Changing Progressivity as a Means of Risk Protection in Investment-Based Social Security

Andrew Samwick
Dartmouth College and NBER
andrew.samwick@dartmouth.edu
December 21, 2006

Abstract: This paper analyzes changes in the progressivity of the Social Security benefit formula as a means of lessening the risk inherent in investment-based Social Security reform. Focusing on a single cohort of workers who will reach their normal retirement age as the Social Security trust fund is projected to be exhausted, it simulates the distribution of benefits subject to both earnings and financial risk in a reformed system in which solvency has been restored and traditional benefits have been augmented by personal retirement accounts. The simulations show that switching from the current benefit formula to the maximally progressive formula—a flat benefit independent of earnings—allows the bottom 30 percent of the earnings distribution to achieve a higher expected utility even with no PRA investments in equity. An additional 30 percent of earners can lessen their equity investments without loss of welfare. Similarly, by using progressive benefit reductions in which replacement rates for lower earnings are reduced by less than those for higher earnings, about half of the equity risk can be eliminated for the lowest earnings decile. Sensitivity tests show that these patterns are robust to different assumptions about risk aversion, the equity premium, and the size of the personal retirement accounts established by the reform.

Keywords:

I thank Mike Hurd and conference participants at the NBER Program on Aging's 2006 Conference on Retirement Research for helpful comments. This research was supported by the U.S. Social Security Administration through grant #10-P-98363-1-03 to the National Bureau of Economic Research as part of the SSA Retirement Research Consortium. The findings and conclusions expressed are solely those of the author and do not represent the views of SSA, any agency of the Federal Government, or the NBER. Any errors are my own.

I. Introduction

Around the globe, traditional pay-as-you-go Social Security systems are facing financial challenges due to demographic changes. With fertility rates at or below replacement levels in developed countries and life expectancy in retirement projected to continue increasing, the ratio of beneficiaries to workers will rise over the coming decades, increasing annual costs relative to income. The imminent retirement of the Baby Boom generation in many developed countries has sharpened attention on the need for reform.[1]

Over the past twelve years, many analysts have proposed that at least some of the financial shortfalls be eliminated through the prefunding of future benefits, in order to ameliorate the increase in pay-as-you-go tax rates on future generations of workers that would otherwise be required.[2] Prefunding can more readily take place in a system of decentralized Personal Retirement Accounts (PRAs) than in the Social Security Trust Fund, particularly when it is desired to exploit the risk-return tradeoff inherent in the equity premium and to separate the incremental saving due to higher Social Security taxes from the rest of the federal government's budget.[3]

The possibility that Social Security benefits paid from personal accounts would be subject to financial risk due to stock return volatility, in turn, has focused attention on ways limit the risk in investment-based Social Security reform. Concerns focus particularly on low-income beneficiaries, for whom Social Security benefits make up a

---

[1] The Social Security Trustees Report 2006 projects an increase in the number of beneficiaries per hundred workers from 30 in 2005 to 49 in 2040 to 53 in 2080 (Table IV.B2). For an international description of the demographic challenge, see World Bank (1994).

[2] The Office of the Chief Actuary at the Social Security Administration has formally analyzed over two dozen proposals. See the memoranda at http://www.ssa.gov/OACT/solvency/ .

[3] See Samwick (1999, 2004) for further discussion of the role of PRAs in prefunding future entitlement benefits.

disproportionate share of their retirement income. Two principal methods have been explored in the recent literature. The first is to offer a guarantee to workers that benefits will not fall below a particular threshold (e.g., 90 percent of scheduled benefits). Feldstein and Ranguelova (2001a,b) demonstrate that such guarantees can be implemented via long-term options on a stock market index in a manner similar to conventional portfolio insurance. The second is to follow common financial planning strategies that reduce the portfolio allocation in equities as an individual approaches retirement. Poterba, Rauh, Venti, and Wise (2006) explore the efficacy of using "life cycle" strategies in this way.

These two mechanisms share the feature that they introduce bonds (preferably as inflation-indexed securities) into the portfolio in order to lessen the exposure to equity risk. However, in doing so, these mechanisms give up the equity premium and thus lose one very important rationale for including PRAs in the reform. In contrast, the analysis below considers an alternative approach based on modifications to the traditional benefit to protect low-earning workers while leaving all workers free to choose their own PRA portfolios. Such an approach may prove to be useful, particularly because any restrictions on the portfolio allocations in the PRAs beyond the determination of which investment choices will be offered are likely to be untenable as the accounts become larger and more popular.

The most direct way to make sure that low-earning workers do not fall into poverty in old age is to increase the progressivity of the benefit formula in the scaled-down version of the traditional system that remains after reform. Doing so would lessen the need to provide insurance against possibly low returns in the PRAs, because low-income retirees would depend less on the PRAs to stay out of poverty. To be sure, there have been discussions of progressive reductions in the traditional benefits as part of a plan to close the financial gap while protecting low-earning workers. This paper adds to the literature by quantifying the effect of such changes to the traditional benefit formula on the need to invest PRAs in equity rather than bonds.

This paper illustrates the link between progressivity and risk using a stylized framework based on simulations of earnings trajectories and portfolio returns. The simulations are based on the projected experience of a cohort of workers corresponding roughly to those born in 1973. To calibrate the simulations, traditional retirement benefits are reduced by 40 percent, an amount comparable to what is projected to be required to restore solvency to the system.[4] The simulations pair reductions in the traditional benefits of this magnitude with PRAs funded by contributions of 2 percent of earnings each year. The main comparisons are between the utility maximizing portfolio allocations to equities across the new configurations of the traditional benefit that are more versus less progressive.

The key finding is that under baseline parameters, the most progressive traditional benefit—a flat benefit independent of earnings—allows the bottom 30 percent of the earnings distribution to achieve a higher expected utility than under the proportional reductions to the current benefit formula even with no investments in equity. An additional 30 percent of earners can lessen their equity investments without loss of

[4] In 2080, the latest year of the projections in the Social Security Trustees Report 2006 (Table IV.B1), the annual gap is 5.38 percentage points of taxable payroll, compared to a cost rate (excluding disability insurance benefits) of 16.27 percentage points of taxable payroll. Thus, the required reduction is 5.38/16.27 = 33 percent. However, this assumes that all benefits—including those of current retirees—can be cut by this amount. The need to protect benefits already in payment would lead to a higher cut to benefits yet to be paid.

welfare relative to those available under the scaled-back current formula. Under more realistic and less extreme changes to the traditional benefit, such as that proposed by Liebman, MacGuineas, and Samwick (2005), about half of the equity risk can be eliminated for the lowest earnings decile, and some equity risk can be eliminated for the bottom six deciles. The optimal allocation to equities in the PRA is not particularly sensitive to the progressivity of the reductions in the traditional benefits—in most simulations, the share in equities increases slightly for low earners and decreases slightly for higher earners with more progressive reductions in the traditional benefits.

The remainder of the paper is organized as follows. Section II lays out the simulation framework for both the traditional benefits and the new system of PRAs. Section III discusses the combinations of PRA asset allocations and reductions in the traditional benefits that will be analyzed. Section IV derives the certainty equivalent measure of expected utility that will be used in the comparisons. Section V presents the baseline results, and Section VI includes sensitivity tests and a comparison to life cycle investment strategies. Section VII concludes.

II. The Simulation Framework

The model used in the analysis focuses on a cohort of workers who should expect to have their traditional benefits reduced at some point when the Social Security system is restored to solvency. Specifically, the analysis simulates the experience of the birth cohort of 1973, who will reach their normal retirement age in 2040, just as the Social Security trust fund is presently projected to be exhausted. Trust Fund exhaustion will necessitate changes to the system, even if they have not been made before that time. The analysis assumes, counterfactually, that the workers have been in the new system since they entered the workforce.

The distribution of earnings at an initial age is assumed to be lognormal, allowing its parameters to be estimated from the mean and median of a sample of data. Kunkel (1996) reports the mean and quartiles of the distribution of earnings by age group for the years 1980-1993, based on a detailed sample of Social Security records. The population of 30 year olds in this analysis is approximated by the 25 – 34 year old cohort in Kunkel's data, and parameters of the lognormal are estimated for each year of Kunkel's data.[5] These parameters are averaged across all the years, and the resulting distribution is scaled up by the growth in the average wage index in Social Security through 2003, the last year for which an estimate of that index is currently available in SSA (2006). To allow for the analysis below to include the distributional consequences of changes to the Social Security benefit formula, the lognormal distribution is approximated by a ten workers who fall at the midpoints of the deciles of that distribution.

For each such worker, earnings evolve over the life cycle due to deterministic changes in permanent income and stochastic shocks to earnings around permanent income. The result of the analysis below is the distribution of simulated benefits, where simulations are conducted with 5,000 independent replicates for each of the ten workers representing the deciles of the initial distribution of earnings. The processes for the growth in permanent income are identical for all replicates of all workers. Permanent income grows each year due the growth in national average wages, approximated here by

[5] The median and mean of a lognormal distribution are given by $\exp(\mu)$ and $\exp(\mu + 0.5*\sigma^2)$, respectively, where exp( ) denotes the exponential function and μ and σ are the mean and standard deviation of the underlying normal distribution. The median therefore identifies μ and the ratio identifies σ. The estimated parameters for the group discussed in the text are {10.2056, 0.5271}.

the average growth rate of Social Security covered wages during the 1952-2003 period, or 1.1 percent per year. Permanent income also follows an age-earnings profile, reflecting changes in individual productivity and hours worked over the life cycle. Each worker is assumed to face the age-earnings profile for the least educated group of workers analyzed by Hubbard, Skinner, and Zeldes (1994).[6] Stochastic deviations from permanent income follow an AR(1) process with a correlation coefficient of $\rho = 0.95$ and a standard deviation of 15 percent.[7] Given these parameters, earnings are backcasted from the initial distribution at age 30 (deterministically, at the average rate of wage growth) to age 21 and then forecasted to age 67.[8]

The Social Security benefit formula depends on the growth in national average wages in two places: to determine the maximum taxable earnings on which payroll taxes are paid and to index each year of earnings for the growth of wages during a worker's career. Since the framework focuses on the deciles of a single age cohort, the growth in the national average wage is approximated by the growth rate of this cohort's average wage over its career. Maximum taxable earnings subject to the payroll tax are projected forward and backward from 2003 (age 30) using this growth rate. With these few assumptions, it is possible to get a reasonable approximation of Social Security benefits by applying the benefit formula to the simulated earnings profiles.

