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EXPLAIN ASSET-PRICING ANOMALIES

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ABSTRACT

Recent studies suggest that the conditional CAPM might hold, period-by-period, and that time-varying betas can explain the failures of the simple, unconditional CAPM. We argue, however, that significant departures from the unconditional CAPM would require implausibly large time-variation in betas and expected returns. Thus, the conditional CAPM is unlikely to explain asset-pricing anomalies like book-to-market and momentum. We test this conjecture empirically by directly estimating conditional alphas and betas from short-window regressions (avoiding the need to specify conditioning information). The tests show, consistent with our analytical results, that the conditional CAPM performs nearly as poorly as the unconditional CAPM.

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1. Introduction

The unconditional CAPM does not describe the cross section of average stock returns. Most prominently, the CAPM does not explain why, over the last forty years, small stocks outperform large stocks, why firms with high book-to-market ratios outperform those with low B/M ratios (the value premium), or why stocks with high returns during the past year continue to outperform those with low past returns (momentum). In this paper, our goal is to understand whether a conditional version of the CAPM might explain these patterns.

Theoretically, it is well known that the conditional CAPM could hold perfectly – that is, conditional alphas are always zero – but that time-variation in beta might lead to unconditional pricing errors (e.g., Jensen, 1968; Dybvig and Ross, 1985; Jagannathan and Wang, 1996). In general, a stock's unconditional alpha will differ from zero if its beta covaries with the market risk premium or with market volatility, as we discuss further below. Put differently, the market portfolio might be conditionally mean-variance efficient in every period yet, at the same time, not on the unconditional mean-variance frontier (e.g., Hansen and Richard, 1987).

Several recent papers argue, in fact, that time variation in beta helps explain the size, B/M, and momentum effects. Zhang (2002) develops a model in which high-B/M stocks are riskiest in bad times, and comovement between betas and the risk premium leads to an unconditional value premium (even though conditional CAPM alphas are exactly zero). Further, Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Petkova and Zhang (2003) show that the betas of small and high-B/M stocks vary over the business cycle in a way that, according to the authors, largely explains why those stocks have positive unconditional alphas (see, also, Avramov and Chordia, 2002; Wang, 2003; Ang and Chen, 2003; Lustig and Van Nieuwerburgh, 2003).

In this paper, we question whether the conditional CAPM can really explain asset-pricing anomalies, either in principle or in practice. The analysis is broken into two parts. First, we argue that if the conditional CAPM truly holds, we should expect to find only small deviations from the unconditional CAPM – much smaller than those observed empirically. Second, we provide direct empirical evidence

that the conditional CAPM does not explain the B/M and momentum effects.

The first point is illustrated quite easily. If market volatility is constant and the conditional CAPM holds, we show that a stock's unconditional alpha is roughly equal to the covariance between its beta and the market risk premium. This covariance is small for empirically-plausible parameters. For example, suppose that the standard deviation of beta is 0.30, about our estimate for a long-short B/M strategy. Then, if beta is perfectly correlated with the risk premium γ_t , the implied unconditional alpha is a modest 0.08% if $\sigma_\gamma = 0.25\%$ and doubles to a still-modest 0.15% if $\sigma_\gamma = 0.50\%$. (The average risk premium is around 0.50% monthly, so $\sigma_\gamma = 0.50\%$ represents large variation through time.) The implied alpha is even smaller if beta is imperfectly correlated with the risk premium or if beta covaries positively with market volatility. Empirically, the alpha of the B/M strategy is 0.59% monthly (std. error, 0.14%), and the alpha of a momentum strategy is 1.01% monthly (std. error, 0.28%), both substantially larger than our estimates for plausible alphas.¹ In short, we argue that observed pricing errors are simply too large to be explained by time variation in beta.

The second part of the paper provides a simple test of the conditional CAPM. Specifically, we directly estimate conditional alphas and betas using short-window regressions. For example, we estimate CAPM regressions every month, quarter, half-year, or year using daily, weekly, or monthly returns, paying special attention to the obvious microstructure issues that affect the estimates (discussed in detail later). The literature has devoted much effort to developing tests of the conditional CAPM, but a problem common to all prior approaches is that they require the econometrician to know the 'right' state variables (e.g., Harvey, 1989; Shanken, 1990; Jagannathan and Wang, 1996; Lettau and Ludvigson, 2001). Cochrane (2001, p. 145) summarizes the issue this way: "Models such as the CAPM imply a conditional linear factor model with respect to *investors'* information sets. The best we can hope to do is test implications conditioned on variables that we observe. Thus, a conditional factor model is *not testable!*" (his

¹ The data are described later. Briefly, the portfolios consist of all NYSE and Amex stocks on CRSP and Compustat from 1964 to 2001. The B/M strategy invests in the top quintile and shorts the bottom quintile of firms ranked by B/M. The momentum strategy invests in the top decile and shorts the bottom decile when stocks are ranked by past 6-month returns.

emphasis). Our methodology gets around this problem because it does not require conditioning information. As long as betas are relatively stable *within* a month or quarter, then simple CAPM regressions estimated over a short window – using no conditioning variables – provide direct estimates of assets’ conditional alphas and betas.

Using the short-window regressions, we obtain time series of conditional alphas and betas for size, B/M, and momentum portfolios from 1964 – 2001. We use the estimates in two ways. First, we study the time-series properties of conditional betas and, as suggested by our earlier discussion, relate these to unconditional deviations from the CAPM. Second, we directly test whether average conditional alphas are zero, as implied by the conditional CAPM. It is useful to note that our tests do not require precise estimates of conditional alphas and betas from individual short-window regressions; the estimates must only be unbiased. Thus, we can estimate the regressions over very short intervals so long as they satisfy standard OLS assumptions.

Our tests suggest that betas vary considerably over time. A nice feature of the short-window regressions is that they allow us to back out the volatility of true conditional betas. Specifically, the variance of estimated betas should equal the variance of true betas plus the variance of sampling error, an estimate of which is provided by the short-window regressions (see, also, Fama and French, 1997). Using this relation, the implied time-series standard deviation of beta is roughly 0.30 for a ‘small minus big’ portfolio, 0.25 for a ‘value minus growth’ portfolio, and 0.60 for a momentum portfolio (the data are described in detail below; see footnote 1 for a brief description). The betas fluctuate over time with variables commonly used to measure business conditions, including past market returns, Tbill rates, aggregate dividend yield, and the term spread (though, interestingly, not with the consumption to wealth ratio of Lettau and Ludvigson, 2001). However, we find no evidence that betas covary with the market risk premium in a way that might explain the portfolios’ unconditional alphas (if anything, the covariances have the wrong sign).

Estimates of conditional alphas provide a more direct test of the conditional CAPM. Average conditional alphas should be zero if the CAPM holds, but instead we find that they are large, statistically

significant, and generally close to the unconditional alphas. The average conditional alpha is around 0.50% for our long-short B/M strategy and around 1.00% for our long-short momentum strategy. (We say ‘around’ because the conditional alphas are estimated several ways; all methods reject the conditional CAPM but their point estimates differ somewhat.) The estimates are more than three standard errors from zero and close to the portfolios’ unconditional alphas, 0.59% and 1.01%, respectively. We do not find a size effect in our data, with conditional and unconditional alphas both close to zero for the ‘small minus big’ strategy.

Overall, the evidence supports our analytical results. Betas vary significantly over time but not enough to explain large unconditional pricing errors. The conditional CAPM performs nearly as poorly as the unconditional CAPM.

Our analysis focuses on the Sharpe-Lintner CAPM, but we believe the conclusions should apply to other models as well: in general, conditioning is unlikely to have a large impact on cross-sectional asset-pricing tests. In intertemporal models, consumption betas and the consumption risk premium must exhibit extreme time-variation in order for a conditional model to significantly outperform an unconditional one. Our tests are difficult to extend directly to the consumption CAPM, because they require high-frequency data, but we provide tentative evidence using the mimicking-portfolio approach of Breeden, Gibbons, and Litzenberger (1989). Specifically, we estimate a consumption-mimicking portfolio by regressing quarterly consumption growth on the Fama and French (1993) factors, either assuming the slopes (i.e., portfolio weights) are constant over time or allowing them to vary with Lettau and Ludvigson’s (2001) consumption-to-wealth ratio. When the mimicking portfolio is used in place of the market portfolio in our tests, we find no evidence that time-varying consumption betas can explain momentum or the value premium.

Our results differ from the conclusions of Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Petkova and Zhang (2003). They suggest that conditioning can be very important in asset-pricing tests. While a full review is beyond the scope of this paper, the key difference with our study is that they focus on cross-sectional regressions rather than time-series intercept tests. Basically, the papers

just test whether the effects of time-varying betas are cross-sectionally correlated with expected returns. To us, this seems a fairly weak test of the conditional CAPM because it ignores important restrictions on the cross-sectional slopes. For example, Petkova and Zhang estimate Fama-MacBeth regressions of returns on stocks' average betas and 'beta-premium' sensitivities (a measure of how beta covaries with the market risk premium). According to the CAPM, the slope on the beta-premium sensitivity should equal the variance of the market risk premium, but the actual estimate appears to be roughly 5 to 10 times too large.² We have similar concerns with the slopes and zero-beta rates in the other two papers, which makes it hard to interpret their results.

The paper is organized as follows. Section 2 analyzes the connection between the conditional and unconditional CAPM. Section 3 introduces the data and describes our testing approach. Section 4 presents the main empirical results, focusing on the Sharpe-Lintner CAPM, and Section 5 explores the consumption CAPM. Section 6 compares our study to other recent papers which test the conditional CAPM. Section 7 concludes.

2. Expected returns and the conditional CAPM

The conditional CAPM does not generally imply a simple unconditional CAPM. In this section, we derive expressions for unconditional alphas and betas when expected returns, volatility, and covariances all change over time. Our goal is not simply to show that the unconditional CAPM fails but, rather, to understand whether the pricing errors might be large enough to explain important asset-pricing anomalies like size, B/M, and momentum.

2.1. Notation and assumptions

Let R_{it} be the excess return on asset i and R_{Mt} be the excess return on the market portfolio in period t (in excess of a possibly time-varying riskfree rate). We impose little structure on the distribution of

² The slope is 0.0003 from 1963 – 2001, whereas Petkova and Zhang estimate that the variance of the market risk premium is around 0.00007 (using point estimates, not adjusted for bias, from their predictive regression in Table 2). We argue that the variance of the risk premium is likely to be even lower, at most 0.000025, which corresponds to a monthly standard deviation of 0.5%.

returns; they are simply assumed to have well-defined conditional and unconditional moments. Conditional moments for period t , given information at $t-1$, are labeled with a t subscript: the conditional expected market return and variance are γ_t and σ_t^2 , and the conditional beta is $\beta_t = \text{cov}_{t-1}(R_{it}, R_{Mt}) / \sigma_t^2$. Likewise, the unconditional market risk premium and variance are γ and σ_M^2 , and the unconditional beta is $\beta^u = \text{cov}(R_{it}, R_{Mt}) / \sigma_M^2$. We sometimes write the conditional beta as $\beta_t = \beta + \eta_t$, where $\beta \equiv E[\beta_t]$ and η_t is the zero-mean, time-varying component (note $\beta^u \neq \beta$). We assume throughout that the conditional CAPM holds, implying $E_{t-1}[R_{it}] = \beta_t \gamma_t$, and that R_{it} is conditionally linearly related to the market return, or $R_{it} = \beta_t R_{Mt} + \varepsilon_t$ with $E_{t-1}[\varepsilon_t | R_{Mt}] = 0$.

