} = \frac{w_t}{\Delta} \begin{pmatrix} \text{cov}(\hat{r}_{t+1,2}, \hat{r}_{t+1,2}) & -\text{cov}(\hat{r}_{t+1,2}, \hat{r}_{t+1,l}) \\ -\text{cov}(\hat{r}_{t+1,l}, \hat{r}_{t+1,2}) & \text{cov}(\hat{r}_{t+1,l}, \hat{r}_{t+1,l}) \end{pmatrix}$$

$$\begin{pmatrix} \text{cov}\left(\hat{r}_{t+1,l}, \sum_{j \geq 0} \Psi^j \hat{y}_{t+1+j}\right) \\ \text{cov}\left(\hat{r}_{t+1,2}, \sum_{j \geq 0} \Psi^j \hat{y}_{t+1+j}\right) \end{pmatrix}$$

where

$$\Delta = \begin{vmatrix} \text{cov}(\hat{r}_{t+1,l}, \hat{r}_{t+1,l}) & \text{cov}(\hat{r}_{t+1,2}, \hat{r}_{t+1,l}) \\ \text{cov}(\hat{r}_{t+1,l}, \hat{r}_{t+1,2}) & \text{cov}(\hat{r}_{t+1,2}, \hat{r}_{t+1,2}) \end{vmatrix}$$

We recall that the domestic and foreign innovation of returns is given by the following equations

$$\begin{aligned} \hat{r}_{t+1,d} &= -(\ell_{t+1} - E_t \ell_{t+1}) - (\pi_{t+1} - E_t \pi_{t+1}) \\ \hat{r}_{t+1,f} &= -(\ell_{t+1}^* - E_t \ell_{t+1}^*) - (\pi_{t+1} - E_t \pi_{t+1}) + (\Delta s_{t+1} - E_t \Delta s_{t+1}) \end{aligned}$$

and define  $\Delta s = \Delta s_{t+1} - E_t \Delta s_{t+1}$ ,  $\pi = \pi_{t+1} - E_t \pi_{t+1}$ ,  $l = l_{t+1} - E_t l_{t+1}$ ,  $y = \sum_{j \geq 0} \Psi^j \hat{y}_{t+1+j}$  and denote  $Cov(x, y) = c(x, y)$  and  $Var(x) = v(x)$ . Expanding equation (B6) we get:

$$\begin{pmatrix} d_d \\ d_f \end{pmatrix} = \frac{w_t}{\Delta} \begin{bmatrix} v(\Delta s - l^* - \pi) c(-l - \pi, y) - c(\Delta s - l^* - \pi, -l - \pi) c(\Delta s - l^* - \pi, y) \\ v(-l - \pi) c(\Delta s - l^* - \pi, y) - c(-l - \pi, \Delta s - l^* - \pi) c(-l - \pi, y) \end{bmatrix}$$

## Appendix C

### *Disaggregated Variance-Covariance Matrix of Innovations*

**Table 7C.1** Variance-covariance matrix: U.S. and domestic currency debt securities

	$Y$	$\pi$	$l_d$	$\Delta s$	$l_e$	$r_e$	$r_d$
$Y$	0.873						
$\pi$	0.108	0.330					
$l_d$	0.013	-0.014	0.020				
$\Delta s$	0.089	0.635	-0.106	10.895			
$l_e$	0.004	-0.006	0.010	-0.078	0.008		
$r_e$	-0.022	0.311	-0.101	10.338	-0.080	10.106	
$r_d$	-0.120	-0.317	-0.006	-0.529	-0.003	-0.210	0.322

**Table 7C.2** Variance-covariance matrix: Japan and domestic currency debt securities

	$Y$	$\pi$	$l_d$	$\Delta s$	$l_e$	$r_e$	$r_d$
$Y$	0.510						
$\pi$	0.072	0.362					
$l_d$	0.028	-0.006	0.022				
$\Delta s$	-0.326	0.181	-0.109	41.692			
$l_e$	-0.016	-0.010	0.004	-0.063	0.006		
$r_e$	-0.381	-0.171	-0.107	41.574	-0.060	41.805	
$r_d$	-0.100	-0.356	-0.017	-0.072	0.006	0.279	0.373

**Table 7C.3** Variance-covariance matrix: Germany and domestic currency debt securities

	$Y$	$\pi$	$l_d$	$\Delta s$	$l_e$	$r_e$	$r_d$
$Y$	0.298						
$\pi$	0.009	0.365					
$l_d$	0.029	-0.010	0.022				
$\Delta s$	-0.259	0.529	-0.258	37.293			
$l_e$	-0.003	0.014	0.007	-0.155	0.008		
$r_e$	-0.264	0.150	-0.255	36.918	-0.177	36.946	
$r_d$	-0.038	-0.355	-0.012	-0.271	-0.021	0.106	0.367

**Table 7C.4****Variance-covariance matrix: Domestic debt and domestic equities**

	$Y$	$\pi$	$l_d$	$\Delta s$	$l_e$	$r_e$	$r_d$
$Y$	0.873						
$\pi$	0.108	0.309					
$l_d$	0.013	-0.014	0.021				
$\Delta s$	0.089	0.635	-0.106	10.895			
$l_e$	0.004	-0.006	0.010	-0.078	0.008		
$r_e$	-0.022	0.311	-0.101	10.338	-0.080	10.320	
$r_d$	-0.120	-0.317	-0.006	-0.529	-0.003	-0.210	0.314

**Table 7C.5****Variance-covariance matrix: Domestic debt and U.S. equities**

	$Y$	$\pi$	$l_d$	$\Delta s$	$l_e$	$r_e$	$r_d$
$Y$	0.298						
$\pi$	0.046	0.329					
$l_d$	0.019	-0.016	0.020				
$\Delta s$	-0.104	0.750	-0.103	10.882			
$l_e$	0.052	-0.680	-0.006	-4.939	59.111		
$r_e$	-0.098	-0.259	-0.093	5.193	54.851	60.304	
$r_d$	-0.065	-0.313	-0.004	-0.647	0.686	0.352	0.317

**Table 7C.6****Variance-covariance matrix: Domestic debt and Japanese equities**

	$Y$	$\pi$	$l_d$	$\Delta s$	$l_e$	$r_e$	$r_d$
$Y$	0.319						
$\pi$	0.057	0.337					
$l_d$	0.023	-0.011	0.022				
$\Delta s$	-0.410	-0.113	-0.136	40.406			
$l_e$	0.290	-0.160	0.231	-8.798	67.711		
$r_e$	-0.178	-0.609	0.106	31.720	59.072	91.402	
$r_d$	-0.080	-0.326	-0.011	0.250	-0.072	0.504	0.337

**Table 7C.7****Variance-covariance matrix: Domestic debt and German equities**

	$Y$	$\pi$	$l_d$	$\Delta s$	$l_e$	$r_e$	$r_d$
$Y$	0.328						
$\pi$	0.014	0.323					
$l_d$	0.033	-0.010	0.022				
$\Delta s$	-0.540	0.809	-0.235	31.201			
$l_e$	0.303	-0.074	-0.087	-37.678	249.916		
$r_e$	-0.252	0.413	-0.312	-7.286	212.312	204.613	
$r_d$	-0.047	-0.313	-0.013	-0.574	0.160	-0.101	0.326

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## Comment Youngsun Koh

This paper discusses a framework for optimal balance sheet management from the tax-smoothing perspective and concludes that the Future Fund

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