In each of the policy scenarios, the traditional benefit is reduced by 40 percent in the aggregate and is augmented by the benefits payable from a PRA. PRA contributions

---

[6] This profile is approximated by having real earnings grow at annual rates of 2.5 percent between ages 21 and 30, 1.7 percent between 31 and 40, 0.5 percent between 41 and 50, and -1.3 percent through age 67. This growth is in addition to the growth in the national average wage.

[7] See Topel and Ward (1992) for other, comparable estimates of the wage process.

[8] Largely because the sample is constructed around a single deterministic age-earnings profile and full employment for each of the earnings deciles, it understates the cross-sectional variation in annual earnings each year. For example, the ratio of the 75th to the 25th percentiles of the earnings distribution at age 50 (or the age group 45-54) in the simulation is 2.59, compared to 3.30 in Kunkel (1996).

are 2 or 3 percent of earnings (depending on the scenario) up to the maximum taxable earnings level. Asset returns are based on the annual total returns in Table 2-5 of Ibboston Associates (2006) for the years 1926 – 2005. Asset classes include large stocks, small stocks, long-term corporate bonds, long-term government bonds, intermediate-term government bonds, and Treasury bills. These returns are further combined in to an equity portfolio (75 percent large stocks and 25 percent small stocks), the corporate bond portfolio, and a government bond portfolio (one third in each of the long-term, intermediate-term, and bills). Each age (e.g., 45) in each of the 5,000 replicates is assigned a random year of returns (e.g., 1973) from this 80-year span. Each of the ten workers, corresponding to the deciles of the initial distribution of earnings, therefore receives the same sequence of return-years. Portfolio allocations are as specified for each scenario. At retirement, PRA balances are converted to inflation-indexed annuities at a real interest rate of 3 percent, matching the long-term bond return in the Trustees Report.[9]

III. Combining Personal Accounts with a Smaller Traditional Benefit

Several approaches to reducing traditional pay-as-you-go benefits are considered, all of which reduce aggregate payouts by 40 percent (because all are designed to restore solvency to the same degree). They differ in the extent to which they protect the benefits of low-earners, whose total retirement incomes are more vulnerable to the financial risk that may come from PRAs. At one extreme is a proportional reduction in the traditional benefits, in which the entire benefit formula is scaled down by 40 percent. This approach

[9] The annuity factor is derived from the Period life table from 2002, available at http://www.ssa.gov/OACT/STATS/table4c6.html. A dollar of PRA balance translates into $1/13.15 in annual inflation-indexed benefits. This figure is the average of the two factors for men (12.3) and women (14.0), respectively.

leaves the progressivity of the traditional benefit unchanged and is referred to as the Proportional Reduction. At the other extreme, the most progressive way to reduce traditional benefits is to pay each beneficiary the same amount, regardless of earnings. In this case, Social Security would play a flat benefit equal to the mean benefit in the system (scaled down by 40 percent). This method is referred to as the Uniform Benefit below.

Between these two extremes lie other possible approaches. One possibility is to use a weighted average of the two extremes. The simulations below consider a Half and Half benefit formula which combines the Proportional Reduction and Uniform Benefit and then divides the total by two. Another approach is to reduce benefits progressively based on features of the current benefit formula. For example, in the reform plan presented by Liebman, MacGuineas, and Samwick (2005), the replacement rates were lowered by 25 percent below the first bend point in the formula (from 90 to 7.5 percent) and 50 percent above the first and second bend points (from 32 and 15 percent to 16 and 7.5 percent).[10]

In a reformed system, PRAs are added to the traditional benefits to help maintain total retirement replacement rates. The asset allocation decision in PRAs in this framework is simply a question of equity relative to bonds. The simulations below consider time-invariant allocations to equity ranging from 0 to 100, effectively assuming annual rebalancing to meet this allocation target.[11] For purposes of comparison, three "life cycle" strategies are also simulated, in which the allocation to stocks averages 50

[10] See SSA (2006) for a description of the Social Security benefit formula. See Goss and Wade (2005) for an evaluation of the Liebman, MacGuineas, and Samwick (2005) plan. Both documents can be found at http://www.nonpartisanssplan.com for reference.

[11] In reality, a worker might choose to vary the allocation to equities over time as a response to realizations of both earnings and investment returns. The assumption of constant allocations throughout the life-cycle greatly simplifies the analysis, in order to focus on the main tradeoff of progressivity in the benefit formula against the need for low-earning workers to exploit the equity premium. The extension to a dynamic programming that solves for the optimal portfolio is a subject for future work.

percent but declines linearly with age at rates of 1.0, 1.5, or 2.0 percentage points per year.

When it evaluates Social Security reform plans, the Office of the Chief Actuary at the Social Security Administration assigns mean returns by asset type. In recent evaluations, such as Goss and Wade (2005), mean returns have been assumed to be 6.2 percent for equity, 3.2 percent for corporate bonds, and 2.7 for government bonds, net of both inflation and a modest 30 basis point administrative cost. The baseline simulations utilize these assumptions. To capture the volatility around the mean, the historical variation in asset returns from 1926 – 2005 reported by Ibbotson Associates (2006) is utilized. Standard deviations are 22.2 percent for equity, 9.2 percent for corporate bonds, and 6.6 percent for government bonds. All simulations preserve these standard deviations but change the mean returns (by the difference between the specified mean return and the mean of the historical data), allowing for potentially lower equity premiums going forward than what SSA's Office of the Actuary has assumed.[12]

IV. Evaluating Risk in Retirement Benefits

In the main simulations, workers are assumed to have constant relative risk aversion (CRRA) utility functions, defined over total retirement benefits, *b*, with risk aversion coefficient γ:

$$u(b) = \frac{b^{1-\gamma}}{1-\gamma}$$

[12] Social Security reform proposals that include PRAs often stipulate that the balance can be bequeathed. Bequests are not modeled in this analysis, but this is not a major problem. Allowing for bequests would simply raise the required contribution rate to the PRA to ensure that the 2 or 3 percentage points specified in the simulations go to fund the annuities.

Expected utility for the worker representing each decile of the initial wage distribution is calculated as the average across 5,000 independently drawn replicates. It therefore encompasses the uncertainty in both portfolio returns and earnings, while also allowing for comparisons across different points in the initial earnings distribution.[13] As a basis for comparison across configurations of the traditional benefit formula and the PRA asset allocation rules, we can calculate the certainty equivalent benefit:

$$b_{CE} = \left[(1-\gamma)E(u(b))\right]^{1/(1-\gamma)} = \left[E\left(b^{1-\gamma}\right)\right]^{1/(1-\gamma)}$$

The certainty equivalent is the retirement benefit that, if received with certainty, would make the individual equally well off as facing the uncertain benefit distribution. For a risk averse individual, the certainty equivalent will be less than the expected benefit level, *E(b)*. A higher certainty equivalent indicates a higher expected utility, and differences in certainty equivalents correspond to risk premiums measured in dollar terms.

By construction, the aggregate expected benefits from the traditional system are identical across all policy scenarios, conditional on the earnings realizations. This is not true within each decile, as some benefit formulas are designed to be more progressive than others and thus provide differential expected benefits to different deciles. Other differences in certainty equivalents across the policy scenarios reflect different exposure to risk, whether through the traditional benefit formula or the PRA investment portfolio, or different expected benefits through the PRA investment portfolio.

[13] Defining the deciles with respect to initial earnings is appropriate in the current framework in which workers are assumed to adopt a single, time-invariant allocation to equities in their PRAs. An alternative approach to doing distributional analysis would use a measure of average lifetime earnings to assign workers to deciles. For example, some workers in the lowest initial earnings decile receive a number of very positive earnings draws and wind up higher in the distribution of lifetime earnings. For comparison, assigning workers to deciles based on their average indexed monthly earnings yields an allocation to deciles with a correlation of 0.83 with the deciles of the initial earnings distribution.

V. Trading off Progressivity and Risk

Figure 1 illustrates the impact of the benefit formula and the equity share of the PRA portfolio on expected benefits. The graph shows the relationship between expected benefits and the equity share in the PRA portfolio for the highest and lowest earnings deciles under three different benefit formulas: Proportional Reduction, Progressive, and Uniform Benefit. The curves for the top decile earner go in that order, and the curves for the bottom decile go in the reverse order. The Proportional Reduction is most generous for the highest decile and least generous for the bottom decile. The Uniform Benefit is the opposite—most generous for the bottom decile and least generous for the top decile. The Progressive benefit reduction actually tracks the Proportional Reduction fairly closely. The Half and Half benefit formula (not shown) would fall exactly between the Proportional Reduction and Uniform Benefit. Because the risk premium on equities is positive, expected benefits increase in all cases with the portfolio share in equities. For workers in the bottom (top) decile, increases in the equity share in the PRA portfolio and increases (decreases) in the progressivity of the traditional benefit formula are two different ways to increase the expected benefit level.

Figure 2 shows the impact of benefit risk on the expected utility of portfolio choices in the PRA. The horizontal axis shows the portfolio share of the PRAs invested in equities, and the vertical axis shows the dollar amount of the expected benefits or expected utility (expressed as a certainty equivalent). The highest curve shows the expected benefits from a traditional benefit based on the current formula, reduced by 40 percent to restore solvency, combined with a PRA funded by contributions of 2 percent of taxable payroll per year. (This is the same curve as the top curve in Figure 1.) The graph is for the highest decile of the earnings distribution. Expected benefits increase slightly faster than linearly with the equity share of the portfolio. With a coefficient of relative risk aversion of 1, the certainty equivalent is increasing with the equity share in the portfolio, but at a decreasing rate. The optimal equity share is therefore 100. As the coefficients of relative risk aversion increase to 3 and 5 in the next two curves, the optimal equity share falls to 80 percent and 60 percent, respectively.[14]

Figures 3 – 6 and Table 1 combine the elements of the first two figures to compare certainty equivalents by earnings decile and equity portfolio share for each of the four possible traditional formulas. Figure 3 shows the results for the lowest earnings decile in the baseline case: PRAs funded by contributions of 2 percent of taxable payroll, a coefficient of relative risk aversion equal to 3, and rates of return on asset classes—equity, corporate bonds, and government bonds—having the values assumed by Goss and Wade (2005) in the Social Security Administration's official scoring of reform proposals: 6.2, 3.2, and 2.7 percent, respectively.