2.2. Unconditional alphas and betas

If the conditional CAPM holds, it is easy to show that $E[R_{it}] = \beta \gamma + \text{cov}(\beta_t, \gamma_t)$. The asset's unconditional alpha, or pricing error, is defined as $\alpha_u = E[R_{it}] - \beta^u \gamma$. Substituting for $E[R_{it}]$ yields

$$\alpha^u = \gamma (\beta - \beta^u) + \text{cov}(\beta_t, \gamma_t). \quad (1)$$

Under some assumptions, discussed below, the unconditional and expected conditional betas are similar, so that α^u is approximately equal to the covariance between beta and the market risk premium. (Jagannathan and Wang, 1996, were the first to emphasize the importance of $\text{cov}(\beta_t, \gamma_t)$ for unconditional tests.) More generally, to find β^u , we need to evaluate the unconditional covariance between R_{Mt} and R_{it} . Using $R_{it} = (\beta + \eta_t)R_{Mt} + \varepsilon_t$, the covariance equals

$$\text{cov}(R_{Mt}, R_{it}) = \beta \sigma_M^2 + E[\eta_t R_{Mt}^2] - \gamma \text{cov}(\eta_t, R_{Mt}). \quad (2)$$

To better understand this equation, break R_{Mt} into three components: $R_{Mt} = \gamma + (\gamma_t - \gamma) + s_t$, where s_t is the unexpected return at time t . Substituting into the expression above, and using the fact that $E[\eta_t] = 0$ and $E_{t-1}[s_t] = 0$, yields:

$$\text{cov}(R_{Mt}, R_{it}) = \beta \sigma_M^2 + \gamma \text{cov}(\eta_t, \gamma_t) + E[\eta_t (\gamma_t - \gamma)^2] + E[\eta_t s_t^2]. \quad (3)$$

Recall that $\beta_t = \beta + \eta_t$ and $\sigma_t^2 = E_{t-1}[s_t^2]$. Therefore, the unconditional beta is

$$\beta^u = \beta + \frac{\gamma}{\sigma_M^2} \text{cov}(\beta_t, \gamma_t) + \frac{1}{\sigma_M^2} \text{cov}[\beta_t, (\gamma_t - \gamma)^2] + \frac{1}{\sigma_M^2} \text{cov}(\beta_t, \sigma_t^2). \quad (4)$$

This expression says that β^u will differ from the expected conditional beta if β_t covaries with the market risk premium (2nd term), if it covaries with $(\gamma_t - \gamma)^2$ (3rd term), or if it covaries with the conditional volatility of the market (last term). Roughly speaking, movement in beta that is correlated with the market risk premium or with market volatility, γ_t or σ_t^2 , raises the unconditional covariance between the stock's return and the market.

For our purposes, the unconditional alpha is more important. Substituting (4) into (1) yields

$$\alpha^u = \left[1 - \frac{\gamma^2}{\sigma_M^2} \right] \text{cov}(\beta_t, \gamma_t) - \frac{\gamma}{\sigma_M^2} \text{cov}[\beta_t, (\gamma_t - \gamma)^2] - \frac{\gamma}{\sigma_M^2} \text{cov}(\beta_t, \sigma_t^2). \quad (5)$$

Eq. (5) provides a very general formula for the unconditional pricing error. It says that, even if the conditional CAPM holds exactly, we should expect to find deviations from the unconditional CAPM if beta covaries with γ_t , $(\gamma_t - \gamma)^2$, or with conditional market volatility. We explore the intuition behind eq. (5) in more detail below.

2.3. Magnitude

The basic message from eq. (5), that time-varying betas can lead to unconditional pricing errors, suggests a possible explanation for asset-pricing anomalies. Indeed, several recent studies argue that size, B/M, and to some extent momentum can be explained largely by movements in beta that are correlated with the risk premium (e.g., Jagannathan and Wang, 1996; Lettau and Ludvigson, 2001; Petkova and Zhang, 2003; Wang, 2003). We use eq. (5) to explore whether time-variation in beta might really be large enough to explain the anomalies.

We should first provide some background. To illustrate the size, B/M, and momentum effects, we form three long-short portfolios using all NYSE and Amex stocks on CRSP / Compustat from 1964 – 2001. The data are described in detail later but, roughly speaking, our size strategy is quintile 1 minus quintile 5 when stocks are sorted by market capitalization, our B/M strategy is quintile 5 minus quintile 1

when stocks are sorted by book-to-market equity, and our momentum strategy is decile 10 minus decile 1 when stocks are sorted by past 6-month returns (all value-weighted). Using monthly returns and the CRSP value-weighted index as the market proxy, the size portfolio has an unconditional alpha of -0.03% (standard error, 0.20%), the B/M portfolio has an unconditional alpha of 0.59% (standard error, 0.14%), and the momentum portfolio has an unconditional alpha of 1.01% (standard error, 0.28%). The size effect is absent in our data, but the B/M and momentum effects are significant and representative of those found in the literature.

Return now to eq. (5). Note that γ^2 / σ_M^2 , in the first term, is the unconditional squared Sharpe ratio of the market. This statistic is small in monthly returns and, for practical purposes, easily ignored: from 1964 – 2001, $\gamma = 0.47\%$ and $\sigma_M = 4.5\%$, so the squared Sharpe ratio is 0.011. Further, for plausible magnitudes of γ_t , the quadratic $(\gamma_t - \gamma)^2$, in the second term, is also quite small. For example, if $\gamma = 0.5\%$ and γ_t varies between 0.0% and 1.0% monthly, then the quadratic term is at most $0.005^2 = 0.000025$. This suggests that the second component of eq. (5) is also negligible. A special case is when γ_t is symmetric and β_t is linearly related to γ_t (e.g., if β_t and γ_t are bivariate normal). Then, the covariance between β_t and $(\gamma_t - \gamma)^2$ is exactly zero regardless of the magnitude of γ_t . Together, these observations suggest the following approximation for α^u :

$$\alpha^u \approx \text{cov}(\beta_t, \gamma_t) - \frac{\gamma}{\sigma_M^2} \text{cov}(\beta_t, \sigma_t^2). \quad (6)$$

Eq. (6) says that the unconditional alpha depends primarily on how β_t covaries with the market risk premium and with market volatility. In a CAPM world, we would expect these two effects to offset: the risk premium and conditional variance should move together, so β_t is likely to covary similarly with both. To be concrete, we consider two special cases of (6):

(1) Constant volatility. If market volatility is constant, an asset's unconditional alpha is

$$\alpha^u \approx \text{cov}(\beta_t, \gamma_t) = \rho \sigma_\beta \sigma_\gamma, \quad (7)$$

Table 1
Deviations from the unconditional CAPM

The table reports the unconditional alpha implied by the conditional CAPM (% monthly). The conditional CAPM is assumed to hold period-by-period, and the asset's beta (β_t) and the expected market risk premium (γ_t) vary over time as indicated in the table. σ_β is the standard deviation of β_t , σ_γ is the standard deviation of γ_t , and ρ is the correlation between β_t and γ_t . Return volatility is assumed to be constant.

	$\rho = 0.6$			$\rho = 1.0$			
	σ_β			σ_β			
	0.3	0.5	0.7	0.3	0.5	0.7	
	Unconditional alpha (%)			Unconditional alpha (%)			
$\sigma_\gamma = 0.1$	0.02	0.03	0.04	$\sigma_\gamma = 0.1$	0.03	0.05	0.07
0.2	0.04	0.06	0.08	0.2	0.06	0.10	0.14
0.3	0.05	0.09	0.12	0.3	0.09	0.15	0.21
0.4	0.07	0.12	0.17	0.4	0.12	0.20	0.28
0.5	0.09	0.15	0.21	0.5	0.15	0.25	0.35

where σ_β and σ_γ are the standard deviations of β_t and γ_t and ρ is their correlation. The pricing error given by (7) is small for empirically-plausible parameters.

Table 1 reports unconditional alphas implied by various combinations of ρ , σ_β , and σ_γ . We consider three values of σ_β – 0.3, 0.5, and 0.7 – which probably span or, more likely, exceed standard deviations encountered in practice.³ Note, for example, that if $\beta = 1.0$ and $\sigma_\beta = 0.5$, a two-standard-deviation interval gives an impressive spread in beta over time from 0.0 to 2.0. We consider five values of σ_γ , from 0.10% to 0.50% monthly. Average γ_t from 1964 – 2001 is 0.47%, using the CRSP value-weighted index, so a standard deviation as high as 0.50% implies very large changes in the expected risk premium (e.g., a two-standard-deviation interval roughly extends from -0.50% to 1.50% monthly, or -6% to 18% annualized).⁴ Finally, we consider two values for ρ , 0.6 and 1.0, the latter providing an upper bound for the pricing error.

The unconditional alphas in Table 1 are generally small relative to empirical estimates for B/M and

³ We offer three observations to support this view: (i) We estimate later that σ_β for our size portfolio is around 0.30, for our B/M portfolio is around 0.25, and for our momentum portfolio is around 0.58; (ii) Fama and French (1997) estimate σ_β for 48 industry portfolios and find an average of 0.12 and a maximum of 0.42; (iii) Fama and French (1992) report unconditional betas for *beta-sorted* portfolios; they find a minimum beta of 0.79, a maximum beta of 1.73, and a cross-sectional standard deviation of 0.31.

⁴ For additional perspective, a regression of NYSE returns on log dividend yield suggests $\sigma_\gamma = 0.30\%$ using point estimates from 1946 – 2000.

momentum portfolios (0.59% and 1.01%, respectively). The implied alphas are typically less than 0.15% monthly, with a maximum of 0.35% for our most extreme combination of $\rho = 1.0$, $\sigma_\beta = 0.7$, and $\sigma_\gamma = 0.50\%$ (which we regard as quite generous). We estimate later that the B/M portfolio has $\sigma_\beta = 0.25$, so Table 1 suggests that time-variation in beta can explain only a small component of the portfolio's alpha, even if β_t and γ_t are perfectly correlated and the risk premium is very volatile. A similar conclusion applies to the momentum strategy, for which we estimate $\sigma_\beta = 0.60$. The bottom line from Table 1 is that the conditional CAPM, with time-varying betas, is unlikely to explain important asset-pricing anomalies like B/M and momentum.

To provide some intuition, Figure 1 plots the unconditional relation between R_i and R_M . The dark curve shows $E[R_i | R_M]$, the predicted return on the asset as a function of the realized market return (the expectation is unconditional). The graph uses the most extreme parameters from Table 1, namely $\rho = 1.0$, $\sigma_\beta = 0.7$, and $\sigma_\gamma = 0.50\%$. Also, for the graph only, we assume that β_t and γ_t are bivariate normal and, conditional on the parameters, returns are normally distributed.