The four curves in Figure 3 correspond to the certainty equivalents as a function of the PRA portfolio share in equity for the Proportional Reduction, Progressive, Half and Half, and Uniform Benefit formulas. In all cases, the highest certainty equivalent occurs at a portfolio share of 100 percent in equities, where the curves intersect the right vertical axis. The differing degree of progressivity across the benefit formulas means that the formulas differ in the level of the certainty equivalents at this optimal portfolio share. With a more progressive traditional benefit, a worker could choose to reduce the equity

[14] The extent of risk aversion can be illustrated by considering how much an individual would pay to avoid a specified risk. Consider a 50-50 chance of having wealth increase or decrease by 25 percent. An individual with a coefficient of relative risk aversion of 3 would pay about 9.1 percent of his wealth to avoid this risk. An investor with log utility (a coefficient of 1) would pay only 3.2 percent, while an investor with a coefficient of relative risk aversion of 5 would pay 13.5 percent.

share—and with it, the volatility of the PRA benefit—while still surpassing the expected utility afforded by a less progressive benefit formula. For example, with the Uniform Benefit and the Half and Half benefit formula, a worker could allocate none of the PRA portfolio to equity and still have a higher certainty equivalent than with the Proportional Reduction benefit formula and a 100 percent allocation to equity. This can be seen in Figure 3 in the greater height of the Uniform Benefit and Half and Half benefit on the *left* vertical axis than the Proportional Reduction achieves on the right vertical axis. For the Progressive formula, an equity share as low as 50 percent is enough to exceed the certainty equivalent generated by the Proportional Reduction and a 100 percent equity share.

These comparisons are summarized in Table 1. The first panel shows the maximum certainty equivalents for each benefit formula (in the columns) and each decile of the earnings distribution (in the rows), where the maximum is chosen over equity shares that are multiples of 5 between 0 and 100. The second panel shows, for each earnings decile and benefit formula, the equity share that gives that maximum certainty equivalent. Finally, the bottom panel shows, for all benefit formulas that are not the Proportional Reduction, the lowest equity share (again, in multiples of 5), that will surpass the maximum certainty equivalent available under the Proportional Reduction. This panel will only have rows for earnings deciles in which this is possible. For example, a Uniform Benefit with an equity share of zero surpasses a Proportional Reduction with any equity share (including the maximum, at 100 percent) for the lowest three earnings deciles.

Figure 4 shows the same relationships for the earnings decile that is fourth from the bottom (roughly the 35th percentile). The curves are in the same order as in Figure 3, and the maximum certainty equivalents continue to occur at portfolio allocations of 100 percent equity. However, the vertical distances between the curves have narrowed, since benefit formulas that have the same average payout but differ in progressivity will redistribute relatively less to the fourth decile than they do to the bottom decile. The maximum certainty equivalent for the Proportional Reduction formula can now be surpassed with equity allocations as low as 30 percent, 50 percent, and 75 percent for the Uniform Benefit, Half and Half, and Progressive benefit formulas, respectively. The bottom panel of Table 1 shows that there is some potential for reducing the required exposure to equity by having a more progressive benefit formula for each of the bottom six deciles, though the potential shrinks at higher deciles.

Figure 5 shows the same curves for the seventh decile (roughly the 65th percentile) of the earnings distribution. The ordering of the curves has now switched, with the Proportional Reduction offering the highest certainty equivalents for each possible equity share, followed by the Progressive, Half and Half, and Uniform Benefit formulas. This is not surprising, as the redistribution toward the lower earning deciles must be paid for by those in higher earning deciles if the reforms have the same aggregate payouts but differ in their progressivity. The optimal equity allocations have fallen slightly, to 95 percent in equity for the Proportional Reduction and Progressive formulas and to 90 percent in equity for the Half and Half and Uniform Benefit formulas. All of the curves are quite close together, indicating very little scope for trading off exposure to equity by switching benefit formulas. Figure 6 shows the curves for the top decile of the earnings distribution. The curves retain the same ordering from Figure 5, but the gaps between the different formulas are now much wider. The optimal share in equity also falls to 75 percent for all four of the benefit formulas.

Figure 7 suggests why the progressivity of the benefit formula is such a powerful tool in comparison to the equity share of the PRA portfolio in affecting workers in the lower earnings deciles. The figure shows the cumulative distribution functions for the four different benefit formulas, holding constant the equity share of the PRA portfolio at 50 percent, for the bottom earnings decile. For any given benefit level, the height of the curve shows the probability of the specified benefit formula generating a benefit level that is at or below the given level. For curves that do not cross, the curve that is everywhere the lowest represents the most preferred benefit formula. As noted above, for this low-earning worker, that is the Uniform Benefit. Indeed, for this benefit formula, all of the variation in benefit levels is due to the variation in asset returns in different scenarios. Moving right to left on the graph, the other benefit formulas lower average benefits and add successively more earnings risk into the benefit distributions. The differences in the lowest benefit amounts across formulas (measured by the horizontal distance between the curves) are quite large. These differences also persist fairly high into the distribution of benefits, disappearing only at the highest benefit levels. Given risk averse workers, the level and likelihood of very low outcomes are of particular concern.

Figure 8 shows the variation in this decile's benefit distributions holding the benefit formula fixed (at Proportional Reduction) while varying the equity share in the portfolio from 0 to 100 percent in increments of 25 percentage points. At the very lowest benefit levels, the differences across the portfolio allocations are quite small in comparison to those shown in Figure 7. (The scales on the axes are identical across the figures.) Low benefit outcomes are primarily due to the factor held constant across the curves—the traditional benefit formula—rather than the factor varying across the curves—the equity share in the PRA portfolio. To the extent that there are differences, both the "All Equity" and "Zero Equity" portfolios have lower minimum benefits than more balanced portfolios. At the low end of the distribution, reducing the equity share from 100 percent does not even generate a lower likelihood of very bad outcomes.

These figures establish the main results of the analysis. Given the assumed average returns on equities and bonds and their historical variation, workers with CRRA utility and a coefficient of relative risk aversion of 3 typically choose very high equity shares in their PRA portfolios, regardless of the formula used to compute the traditional benefit. However, switching from a proportional reduction in the traditional benefits to any of the three more progressive benefit formulas increases the traditional benefits going to the bottom six deciles of the earnings distribution. This increase in traditional benefits gives the worker room to lower the equity share in the PRA portfolio while still achieving the same certainty equivalent available with the optimal equity share in the PRA under the proportionally reduced benefit. In the case of the maximally progressive benefit formula, in which the traditional benefit is a uniform benefit unrelated to the worker's earnings, the equity share could fall to zero for the lowest three deciles. Higher deciles or less extreme changes to the progressivity of the benefit formula result in somewhat smaller possible reductions in equity exposure.

VI. Sensitivity Tests

In this section, the robustness of the main results is assessed by varying the degree of risk aversion, the constancy of the coefficient of relative risk aversion, the equity premium, and the size of the PRAs measured by the annual contributions as a percentage of earnings. More risk aversion, declining relative risk aversion, a lower equity premium, and larger PRAs generally reduce the optimal portfolio allocations in equities and slightly compress the difference in the allocations across configurations of the traditional benefit that achieve the same certainty equivalent. This section concludes with a discussion of life cycle portfolio strategies.

Risk Aversion

The baseline choice of the coefficient of risk aversion is consistent with assumptions found in the literature on insurance and risk. Table 2 repeats the analysis of Table 1 for a higher coefficient of relative risk aversion equal to 5. The first consequence of higher relative risk aversion is that all of the certainty equivalents in the top panel of Table 2 are lower than their counterparts in Table 1. Consistent with Figure 2, a worker with higher risk aversion would pay a greater risk premium to avoid a given risk. The next panel of Table 2 shows that the workers seek to avoid this risk by reducing their equity shares in the PRA portfolio.[15] For example, with the Proportional Reduction, optimal equity shares are 95 percent in the lowest earnings decile, falling to 60 percent by the highest earnings decile.

[15] In other words, the certainty equivalents would be even lower if the workers were constrained to hold the equity shares at the levels in the middle panel of Table 1.

As shown in the bottom panel of the table, changes in the progressivity of the traditional benefit allow for reductions in equity exposure in the PRA portfolio that are comparable to those for the less risk averse workers in Table 1. For example, it is still the case that the bottom six earnings deciles have room to lower their equity exposure with more progressive traditional benefit formulas. In addition, the allowable percentage point reductions in the equity shares are similar. For example, with a Uniform Benefit, the bottom four deciles can now eliminate their equity exposure entirely. With the Progressive benefit formula, the equity share for the bottom earnings decile can fall from 95 to 25 percent without a loss in expected utility. Thus, the main results are robust to a higher coefficient of relative risk aversion.

Declining Relative Risk Aversion

The results in the middle panels of Tables 1 and 2 show that the optimal allocation to equity declines at higher earnings deciles. This pattern arises due to the maintained assumption in the simulations that workers have no other sources of retirement income apart from the traditional benefit and the PRA. Because even the current Social Security formula is progressive, workers in lower earnings deciles have a greater proportion of their retirement benefits insulated from investment risk. With a homothetic expected utility function, this enables lower earning workers to take on more equity risk in their PRA portfolios.[16]

[16] This assumption also generates the tendency for more progressive benefit formulas to have higher optimal allocations to equity for the bottom earnings deciles and lower optimal allocations to equity for the top earnings deciles. Greater progressivity results in more non-PRA benefits at low earnings deciles and less non-PRA benefits, relative to lifetime earnings, at high earnings deciles.

This pattern is counterfactual—in reality, investment allocations to equity rise dramatically with earnings.[17] One way to make the simulations more consistent with observed investment behavior is to modify the expected utility function to exhibit declining, rather than constant, relative risk aversion. The simplest such modification to make is to introduce a "subsistence level" of retirement benefit into the utility function, via the parameter k in:

$$u(b) = \frac{(b-k)^{1-\gamma}}{1-\gamma}$$

Note that $k = 0$ corresponds to CRRA utility and that with k greater than zero, utility is not defined for retirement benefit levels below k. For retirement benefit levels above k, risk is perceived relative to the subsistence level. Since low-earning deciles have benefits closest to this subsistence level, they will lower their equity allocations relative to the CRRA case. The certainty equivalent for this DRRA expected utility function is given by:

$$b_{CE} = k + \left[(1-\gamma)E(u(b))\right]^{1/(1-\gamma)} = k + \left[E\left((b-k)^{1-\gamma}\right)\right]^{1/(1-\gamma)}$$

Tables 3 and 4 repeat the analyses in Tables 1 and 2 using this DRRA expected utility function. The subsistence level is assumed to be $10,000, which is close to the minimum benefit for the lowest earning decile shown in Figure 7. The top panels of the tables show that the certainty equivalents are lower when expected utility exhibits

[17] See, for example, the tabulations in Bucks, Kennickell, and Moore (2004) or the multivariate estimates in Poterba and Samwick (2003), both based on data from the Surveys of Consumer Finances.

declining rather than constant relative risk aversion.[18] The middle panels of the tables show that optimal equity allocations are lower with declining relative risk aversion.

However, comparisons of the changes in the optimal equity allocations by earnings decile and across traditional benefit formulas relative to the CRRA case are not straightforward. For example, with $\gamma = 3$, equity shares with a Proportional Reduction in the traditional benefit fall from 75 to 65 percent over the earnings deciles, compared to a decline from 100 to 80 percent in the CRRA case, indicating less sensitivity to earnings decile. However, with a Uniform Benefit, they fall from 95 to 60 percent over the earnings deciles, compared to a decline from 100 to 75 percent in the CRRA case, indicating more sensitivity to earnings decile. Similar results hold for the higher risk aversion in Table 4 and in the differences across columns in the respective cases.

Nonetheless, the bottom panels of the tables show that changing from a Proportional Reduction to a more progressive benefit formula can lessen equity exposure by as much or more than in the CRRA case. For example, with $\gamma = 3$, the bottom six deciles can again have their equity exposure reduced. With a Uniform Benefit, the bottom four deciles can reduce equity exposure to zero without falling behind the Proportional Reduction. The sixth decile can lower its equity share from 70 to 35 percent, compared to a reduction from 95 to 80 percent in the CRRA case shown in Table 1. With the Progressive formula, the bottom decile can reduce its equity exposure down to zero and the sixth decile can reduce its equity share from 75 to 55 percent (compared to a reduction from 95 to 90 percent in the CRRA case). The results in Table 4 at higher risk aversion levels are even more pronounced. Thus, the main results shown in the

[18] The degree of relative risk aversion for any expected utility function is given by $-b*u''( )/u'( )$. For the DRRA utility function, this expression is $\gamma*b/(b-k)$, which is equal to the constant γ for $k = 0$ and is declining toward γ when $k > 0$.

previous section are robust to and strengthened by a switch to an expected utility function that exhibits declining rather than constant relative risk aversion.

Lower Equity Premium

The sustainability of the premium that has existed to investments in equities historically has been the subject of considerable debate. Particularly in the case of financial market returns, past performance may be an unreliable guide to future outcomes. For example, if over the past 30 years, systematic risk in the stock market fell, then the appropriate rate of return to assume going forward would be lower. However, during this period of time that risk fell, the reduction in risk would generate a contemporaneous increase in equity prices. These high holding period returns would arise precisely because future ex ante returns had fallen and would thus be a poor guide to forecasting those future returns.[19]

In light of such considerations, Table 5 reports the results of simulations in which the expected return on equities is lowered from 6.2 percent to 4.7 percent. PRA contributions remain 2 percent of earnings per year, and the comparisons are shown for a CRRA utility function with a relative risk aversion coefficient of 3. As expected, the 150 basis point reduction in the equity premium lowers the certainty equivalents for all earnings deciles and benefit formulas, shown in the top panel. The lower equity premium also shifts the optimal portfolio allocation to equity lower. For the Proportional Reduction, equity shares range from 85 to 55 percent, compared to 100 to 80 percent in

[19] For a discussion of the issues associated with choosing a real return on stocks for the long term, see the papers by John Campbell, Peter Diamond, and John Shoven in Social Security Advisory Board (2001).

Table 1. For the Uniform Benefit, equity shares range from 95 to 50 percent, compared to 100 to 75 percent in Table 1.

With a lower equity premium, there is greater scope for changes in the progressivity of the benefit formula to substitute for higher equity allocations. The bottom panel of Table 5 shows that with a Uniform Benefit, the bottom four deciles can reduce their equity shares to zero to keep pace with the optimal allocations of 75 to 85 percent in the Proportional Reduction case. The sixth decile can reduce its equity share to 45 percent from 65 percent. In Table 1, with the higher equity premium, this decile could reduce its equity share only to 80 percent from 95 percent. Possible reductions in equity exposure for other benefit formulas are smaller than with the Uniform Benefit formula but similarly larger than their counterparts with the higher equity premium in Table 1. Thus, the main results in the previous section are robust and even strengthened in the presence of a lower equity premium.

Larger Personal Retirement Accounts

Compared to the investment-based reform plans that have been proposed (see footnote 2), a PRA funded by only a 2 percent contribution is fairly small. The ability of progressivity in the traditional benefit to offset financial risk in the PRAs depends on the relative size of the two benefits. To investigate this dependence and extend the analysis to cover more of the range of proposed reforms, Table 6 presents the results of simulations in which the annual PRA contribution is increased from 2 to 3 percent of earnings. The certainty equivalents in the top panel are all naturally higher than their counterparts in Table 1, since the additional 1-percent contributions are not accounted for by reduced consumption elsewhere in this framework. The middle panel of the table shows that optimal equity allocations are slightly lower with the larger PRAs. As the PRAs get larger relative to the traditional benefit, workers seek to mitigate their risk exposure through lower allocations to equity.

The bottom panel shows that the ability to offset equity exposure through more progressive traditional benefit formulas is slightly lower or higher, depending on the earnings decile and benefit formula. With a Uniform Benefit, the bottom two deciles can reduce their equity shares to zero to keep pace with the optimal allocations of 95 to 100 percent in the Proportional Reduction case. In Table 1, with the smaller PRAs, the bottom three deciles could eliminate all equity exposure. The sixth decile can reduce its equity share to 65 percent from 85 percent, compared to a reduction to 80 percent from 95 percent in Table 1. For the Progressive benefit formula, reductions in equity exposure relative to the Proportional Reduction formula are comparable to those in Table 1. Because the increased size of the PRA reduces the optimal equity allocations for all of the traditional benefit formulas, the differences relative to the differences for the smaller PRAs are not particularly large.

Life Cycle Portfolios

As noted above, prior studies have analyzed the use of life cycle investment strategies to mitigate financial risk in PRAs. Figure 9 compares a portfolio with an age-invariant allocation of 50 percent to equity with three life cycle strategies that shift from equity to bonds as retirement approaches. The first starts at a 95 percent equity share and decreases 2 percentage points per year, reaching 5 percent on the eve of retirement. The second starts at an 83.75 percent equity share and decreases 1.5 percentage points per year, reaching 16.25 percent on the eve of retirement. The third starts at a 72.5 percent equity share and decreases by 1 percentage point per year, reaching 27.5 percent on the eve of retirement. All three strategies are centered on a 50 percent equity share, based on the simple average of the allocation rules by age. The figure pertains to the lowest earnings decile and shows the cumulative distribution functions for each of the four investment options.

There are two important features of the graph. First, the curves all lie virtually on top of each other. There cannot be much of an improvement in expected utility by switching to a life cycle strategy that results in a distribution of benefits that is so similar to the age-invariant portfolio allocation. Second, the life cycle strategies lie above the age-invariant portfolio for all but the very lowest percentiles of the distributions, the more so the greater the decline in the equity allocation with age. The reason is that the life cycle strategies do not have the same expected benefits as the age-invariant portfolio, because the life cycle strategies focus the high-equity allocations on the early years, when many years of contributions are yet to be made.

Thus, life cycle strategies may be desirable, but this is so in the current context primarily because they serve to reduce the overall level of equity exposure. This may be a desirable goal—for example, if the equity premium is low enough or volatility of returns is high enough—but it can be achieved more straightforwardly with a simple reduction in the age-invariant portfolio share in equities given the parameters used in the simulations above.

VII. Conclusions

Policy makers seeking to design investment-based Social Security reform proposals have wrestled with the issue of how much financial risk is appropriate for individuals to bear. Suggested methods of alleviating risk have focused on strategies that amount to requiring more bonds relative to equity in the Personal Retirement Accounts, whether through the purchase of guarantees or life cycle investment strategies. It is worth emphasizing that most of the simulations in this paper suggest fairly high optimal allocations to equities, particularly by those in the lowest deciles of the earnings distribution. Direct restrictions on equity holding in PRAs are likely to prove unpopular, particularly among those whose opportunities are most broadened by the chance to invest their mandatory contributions in equities. This paper suggests another possibility for alleviating the consequences of financial risk; namely, increasing the progressivity of the traditional benefit. Doing so insulates workers in the lower part of the benefit distribution against possibly adverse shocks to financial returns without constraining them to not invest in equities.

The main simulations in the paper compare proportional reductions in traditional benefits with more progressive reductions. The key finding is that under baseline parameters, the most progressive traditional benefit—a flat benefit independent of earnings—allows the allocation to equities to be reduced to zero for the lowest three earnings deciles relative to the optimal allocation when the traditional benefits are reduced proportionately based on the current formula. The next three deciles are able to achieve some reduction in equity exposure as well. Under less extreme changes to the traditional benefit, such as that proposed by Liebman, MacGuineas, and Samwick (2005), the allocation to equities can be decreased by half for the lowest earnings decile and by smaller fractions for an additional five deciles. Sensitivity tests show that optimal allocations to equities typically decrease with higher risk aversion, declining risk aversion, a lower equity premium, or larger accounts, but the general pattern of results persists and in some cases allows for greater equity reduction through higher progressivity in the traditional benefit formula.

The results in this paper suggest two avenues for further research. First, the present analysis used a very stylized model of the initial earnings distribution and its evolution over time to simulate the distribution of future benefits. Actual data and more sophisticated time-series estimates could be incorporated. Second, the present analysis focused on time-invariant portfolio allocations in the PRAs, which were further assumed to be the worker's only source of investment wealth. While the latter might be a reasonable approximation for the lowest earning households, higher earning households are likely to have existing holdings of equities that make the portfolio allocation decision in the PRA less consequential. Extending the current framework to allow for optimal, age-dependent portfolio allocations and for saving in accounts other than the PRAs would provide better estimates of the extent to which greater progressivity can protect low earners from investment risk and of the size of the welfare costs paid by higher earners for providing this protection.

References

The Board of Trustees, Federal Old-Age and Survivors Insurance and Federal Disability Insurance Trust Funds (2006). *Annual Report*. Washington: U.S. Government Printing Office. [Cited as "Social Security Trustees Report 2006."] http://www.ssa.gov/OACT/TR/TR06/tr06.pdf

Bucks, Brian K., Arthur B. Kennickell, and Kevin B. Moore (2006). "Recent Changes in U.S. Family Finances: Evidence from the 2001 and 2004 Survey of Consumer Finances," *Federal Reserve Bulletin*, Vol. 92 (February 2006): A1-A38. http://www.federalreserve.gov/pubs/oss/oss2/2004/bull0206.pdf

Feldstein, Martin S. and Elena Ranguelova (2001a). "Accumulated Pension Collars: A Market Approach to Reducing The Risk of Investment-Based Social Security Reform," in James M. Poterba (ed.) *Tax Policy and the Economy 2000*. Cambridge: MIT Press.