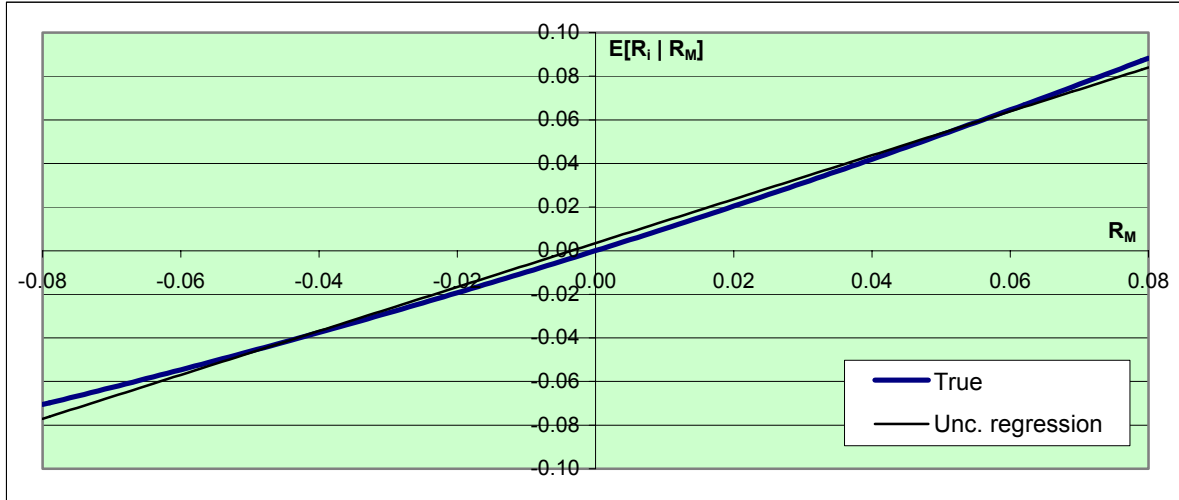
The graph shows that comovement in beta and the risk premium induces a slight convexity in the relation between R_i and R_M (conditionally, R_i and R_M are linearly related). The reason is that beta tends to be high when the market return is high. Changes in beta have only a small impact on the graph because they are very weakly correlated with *realized* market returns (even though beta is perfectly correlated with *expected* market returns).

The figure illustrates why the asset's unconditional alpha is positive, and also why it is small. In particular, α^u is the intercept in a simple *linear* regression of R_i on R_M , depicted by the thin line in Figure 1. Since the true relation is convex and passes through zero (if the conditional CAPM holds), the intercept in the linear regression must be positive. Again, however, the unconditional alpha is small because changes in beta induce only slight convexity in the relation between R_i and R_M . (The effects are reversed, of course, when beta and the risk premium are negatively correlated; the true relation is concave and the unconditional alpha is negative.)

Figure 1

The unconditional relation between R_i and R_M

The figure shows the excess return predicted on stock i as a function of the market excess return. The dark line shows the true $E[R_i | R_M]$ and the thin line shows the unconditional linear regression of R_i on R_M . Returns are conditionally normally distributed and the conditional CAPM is assumed to hold period-by-period. Beta (β_t) and the expected risk premium (γ_t) vary over time: β_t and γ_t are perfectly correlated, β_t has mean 1.0 and standard deviation 0.7, and γ_t has mean 0.5% and standard deviation 0.5%. The conditional standard deviation of the market return is 5% (constant).



(2) *Constant risk aversion.* The analysis above assumes that market volatility is constant, but we would expect that time-varying volatility would strengthen the conclusions: Eq. (6) shows that unconditional alphas are increasing in $\text{cov}(\beta_t, \gamma_t)$ but decreasing in $\text{cov}(\beta_t, \sigma_t^2)$. Thus, if the risk premium and volatility move together, the impact of time-varying volatility would tend to offset the impact of the risk premium. The connection between γ_t and σ_t^2 is difficult to estimate, since returns are so noisy, but there is strong indirect evidence that the relation is positive (e.g., French, Schwert, and Stambaugh, 1987; Campbell and Hentschel, 1992).

As a simple illustration, we adopt Merton's (1980) model of the conditional risk premium. Merton suggests that, if preferences are stable and hedging demands are not too important, the risk premium can be approximated by $\gamma_t = \lambda \sigma_t^2$, where λ is the relative risk aversion of the representative investor. Substituting into eq. (6), and using the fact that $\lambda = \gamma / E[\sigma_t^2]$ and $\sigma_M^2 = \sigma_\gamma^2 + E[\sigma_t^2]$, the unconditional

alpha simplifies to:

$$\alpha^u \approx \left[\frac{\sigma_\gamma^2}{\sigma_M^2} \right] \text{cov}(\beta_t, \gamma_t). \quad (8)$$

The expression in brackets equals the R^2 in a predictive regression for returns, while the covariance term is the unconditional alpha when only the risk premium varies (see eq. 7). A predictive regression R^2 using monthly returns is typically around 1% or less, so this model suggests that unconditional pricing errors will be very close to zero – i.e., divide the implied alphas in Table 1 by 100. The model is clearly special, but it serves to illustrate that time-varying volatility is likely to strengthen our basic conclusion: unconditional pricing errors implied by the conditional CAPM are simply too small to explain significant asset-pricing anomalies.

3. Testing the conditional CAPM

The analysis above relies, in part, on subjective judgments about what constitute ‘reasonable’ parameter values. Some readers will undoubtedly disagree with our assumptions. Therefore, in the remainder of the paper, we estimate some of the parameters and provide a simple direct test of the conditional CAPM.

3.1. Methodology

The basic framework for our tests is standard. We focus on time-series CAPM regressions for a handful of stock portfolios (described below):

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it}, \quad (9)$$

where R_{it} is the excess return on portfolio i and R_{Mt} is the excess return on the market. The CAPM predicts, of course, that α_i is zero.

For unconditional tests, we estimate (9) using the full time series of returns for each portfolio, restricting α_i and β_i to be constant. For conditional tests, a common approach is to model betas as functions of observed macroeconomic variables (e.g., Shanken, 1990; Ferson and Schadt, 1996; Lettau

and Ludvigson, 2001), but this approach is strictly valid only if the econometrician knows the full set of state variables available to investors. (The same criticism applies to alternative tests proposed in the literature; see Cochrane, 2001, for a review.)

Our tests use a different methodology that does not require conditioning information: we directly estimate conditional alphas and betas using short-window regressions. That is, rather than estimate (9) once, using the full time series of returns, we estimate it separately every, say, quarter using daily or weekly returns. By doing so, we get a direct estimate of each quarter's conditional alpha and beta. The implicit assumption here is that beta is relatively stable *within* the quarter, so that each regression can simply treat it as constant. This seems like a fairly mild assumption. Empirical tests often assume beta is stable for five or more *years*, and studies that allow beta to vary as a function of macroeconomic variables typically use very persistent series, like Tbill rates and dividend yield, implying that betas also change quite slowly. For robustness, we estimate regressions over a variety of interval lengths – monthly, quarterly, semiannually, and yearly – and using returns measured daily, weekly, or monthly. Also, we pay special attention to problems caused by nonsynchronous prices that might bias the estimates (discussed further below).

The procedure above generates time series of conditional alpha and beta estimates for each portfolio. We use the estimates in two ways. First, we study the time-series properties of beta and, as suggested by Section 2, relate these to the portfolio's implied unconditional alpha: we estimate how volatile betas are and how they correlate with business conditions and the market risk premium. Next, we directly test whether the average conditional alphas are zero, as implied by the conditional CAPM. Neither test requires that conditional alphas and betas are estimated precisely in individual short-window regressions. The fact that each regression uses a relatively small number of data points will not mean that our tests have low power.

3.2. *Microstructure issues*

Tests of the CAPM, and other factor models, nearly always use monthly returns. We use daily or

weekly returns because the regressions are estimated over very short windows – quarterly and semiannually in most tests. In principle, betas will be estimated more precisely using higher-frequency data, just as Merton (1980) observed for variances. In practice, using daily and weekly returns creates at least two problems.

First, ignoring microstructure issues, betas estimated for different return horizons will differ slightly because of compounding (Levhari and Levy, 1977; Handa, Kothari, and Wasley, 1989). Suppose that daily returns, R_i , are IID and let compounded N -day returns equal $R_i(N) = \prod_i(1+R_i) - 1$. The beta for compounded returns is

$$\beta_i(N) = \frac{E[(1+R_i)(1+R_M)]^N - E[1+R_i]^N E[1+R_M]^N}{E[(1+R_M)^2]^N - E[1+R_M]^{2N}}. \quad (10)$$

Eq. (10) implies that beta depends on the horizon: $\beta_i(N)$ increases in N if $\beta_i(1) > 1$ but decreases if $\beta_i(1) < 1$ (see Levhari and Levy). However, this effect is tiny and we ignore it in the remainder of the paper. For example, if the market return has mean 0.5% and standard deviation 5% monthly, then a stock with a daily beta of 1.300 would have a monthly beta of 1.302.

Second, and more importantly, nonsynchronous prices can have a big impact on short-horizon betas. Lo and MacKinlay (1990) show that small stocks react with a significant (week or more) delay to common news, so a daily or weekly beta will miss much of the small stock covariance with market returns. To mitigate the problem, all of our tests use value-weighted portfolios and exclude NASDAQ stocks. Also, following Dimson (1979), we include both current and lagged market returns in the regressions, estimating beta as the sum of the slopes on all lags (alpha is still just the intercept). For daily returns, we include four lags of market returns, imposing the constraint that lags 2 – 4 have the same slope to reduce the number of parameters:

$$R_{i,t} = \alpha_i + \beta_{i0} R_{M,t} + \beta_{i1} R_{M,t-1} + \beta_{i2} [(R_{M,t-2} + R_{M,t-3} + R_{M,t-4})/3] + \varepsilon_{i,t}. \quad (11)$$

The daily beta is then $\beta_i = \beta_{i0} + \beta_{i1} + \beta_{i2}$. (Adding a few more lags does not affect our results.) For weekly returns, we include two lags of market returns:

$$R_{i,t} = \alpha_i + \beta_{i0} R_{M,t} + \beta_{i1} R_{M,t-1} + \beta_{i2} R_{M,t-2} + \varepsilon_{i,t}, \quad (12)$$

where the weekly beta is again $\beta_i = \beta_{i0} + \beta_{i1} + \beta_{i2}$. To increase precision, we estimate (12) using overlapping returns (i.e., consecutive observations overlap by four days). Finally, although not the focus of our tests, we estimate monthly betas including one lag of market returns:

$$R_{i,t} = \alpha_i + \beta_{i0} R_{M,t} + \beta_{i1} R_{M,t-1} + \varepsilon_{i,t}, \quad (13)$$

where the monthly beta is $\beta_i = \beta_{i0} + \beta_{i1}$. As discussed below, Dimson betas are not a perfect solution, but our results do not seem to be driven by measurement problems. Indeed, unconditional alphas estimated by (11) – (13) are nearly identical for our test portfolios.