Feldstein, Martin S. and Elena Ranguelova (2001b). "Individual Risk in an Investment-Based Social Security System," *American Economic Review*, Vol. 91, No. 4 (September): 1116-25.

Goss, Stephen C. and Alice H. Wade (2005). "Estimated Financial Effects of 'A Nonpartisan Approach to Reforming Social Security – A Proposal Developed by Jeffrey Liebman, Maya MacGuineas and Andrew Samwick.'" Manuscript, Social Security Administration, November 17. http://www.ssa.gov/OACT/solvency/Liebman_20051117.pdf

Hubbard, R. Glenn, Jonathan S. Skinner, and Stephen P. Zeldes (1994). "The Importance of Precautionary Motives in Explaining Individual and Aggregate Saving," *Carnegie Rochester Conference Series on Public Policy*, Vol. 40, 59 – 125.

Ibbotson Associates (2006). *Stocks, Bonds, Bills, and Inflation Yearbook 2006*. Chicago: Ibbotson Associates.

Kunkel, Jeffrey L. (1996). "Frequency Distribution of Wage Earners by Wage Level," *Actuarial Note, No. 135*. Washington: Social Security Administration, Office of the Actuary, July. http://www.ssa.gov/OACT/NOTES/pdf_notes/note135.pdf

Liebman, Jeffrey, Maya MacGuineas, and Andrew Samwick (2005). "Nonpartisan Social Security Reform Plan," Manuscript, Harvard University, December. http://www.ksg.harvard.edu/jeffreyliebman/lms_nonpartisan_plan_description.pdf

Poterba, James, Joshua Rauh, Steven Venti, and David Wise (2006). "Lifecycle Asset Allocation Strategies and the Distribution of 401(k) Retirement Wealth," National Bureau of Economic Research, Working Paper No. 11974, January. http://www.nber.org/papers/w11974

Poterba, James and Andrew A. Samwick (2003). "Taxation and Household Portfolio Composition: U.S. Evidence from the 1980s and 1990s," *Journal of Public Economics*, Vol. 87 (January): 5-38.

Samwick, Andrew A. (1999). "Social Security Reform in the United States." *National Tax Journal*, 52 (December), 819 – 842.

Samwick, Andrew A. (2004). "Social Security Reform: The United States in 2002." in Einar Overbye and Peter A. Kemp (eds.) *Pensions: Challenges and Reform*. Aldershot: Ashgate Publishing Limited, 53-69.

Social Security Administration, Office of Policy (2006). *Annual Statistical Supplement to the Social Security Bulletin, 2005*. Washington: Social Security Administration, February. [Cited as SSA (2006)] http://www.socialsecurity.gov/policy/docs/statcomps/supplement/2005/

Social Security Advisory Board (2001). "Estimating the Real Rate of Return on Stocks Over the Long Term," August. http://www.ssab.gov/Publications/Financing/estimated%20rate%20of%20return.pdf

Topel, Robert H. and Michael P. Ward (1992). "Job Mobility and the Careers of Young Men," *The Quarterly Journal of Economics*, Vol. 107, No. 2. (May): 439-479.

World Bank (1994). *Averting the Old Age Crisis: Policies to Protect the Old and Promote Growth*. New York: Oxford University Press.

Table 1: Optimal Portfolio Shares in Equity, Baseline Case

	Highest Certainty Equivalent			
Decile	Proportional	Progressive	Half and Half	Uniform
1	16362	16948	18288	20151
2	18373	18817	19819	21185
3	19862	20194	20934	21925
4	21236	21466	21968	22621
5	22571	22700	22974	23307
6	23914	23950	24011	24044
7	25395	25333	25170	24888
8	27212	27035	26606	25946
9	29587	29268	28509	27375
10	33956	33381	32029	30058
	Optimal Equity Share of PRA Portfolio			
Decile	Proportional	Progressive	Half and Half	Uniform
1	100	100	100	100
2	100	100	100	100
3	100	100	100	100
4	100	100	100	100
5	100	100	100	100
6	95	95	95	95
7	95	95	90	90
8	90	90	90	85
9	85	85	85	85
10	80	80	80	75
	Lowest Equity Share with Higher Expected Utility than Proportional			
Decile		Progressive	Half and Half	Uniform
1		50	0	0
2		60	15	0
3		70	35	0
4		75	50	30
5		80	65	50
6		90	80	80

Notes:
1) PRAs are funded by 2 percent contributions.
2) Equity returns average 6.2 percent (net of inflation and administrative costs).
3) Utility is Constant Relative Risk Aversion, with a coefficient of 3.

Table 2: Optimal Portfolio Shares in Equity, Higher Risk Aversion

	Highest Certainty Equivalent			
Decile	Proportional	Progressive	Half and Half	Uniform
1	15808	16419	17801	19705
2	17582	18059	19124	20547
3	18927	19298	20111	21168
4	20173	20447	21032	21762
5	21405	21578	21934	22349
6	22622	22706	22858	22982
7	23966	23952	23885	23703
8	25648	25515	25179	24630
9	27787	27519	26869	25866
10	31823	31297	30054	28223
	Optimal Equity Share of PRA Portfolio			
Decile	Proportional	Progressive	Half and Half	Uniform
1	95	95	100	100
2	85	85	90	90
3	85	85	85	85
4	80	80	80	80
5	75	75	75	75
6	75	75	75	75
7	70	70	70	70
8	70	70	70	65
9	65	65	65	60
10	60	60	60	55
	Lowest Equity Share with Higher Expected Utility than Proportional			
Decile		Progressive	Half and Half	Uniform
1		25	0	0
2		35	0	0
3		45	10	0
4		50	25	0
5		55	35	20
6		60	50	45

Notes:
1) PRAs are funded by 2 percent contributions.
2) Equity returns average 6.2 percent (net of inflation and administrative costs).
3) Utility is Constant Relative Risk Aversion, with a coefficient of 5.

Table 3: Optimal Portfolio Shares in Equity, Declining Relative Risk Aversion

	Highest Certainty Equivalent			
Decile	Proportional	Progressive	Half and Half	Uniform
1	15199	15946	17516	19558
2	17010	17605	18842	20398
3	18423	18894	19856	21028
4	19746	20101	20809	21634
5	21094	21320	21760	22240
6	22386	22510	22724	22893
7	23858	23862	23819	23648
8	25691	25555	25206	24626
9	28002	27711	27014	25941
10	32366	31796	30454	28472

	Optimal Equity Share of PRA Portfolio			
Decile	Proportional	Progressive	Half and Half	Uniform
1	75	80	90	95
2	75	75	80	85
3	75	75	80	80
4	75	75	75	80
5	75	75	75	75
6	70	70	75	70
7	70	70	70	70
8	70	70	70	65
9	70	70	65	65
10	65	65	60	60

	Lowest Equity Share with Higher Expected Utility than Proportional		
Decile	Progressive	Half and Half	Uniform
1	0	0	0
2	20	0	0
3	30	0	0
4	40	15	0
5	50	30	15
6	55	45	35

Notes:
1) PRAs are funded by 2 percent contributions.
2) Equity returns average 6.2 percent (net of inflation and administrative costs).
3) Utility is Declining Relative Risk Aversion,
with a coefficient of 3 and susbsitence level of 10000.

Table 4: Optimal Portfolio Shares in Equity, Higher and Declining Relative Risk Aversion

	Highest Certainty Equivalent			
Decile	Proportional	Progressive	Half and Half	Uniform
1	14533	15356	17018	19109
2	16100	16791	18166	19822
3	17324	17913	19053	20358
4	18510	18986	19891	20872
5	19783	20120	20749	21391
6	20898	21154	21585	21939
7	22305	22414	22564	22577
8	24063	24010	23827	23418
9	26088	25889	25377	24524
10	30199	29705	28529	26748

	Optimal Equity Share of PRA Portfolio			
Decile	Proportional	Progressive	Half and Half	Uniform
1	55	60	65	75
2	50	55	60	65
3	55	55	60	60
4	50	55	55	60
5	55	55	55	55
6	50	50	50	50
7	50	50	50	50
8	50	50	50	45
9	50	50	50	45
10	45	45	45	40

	Lowest Equity Share with Higher Expected Utility than Proportional		
Decile	Progressive	Half and Half	Uniform
1	0	0	0
2	0	0	0
3	10	0	0
4	15	0	0
5	25	0	0
6	30	10	0
7	40	30	30

Notes:
1) PRAs are funded by 2 percent contributions.
2) Equity returns average 6.2 percent (net of inflation and administrative costs).
3) Utility is Declining Relative Risk Aversion,
with a coefficient of 5 and susbsitence level of 10000.

Table 5: Optimal Portfolio Shares in Equity, Lower Equity Returns

Decile	Proportional	Progressive	Half and Half	Uniform
	Highest Certainty Equivalent			
1	15473	16045	17355	19178
2	17265	17694	18662	19984
3	18596	18916	19628	20580
4	19820	20040	20522	21147
5	21023	21146	21407	21723
6	22220	22255	22312	22340
7	23549	23490	23334	23061
8	25162	24994	24588	23963
9	27275	26972	26256	25192
10	31185	30639	29366	27510

Decile	Proportional	Progressive	Half and Half	Uniform
	Optimal Equity Share of PRA Portfolio			
1	85	90	90	95
2	80	80	80	85
3	75	75	75	75
4	75	75	75	75
5	70	70	70	70
6	65	65	65	65
7	65	65	65	60
8	60	60	60	60
9	60	60	55	55
10	55	55	50	50

Decile	Progressive	Half and Half	Uniform
	Lowest Equity Share with Higher Expected Utility than Proportional		
1	0	0	0
2	15	0	0
3	25	0	0
4	35	5	0
5	45	25	5
6	55	50	45

Notes:
1) PRAs are funded by 2 percent contributions.
2) Equity returns average 4.7 percent (net of inflation and administrative costs).
3) Utility is Constant Relative Risk Aversion, with a coefficient of 3.

Table 6: Optimal Portfolio Shares in Equity, Larger PRAs

Decile	Proportional	Progressive	Half and Half	Uniform
	Highest Certainty Equivalent			
Decile	Proportional	Progressive	Half and Half	Uniform
1	17628	18231	19608	21522
2	20000	20460	21500	22921
3	21780	22126	22898	23936
4	23440	23681	24213	24908
5	25066	25205	25503	25874
6	26725	26771	26854	26921
7	28569	28515	28371	28118
8	30872	30698	30279	29639
9	33896	33578	32824	31702
10	39522	38941	37579	35590
	Optimal Equity Share of PRA Portfolio			
Decile	Proportional	Progressive	Half and Half	Uniform
1	100	100	100	100
2	95	95	100	100
3	95	95	95	95
4	90	90	90	90
5	85	85	85	85
6	85	85	85	85
7	80	80	80	80
8	80	80	80	75
9	75	75	75	75
10	70	70	70	65
	Lowest Equity Share with Higher Expected Utility than Proportional			
Decile		Progressive	Half and Half	Uniform
1		50	0	0
2		60	25	0
3		65	35	10
4		70	50	30
5		70	60	45
6		80	70	65

Notes:
1) PRAs are funded by 3 percent contributions.
2) Equity returns average 6.2 percent (net of inflation and administrative costs).
3) Utility is Constant Relative Risk Aversion, with a coefficient of 3.

Figure 1: Expected Benefits by Benefit Formula and Equity Share

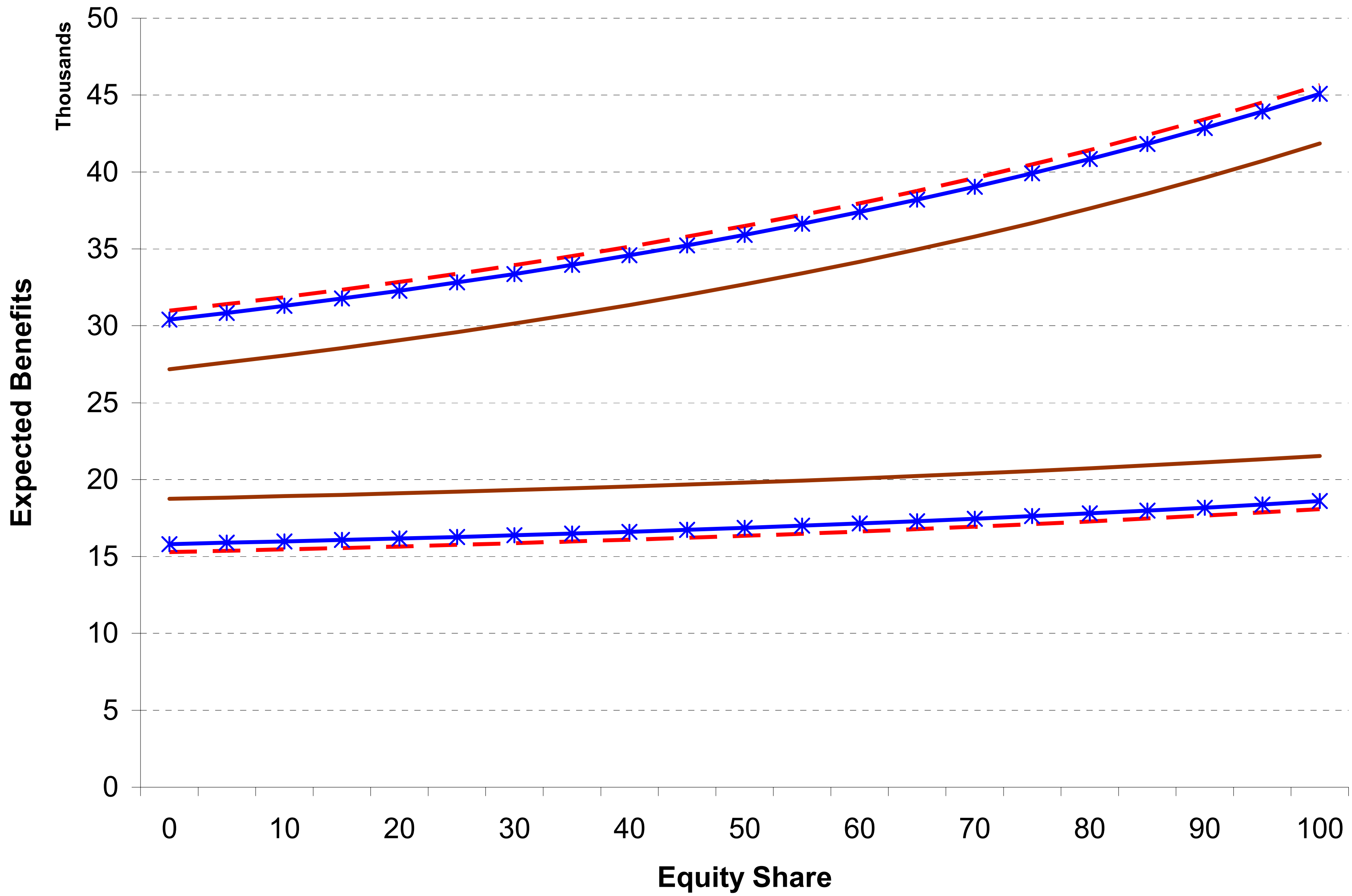

Figure 2: Expected Benefits and Certainty Equivalents, Top Decile, Baseline Case

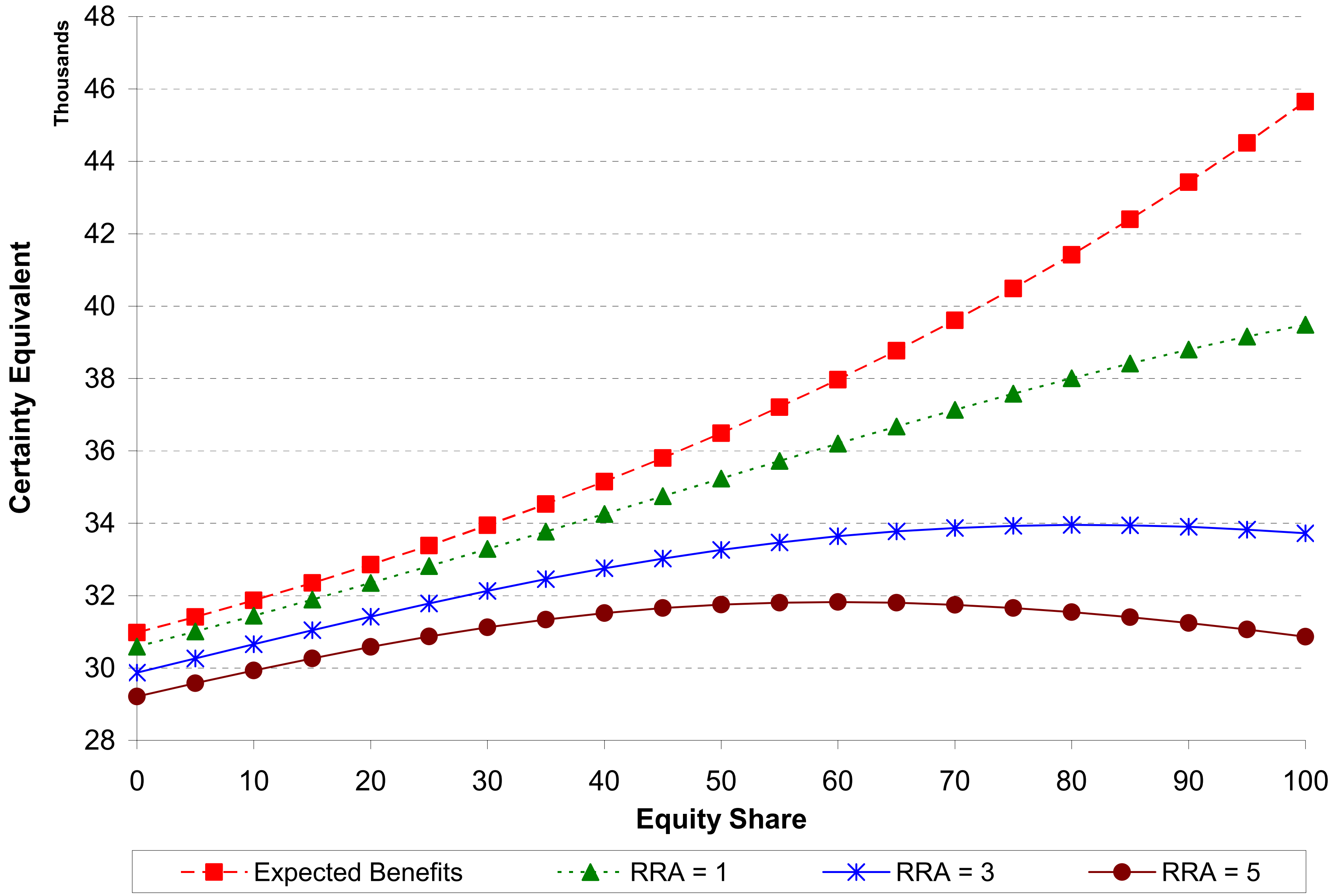

Figure 3: Certainty Equivalents by Benefit Formulas and Equity Shares, Baseline Case, Decile 1