3.3. *The data*

The tests use returns on size, B/M, and momentum portfolios from July 1964 – June 2001. Prices and returns come from the CRSP daily stock file, while book values come from the merged CRSP / Compustat database. The portfolios are value-weighted and contain only NYSE and Amex stocks, excluding ADRs, REITs, and primes and scores.

The size and B/M portfolios are similar to those of Fama and French (1993). In June of every year, we form 25 size-B/M portfolios based on the intersection of five size and five B/M portfolios, with breakpoints given by NYSE quintiles. Size is the market value of equity at the end of June, while B/M is the ratio of book equity in the prior fiscal year (common equity plus balance sheet deferred taxes) to market equity at the end of December. Our tests are then based on six combinations of the 25 size-B/M portfolios: ‘Small’ is the average of the five portfolios in the lowest size quintile, ‘Big’ is the average of the five portfolios in the highest size quintile, and ‘S-B’ is their difference. Similarly, ‘Growth’ is the average of the five portfolios in the low-B/M quintile, ‘Value’ is the average of the five portfolios in the high-B/M quintile, and ‘V-G’ is their difference. Our ‘S-B’ and ‘V-G’ portfolios are much like Fama and French’s SMB and HML factors, except that we exclude NASDAQ stocks and start with 25 basis portfolios (rather than six).

The momentum portfolios are constructed separately using all stocks on CRSP with the required data (i.e., not restricted to Compustat firms). We sort stocks every month into deciles based on past 6-month returns and hold the portfolios for overlapping 6-month periods, as in Jegadeesh and Titman (1993). (This means, in effect, that one-sixth of each portfolio is rebalanced every month.) Again, the tests focus on a subset of the 10 portfolios: ‘Losers’ is the return on the bottom decile, ‘Winners’ is the return of the top decile, and ‘W-L’ is their difference.

The tests use returns compounded over three horizons: daily, weekly, and monthly. Weekly returns are calculated by compounding daily returns over five-day intervals. (For long-short strategies, we compound each side of the strategy and then difference.) We use five-day windows, not calendar weeks, in part because they are easily aligned with calendar quarters and in part because the changing number of trading days in a week (sometimes as few as three) would complicate some of the tests. For simplicity, we refer to the series simply as ‘weekly’ returns. Monthly returns are calculated in the standard way, compounding within calendar months.

Our market proxy is the excess return on the CRSP value-weighted index (all stocks), compounded weekly and monthly in the same manner as the other portfolios. The tests use excess returns on all portfolios, net of the one-month Treasury Bill rate.

To set the stage, Table 2 reports summary statistics for the size, B/M, and momentum portfolios from 1964 – 2001. Panel A shows average excess returns measured daily, weekly, and monthly. The estimates are all expressed in percent monthly; the daily estimates are multiplied by 21 (trading days per month) and the weekly estimates are multiplied by 21/5. Average returns exhibit the usual cross-sectional patterns: small stocks outperform large stocks (0.71% vs. 0.50% using monthly returns), high-B/M stocks outperform low-B/M stocks (0.88% vs. 0.41%), and winners outperform losers (0.91% vs. 0.01%). Estimates of average returns are always lowest using daily returns and highest using monthly returns. A very small portion of this pattern could be attributed to compounding, but it more likely reflects positive autocorrelation in daily returns. In particular, if daily returns are IID, monthly expected returns would be $\mu_{\text{mon}} = E[\prod_i(1+R_i)] - 1 = (1 + \mu_{\text{day}})^{21} - 1$, essentially identical to $21 \times \mu_{\text{day}}$. However, the expected

Table 2**Summary statistics for size, B/M, and momentum portfolios, 1964 – 2001**

The table reports average returns and unconditional CAPM regressions for size, B/M, and momentum portfolios. The estimates are obtained three ways, using daily, weekly, or monthly returns. Alphas and betas are adjusted for non-synchronous trading as described in the text. Average returns and alphas are expressed in percent monthly; the daily estimates are multiplied by 21 (trading days per month) and the weekly estimates are multiplied by 21/5. The portfolios are formed using all NYSE and Amex stocks on CRSP/Compustat. We begin with 25 size-B/M portfolios and 10 return-sorted portfolios, all value weighted. The size-B/M portfolios are formed from the intersection of five size (market equity) and five B/M (book equity divided by market equity) portfolios, with breakpoints determined by NYSE quintiles. ‘Small’ is the average of the five low-market-cap portfolios, ‘Big’ is the average of the five high-market-cap portfolios, and ‘S-B’ is their difference. ‘Growth’ is the average of the five low-B/M portfolios, ‘Value’ is the average of the five high-B/M portfolios, and ‘V-G’ is their difference. The return-sorted portfolios are formed independently by ranking stocks based on past 6-month returns. ‘Losers’ is the bottom decile, ‘Winners’ is the top decile, and ‘W-L’ is their difference.

		Size			B/M			Momentum		
		Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
<i>Panel A: Excess returns</i>										
Avg.	Day	0.57	0.49	0.08	0.32	0.81	0.49	-0.10	0.87	0.97
	Wk	0.63	0.50	0.13	0.37	0.84	0.47	-0.04	0.91	0.95
	Month	0.71	0.50	0.21	0.41	0.88	0.47	0.01	0.91	0.90
Std error	Day	0.28	0.20	0.19	0.27	0.23	0.13	0.33	0.28	0.26
	Wk	0.26	0.18	0.18	0.26	0.22	0.12	0.30	0.26	0.25
	Month	0.34	0.19	0.23	0.30	0.26	0.16	0.35	0.28	0.27
<i>Panel B: Unconditional alphas</i>										
Est.	Day	0.09	0.10	-0.01	-0.21	0.39	0.60	-0.64	0.35	0.99
	Wk	0.05	0.10	-0.05	-0.22	0.37	0.59	-0.66	0.37	1.03
	Month	0.07	0.11	-0.03	-0.20	0.39	0.59	-0.63	0.38	1.01
Std error	Day	0.15	0.06	0.17	0.10	0.12	0.12	0.18	0.13	0.26
	Wk	0.14	0.06	0.16	0.09	0.11	0.11	0.17	0.12	0.25
	Month	0.18	0.07	0.20	0.11	0.13	0.14	0.19	0.13	0.28
<i>Panel C: Unconditional betas</i>										
Est.	Day	1.07	0.87	0.20	1.18	0.94	-0.25	1.22	1.17	-0.06
	Wk	1.25	0.86	0.39	1.27	1.03	-0.24	1.33	1.16	-0.17
	Month	1.34	0.83	0.51	1.30	1.05	-0.25	1.36	1.14	-0.22
Std error	Day	0.03	0.01	0.03	0.02	0.03	0.02	0.03	0.02	0.05
	Wk	0.03	0.01	0.04	0.02	0.03	0.03	0.04	0.03	0.06
	Month	0.05	0.02	0.06	0.03	0.04	0.04	0.06	0.04	0.08

compounded return is higher if daily returns are positively autocorrelated (i.e., the expectation in the last sentence would have additional covariance terms). This argument likely explains why average daily and monthly returns are most different for small stocks.

Panel B show unconditional alphas for the portfolios (percent monthly). The estimates are remarkably similar for the three return horizons. Focusing on the long-short portfolios, S-B has a daily alpha of -0.01% and a monthly alpha of -0.03% , V-G has a daily alpha of 0.60% and a monthly alpha of 0.59% , and W-L has a daily alpha of 0.99% and a monthly alpha of 1.01% . Thus, after adjusting for risk, the size effect is absent in our data but the B/M and momentum effects are strong (using monthly returns, the latter two are about 4 standard errors from zero).

The contrast between Panels A and B is interesting: excess returns increase with the return horizon but alphas do not. The betas in Panel C show why: betas increase, roughly speaking, at the same rate as excess returns, so the net effect is that alphas are constant across horizons (i.e., $\alpha_i = E[R_i] - \beta_i E[R_M]$ is fairly constant). Thus, nonsynchronous prices have important effects on excess returns and betas but has little impact on tests of the CAPM for our portfolios. In general, betas tend to increase modestly with the return horizon, except for small stocks where the effect is stronger. Focusing on the long-short portfolios, S-B has a daily beta of 1.07 and a monthly beta of 1.34, V-G has both daily and monthly betas of -0.25 , and W-L has a daily beta of -0.06 and a monthly beta of -0.22 . (Weekly betas tend to be close to monthly betas.) Nonsynchronous prices have almost no impact on the long-short B/M strategy and only a small impact on the momentum strategy.

4. Empirical results

We now turn to the main empirical results. The analysis in Section 2 showed that, if the conditional CAPM holds, time-variation in betas should explain assets' unconditional alphas. Thus, we begin by studying the time-series properties of beta, which should be of interest beyond their implications for the conditional CAPM (see, e.g., Franzoni, 2002). We then provide a direct test of the CAPM, asking whether conditional alphas are zero.

The main inputs for the empirical tests are the time series of conditional alpha and beta estimates from the short-window regressions (see Section 3). We have explored a variety of window lengths and return horizons, but the tests focus on alphas and betas estimated four ways: (i) quarterly using daily

returns; (ii) semiannually using both daily (Semiannual 1) and weekly (Semiannual 2) returns; and (iii) annually using monthly returns. The estimates are corrected for nonsynchronous trading using the methodology in Section 3.2.

4.1. Time-variation in betas

Table 3 reports summary statistics for the conditional betas, and Figure 2 plots the time series of Semiannual 1 estimates (based on daily returns) for the three long-short strategies. The key message is that betas vary considerably over time. Much of the variability seems to be due to changes in true conditional betas, not estimation error.

Average betas, in panel A of Table 3, are generally close to the unconditional betas in Table 2. Except for small stocks, they are similar for the different estimation methods. Focusing on betas estimated semiannually from weekly returns, S-B has an average conditional beta of 0.32 (vs. an unconditional beta of 0.39), V-G has an average beta of -0.19 (vs. an unconditional beta of -0.24), and W-L has an average beta of -0.14 (vs. an unconditional beta of -0.17).

More importantly, Panels C and D and Figure 2 indicate that betas fluctuate significantly over time. Panel C shows that, except for the Big and Growth portfolios, the standard deviation of *estimated* betas is typically greater than 0.30 and, for momentum portfolios, often higher than 0.40. Of course, some of the variability is due to sampling error, so we focus more on the implied variability of true betas. Specifically, we can think of the estimated betas as $b_t = \beta_t + e_t$, where β_t is the true conditional beta and e_t is sampling error. As long as the short-window regressions satisfy standard OLS assumptions, β_t and e_t will be uncorrelated, so:

$$\text{var}(b_t) = \text{var}(\beta_t) + \text{var}(e_t), \tag{14}$$

where $\text{var}(b_t)$ is the variance of estimated betas and $\text{var}(e_t)$ is the average squared standard error from the regressions (see Fama and French, 1997). We solve (14) for $\text{var}(\beta_t)$ to obtain the implied variability of true betas, reported in Panel D.