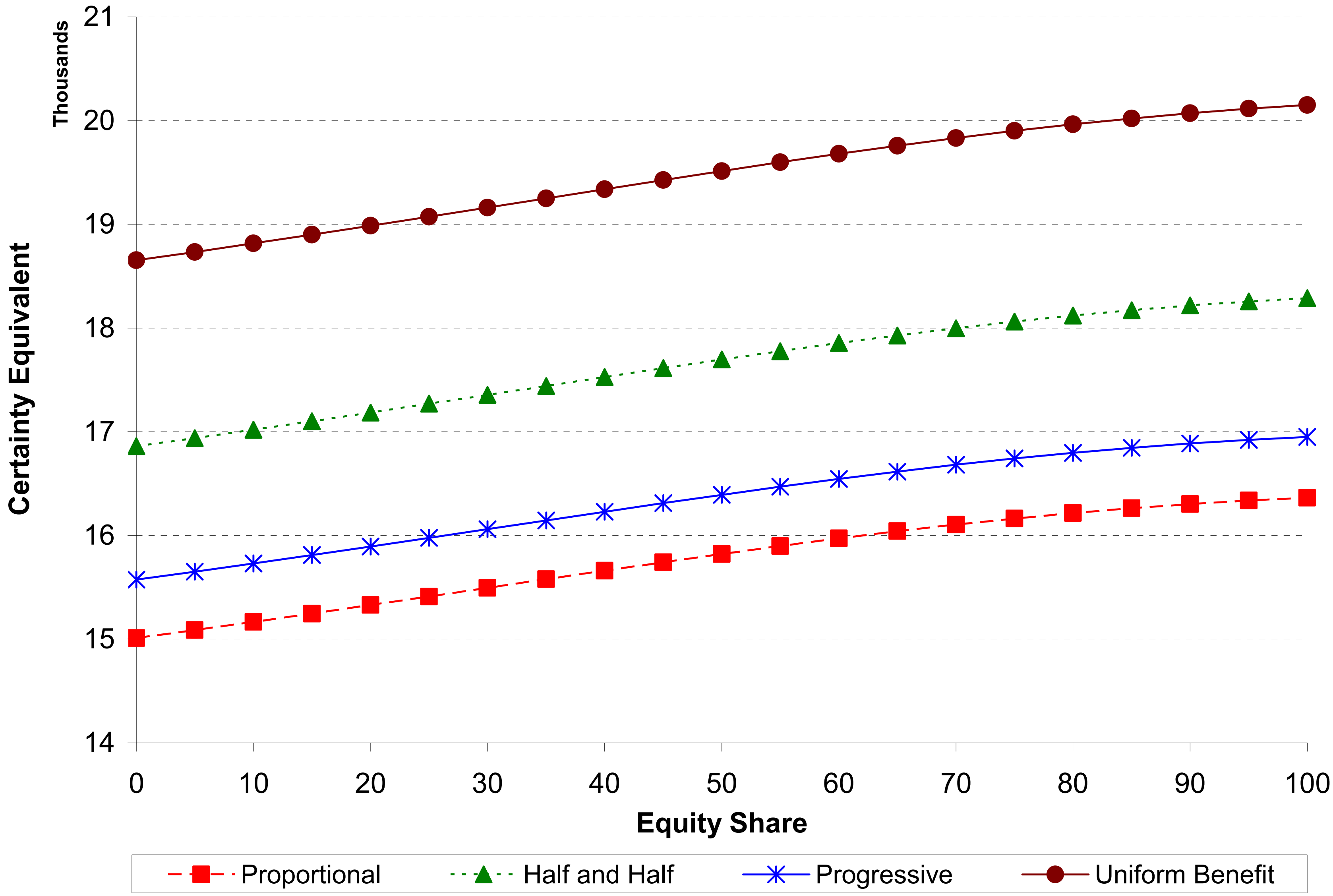

Figure 4: Certainty Equivalents by Benefit Formulas and Equity Shares, Baseline Case, Decile 4

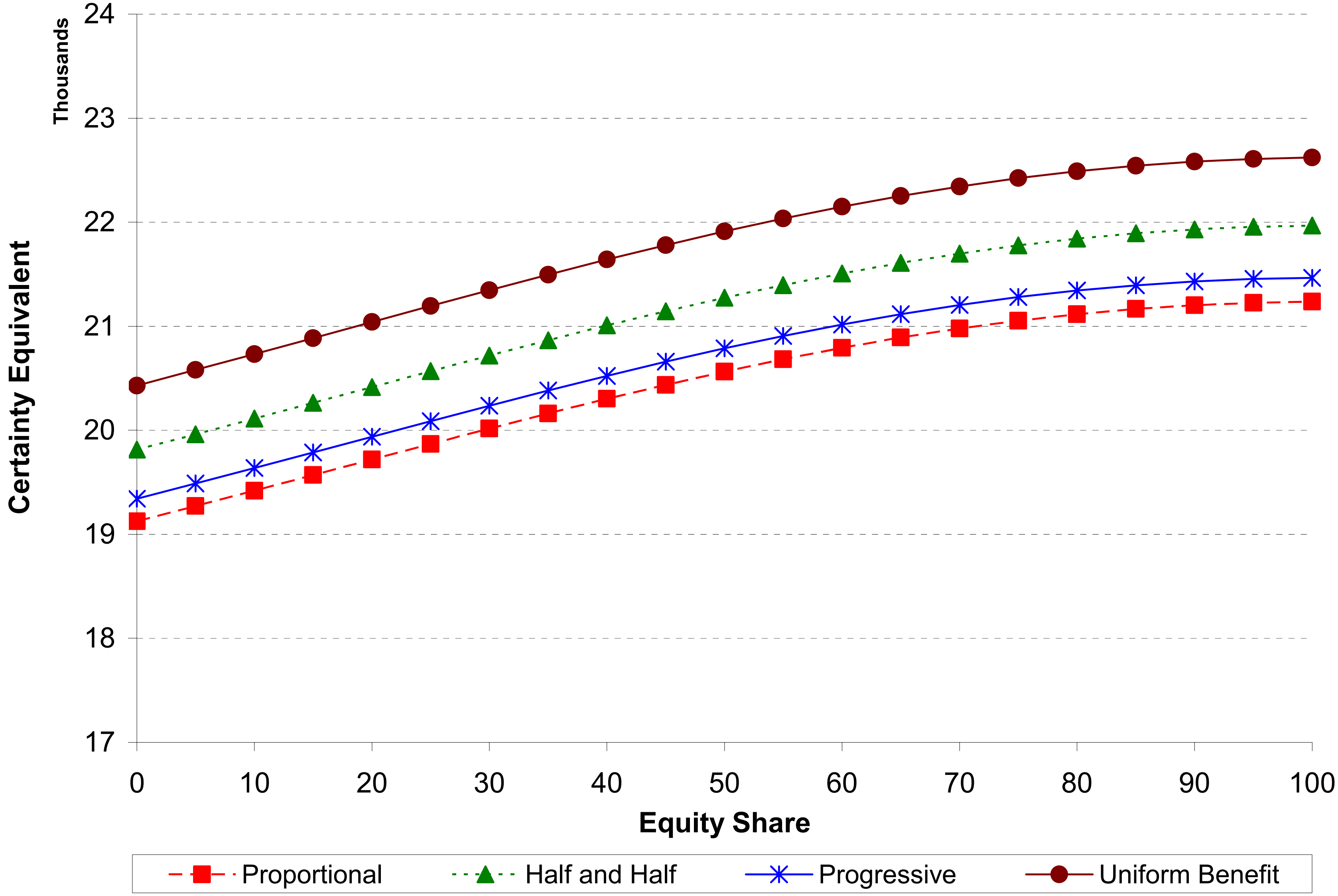

Figure 5: Certainty Equivalents by Benefit Formulas and Equity Shares, Baseline Case, Decile 7

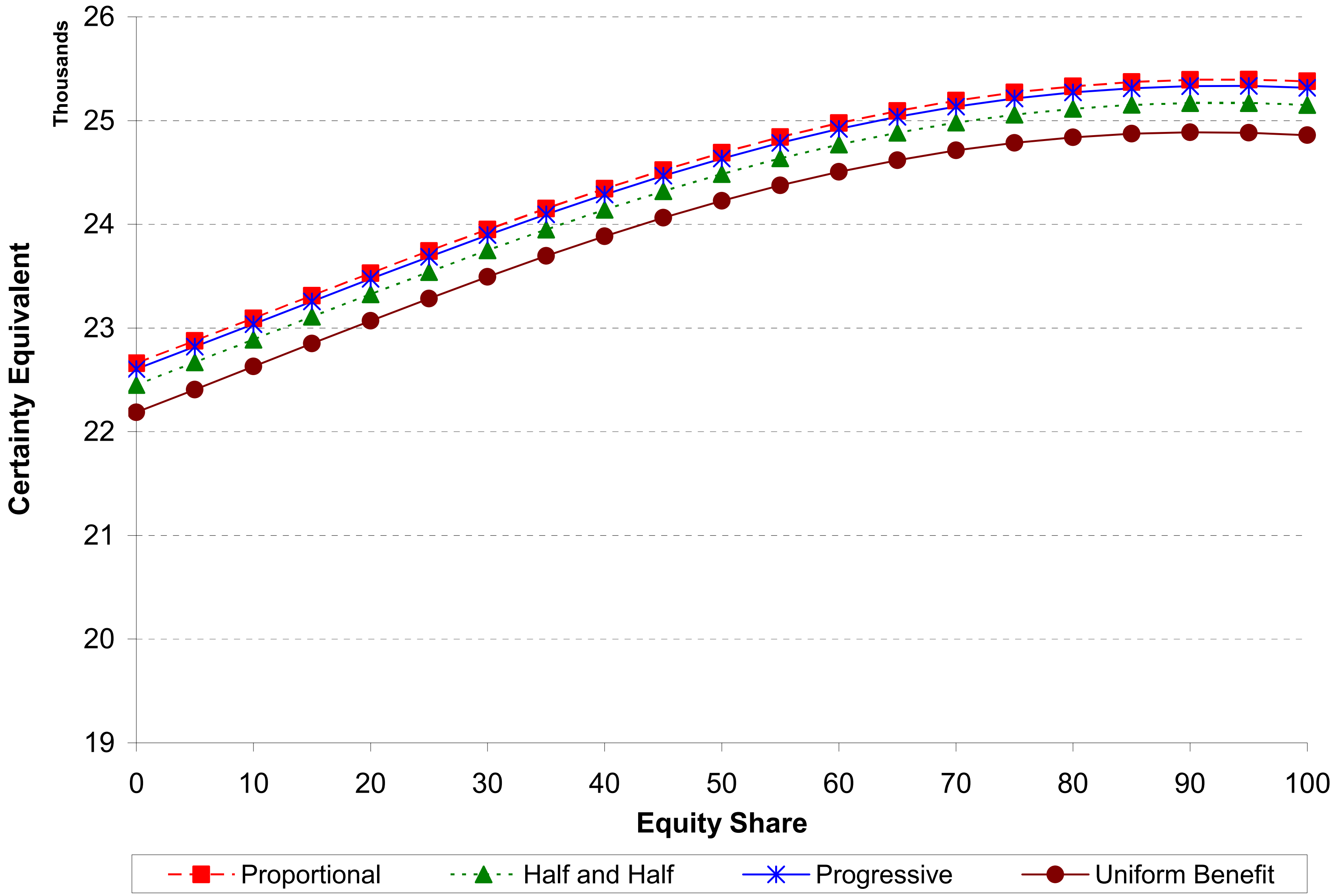

Figure 6: Certainty Equivalents by Benefit Formulas and Equity Shares, Baseline Case, Decile 10

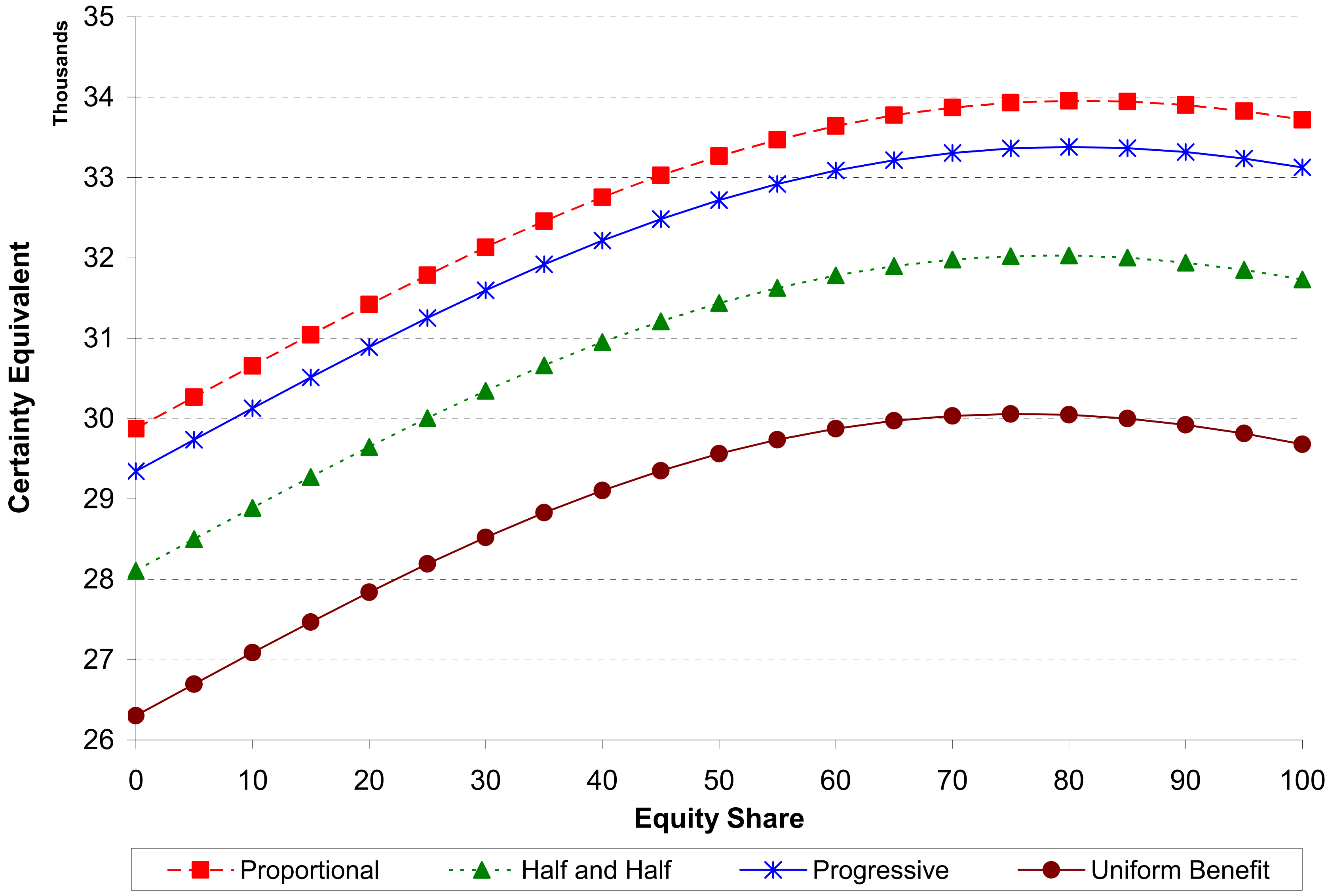

Figure 7: CDFs for Benefits, 2% PRAs, 50% Equity Shares, Decile 1

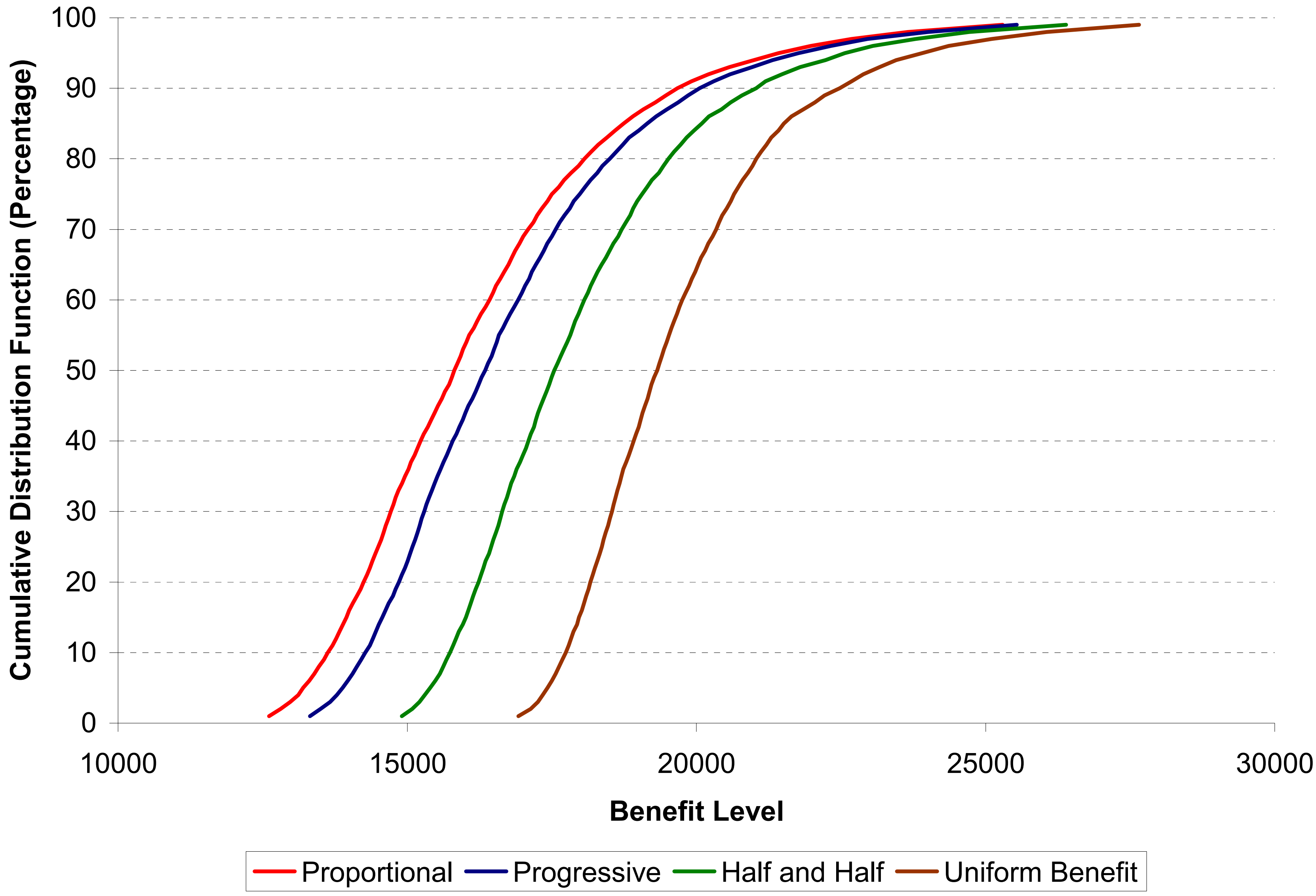

Figure 8: CDFs for Benefits, 2% PRAs, Proportional Reduction, Decile 1

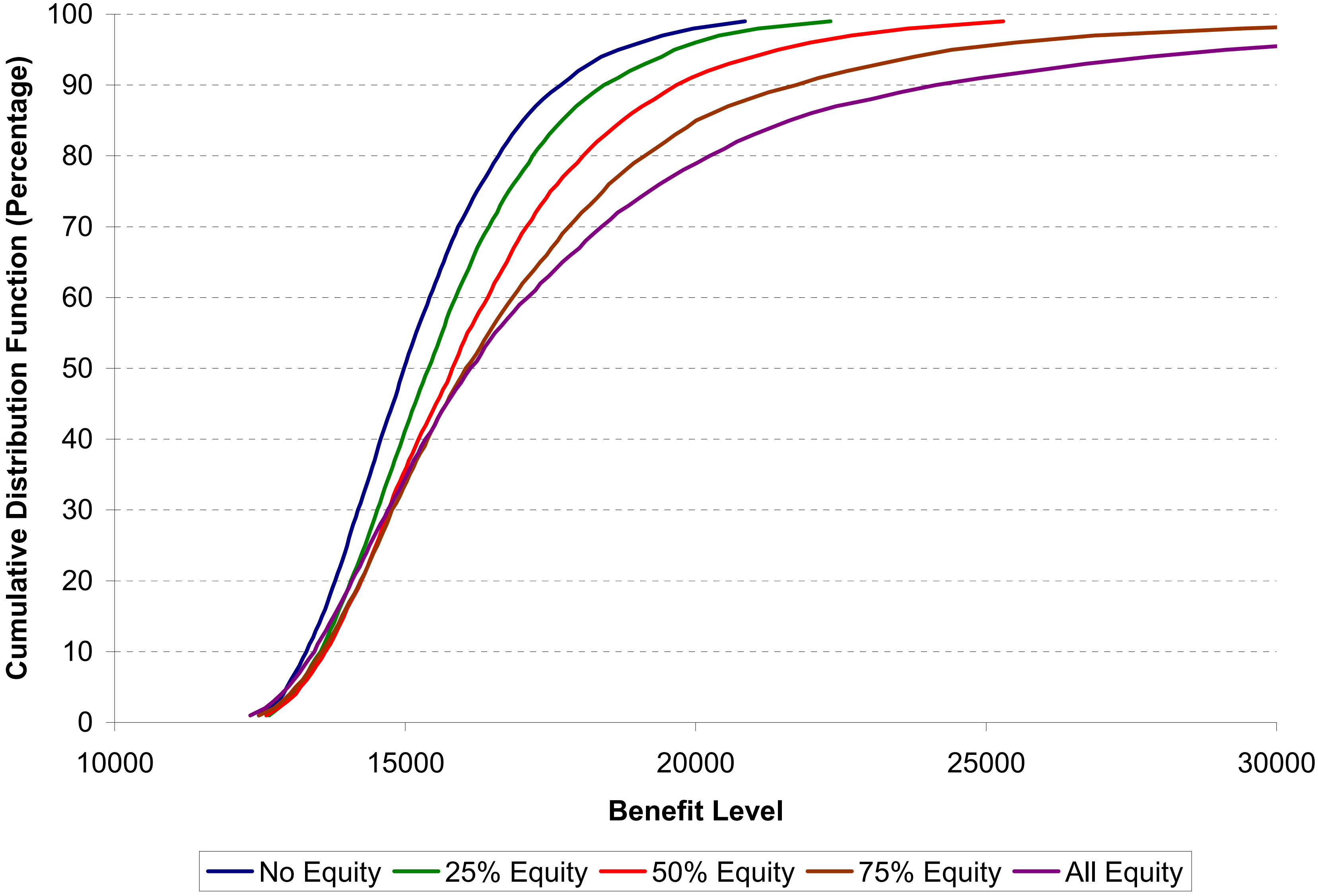

Figure 9: CDFs for Benefits, Life Cycle Allocations, Baseline, Proportional Reduction, Decile 1