The volatility of betas remains substantial even after removing sampling error. Focusing on the

Table 3
Time-variation in betas, 1964 – 2001

The table reports summary statistics for the conditional betas of size, B/M, and momentum portfolios. Betas are estimated quarterly using daily returns, semiannually using daily and weekly returns, and annually using monthly returns. They are adjusted for non-synchronous trading as described in the text. The table reports the time-series mean and standard deviation of the beta estimates (Panels A and C), the average standard error of beta from the short-window regressions (Panel B), and the implied time-series standard deviation of true betas (Panel D). The portfolios are formed using all NYSE and Amex stocks on CRSP/Compustat. We begin with 25 size-B/M portfolios and 10 return-sorted portfolios, all value weighted. The size-B/M portfolios are formed from the intersection of five size (market value of equity) and five B/M (book equity divided by market equity) portfolios, with breakpoints given by NYSE quintiles. ‘Small’ is the average of the five small portfolios, ‘Big’ is the average of the five large portfolios, and ‘S-B’ is their difference. Similarly, ‘Growth’ is the average of the five low-B/M portfolios, ‘Value’ is the average of the five high-B/M portfolios, and ‘V-G’ is their difference. The return-sorted portfolios are formed independently by ranking stocks based on past 6-month returns. ‘Losers’ is the bottom decile, ‘Winners’ is the top decile, and ‘W-L’ is their difference.

	Size			B/M			Momentum		
	Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
<i>Panel A: Average betas</i>									
Quarterly ^a	1.03	0.93	0.10	1.17	0.98	-0.19	1.19	1.24	0.05
Semiannual 1	1.07	0.93	0.14	1.19	0.99	-0.20	1.20	1.24	0.05
Semiannual 2	1.23	0.91	0.32	1.25	1.06	-0.19	1.33	1.19	-0.14
Annual	1.49	0.83	0.66	1.36	1.17	-0.19	1.38	1.24	-0.14
<i>Panel B: Average std error^b</i>									
Quarterly	0.13	0.06	0.17	0.10	0.10	0.12	0.17	0.14	0.24
Semiannual 1	0.09	0.04	0.12	0.07	0.07	0.08	0.12	0.09	0.17
Semiannual 2	0.16	0.07	0.20	0.11	0.12	0.14	0.20	0.15	0.29
Annual	0.36	0.13	0.42	0.22	0.24	0.30	0.40	0.28	0.57
<i>Panel C: Std deviation of estimated betas</i>									
Quarterly	0.35	0.15	0.38	0.22	0.30	0.28	0.41	0.37	0.68
Semiannual 1	0.31	0.13	0.32	0.19	0.29	0.25	0.33	0.32	0.58
Semiannual 2	0.35	0.13	0.38	0.20	0.33	0.33	0.44	0.36	0.71
Annual	0.54	0.14	0.56	0.27	0.46	0.41	0.52	0.44	0.83
<i>Panel D: Implied std deviation of true betas^c</i>									
Quarterly	0.32	0.13	0.33	0.19	0.28	0.25	0.36	0.33	0.63
Semiannual 1	0.29	0.12	0.30	0.18	0.28	0.24	0.30	0.30	0.55
Semiannual 2	0.31	0.10	0.32	0.16	0.31	0.29	0.36	0.32	0.62
Annual	0.35	--	0.25	0.04	0.37	0.19	0.19	0.29	0.52

^a Quarterly and Semiannual 1 betas are estimated from daily returns, Semiannual 2 betas are estimated from weekly returns, and Annual betas are estimated from monthly returns.

^b Average standard error from the quarterly, semi-annual, or annual regressions, not the standard error of the average.

^c The implied variance of true betas equals $\text{var}(b_i) - \overline{\text{var}(e_i)}$, the difference between the variance of estimated betas and the average variance of the sampling error in b_i (from the regressions). The standard deviation is undefined for Big using annual windows / monthly returns because the implied variance is negative.

Figure 2
Conditional betas, 1964 – 2001

The figure plots conditional betas for size, B/M, and momentum portfolios. The dark line is the point estimate and the light lines indicate a two-standard-deviation confidence interval. Betas are estimated semiannually (non-overlapping windows) using daily returns. The notation .1 and .2 denote the first and second halves of the year. The portfolios are formed using all NYSE and Amex stocks on CRSP/Compustat. We begin with 25 size-B/M portfolios and 10 return-sorted portfolios, all value weighted. The size-B/M portfolios are formed from the intersection of five size (market equity) and five B/M (book equity divided by market equity) portfolios, with breakpoints given by NYSE quintiles. ‘Small’ is the average of the five small portfolios and ‘Big’ is the average of the five large portfolios. Similarly, ‘Growth’ is the average of the five low-B/M portfolios and ‘Value’ is the average of the five high-B/M portfolios. The return-sorted portfolios are formed by ranking stocks based on past 6-month returns. ‘Losers’ is the bottom decile and ‘Winners’ is the top decile.



long-short strategies, S-B's beta has a standard deviation around 0.30, V-G's beta has a standard deviation around 0.25, and W-L's beta has a standard deviation around 0.60. These estimates are economically large and reveal striking time-heterogeneity in the portfolios (see also Franzoni, 2002). The fluctuations can be seen most easily in Figure 2. In particular, S-B's beta varies from a high of 1.02 (t-statistic, 7.22) in 1966 to a low of -0.64 (t-statistic, -5.51) in 1989. The B/M strategy's beta reaches a maximum of 0.54 (t-statistic, 5.13) in 1976 before falling to a minimum of -0.99 (t-statistic, -11.39) just six years later. The momentum strategy's beta is the most volatile, which is not surprising given that the strategy almost certainly has the highest turnover. Figure 2 shows that W-L's beta varies from a high of 2.25 (t-statistic, 8.98) to a low of -1.51 (t-statistic, -4.47).

In Section 2, we showed that if the conditional CAPM holds and market volatility is constant (or uncorrelated with beta), a portfolio's unconditional alpha is approximately equal to $\text{cov}(\beta_t, \gamma_t) = \rho \sigma_\beta \sigma_\gamma$, where γ_t is the market risk premium. At the time, we considered values of σ_β ranging from 0.3 to 0.7 to illustrate that implied alphas are relatively small for 'plausible' parameters (see Table 1). This range seems reasonable given the results in Table 3.

The time-series plots show that betas sometimes change considerably from one year to the next but, in general, exhibit a fair degree of persistence. Table 4 looks more carefully at time-variation in betas, exploring the persistence of beta and the correlation between beta and several commonly used state variables. The state variables are lagged relative to beta (i.e., known prior to the beta estimation window), so the correlations are predictive. $R_{M,-6}$ is the past 6-month return on the market portfolio (the six month period matches the formation period for momentum portfolios); TBILL is the one-month Tbill rate; DY is the annual dividend yield on the value-weighted NYSE index, measured over rolling 12-month windows; TERM is the yield spread between 10-year and 1-year Tbonds; and CAY is the consumption to wealth ratio of Lettau and Ludvigson (2001), which roughly measures whether stock prices are high or low relative to aggregate consumption (see their paper for details). The portfolios' lagged betas, β_{t-1} , are included to test for persistence. In Table 4, we focus on betas estimated semiannually using daily returns,

the same as those used in Figure 2.

Panel A reports the correlation between betas and the state variables. The first row shows that betas are persistent but that autocorrelations are far from one. The autocorrelations are between 0.45 and 0.68 for most of the raw portfolios and a bit lower, between 0.37 and 0.51, for the long-short strategies.⁵ Momentum portfolios have the least persistent betas, again, presumably reflecting the high turnover of the portfolios. Momentum betas are also highly correlated with past market returns. Winner betas increase (correlation of 0.47) and Loser betas decrease (correlation of -0.53) after the market does well. This pattern is intuitive: we would expect that the Winner portfolio becomes weighted towards high-beta stocks when the market goes up, since those stocks tend to perform well (Ball, Kothari, and Shanken, 1995; Grundy and Martin, 2001).

Panel B studies the joint explanatory power of our business-condition proxies. For the regressions, the state variables (including lagged betas) are scaled by their standard deviations, so the slopes can be interpreted as the change in beta predicted by a one-standard-deviation change in the state variable.

The results indicate that betas vary significantly over the business cycle. The slopes on TBILL, DY, and TERM are significant for many portfolios, while $R_{M,-6}$ is significant only for the momentum portfolios (as discussed above). Small, Value, and Winner stocks tend to have high betas when Tbill rates and the term spread are low (t-statistics ranging from -2.40 to -3.41) and when DY is high (t-statistics between 2.82 and 3.64). Economically, the slopes are quite large. A one-standard-deviation increase in TBILL or TERM is associated with a -0.08 to -0.14 drop in the portfolios' betas, while a one-standard-deviation increase in DY is associated with a 0.11 to 0.16 rise in the portfolios' betas. DY is also positively related to Big and Growth beta but the relations are weaker (slopes of 0.05 and 0.06, respectively). The connection between past market returns and momentum betas is also quite strong. For

⁵ Under the null that the autocorrelations are zero, the standard error of the estimates is roughly $1/\sqrt{T} = 0.116$, where T is 74 semiannual periods. Note, also, that sampling error in estimated betas would tend to push the autocorrelations toward zero. Specifically, if sampling error is serially uncorrelated (as implied by OLS) and unrelated to future betas, then the autocorrelation of estimated betas (b_t) equals the autocorrelation of true betas (β_t) multiplied by $\text{var}(\beta_t) / \text{var}(b_t) < 1$. Comparing the Semiannual 1 variances in Panels C and D of Table 3, the attenuation bias is small for our data.

Table 4
Predicting conditional betas, 1964 – 2001

The table reports the correlation between state variables and the conditional betas of size, B/M, and momentum portfolios. Betas are estimated semiannually using daily returns. The state variables are lagged relative to the beta estimates. β_{t-1} is the portfolio's lagged beta; $R_{M,-6}$ is the past 6-month market return; TBILL is the one-month Tbill rate; DY is the log dividend yield on the value-weighted NYSE index; TERM is the yield spread between 10-year and 1-year Tbonds; CAY is the consumption to wealth ratio of Lettau and Ludvigson (2001). Panel A reports simple correlations between estimated conditional betas and the state variables, and Panel B reports slope estimates when betas are regressed on all of the state variables together. The portfolios are formed using all NYSE and Amex stocks on CRSP/Compustat. We begin with 25 size-B/M portfolios and 10 return-sorted portfolios, all value weighted. The size-B/M portfolios are formed from the intersection of five size (market value of equity) and five B/M (book equity divided by market equity) portfolios, with breakpoints given by NYSE quintiles. 'Small' is the average of the five small portfolios, 'Big' is the average of the five large portfolios, and 'S-B' is their difference. Similarly, 'Growth' is the average of the five low-B/M portfolios, 'Value' is the average of the five high-B/M portfolios, and 'V-G' is their difference. The return-sorted portfolios are formed independently by ranking stocks based on past 6-month returns. 'Losers' is the bottom decile, 'Winners' is the top decile, and 'W-L' is their difference.

	Size			B/M			Momentum		
	Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
<i>Panel A: Correlation between betas and state variables</i>									
β_{t-1}	0.55	0.68	0.43	0.58	0.67	0.51	0.30	0.45	0.37
$R_{M,-6}$	-0.05	-0.01	-0.05	-0.18	0.00	0.14	-0.53	0.47	0.56
TBILL	-0.04	0.11	-0.08	0.15	-0.12	-0.25	0.14	-0.25	-0.21
DY	0.22	0.64	-0.04	0.37	0.40	0.18	0.13	-0.12	-0.14
TERM	-0.20	0.19	-0.27	-0.12	0.01	0.10	-0.01	-0.08	-0.04
CAY	-0.12	0.50	-0.31	-0.01	0.17	0.20	0.09	-0.09	-0.10
<i>Panel B: Betas regressed on the state variables^a</i>									
Slope estimate									
β_{t-1}	0.12	0.05	0.11	0.10	0.12	0.08	0.10	0.15	0.22
$R_{M,-6}$	0.05	-0.01	0.04	0.02	0.04	0.04	-0.19	0.20	0.39
TBILL	-0.13	-0.02	-0.11	-0.03	-0.14	-0.13	0.09	-0.14	-0.24
DY	0.14	0.05	0.09	0.06	0.16	0.10	-0.07	0.11	0.19
TERM	-0.10	0.00	-0.10	-0.02	-0.08	-0.07	0.07	-0.11	-0.19
CAY	-0.05	0.02	-0.08	-0.03	-0.01	0.03	0.00	-0.01	-0.01
t-statistic									
β_{t-1}	3.53	3.99	2.83	4.24	3.88	2.62	3.03	5.31	4.49
$R_{M,-6}$	1.52	-0.45	1.17	0.73	1.58	1.41	-5.63	7.25	7.63
TBILL	-2.56	-1.39	-2.09	-1.06	-3.19	-2.98	1.79	-3.41	-3.22
DY	2.82	3.05	1.74	2.10	3.64	2.65	-1.50	2.87	2.65
TERM	-2.40	-0.25	-2.21	-0.81	-2.40	-1.99	1.60	-3.07	-2.81
CAY	-1.32	1.86	-1.81	-1.34	-0.17	0.98	0.07	-0.22	-0.13
Adj R ²	0.37	0.60	0.26	0.34	0.52	0.32	0.35	0.56	0.53

^a The state variables – including lagged beta – are scaled by their standard deviations. The slopes can be interpreted as the predicted change in beta associated with a one-standard-deviation change in the state variable. Bold denotes estimates greater than 1.99 standard errors from zero.

the long-short portfolio, W-L, beta increases by 0.39 if past 6-month market returns are one-standard-deviation above average.

It is interesting to note that CAY, the consumption-to-wealth ratio of Lettau and Ludvigson (2001), shows little relation to betas. Lettau and Ludvigson suggest that CAY captures important information about the market and consumption betas of value stocks. The correlations in Panel A provide some evidence that CAY is positively related to Value betas (correlation of 0.17), but the regression slopes in Panel B are close to zero for all portfolios.

In summary, Tables 3 and 4 show that size, B/M, and momentum betas vary considerably over time. Betas are somewhat persistent but exhibit remarkably large, high-frequency fluctuations. (As a benchmark, our non-return state variables are generally much more persistent, with semiannual autocorrelations ranging from 0.64 to 0.97.) Betas also fluctuate with state variables that prior research shows capture information about business conditions and, to some extent, the market risk premium (e.g., Fama and Schwert, 1977; Fama and French, 1989).

4.2. Beta and the market risk premium

It is useful to return now to the analysis in Section 2. If the conditional CAPM holds and market volatility is constant, a portfolio's unconditional alpha is approximately equal to $\text{cov}(\beta_t, \gamma_t)$, where γ_t is the risk premium. Table 4 provides indirect evidence that betas might, indeed, covary with the risk premium, suggesting that time-variation in betas could help explain the unconditional alphas of size, B/M, and momentum portfolios. We have argued that $\text{cov}(\beta_t, \gamma_t)$ is likely to be small, but that conclusion relies on our guess about plausible variation in γ_t . In this section, we directly estimate the covariance between betas and the market risk premium.

We consider two ways of estimating $\text{cov}(\beta_t, \gamma_t)$. Our first estimate is simply $\text{cov}(b_t, R_{Mt})$, where we have replaced the true conditional beta with our estimate, b_t , and have replaced the risk premium with the realized market return, R_{Mt} . The logic here is that, under the assumptions of OLS, sampling error in beta should be uncorrelated with market returns, so the covariance between b_t and R_{Mt} provides an unbiased

estimate of $\text{cov}(\beta_t, \gamma_t)$:

$$\text{cov}(b_t, R_{Mt}) = \text{cov}(\beta_t + e_t, \gamma_t + s_t) = \text{cov}(\beta_t, \gamma_t), \quad (15)$$

which uses the fact that s_t must be uncorrelated with β_t . Eq. (15) is necessarily true if returns are conditionally normally distributed, but it does not have to hold for alternative (nonlinear) distributions. For example, Ang and Chen (2002) show that stocks covary more strongly in down markets, suggesting that, for some stocks, e_t and s_t will be negatively correlated. (This problem is likely to be more severe for shorter estimation windows, in which a few large returns can have a big impact on the estimates.) Therefore, this first estimator should be interpreted with caution; we report it primarily as a benchmark rather than as an perfect estimate of $\text{cov}(\beta_t, \gamma_t)$.

Our second estimate uses the predictive regressions described above (in Table 4). In particular, the estimator is given by $\text{cov}(b_t^*, R_{Mt})$, where b_t^* is the fitted value from the regression of b_t on the state variables and its own lag. Because the predictor variables are all known at the beginning of the period, it must be the case that

$$\text{cov}(b_t^*, R_{Mt}) = \text{cov}(b_t^*, \gamma_t). \quad (16)$$

The estimator will equal $\text{cov}(\beta_t, \gamma_t)$ if the error in b_t^* is uncorrelated with the market risk premium, i.e., if $\text{cov}(\gamma_t, \beta_t - b_t^*) = 0$. This requires that the state variables do a good job capturing *either* time-variation in the risk premium or time-variation in betas (one is necessary, not both). Thus, unlike elsewhere in the paper, this test depends on whether we know the ‘right’ state variables. Table 4 shows that the instruments do capture a significant fraction of movements in betas – the regression R^2 range from 0.26 to 0.60 – but there clearly remains a large component unexplained. Thus, we again interpret the results with caution, although we have no particular reason to believe that the unexplained component of beta is correlated with γ_t .⁶

⁶ The fitted value b_t^* comes from a first-stage regression of b_t on the state variables. The results are identical if we reverse the procedure, first regressing R_{Mt} on the state variables and then using the fitted (expected) returns to estimate the covariance with b_t .

Table 5 reports estimates for the size, B/M, and momentum portfolios. To calculate the covariances, market returns are expressed in percent monthly, so the numbers can be interpreted as the unconditional monthly alphas implied by the conditional CAPM under the assumption that market volatility is constant (i.e., $\alpha^u \approx \text{cov}(\beta_t, \gamma_t)$).⁷

Panel A shows results for the first estimator, equal to the covariance between estimated betas and contemporaneous market returns. The results provide no evidence that time-varying betas can salvage the CAPM: the implied alphas are either close to zero or have the wrong sign. Small, Growth, and Winner betas all covary negatively with market returns, with estimates ranging from -0.07% to -0.32% for quarterly and semiannual estimates of beta (the covariances for annual betas are close to zero). Large-stock betas, in contrast, covary positively with market returns, but the estimates are economically small (0.07%). For the long-short portfolios, S-B and W-L betas are higher in down markets, while V-G betas show little relation to market returns. Thus, Panel A provides no evidence that conditional betas covary with the risk premium in a way that can explain the unconditional alphas observed for B/M and momentum portfolios.

Again, we caution that the estimates in Panel A likely reflect correlation between sampling error in b_t and unexpected market returns. Our second estimator, in Panel B, gets around this problem. It equals the covariance between *predicted* betas and market returns, which will be a better estimator if our state variables do a good job capturing time-variation in beta or time-variation in the market risk premium. The estimates in Panel B are almost always closer to zero than those in Panel A, but the covariances typically have the same sign. Focusing on the long-short portfolios, S-B's and W-L's predicted betas still covary negatively with market returns, but now only the estimate for the size strategy is significant. The quarterly and semiannual estimates are between -0.05% and -0.14% . The B/M portfolios' predicted betas show no relation to market returns, with estimates between -0.02% and 0.03% (standard errors of

⁷ Standard errors are obtained by regressing market returns on estimated betas (Panel A) or predicted betas (Panel B), scaling the independent variable so that the slope equals a simple covariance (i.e., dividing by the variance of the independent variable). Implicitly, then, the standard errors in Table 5 condition on the sample variance of estimated or predicted betas.

Table 5
Conditional betas and the market risk premium, 1964 – 2001

The table reports the covariance between market returns and the conditional betas on size, B/M, and momentum portfolios. Betas are estimated quarterly using daily returns, semiannually using daily and weekly returns, and annually using monthly returns. The market portfolio is the CRSP value-weighted index. Excess returns on the index are measured over the same window as betas (e.g., quarterly betas covary with quarterly returns), but the numbers are all expressed in percent monthly (e.g., the quarterly covariance is divided by 3). Panel A reports the market's covariance with contemporaneously estimated betas and Panel B reports the market's covariance with predicted betas, taken from the regression of estimated betas on lagged state variables (see Table 4). The size, B/M, and momentum portfolios are formed using all NYSE and Amex stocks on CRSP/Compustat.

		Size			B/M			Momentum		
		Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
<i>Panel A: Covariance between estimated betas and market returns^a</i>										
Est.	Qtr	-0.32	0.07	-0.39	-0.20	-0.12	0.09	0.16	-0.23	-0.38
	Semi 1	-0.17	0.07	-0.24	-0.14	-0.03	0.11	-0.03	-0.07	-0.04
	Semi 2	-0.12	0.07	-0.19	-0.10	-0.03	0.07	0.15	-0.18	-0.33
	Annual	0.06	0.03	0.03	-0.03	0.01	0.04	-0.08	0.11	0.20
Std err.	Qtr	0.08	0.03	0.08	0.05	0.07	0.06	0.09	0.08	0.16
	Semi 1	0.07	0.03	0.07	0.04	0.07	0.06	0.08	0.07	0.13
	Semi 2	0.08	0.03	0.08	0.04	0.08	0.07	0.10	0.08	0.15
	Annual	0.12	0.03	0.13	0.06	0.10	0.09	0.12	0.10	0.19
<i>Panel B: Covariance between predicted betas and market returns</i>										
Est.	Qtr	-0.06	0.04	-0.09	-0.01	-0.02	0.02	0.06	-0.05	-0.12
	Semi 1	-0.07	0.03	-0.10	-0.02	-0.02	0.01	0.05	-0.07	-0.12
	Semi 2	-0.04	0.02	-0.05	0.00	-0.01	0.00	0.07	-0.08	-0.14
	Annual	0.03	0.01	0.02	0.00	0.01	0.03	0.05	-0.03	-0.08
Std err.	Qtr	0.04	0.02	0.04	0.03	0.05	0.04	0.05	0.06	0.10
	Semi 1	0.05	0.02	0.04	0.03	0.05	0.04	0.05	0.06	0.10
	Semi 2	0.04	0.02	0.03	0.02	0.05	0.04	0.06	0.05	0.10
	Annual	0.05	0.02	0.05	0.03	0.06	0.04	0.06	0.05	0.09

^a Quarterly and Semiannual 1 betas are estimated from daily returns, Semiannual 2 betas are estimated from weekly returns, and Annual betas are estimated from monthly returns.

0.02% to 0.05%).

In short, although betas vary considerably over time, they do not seem to covary strongly with the market risk premium. As a result, unconditional pricing errors implied by the conditional CAPM are much smaller than those actually observed for B/M and momentum portfolios. Using the estimates in Panel B, the conditional CAPM predicts that V-G should have an unconditional alpha of 0.01%, a tiny fraction of the actual alpha, 0.59% (see Table 2). W-L should have an alpha of -0.12%, small and

opposite in sign to the actual alpha, 1.03%. Thus, time-variation in beta does not seem to explain deviations from the unconditional CAPM.

4.3. Beta and market volatility

The results above focus on the covariance between beta and the market risk premium. In Section 2, we showed that covariance with market volatility could also be important. With time-varying volatility, the unconditional alpha is (eq. 6):

$$\alpha^u \approx \text{cov}(\beta_t, \gamma_t) - \frac{\gamma}{\sigma_M^2} \text{cov}(\beta_t, \sigma_t^2). \quad (17)$$

In untabulated results, we find that the last term in (17), which captures the impact of time-varying volatility, is economically quite small. To estimate $\text{cov}(\beta_t, \sigma_t^2)$, we calculate conditional market volatility much like we do betas, using daily, weekly or monthly returns over short windows. (The estimate includes an autocorrelation adjustment following the approach of French, Schwert, and Stambaugh, 1987; we use de-meaned returns to calculate variance, while they use raw returns, but the methodology is otherwise similar.) We then estimate the covariance between market volatility and both estimated and predicted betas, just as we did with realized market returns in Section 4.2. From 1964 – 2001, the estimates of $\text{cov}(\beta_t, \sigma_t^2)$ are between -0.02 and 0.02 for every portfolio (percent monthly; standard errors of 0.01 – 0.02). Multiplying by $\gamma / \sigma_M^2 = (0.0047 / 0.045^2) = 2.32$, the covariance between betas and market volatility has an economically small impact on the unconditional alpha, at most +/-0.05% monthly. Thus, accounting for time-varying market volatility does little to improve the performance of the conditional CAPM.

4.4. Conditional alphas

The tests above show that the unconditional alphas of B/M and momentum portfolios are inconsistent with the conditional CAPM. We now directly test whether *conditional* alphas are zero. Unlike prior tests in the literature, our short-horizon regressions allow us to test the conditional CAPM

Table 6
Average conditional alphas, 1964 – 2001

The table reports average conditional alphas for size, B/M, and momentum portfolios (monthly, in percent). Alphas are estimated quarterly using daily returns, semiannually using daily and weekly returns, and annually using monthly returns. The portfolios are formed using all NYSE and Amex stocks on CRSP/ Compustat. Bold denotes estimates greater than two standard errors from zero.

	Size			B/M			Momentum		
	Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
<i>Average conditional alpha (%)</i>									
Quarterly ^a	0.42	0.00	0.42	-0.01	0.49	0.50	-0.79	0.55	1.33
Semiannual 1	0.26	0.00	0.26	-0.08	0.40	0.47	-0.61	0.39	0.99
Semiannual 2	0.16	0.01	0.15	-0.12	0.36	0.48	-0.83	0.53	1.37
Annual	-0.06	0.08	-0.14	-0.20	0.32	0.53	-0.56	0.21	0.77
<i>Standard error</i>									
Quarterly	0.20	0.06	0.22	0.12	0.14	0.14	0.20	0.13	0.26
Semiannual 1	0.21	0.06	0.23	0.12	0.14	0.15	0.19	0.14	0.25
Semiannual 2	0.21	0.06	0.23	0.14	0.15	0.16	0.20	0.15	0.27
Annual	0.26	0.07	0.29	0.16	0.17	0.14	0.21	0.17	0.29

^a Quarterly and Semiannual 1 alphas are estimated from daily returns, Semiannual 2 alphas are estimated from weekly returns, and Annual alphas are estimated from monthly returns.

without knowing the right state variables, so long as betas are relatively stable within the estimation window (quarterly or semiannually).

Table 6 reports average conditional alphas for the portfolios. Except for small stocks, the estimates are close to the unconditional alphas in Table 2 and provide strong evidence against the conditional CAPM. V-G's and W-L's average alphas are economically large and statistically significant. Depending on the estimation method, V-G's average conditional alpha is between 0.47% and 0.53% (t-statistics of 3.05 to 3.65), compared with an unconditional alpha around 0.59%. W-L's average alpha shows more dispersion, ranging from 0.77% to 1.37% (t-statistics of 2.66 to 5.12), but the estimates are in line with an unconditional alpha of about 1.00%. In short, the conditional CAPM performs about as poorly as the unconditional CAPM.

The alpha estimates are consistent with our analysis in Table 5. In that table, we show how time-varying betas would affect unconditional alphas if market volatility is constant. In fact, the estimates in

Table 5 are very close to the difference between the unconditional alphas and average conditional alphas of the portfolios (Tables 2 and 6). For example, the covariance between V-G betas and the risk premium (quarterly estimates) is 0.09%, roughly equal to the unconditional alpha, 0.60%, minus the average conditional alpha, 0.50%.

5. The consumption CAPM

So far, our analysis has focused on the Sharpe-Lintner CAPM. But as discussed in the introduction, we believe the conclusions should apply to other models as well: in general, conditioning seems unlikely to have a large impact on cross-sectional asset-pricing tests. This section briefly explores the performance of the consumption CAPM.

Analytically, the basic argument is easy to extend to the consumption CAPM: consumption betas and the consumption risk premium must have large time-variation in order for a conditional model to significantly outperform an unconditional one. The problem is that we do not have good intuition about the price of consumption risk, or how it varies over time, largely because of the poor empirical performance of the consumption CAPM.⁸

Empirically, our tests cannot be extended directly to the consumption CAPM because they require high-frequency data. We can, however, provide at least tentative evidence using the mimicking-portfolio approach of Breeden, Gibbons, and Litzenberger (1989). They observe that, in empirical tests, the portfolio which has the highest correlation with consumption can be used in place of consumption itself; betas with respect to the maximally-correlated portfolio (MCP) are proportional to betas with consumption, so both should explain expected returns. For our purposes, the mimicking-portfolio

⁸ To illustrate, consider three ways to estimate the unconditional price of consumption-beta risk, denoted λ . From a theoretical standpoint, λ should be roughly $\phi \sigma_c^2$ if investors have time-additive utility, where ϕ is aggregate relative risk aversion and σ_c^2 is the variance of consumption growth (see Cochrane, 2001). The variance of consumption growth is around 0.03% annually during our sample, so the price of consumption risk would be 0.15% annually if the risk aversion coefficient is 5. Second, in cross-sectional regressions using 25 size-B/M portfolios, Lettau and Ludvigson (2001) estimate that λ equals 0.88% (standard error, 0.77%; see row 1 of their Table 3). Finally, the CRSP value weighted index has a consumption beta of 1.9 and an annual excess return of 5.6% during our sample, which suggest that λ equals 2.95% annually.

approach is convenient because, given an estimate of the MCP, we can simply repeat our earlier tests using the MCP in place of the market portfolio.

Following Breeden et al., we estimate the MCP by regressing quarterly consumption growth on a given set of assets. This regression finds the linear combination of the assets that has the highest correlation with consumption, implying that the slopes are proportional to the MCP portfolio weights. The key difficulty here is to choose the right set of assets for the regression (with only 148 quarterly data points, we have to limit the set of assets in some way). Since the MCP will be mean-variance efficient if the consumption CAPM holds, we use the Fama and French (1993) factors – R_M , SMB, and HML – guided by evidence that they do a good job spanning the tangency portfolio. Limiting the set of assets to a few that (nearly) span the tangency portfolio should give the consumption CAPM the best chance of working (indeed, if the tangency portfolio was the only asset used in the MCP regression, the consumption CAPM would appear to work perfectly).

Breeden et al. test the unconditional CAPM, so it makes sense that they estimate the MCP using a regression with constant slopes. We use this approach in some tests, but since we are interested in the conditional CAPM, we also estimate a dynamic mimicking portfolio allowing the slopes (i.e., portfolio weights) to change over time:

$$c_t = a_0 + a_1 z_t + (\varphi_{m,0} + \varphi_{m,1} z_{t-1}) R_M + (\varphi_{s,0} + \varphi_{s,1} z_{t-1}) SMB + (\varphi_{h,0} + \varphi_{h,1} z_{t-1}) HML + e_t, \quad (18)$$

where z_t is a state variable (or variables) that captures time-variation in the MCP weights. The reported results use Lettau and Ludvigson's (2001) consumption-to-wealth ratio, CAY, as the state variable. Lettau and Ludvigson find that consumption betas of size and B/M portfolios vary over time with CAY, and it seems likely that the MCP weights of SMB and HML will do so as well. Though not tabulated, we find similar results when Tbill rates, the default and term spreads, and aggregate dividend yield are used together in place of CAY.

Results. In the first-stage regressions, estimating the MCP, consumption is measured as per capita consumption of nondurables and services (see Lettau and Ludvigson, 2001, for a description of the consumption and CAY series). Unconditionally, the three factors are quite weakly correlated with

consumption, with a regression R^2 of 0.00. The estimated MCP seems reasonable, however, with weights of 0.36, 0.36, and 0.28 on R_M , SMB, and HML (standard errors of 0.38 to 0.50), and the MCP has a higher Sharpe ratio than the market (0.27 vs. 0.17 quarterly). In the dynamic regression (18), the Fama-French factors do a better job explaining consumption growth, with an R^2 of 0.15. The average slope on R_M is marginally significant (t-statistic of 1.81), and the slopes on both R_M and SMB seem to vary over time with CAY (t-statistics of -3.11 and 1.95 , respectively).

Given an estimated MCP, we test the consumption CAPM by substituting the MCP in place of the market portfolio in the regressions described in Sections 3 and 4 (the test portfolios remain the same). Table 7 reports the unconditional alphas from the full-sample regressions and average conditional alphas from the short-window regressions. Panel A uses the constant-weight mimicking portfolio and Panel B uses the dynamic mimicking portfolio.

Not surprisingly, the unconditional consumption CAPM cannot explain B/M or momentum. V-G's unconditional alpha is between 0.45% and 0.55% using either the constant-weight or dynamic MCP, and always more than two standard errors from zero. W-L's alpha is between 0.92% and 1.09%, and also statistically significant. These estimates are close to those for the simple CAPM. The main difference with our earlier CAPM results is that small stocks seem to underperform large stocks in Panel A. Also, alphas on each side of the long-short strategies differ substantially from the simple CAPM estimates, lower everywhere in Panel A but higher in Panel B.

Average conditional alphas are generally similar to unconditional alphas. For V-G, the conditional alphas are somewhat smaller, between 0.32% and 0.42%, but remain more than two standard errors from zero. For W-L, the conditional alphas show large variability across the different estimation methods (dynamic vs. constant-weight MCP; daily vs. monthly returns), ranging from 0.77% up to 1.71%, but are always more than three standard errors from zero (with one exception, which has a t-statistic of 2.44). These results provide no evidence that the consumption CAPM, even allowing for time-varying betas, can explain the B/M and momentum effects. Overall, they support our argument that conditioning is unlikely to have much impact on asset-pricing tests.

Table 7
Consumption CAPM: Alphas, 1964 – 2001

The table reports unconditional alphas and average conditional alphas for the consumption CAPM (% monthly). Unconditional alphas are estimated from full-sample regressions using daily, weekly, or monthly returns. Conditional alphas are estimated quarterly using daily returns, semiannually using daily and weekly returns, and annually using monthly returns. The tests are like those in Tables 2 and 5, but a consumption-mimicking portfolio is used in place of the market portfolio. The mimicking portfolio is estimated by regressing quarterly consumption growth on the Fama and French factors (R_M , SMB, and HML); the slopes can be thought of as portfolio weights that give the combination of R_M , SMB, and HML with the highest correlation with consumption growth. We use these weights, together with daily returns on the three factors, to construct the mimicking portfolio. In panel A, the mimicking portfolio has constant weights throughout the sample. In panel B, the weights change dynamically, quarter to quarter, as linear functions of CAY. The size, B/M, and momentum portfolios are formed using all NYSE and Amex stocks on CRSP/Compustat. Bold denotes estimates more than two standard errors from zero.

		Size			B/M			Momentum		
		Small	Big	S-B	Grwth	Value	V-G	Losers	Winrs	W-L
<i>Panel A: Constant-weight mimicking portfolio</i>										
Unconditional alphas										
Est.	Day	-0.39	0.03	-0.42	-0.49	0.05	0.55	-0.90	0.17	1.07
	Wk	-0.37	0.05	-0.42	-0.44	0.07	0.51	-0.87	0.22	1.09
	Month	-0.33	0.05	-0.39	-0.40	0.10	0.50	-0.83	0.24	1.07
Average conditional alphas ^a										
Est.	Qtr	-0.31	0.05	-0.36	-0.36	0.02	0.37	-1.23	0.48	1.71
	Semi 1	-0.34	-0.04	-0.30	-0.41	0.01	0.42	-1.02	0.19	1.22
	Semi 2	-0.41	0.10	-0.50	-0.33	0.08	0.41	-0.92	0.35	1.26
	Annual	-0.44	0.18	-0.62	-0.27	0.12	0.39	-0.63	0.19	0.82
Std err.	Qtr	0.10	0.14	0.16	0.17	0.07	0.16	0.24	0.19	0.27
	Semi 1	0.11	0.14	0.17	0.18	0.07	0.18	0.22	0.19	0.24
	Semi 2	0.12	0.14	0.17	0.19	0.07	0.19	0.21	0.20	0.24
	Annual	0.10	0.16	0.20	0.16	0.08	0.17	0.20	0.21	0.22
<i>Panel B: Dynamic mimicking portfolio</i>										
Unconditional alphas										
Est.	Day	0.50	0.45	0.05	0.26	0.75	0.49	-0.16	0.81	0.96
	Wk	0.53	0.45	0.08	0.30	0.76	0.46	-0.12	0.83	0.96
	Month	0.59	0.44	0.16	0.33	0.78	0.45	-0.09	0.83	0.92
Average conditional alphas ^a										
Est.	Qtr	0.59	0.45	0.13	0.41	0.73	0.32	-0.21	0.98	1.19
	Semi 1	0.53	0.44	0.09	0.37	0.72	0.36	-0.04	0.84	0.88
	Semi 2	0.47	0.27	0.20	0.25	0.59	0.34	-0.22	0.82	1.04
	Annual	0.49	0.29	0.20	0.23	0.61	0.37	-0.19	0.58	0.77
Std err.	Qtr	0.33	0.21	0.23	0.30	0.27	0.14	0.38	0.28	0.30
	Semi 1	0.34	0.23	0.24	0.32	0.28	0.15	0.40	0.28	0.29
	Semi 2	0.35	0.21	0.24	0.32	0.28	0.15	0.40	0.27	0.30
	Annual	0.39	0.18	0.29	0.33	0.30	0.16	0.37	0.26	0.31

^a Quarterly and Semiannual 1 alphas are estimated from daily returns, Semiannual 2 alphas are estimated from weekly returns, and Annual alphas are estimated from monthly returns.

6. Comparison with other studies

Our empirical results, and generally skeptical view of conditioning, stand in stark contrast to the conclusions of Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Petkova and Zhang (2003). They find that conditioning dramatically improves the performance of both the simple and consumption CAPMs. Their studies have been influential, so it might be worthwhile to offer a few observations on why our conclusions differ.

The papers clearly differ from ours in many ways, but a key distinction is that they focus on cross-sectional regressions instead of the time-series intercept tests that we emphasize. As such, the papers test only the qualitative implications of the conditional CAPM, that the effects of time-varying betas are cross-sectionally correlated with expected returns. They do not fully test the conditional CAPM, which imposes additional, important restrictions on the cross-sectional slopes (the restrictions are implicit in our intercept-based tests).

This point can be seen most easily in the context of the simple CAPM. A full test is whether expected returns are linear in conditional betas, $E_{t-1}[R_t] = \beta_t \gamma_t$, with the slope equal to the market risk premium. However, Jagannathan and Wang (1996), Lettau and Ludvigson (2001), and Petkova and Zhang (2003) take expectations and focus, in different ways, on the implied unconditional relation, $E[R_t] = \beta \gamma + \text{cov}(\beta_t, \gamma_t)$. In a cross-sectional regression, the slope on β should equal the unconditional risk premium and the slope on $\text{cov}(\beta_t, \gamma_t)$ should equal one, but the papers instead treat them as free parameters. We think this explains why they find conditioning to be so important; in particular, the estimated slopes on $\text{cov}(\beta_t, \gamma_t)$ are almost certainly much larger than one.⁹ If so, their results actually provide evidence *against* the conditional CAPM.

We illustrated this point earlier using Petkova and Zhang (2003). Here we offer an example from

⁹ The papers don't estimate the equation $E[R_t] = \beta \gamma + \text{cov}(\beta_t, \gamma_t)$ directly. Petkova and Zhang come closest, replacing $\text{cov}(\beta_t, \gamma_t)$ by $\phi = \text{cov}(\beta_t, \gamma_t) / \sigma_\gamma^2$. Jagannathan and Wang show that β and $\text{cov}(\beta_t, \gamma_t)$ are linearly related to β^u and $\text{cov}(R_t, \gamma_t) / \sigma_\gamma^2$, and use the latter two in the tests. Lettau and Ludvigson motivate their tests differently, but under the assumption that conditional betas are linear in CAY, their regressions use the average consumption beta and a variable that is proportional to $\text{cov}(\beta_t, \gamma_t)$.

Lettau and Ludvigson (2001). If we assume that consumption betas (β_i) are linear in CAY and the zero-beta rate is constant, their Table 3 shows returns regressed on β and $\delta \equiv \text{cov}(\beta_t, \text{CAY}_{t-1}) / \text{var}(\text{CAY})$. In this context, the slope on β should be the average consumption-beta risk premium, λ , and the slope on δ should equal $\text{cov}(\lambda_t, \text{CAY}_{t-1})$. Lettau and Ludvigson find that λ is close to zero, but the slope on δ is around 0.06% or 0.07% quarterly depending on the specification. In principle, we could test whether this slope equals $\text{cov}(\lambda_t, \text{CAY}_{t-1})$, but the statistics reported in the paper are insufficient to do so. Here we simply note that the estimates seem huge, implying that σ_λ is greater than 3.2% quarterly (if slope = $\text{cov}(\lambda_t, \text{CAY}_{t-1}) < \sigma_\lambda \sigma_{\text{cay}}$, re-arranging yields $\sigma_\lambda > \text{slope} / \sigma_{\text{cay}} = 0.0006 / 0.019 = 0.032$). In other words, their estimates say that the risk premium is on average close to zero but has enormous volatility (and, since λ_t must be positive, it must also have enormous skewness). These facts are difficult to reconcile – quantitatively – with the consumption CAPM.

On a related note, the cross-sectional R^2 's reported by Jagannathan and Wang (1996) and Lettau and Ludvigson (2001) should be interpreted with caution. Both papers report striking improvements in explanatory power when moving from unconditional to conditional tests. The impact of the studies, in fact, seems to come largely from the dramatic increase in R^2 , nicely illustrated by their figures showing predicted returns plotted on actual returns.

However, we don't find these R^2 's very informative, for three related reasons: (1) As discussed above, the papers ignore key restrictions on the cross-sectional slopes; the R^2 's would likely drop significantly if the restrictions were imposed. (2) Returns on size and B/M portfolios can be traced to three common factors (time-series R^2 's above 90%), and that betas on the factors explain most of the cross-sectional variation in expected returns. In this setting, one can show that almost any multi-factor model will produce a high cross-sectional R^2 . (3) The papers don't report standard errors or confidence intervals for the R^2 , and simulations show that it is easy to find a high sample R^2 even when the population R^2 is zero. For example, we simulate the two-pass regressions of Lettau and Ludvigson (2001, Table 3) using historical returns and consumption but substituting a randomly generated state variable in place of CAY.

In 10,000 simulations, the median R^2 is 0.43, and the 5th and 95th percentiles are 0.12 and 0.72, respectively (compared with a reported 0.66). In short, despite its increasing use, the cross-sectional R^2 doesn't seem to be a very meaningful metric.¹⁰

7. Conclusion

The main point of the paper is easily summarized: the conditional CAPM cannot explain asset-pricing anomalies like B/M or momentum. Analytically, if the conditional CAPM holds, deviations from the unconditional CAPM depend on the covariances among betas, the market risk premium, and market volatility. We argue that, for plausible parameters, the covariances are simply too small to explain large unconditional pricing errors.

The empirical tests support this view. We use short-window regressions to directly estimate conditional alphas and betas for size, B/M, and momentum portfolios from 1964 – 2001. This methodology gets around the problem, common to all prior tests, that the econometrician cannot observe investors' information sets. We find that betas vary considerably over time, with relatively high-frequency changes from year to year, but not enough to generate significant unconditional pricing errors. Indeed, there is little evidence that betas covary with the market risk premium in a way that might explain the alphas of B/M and momentum portfolios. Most important, conditional alphas are large and significant, in direct violation of the conditional CAPM.

¹⁰ Roll and Ross (1994) and Kandel and Stambaugh (1995) reach the same conclusion about the cross-sectional R^2 in simple CAPM regressions. They show that the population OLS R^2 says very little about the location of the market proxy in mean-variance space.

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